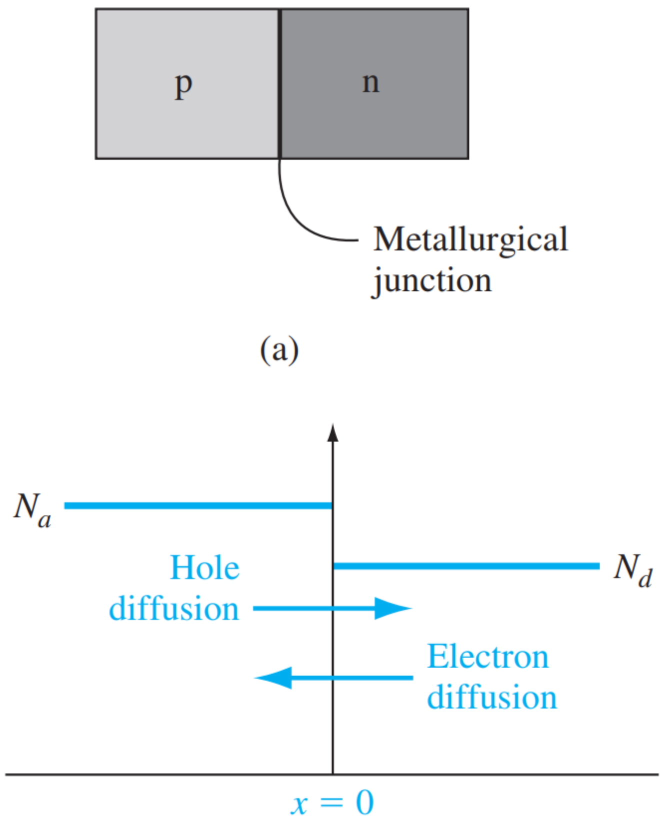
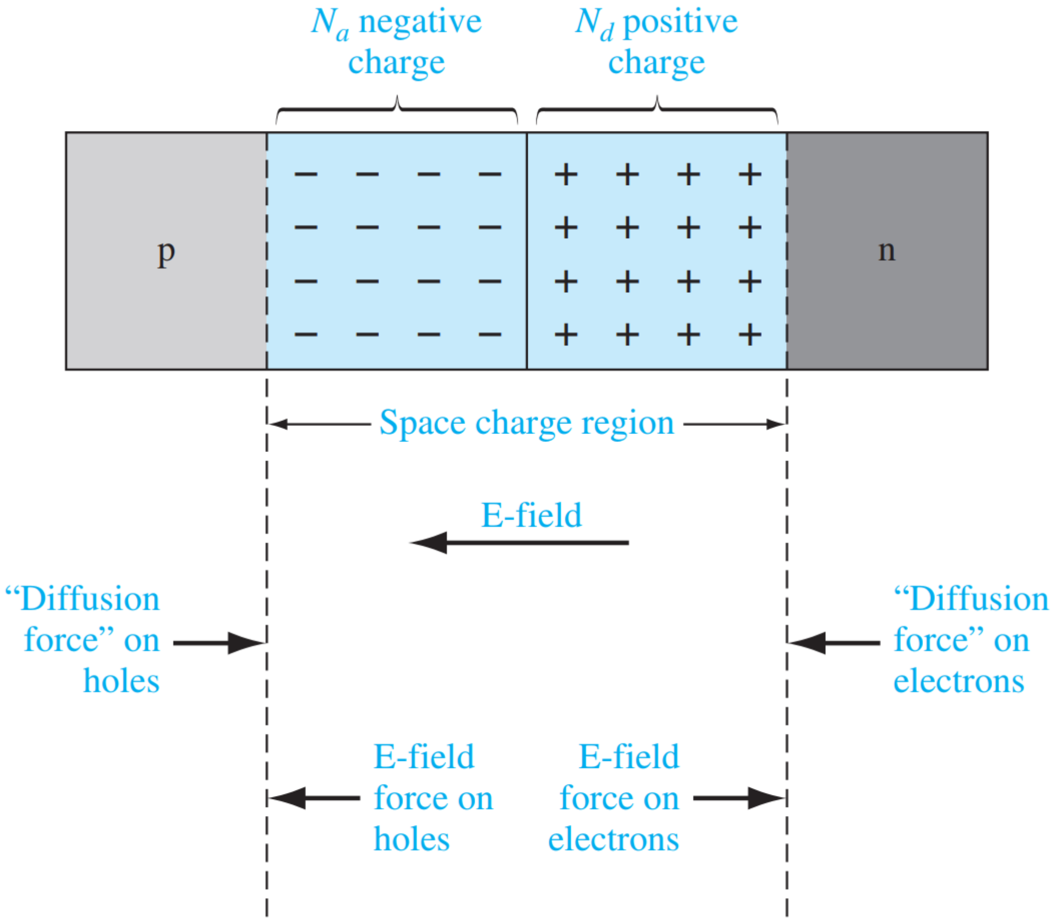
Chapter 7

