Chapter 5

Carrier Transport Phenomena

學習重點

1. 熱擾 (或熱移動 thermal motion)
2. 載子受電場影響產生之移動，稱為飄移，其產生之電流稱飄移電流
3. 載子濃度不均產生之移動，稱為擴散，由高濃度向低濃度擴散，其產生之電流稱擴散電流
4. 飄移程度：受電場影響之移動快慢，用載子遷移率描述
5. 擴散現象：用擴散常數描述
6. 愛因斯坦關係：熱平衡下遷移率與擴散常數之間的關係
7. 霍爾效應及其應用

Thermal motion

- Even without an applied electric field, carriers are not at rest but possess finite kinetic energies

$$Average\ electron\ kinetic\ energy = \frac{total\ kinetic\ energy}{number\ of\ electrons} = \frac{\int f(E) * D(E) * (E - E_c)\, dE}{\int f(E) * D(E)\, dE}$$

$$Theral\ velocity = v_{th} = \sqrt{\frac{3kT}{m}}$$

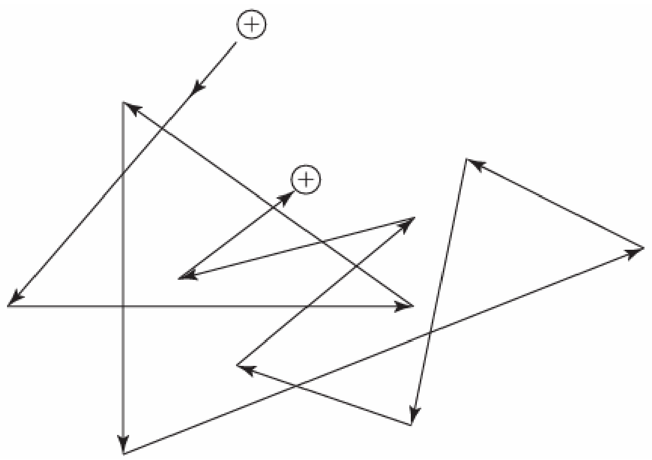
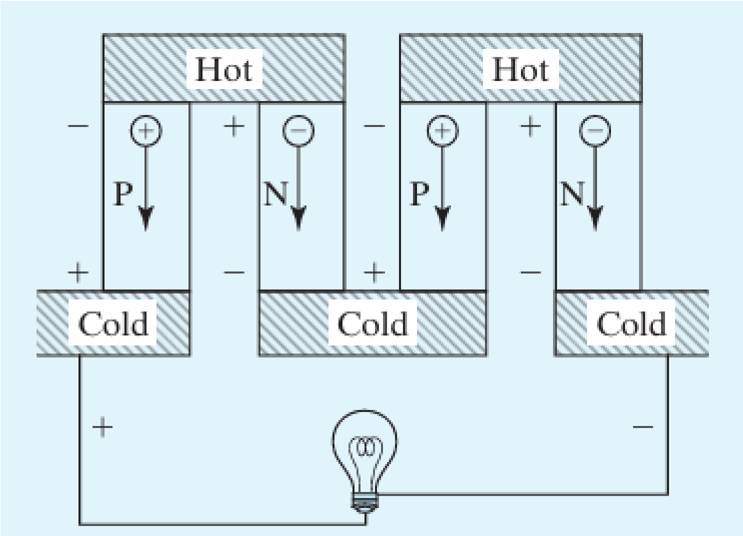


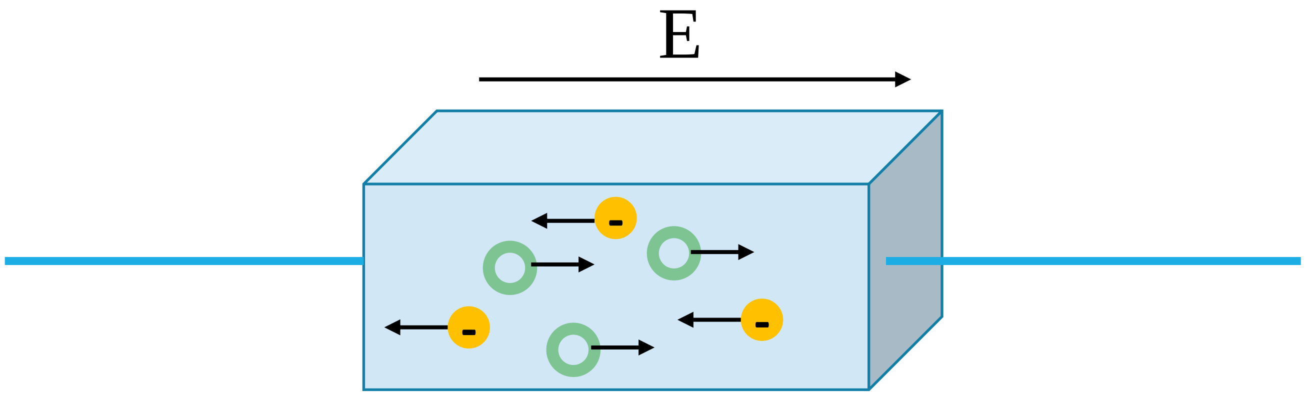
FIGURE 2–1 The thermal motion of an electron or a hole changes direction frequently by scattering off imperfections in the semiconductor crystal.



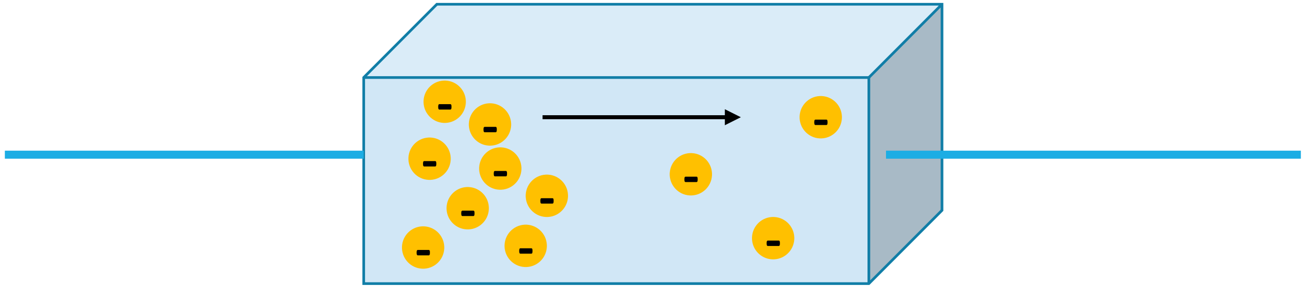
傳導 Transport

傳導: 材料裡面的電子或電洞之淨流動

- 飄移 Drift: 因電場而引起的淨流動



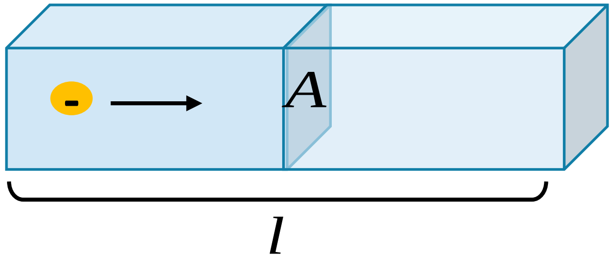
- 擴散 Diffusion: 因濃度不均勻而引起的淨流動



載子飄移

Drift Current Density 飄移電流密度 (單位面積流過的電流)

$$J = \frac{I}{A} = \rho v$$



電子 $$J_e = \rho v_{dn} = (-e) n v_{dn}$$

電洞 $$J_h = \rho v_{dp} = e p v_{dp}$$

ρ 電荷密度 C/cm^3

v_d 飄移速度 cm/sec

n 電子密度 $1/cm^3$

p 電洞密度 $1/cm^3$

遷移率 Mobility

- Mobility: 飄移速度與施加電場強度之間的關係常數

電子 $v_{dn} = -\mu_n \mathrm{E}$

電洞 $v_{dp} = \mu_p \mathrm{E}$

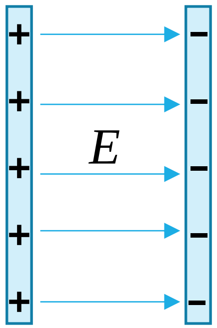


Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

- 總飄移電流密度

$$J = (-e)nv_{dn} + epv_{dp}$$

$$= (-e)n(-\mu_n \mathrm{E}) + ep(\mu_p \mathrm{E})$$

$$= \underbrace{e(n\mu_n + p\mu_p)}_{\text{電導率}}\mathrm{E} = \sigma \mathrm{E}$$

Example 5.1

Objective: Calculate the drift current density in a semiconductor for a given electric field.

