ELEC 5360 Homework 4

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1.

The log-likelihood function is given by

$$\begin{aligned}\Lambda_L(\tau) &= \Re[\frac{1}{N_0}\int_{T_0} r(t)s_l^*(t,\tau)dt] \\ &= \Re[\frac{1}{N_0}\int_{T_0} r(t)\sum_n I_n g^*(t-nT-\tau)dt] \\ &= \Re[\sum_n I_n y_n(\tau)]\end{aligned}$$

where $y_n = \frac{1}{N_0}\int_{T_0} r(t)g^*(t-nT-\tau)dt$

In order to maximize the likelihood function, we set

$$\frac{d\Lambda_L(\tau)}{d\tau} = \Re[\sum_n I_n \frac{d}{d\tau}y_n(\tau)] = 0$$

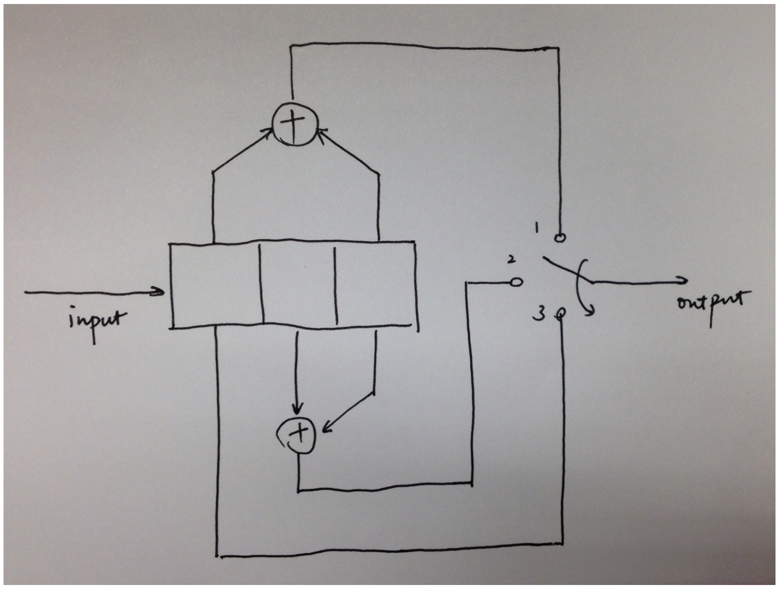
Further simply the expression,

$$\Re[\sum_n I_n \frac{d}{d\tau}y_n(\tau)] = \sum_n \Re[I_n]\Re[\frac{d}{d\tau}y_n(\tau)] - \sum_n \Im[I_n]\Im[\frac{d}{d\tau}y_n(\tau)]$$

Therefore we obtain,

$$\sum_n \Re[I_n]\Re[\frac{d}{d\tau}y_n(\tau)] = \sum_n \Im[I_n]\Im[\frac{d}{d\tau}y_n(\tau)]$$

2. (a)

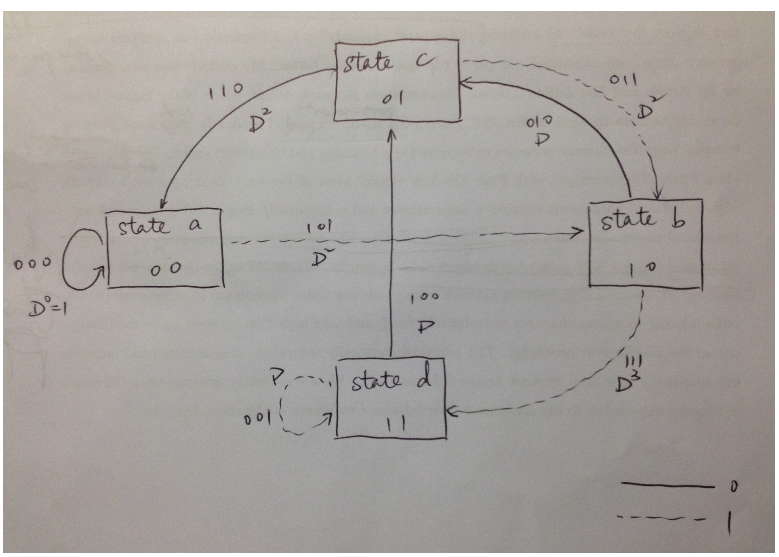


(b)

The code rate is 1/3.

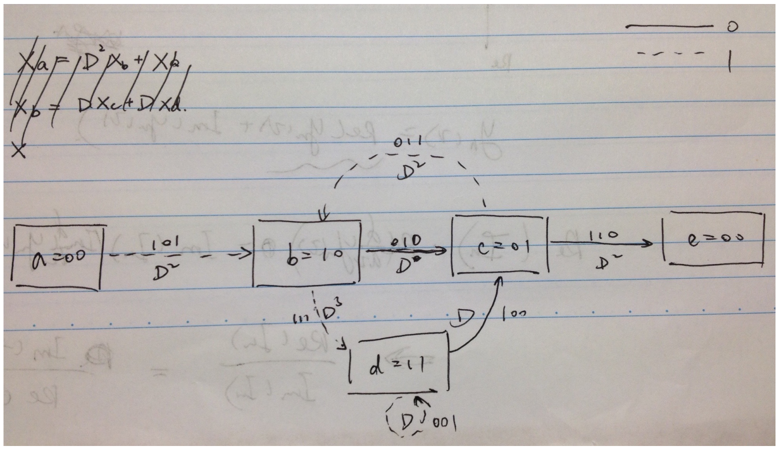
The constrain length is 3.