The pn Junction

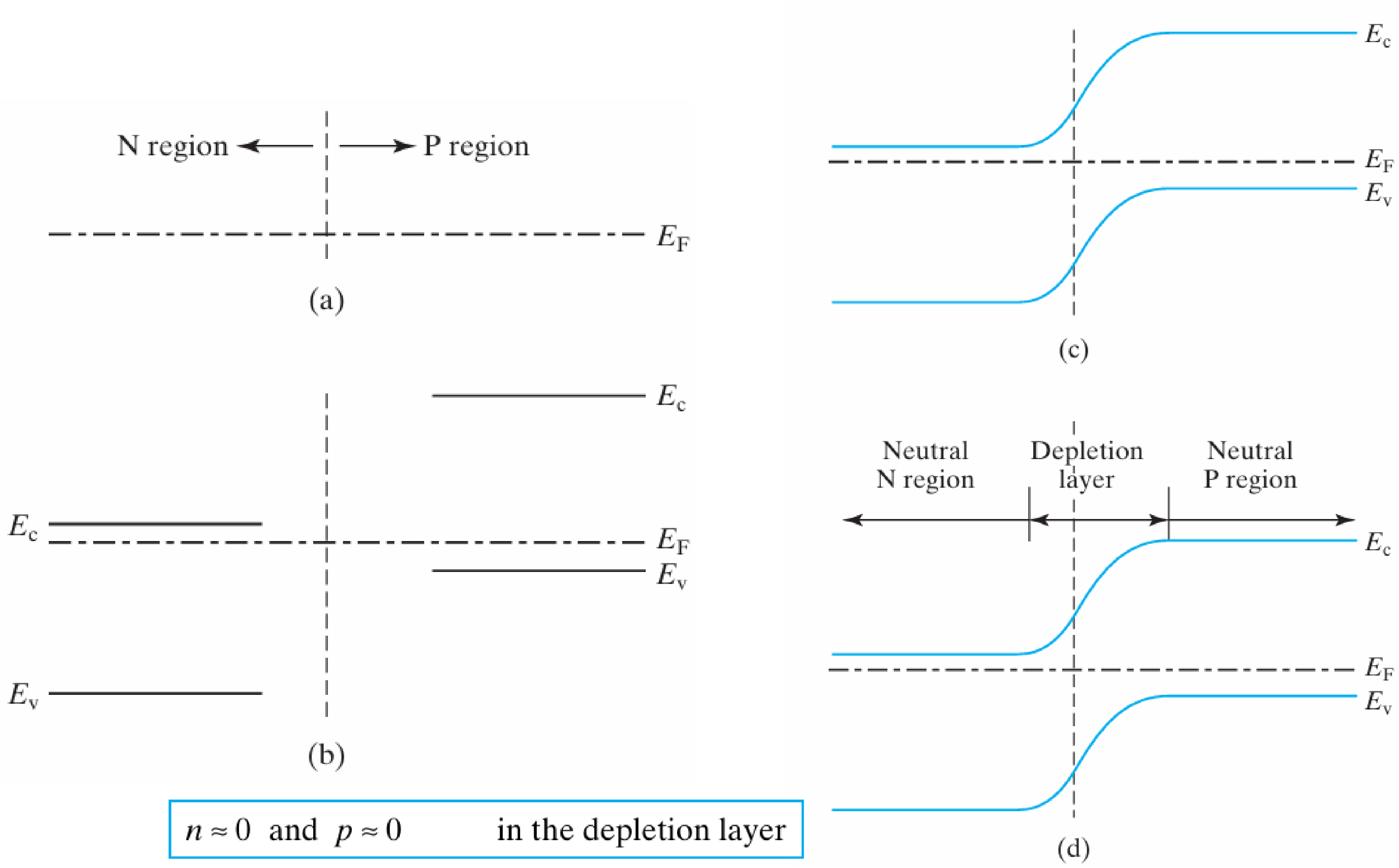
Outline

Basic Structure of pn Junction





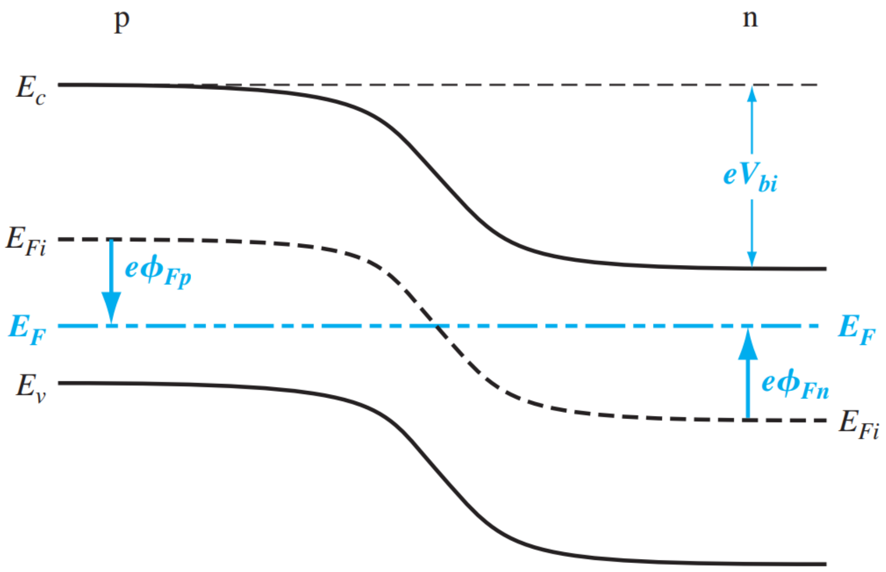
平衡時 Zero Applied Bias



$n \approx 0$ and $p \approx 0$ in the depletion layer

平衡時 Zero Applied Bias

平衡系統內，費米能階必處處相等



$$N_d \approx n_0 = n_i \exp\left(\frac{E_F - F_{Fi}}{kT}\right) \Rightarrow \phi_{Fn} = -\frac{kT}{e}\ln\left(\frac{N_d}{n_i}\right)$$

$$N_a \approx p_0 = n_i \exp\left(\frac{E_{Fi} - F_F}{kT}\right) \Rightarrow \phi_{Fp} = \frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right)$$

內建位能屏障 Built-in Potential Barrier

$$V_{bi} = \left|\phi_{Fp}\right| + \left|\phi_{Fn}\right| \Rightarrow V_{bi} = \frac{kT}{e}\ln\left(\frac{N_d N_a}{n_i^2}\right)$$

Example 7.1

Objective: Calculate the built-in potential barrier in a pn junction.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

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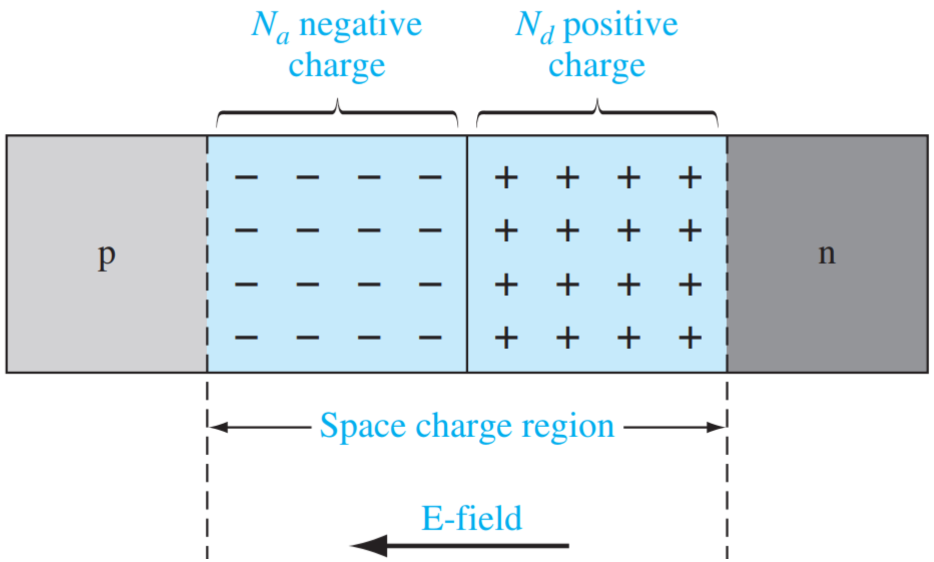
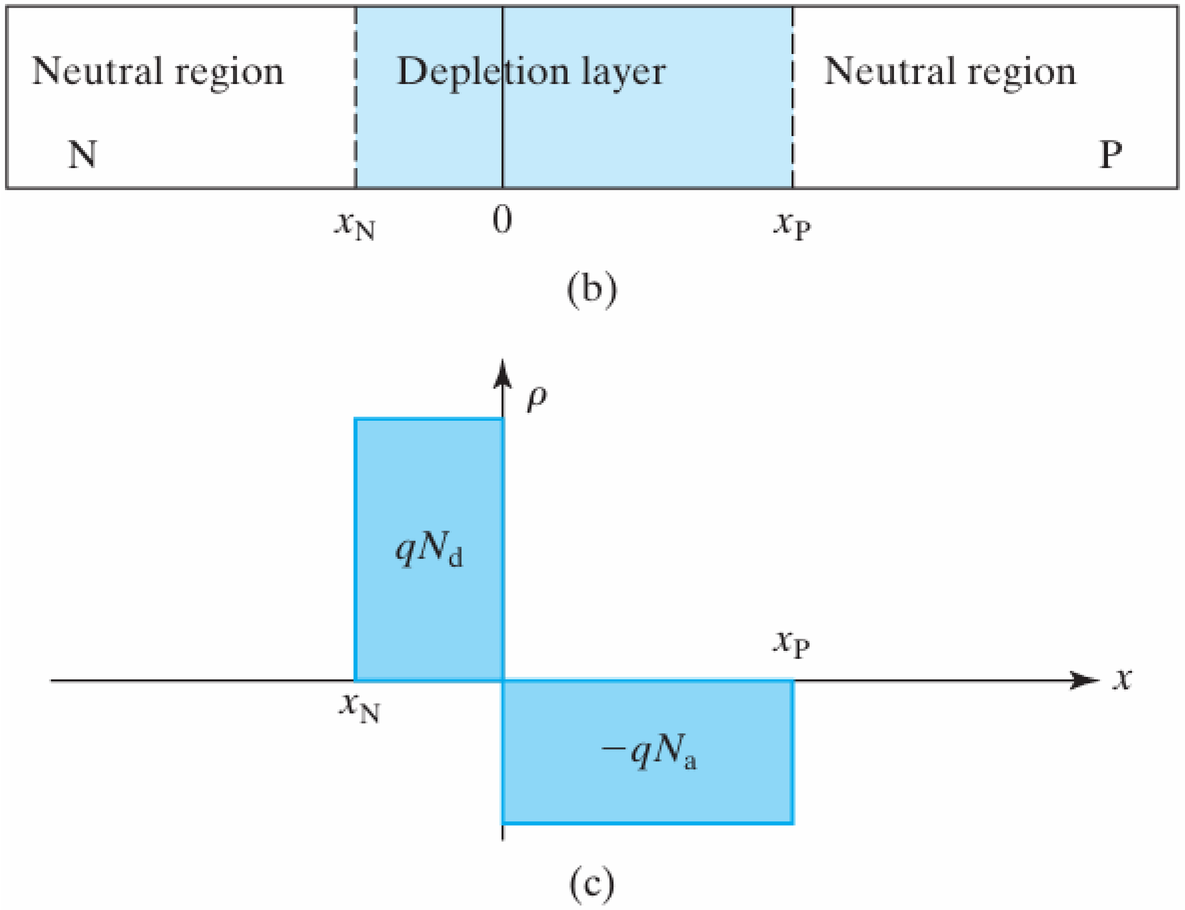
Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left[\frac{(2 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.713 \text{ V}$$