Consider a gallium arsenide sample at $T = 300$ K with doping concentrations of $N_a = 0$ and $N_d = 10^{16}\ \text{cm}^{-3}$. Assume complete ionization and assume electron and hole mobilities given in Table 5.1. Calculate the drift current density if the applied electric field is $E = 10$ V/cm.

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$$n = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + n_i^2} \approx 10^{16} \Rightarrow p = \frac{n_i^2}{n} = \frac{\left(1.8\times10^{6}\right)^2}{10^{16}} = 3.24\times10^{-4}$$

$$J = e\left(n\mu_n + p\mu_p\right)\mathrm{E} \approx en\mu_n\mathrm{E} = \left(1.6\times10^{-19}\right)\left(10^{16}\right)\left(8500\right)\left(10\right) = 136 \quad \mathrm{A/cm^2}$$

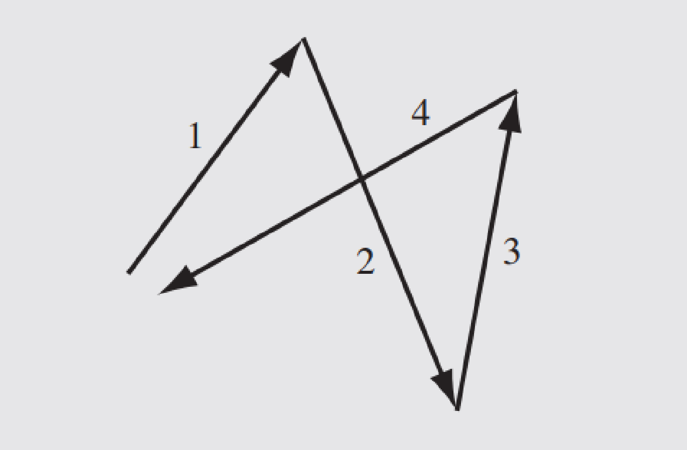
遷移率 Mobility

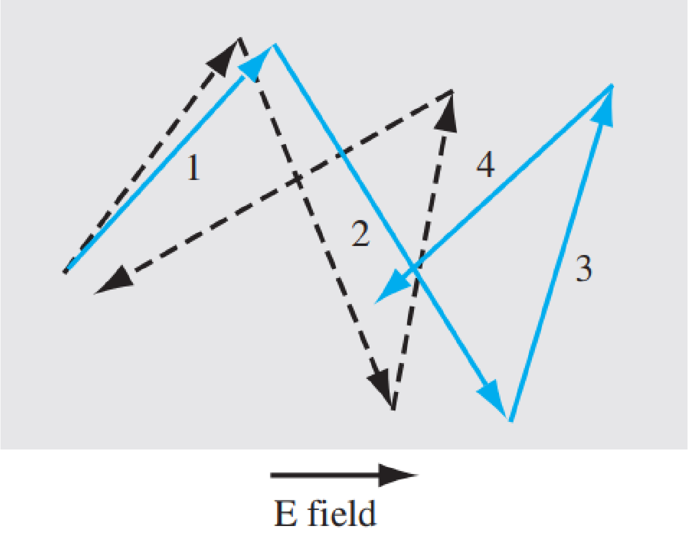
牛頓第二定律 $\quad F = ma = m\dfrac{v_d}{\tau}$

電場給電荷的力 $\quad F = e\mathrm{E}$

$$\begin{cases} F = ma = m\dfrac{v_d}{\tau} \\ F = e\mathrm{E} \end{cases} \Rightarrow v_d = \frac{e\tau}{m}E \Rightarrow \begin{cases} \mu_n = \dfrac{e\tau_{cn}}{m_{cn}^*} \\ \mu_p = \dfrac{e\tau_{cp}}{m_{cp}^*} \end{cases}$$

τ: 平均碰撞週期，每經過τ就會產生一次碰撞





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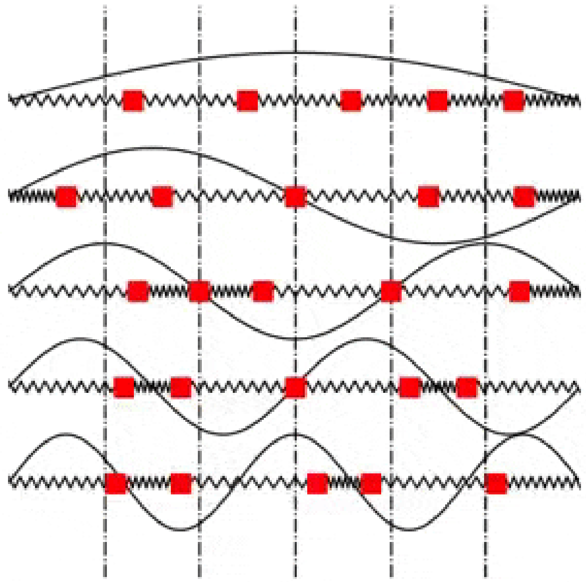
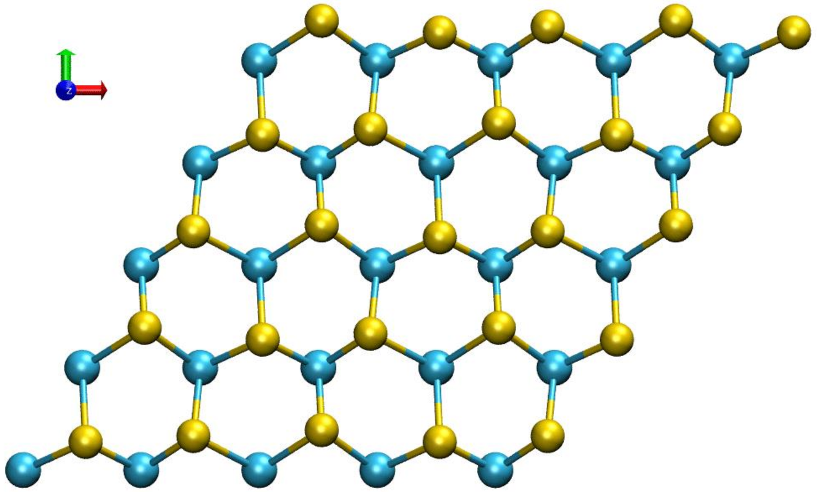
聲子散射 Phonon scattering

