MA3227 Numerical Analysis II

Lab Session 5

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3 Generic two-stage Runge-Kutta method

Using $y(0) = y_0$, $\dot{y}(0) = f(y_0)$ and $\ddot{y}(0) = f'(y_0) f(y_0)$, we obtain

$$\tilde{y}(t) = y_0 + f(y_0) w_1 t + f(y_0 + f(y_0) \theta t) w_2 t$$

$$= y_0 + f(y_0) w_1 t + f(y_0) w_2 t + f'(y_0) f(y_0) \theta w_2 t^2 + \mathcal{O}(t^3)$$

$$= y(0) + (w_1 + w_2) \dot{y}(0) t + \theta w_2 \ddot{y}(0) t^2 + \mathcal{O}(t^3).$$

Matching the coefficients with the Taylor series for y(t), we obtain

$$w_1 + w_2 = 1, \qquad \theta \, w_2 = \frac{1}{2}$$

which yields

$$w_1 = 1 - \frac{1}{2\theta}, \qquad w_2 = \frac{1}{2\theta}$$

as claimed.