

Algorithm Library

Liu Yang

November 7, 2018

Contents

1	字符串	5
1.1	KMP	5
1.2	AC 自动机	6
2	动态规划	8
2.1	最长不下降子序列	8
2.2	最长公共子序列	8
2.3	背包	9
3	数据结构	10
3.1	树状数组	10
3.2	线段树	11
3.2.1	线段树-Array	11
3.2.2	线段树-Struct	13
3.3	伸展树 (Splay Tree)	16
3.3.1	Splay-维护二叉查找树	16
3.3.2	Splay-维护数列	20
3.4	字典树 (Trie Tree)	24
3.5	Dfs 序	26
3.6	最近公共祖先	27
3.6.1	在线 LCA	27
3.6.2	离线 LCA	29
4	图论	31
4.1	最小生成树	31
4.1.1	Prim-邻接表	31
4.1.2	Kruskal	33
4.2	最短路	34
4.2.1	Bellman-Ford(判负环)	34
4.2.2	Dijkstra-邻接表	35
4.2.3	Dijkstra-堆优化-邻接表	37
4.2.4	Dijkstra-堆优化-链式前向星	38
4.2.5	Spfa-邻接表	39
4.2.6	Floyd	41
4.3	第 K 短路	41
4.3.1	A* 算法-链式前向星	41
4.4	二分图匹配	44
4.4.1	匈牙利算法-链式前向星	44
4.5	最大流	45
4.5.1	Ford-Fulkerson-邻接矩阵	45
4.5.2	Dinic-邻接矩阵	47
4.5.3	Dinic-链式前向星	48

4.6	费用流	51
4.6.1	最小费用最大流-Spfa	51
5	计算几何	53
5.0.1	计算几何	53
6	数论	57
6.1	素数	57
6.2	母函数	57
6.3	快速乘 + 快速幂	58
6.4	卡特兰	59
6.5	斯特林	59
6.6	错排	59
6.7	斐波那契-矩阵快速幂	60
6.8	逆元	61
6.8.1	逆元-扩展欧几里得	61
6.8.2	逆元-递推	62
6.8.3	阶乘逆元	62
6.9	欧拉函数	63
6.9.1	欧拉函数-单独求解	63
6.9.2	欧拉函数-筛法	64
6.9.3	欧拉函数-线性筛	64
7	其他	65
7.1	尼姆博弈	65
7.2	闰年	66
7.3	阶乘-万进制数组模拟	66
7.4	读写挂	67
7.5	vim	72
8	Yiguang Li	75
8.1	匈牙利算法	75
8.2	KM 算法求最佳匹配	76
8.3	HK 算法求最大匹配	77
8.4	Tarjan 求连通分量	78
8.5	Tarjan 求割点和桥	80
8.6	字符串最大最小表示法	82
8.7	扩展 KMP	83
8.8	Manacher 求回文	84
8.9	树链剖分	85
8.10	字典树	86
8.11	卢卡斯	87

9 Hongliang Deng	91
9.1 树的重心	91
9.2 一般的 SG 函数	94
9.3 复杂的 SG 函数	96
9.4 多重集合的排列组合	97
9.5 莫比乌斯反演	97

1 字符串

1.1 KMP

```
#include <bits/stdc++.h>
```

```
// 对模式串 Pattern 计算 Next 数组
```

```
void KMPPre(string Pattern, vector<int> &Next) {  
    int i = 0, j = -1;  
    Next[0] = -1;  
    int Len = int(Pattern.length());  
    while (i != Len) {  
        if (j == -1 || Pattern[i] == Pattern[j]) {  
            Next[++i] = ++j;  
        }  
        else {  
            j = Next[j];  
        }  
    }  
}
```

```
// 优化对模式串 Pattern 计算 Next 数组
```

```
void PreKMP(string Pattern, vector<int> &Next) {  
    int i, j;  
    i = 0;  
    j = Next[0] = -1;  
    int Len = int(Pattern.length());  
    while (i < Len) {  
        while (j != -1 && Pattern[i] != Pattern[j]) {  
            j = Next[j];  
        }  
        if (Pattern[++i] == Pattern[++j]) {  
            Next[i] = Next[j];  
        }  
        else {  
            Next[i] = j;  
        }  
    }  
}
```

```
// 利用预处理 Next 数组计数模式串 Pattern 在主串 Main 中出现次数
```

```
int KMPCount(string Pattern, string Main) {  
    int PatternLen = int(Pattern.length()), MainLen =  
        ↪ int(Main.length());
```

```
vector<int> Next(PatternLen + 1, 0);
//PreKMP(Pattern, Next);
KMPPre(Pattern, Next);
int i = 0, j = 0;
int Ans = 0;
while (i < MainLen) {
    while (j != -1 && Main[i] != Pattern[j]) {
        j = Next[j];
    }
    i++; j++;
    if (j >= PatternLen) {
        Ans++;
        j = Next[j];
    }
}
return Ans;
}
```

1.2 AC 自动机

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 子节点记录数组
int Son[maxn][26];
int Val[maxn];
// 失配指针 Fail 数组
int Fail[maxn];
// 节点数量
int Tot;

// Trie Tree 初始化
void TrieInit() {
    Tot = 0;
    memset(Son, 0, sizeof(Son));
    memset(Val, 0, sizeof(Val));
    memset(Fail, 0, sizeof(Fail));
}

// 计算字母下标
int Pos(char X) {
    return X - 'a';
}
```

```
// 向 Trie Tree 中插入 Str 模式字符串
void Insert(string Str) {
    int Cur = 0, Len = int(Str.length());
    for (int i = 0; i < Len; ++i) {
        int Index = Pos(Str[i]);
        if (!Son[Cur][Index]) {
            Son[Cur][Index] = ++Tot;
        }
        Cur = Son[Cur][Index];
    }
    Val[Cur]++;
}

// Bfs 求得 Trie Tree 上失配指针
void GetFail() {
    queue<int> Que;
    for (int i = 0; i < 26; ++i) {
        if (Son[0][i]) {
            Fail[Son[0][i]] = 0;
            Que.push(Son[0][i]);
        }
    }
    while (!Que.empty()) {
        int Cur = Que.front(); Que.pop();
        for (int i = 0; i < 26; ++i) {
            if (Son[Cur][i]) {
                Fail[Son[Cur][i]] = Son[Fail[Cur]][i];
                Que.push(Son[Cur][i]);
            }
            else {
                Son[Cur][i] = Son[Fail[Cur]][i];
            }
        }
    }
}

// 询问 Str 中出现的模式串数量
int Query(string Str) {
    int Len = int(Str.length());
    int Cur = 0, Ans = 0;
    for (int i = 0; i < Len; ++i) {
        Cur = Son[Cur][Pos(Str[i])];
    }
}
```

```
        for (int j = Cur; j && ~Val[j]; j = Fail[j]) {
            Ans += Val[j];
            Val[j] = -1;
        }
    }
    return Ans;
}
```

2 动态规划

2.1 最长不下降子序列

```
#include <bits/stdc++.h>

// 最长不下降子序列 (LIS), Num: 序列
int LIS(std::vector<int> &Num) {
    int Ans = 1;
    // Last[i] 为长度为 i 的不下降子序列末尾元素的最小值
    std::vector<int> Last(int(Num.size()) + 1, 0);
    Last[1] = Num[1];
    for (int i = 2; i <= int(Num.size()); ++i) {
        if (Num[i] >= Last[Ans]) {
            Last[++Ans] = Num[i];
        }
        else {
            int Index = std::upper_bound(Last.begin() + 1,
                Last.end(), Num[i]) - Last.begin();
            Last[Index] = Num[i];
        }
    }
    // 返回结果
    return Ans;
}
```

2.2 最长公共子序列

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// Dp[i][j]: Str1[1]~Str1[i] 和 Str2[1]~Str2[j] 对应的公共子序列
//    ↪ 长度
int Dp[maxn][maxn];
```



```
// 最长公共子序列 (LCS)
void LCS(std::string Str1, std::string Str2) {
    for (int i = 0; i < int(Str1.length()); ++i) {
        for (int j = 0; j < int(Str2.length()); ++j) {
            if (Str1[i] == Str2[j]) {
                Dp[i + 1][j + 1] = Dp[i][j] + 1;
            }
            else {
                Dp[i + 1][j + 1] = std::max(Dp[i][j + 1], Dp[i
↪ + 1][j]);
            }
        }
    }
}
```

2.3 背包

```
#include <bits/stdc++.h>

const int maxn = "Edit";

int Dp[maxn];
// NValue: 背包容量, NKind: 总物品数
int NValue, NKind;

// 01 背包, 代价为 Cost, 获得的价值为 Weight
void ZeroOnePack(int Cost, int Weight) {
    for (int i = NValue; i >= Cost; --i) {
        Dp[i] = std::max(Dp[i], Dp[i - Cost] + Weight);
    }
}

// 完全背包, 代价为 Cost, 获得的价值为 Weight
void CompletePack(int Cost, int Weight) {
    for (int i = Cost; i <= NValue; ++i) {
        Dp[i] = std::max(Dp[i], Dp[i - Cost] + Weight);
    }
}

// 多重背包, 代价为 Cost, 获得的价值为 Weight, 数量为 Amount
void MultiplePack(int Cost, int Weight, int Amount) {
    if (Cost * Amount >= NValue) {
        CompletePack(Cost, Weight);
    }
}
```

```
    else {
        int k = 1;
        while (k < Amount) {
            ZeroOnePack(k * Cost, k * Weight);
            Amount -= k;
            k <= 1;
        }
        ZeroOnePack(Amount * Cost, Amount * Weight);
    }
}
```

