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What's Up?

The MPCORB file from the Minor Planet Center (MPC) provides orbital elements for all known asteroids. Sometimes it is useful to be able to scan the file to find out what might be observable currently – What's Up?

Introduction

The Minor Planet Center (MPC) suffered a major systems failure on 5th June 2019 as a result of which they stopped producing some files I had been using to decide on future observing targets. In particular the "Dates of Last Observations of NEOs" was a good starting point for a list of currently observable Near Earth Objects. Two months later the files had not returned and the data in the 5th June files had become completely out of date and useless. I therefore set out to generate my own "What's Up?" list.

The MPCORB file contains over 800,000 objects most of which are of no interest. It also has an epoch (the effective date of the orbital elements) that can be months away from today, and that is no use for predicting the positions of close-passing objects.

However, the MPC also produces a subset of MPCORB called NEAp01.txt that contains only Near Earth Objects (about 20,000 of them) and has an epoch of 00:00 UTC tomorrow. This is the file I can work with. The process is to:

- Calculate the current heliocentric ecliptic position of each object.
- Calculate solar elongation and phase angle as seen from Earth.
- Calculate the current apparent magnitude based on H and G magnitudes.
- Produce a short list based on acceptable elongation and magnitude.

The input

The orbital elements are read from the file NEAp01.txt that contains the elements of all Near Earth Asteroids at epoch "tomorrow 00:00 UTC".

The elements obtained from the file are:

- t_0 The epoch of the elements. The exact date-time when these elements are correct.
- a Semi-major axis. Average distance from the Sun.
- *e* Eccentricity. Shape of the ellipse.
- i Inclination. Angle between the plane of the orbit and the ecliptic.
- Ω Longitude of the ascending node. Points to the position in the orbit where the object rises up through the plane of the ecliptic.
- ω Argument of perihelion. Angle from Ω to the position in the orbit where the object is closest to the Sun.

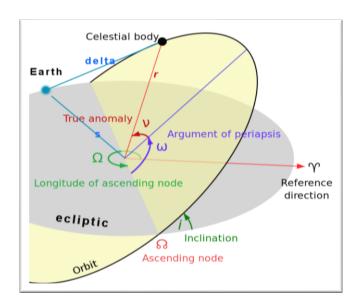
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 M_0 Mean anomaly. Angle of how far the object is round its orbit at the time of the epoch, and assuming it travels round the orbit at a steady angular rate of mean daily motion (which it does not!).

Mean daily motion. Daily angular progress round the orbit if it were to go at a steady rate (which it does not!)

Also obtained from the file are:

- H Absolute magnitude. Hypothetical magnitude if placed one AU from Earth and one AU from Sun and seen with 100% phase (a condition which is physically impossible!)
- G Slope parameter. A measure of how quickly the apparent magnitude changes as the viewing phase angle changes.



Orbital Elements

The calculations

Position in orbit

The time difference between "now" and the epoch is established and used to calculate the mean anomaly "now".

$$M_{now} = M_0 + n(t_{now} - t_0)$$

But this mean anomaly is the position in the orbit if the object moved at a steady angular speed. We need to convert it to a true anomaly that takes into account the variation in speed as it moves round its orbit. To do that we must first get the eccentric anomaly E. (E is the first step in constructing a geometric figure that will lead us to the objects true position <u>as described here</u>). The eccentric anomaly is related to the mean anomaly by Kepler's equation:

$$M = E - e \sin E$$

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Unfortunately, this equation has no closed solution, you cannot calculate E from a given value of M. A numerical solution is required and I normally use the Newton-Raphson method (see below).

