Homework 5 - Simulation of a Clamped Thin Beam Using a Discrete Plate Model

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1 Overview

In this homework, the static deformation of a thin cantilever beam is computed using the discrete plate model. The beam has length $l = 0.1$ m, width $w = 0.01$ m, and thickness $h = 0.002$ m. The left edge $(x = 0)$ is fully clamped, and gravity acts in the negative z-direction.

The goal is to compute the downward tip displacement

$$\delta_{\text{plate}}(t) = z_{\text{tip}}(t) - z_{\text{tip}}(0),$$

examine its evolution in time, extract the steady-state value, and compare it with the classical Euler–Bernoulli prediction

$$\delta_{EB} = \frac{ql^4}{8YI}, \quad q = \rho A g, \ A = wh, \ I = \frac{wh^3}{12}.$$

2 Deformation Snapshots

The mesh consists of two rows of ten nodes connected by triangular elements. The four leftmost nodes are fixed, enforcing the clamped boundary condition. Figures 1, 2, and 3 show the deformed configuration at $t = 0.001$ s, 3.001 s, and 9.001 s.
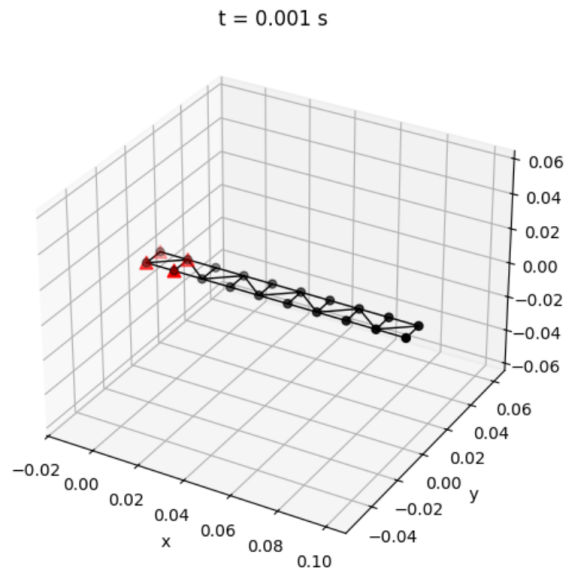

Figure 1: Plate configuration at $t = 0.001$ s, immediately after gravity is applied.
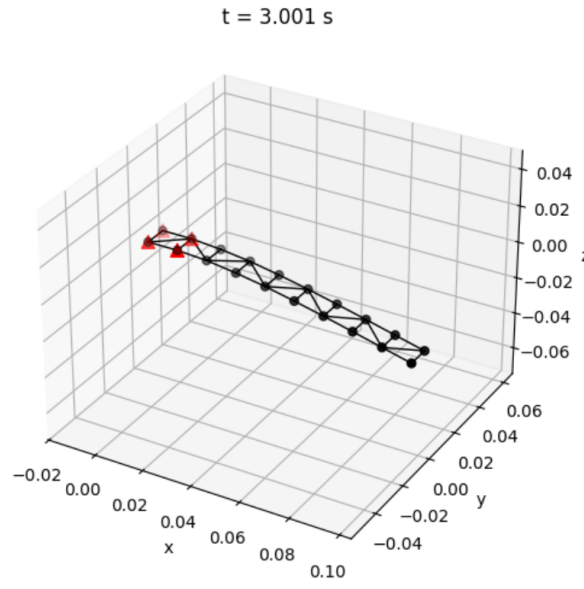
Figure 2: Intermediate deformation at $t = 3.001$ s.

The deformation shape is consistent with the expected bending mode of a cantilever beam under uniform loading.

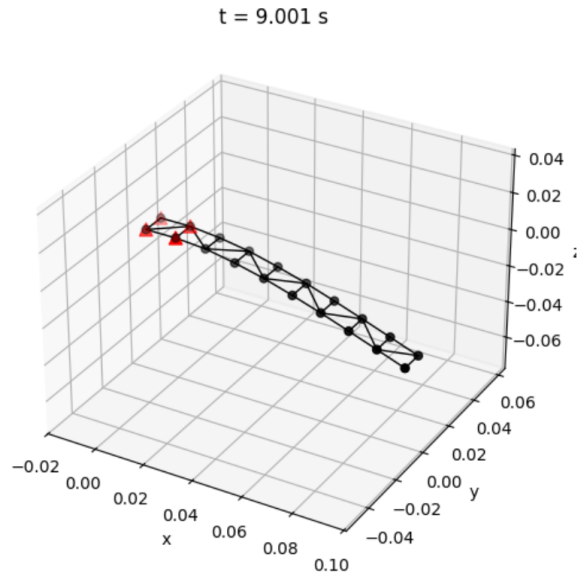Figure 3: Near-steady deformation at $t = 9.001$ s.

3   Tip Displacement vs Time

The end-tip displacement $\delta_{\text{plate}}(t)$ is plotted in Figure 4. The plate deflects downward smoothly and approaches a steady-state value.
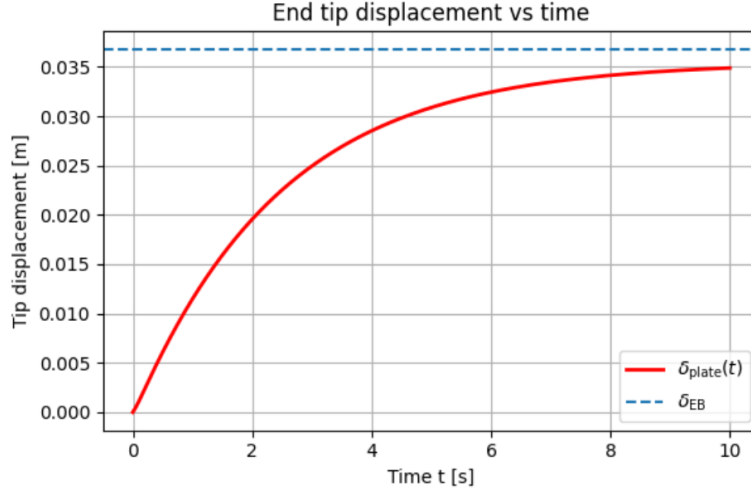

Figure 4: Time history of the end-tip displacement $\delta_{\text{plate}}(t)$ compared with Euler–Bernoulli prediction δ_{EB}.

From the plot, the numerical steady displacement is approximately

$$\delta_{\text{plate, final}} \approx 0.035 \text{ m}.$$

4   Comparison With Euler–Bernoulli Theory

Using the beam formula

$$\delta_{EB} = \frac{ql^4}{8YI},$$

and the given parameters ($\rho = 1000$ kg/m^3, $Y = 10^7$ Pa), the theoretical value is

$$\delta_{EB} \approx 0.037 \text{ m}.$$

The normalized difference is therefore

$$\frac{|\delta_{\text{plate, final}} - \delta_{EB}|}{\delta_{EB}} \approx \frac{|0.035 - 0.037|}{0.037} \approx 5\%.$$

This level of agreement is reasonable given the coarse mesh (two rows of nodes) and the difference between the plate model and beam theory. The plate simulation captures the correct magnitude of the static sag and converges smoothly toward the equilibrium configuration.

5   Conclusion

The discrete plate model successfully predicts the downward deflection of the clamped beam under self-weight. The simulated steady-state tip displacement agrees with Euler–Bernoulli theory within approximately 5%. The results confirm that even a simple triangular discretization can reproduce the essential bending behavior of a cantilever beam.