Lab Session 5

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3 Generic two-stage Runge-Kutta method

Using $y(0) = y_0$, $\dot{y}(0) = f(y_0)$ and $\ddot{y}(0) = f'(y_0)\,f(y_0)$, we obtain

$$
\begin{aligned}
\tilde{y}(t) &= y_0 + f(y_0)\,w_1\,t + f\big(y_0 + f(y_0)\,\theta\,t\big)\,w_2\,t \\
&= y_0 + f(y_0)\,w_1\,t + f(y_0)\,w_2\,t + f'(y_0)\,f(y_0)\,\theta\,w_2\,t^2 + \mathcal{O}(t^3) \\
&= y(0) + (w_1 + w_2)\,\dot{y}(0)\,t + \theta\,w_2\,\ddot{y}(0)\,t^2 + \mathcal{O}(t^3).
\end{aligned}
$$

Matching the coefficients with the Taylor series for $y(t)$, we obtain

$$
w_1 + w_2 = 0, \qquad \theta\,w_2 = \tfrac{1}{2}
$$

which yields

$$
w_1 = 1 - \tfrac{1}{2\theta}, \qquad w_2 = \tfrac{1}{2\theta}
$$

as claimed.