ELEC 5360 Fall 2014
Homework 3

Due on Oct. 16, 2014, before class

1. **4-QAM**

Consider 4-QAM with the four signal points $u = \pm a \pm ia$. Assume Gaussian noise with spectral density $N_0/2$ per dimension.

(a) Sketch the signal set and the ML decision regions for the received complex sample value v. Find the exact probability of error (in terms of the Q function) for this signal set using ML detection.

(b) Consider 4-QAM as two 2-PAM systems in parallel. That is, a ML decision is made on $\mathrm{Re}(u)$ from $\mathrm{Im}(v)$ and a decision is made on $\mathrm{Im}(u)$ from $\mathrm{Im}(v)$. Find the error probability (in terms of the Q function) for the ML decision on $\mathrm{Re}(u)$ and similarly for the decision on $\mathrm{Im}(u)$

(c) Explain the difference between what has been called an error in part (a) and what has been called an error in part (b).

(d) Derive the QAM error probability directly from the PAM error probability.

2. **Union bound**

Consider the modulation scheme as shown in Fig. 1. The AWGN channel is assumed with power spectral density $N_0/2$. Please answer the following questions.

(a) Assuming equally likely symbols, what is the optimal detector and what is the decision region for each symbol? Plot these regions.

(b) What is the pair-wise error probability $P_e(s_1|s_2)$, i.e., the probability of detecting s_1 while transmitting s_2? What is $P_e(s_6|s_2)$?

(c) Derive the union bound for $P(\text{error}|s_2)$.

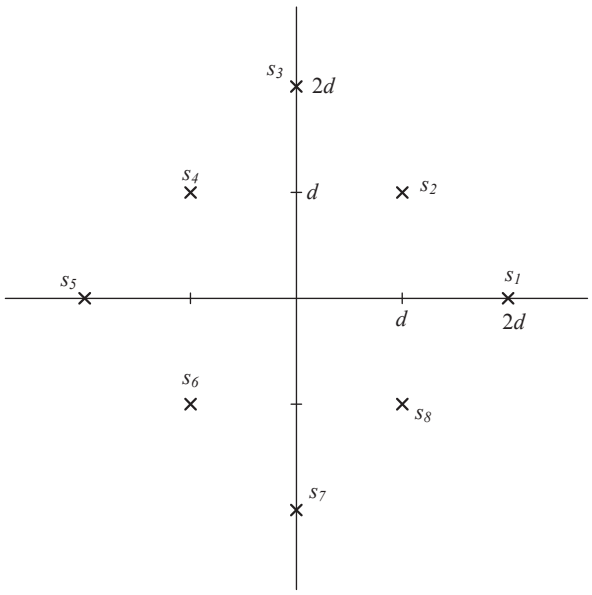

Figure 1: A modulation scheme.

3. **Matlab experiments**

Compare the performance of the following modulation techniques: 4-PAM, 4-QAM and 16-QAM. Simulate them with Gray labeling over an AWGN channel.

(a) Plot the symbol error probability P_s versus the symbol energy-to-noise ratio E_s/N_0 for each of them, and also plot the results obtained by error probability expressions derived in the lecture.

(b) Plot the average bit error probability P_b versus the bit energy-to-noise ratio E_b/N_0 for each of them.

Please provide the MATLAB codes for (a) and (b). (Note: An illustration for the signal constellation with Gray labeling is shown in Figure 2. Also, you can use "semilogy" to plot the curves in MATLAB.)

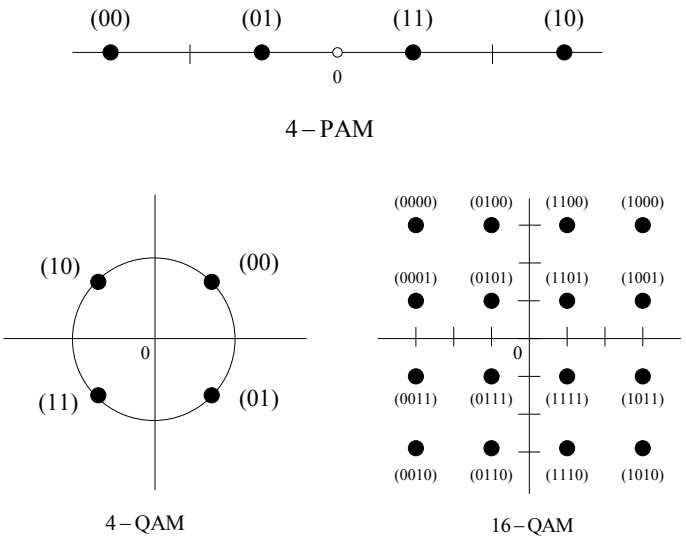

Figure 2: An illustration for the signal constellation with Gray labeling.