電荷密度分布

理想假設：電荷均勻分布且空乏區邊界陡峭

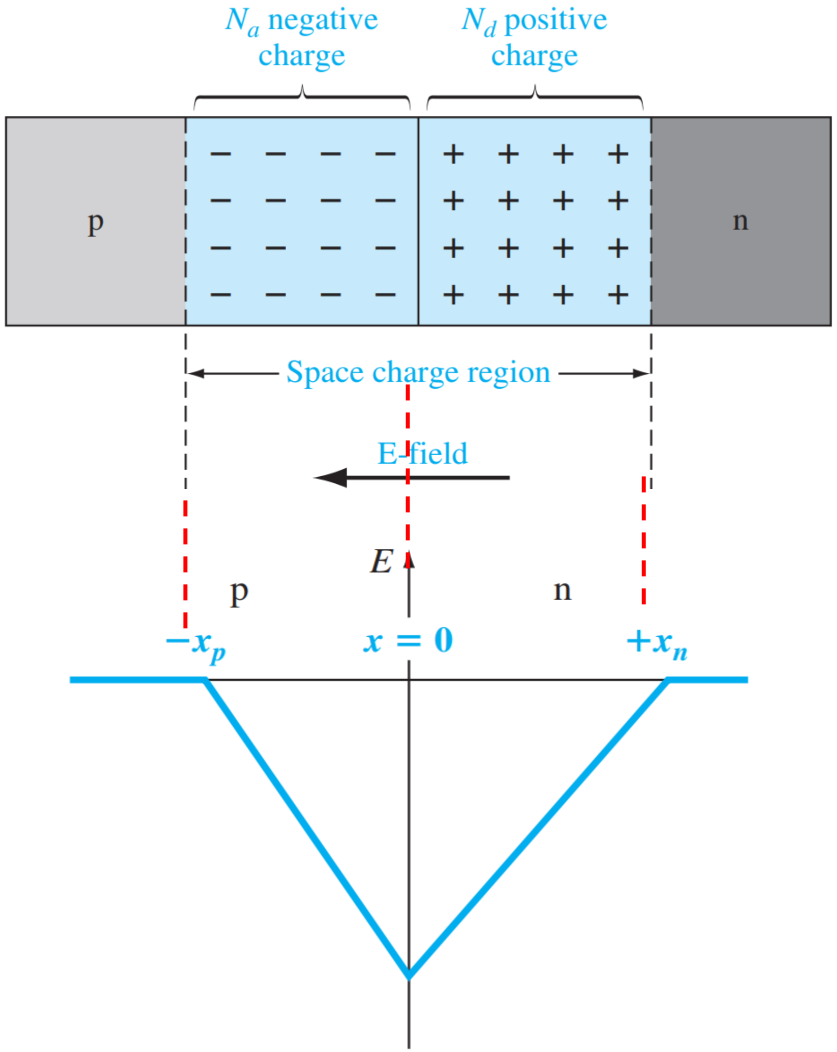




電場大小分布

由高斯定律可求得電場大小分布

$$\boxed{\frac{dE}{dx}=\frac{\rho}{\varepsilon}}$$

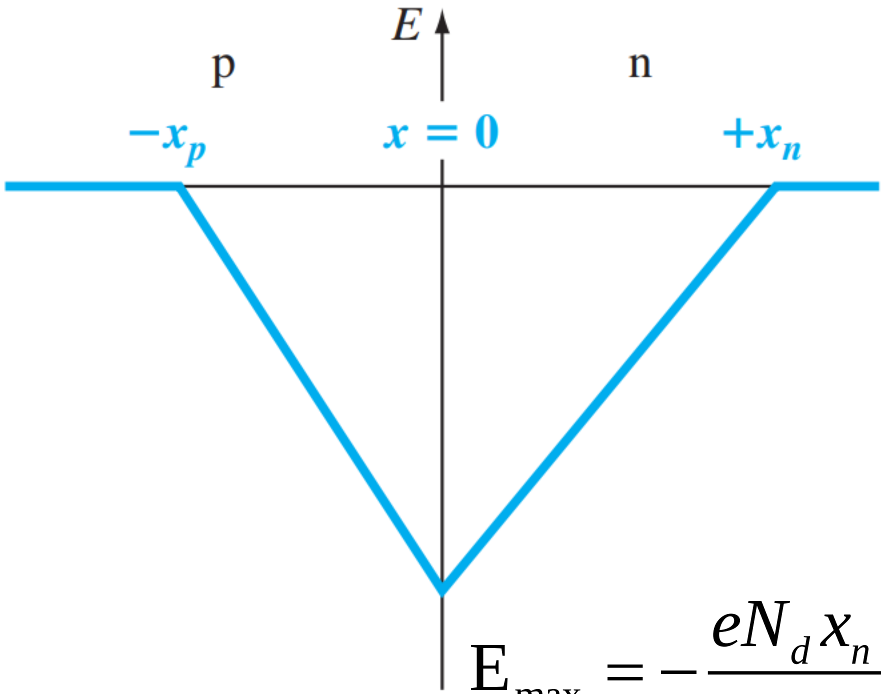


P區 $$E=-\frac{eN_a}{\varepsilon_s}\left(x+x_p\right)$$

N區 $$E=\frac{eN_d}{\varepsilon_s}\left(x-x_n\right)$$

電場最大值

在 x=0 位置，左右兩邊電場必相等



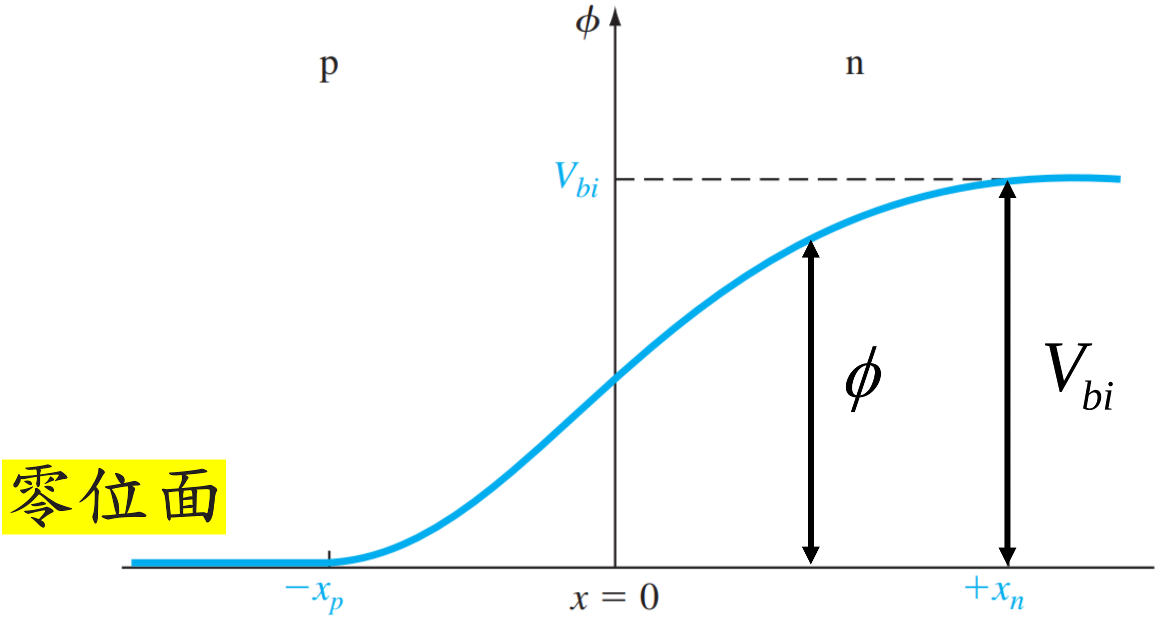
$$E_{max} = -\frac{eN_d x_n}{\varepsilon_s}$$

$$E_{max} = -\frac{eN_a x_p}{\varepsilon_s} = -\frac{eN_d x_n}{\varepsilon_s}$$

$$\Rightarrow N_a x_p = N_d x_n$$ 重要關係

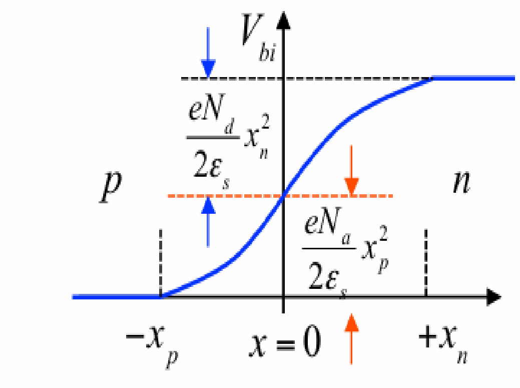
電位分布

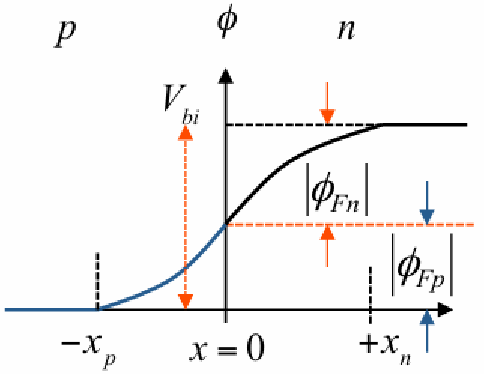
由電場的定義可求得電位 $\boxed{E=-\dfrac{d\phi}{dx}}$



P區 $\phi=\dfrac{eN_a}{\varepsilon_s}\left(\dfrac{x^2}{2}+x_p\cdot x+\dfrac{x_p^2}{2}\right)$

N區 $\phi=\dfrac{eN_d}{\varepsilon_s}\left(-\dfrac{x^2}{2}+x_n\cdot x\right)+\dfrac{eN_a x_p^2}{2\varepsilon_s}$





$V_{bi}=|\phi_{Fp}|+|\phi_{Fn}|\Rightarrow V_{bi}=\dfrac{kT}{e}\ln\left(\dfrac{N_dN_a}{n_i^2}\right)$

空乏區寬度與電場最大值

由右邊兩式可求出空乏區寬度

$$\phi(x_n) = V_{bi} = \frac{e}{2\varepsilon_s}\left(N_d + \frac{N_d^2}{N_a}\right)x_n^2$$

$$N_a x_p = N_d x_n$$

$$\left\{\begin{aligned} x_n &= \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_a}{N_d}\frac{1}{N_a+N_d}} \\ x_p &= \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}} \\ \mathrm{E}_{\max} &= -\frac{eN_d x_n}{\varepsilon_s} \end{aligned}\right. \Rightarrow \left\{\begin{aligned} W &= x_n + x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a+N_d}{N_a N_d}\right]} \\ \mathrm{E}_{\max} &= -\frac{2V_{bi}}{W} \end{aligned}\right.$$

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Example 7.2

Objective: Calculate the space charge width and electric field in a pn junction for zero bias.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}.