3 数据结构

3.1 树状数组

```
#include <bits/stdc++.h>
#define lowbit(x) (x & (-x))

const int maxn = "Edit";

// 树状数组
int C[maxn];

// 更新树状数组信息
void Update(int X, int Val) {
    while (X < maxn) {
        C[X] += Val;
        X += lowbit(X);
    }
}

// 求和
int GetSum(int X) {
    int Res = 0;
    while (X > 0) {
        Res += C[X];
        X -= lowbit(X);
    }
    return Res;
}
```

3.2 线段树

3.2.1 线段树-Array

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// Sum: 线段树信息 (此模板为求和), Lazy: 惰性标记
int Sum[maxn << 2], Lazy[maxn << 2];

// 更新节点信息, 这里是求和
void PushUp(int Root) {
    Sum[Root] = Sum[Root << 1] + Sum[Root << 1 | 1];
}

// 下推标记函数, LeftNum, RightNum: 分别为左右子树的数字数量
void PushDown(int Root, int LeftNum, int RightNum) {
    if (Lazy[Root]) {
        // 下推标记
        Lazy[Root << 1] += Lazy[Root];
        Lazy[Root << 1 | 1] += Lazy[Root];
        // 根据惰性标修改子节点的值
        Sum[Root << 1] += Lazy[Root] * LeftNum;
        Sum[Root << 1 | 1] += Lazy[Root] * RightNum;
        // 清除本节点惰性标记
        Lazy[Root] = 0;
    }
}

// 建树, Left、Right: 当前节点区间, Root: 当前节点编号
void Build(int Left, int Right, int Root) {
    Lazy[Root] = 0;
    // 到达叶子节点
    if (Left == Right) {
        scanf("%d", &Sum[Root]);
        return;
    }
    int Mid = (Left + Right) >> 1;
    // 左子树
    Build(Left, Mid, Root << 1);
    // 右子树
    Build(Mid + 1, Right, Root << 1 | 1);
    // 更新信息
```

```

    PushUp(Root);
}

// 单点修改, Pos: 修改点位置, Value: 修改值, Left、Right: 当前区
↪ 间, Root: 当前节点编号
void PointUpdate(int Pos, int Value, int Left, int Right, int
↪ Root) {
    // 修改叶子节点
    if (Left == Right) {
        Sum[Root] += Value;
        return;
    }
    int Mid = (Left + Right) >> 1;
    // 根据条件判断调用左子树还是右子树
    if (Pos <= Mid) {
        PointUpdate(Pos, Value, Left, Mid, Root << 1);
    }
    else {
        PointUpdate(Pos, Value, Mid + 1, Right, Root << 1 |
↪ 1);
    }
    // 子节点更新后更新此节点
    PushUp(Root);
}

// 区间修改, OperateLeft、OperateRight: 操作区间, Left、Right:
↪ 当前区间, Root: 当前节点编号
void IntervalUpdate(int OperateLeft, int OperateRight, int
↪ Value, int Left, int Right, int Root) {
    // 若本区间完全在操作区间内
    if (OperateLeft <= Left && OperateRight >= Right) {
        Sum[Root] += Value * (Right - Left + 1);
        // 增加惰性标记, 表示本区间 Sum 正确, 但子区间仍需要根据
↪ 惰性标记调整更新
        Lazy[Root] += Value;
        return;
    }
    int Mid = (Left + Right) >> 1;
    // 下推标记
    PushDown(Root, Mid - Left + 1, Right - Mid);
    // 根据条件判断调用左子树还是右子树
    if (OperateLeft <= Mid) {
        IntervalUpdate(OperateLeft, OperateRight, Value, Left,
↪ Mid, Root << 1);
    }
}

```

```
    }
    if (OperateRight > Mid) {
        IntervalUpdate(OperateLeft, OperateRight, Value, Mid +
            ↪ 1, Right, Root << 1 | 1);
    }
    // 更新当前节点信息
    PushUp(Root);
}

// 区间查询, OperateLeft、OperateRight: 操作区间, Left、Right:
↪ 当前区间, Root: 当前节点编号
int Query(int OperateLeft, int OperateRight, int Left, int
    ↪ Right, int Root) {
    // 区间内直接返回
    if (OperateLeft <= Left && OperateRight >= Right) {
        return Sum[Root];
    }
    int Mid = (Left + Right) >> 1;
    // 下推标记
    PushDown(Root, Mid - Left + 1, Right - Mid);
    // 叠加结果
    int Ans = 0;
    if (OperateLeft <= Mid) {
        Ans += Query(OperateLeft, OperateRight, Left, Mid,
            ↪ Root << 1);
    }
    if (OperateRight > Mid) {
        Ans += Query(OperateLeft, OperateRight, Mid + 1,
            ↪ Right, Root << 1 | 1);
    }
    // 返回结果
    return Ans;
}
```

3.2.2 线段树-Struct

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 线段树节点
struct Node {
    int Left, Right;
    int Lazy, Tag;
```

```

    int Sum;
};

Node SegmentTree[maxn << 2];

// 更新节点信息
void PushUp(int Root) {
    SegmentTree[Root].Sum = SegmentTree[Root << 1].Sum +
        ↪ SegmentTree[Root << 1 | 1].Sum;
}

// 建树, Left、Right: 当前节点区间, Root: 当前节点编号
void Build(int Left, int Right, int Root) {
    SegmentTree[Root].Left = Left;
    SegmentTree[Root].Right = Right;
    SegmentTree[Root].Lazy = 0;
    SegmentTree[Root].Tag = 0;
    // 叶子节点
    if (Left == Right) {
        scanf("%d", &SegmentTree[Root].Sum);
        return;
    }
    // 左右子树
    int Mid = (Left + Right) >> 1;
    Build(Left, Mid, Root << 1);
    Build(Mid + 1, Right, Root << 1 | 1);
    // 更新
    PushUp(Root);
}

// 单点更新, Pos: 修改点位置, Value: 修改值, Root: 当前节点编号
void PointUpdate(int Pos, int Value, int Root) {
    SegmentTree[Root].Sum += Value;
    if (SegmentTree[Root].Left == Pos &&
        ↪ SegmentTree[Root].Right == Pos) {
        return;
    }
    int Mid = (SegmentTree[Root].Left +
        ↪ SegmentTree[Root].Right) >> 1;
    if (Pos <= Mid) {
        PointUpdate(Pos, Value, Root << 1);
    }
    else {
        PointUpdate(Pos, Value, Root << 1 | 1);
    }
}

```

```

    }
    PushUp(Root);
}

// 区间修改, Left、Right: 修改区间, Value: 修改值, Root: 当前节点
// 编号
void IntervalUpdate(int Left, int Right, int Value, int Root)
{
    if (SegmentTree[Root].Left == Left &&
        SegmentTree[Root].Right == Right) {
        SegmentTree[Root].Lazy = 1;
        SegmentTree[Root].Tag = Value;
        SegmentTree[Root].Sum = (Right - Left + 1) * Value;
        return;
    }
    int Mid = (SegmentTree[Root].Left +
        SegmentTree[Root].Right) >> 1;
    // 下推更新
    if (SegmentTree[Root].Lazy == 1) {
        SegmentTree[Root].Lazy = 0;
        IntervalUpdate(SegmentTree[Root].Left, Mid,
            SegmentTree[Root].Tag, Root << 1);
        IntervalUpdate(Mid + 1, SegmentTree[Root].Right,
            SegmentTree[Root].Tag, Root << 1 | 1);
        SegmentTree[Root].Tag = 0;
    }
    if (Right <= Mid) {
        IntervalUpdate(Left, Right, Value, Root << 1);
    }
    else if (Left > Mid) {
        IntervalUpdate(Left, Right, Value, Root << 1 | 1);
    }
    else {
        IntervalUpdate(Left, Mid, Value, Root << 1);
        IntervalUpdate(Mid + 1, Right, Value, Root << 1 | 1);
    }
    PushUp(Root);
}

// 区间查询, Left、Right: 查询区间, Root: 当前节点编号
int Query(int Left, int Right, int Root) {
    if (Left == SegmentTree[Root].Left && Right ==
        SegmentTree[Root].Right) {
        return SegmentTree[Root].Sum;
    }

```

```
}
int Mid = (SegmentTree[Root].Left +
    ↪ SegmentTree[Root].Right) >> 1;
if (Right <= Mid) {
    return Query(Left, Right, Root << 1);
}
else if (Left > Mid) {
    return Query(Left, Right, Root << 1 | 1);
}
else {
    return Query(Left, Mid, Root << 1) + Query(Mid + 1,
        ↪ Right, Root << 1 | 1);
}
}
```

3.3 伸展树 (Splay Tree)

3.3.1 Splay-维护二叉查找树

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct SplayTree {
    // Root:Splay Tree 根节点
    int Root, Tot;
    // Son[i][0]:i 节点的左孩子, Son[i][1]:i 节点的右孩子
    int Son[maxn][2];
    // Pre[i]:i 节点的父节点
    int Pre[maxn];
    // Val[i]:i 节点的权值
    int Val[maxn];
    // Size[i]: 以 i 节点为根的 Splay Tree 的节点数 (包含自身)
    int Size[maxn];
    // Cnt[i]: 节点 i 的权值的出现次数
    int Cnt[maxn];

    void PushUp(int X) {
        Size[X] = Size[Son[X][0]] + Size[Son[X][1]] + Cnt[X];
    }

    // 判断 X 节点是其父节点的左孩子还是右孩子
    bool Self(int X) {
        return X == Son[Pre[X]][1];
    }
};
```



```

}

void Clear(int X) {
    Son[X][0] = Son[X][1] = Pre[X] = Val[X] = Size[X] =
    ↪ Cnt[X] = 0;
}

// 旋转
void Rotate(int X) {
    int Fa = Pre[X], FaFa = Pre[Fa], XJ = Self(X);
    Son[Fa][XJ] = Son[X][XJ ^ 1];
    Pre[Son[Fa][XJ]] = Pre[X];
    Son[X][XJ ^ 1] = Pre[X];
    Pre[Fa] = X;
    Pre[X] = FaFa;
    if (FaFa) {
        Son[FaFa][Fa == Son[FaFa][1]] = X;
    }
    PushUp(Fa);
    PushUp(X);
}