1. 在高於絕對零度時，半導體中的原子會在相對其晶格位置作隨機振動。此振動會破壞完美週期位能函數。
2. 當溫度升高，原子震動升高，電子與聲子碰撞的機率升高

$$\mu_L \propto \left(\frac{1}{T}\right)^{1.5}$$

溫度愈高，mobility愈差



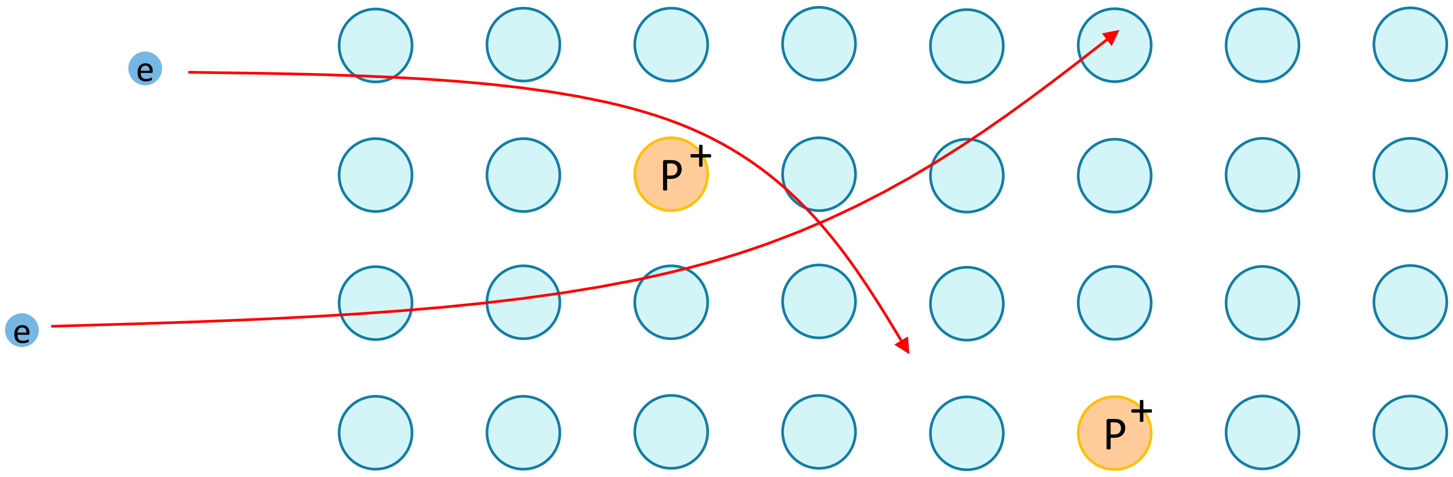


游離雜質散射 Ionized impurity scattering

1. 受到晶格中帶電離子的庫侖力影響，進而產生的碰撞
2. 考慮離子時，載子的遷移率:

$$N_I = Nd^+ + Na^-$$

$$\mu_I \propto \frac{T^{1.5}}{N_I} \longrightarrow \text{離子濃度}$$



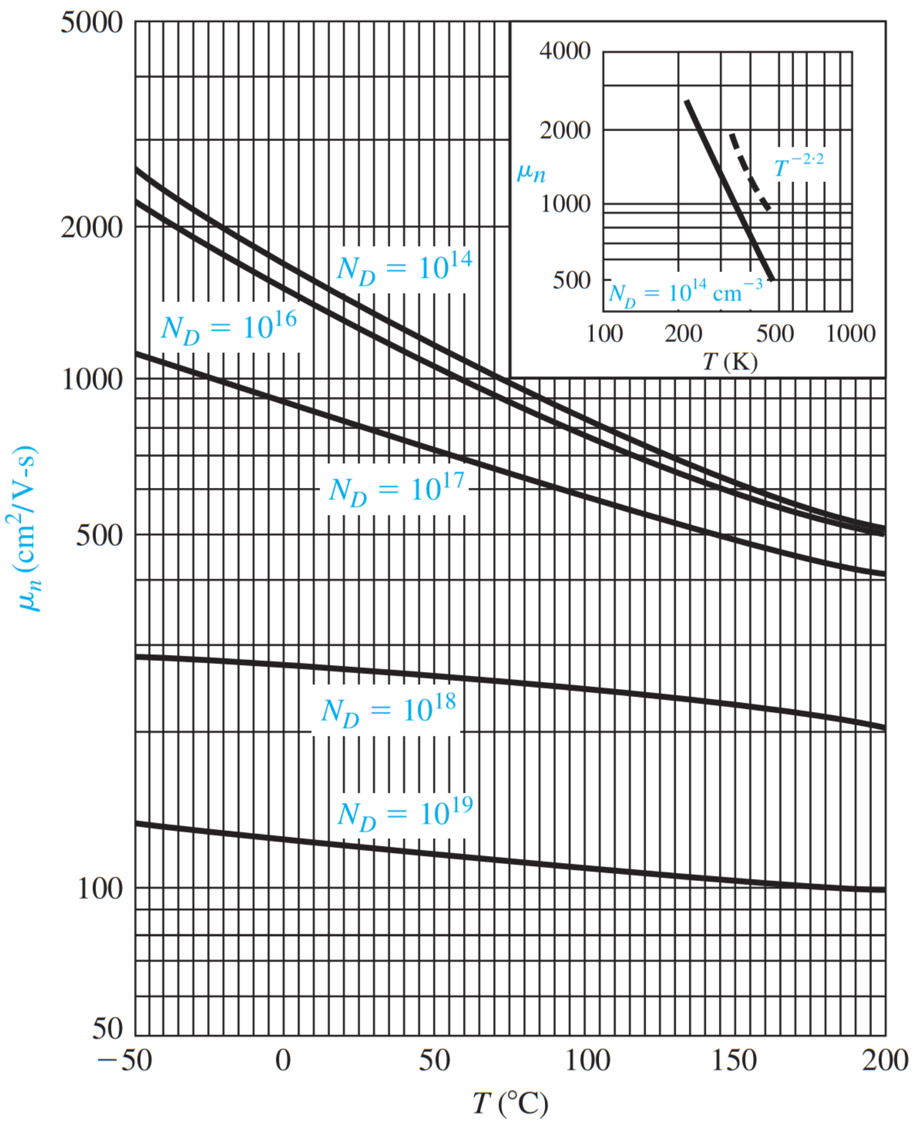
等效遷移率

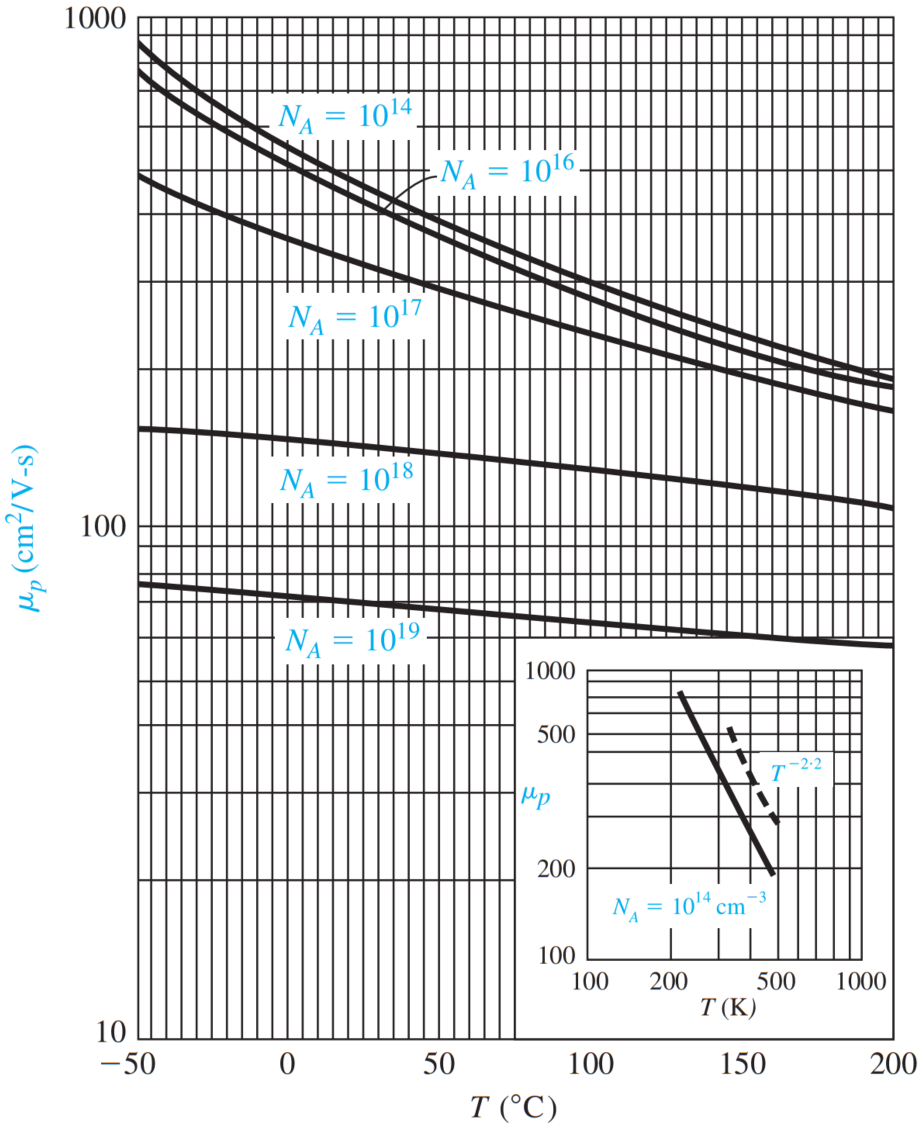
Effective Mobility $$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I} + \frac{1}{\mu_A} + \cdots$$

