ELEC 5360 Fall 2014
Homework 2

Due on Oct. 9, 2014, before class

1. **Lloyd-Max algorithm**
Run *one iteration* of the Lloyd-Max algorithm for the design of an optimal $M = 4$ level quantizer for a random variable with triangular pdf given by

$$f_X(x) = \begin{cases} -2x+2, & 0 < x \leq 1, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Initialize the four representation points as $a_1 = 0.2, a_2 = 0.6, a_3 = 0.7, a_4 = 0.9$. Denote the endpoints as $b_0 < b_1 < b_2 < b_3 < b_4$ with $b_0 = 0$ and $b_4 = +\infty$. Find the values of b_1, b_2, b_3, and the representation points a_1, a_2, a_3, a_4, in the next iteration.

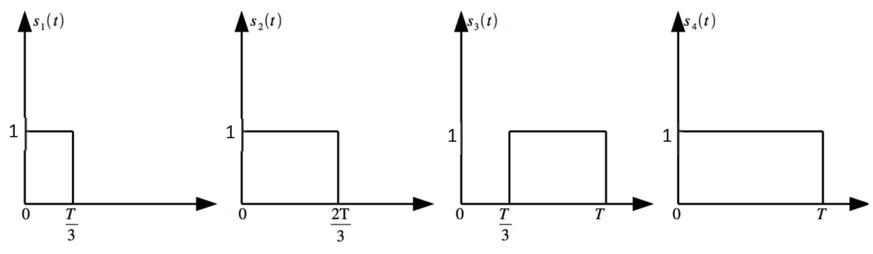
2. **Orthogonal expansion**
Consider the following three waveforms f_1, f_2, f_3:

$$f_1(t) = 1/2, 0 \leq t \leq 4; \quad f_2(t) = \begin{cases} 1/2, & 0 \leq t \leq 2, \\ -1/2, & 2 < t \leq 4; \end{cases} \quad f_3(t) = \begin{cases} 1/2, & 0 \leq t \leq 1, \\ -1/2, & 1 < t \leq 2, \\ 1/2, & 2 < t \leq 3, \\ -1/2, & 3 < t \leq 4. \end{cases} \tag{2}$$

(a) Show that these waveforms are *orthonormal*.
(b) Express the waveform $x(t)$ as a linear combination of $f_n(t), n = 1, 2, 3$, if

$$x(t) = \begin{cases} -1, & 0 \leq t \leq 1, \\ 1, & 1 < t \leq 3, \\ -1, & 3 < t \leq 4. \end{cases} \tag{3}$$

3. **Gram-Schmidt orthogonalization procedure**
Consider the four signal $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$:



(a) Using the Gram-Schmidt orthogonalization procedure, find an orthonormal basis for this set of signals.
(b) Express each signal in terms of the orthonormal basis functions found in part (a).

4. **Nyquist criterion**
Consider a PAM baseband system in which the modulator is defined by a signal interval T and a waveform $p(t)$, the channel is defined by a filter $h(t)$, and the receiver is defined by filter $q(t)$, which is sampled at T-spaced intervals. The received waveform, after the receiver filter $q(t)$, is then given by $r(t) = \sum_k u_k g(t - kT)$, where $g(t) = p(t) * h(t) * q(t)$.
(a) What property must $g(t)$ have so that $r(kT) = u_k$ for all k and for all choices of input $\{u_k\}$? What is the Nyquist criterion for $\hat{g}(f)$?

(b) Now assume that $T = 1/2$ and that $p(t)$, $h(t)$, $q(t)$ and all their Fourier transforms are restricted to be real. Assume further that $\hat{p}(f)$ and $\hat{h}(f)$ are given by

$$\hat{p}(f) = \begin{cases} 1, & |f| \le 0.5, \\ 1.5 - |f|, & 0.5 < |f| \le 1.5, \\ 0, & |f| > 1.5; \end{cases} \qquad \hat{h}(f) = \begin{cases} 1, & |f| \le 0.6, \\ 0, & |f| \ge 0.6. \end{cases} \tag{4}$$

Is it possible to choose a receiver filter transform $\hat{q}(f)$ so that there is no intersymbol interference? If so, give such a $\hat{q}(f)$.

(c) Find the conditions on $\hat{h}(f)\hat{q}(f)$ under which intersymbol interference can be avoided by proper choice of $\hat{p}(f)$.

5. **White Gaussian noise**

Let $\{Z(t)\}$ be a white Gaussian noise process of spectral density $N_0/2$. [Hint: a linear functional of $\{Z(t)\}$, given as $\int Z(t)g(t)dt$ is a Gaussian random variable.]

(a) Let $\{\phi_i(t)\}$ be a countable set of real orthonormal functions. Find the distribution of $V_i = \int \phi_i(t)Z(t)dt$.

(b) Let $Y = \int_0^T Z(t)dt$. Find the probability density of Y.