// 旋转 X 节点到根节点
void Splay(int X) {
    for (int i = Pre[X]; i = Pre[X]; Rotate(X)) {
        if (Pre[i]) {
            Rotate(Self(X) == Self(i) ? i : X);
        }
    }
    Root = X;
}

// 插入数 X
void Insert(int X) {
    if (!Root) {
        Val[++Tot] = X;
        Cnt[Tot]++;
        Root = Tot;
        PushUp(Root);
        return;
    }
    int Cur = Root, F = 0;
    while (true) {
        if (Val[Cur] == X) {

```

```

        Cnt[Cur]++;
        PushUp(Cur);
        PushUp(F);
        Splay(Cur);
        break;
    }
    F = Cur;
    Cur = Son[Cur][Val[Cur] < X];
    if (!Cur) {
        Val[++Tot] = X;
        Cnt[Tot]++;
        Pre[Tot] = F;
        Son[F][Val[F] < X] = Tot;
        PushUp(Tot);
        PushUp(F);
        Splay(Tot);
        break;
    }
}
}

```

// 查询 x 的排名

```

int Rank(int X) {
    int Ans = 0, Cur = Root;
    while (true) {
        if (X < Val[Cur]) {
            Cur = Son[Cur][0];
        }
        else {
            Ans += Size[Son[Cur][0]];
            if (X == Val[Cur]) {
                Splay(Cur);
                return Ans + 1;
            }
            Ans += Cnt[Cur];
            Cur = Son[Cur][1];
        }
    }
}

```

// 查询排名为 x 的数

```

int Kth(int X) {
    int Cur = Root;
    while (true) {

```

```
        if (Son[Cur][0] && X <= Size[Son[Cur][0]]) {
            Cur = Son[Cur][0];
        }
        else {
            X -= Cnt[Cur] + Size[Son[Cur][0]];
            if (X <= 0) {
                return Val[Cur];
            }
            Cur = Son[Cur][1];
        }
    }
}

/*
 * 在 Insert 操作时 X 已经 Splay 到根了
 * 所以 X 的前驱就是 X 的左子树的最右边的节点
 * 后继就是 X 的右子树的最左边的节点
 */

// 求前驱
int GetPath() {
    int Cur = Son[Root][0];
    while (Son[Cur][1]) {
        Cur = Son[Cur][1];
    }
    return Cur;
}

// 求后继
int GetNext() {
    int Cur = Son[Root][1];
    while (Son[Cur][0]) {
        Cur = Son[Cur][0];
    }
    return Cur;
}

// 删除值为 X 的节点
void Delete(int X) {
    // 将 X 旋转到根
    Rank(X);
    if (Cnt[Root] > 1) {
        Cnt[Root]--;
    }
}
```

```
        PushUp(Root);
        return;
    }
    if (!Son[Root][0] && !Son[Root][1]) {
        Clear(Root);
        Root = 0;
        return;
    }
    if (!Son[Root][0]) {
        int Temp = Root;
        Root = Son[Root][1];
        Pre[Root] = 0;
        Clear(Temp);
        return;
    }
    if (!Son[Root][1]) {
        int Temp = Root;
        Root = Son[Root][0];
        Pre[Root] = 0;
        Clear(Temp);
        return;
    }
    int Temp = GetPath(), Old = Root;
    Splay(Temp);
    Pre[Son[Old][1]] = Temp;
    Son[Temp][1] = Son[Old][1];
    Clear(Old);
    PushUp(Root);
}
};
```

3.3.2 Splay-维护数列

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// Root:Splay Tree 根节点
int Root, Tot;
// Son[i][0]:i 节点的左孩子, Son[i][1]:i 节点的右孩子
int Son[maxn][2];
// Pre[i]:i 节点的父节点
int Pre[maxn];
// Val[i]:i 节点的权值
```

```
int Val[maxn];
// Size[i]: 以 i 节点为根的 Splay Tree 的节点数 (包含自身)
int Size[maxn];
// 惰性标记数组
bool Lazy[maxn];

void PushUp(int X) {
    Size[X] = Size[Son[X][0]] + Size[Son[X][1]] + 1;
}

void PushDown(int X) {
    if (Lazy[X]) {
        std::swap(Son[X][0], Son[X][1]);
        if (Son[X][0]) {
            Lazy[Son[X][0]] ^= 1;
        }
        if (Son[X][1]) {
            Lazy[Son[X][1]] ^= 1;
        }
        Lazy[X] = 0;
    }
}

// 判断 X 节点是其父节点的左孩子还是右孩子
bool Self(int X) {
    return Son[Pre[X]][1] == X;
}

// 旋转节点 X
void Rotate(int X) {
    int Fa = Pre[X], FaFa = Pre[Fa], XJ = Self(X);
    PushDown(Fa); PushDown(X);
    Son[Fa][XJ] = Son[X][XJ ^ 1];
    Pre[Son[Fa][XJ]] = Pre[X];
    Son[X][XJ ^ 1] = Pre[X];
    Pre[Fa] = X;
    Pre[X] = FaFa;
    if (FaFa) {
        Son[FaFa][Fa == Son[FaFa][1]] = X;
    }
    PushUp(Fa); PushUp(X);
}

// 旋转 X 节点到节点 Goal
```

```

void Splay(int X, int Goal = 0) {
    for (int Cur = Pre[X]; (Cur = Pre[X]) != Goal; Rotate(X))
        ↪ {
            PushDown(Pre[Cur]); PushDown(Cur); PushDown(X);
            if (Pre[Cur] != Goal) {
                if (Self(X) == Self(Cur)) {
                    Rotate(Cur);
                }
                else {
                    Rotate(X);
                }
            }
        }
    if (!Goal) {
        Root = X;
    }
}

```

// 获取以 R 为根节点 *Splay Tree* 中的第 K 大个元素在 *Splay Tree*
 ↪ 中的位置

```

int Kth(int R, int K) {
    PushDown(R);
    int Temp = Size[Son[R][0]] + 1;
    if (Temp == K) {
        return R;
    }
    if (Temp > K) {
        return Kth(Son[R][0], K);
    }
    else {
        return Kth(Son[R][1], K - Temp);
    }
}

```

// 获取 *Splay Tree* 中以 X 为根节点子树的最小值位置

```

int GetMin(int X) {
    PushDown(X);
    while (Son[X][0]) {
        X = Son[X][0];
        PushDown(X);
    }
    return X;
}

```

```
// 获取 Splay Tree 中以  $x$  为根节点子树的最大值位置
int GetMax(int X) {
    PushDown(X);
    while (Son[X][1]) {
        X = Son[X][1];
        PushDown(X);
    }
    return X;
}

// 求节点  $x$  的前驱节点
int GetPath(int X) {
    Splay(X, Root);
    int Cur = Son[Root][0];
    while (Son[Cur][1]) {
        Cur = Son[Cur][1];
    }
    return Cur;
}

// 求节点  $y$  的后继节点
int GetNext(int X) {
    Splay(X, Root);
    int Cur = Son[Root][1];
    while (Son[Cur][0]) {
        Cur = Son[Cur][0];
    }
    return Cur;
}

// 翻转 Splay Tree 中 Left~Right 区间
void Reverse(int Left, int Right) {
    int X = Kth(Root, Left), Y = Kth(Root, Right);
    Splay(X, 0);
    Splay(Y, X);
    Lazy[Son[Y][0]] ^= 1;
}

// 建立 Splay Tree
void Build(int Left, int Right, int Cur) {
    if (Left > Right) {
        return;
    }
}
```

```
    }
    int Mid = (Left + Right) >> 1;
    Build(Left, Mid - 1, Mid);
    Build(Mid + 1, Right, Mid);
    Pre[Mid] = Cur;
    Val[Mid] = Mid - 1;
    Lazy[Mid] = 0;
    PushUp(Mid);
    if (Mid < Cur) {
        Son[Cur][0] = Mid;
    }
    else {
        Son[Cur][1] = Mid;
    }
}

// 输出 Splay Tree
void Print(int Cur) {
    PushDown(Cur);
    if (Son[Cur][0]) {
        Print(Son[Cur][0]);
    }
    // 哨兵节点判断
    if (Val[Cur] != -INF && Val[Cur] != INF) {
        printf("%d ", Val[Cur]);
    }
    if (Val[Son[Cur][1]]) {
        Print(Son[Cur][1]);
    }
}
```

3.4 字典树 (Trie Tree)

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Trie {
    // Trie Tree 节点
    int Son[maxn][26];
    // Trie Tree 节点数量
    int Tot;

    // 字符串数量统计数组
```



```
int Cnt[maxn];

// Trie Tree 初始化
void TrieInit() {
    Tot = 0;
    memset(Cnt, 0, sizeof(Cnt));
    memset(Son, 0, sizeof(Son));
}

// 计算字母下标
int Pos(char X) {
    return X - 'a';
}

// 向 Trie Tree 中插入字符串 Str
void Insert(string Str) {
    int Cur = 0, Len = int(Str.length());
    for (int i = 0; i < Len; ++i) {
        int Index = Pos(Str[i]);
        if (!Son[Cur][Index]) {
            Son[Cur][Index] = ++Tot;
        }
        Cur = Son[Cur][Index];

        Cnt[Cur]++;
    }
}

// 查找字符串 Str, 存在返回 true, 不存在返回 false
bool Find(string Str) {
    int Cur = 0, Len = int(Str.length());
    for (int i = 0; i < Len; ++i) {
        int Index = Pos(Str[i]);
        if (!Son[Cur][Index]) {
            return false;
        }
        Cur = Son[Cur][Index];
    }
    return true;
}

