Self-interacting Boson stars

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Abstract

Current development in cosmology suggested that scalar fields may help to solve the dark matter mystery by having the scalar particles to form condensed objects. One example is Boson star, a hypothetical compact object that could be constructed with a complex scalar field coupled to gravity and the self repulsive interaction between particles. In this study by assuming the standard outgoing wave boundary condition of the scalar field and the Schwarzchild-like metric, the initial state and the evolution of a self-gravitating Boson star with self-interaction are evaluated by solving the Einstein-Klein-Gordan (EKG) system of equations with spherical symmetry. The stability of boson stars and the dynamical collapse of unstable stars to black holes are discussed. Even the code is not yet well tested, some results are presented here.

Introduction

Suppose the metric is Schwarzchild-like:

$$ds^2 = -\alpha^2(r,t)dt^2 + a^2(r,t)dr^2 + r^2 d\Omega^2$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$.
The potential with self-interacting constant λ:

$$V(|\phi|^2) = \frac{1}{2}m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

For Boson star, it has a complex scalar field, i.e. $\phi = \phi_1 + i\phi_2$. Theoretically, the scalar field is harmonic in time, which means that $\phi(r,t) = \phi_0(r)e^{i\omega t} = \phi_0(r)\cos\omega t + i\phi_0(r)\sin\omega t$.

Solving the EKG equations for the initial states where α and a are assumed to be independent of coordinate time t:

$$\frac{\partial_r a}{a} = \frac{1-a^2}{2r} + \frac{\kappa_0}{4}r[(\frac{\omega a\phi_0}{\alpha})^2 + \Phi^2 + a^2\phi_0^2(m^2 + \frac{1}{2}\lambda\phi_0^2)]$$

$$\frac{\partial_r \alpha}{\alpha} = \frac{a^2-1}{r} + \frac{\partial_r a}{a} - \frac{\kappa_0}{2}ra^2\phi_0^2(m^2 + \frac{1}{2}\lambda\phi_0^2)$$

$$\partial_r\Phi = \Phi(\frac{\partial_r a}{a} - \frac{\partial_r\alpha}{\alpha} - \frac{2}{r}) + a^2\phi_0(m^2 + \lambda\phi_0^2 - (\frac{\omega}{\alpha})^2)$$

$$\partial_r\phi_0 = \Phi$$

The boundary conditions are $a(0) = 1$, $\phi_0(0) = k$, $\phi_0(\infty) = 0$, $\Phi(0) = \Phi(\infty) = 0$ and $\alpha(r_\infty) = \frac{1}{a(r_\infty)}$, where k is chosen initially.

Define new variables $\Phi_i = \partial_r\phi_i$ and $\pi_i = \frac{a}{\alpha}\partial_t\phi_i$. Then, the evolution equations become:

$$\partial_t\phi_i = \frac{\alpha}{a}\pi_i \ (for\ i = 1,2)$$

$$\partial_t\Phi_i = \partial_r(\frac{\alpha}{a}\pi_i) \ (for\ i = 1,2)$$

$$\partial_t\pi_i = \frac{\partial}{\partial r^3}(r^2\frac{\alpha}{a}\Phi_i) - 2a\alpha\phi_i\frac{dV}{d|\phi_i|^2} \ (for\ i = 1,2)$$

$$\frac{\partial_r a}{a} = \frac{1-a^2}{2r} + \frac{\kappa_0}{4}r(\pi_1^2 + \pi_2^2 + \Phi_1^2 + \Phi_2^2 + 2a^2V)$$

$$\frac{\partial_r\alpha}{\alpha} = \frac{a^2-1}{r} + \frac{\partial_r a}{a} - \kappa_0 ra^2V$$

Boundary conditions and initial conditions:

$$\begin{cases} \partial_r\pi_i(r_\infty) + \partial_t\pi_i(r_\infty) + \frac{\pi_i(r_\infty)}{r_\infty} = 0 \\ \partial_r\phi_i(r_\infty) + \partial_t\phi_i(r_\infty) + \frac{\phi_i(r_\infty)}{r_\infty} = 0 \\ \Phi_i(r_\infty) + \pi_i(r_\infty) + \frac{\phi_i(r_\infty)}{r_\infty} = 0 \end{cases}$$

$$\begin{cases} \phi_1(0,r) = \phi_0(r)\ , \phi_2(0,r) = 0 \\ \Phi_1(0,r) = \Phi(r)\ , \Phi_2(0,r) = 0 \\ \pi_1(0,r) = 0\ , \pi_2(0,r) = \frac{\omega a(r)\phi_0(r)}{\alpha(r)} \\ a(0,r) = a(r) \\ \alpha(0,r) = \alpha(r) \end{cases}$$