聲子　　離子　　合金

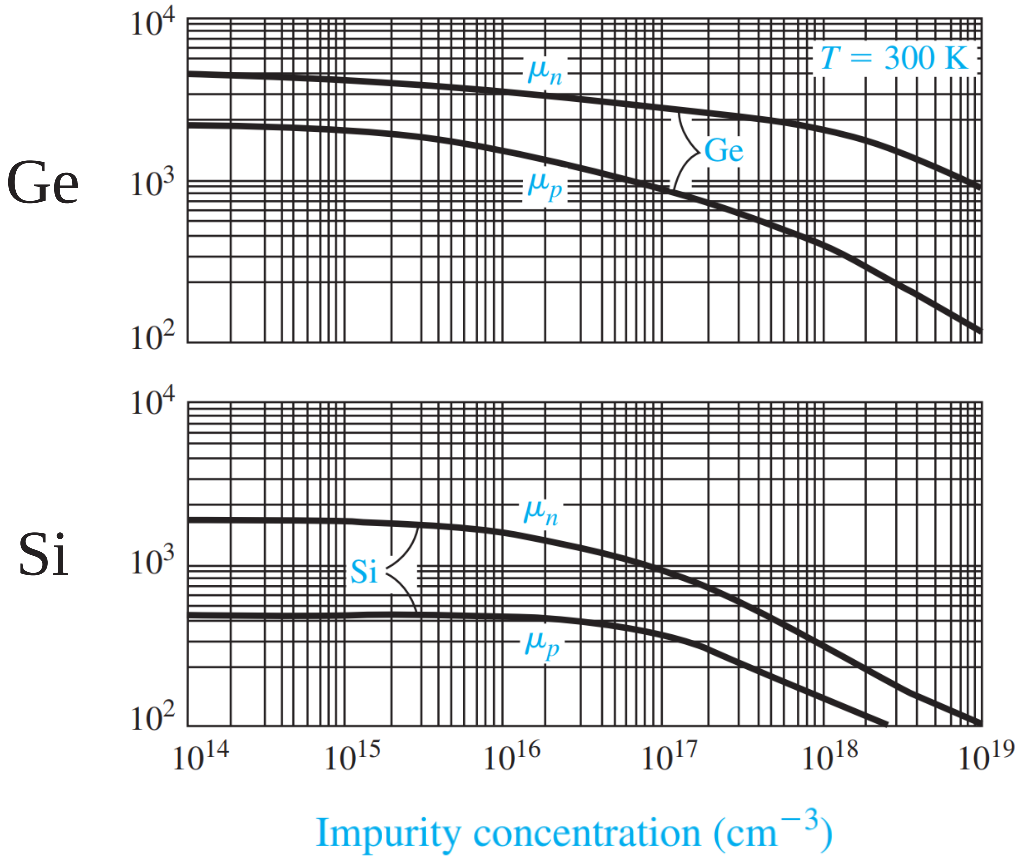
1. Lattice scattering
2. Ionized impurity scattering
3. Alloy scattering (GaAs)
4. Inelastic scattering
5. Electron–electron scattering

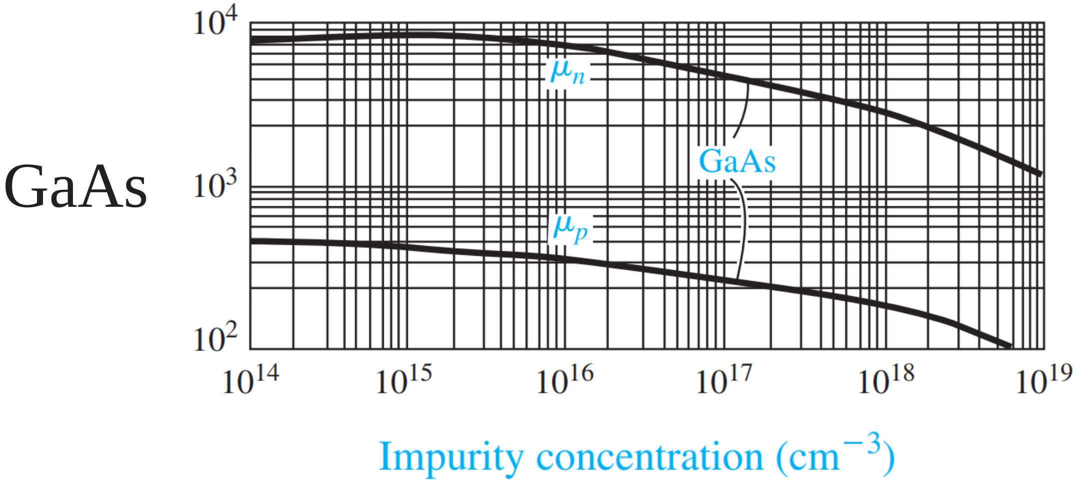
Mobility vs T (Extrinsic Silicon)





Mobility vs Doping Level (T=300 K)





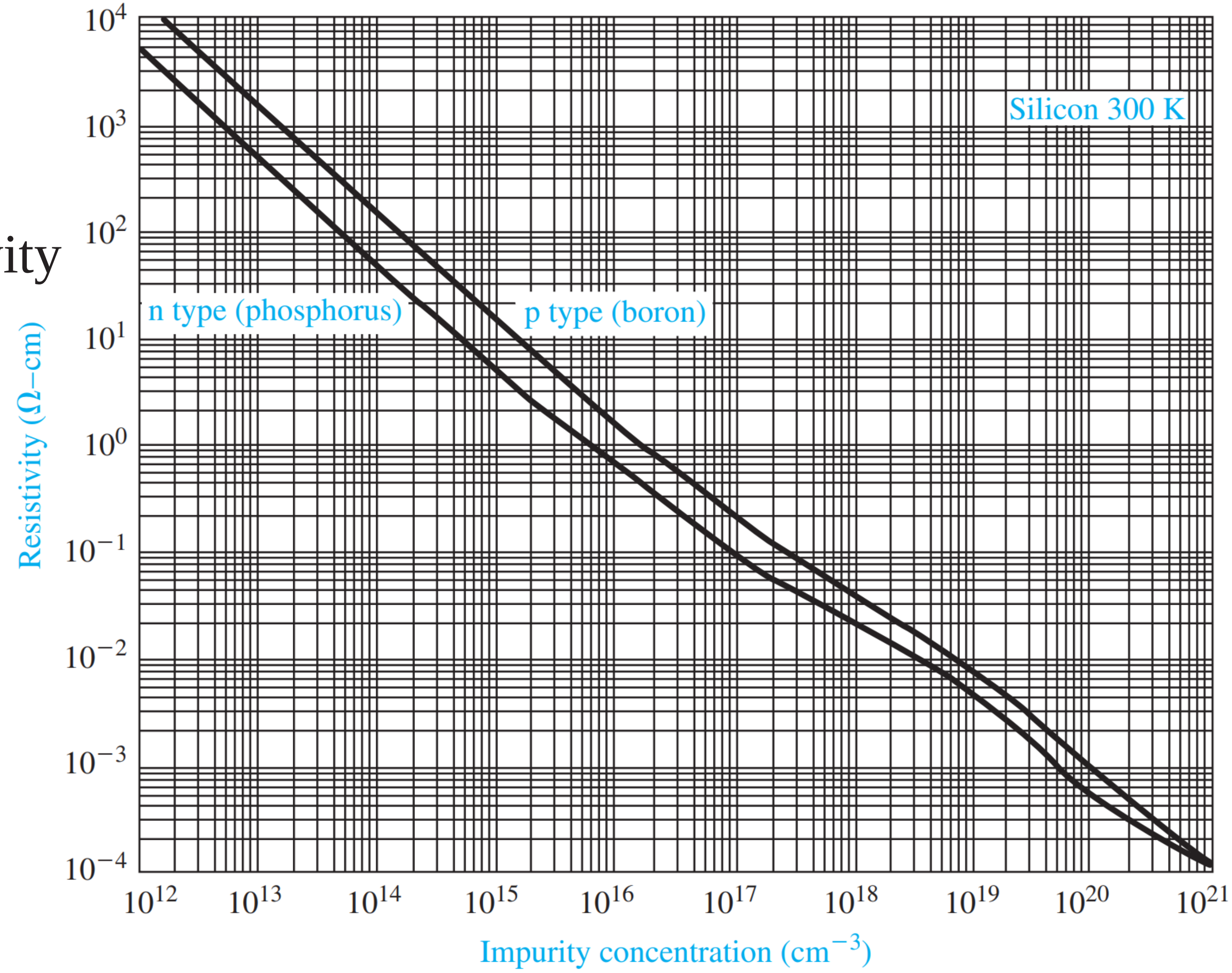
Resistivity vs Doping Level (Silicon)

