Literature Review on Radar Detection with Consideration of Scan Time

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***Abstract*—Detection Rate, a metric proposed by Grossi, Lops and Venturino [1], was suggested to be a better optimization metric in radar detection. A simulation of three study cases utilizing Likelihood Ratio Test, Maximum Likelihood estimation and Neyman-Pearson Lemma to maximize the detection rate of a single target under the constraint of false alarm rate was conducted. The results verified that the claim is true under the three cases.**

***Index Terms*—detection problem, radar, likelihood ratio test, time constraint, detection time, detection rate, false alarm rate, simulation**

I. Introduction

The detection problem has been a crucial research area in signal processing, and has been widely used across different fields. It has been used in radar for decades, and according to the specifications of each radar, the radar is configured to maximize the probability of detection under some constraints. Neyman-Pearson Lemma by Neyman and Pearson [3], for example, minimizes the probability of false alarm under a certain threshold to maximize the probability of detection under the likelihood ratio test. Longer exposure time to the target would also increase probability of detection, and decrease the probability of false alarm. But, intuitively, we would then realize that increased scan time would increase our probability of detection to a certain point where afterwards would only yield diminishing return, and in most use cases, not a feasible choice.

Therefore, Grossi, Lops and Venturino proposed that time should also be a constraint under the optimization rule and reintroduced the Neyman-Pearson Lemma with the constraining false alarm rate instead of the false alarm probability [1], to maximize the detection rate instead of the detection probability. These two terms are defined as follows.

For a radar with $T > 0$ seconds required to observe a target, M scanned directions and $N \in \mathbb{N}$ data samples collected for one target as defined by Grossi, Lops and Venturino [1], the total scan time of this radar would be

$$T_s = MNT \tag{1}$$

Then, the detection rate is defined as

$$\text{DR} = \frac{P_\text{d}}{T_s} \tag{2}$$

where P_d is the probability of detection of a target. Likewise, the false alarm rate is defined as

$$\text{FAR} = \frac{P_\text{fa}}{T_s} \tag{3}$$

where P_fa is the probability of false alarm of a target. Then, according to the revised Neyman-Pearson Lemma, the optimization rule becomes

$$\max_{N,\delta} \frac{P_\text{d}(N,\delta)}{N} \text{ s.t. } \frac{P_\text{fa}(N,\delta)}{N} \leq A \tag{4}$$

where δ is the decision rule by the Likelihood Ratio Test, and A is the maximum false alarm rate threshold.

This paper is written in conjunction to a course requirement, where two figures are replicated from a paper of choice. The two figures that are being replicated would also be included in the Result section.

II. Simulation Setup

This section explains the setup of the simulation used to explore, and thus verify the effect of optimizing against scan time of radar. The signal received by the radar r_n is modelled as follows:

$$r_n = \begin{cases} w_n + s_n, & \text{under } H_1 \\ w_n, & \text{under } H_0 \end{cases} \tag{5}$$

where $\{s_n\}$ is the target response signal without noise, modeled as complex circularly-symmetric Gaussian random variable with zero mean and variance ε, and $\{w_n\}$ is the noise signal, modeled as independent and identically distributed complex circularly-symmetric Gaussian random variable with zero mean and variance σ^2. This signal corresponds to the Swerling I fluctuation model [4]. The hypothesis H_0 is defined as target is absent, and hypothesis H_1 is defined as target is present. SNR per data sample ρ is defined as

$$\rho = \frac{\varepsilon}{\sigma^2} \tag{6}$$

A. Case I

For the best case scenario, we assume that the noise variance σ^2 is known. Then, for a given N, the decision rule is in the form of:

$$\delta_N(r_N) = \begin{cases} 1, & \text{if } \frac{1}{N\sigma^2}|\sum_{n=1}^{N} r_n|^2 > \gamma_1 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where $\gamma_1 = -\ln(AN)$ is the detection threshold [1].

B. Case II

Most of the time, the noise variance is unknown and possibly fluctuating. For such, assume that secondary data samples of the noise $\{x_l\}$ with size L are available and the variance is constant over that short period of time. Then, the maximum likelihood estimate of the variance $\hat{\sigma^2}$ would be

$$\hat{\sigma^2} = \frac{1}{L}\sum_{l=1}^{L}|x_l|^2 \tag{8}$$

where it is also an unbiased estimator of the variance [2]. With the estimator $\hat{\sigma^2}$ we are able to plug it back into (7) to obtain the following decision rule:

$$\delta_N(r_N) = \begin{cases} 1, & \text{if } \frac{\frac{1}{N}|\sum_{n=1}^{N} r_n|^2}{\frac{1}{L}\sum_{l=1}^{L}|x_l|^2} > \gamma_2 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $\gamma_2 = L((AN)^{-1/L} - 1)$ is the detection threshold [1].