// 查询字典树中以 Str 为前缀的字符串数量
int PathCnt(string Str) {
```

```
    int Cur = 0, Len = int(Str.length());
    for (int i = 0; i < Len; ++i) {
        int Index = Pos(Str[i]);
        if (!Son[Cur][Index]) {
            return 0;
        }
        Cur = Son[Cur][Index];
    }
    return Cnt[Cur];
}
};
```

3.5 Dfs 序

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 链式前向星建图
struct Link {
    int V, Next;
};

Link edges[maxn << 1];
int Head[maxn];
int Tot = 0;

void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

void AddEdge(int U, int V) {
    edges[++Tot] = Link {V, Head[U]};
    Head[U] = Tot;
    edges[++Tot] = Link {U, Head[V]};
    Head[V] = Tot;
}

int Cnt;
int InIndex[maxn], OutIndex[maxn];

// Dfs 序
void DfsSequence(int Node, int Pre) {
```

```
Cnt++;
InIndex[Node] = Cnt;
for (int i = Head[U]; i != -1; i = edges[i].Next) {
    if (edges[i].V != Pre) {
        DfsSequence(edges[i].V, Node);
    }
}
OutIndex[U] = Cnt;
}
```

3.6 最近公共祖先

3.6.1 在线 LCA

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 节点深度
int Rmq[maxn << 1];

struct ST {
    // 最小值对应下标
    int Dp[maxn << 1][20];
    // RMQ 初始化
    void init(int N) {
        for (int i = 1; i <= N; ++i) {
            Dp[i][0] = i;
        }
        for (int j = 1; (1 << j) <= N; ++j) {
            for (int i = 1; i + (1 << j) - 1 <= N; ++i) {
                Dp[i][j] = Rmq[Dp[i][j - 1]] < Rmq[Dp[i + (1
                    ↪ << (j - 1))][j - 1]] ? Dp[i][j - 1] : Dp[i
                    ↪ + (1 << (j - 1))][j - 1];
            }
        }
    }
    // RMQ 查询
    int Query(int A, int B) {
        if (A > B) {
            std::swap(A, B);
        }
        int K = int(log2(B - A + 1));
```

```
        return Rmq[Dp[A][K]] <= Rmq[Dp[B - (1 << K) + 1][K]] ?
            ↪ Dp[A][K] : Dp[B - (1 << K) + 1][K];
    }
};

// 边
struct Link {
    int V, Next;
};

// 链式前向星存树边图
Link edges[maxn << 1];
int Head[maxn];
int Tot;

// 深搜遍历顺序
int Vertex[maxn << 1];
// 节点在深搜中第一次出现的位置
int First[maxn];
// 遍历节点数量
int Cnt;
ST St;

// 链式前向星存图初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 链式前向星存图添加一条由 U 至 V 的边
void AddEdge(int U, int V) {
    edges[Tot] = Link {V, Head[U]};
    Head[U] = Tot++;
}

// 深搜, U: 当前搜索节点, Pre:U 的前驱节点, Depth: 树上深度
void Dfs(int U, int Pre, int Depth) {
    Vertex[++Cnt] = U;
    Rmq[Cnt] = Depth;
    First[U] = Cnt;
    for (int i = Head[U]; i != -1; i = edges[i].Next) {
        int V = edges[i].V;
        if (V == Pre) {
```

```
        continue;
    }
    Dfs(V, U, Depth + 1);
    Vertex[++Cnt] = U;
    Rmq[Cnt] = Depth;
}
}

// LCA 查询前的初始化, Root: 根节点, NodeNum: 节点数量
void LCA_Init(int Root, int NodeNum) {
    Cnt = 0;
    Dfs(Root, Root, 0);
    St.init(2 * NodeNum - 1);
}

// 查询节点 U 和节点 V 的 LCA
int Query_LCA(int U, int V) {
    return Vertex[St.Query(First[U], First[V])];
}
```

3.6.2 离线 LCA

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 树边
struct Edge {
    int V, Next;
};

// 询问
struct Query {
    int Q, Next;
    int Index;
};

// 并查集数组
int Pre[maxn << 2];
// 树边
Edge edges[maxn << 2];
int Head[maxn];
int Tot;
// 询问
```

```
Query querys[maxn << 2];
int QHead[maxn];
int QTot;
// 访问标记
int Vis[maxn];
int Ancestor[maxn];
// 结果
int Answer[maxn];

// 并查集查找
int Find(int X) {
    int R = X;
    while (Pre[R] != -1) {
        R = Pre[R];
    }
    return R;
}

// 并查集合并
void Join(int U, int V) {
    int RU = Find(U);
    int RV = Find(V);
    if (RU != RV) {
        Pre[RU] = RV;
    }
}

// 添加树边
void AddEdge(int U, int V) {
    edges[Tot] = Edge {V, Head[U]};
    Head[U] = Tot++;
}

// 添加询问
void AddQuery(int U, int V, int Index) {
    querys[QTot] = Query {V, QHead[U], Index};
    QHead[U] = QTot++;
    querys[QTot] = Query {U, QHead[V], Index};
    QHead[V] = QTot++;
}

// 初始化
void Init() {
    Tot = 0;
```

```
memset(Head, -1, sizeof(Head));
QTot = 0;
memset(QHead, -1, sizeof(QHead));
memset(Vis, false, sizeof(Vis));
memset(Pre, -1, sizeof(Pre));
memset(Ancestor, 0, sizeof(Ancestor));
}

// LCA 离线 Tarjan 算法
void Tarjan(int Node) {
    Ancestor[Node] = Node;
    Vis[Node] = true;
    for (int i = Head[Node]; i != -1; i = edges[i].Next) {
        if (Vis[edges[i].V]) {
            continue;
        }
        Tarjan(edges[i].V);
        Join(Node, edges[i].V);
        Ancestor[Find(Node)] = Node;
    }
    for (int i = QHead[Node]; i != -1; i = queries[i].Next) {
        if (Vis[queries[i].Q]) {
            Answer[queries[i].Index] =
                Ancestor[Find(queries[i].Q)];
        }
    }
}
```

4 图论

4.1 最小生成树

4.1.1 Prim-邻接表

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Link {
    // V: 连接点, Dis: 边权
    int V, Dis;
    Link(int _V = 0, int _Dis = 0): V(_V), Dis(_Dis) {}
};
```

```
// N: 顶点数, E: 边数
int N, E;
// 松弛更新权值数组
int Dis[maxn];
// 访问标记数组
int Vis[maxn];
// 邻接表
std::vector<Link> Adj[maxn];

// 建图加边, U, V: 顶点, Weight: 权值
void AddEdge(int U, int V, int Weight) {
    Adj[U].push_back(Link (V, Weight));
    // 无向图反向建边
    Adj[V].push_back(Link (U, Weight));
}

// Prim 算法
int Prim(int Start) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, 0, sizeof(Vis));
    Dis[Start] = 0;
    int Res = 0;
    for (int i = 1; i <= N; ++i) {
        // 选择距已生成树权值最小的顶点
        int U = -1, Min = INF;
        for (int j = 1; j <= N; ++j) {
            if (!Vis[j] && Dis[j] < Min) {
                U = j;
                Min = Dis[j];
            }
        }
        // 更新、标记
        Vis[U] = 1;
        Res += Min;
        // 松弛
        for (int j = 0; j < int(Adj[U].size()); ++j) {
            int V = Adj[U][j].V;
            if (!Vis[V] && Adj[U][j].Dis < Dis[V]) {
                Dis[V] = Adj[U][j].Dis;
            }
        }
    }
    // 返回结果
```



```
    return Res;
}
```

4.1.2 Kruskal

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Edge {
    int U, V, Dis;
};

// N: 顶点数, E: 边数, Pre 并查集
int N, E, Pre[maxn];
// edges: 边
Edge edges[maxn];

void Init() {
    // 并查集初始化
    for (int i = 0; i <= N; ++i) {
        Pre[i] = i;
    }
}

// 并查集查询
int Find(int X) {
    int R = X;
    while (Pre[R] != R) {
        R = Pre[R];
    }
    return R;
}

// 并查集合并
void Join(int X, int Y) {
    int XX = Find(X);
    int YY = Find(Y);
    if (XX != YY) {
        Pre[XX] = YY;
    }
}

// Kruskal 算法
```

```
int Kruskal() {
    // 贪心排序
    std::sort(edges + 1, edges + E + 1);
    Init();
    int Res = 0;
    // 选边计算
    for (int i = 1; i <= E; ++i) {
        Edge Temp = edges[i];
        if (Find(Temp.U) != Find(Temp.V)) {
            Join(Temp.U, Temp.V);
            Res += Temp.Dis;
        }
    }
    return Res;
}
```

4.2 最短路

4.2.1 Bellman-Ford(判负环)

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Link {
    // U, V: 顶点, Dis: 边权
    int U, V;
    int Dis;
};
// 松弛更新数组
int Dis[maxn];
// 边
std::vector<Link> edges;

// Bellman_Ford 算法判断是否存在负环回路
bool BellmanFord(int Start, int N) {
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    // 最多做 N-1 次
    for (int i = 1; i < N; ++i) {
        bool flag = false;
        for (int j = 0; j < int(edges.size()); ++j) {
```