In this study, we choose our units such that $\kappa_0 = 8\pi$ and $m = 1$.
Mass of Boson star:

$$M = \frac{r_\infty}{2}(1 - \frac{1}{a^2(\infty)})$$

Binding energy $E_b = M - Nm$, where

$$N = \int \frac{i}{2}\sqrt{-g}g^{0\nu}[\phi^*\partial_\nu\phi - \phi\partial_\nu\phi^*]\,d^3x$$

is the number of particle (or bosonic number).

Initial data

We define a new variable $\Lambda = \frac{\lambda}{4\pi}$. The Initial state equations are solved by the standard scheme of Fourth order Runge-Kutta, shooting method is applied simultaneously to find ω.

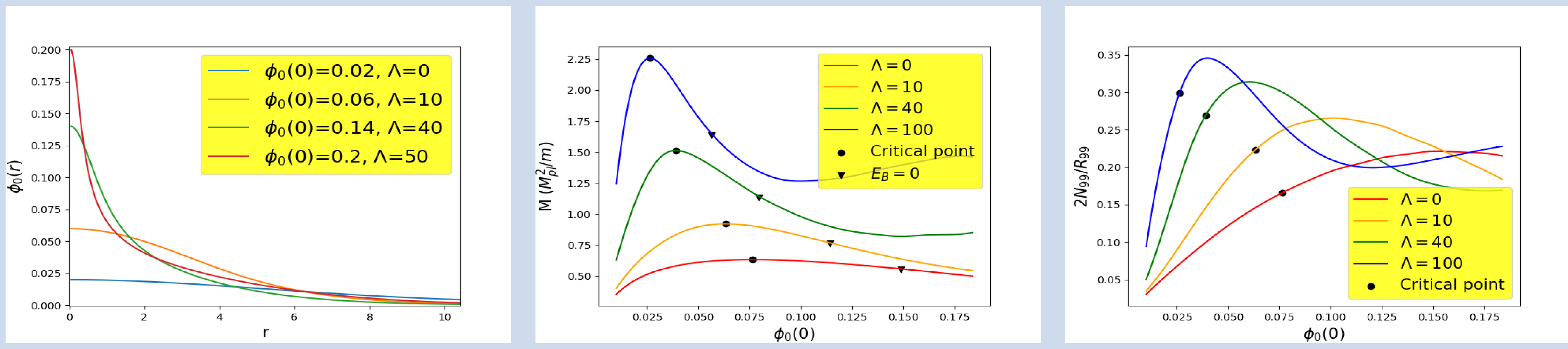

Figure 1. The left graph shows some solutions of $\phi_0(r)$. The middle graph shows the mass of boson star against different $\phi_0(0)$. The rightmost graph shows the compactness of each solution.
The leftmost graph shows that $\phi_0(r)$ is always a Gaussian-like function. In the middle graph, the inverted triangular bullets are the points that $E_b = 0$, the circular bullets represent the maximum mass for each Λ. Configurations located on the left-hand side of the circular bullet are stable boson stars for each constant Λ line; while configurations on the right-hand side are unstable. Specifically, every configuration between the circular bullet and the inverted triangular bullet will collapse to a black hole even for a very small perturbation. The rightmost graph shows the compactness of each solution, by defining N_{99} as the 99% of the number of particles and R_{99} is the radius that gives N_{99}.

Time evolution

The Evolution equations are solved by Partially Implicit Runge-Kutta (PIRK) schemes. More specifically, the PIRK3 scheme is used to evaluate $\phi_i(r,t)$, $\Phi_i(r,t)$ and $\pi_i(r,t)$, while the PIRK2 scheme is used to evaluate $a(r,t)$, $\alpha(r,t)$. Spatial derivatives are evaluated by the central finite-difference method and the rightward finite-difference method for the outer boundary.