C. Case III

This case investigates the effect of an ambiguous Doppler shift in the signal, where $s_n = se^{-j2\pi\nu n}$ is the signal under the unknown Doppler shift in phase of $\nu \in [0, 1)$. Assume noise variance σ^2 is known, a maximum likelihood estimator of ν is used for the General Likelihood Ratio Test, which implies the following decision rule:

$$\delta_N(r_N) = \begin{cases} 1, & \text{if } \max\limits_{\nu\in[0,1)} \frac{1}{N\sigma^2}|\sum_{n=1}^{N} r_n e^{-j2\pi\nu n}|^2 > \gamma_3 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

where $\gamma_3 = -\ln(1 - (1 - AN)^{(1-N)})$ is the detection threshold [1].

III. Result and Discussion

The simulation is done on Matlab R2020b according to the setup as defined above, as well as the following assumptions. The time required to observe an object $T = 1$ms. The scanned directions of the radar $M = 60$. The range of the number of data samples $N \in [4, 150]$. Case I and II each has a trial size of 10^4, and case III, due to the heavy computation of the General Likelihood Ratio Test, has a reduced trial size of 3×10^3. The maximum false alarm rate level A' is defined as $\frac{A}{MT}$. Case I uses a maximum false alarm rate level of $= 2.78 \times 10^-7$ fa/s, and Case II and II uses $A' = 5 \times 10^-7$ fa/s.

A. Case I

From Fig. 1, it is clear that in this case, the optimal number of scans needed to detect is small, and shows the best case scenario of this detection problem out of the three cases. It is observed that the detection rate decreases as the number of scans increases after an optimal point, and more scans of the target would not yield more information.

When compared to Fig. 2, the same figure generated by Grossi, Lops and Venturino under the same parameters but with their closed-form solution [1], it is clear that the solution of optimized number of data samples N is close and the trend is identical.

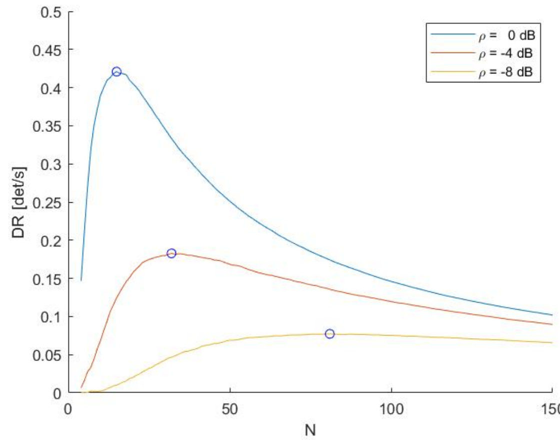

Fig. 1. DR versus number of data samples for different values of SNR per data sample in Case I. The circle denotes the maximum value of DR.

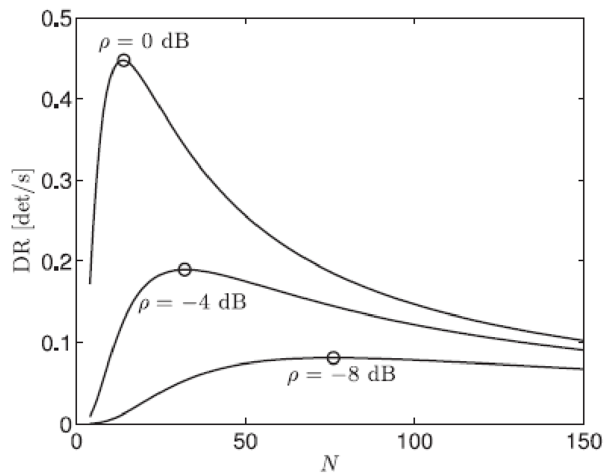

Fig. 2. Generated by Grossi, Lops and Venturino using the same parameters in Case I with their closed-form solution [1].