```
        if (Dis[edges[j].V] > Dis[edges[j].U] +
            ↪ edges[j].Dis) {
            Dis[edges[j].V] = Dis[edges[j].U] +
            ↪ edges[j].Dis;
            flag = true;
        }
    }
    // 没有负环回路
    if (!flag) {
        return true;
    }
}
// 有负环回路
for (int j = 0; j < int(edges.size()); ++j) {
    if (Dis[edges[j].V] > Dis[edges[j].U] + edges[j].Dis)
        ↪ {
            return false;
        }
}
// 没有负环回路
return true;
}
```

4.2.2 Dijkstra-邻接表

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Link {
    // V: 连接点, Dis: 边权
    int V, Dis;
    Link(int _V = 0, int _Dis = 0): V(_V), Dis(_Dis) {}
};

// N: 顶点数, E: 边数
int N, E;
// 松弛更新数组
int Dis[maxn];
// 访问标记数组
bool Vis[maxn];
// 邻接表
std::vector<Link> Adj[maxn];
```

```
// 建图加边, U V: 顶点, Weight: 权值
void AddEdge(int U, int V, int Weight) {
    Adj[U].push_back(Link (V, Weight));
    // 无向图反向建边
    Adj[V].push_back(Link (U, Weight));
}

// Dijkstra 算法
int Dijkstra(int Start, int End) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, 0, sizeof(Vis));
    Dis[Start] = 0;
    for (int i = 1; i <= N; ++i) {
        // 选择距起点权值和最小的顶点
        int U = -1, Min = INF;
        for (int j = 1; j <= N; ++j) {
            if (!Vis[j] && Dis[j] < Min) {
                U = j;
                Min = Dis[j];
            }
        }
        // 查询失败, 两点不相连
        if (U == -1) {
            return -1;
        }
        // 寻找到最短路
        else if (U == End) {
            return Dis[End];
        }
        // 标记
        Vis[U] = 1;
        // 松弛
        for (int j = 0; j < int(Adj[U].size()); ++j) {
            int V = Adj[U][j].V;
            if (!Vis[V] && Dis[U] + Adj[U][j].Dis < Dis[V]) {
                Dis[V] = Dis[U] + Adj[U][j].Dis;
            }
        }
    }
}
```

4.2.3 Dijkstra-堆优化-邻接表

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Link {
    // V: 连接点, Dis: 边权
    int V, Dis;
    Link(int _V = 0, int _Dis = 0): V(_V), Dis(_Dis) {}
};

// N: 顶点数, E: 边数
int N, E;
// 松弛更新数组
int Dis[maxn];
// 邻接表
std::vector<Link> Adj[maxn];

// 建图加边, U V: 顶点, Weight: 权值
void AddEdge(int U, int V, int Weight) {
    Adj[U].push_back(Link (V, Weight));
    // 无向图反向建边
    Adj[V].push_back(Link (U, Weight));
}

// Dijkstra 堆优化算法
void Dijkstra(int Start) {
    std::priority_queue<std::pair<int, int>,
        ⇨ std::vector<std::pair<int, int> >,
        ⇨ std::greater<std::pair<int, int> > > Que;
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    Que.push(std::make_pair(0, Start));
    while (!Que.empty()) {
        std::pair<int, int> Keep = Que.top();
        Que.pop();
        int V = Keep.second;
        if (Dis[V] < Keep.first) {
            continue;
        }
        for (int i = 0; i < int(Adj[V].size()); ++i) {
            Link Temp = Adj[V][i];
```

```
        if (Dis[Temp.V] > Dis[V] + Temp.Dis) {
            Dis[Temp.V] = Dis[V] + Temp.Dis;
            Que.push(std::make_pair(Dis[Temp.V], Temp.V));
        }
    }
}
```

4.2.4 Dijkstra-堆优化-链式前向星

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const int INF = "Edit";

// 边
struct Link {
    // V: 连接点, Weight: 权值, Next: 上一条边的编号
    int V, Weight, Next;
};

// 边, 一定要开到足够大
Link edges[maxn << 1];
// Head[i] 为点 i 上最后一条边的编号
int Head[maxn];
// 增加边时更新编号
int Tot;
// 松弛更新数组, 最短路
int Dis[maxn];

// 链式前向星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 添加一条 U 至 V 权值为 Weight 的边
void AddEdge(int U, int V, int Weight) {
    edges[Tot] = Link (V, Weight, Head[U]);
    Head[U] = Tot++;
}

// 最短路优化堆排序规则
struct Cmp {
```

```
    bool operator() (const int &A, const int &B) {
        return Dis[A] > Dis[B];
    }
};

// N: 顶点数, E: 边数
int N, E;

// Dijkstra 算法, Start: 起点
void Dijkstra(int Start) {
    std::priority_queue<int, std::vector<int>, Cmp> Que;
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.top(); Que.pop();
        for (int i = Head[U]; ~i; i = edges[i].Next) {
            if (Dis[edges[i].V] > Dis[U] + edges[i].Weight) {
                Dis[edges[i].V] = Dis[U] + edges[i].Weight;
                Que.push(edges[i].V);
            }
        }
    }
}
```

4.2.5 Spfa-邻接表

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Link {
    // V: 连接点, Dis: 边权
    int V, Dis;
};

// N: 顶点数, E: 边数
int N, E;
// 访问标记数组
bool Vis[maxn];
// 每个点的入队列次数
int Cnt[maxn];
```

```
// 最短路数组
int Dis[maxn];
// 邻接表
std::vector<Link> Adj[maxn];

// 建图加边,  $u$   $v$  之间权值为  $Weight$  的边
void AddEdge (int U, int V, int Weight) {
    Adj[U].push_back(Link (V, Weight));
    // 无向图建立反向边
    Adj[V].push_back(Link (U, Weight));
}

// SPFA 算法,  $Start$ : 起点
bool SPFA(int Start) {
    memset(Vis, false, sizeof(Vis));
    memset(Dis, INF, sizeof(Dis));
    memset(Cnt, 0, sizeof(Cnt));
    Vis[Start] = true;
    Dis[Start] = 0;
    Cnt[Start] = 1;
    std::queue<int> Que;
    while (!Que.empty()) {
        Que.pop();
    }
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.front();
        Que.pop();
        Vis[U] = false;
        for (int i = 0; i < int(Adj[U].size()); ++i) {
            int V = Adj[U][i].V;
            if (Dis[V] > Dis[U] + Adj[U][i].Dis) {
                Dis[V] = Dis[U] + Adj[U][i].Dis;
                if (!Vis[V]) {
                    Vis[V] = true;
                    Que.push(V);
                    //  $Cnt[i]$  为  $i$  顶点入队列次数, 用来判定是否
                    //  $\rightarrow$  存在负环回路
                    if (++Cnt[V] > N) {
                        return false;
                    }
                }
            }
        }
    }
}
```



```
    }  
    }  
    return true;  
}
```

4.2.6 Floyd

```
#include <bits/stdc++.h>  
  
const int maxn = "Edit";  
  
// N: 顶点数  
int N;  
// Dis[i][j] 为 i 点到 j 点的最短路  
int Dis[maxn][maxn];  
  
// Floyd 算法  
void Floyd() {  
    for (int k = 1; k <= N; ++k) {  
        for (int i = 1; i <= N; ++i) {  
            for (int j = 1; j <= N; ++j) {  
                Dis[i][j] = std::min(Dis[i][j], Dis[i][k] +  
                    ↪ Dis[k][j]);  
            }  
        }  
    }  
}
```

4.3 第 K 短路

4.3.1 A* 算法-链式前向星

```
#include <bits/stdc++.h>  
  
const int INF = "Edit";  
const int maxn = "Edit";  
  
struct Link {  
    int V, Weight, Next;  
};  
  
Link edges[maxn << 1];  
int Head[maxn];  
int Tot;  
// 反向边
```

```
Link Reverseedges[maxn << 1];
int ReverseHead[maxn];
int ReverseTot;

// 链式前向星存图初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
    ReverseTot = 0;
    memset(ReverseHead, -1, sizeof(ReverseHead));
}

// 加边建图
void AddEdge(int U, int V, int Weight) {
    edges[Tot] = Link {V, Weight, Head[U]};
    Head[U] = Tot++;
    // 用反向边另建图
    Reverseedges[ReverseTot] = Link {U, Weight,
        ↪ ReverseHead[V]};
    ReverseHead[V] = ReverseTot++;
}

int Dis[maxn];

struct Cmp {
    bool operator() (const int &A, const int &B) {
        return Dis[A] > Dis[B];
    }
};

// 利用反向边图求各点到终点的最短路
void Dijkstra(int Start) {
    priority_queue<int, vector<int>, Cmp> Que;
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.top(); Que.pop();
        for (int i = ReverseHead[U]; i != -1; i =
            ↪ Reverseedges[i].Next) {
            if (Dis[Reverseedges[i].V] > Dis[U] +
                ↪ Reverseedges[i].Weight) {
                Dis[Reverseedges[i].V] = Dis[U] +
                    ↪ Reverseedges[i].Weight;
            }
        }
    }
}
```

```
        Que.push(Reverseedges[i].V);
    }
}

}

}

struct AStarNode {
    int F, G, Point;
    // A* 核心:  $F=G+H(Point)$ , 这里  $H(Point)=Dis[Point]$ 
    bool operator < (const AStarNode &A) const {
        if (F == A.F) {
            return G > A.G;
        }
        return F > A.F;
    }
};

// A* 算法求起点 Start 到终点 End 的第 K 短路
int AStar(int Start, int End, int K) {
    int Cnt = 0;
    priority_queue<AStarNode> Que;
    // 注意特盘相同点是否算最短路
    if (Start == End) {
        K++;
    }
    // 起点与终点不连通
    if (Dis[Start] == INF) {
        return -1;
    }
    Que.push(AStarNode {Dis[Start], 0, Start});
    while (!Que.empty()) {
        AStarNode Keep = Que.top(); Que.pop();
        if (Keep.Point == End) {
            Cnt++;
            if (Cnt == K) {
                // 返回第 K 短路长度
                return Keep.G;
            }
        }
        for (int i = Head[Keep.Point]; i != -1; i =
            ↪ edges[i].Next) {
            AStarNode Temp;
            Temp.Point = edges[i].V;
```