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Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}\ \text{cm}^{-3}$ and $N_d = 10^{15}\ \text{cm}^{-3}$.

$$V_{bi} = 0.0259 \ln\left(\frac{N_d N_a}{n_i^2}\right) = 0.635 \quad \text{V}$$

$$W = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}}\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$= 0.951 \times 10^{-4}\ \text{cm} = 0.951\ \mu\text{m}$$

$$\mathrm{E}_{\max} = -\frac{2V_{bi}}{W} = -\frac{2 \times 0.635}{0.951 \times 10^{-4}} = -1.34 \times 10^{4} \quad \text{V/cm}$$

空乏區寬度(重度參雜)

$N_a x_p = N_d x_n$

空乏區寬度: $W = xn + xp = \sqrt{\frac{2\varepsilon_s V_{bi}}{q} * (\frac{1}{N_a} + \frac{1}{N_d})}$

If $N_a >> N_d$, as in a P$^+$N junction,

$$\boxed{W_{\text{dep}} \approx \sqrt{\frac{2\varepsilon_s \phi_{\text{bi}}}{qN_{\text{d}}}}} \approx |x_{\text{N}}|$$

If $N_d >> N_a$, as in an N$^+$P junction,

$$\boxed{W_{\text{dep}} \approx \sqrt{\frac{2\varepsilon_s \phi_{\text{bi}}}{qN_{\text{a}}}}} \approx |x_{\text{P}}|$$

空乏區寬度(重度參雜)

EXAMPLE 4–1 A P$^+$N junction has $N_a = 10^{20}\text{cm}^3$ and $N_d = 10^{17}\text{cm}^{-3}$. What is (a) the built-in potential, (b) W_{dep}, (c) x_N, and (d) x_P?

SOLUTION:

a. Using Eq. (4.1.2),

$$\phi_{bi} = \frac{kT}{q}\ln\frac{N_d N_a}{n_i^2} \approx 0.026\ \text{V}\ln\frac{10^{20}\times 10^{17}\text{cm}^{-6}}{10^{20}\text{cm}^{-6}} \approx 1\ \text{V}$$

b. Using Eq. (4.2.9),

$$W_{dep} \approx \sqrt{\frac{2\varepsilon_s\phi_{bi}}{qN_d}} = \left(\frac{2\times 12\times 8.85\times 10^{-14}\times 1}{1.6\times 10^{-19}\times 10^{17}}\right)^{1/2}$$

$$= 1.2\times 10^{-5}\text{cm} = 0.12\ \mu\text{m} = 120\ \text{nm} = 1200\ \text{Å}$$

c. In a P$^+$N junction, nearly the entire depletion layer exists on the N-side.

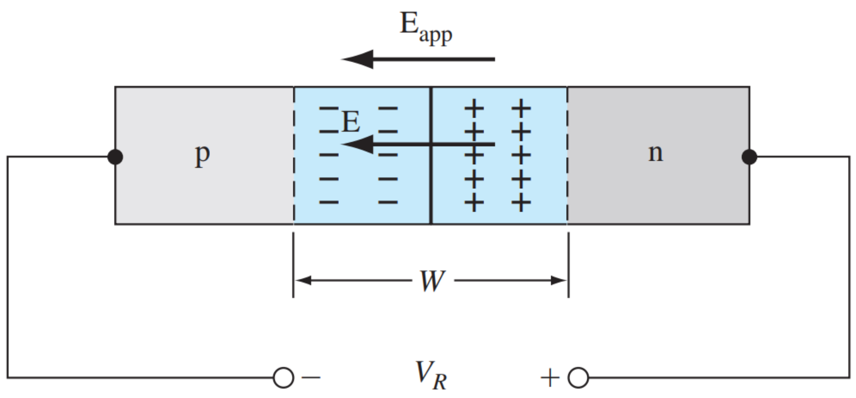
$$|x_N| \approx W_{dep} = 0.12\ \mu\text{m}$$

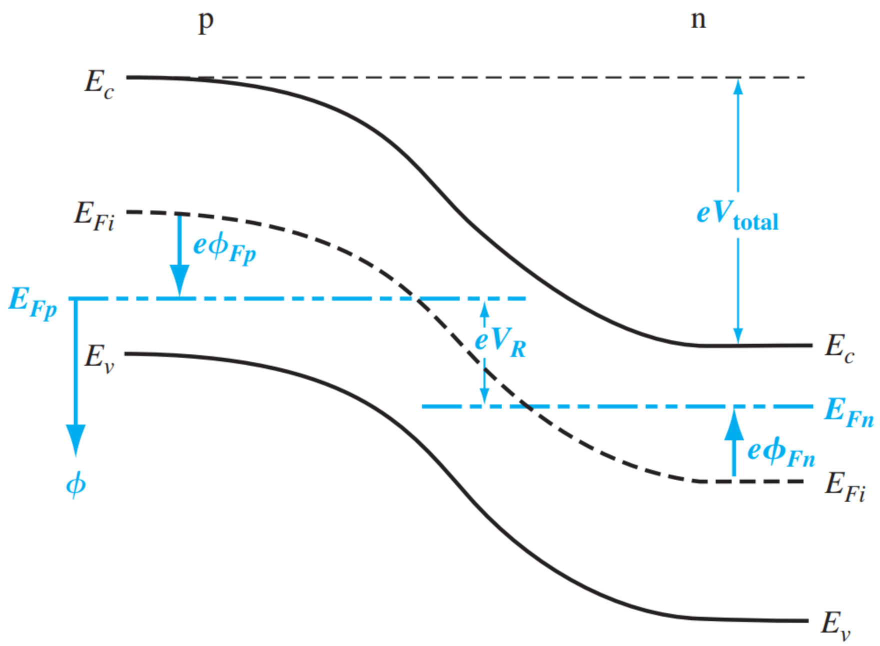
d. Using Eq. (4.2.5),

$$|x_P| = |x_N|N_d/N_a = 0.12\ \mu\text{m}\times 10^{17}\text{cm}^{-3}/10^{20}\text{cm}^{-3} = 1.2\times 10^{-4}\ \mu\text{m}$$

$$= 1.2\ \text{Å} \approx 0$$

逆向偏壓 Reverse Applied Bias



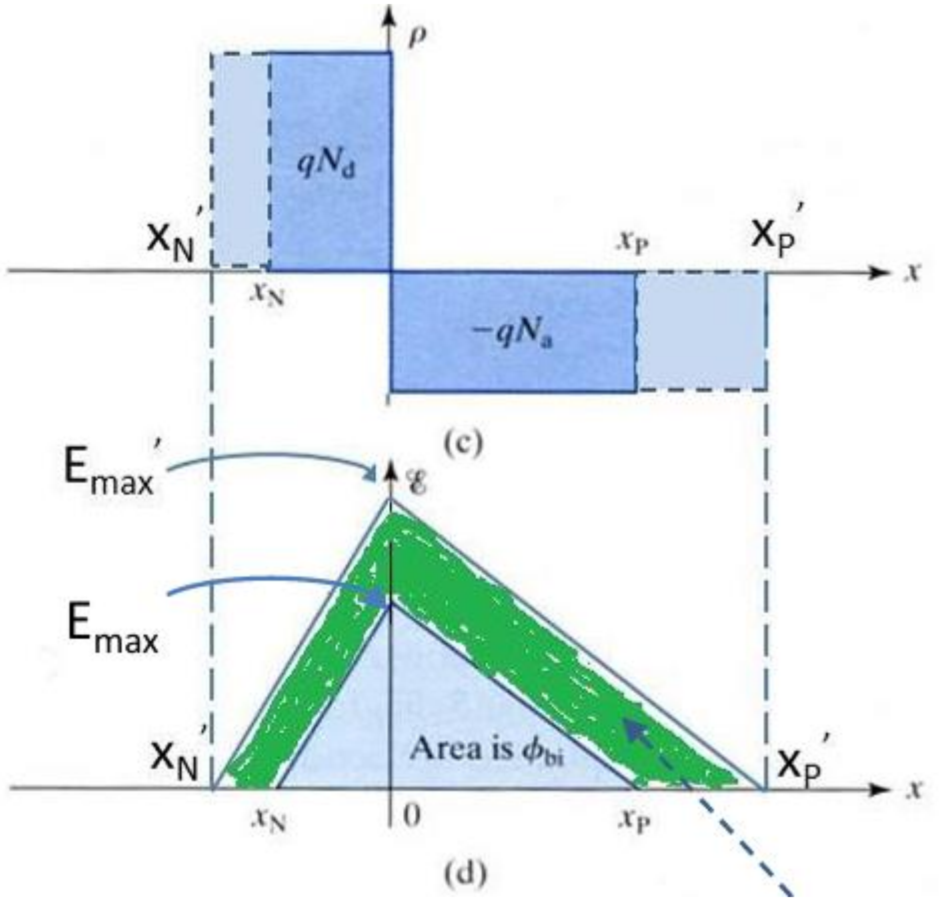


$$V_{total} = |\phi_{Fp}| + |\phi_{Fn}| + V_R$$

$$\Rightarrow W = x_n + x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right]}$$

$$\Rightarrow \mathrm{E}_{\max} = -\frac{2(V_{bi} + V_R)}{W}$$

逆向偏壓 Reverse Applied Bias



- The additional area is equal to applied reverse bias voltage
- There will be harder to let the p-n junction conduct
- If the reverse bias voltage still increase until $E'_{max} > E_{crit}$, the p-n junction will breakdown called junction breakdown
- So, if we still need to applied reverse bias voltage but don’t want to increase the electric field, how can we do ?

$$E = \frac{\rho}{\varepsilon} = \frac{q * Nd * xn}{\varepsilon} = \frac{q * Na * xp}{\varepsilon}$$

Example 7.3

Objective: Calculate the width of the space charge region in a pn junction when a reverse-biased voltage is applied.

Again consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Assume that $n_i = 1.5 \times 10^{10}$ cm^{-3} and $V_R = 5$ V.