B. Case II

The secondary data samples of noise $\{x_l\}$ has size $L = 10$ in this setup. In Fig. 3, the optimal number of scans needed to detect is much larger with a smaller SNR per data point, and shows the common scenario of this detection problem out of the three cases. The trend is also clear that the detection rate decreases as the number of scans increases after an optimal point. Note that if we increase the number of secondary data samples of the noise L, the detection rate would increase to resemblance closer to Fig. 1.

When compared to Fig. 4, the same figure generated by Grossi, Lops and Venturino under the same parameters but with their closed-form solution [1], it is clear that the solution of optimized number of data samples N is close and the trend is identical.

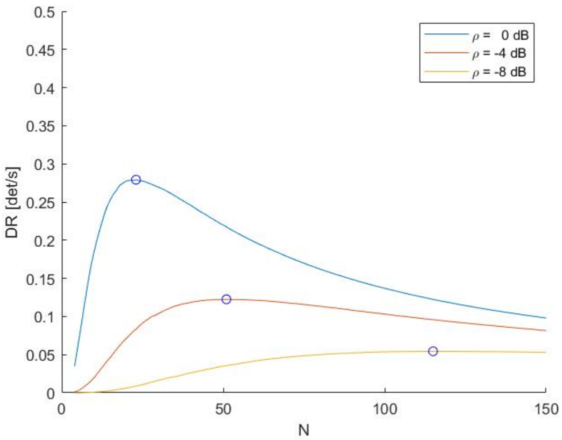

Fig. 3. DR versus number of data samples for different values of SNR per data sample in Case II. The circle denotes the maximum value of DR.

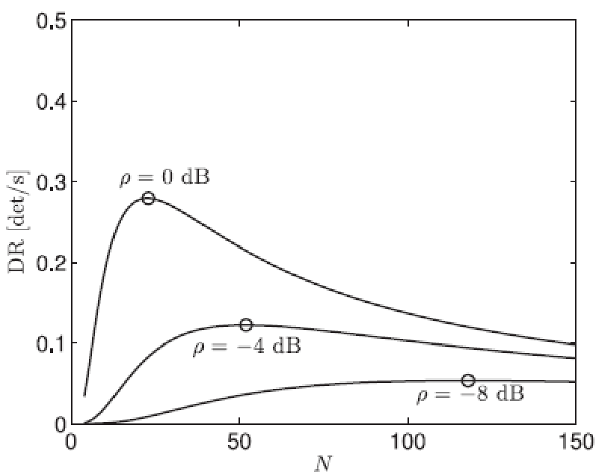

Fig. 4. Generated by Grossi, Lops and Venturino using the same parameters in Case II with their closed-form solution [1].

C. Case III

With a known noise variance σ^2, the curve in Fig. 5 has a much higher detection rate compared to Fig. 3. Once again, we observe the same trend that the detection rate decreases as the number of scans increases after an optimal point. The roughness of the trend comes from the smaller number of trials of the simulation, as calculating the General Likelihood Ratio Test consume more computation power than the first two cases.

This figure is not generated to replicate any figures presented in Grossi, Lops and Venturino's paper, only as a comparison to Fig. 1 and Fig. 3. This case is discussed as Case IV under Grossi, Lops and Venturino's paper [1].

IV. Conclusion

The simulation result verifies the result that an optimal scan time exists for a Detection rate problem. Detection rate and False alarm rate would provide a better metric in calculating radar configuration parameters under different

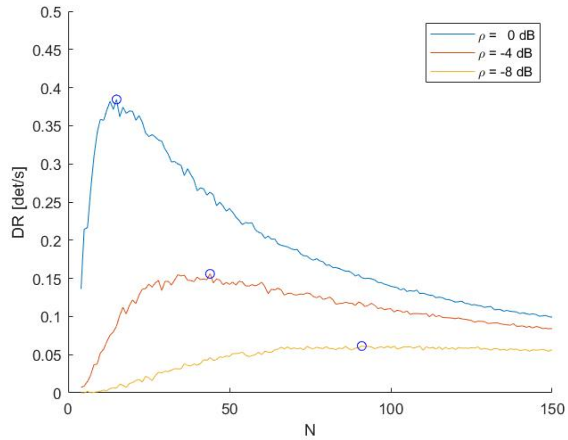

Fig. 5. DR versus number of data samples for different values of SNR per data sample in Case III. The circle denotes the maximum value of DR.

conditions. Several analyses were published to investigate the effect of different assumptions, like different Swerling models for different types of signals [2], [4], and has produced similar conclusions that an optimization against False alarm rate to maximize Detection rate is a valid metric under certain cases.

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