```
        Temp.G = Keep.G + edges[i].Weight;
        Temp.F = Temp.G + Dis[Temp.Point];
        Que.push(Temp);
    }
}
return -1;
}
```

4.4 二分图匹配

4.4.1 匈牙利算法-链式前向星

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Link {
    int V, Next;
};

Link edges[maxn << 1];
int Head[maxn];
int Tot;

// 链式前向星存图初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 加边建图
void AddEdge(int U, int V) {
    edges[Tot] = Link {V, Head[U]};
    Head[U] = Tot++;
}

// 匹配左顶点数
int N;
// 右顶点匹配左顶点编号
int Linker[maxn];
// 右顶点匹配标记
bool Vis[maxn];

// 深度优先搜索增广路经
```

```
bool Dfs(int U) {
    for (int i = Head[U]; i != -1; i = edges[i].Next) {
        if (!Vis[edges[i].V]) {
            Vis[edges[i].V] = true;
            if (Linker[edges[i].V] == -1 ||
                ↪ Dfs(Linker[edges[i].V])) {
                Linker[edges[i].V] = U;
                return true;
            }
        }
    }
    return false;
}

// 匈牙利算法
int Hungary() {
    int Ans = 0;
    memset(Linker, -1, sizeof(Vis));
    // 枚举左顶点
    for (int i = 1; i <= N; ++i) {
        memset(Vis, false, sizeof(Vis));
        if (Dfs(i)) {
            Ans++;
        }
    }
    return Ans;
}
```

4.5 最大流

4.5.1 Ford-Fulkerson-邻接矩阵

```
#include <bits/stdc++.h>
// 正无穷
const int INF = "Edit";
const int maxn = "Edit";

// N: 顶点数, E: 边数
int N, E;
// 访问标记数组
bool Vis[maxn];
// 邻接矩阵
int Adj[maxn][maxn];
```

```
// Dfs 搜索增广路径, Vertex: 当前搜索顶点, End: 搜索终点,
↪ NowFlow: 当前最大流量
int Dfs(int Vertex, int End, int NowFlow) {
    // 搜索到终点结束
    if (Vertex == End) {
        return NowFlow;
    }
    // 标记访问过的顶点
    Vis[Vertex] = true;
    // 枚举寻找顶点
    for (int i = 1; i <= N; ++i) {
        if (!Vis[i] && Adj[Vertex][i]) {
            int FindFlow = Dfs(i, End, NowFlow <
                ↪ Adj[Vertex][i] ? NowFlow : Adj[Vertex][i]);
            if (!FindFlow) {
                continue;
            }
            // 找到增广路径后更新邻接矩阵残留网
            Adj[Vertex][i] -= FindFlow;
            Adj[i][Vertex] += FindFlow;
            // 返回搜索结果
            return FindFlow;
        }
    }
    // 未找到增广路径, 搜索失败
    return false;
}

// Ford-Fulkersonsone 算法, Start: 起点, End: 终点
int FordFulkerson(int Start, int End) {
    // MaxFlow: 最大流, Flow: 搜索到的增广路径最大流
    int MaxFlow = 0, Flow = 0;
    memset(Vis, false, sizeof(Vis));
    // 搜索增广路径
    while (Flow = Dfs(Start, End, INF)) {
        MaxFlow += Flow;
        memset(Vis, false, sizeof(Vis));
    }
    // 返回结果
    return MaxFlow;
}
```

4.5.2 Dinic-邻接矩阵

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// N: 顶点数, E: 边数
int N, E;
// 分层数组
int Depth[maxn];
// 邻接矩阵
int Adj[maxn][maxn];

// Bfs 搜索分层图, Start: 起点, End: 终点
bool Bfs(int Start, int End) {
    std::queue<int> Que;
    memset(Depth, -1, sizeof(Depth));
    Depth[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int Vertex = Que.front();
        Que.pop();
        for (int i = 1; i <= N; ++i) {
            if (Depth[i] == -1 && Adj[Vertex][i]) {
                // 分层编号
                Depth[i] = Depth[Vertex] + 1;
                Que.push(i);
            }
        }
    }
    return Depth[End] > 0;
}

// Dfs 搜索增广路径, Vertex: 当前搜索顶点, End: 终点, NowFlow: 当前最大流量
int Dfs(int Vertex, int End, int NowFlow) {
    // 搜索到终点结束
    if (Vertex == End) {
        return NowFlow;
    }
    int FindFlow = 0;
    // 枚举顶点
    for (int i = 1; i <= N; ++i) {
```

```
        if (Adj[Vertex][i] && Depth[i] == Depth[Vertex] + 1) {
            FindFlow = Dfs(i, End, std::min(NowFlow,
                ↪ Adj[Vertex][i]));
            if (FindFlow) {
                // 找到增广路径后更新邻接矩阵残留网
                Adj[Vertex][i] -= FindFlow;
                Adj[i][Vertex] += FindFlow;
                // 返回搜索结果
                return FindFlow;
            }
        }
        // 炸点优化
        if (!FindFlow) {
            Depth[Vertex] = -2;
        }
        // 未找到增广路径
        return false;
    }

    // Dinic 算法, Start: 起点, End: 终点
    int Dinic(int Start, int End) {
        // MaxFlow: 最大流
        int MaxFlow = 0;
        // 分层搜索增广路径直至终点无法分层
        while (Bfs(Start, End)) {
            MaxFlow += Dfs(Start, End, INF);
        }
        // 返回结果
        return MaxFlow;
    }
```

4.5.3 Dinic-链式前向星

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Link {
    // V: 连接点, Weight: 权值, Next: 上一条边的编号
    int V, Weight, Next;
};
```



```
// 边，一定要开到足够大
Link edges[maxn << 1];
// Head[i] 为点 i 上最后一条边的编号
int Head[maxn];
// 增加边时更新编号
int Tot;
// N: 顶点数, E: 边数
int N, E;
// Bfs 分层深度
int Depth[maxn];
// 当前弧优化
int Current[maxn];

// 链式向前星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 添加一条由 U 至 V 权值为 Weight 的边
void AddEdge(int U, int V, int Weight, int ReverseWeight = 0)
↪ {
    edges[Tot] = Link (V, Weight, Head[U]);
    Head[U] = Tot++;
    // 反向建边
    edges[Tot] = Link (U, ReverseWeight, Head[V]);
    Head[V] = Tot++;
}

// Bfs 搜索分层图, Start: 起点, End: 终点
bool Bfs(int Start, int End) {
    memset(Depth, -1, sizeof(Depth));
    std::queue<int> Que;
    Depth[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int Vertex = Que.front();
        Que.pop();
        for (int i = Head[Vertex]; i != -1; i = edges[i].Next)
            ↪ {
                if (Depth[edges[i].V] == -1 && edges[i].Weight >
                    ↪ 0) {
                    Depth[edges[i].V] = Depth[Vertex] + 1;
                }
            }
    }
}
```

```

        Que.push(edges[i].V);
    }
}
}
return Depth[End] != -1;
}

// Dfs 搜索增广路径, Vertex: 当前搜索顶点, End: 终点, NowFlow: 当前最大流
↪ 前最大流
int Dfs(int Vertex, int End, int NowFlow) {
    // 搜索到终点或者可用当前最大流为 0 返回
    if (Vertex == End || NowFlow == 0) {
        return NowFlow;
    }
    // UsableFlow: 可用流量, 当达到 NowFlow 时不可再增加,
    ↪ FindFlow: 递归深搜到的最大流
    int UsableFlow = 0, FindFlow;
    // &i=Current[Vertex] 为当前弧优化, 每次更新 Current[Vertex]
    for (int &i = Current[Vertex]; i != -1; i = edges[i].Next)
        ↪ {
            if (edges[i].Weight > 0 && Depth[edges[i].V] ==
                ↪ Depth[Vertex] + 1) {
                FindFlow = Dfs(edges[i].V, End, std::min(NowFlow -
                    ↪ UsableFlow, edges[i].Weight));
                if (FindFlow > 0) {
                    edges[i].Weight -= FindFlow;
                    // 反边
                    edges[i ^ 1].Weight += FindFlow;
                    UsableFlow += FindFlow;
                    if (UsableFlow == NowFlow) {
                        return NowFlow;
                    }
                }
            }
        }
    }
    // 炸点优化
    if (!UsableFlow) {
        Depth[Vertex] = -2;
    }
    return UsableFlow;
}

// Dinic 算法, Start: 起点, End: 终点

```

```
int Dinic(int Start, int End) {
    int MaxFlow = 0;
    while (Bfs(Start, End)) {
        // 当前弧优化
        for (int i = 1; i <= N; ++i) {
            Current[i] = Head[i];
        }
        MaxFlow += Dfs(Start, End, INF);
    }
    // 返回结果
    return MaxFlow;
}
```

4.6 费用流

4.6.1 最小费用最大流-Spfa

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Link {
    // V: 连接点, Flow: 流量, Cost: 费用
    int V, Cap, Cost, Flow, Next;
};

// N: 顶点数, E: 边数
int N, E;
int Head[maxn];
// 前驱记录数组
int Path[maxn];
int Dis[maxn];
// 访问标记数组
bool Vis[maxn];
int Tot;
// 链式前向星
Link edges[maxn];

// 链式前向星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}
```

```

}

// 建图加边, U, V 之间建立一条费用为 Cost 的边
void AddEdge(int U, int V, int Cap, int Cost) {
    edges[Tot] = Link {V, Cap, Cost, 0, Head[U]};
    Head[U] = Tot++;
    edges[Tot] = Link {U, 0, -Cost, 0, Head[V]};
    Head[V] = Tot++;
}

// SPFA 算法, Start: 起点, End: 终点
bool SPFA(int Start, int End) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, false, sizeof(Vis));
    memset(Path, -1, sizeof(Path));
    Dis[Start] = 0;
    Vis[Start] = true;
    std::queue<int> Que;
    while (!Que.empty()) {
        Que.pop();
    }
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.front();
        Que.pop();
        Vis[U] = false;
        for (int i = Head[U]; i != -1; i = edges[i].Next) {
            int V = edges[i].V;
            if (edges[i].Cap > edges[i].Flow && Dis[V] >
                Dis[U] + edges[i].Cost) {
                Dis[V] = Dis[U] + edges[i].Cost;
                Path[V] = i;
                if (!Vis[V]) {
                    Vis[V] = true;
                    Que.push(V);
                }
            }
        }
    }
    return Path[End] != -1;
}

```