Having a value for E we now can find the object's coordinates in the plane of its own orbit.

$$x_1 = a(\cos E - e)$$
$$y_1 = a\sin E\sqrt{(1 - e^2)}$$

We can now get the object's current distance from the Sun r and true anomaly v (angular distance round the orbit from perihelion).

$$r = \sqrt{(x^2 + y^2)}$$
$$v = \operatorname{atan2}(x, y)$$

Heliocentric Ecliptic coordinates

We must convert the coordinates into a system that matches that used to describe the position of Earth. We could use heliocentric ecliptic or heliocentric equatorial. I use the first of these.

$$x = r(\cos\Omega\cos(v + \omega) - \sin\Omega\sin(v + \omega)\cos i)$$
$$y = r(\sin\Omega\cos(v + \omega) + \cos\Omega\sin(v + \omega)\cos i)$$
$$z = r(\sin(v + \omega)\sin i)$$

Position of Earth

We now need the position of Earth in the same heliocentric ecliptic coordinates. We could use the same procedure applied to Earth's orbital elements but it's not so easy to get hold of the Earth's elements at today's epoch. I choose to use $\underline{VSOP97}$ data, which claims to be accurate to within \pm 1 arcsecond. Assistance with this approach is available from this site which will generate the required code in a selection of programming languages.

We can now obtain the object's distance from Earth (usually called delta Δ) and Earth's distance from the Sun (s). (We could assume the Earth is 1 AU from the Sun, but it does vary and this way is more accurate.)

$$\Delta = \sqrt{((x - x_{Earth})^2 + (y - y_{Earth})^2 + (z - z_{Earth}))}$$

$$s = \sqrt{(x_{Earth}^2 + y_{Earth}^2 + z_{Earth}^2)}$$

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Phase and Elongation

We now have the lengths of all three sides of the triangle formed by object-Earth-Sun and can solve the angles. Of particular interest are

- Phase angle α the Earth-object-Sun angle that tells us the phase of illumination of the object's surface. At a large phase angle we will see it only partially illuminated and therefore faint.
- Solar elongation β the Sun-Earth-object angle which is the apparent angular distance of the object from the Sun. We would not normally try to observe anything less than about 60 degrees from the Sun soon after sunset or before sunrise.

$$\alpha = \cos^{-1}((\Delta^2 + s^2 - r^2)/(2\Delta s))$$

$$\beta = \cos^{-1}((r^2 + \Delta^2 - s^2)/(2\Delta r))$$

At this point we can discard any objects that have β less than an acceptable elongation.

Magnitude

We will want to ensure the object has a magnitude that will make it a potential target for a Slooh telescope.

The absolute magnitude of the object when 1AU from the Sun and 1AU from the Earth is H. So the magnitude V(0) at other distances, and assuming zero phase angle, would be:

$$V(0) = H + 5\log_{10}(\Delta r)$$

It has been long known that the apparent magnitude of asteroids does not fall off in a linear way compared with their percentage of illumination as phase angle increases. They tend to be brighter than expected at small phase angles and dimmer at large phase angles. The IAU adopted a value called the Phase Parameter (G) in 1985 to help describe this behaviour. G has been measured for some well known objects, but in most cases it is assumed G=0.15.

The change in magnitude at phase angle α is given as:

$$V(\alpha) = V(0) - 2.5 \log_{10}((1 - G)\phi_1 + G\phi_2)$$

Where:

$$\phi_1 = exp\left(-3.33\left(\tan\frac{\alpha}{2}\right)^{0.63}\right)$$

$$\phi_2 = exp\left(-1.87\left(\tan\frac{\alpha}{2}\right)^{1.23}\right)$$

We can now discard any objects where the apparent magnitude $V(\alpha)$ is beyond the capability of our telescopes.

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Conclusion

The results of the above can yield a list of potentially observable NEOs but does not tell us if a particular object is visible from a particular observatory at a particular time. Maybe a further note on that later. In practice, most of the objects will be visible at Teide or Chile some time during the night.

Newton Raphson Method

This is a common tool for finding the roots of an equation numerically. This version is taken from Boulet (1991, p321). We looking to obtain the value of E given the values of e and e when e in e in e.

An approximate initial solution is taken to be E=M.

We estimate the error. If E is correct we will end up with an error approaching zero.

$$err = E - e \sin E - M$$

If the error is more than an acceptable limit then *E* is adjusted.

$$E \leftarrow E - (1 - e \cos E)$$

The process is repeated until the error is below a suitable limit (I use 10⁻⁸) as none of the input parameters have a better precision.