ELEC 5360 Fall 2014
Homework 4

Due on Nov. 13, 2014, before class

1. **Synchronization**
Determine the ML estimate of the time delay τ for the QAM signal of the form

$$s(t) = \mathrm{Re}[s_l(t;\tau)e^{j2\pi f_c t}], \tag{1}$$

where

$$s_l(t;\tau) = \sum_n I_n g(t - nT - \tau), \tag{2}$$

and $\{I_n\}$ is a sequence of complex-valued data. Assume decision-directed estimation, and the log-likelihood function is given by

$$\Lambda_L(\tau) = \mathrm{Re}\left[\frac{1}{N_0}\int_{T_0} r(t)s_l^*(t;\tau)dt\right]. \tag{3}$$

2. **Convolutional Code**
Consider the encoder of a convolutional code, specified by the generator sequences:

$$g_1 = [101], g_2 = [011], g_3 = [100]. \tag{4}$$

(a) Draw the convolutional encoder.
(b) What is the code rate? What is the constraint length?
(c) Draw the state diagram and label the state diagram with distance D.
(d) Derive the transfer function.
(e) What is the minimum free distance d_{free} of the code?

3. OFDM is a special type of multicarrier modulation in which the subcarriers of the corresponding subchannels are mutually orthogonal. Now you need to verify this orthogonality. Assume we have N subbands (or subchannels). With each subband (or subchannel), we associate a sinusoidal carrier signal of the form

$$s_k(t) = \cos(2\pi f_k t + \phi_k), k = 0, 1, \ldots, N-1, \tag{5}$$

where f_k is the mid frequency in the k-th subchannel. When the subcarrier spacing is $1/T$ and symbol rate in each subchannel is also $1/T$, please verify that

$$\int_0^T \cos(2\pi f_k t + \phi_k)\cos(2\pi f_j t + \phi_j)dt = 0, \forall k \neq j. \tag{6}$$

where $\{f_k\}$ are independent of the values of the signal phrases $\{\phi_k\}, \forall k$. Assume that $f_k + f_j \gg 1$.

4. **OFDM System**
LTE downlink is based on OFDM. Consider the normal subcarrier space $\Delta f = 15$ kHz, and assume the transmission bandwidth is 20 MHz, corresponding to FFT size 2048, so the sampling time is $T_s = \frac{1}{\Delta f \times 2048}$. LTE defines two types of cyclic prefix (CP): the normal CP with 144 samples and the extended CP with 512 samples. Different CPs are defined for different application scenarios.
(a) What is the maximum possible symbol rate on each subcarrier?
(b) What is the OFDM symbol duration for LTE?
(c) Why do we need CP in OFDM?
(d) What is the power loss with normal CP and extended CP, respectively?
(e) If the delay spread in the operating environment is 7 μs, which type of CP should we use?