$$J = e\left(n\mu_n + p\mu_p\right)\mathrm{E} = \sigma \mathrm{E}$$

Conductivity
電導率

$$\rho = \frac{1}{\sigma} = \frac{1}{e\left(n\mu_n + p\mu_p\right)}$$

Resistivity
電阻率



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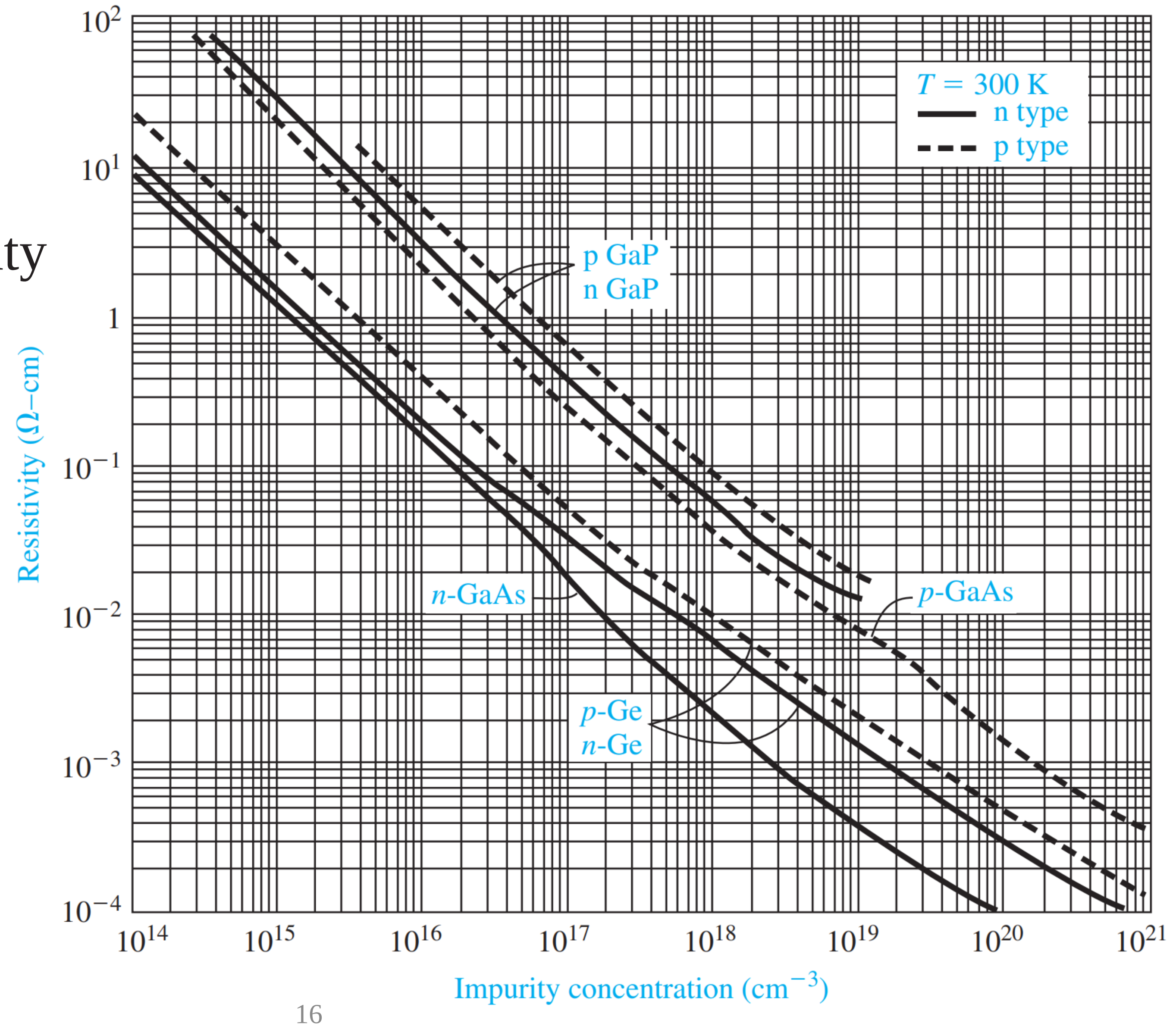
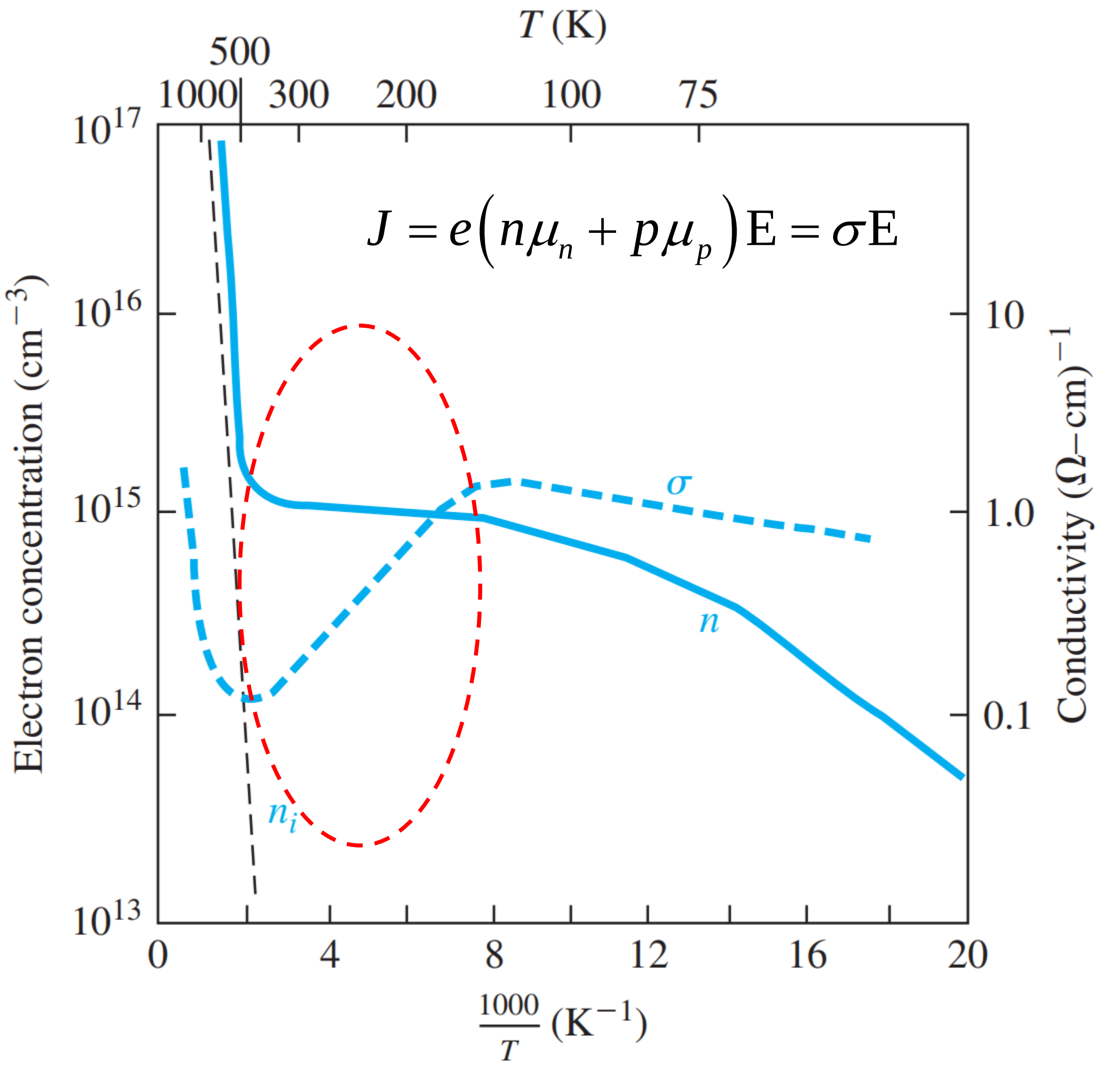


