MA3227 Numerical Analysis II

Tutorial 1: Big O Notation

Algebraic and exponential scaling

For each of the following functions $f(n)$, determine whether $f(n)$ scales algebraically or exponentially in the limit $n \to \infty$ by plotting $f(n)$ on appropriate axes. If $f(n)$ scales algebraically, then also determine the order of scaling by comparing $f(x)$ against n^p.

1. Fibonacci sequence: $f(n) = \begin{cases} 1 & \text{if } n = 1 \text{ or } n = 2, \\ f(n-1) + f(n-2) & \text{otherwise.} \end{cases}$
2. Triangular loop: $f(n) = \sum_{i=1}^{n} \sum_{j=1}^{i} 1.$
3. Geometric series: $f(n) = \sum_{k=1}^{n} 2^k.$
4. Recursive: $f(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2\, f(\frac{n}{2}) + 1 & \text{otherwise.} \end{cases}$
   Assume that $n = 2^k$ is a power of 2.

Other types of scaling (advanced)

Figure out a way to demonstrate the scaling of the following functions in the limit $n \to \infty$.

1. $f(n) = \begin{cases} 1 & \text{if } n = 1, \\ f(\frac{n}{2}) + 1 & \text{otherwise.} \end{cases}$ *Hint*: $f(n)$ scales logarithmically.
   Assume that $n = 2^k$ is a power of 2.
2. $f(n) = \exp(\sqrt{n})$ *Hint*: $f(n)$ scales super-algebraically but sub-exponentially.