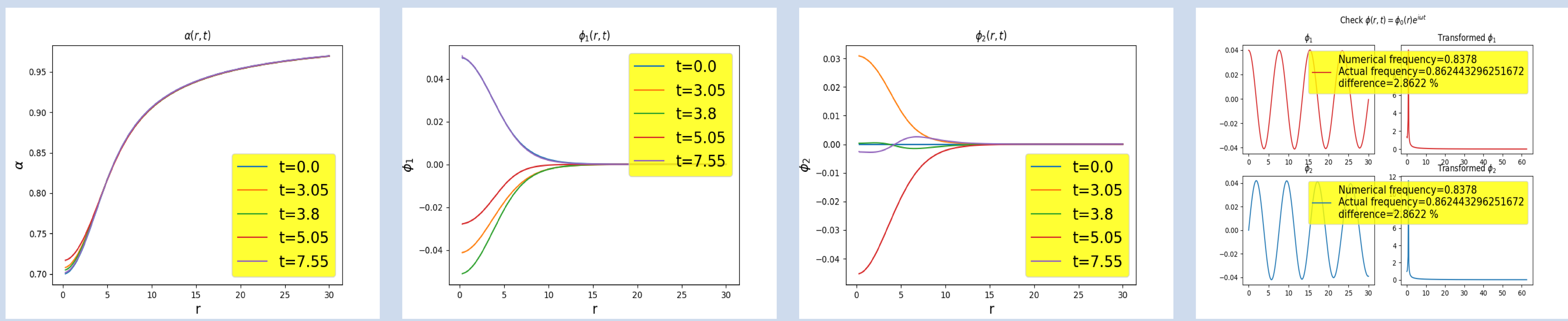

Figure 2. A stable model: $\phi(0,0) = 0.05$, $\Lambda = 10$.
The above graphs show the evolution of a stable boson star. As time evolves, there is only a little perturbation on the lapse function $\alpha(r,t)$. Plots of $\phi_1(r,t)$ and $\phi_2(r,t)$ also show the desired behavior by having harmonic oscillations in time. The oscillation frequency can be obtained by performing a Fourier transform and agrees with the expected value ω.

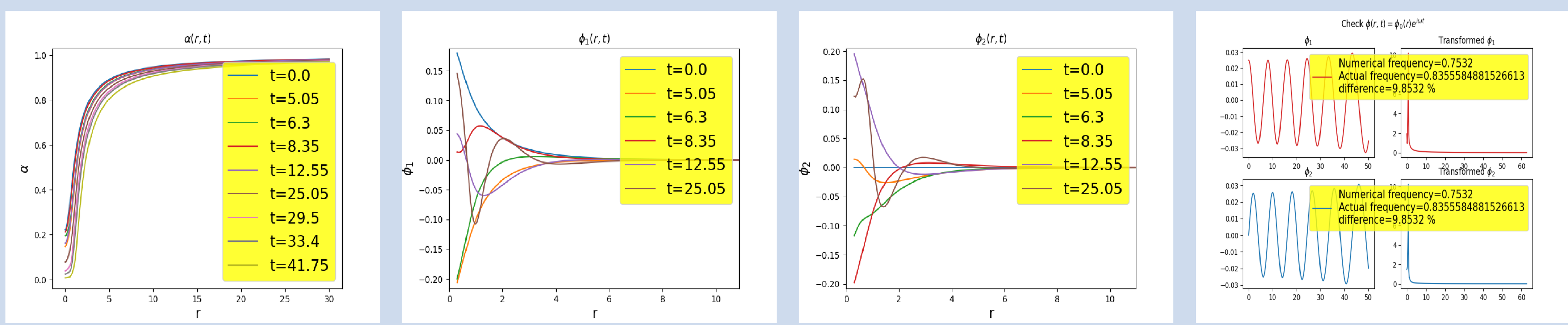

Figure 3. An unstable model: $\phi(0,0) = 0.2$, $\Lambda = 10$.
The above graphs show the evolution of an unstable boson star. As time evolves, the central value of the lapse function $\alpha(r,t)$ is generally collapsing to zero, implying that the boson star is collapsing to a black hole. Besides, the plots of $\phi_1(r,t)$ and $\phi_2(r,t)$ show a harmonic relation with time.

Conclusion and Acknowledgements

We study the static properties and dynamics of boson stars by solving the EKG equations numerically in spherically symmetric spacetimes. In particular, the stability of stable boson stars and the dynamical collapse of unstable configurations are studied.

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