MA3227 Numerical Analysis II

Lecture 0: Introduction

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Introduction

Module outline

- ▶ Big O notation for talking about runtime and convergence (week 1)
- ▶ Bisection and Newton methods for nonlinear equations (week 2)
- ▶ Finite difference method for partial differential equations (week 3)
- ▶ LU factorisation and Krylov methods for sparse linear systems (weeks 4-6)
- ▶ Runge Kutta Methods for ordinary differential equations (weeks 7-9)
- ▶ Monte Carlo Methods for computing intractable sums and integrals (weeks 10-12)

Introduction

Partial differential equations and sparse linear systems
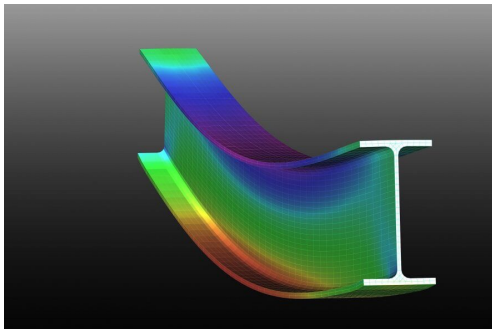
Challenges:

- ▶ Devise a mathematical model for phenomena like bending beams.
- ▶ Translate model into a computationally tractable form.
- ▶ Solve the resulting linear system efficiently.

Introduction

Ordinary differential equations

Challenges:

▶ Devise a mathematical model for phenomena like rocket trajectories.

▶ Translate model into a computationally tractable form.

▶ Do step 2 as efficiently as possible.

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https://pixabay.com/photos/rocket-launch-night-trajectory-693236/

Introduction

Monte Carlo Methods

Assume we know the following.

▶ There is a single COVID patient in Singapore on day 0.

▶ Every patient infects someone with probability p_1 on any given day.

▶ Every patient recovers with probability p_2 on any given day.

What is the probability that the disease will infect 100 people?

This question could in principle be answered as follows.

▶ Go through every possible scenario for the epidemic.

▶ Sum the probabilities of all those scenarios where the disease died out quickly.

Problem: The number of scenarios is way to large!

Alternative approach: Play out a random sample of scenarios and compute the fraction of scenarios where the disease spreads widely.

See `pandemic()` for a demonstration of this approach.

Introduction

Monte Carlo Methods (continued)

The `pandemic()` function answers a complicated mathematical question essentially by flipping coins. It therefore has some resemblance to gambling and is hence called the *Monte Carlo method* in reference to the casino of the same name.

Fun fact: If the Monte Carlo idea had been developed in Singapore, it might have been called the Marina Bay Sands method instead.