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Again consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Assume that $n_i = 1.5 \times 10^{10}$ cm^{-3} and $V_R = 5$ V.

$$W = \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 5)}{1.6 \times 10^{-19}}\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$W = 2.83 \times 10^{-4} \text{ cm} = 2.83\ \mu\text{m}$$

Example 7.4

Objective: Design a pn junction to meet maximum electric field and voltage specifications.

Consider a silicon pn junction at $T = 300$ K with a p-type doping concentration of $N_a = 2 \times 10^{17}$ cm^{-3}. Determine the n-type doping concentration such that the maximum electric field is $|E_{max}| = 2.5 \times 10^5$ V/cm at a reverse-biased voltage of $V_R = 25$ V.

Example 7.4

Objective: Design a pn junction to meet maximum electric field and voltage specifications.

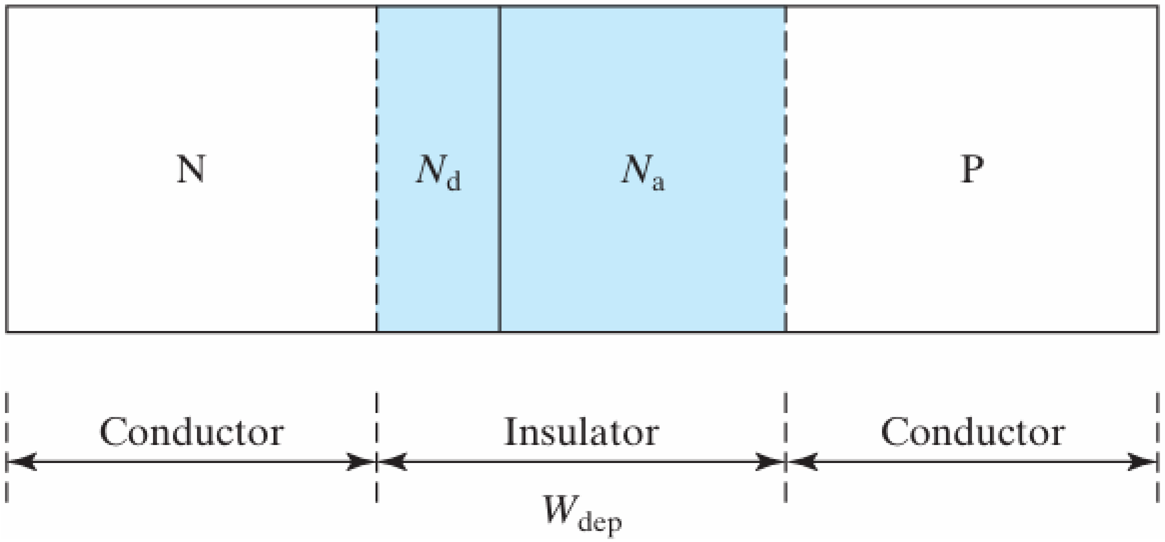
Consider a silicon pn junction at $T = 300$ K with a p-type doping concentration of $N_a = 2 \times 10^{17}$ cm^{-3}. Determine the n-type doping concentration such that the maximum electric field is $|E_{max}| = 2.5 \times 10^5$ V/cm at a reverse-biased voltage of $V_R = 25$ V.

$$|E_{max}| \cong \left\{ \frac{2eV_R}{\epsilon_s} \left(\frac{N_a N_d}{N_a + N_d} \right) \right\}^{1/2}$$

$$2.5 \times 10^5 = \left\{ \frac{2(1.6 \times 10^{-19})(25)}{(11.7)(8.85 \times 10^{-14})} \left[\frac{(2 \times 10^{17})N_d}{2 \times 10^{17} + N_d} \right] \right\}^{1/2}$$

$$N_d = 8.43 \times 10^{15} \text{ cm}^{-3}$$

接面電容 Junction Capacitance



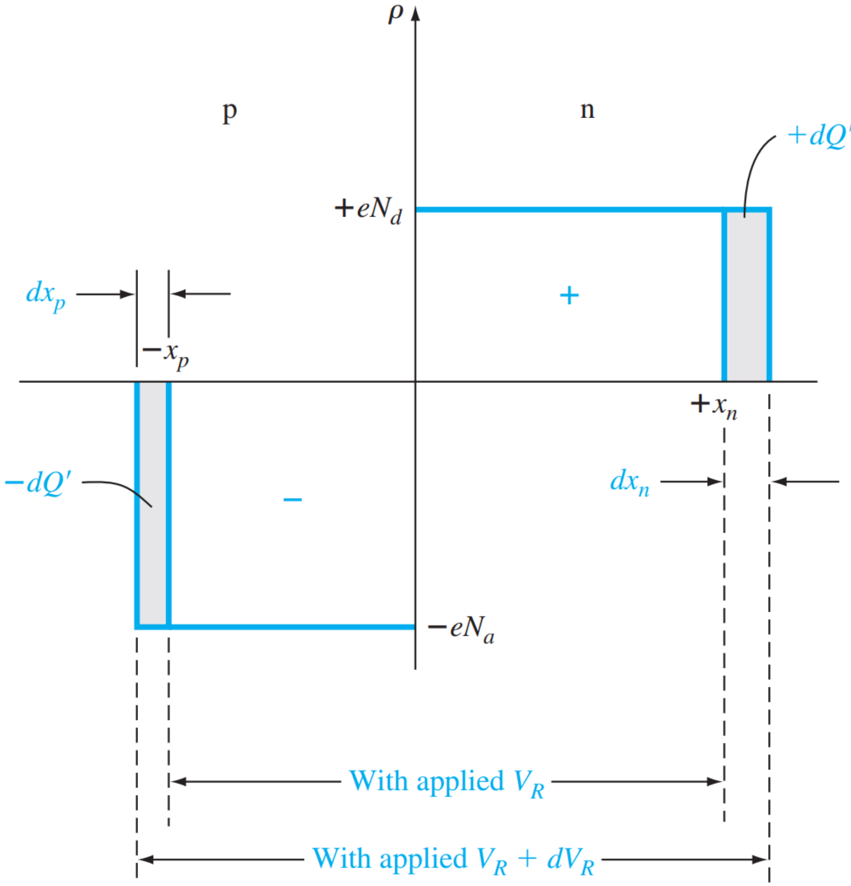
Depletion-layer capacitance $C_{dep} = \frac{A * \varepsilon_s}{W_{dep}}$

A: represent the junction area

W_{dep}: represent the depletion region width

接面電容 Junction Capacitance

- 當施加逆向偏壓時，會改變空乏區的寬度(電荷量改變)
- 定義 $C' = \dfrac{dQ'}{dV_R}$ $,Q' = \dfrac{Q}{A}$ 單位面積的電荷量



$$Q' = eN_d x_n = eN_d \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \frac{N_a}{N_d} \frac{1}{N_a + N_d}}$$

$$\frac{dQ'}{dV_R} = \frac{\varepsilon_s}{W}$$

$$\boxed{C' = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}}$$

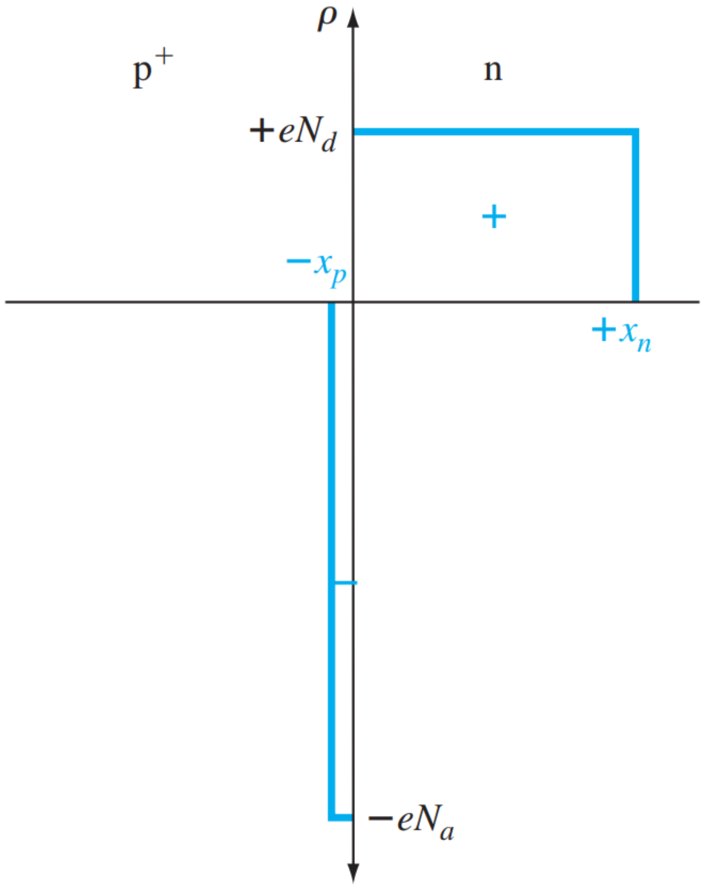
單邊接面 One-Sided Junctions

- 對於一個介面，其中一邊的濃度遠大於另一邊(20倍以上)，例 p^+n
- 空乏區主要落在濃度小的那一側

$$x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_a}{N_d}\frac{1}{N_a+N_d}} \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{1}{N_d}}$$

$$x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}} \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_d}{N_a^2}} \approx 0$$