```

// 最小费用最大流, Start: 起点, End: 终点, Cost: 最小费用
int MinCostMaxFlow(int Start, int End, int &MinCost) {

```

```
int MaxFlow = 0;
MinCost = 0;
while (SPFA(Start, End)) {
    int Min = INF;
    for (int i = Path[End]; i != -1; i = Path[edges[i ^
        ↪ 1].V]) {
        if (edges[i].Cap - edges[i].Flow < Min) {
            Min = edges[i].Cap - edges[i].Flow;
        }
    }
    for (int i = Path[End]; i != -1; i = Path[edges[i ^
        ↪ 1].V]) {
        edges[i].Flow += Min;
        edges[i ^ 1].Flow -= Min;
        MinCost += edges[i].Cost * Min;
    }
    MaxFlow += Min;
}
// 返回最大流
return MaxFlow;
}
```

5 计算几何

5.0.1 计算几何

```
#include <bits/stdc++.h>

const double eps = "Edit";

int Sgn(double X) {
    if (fabs(X) < eps) {
        return 0;
    }
    if (X < 0) {
        return -1;
    }
    else {
        return 1;
    }
}

// 点
struct Point {
```

```
// X: 横坐标, Y: 纵坐标
double X, Y;

void input() {
    scanf("%lf%lf", &X, &Y);
}

// 减法
Point operator - (const Point &B) const {
    return Point (X - B.X, Y - B.Y);
}

// 点积
double operator * (const Point &B) const {
    return X * B.X + Y * B.Y;
}

// 叉积
double operator ^ (const Point &B) const {
    return X * B.Y - Y * B.X;
}
};

// 两点间距离
double Distance(Point A, Point B) {
    return sqrt((B - A) * (B - A));
}

// 线段
struct Segment {
    Point S, T;

    void Input() {
        S.Input();
        T.Input();
    }

    // 向量叉积
    double operator ^ (const Segment &B) const {
        return (T - S) ^ (B.T - B.S);
    }
};

// 判断线段 A、B 是否相交
```

```

bool IsIntersect(Segment A, Segment B) {
    return
        max(A.S.X, A.T.X) >= min(B.S.X, B.T.X) &&
        max(B.S.X, B.T.X) >= min(A.S.X, A.T.X) &&
        max(A.S.Y, A.T.Y) >= min(B.S.Y, B.T.Y) &&
        max(B.S.Y, B.T.Y) >= min(A.S.Y, A.T.Y) &&
        (A ^ Segment {A.S, B.S}) * (A ^ Segment {A.S,
        ↪ B.T}) <= 0 &&
        (B ^ Segment {B.S, A.S}) * (B ^ Segment {B.S,
        ↪ A.T}) <= 0;
}

// 判断线段 A 所在直线与线段 B 是否相交
bool IsIntersect(Segment A, Segment B) {
    return Sgn((B.S - A.T) ^ (A.S - A.T)) * Sgn((B.T - A.T) ^
    ↪ (A.S - A.T)) <= 0;
}

// 判断线段 (直线) A、B 是否平行
bool Parallel(Segment A, Segment B) {
    return Sgn((A.S - A.T) ^ (B.S - B.T)) == 0;
}

// 判断直线 A、B 是否相交
bool IsIntersect(Segment A, Segment B) {
    return !Parallel(A, B) || (Parallel(A, B) && !(Sgn((A.S -
    ↪ B.S) ^ (B.T - B.S)) == 0));
}

// 求直线 A、B 交点
Point IntersectionPoint(Segment A, Segment B) {
    double X = (B.T - B.S) ^ (A.S - B.S), Y = (B.T - B.S)
    ↪ ^ (A.T - B.S);
    return Point {(A.S.X * Y - A.T.X * X) / (Y - X),
    ↪ (A.S.Y * Y - A.T.Y * X) / (Y - X)};
}

// 凸包, points: 所有点, 返回凸包总长度
double ConvexHull(std::vector<Point> points) {
    int N = int(points.size());
    // 特判点数小于等于 2 的情况
    if (N == 1) {
        return 0;
    }
}

```

```
}
else if (N == 2) {
    return Distance(points[0], points[1]);
}
// 查找最左下角的基准点
int Basic = 0;
for (int i = 0; i < N; ++i) {
    if (points[i].Y > points[Basic].Y ||
        (points[i].Y == points[Basic].Y && points[i].X <
         points[Basic].X)) {
        Basic = i;
    }
}
std::swap(points[0], points[Basic]);
// 对其它点进行极角排序
std::sort(points.begin() + 1, points.end(), [&] (const
    Point &A, const Point &B) {
    double Temp = (A - points[0]) ^ (B - points[0]);
    if (Temp > 0) {
        return true;
    }
    else if (!Temp && Distance(A, points[0]) < Distance(B,
        points[0])) {
        return true;
    }
    return false;
});
// 凸包选点
std::vector<Point> Stack;
Stack.push_back(points[0]);
for (int i = 2; i < N; ++i) {
    while (Stack.size() >= 2 && ((Stack.back() -
        Stack[int(Stack.size()) - 2]) ^ (points[i] -
        Stack[int(Stack.size()) - 2])) <= 0) {
        Stack.pop_back();
    }
}
Stack.push_back(points[0]);
// 计算总长
double Ans = 0;
for (int i = 1; i < int(Stack.size()); ++i) {
    Ans += Distance(Stack[i], Stack[i - 1]);
}
// 返回结果
```



```
    return Ans;
}
```

6 数论

6.1 素数

```
#include <bits/stdc++.h>

const int maxn = "Edit";

bool IsPrime[maxn];

void PrimeInit() {
    memset(IsPrime, true, sizeof(IsPrime));
    IsPrime[0] = IsPrime[1] = false;
    for (long long i = 2; i < maxn; ++i) {
        if (!IsPrime[i]) {
            for (long long j = i * i; j < maxn; j += i) {
                IsPrime[j] = false;
            }
        }
    }
}
```

6.2 母函数

```
#include <bits/stdc++.h>

const int maxn = "Edit";

void GeneratingFunction() {
    int n;
    int c1[maxn], c2[maxn];
    scanf("%d", &n);
    for (int i = 0; i < maxn; ++i) {
        c1[i] = 1;
        c2[i] = 0;
    }
    // c1[i] 为  $x^i$  的系数
    // c2 为中间变量
    for (int i = 2; i <= n; ++i) {
        for (int j = 0; j <= n; ++j) {
            for (int k = 0; k + j <= n; k += i) {
```

```
        c2[j + k] += c1[i];
    }
}
for (int j = 0; j <= n; ++j) {
    c1[j] = c2[j];
    c2[j] = 0;
}
}
```

6.3 快速乘 + 快速幂

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;

// 快速乘求  $A*B\%mod$ 
long long QuickMul(long long A, long long B) {
    long long Ans = 0;
    while (B) {
        if (B & 1) {
            Ans = (Ans + A) % mod;
        }
        A = (A + A) % mod;
        B >>= 1;
    }
    return Ans;
}

// 快速幂求  $A^B\%mod$ 
long long QuickPow(long long A, long long B) {
    long long Ans = 1;
    while (B) {
        if (B & 1) {
            Ans = QuickMul(Ans, A) % mod;
        }
        A = QuickMul(A, A) % mod;
        B >>= 1;
    }
    return Ans;
}
```

6.4 卡特兰

```
#include <bits/stdc++.h>

const int maxn = "Edit";

long long Catalan[maxn];

// 递推求卡特兰数
void CatalanInit() {
    memset(Catalan, 0, sizeof(Catalan));
    Catalan[0] = Catalan[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Catalan[i] = Catalan[i - 1] * (4 * i - 2) / (i + 1);
    }
}
```

6.5 斯特林

```
#include <bits/stdc++.h>

const double pi = acos(-1.0);
const double e = 2.718281828459;

int Stirling(int x) {
    if (x <= 1) {
        return 1;
    }
    return int(ceil(log10(2 * pi * x) / 2 + x * log10(x /
        ↪ e)));
}
```

6.6 错排

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const int mod = 1e9 + 7;

// Staggered: 错排数
long long Staggered[maxn];

// 求错排数
void StaggeredInit() {
    Staggered[1] = 0;
```

```
    Staggered[2] = 1;
    // 递推求错排数
    for (int i = 3; i < maxn; ++i) {
        Staggered[i] = (i - 1) * (Staggered[i - 1] +
            ↪ Staggered[i - 2]) % mod;
    }
}
```

6.7 斐波那契-矩阵快速幂

```
#include <bits/stdc++.h>
```

```
const int mod = 1e9 + 7;
```

```
// 矩阵结构体
```

```
struct Matrix {
```

```
    // 矩阵
```

```
    long long Mat[2][2];
```

```
    Matrix() {}
```

```
    // 重载矩阵乘法
```

```
    Matrix operator * (Matrix const &A) const {
```

```
        Matrix Res;
```

```
        memset(Res.Mat, 0, sizeof(Res.Mat));
```

```
        for (int i = 0; i < 2; ++i) {
```

```
            for (int j = 0; j < 2; ++j) {
```

```
                for (int k = 0; k < 2; ++k) {
```

```
                    Res.Mat[i][j] = (Res.Mat[i][j] + Mat[i][k]
```

```
                        ↪ * A.Mat[k][j] % mod) % mod;
```

```
                }
```

```
            }
```

```
        }
```

```
        return Res;
```

```
    }
```

```
};
```

```
// 重载矩阵快速幂
```

```
Matrix operator ^ (Matrix Base, long long K) {
```

```
    Matrix Res;
```

```
    memset(Res.Mat, 0, sizeof(Res.Mat));
```

```
    Res.Mat[0][0] = Res.Mat[1][1] = 1;
```

```
    while (K) {
```

```
        if (K & 1) {
```

```
            Res = Res * Base;
```

```
        }
```

```
        Base = Base * Base;
        K >>= 1;
    }
    return Res;
}

// 斐波那契数列中第  $x$  项
long long Fib(long long X) {
    Matrix Base;
    Base.Mat[0][0] = Base.Mat[1][0] = Base.Mat[0][1] = 1;
    Base.Mat[1][1] = 0;
    return (Base ^ X).Mat[0][1];
}
```

6.8 逆元

6.8.1 逆元-扩展欧几里得