(c)



(d)

Split the state a to *start* state a and *end* state e as shown below.



Then we obtain

$$X_b = D^2 X_a + D^2 X_c$$
$$X_c = D X_b + D X_d$$
$$X_d = D^3 X_b + D X_d$$
$$X_e = D^2 X_c$$

Simplify the set of equations, we will get

$$X_e = D^2 (D X_b + D X_d)$$
$$X_d = D^3/(1 - D) X_b$$
$$D^2 X_a = X_b - D^2 (D X_b + D X_d)$$

Then

$$\begin{aligned}
\frac{X_e}{X_a} &= \frac{(D^3 + D^6/(1-D))D^2}{1 - D^3 + D^6/(1-D)} \\
&= \frac{D^5(1-D) + D^8}{(1-D^3)(1-D) + D^6} \\
&= \frac{D^5 - D^6 + D^8}{1 - D - D^3 + D^4 + D^6}
\end{aligned}$$

(e)

From last equation, we obtain

$$\frac{X_e}{X_a} = (D^5 - D^6 + D^8)[1 + (D + D^3 - D^4 - D^6) + (D + D^3 - D^4 - D^6)^2 + ...]$$

Therefore the minimum free distance d_{free} of the code is 5.

3.

Notice that $f_k - f_j = n/T$ for $n = 1, 2, ..., N-1$ (as $f_k \neq f_j$), also under the assumption that $f_k + f_j \gg 1$, Then

$$\begin{aligned}
&\int_0^T cos(2\pi f_k t + \phi_k) cos(2\pi f_j t + \phi_j) dt \\
&= \frac{1}{2}\int_0^T cos(2\pi f_k t + \phi_k + 2\pi f_j t + \phi_j) dt + \frac{1}{2}\int_0^T cos(2\pi f_k t + \phi_k - 2\pi f_j t - \phi_j) dt \\
&= \frac{1}{2}\int_0^T cos(2\pi (f_k + f_j) t + \phi_k + \phi_j) dt + \frac{1}{2}\int_0^T cos(2\pi nt/T + \phi_k - \phi_j) dt \\
&= \frac{1}{2\pi \frac{n}{T}} \frac{1}{2} sin(2\pi \frac{nt}{T} + \phi_k - \phi_j)\Big|_0^T + \frac{1}{2\pi(f_k + f_j)} \frac{1}{2} sin(2\pi(f_k + f_j)t + \phi_k + \phi_j)\Big|_0^T
\end{aligned}$$

Notice that

$$sin(2\pi \frac{nt}{T} + \phi_k - \phi_j)\Big|_0^T = sin(2\pi n + \phi_k - \phi_j) - sin(\phi_k - \phi_j) = 0$$

and

$$\frac{1}{2\pi(f_k + f_j)} \simeq 0 \text{ for large } f_k + f_j$$

Hence we get $\int_0^T cos(2\pi f_k t + \phi_k) cos(2\pi f_j t + \phi_j) dt = 0$ and therefore subcarriers of the corresponding sub-channels are mutually orthogonal.

4. (a)

If we dont account the cyclic prefix, the maximum symbol rate for each subcarrier relates to the Δf. So the ideal maximum symbol rate will be 15k symbol/sec. However, in this specific case, taking the CP in to consideration, using the least amount of CP, we get the maximum symbol rate $(\text{size}(FFT) - \text{size}(CP))/(\text{size}(FFT) \times T_{sample}) = (2048 - 144)/(2048/(\Delta f \times 2048)) =$ 13.95k symbol/sec

(b)

The symbol duration is given by $T = NT_{tb}$, where $T_t b = 1/f_{tb}$ and $N = 20$ MHz/15 kHz. Plugging the numbers in, we get 20 MHz/15 kHz/20 MHz $= 1/15 \approx 0.0667$ sec.

(c)

One view of the need for CP in OFDM is, in the guarding interval preventing the inter-OFDM system-interference, we may as well arising something to further prevent information loss. More formally, in the channel matrix point of view, cyclic prefix will lead to a circular matrix which can further leads to diagonal matrix decomposition, this help simplify the matrix a lot.

(d)

The power loss can be defined as the ratio between redundant information propagated and symbol transmitted.

Therefore, for normal CP, we get 144/2048, for extended CP, 512/2048

(e)

We need to require the guarding interval spread wider than the delay, namely $CP/(\text{size}(FFT)\Delta f) \geq 7\mu s$, where $\Delta f = 15$ kHz. Then we get $CP \geq 215.04$. Therefore the extended 512 CP is needed.