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a+N_d}{N_a N_d}\right]} \approx \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{1}{N_d}}$$



單邊接面 One-Sided Junctions

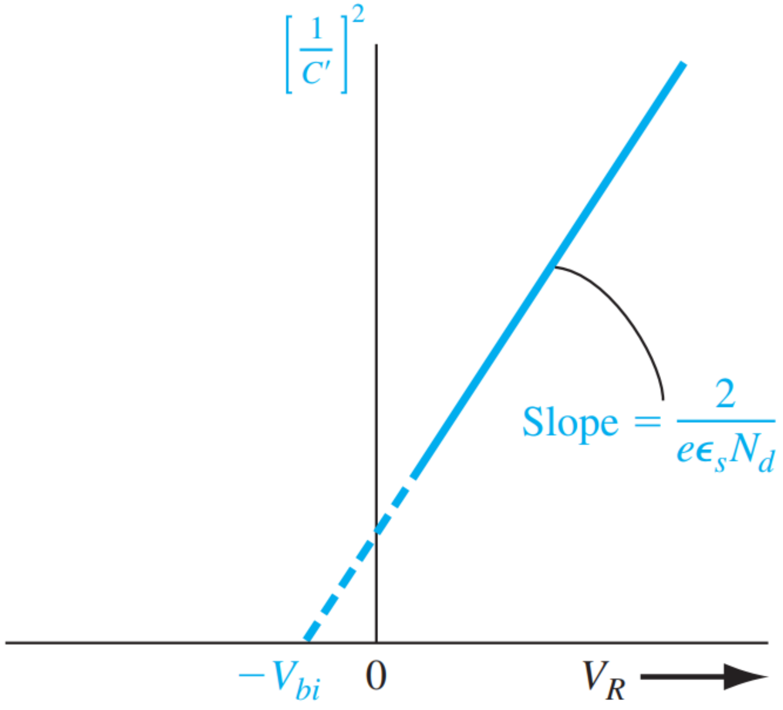
- 利用施加一小訊號，可以透過儀器測量到 Junction Capacitance
- 進而得到材料的內建電位差 V_{bi} 以及摻雜濃度

$$C' = \frac{dQ'}{dV_R} \approx \sqrt{\frac{\varepsilon_s e N_d}{2\left(V_{bi}+V_R\right)}}$$

$$\left(\frac{1}{C'}\right)^2 = \frac{2\left(V_{bi}+V_R\right)}{\varepsilon_s e N_d}$$

實驗變量

可量測到



Example 7.5

Objective: Calculate the junction capacitance of a pn junction.

Consider the same pn junction as that in Example 7.3. Again assume that $V_R = 5$ V.

$$C' = \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{16})(10^{15})}{2(0.635 + 5)(10^{16} + 10^{15})} \right\}^{1/2}$$

$$C' = 3.66 \times 10^{-9} \text{ F/cm}^2$$

If the cross-sectional area of the pn junction is, for example, $A = 10^{-4}$ cm^2, then the total junction capacitance is

$$C = C' \cdot A = 0.366 \times 10^{-12} \text{ F} = 0.366 \text{ pF}$$

Example 7.6

Objective: Determine the impurity doping concentrations in a p^+n junction given the parameters from Figure 7.11.

Assume that the intercept and the slope of the curve in Figure 7.11 are $V_{bi} = 0.725$ V and $6.15 \times 10^{15}\ (\text{F/cm}^2)^{-2}\ (\text{V})^{-1}$, respectively, for a silicon p^+n junction at $T = 300$ K.

$$N_d = \frac{2}{e\,\epsilon_s} \cdot \frac{1}{slope} = \frac{2}{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(6.15 \times 10^{15})}$$

$$N_d = 1.96 \times 10^{15}\ \text{cm}^{-3}$$

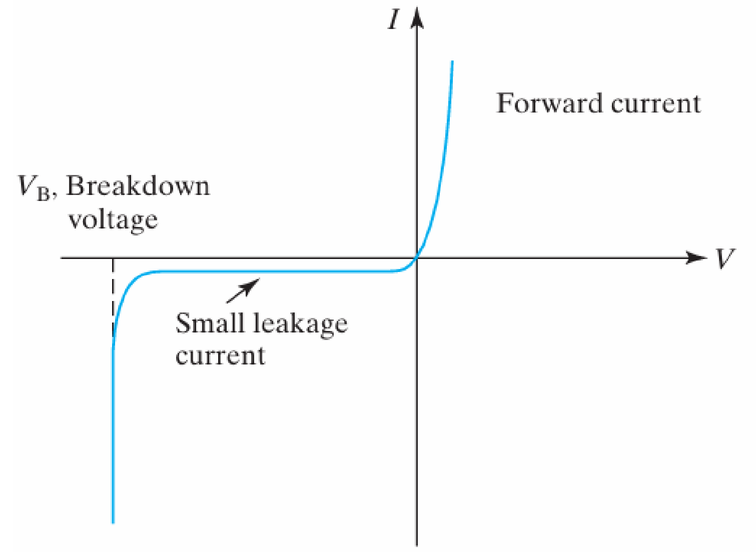
From the expression for V_{bi}, which is

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

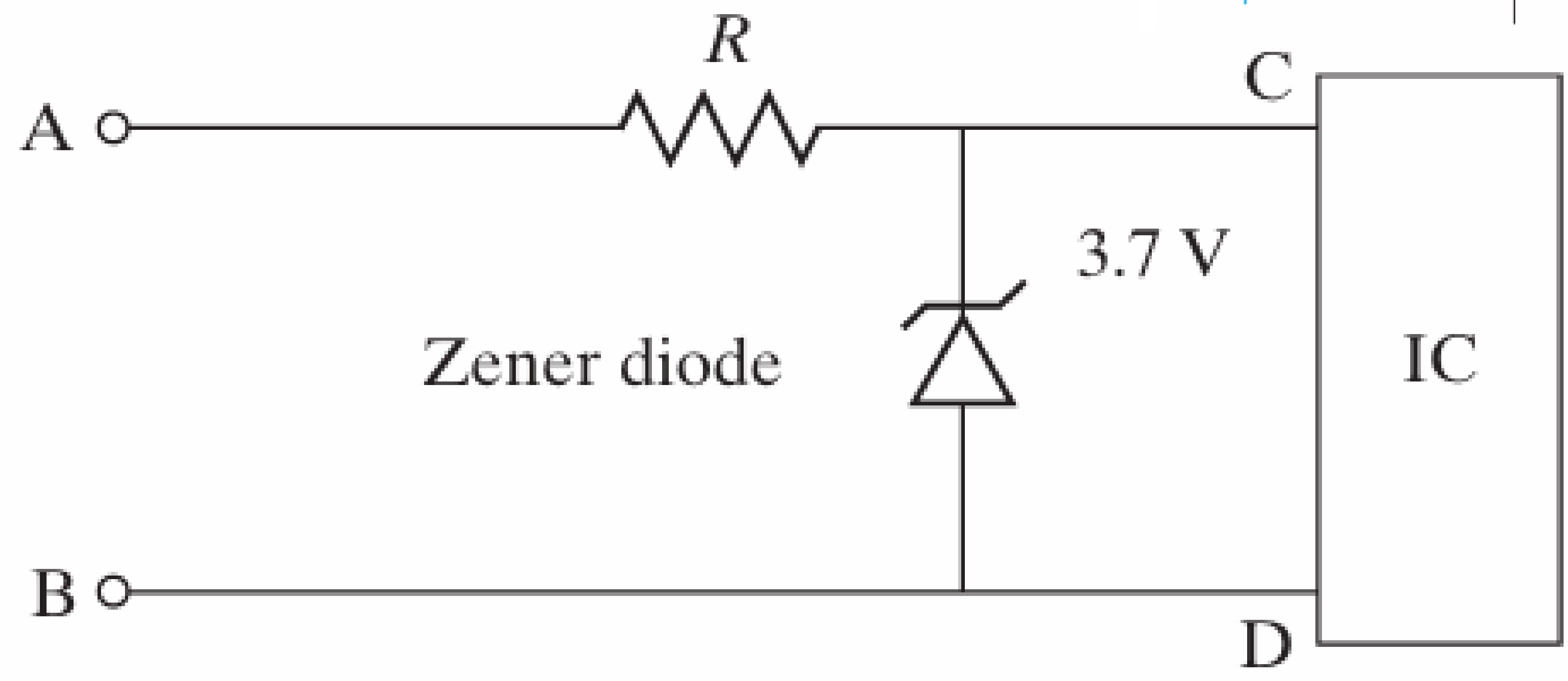
we can solve for N_a as

$$N_a = \frac{n_i^2}{N_d} \exp\left(\frac{V_{bi}}{V_t}\right) = \frac{(1.5 \times 10^{10})^2}{1.963 \times 10^{15}} \exp\left(\frac{0.725}{0.0259}\right) = 1.64 \times 10^{17}\ \text{cm}^{-3}$$