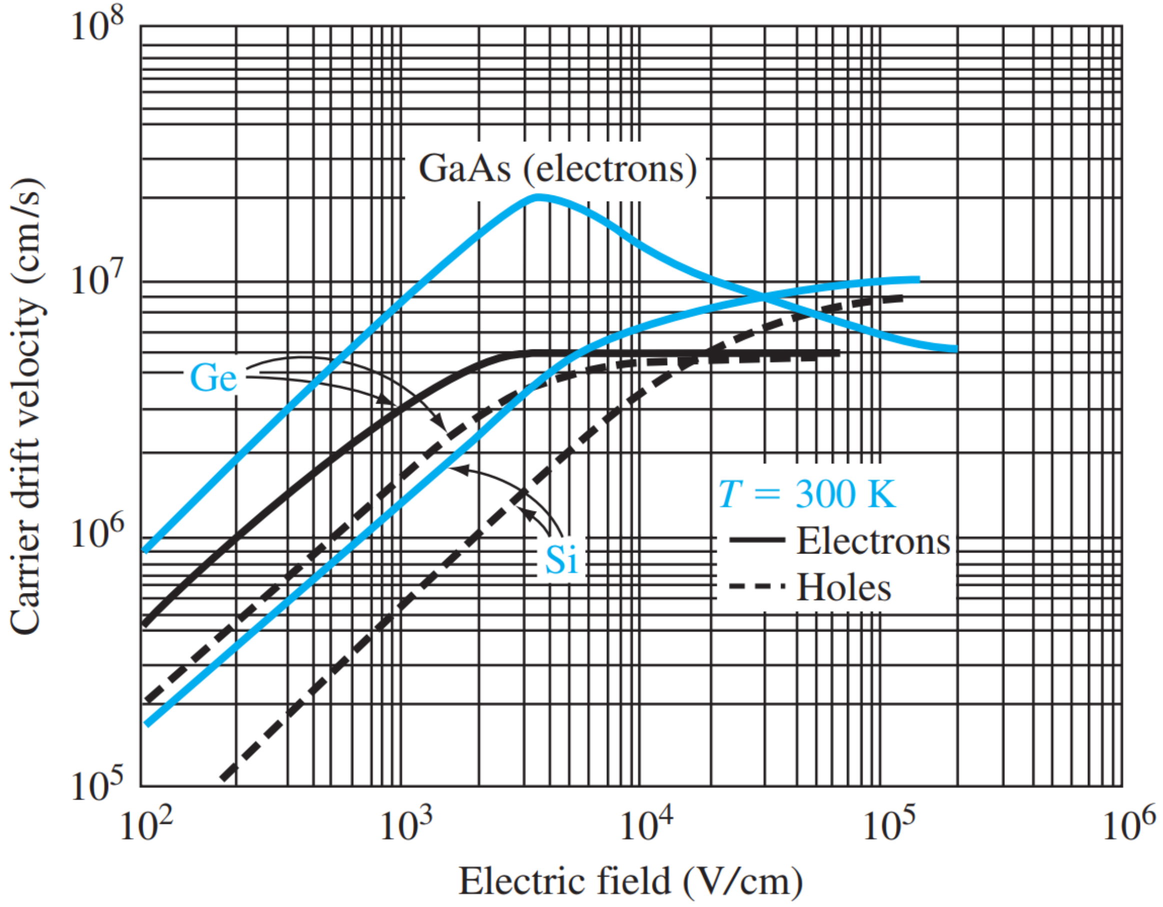
Conductivity vs T^{-1}

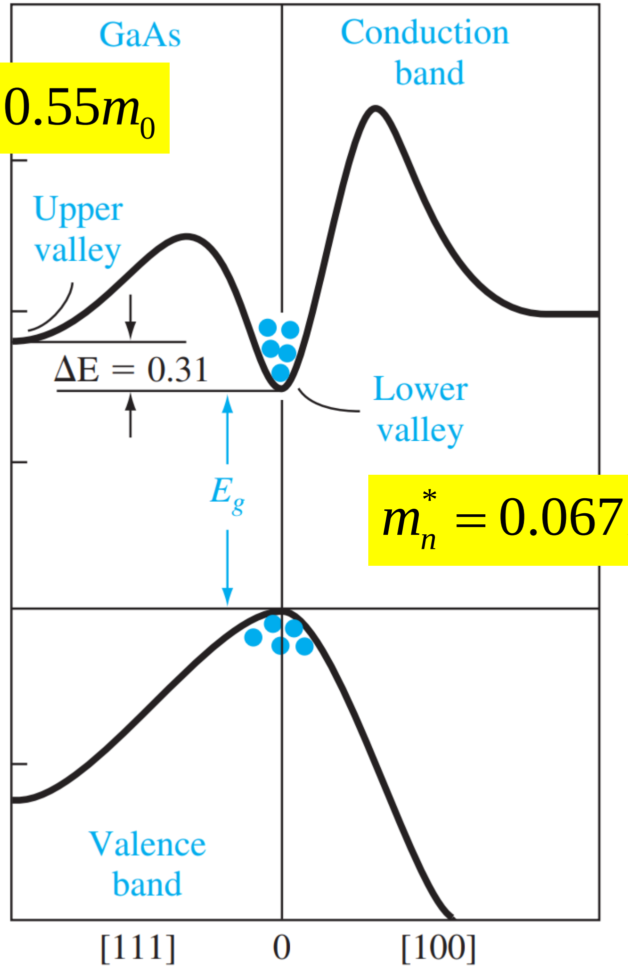


Velocity Saturation (飽和)

線性區 $v_d = \mu E$

飽和區 $\approx 10^7$ cm/s





$m_n^* = 0.55m_0$

$m_n^* = 0.067m_0$

載子擴散

通量 (每單位面積流過的量)

$$F = n\, v_{th}$$

n　載子濃度，隨位置變化

v_{th} 平均熱效應速度

在位置 $x=0$ ，載子往左流到 $x=-l$

在位置 $x=l$ ，載子往左流到 $x=0$

$$F = \frac{1}{2} n_{-l} v_{th} - \frac{1}{2} n_l v_{th} = \frac{1}{2} v_{th} \left(n_{-l} - n_l \right)$$

