## MA3227 Numerical Analysis II

## **Tutorial 2: Nonlinear Equations**

## Bisection and Newton's methods in practice

Use the bisection and Newton methods implemented in the Roots package to determine the unique real root of the function

$$f(x) = 4 - 3x + 2x^2 - x^3.$$

To do so, you will have to determine an initial bracketing interval  $[a_0, b_0]$  and an initial guess  $x_0$ . These quantities can be easily read off from a plot of f(x) over a suitable range of x-values.

## Newton's method for computing complex square roots

Write a Julia function square\_root(w) which uses Newton's method to compute complex square roots

$$x + iy = \sqrt{u + iv}$$
  $\iff$   $(x + iy)^2 = u + iv.$ 

To do so, you must translate the equation on the right into a  $2 \times 2$  system of nonlinear equations f(x) = 0, manually compute the Jacobian  $\nabla f(x)$  and then implement the Newton iteration

$$x_{k+1} = x_k - \nabla f(x_k)^{-1} f(x_k)$$

in Julia (you can use df(xk) \ f(xk) to solve the linear system).

Can you observe the quadratic convergence of Newton's method?

Algorithmic details:

- Use  $(x_0, y_0) = (u, v)$  as initial guess.
- Terminate the iteration once  $|(x + \iota y)^2 (w + \iota v)| \le 10 \, \text{eps}$  ()  $|x + \iota y|$ .
- Throw an error if the iteration does not terminate after 20 steps. You can do so using the command error("[error message]").

Julia hints:

- The imaginary unit is called im. (Example: im^2 -> -1+0im)
- You can obtain the real and imaginary parts of a complex number z using real(z) and imag(z), respectively. (Example: real(1+2im) -> 1, imag(1+2im) -> 2)