穩壓與崩潰



I
V
Forward current
V_B, Breakdown voltage
Small leakage current



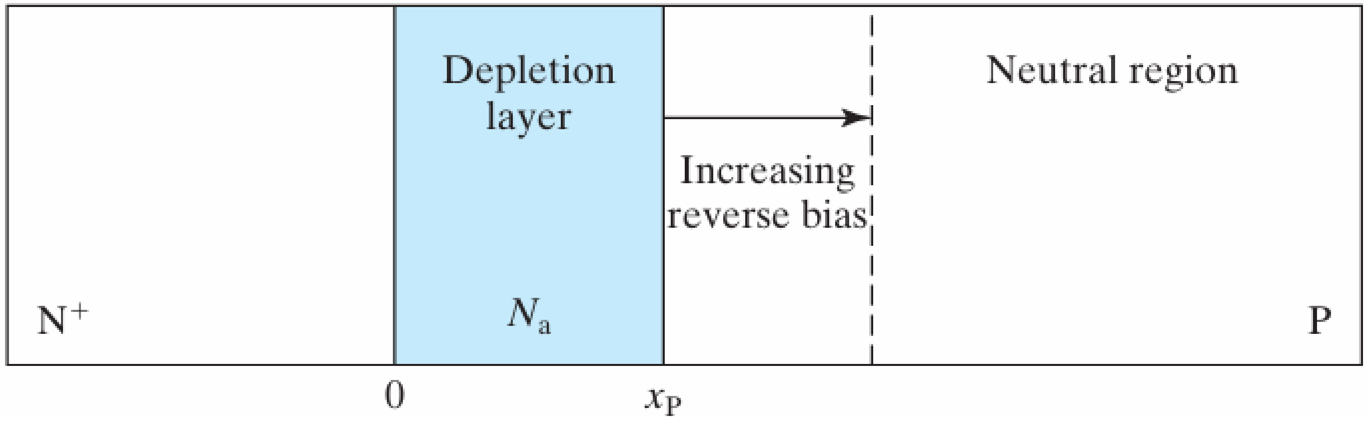
R
C
A
3.7 V
Zener diode
IC
B
D

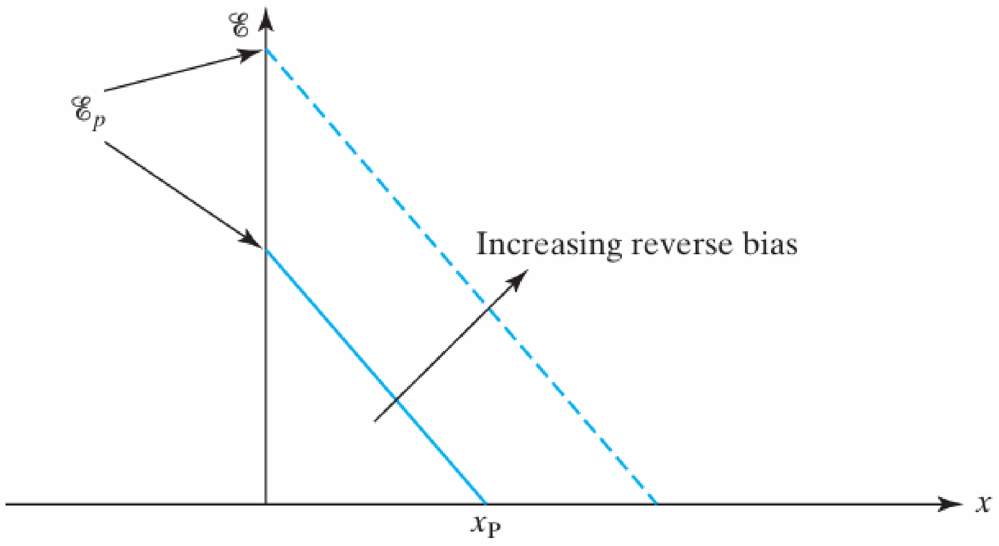
崩潰 Junction Breakdown

Peak Electric Field

- Junction breakdown occurs when the peak electric field in the PN junction reaches a critical value

$$\mathscr{E}_p = \mathscr{E}(0) = \left[\frac{2qN}{\varepsilon_s}(\phi_{\text{bi}} + |V_r|)\right]^{1/2}$$

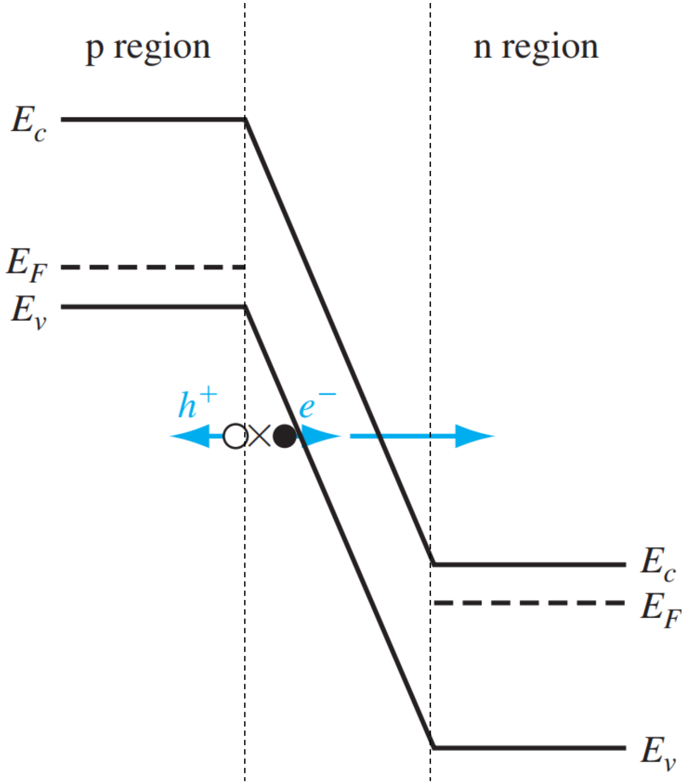




崩潰 Junction Breakdown

齊納效應 (Zener effect)

- p 區和 n 區均以高濃度摻雜，空乏區很窄
- p 區價帶電子容易直接穿隧至 n 區傳導帶



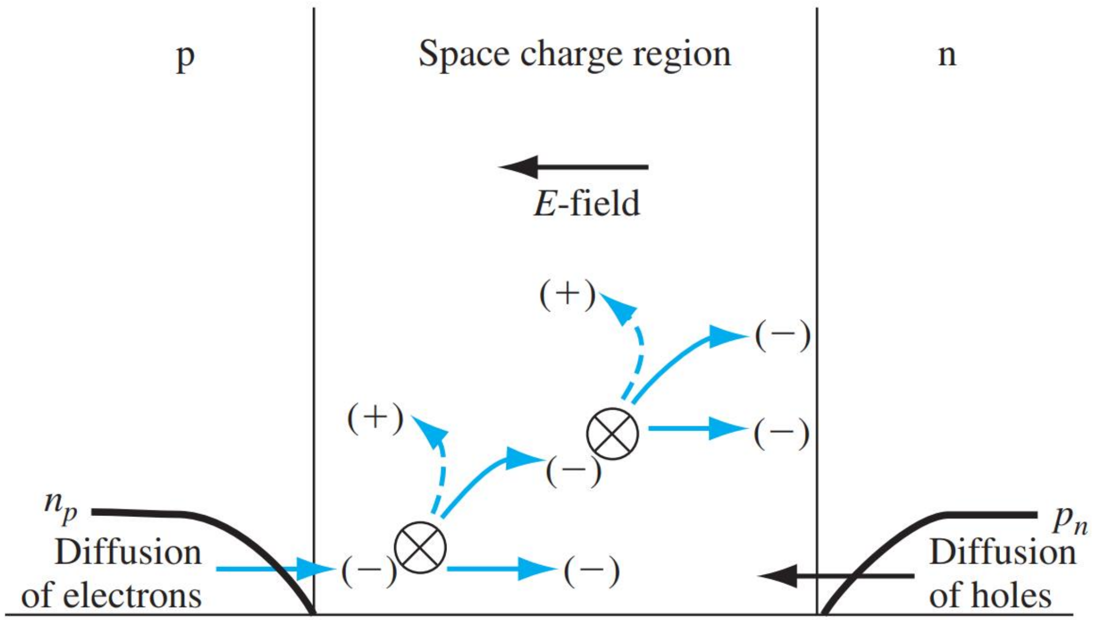
$$W=\sqrt{\frac{2\varepsilon_s\left(V_{bi}+V_R\right)}{e}\left[\frac{N_a+N_d}{N_aN_d}\right]}$$

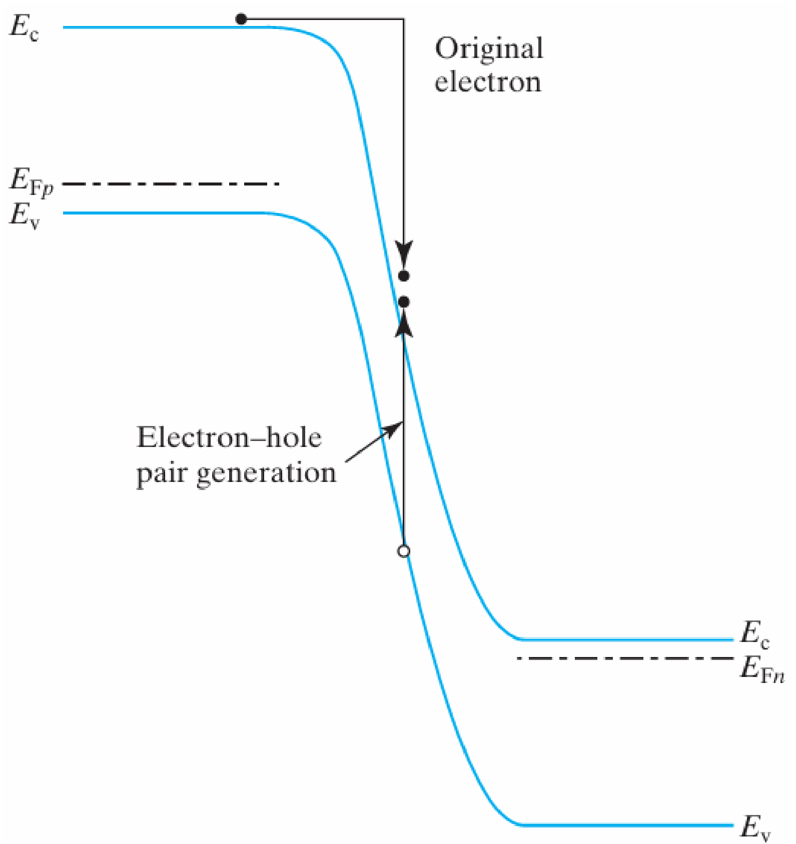
$$N_a\uparrow N_d\uparrow\Rightarrow W\downarrow$$

