High/low Rank Regulated Bayesian Null-Stream Formulation to Probe for Alternative Gravitational Wave Polarizations

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Gravitational Wave Polarization

Gravitational Wave Polarization

- General relativity predicts tensor mode polarization only.

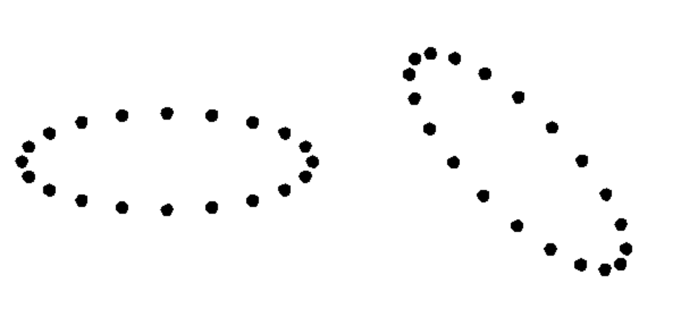


Figure 1: The effect on a ring of free-falling test particles of a GW in $+$ tensor mode, and $\times$ tensor mode.[2]

[2] https://en.wikipedia.org/wiki/Gravitational_wave

Gravitational Wave Polarization

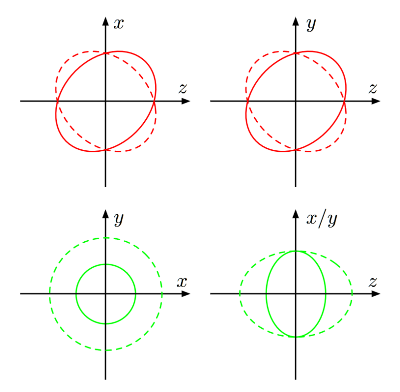


Figure 2: The effect on a ring of free-falling test particles of a GW in vector X mode (top left), vector Y mode (top right), breathing mode (bottom left) and longitudinal mode (bottom right).[4]

- General metric theory of relativity predicts six possible modes

[4] `https://vixra.org/abs/1103.0109`

Gravitational Wave Polarization

- Studying GW polarization can be a tool to test the validity of general relativity

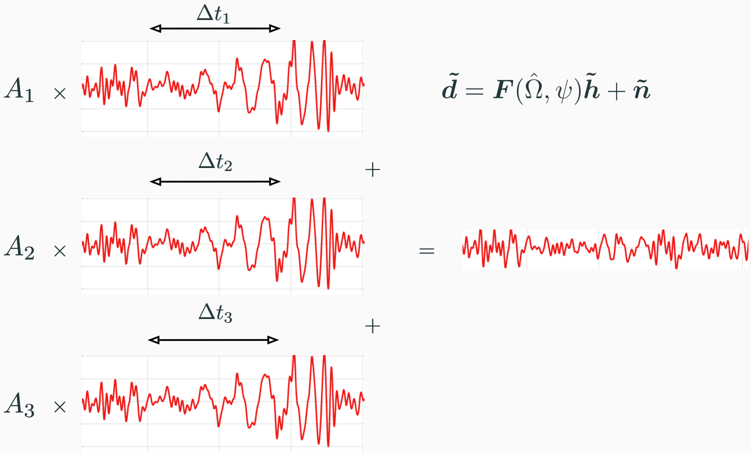
Null-Stream-based Approach

Why Null Stream?

- Projection-based method
- Does not require waveform template (also called Model-independent)

Why Null Stream?

Note that if we have 3 or more detectors, we can construct a null projector to cancel the signal content without having to assume the form of the wave.



$$\tilde{\boldsymbol{d}} = \boldsymbol{F}(\hat{\Omega}, \psi)\tilde{\boldsymbol{h}} + \tilde{\boldsymbol{n}}$$

Why Null Stream?

- The null projector is computed from the noise-weighed antenna response function $\boldsymbol{F}_{\boldsymbol{w}}^{+,\times} = [\boldsymbol{F}_{\boldsymbol{w}}^{+}(\hat{\Omega}), \boldsymbol{F}_{\boldsymbol{w}}^{\times}(\hat{\Omega})]$
- Noise-weighted:

$$\boldsymbol{F}_{\boldsymbol{w}}^{+,\times}(\hat{\Omega}, k) = \frac{\boldsymbol{F}^{+,\times}(\hat{\Omega})}{\sqrt{\frac{N}{2}\boldsymbol{S}[k]}}$$

- Null projector:

$$\boldsymbol{P}^{null} = \boldsymbol{I} - \boldsymbol{F}(\boldsymbol{F}^{\dagger}\boldsymbol{F})^{-1}\boldsymbol{F}^{\dagger}$$

- Only depends on the sky position and the geometry of the detectors.
- Does NOT depend on the waveform.

Polarization Study

- Different polarization hypothesis $\boldsymbol{H}$ imply different antenna response functions $\boldsymbol{F}$, for example:
  - $\boldsymbol{H}_{tensor} \rightarrow \boldsymbol{F} = [\boldsymbol{f}_{+}\ \boldsymbol{f}_{\times}]$
  - $\boldsymbol{H}_{vector} \rightarrow \boldsymbol{F} = [\boldsymbol{f}_{x}\ \boldsymbol{f}_{y}]$
  - $\boldsymbol{H}_{scalar} \rightarrow \boldsymbol{F} = [\boldsymbol{f}_{b}\ \boldsymbol{f}_{l}] = [\boldsymbol{f}_{b}]$
- Construct null projector corresponding to the hypotheses.
- Degree of Freedom from scalar hypothesis is 1 less since $\boldsymbol{f}_b$ and $\boldsymbol{f}_l$ are collinear.