$$= -v_{th} l \frac{dn}{dx} = -D \frac{dn}{dx}$$

D :擴散常數

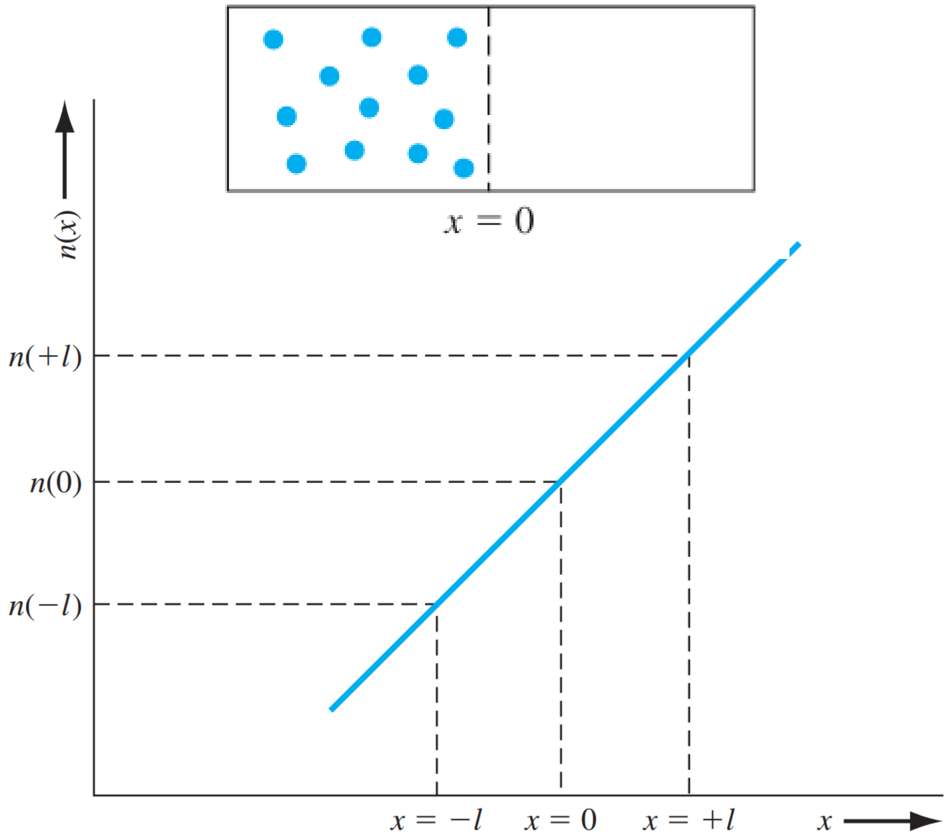
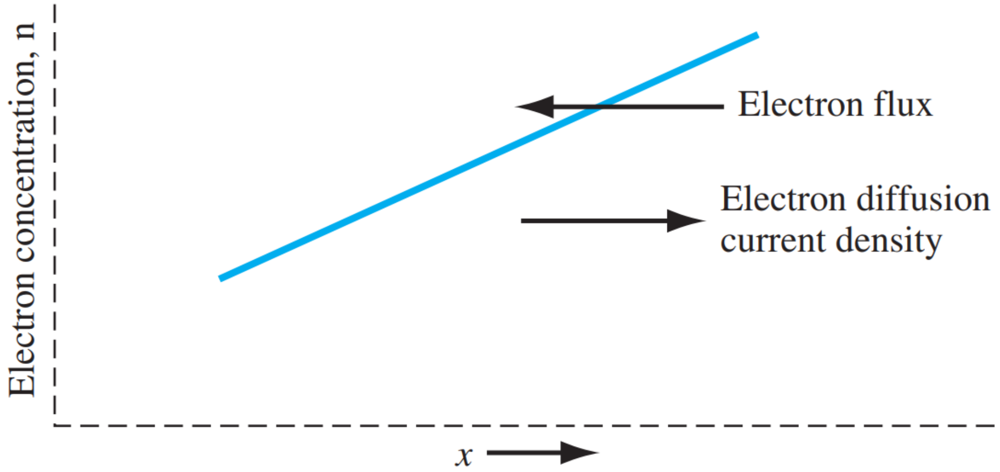


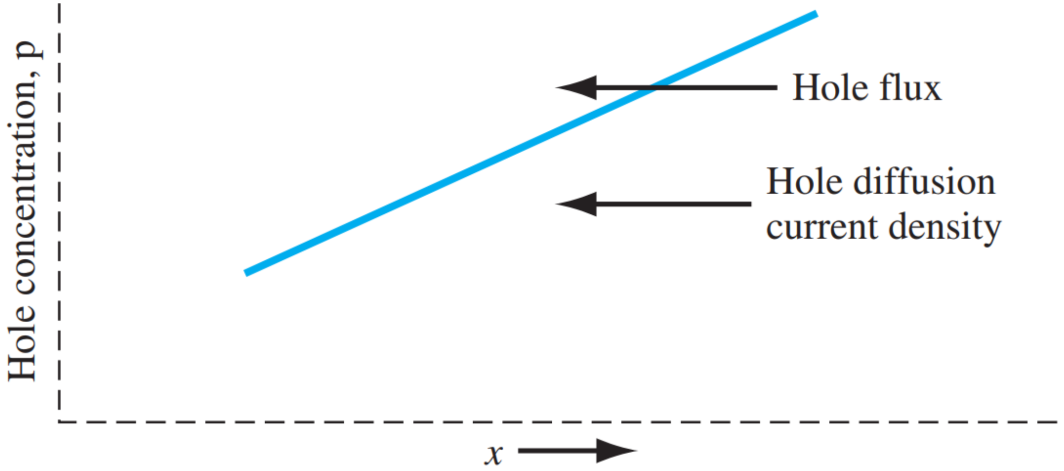
Figure 5.10 | Electron concentration versus distance.

載子擴散

$$J_n = (-e)F = eD_n \frac{dn}{dx}$$



$$J_p = eF = -eD_p \frac{dn}{dx}$$



Example 5.5

Objective: Calculate the diffusion current density given a density gradient.

Assume that, in an n-type gallium arsenide semiconductor at $T = 300$ K, the electron concentration varies linearly from 1×10^{18} to 7×10^{17} cm^{-3} over a distance of 0.10 cm. Calculate the diffusion current density if the electron diffusion coefficient is $D_n = 225$ cm^2/s.

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$$J_{n|dif} = eD_n \frac{dn}{dx} \approx eD_n \frac{\Delta n}{\Delta x}$$

$$= (1.6 \times 10^{-19})(225)\left(\frac{1 \times 10^{18} - 7 \times 10^{17}}{0.10}\right) = 108 \text{ A/cm}^2$$

總電流密度

總電流密度 = 飄移(drift) + 擴散(diffusion)

1-D

$$J = eF = en\mu_n \mathrm{E} + ep\mu_p \mathrm{E} + eD_n \frac{dn}{dx} - eD_p \frac{dn}{dx}$$

3-D

$$J = eF = en\mu_n \mathrm{E} + ep\mu_p \mathrm{E} + eD_n \nabla n - eD_p \nabla p$$

Graded Impurity Distribution

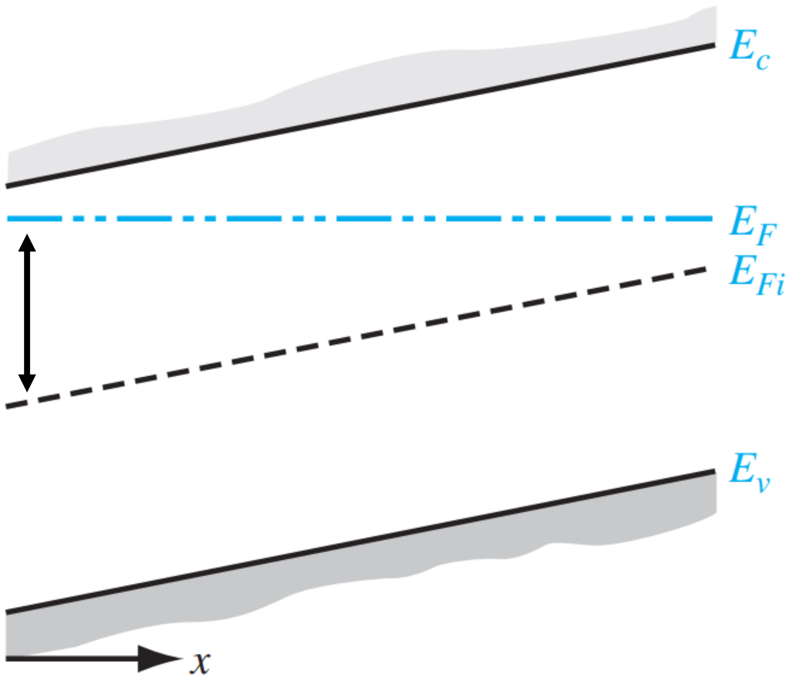
- 雜質摻雜濃度不均勻，而是與位置相關
- 因擴散產生電流表示必存在一個內部的感應電場

電子濃度

$$n_o = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) \approx N_d$$

感應電場

$$\mathrm{E}_x = -\frac{kT}{e}\frac{1}{N_d}\frac{dN_d}{dx}$$



平衡時，費米能階必處處相等

Example 5.6

Objective: Determine the induced electric field in a semiconductor in thermal equilibrium, given a linear variation in doping concentration.