崩潰 Junction Breakdown

雪崩效應 (Avalanche effect)

- p 區的電子為少數載子，因擴散效應而有微量的流動
- 當偏壓愈加愈大時，空乏區內的電場極高
- 原本微量的電子進入空乏區後，在極高電場的加速下撞出更多電子電洞對，造成雪崩式效應。

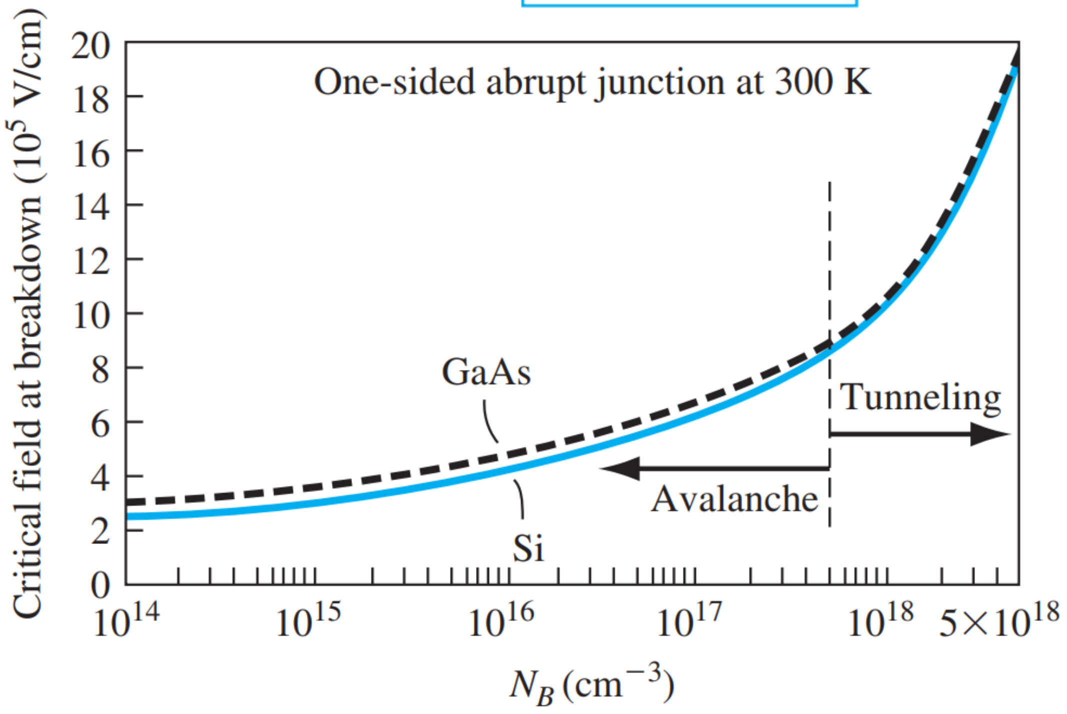


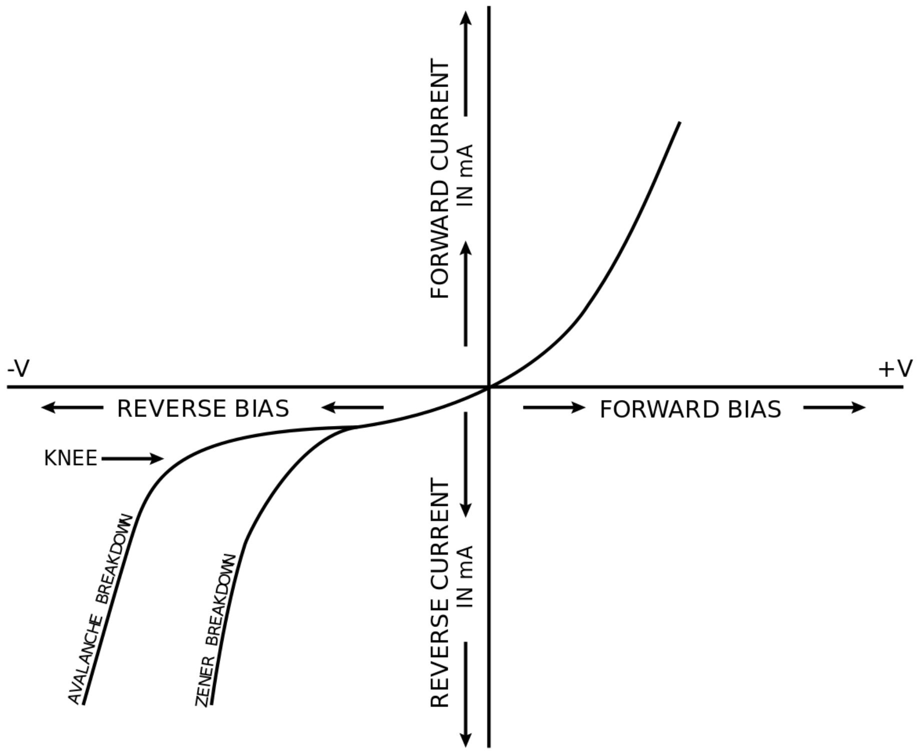


崩潰電壓

- 逆向偏壓時，齊納效應和雪崩效應均會發生
- 當摻雜濃度較高時，主要由齊納效應主宰。

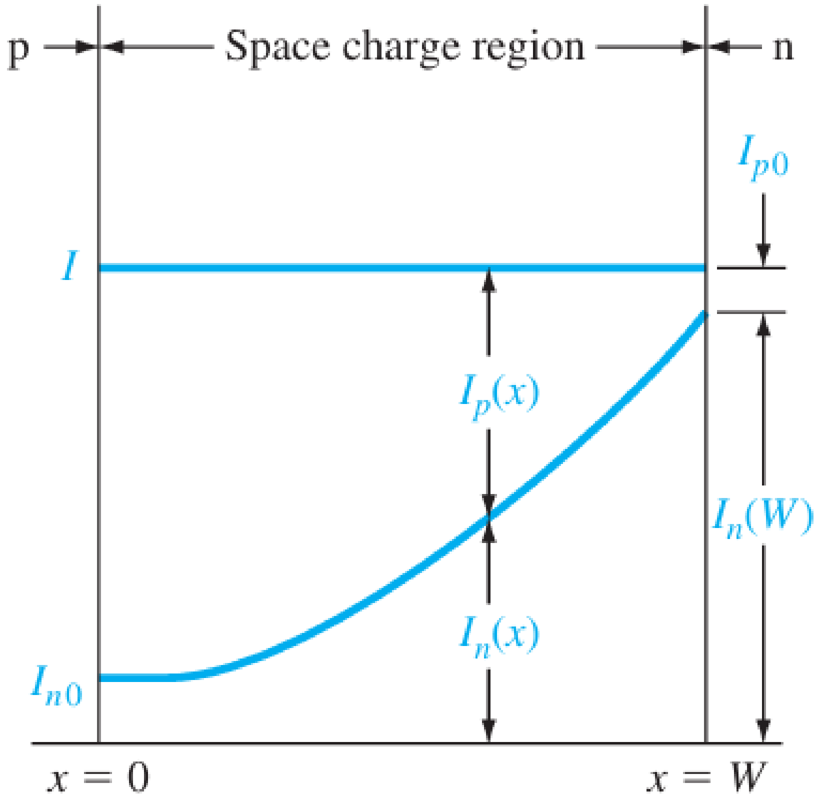
崩潰電壓 $\boxed{V_B = \frac{\epsilon_s E_{crit}^2}{2eN_B}}$





高壓避免崩潰

For the avalanche breakdown → $I_n(W) = M_n I_{n0}$



$$1 - \frac{1}{M_n} = \int_0^W \alpha\, dx$$ → $\alpha(E) = \alpha_0 e^{-H/E}$

$$e^{-(\text{energy barrier})/\text{mean energy}}$$

崩潰電壓 $$\boxed{V_B = \frac{\epsilon_s E_{crit}^2}{2eN_B}}$$

How to avoid the breakdown when we need the device work for high-voltage application, e.g. EV ?

比較

特徵	Zener效應	雪崩效應
摻雜濃度	較高	較低
電流機制	穿隧效應	在高反向電場下，載子獲得足夠能量以撞擊晶格，產生新的載子。
臨界電場	較大	較小
崩潰電壓	較小	較大
應用場景	穩壓電源、電壓參考源	瞬態保護、峰值檢測、高壓發生器等