Polarization Study

- Approximation is done to match the degrees of freedom in antenna response functions
- Rank reduction (by SVD):

$$F_w^{D\times 2} \approx F_w^{D\times 1}$$

- Rank promotion (by adding an orthogonal vector):

$$F_w^{D\times 1} \approx F_w^{D\times 2}$$

BANTAM(BAyesian Null sTreAM): A pipeline designed for null stream analysis.

- Set the Power Spectral Density (PSD).
- We used Advanced LIGO designed sensitivity and Advanced Virgo designed sensitivity.
- Set injection polarizations modes and hypotheses modes.

Output:

- Log-evidence for each model
- Signal-to-Noise Ratio (SNR)

Polarization Study

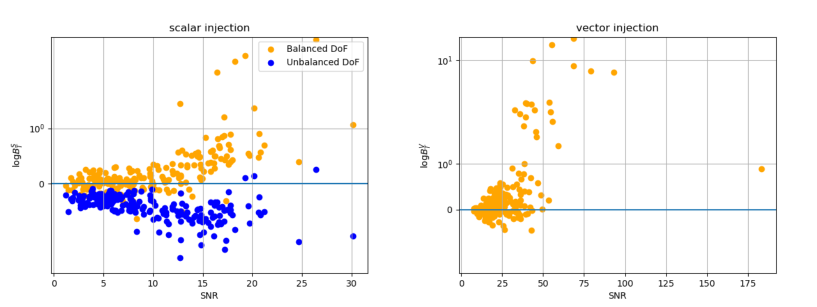


Figure 3: $\log B$ vs SNR, scalar and vector mode, 200 injections each

$$\text{log-Bayes-factor: } \log B^{M_2}_{M_1} = \log \frac{\int p(d|\vec{\theta};M_2)p(\vec{\theta})d\vec{\theta}}{\int p(d|\vec{\theta};M_1)p(\vec{\theta})d\vec{\theta}}$$

- Both show a generally correct trend
- Still have biases
- Use *machine learning* to reduce the biases

Machine learning

BANTAM
- Tensor x 200
- Scalar x 200
- Vector x 200

log-evidence

tensor_N_0	tensor_R_1	vector_N_0	vector_R_1	scalar_N_0	scalar_P_1	SNR
9.123	8.342	-2.342	-0.921	-3.023	-2.983	3.259
8.324	4.342	3.433	1.232	9.126	6.385	9.136
...	...	...	...	...	...	...

Remove SNR < 8 samples

log-Bayes-factor

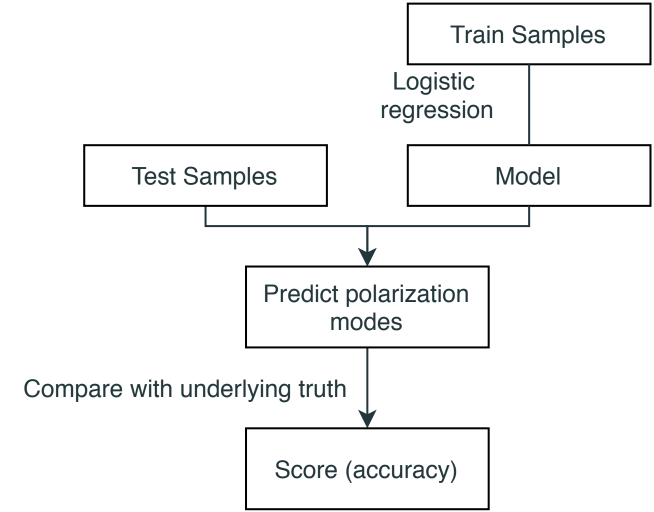
tensor_N_0 - vector_N_0	tensor_R_1 - scalar_N_0	tensor_R_1 - vector_R_1	...	SNR
0.352	-0.923	-1.362	...	19.923
-0.923	-1.342	1.433	...	23.142
0.355	5.242	-19.244	...	9.511
...	...	...	...	14.299

80% → Train Samples

20% → Test Samples

N: Normal DoF, R: Reduced DoF, P: Promoted DoF

Machine learning



Preliminary Results

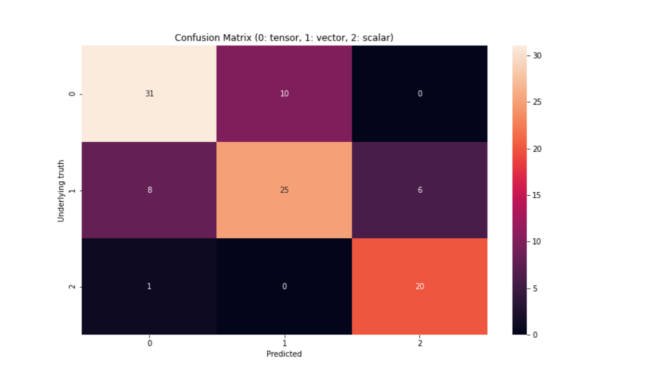


Figure 4: Confusion matrix classifying tensor, vector, scalar modes from test samples, score: 0.752.

0: Tensor, 1: Vector, 2: Scalar

Conclusion

- By using null stream analysis and Bayes-factors from rank reduced/promoted antenna response function hypotheses, constraint by the difference in degree of freedom can be relaxed.
- We demonstrate the feasibility to apply machine learning to probe for non-tensorial modes with the approximated log-Bayes-factors

Potential Improvements

- Use more generic waveforms eg. sine-gaussian wavelet to simulate non-tensorial signals instead of projecting tensorial functions onto non-tensorial spaces.
- Use other models to classify the GW events. (like Random Forest classifier/neural network)
- More train samples.