Assume that the donor concentration in an n-type semiconductor at $T = 300$ K is given by

$$N_d(x) = 10^{16} - 10^{19}x \qquad (\text{cm}^{-3})$$

where x is given in cm and ranges between $0 \le x \le 1\ \mu\text{m}$

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$$\frac{dN_d(x)}{dx} = -10^{19} \qquad \text{E}_x = \frac{-(0.0259)(-10^{19})}{(10^{16} - 10^{19}x)}$$

The Einstein Relation

一個材料系統內部達熱平衡時，無任何淨電流

$$J_n = en\mu_n \mathrm{E}_x + eD_n \frac{dn}{dx} = 0$$

擴散感應電場

$$\mathrm{E}_x = -\frac{kT}{e}\frac{1}{N_d}\frac{dN_d}{dx}$$

$$\Rightarrow -eN_d\mu_n \frac{kT}{e}\frac{1}{N_d}\frac{dN_d}{dx} + eD_n\frac{dN_d}{dx} = 0$$

$$\Rightarrow \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

Table 5.2 | Typical mobility and diffusion coefficient values at $T = 300$ K ($\mu = \text{cm}^2/\text{V-s}$ and $D = \text{cm}^2/\text{s}$)

	μ_m	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

Nonuniform doping → Diffusion current → Charge separation

↓

Balanced current ← Drift current ← Induced field

Example 5.7

Objective: Determine the diffusion coefficient given the carrier mobility.

Assume that the mobility of a particular carrier is 1000 $cm^2/V\text{-}s$ at $T = 300$ K.

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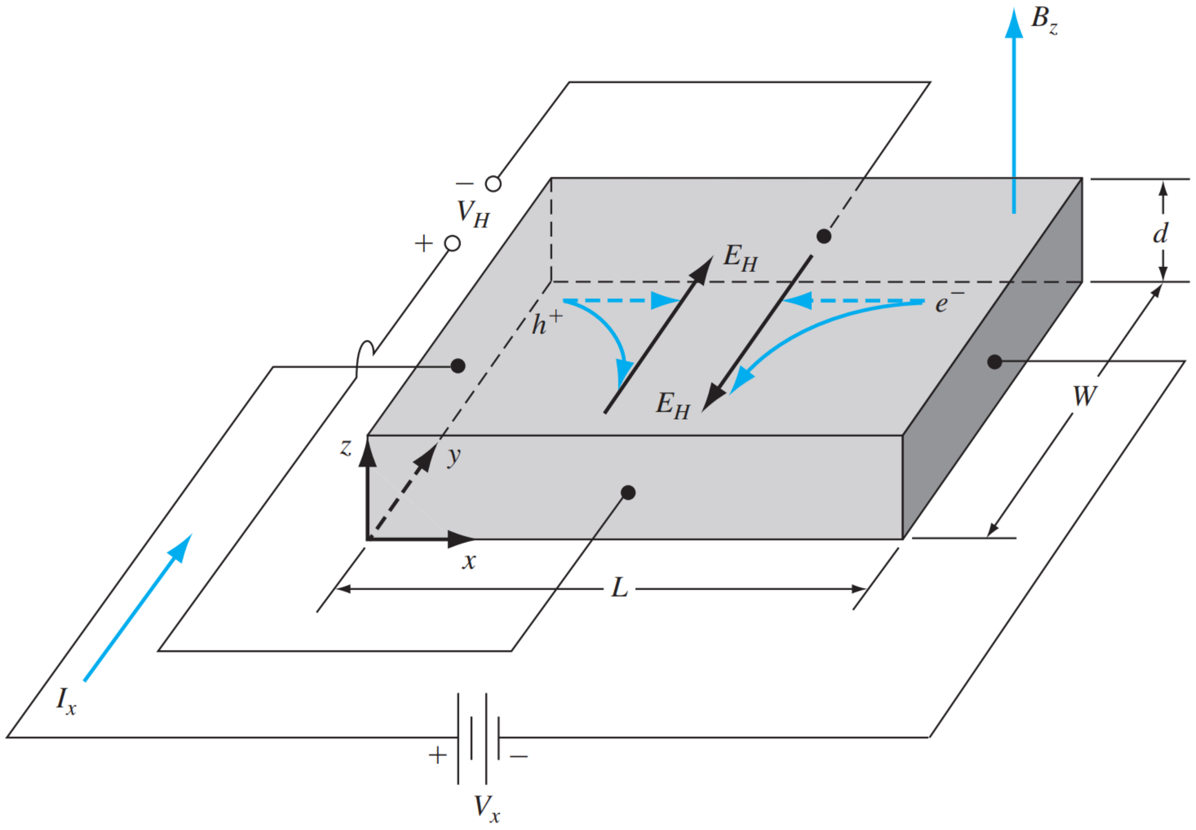
Assume that the mobility of a particular carrier is 1000 $cm^2/V\text{-}s$ at $T = 300$ K.

$$D = \left(\frac{kT}{e}\right)\mu = (0.0259)(1000) = 25.9 \text{ cm}^2/\text{s}$$

霍爾效應(測量載子濃度和遷移率)



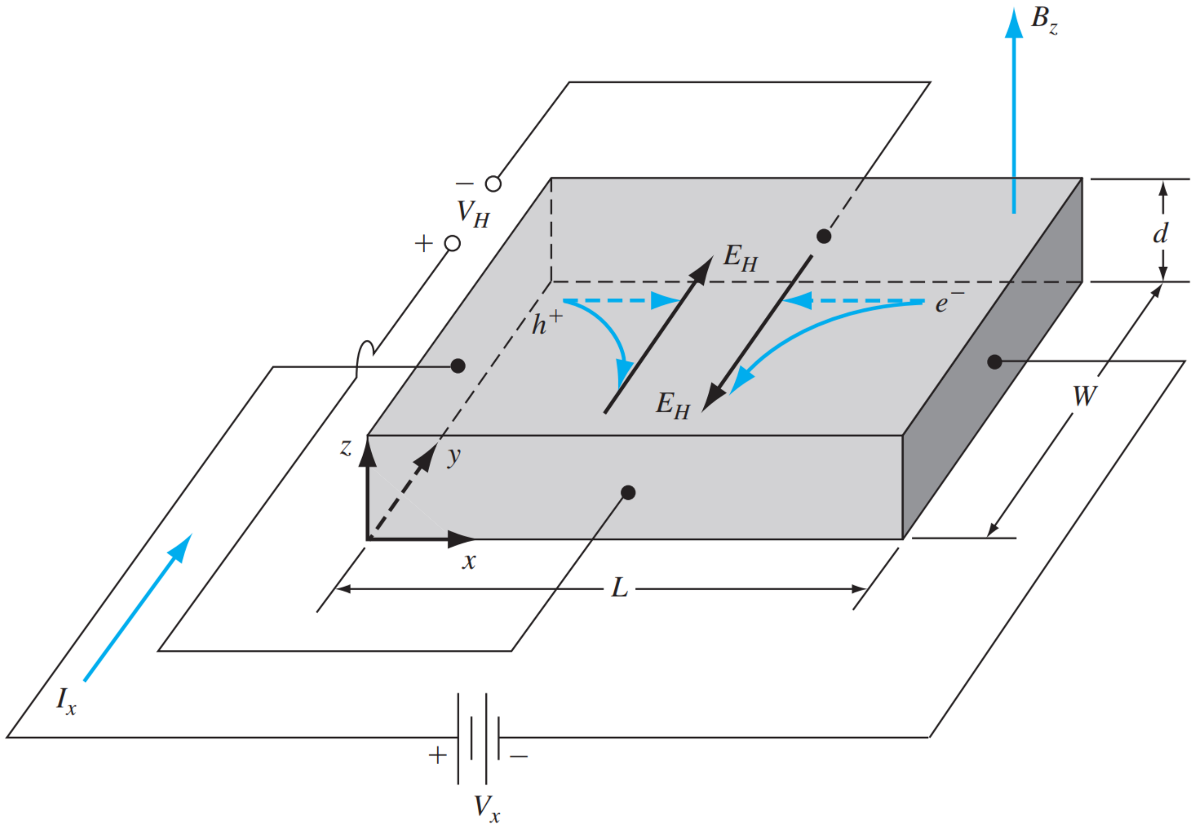
$$F = q\left[\vec{E} + \vec{v} \times \vec{B}\right] = 0$$

$$E_H = v_x B_z \Rightarrow \frac{V_H}{W} = \frac{J_x}{ep} B_z$$

$$p = \frac{J_x B_z W}{eV_H} = \frac{I_x B_z}{eV_H d}$$

濃度

霍爾效應(測量載子濃度和遷移率)



$$J_x = ep\mu_p \mathrm{E}_x$$

$$\mu_p = \frac{J_x}{ep\mathrm{E}_x} = \frac{I_x L}{epV_x W d}$$

遷移率

Hall measurement:

- N or P-type
- Majority carrier concentration
- Majority carrier mobility
- Conductivity and resistivity