```
#include <bits/stdc++.h>

// 扩展欧几里得,  $A*X+B*Y=D$ 
long long ExtendGcd(long long A, long long B, long long &X,
    ↪ long long &Y) {
    // 无最大公约数
    if (A == 0 && B == 0) {
        return -1;
    }
    if (B == 0) {
        X = 1;
        Y = 0;
        return A;
    }
    long long D = ExtendGcd(B, A % B, Y, X);
    Y -= A / B * X;
    return D;
}

// 逆元,  $AX = 1(mod M)$ 
long long Inv(long long A, long long N) {
    long long X, Y;
    long long D = ExtendGcd(A, N, X, Y);
    if (D == 1) {
        return (X % N + N) % N;
    }
}
```

```
    else {  
        return -1;  
    }  
}
```

6.8.2 逆元-递推

```
#include <bits/stdc++.h>  
  
const int mod = 1e9 + 7;  
const int maxn = "Edit";  
  
long long Inv[maxn];  
  
// 递推求逆元  
void InvInit() {  
    Inv[1] = 1;  
    for (int i = 2; i < maxn; ++i) {  
        Inv[i] = (mod - mod / i) * Inv[mod % i] % mod;  
    }  
}
```

6.8.3 阶乘逆元

```
#include <bits/stdc++.h>  
  
const int mod = 1e9 + 7;  
const int maxn = "Edit";  
  
// 快速乘  
long long QuickMul(long long A, long long B) {  
    long long Ans = 0;  
    while (B) {  
        if (B & 1) {  
            Ans = (Ans + A) % mod;  
        }  
        A = (A + A) % mod;  
        B >>= 1;  
    }  
    return Ans;  
}  
  
// 快速幂  
long long QuickPow(long long A, long long B) {
```

```

    long long Ans = 1;
    while (B) {
        if (B & 1) {
            Ans = QuickMul(Ans, A) % mod;
        }
        A = QuickMul(A, A) % mod;
        B >>= 1;
    }
    return Ans;
}

// Factorial: 阶乘, FactorialInv: 阶乘逆元
long long Factorial[maxn], FactorialInv[maxn];

// 求阶乘逆元
void FactorialInvInit() {
    // 求阶乘
    Factorial[0] = 0;
    Factorial[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Factorial[i] = (Factorial[i - 1] * i) % mod;
    }
    // 飞马小定理求最大值阶乘逆元
    FactorialInv[maxn - 1] = QuickPow(Factorial[maxn - 1], mod
    ↪ - 2);
    // 递推求阶乘逆元
    for (int i = maxn - 2; i >= 0; --i) {
        FactorialInv[i] = (FactorialInv[i + 1] * (i + 1)) %
        ↪ mod;
    }
}

```

6.9 欧拉函数

6.9.1 欧拉函数-单独求解

```

#include <bits/stdc++.h>

// 单独求解欧拉函数
int Phi(int X) {
    int Ans = X;
    for (int i = 2; i <= int(sqrt(X)); ++i) {
        if (!(X % i)) {
            Ans = Ans / i * (i - 1);
        }
    }
}

```

```
        while (!(X % i)) {
            X /= i;
        }
    }
    if (X > 1) {
        Ans = Ans / X * (X - 1);
    }
    return Ans;
}
```

6.9.2 欧拉函数-筛法

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 欧拉函数
int Phi[maxn];

// 筛法求欧拉函数
void Euler() {
    for (int i = 1; i < maxn; ++i) {
        Phi[i] = i;
    }
    for (int i = 2; i < maxn; i += 2) {
        Phi[i] /= 2;
    }
    for (int i = 3; i < maxn; i += 2) {
        if (Phi[i] == i) {
            for (int j = i; j < maxn; j += i) {
                Phi[j] = Phi[j] / i * (i - 1);
            }
        }
    }
}
```

6.9.3 欧拉函数-线性筛

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 素数标记
```



```
bool IsPrime[maxn];
// 欧拉函数
int Phi[maxn];
// 素数
int Prime[maxn];
// 素数个数
int Tot;

// 同时求得欧拉函数和素数表
void PhiPrime() {
    memset(IsPrime, false, sizeof(IsPrime));
    Phi[1] = 1;
    Tot = 0;
    for (int i = 2; i < maxn; ++i) {
        if (!IsPrime[i]) {
            Prime[Tot++] = i;
            Phi[i] = i - 1;
        }
        for (int j = 0; j < Tot; ++j) {
            if (i * Prime[j] > maxn) {
                break;
            }
            IsPrime[i * Prime[j]] = true;
            if (!(i % Prime[j])) {
                Phi[i * Prime[j]] = Phi[i] * Prime[j];
                break;
            }
            else {
                Phi[i * Prime[j]] = Phi[i] * (Prime[j] - 1);
            }
        }
    }
}
```

7 其他

7.1 尼姆博弈

```
#include <bits/stdc++.h>

// 尼姆博弈
bool Nim(std::vector<int> Num) {
    int Ans = 0;
    for (int i = 0; i < int(Num.size()); ++i) {
```

```
        Ans ^= Num[i];
    }
    // ans 不为零则先手赢，否则为后手赢
    return Ans != 0 ? true : false;
}
```

7.2 闰年

```
#include <bits/stdc++.h>

inline int Leep(int Year) {
    return (!(Year % 4) && (Year % 100)) || !(Year % 400);
}
```

7.3 阶乘-万进制数组模拟

```
#include <bits/stdc++.h>

void Factorial() {
    int res[10010];
    int Book = 1;
    int BaoFour = 0;
    res[Book] = 1;
    int n;
    scanf("%d", &n);
    // 乘法计算
    for (int i = 1; i <= n; ++i) {
        BaoFour = 0;
        for (int j = 1; j <= Book; ++j) {
            res[j] = res[j] * i + BaoFour;
            BaoFour = res[j] / 10000;
            res[j] = res[j] % 10000;
        }
        if (BaoFour > 0) {
            res[++Book] += BaoFour;
        }
    }
    printf("%d", res[Book]);
    // 补零输出
    for (int i = Book - 1; i > 0; --i) {
        if (res[i] >= 1000) {
            printf("%d", res[i]);
        }
        else if (res[i] >= 100) {
```

```
        printf("0%d",res[i]);
    }
    else if (res[i] >= 10) {
        printf("00%d",res[i]);
    }
    else {
        printf("000%d",res[i]);
    }
}
putchar('\n');
}
```

7.4 读写挂

```
#include <bits/stdc++.h>
```

```
// 普通读入挂
```

```
template <class T>
inline bool read(T &ret) {
    char c;
    int sgn;
    if (c = getchar(), c == EOF) {
        return false;
    }
    while (c != '-' && (c < '0' || c > '9')) {
        c = getchar();
    }
    sgn = (c == '-') ? -1 : 1;
    ret = (c == '-') ? 0 : (c - '0');
    while (c = getchar(), c >= '0' && c <= '9') {
        ret = ret * 10 + (c - '0');
    }
    ret *= sgn;
    return true;
}
```

```
// 普通输出挂
```

```
template <class T>
inline void out(T x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) {
```

```
        out(x / 10);
    }
    putchar(x % 10 + '0');
}
```

// 牛逼读入挂

```
namespace fastIO {
    const int MX = 4e7;
    char buf[MX];
    int c, sz;
    void begin() {
        c = 0;
        sz = fread(buf, 1, MX, stdin);
    }
    template <class T>
    inline bool read(T &t) {
        while (c < sz && buf[c] != '-' && (buf[c] < '0' ||
            ↪ buf[c] > '9')) {
            c++;
        }
        if (c >= sz) {
            return false;
        }
        bool flag = 0;
        if (buf[c] == '-') {
            flag = 1;
            c++;
        }
        for (t = 0; c < sz && '0' <= buf[c] && buf[c] <= '9';
            ↪ ++c) {
            t = t * 10 + buf[c] - '0';
        }
        if (flag) {
            t = -t;
        }
        return true;
    }
}
```

// 超级读写挂

```
namespace IO{
    #define BUF_SIZE 100000
    #define OUT_SIZE 100000
```

```

#define ll long long
//fread->read

bool IOerror=0;
inline char nc(){
    static char
    ↪ buf[BUF_SIZE],*p1=buf+BUF_SIZE,*pend=buf+BUF_SIZE;
    if (p1==pend){
        p1=buf; pend=buf+fread(buf,1,BUF_SIZE,stdin);
        if (pend==p1){IOerror=1;return -1;}
        //printf("IO error!\n");system("pause");for
        ↪ (;;);exit(0);}
    }
    return *p1++;
}
inline bool blank(char ch){return ch=='
    ↪ ' ||ch=='\n' ||ch=='\r' ||ch=='\t';}
inline void read(int &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}
inline void read(ll &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}
inline void read(double &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (ch=='.'){
        double tmp=1; ch=nc();
        for
        ↪ (;ch>='0'&&ch<='9';ch=nc())tmp/=10.0,x+=tmp*(ch-'0');
    }
}

```

```

    }
    if (sign)x=-x;
}
inline void read(char *s){
    char ch=nc();
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    for (;!blank(ch)&&!IOerror;ch=nc())*s++=ch;
    *s=0;
}
inline void read(char &c){
    for (c=nc();blank(c);c=nc());
    if (IOerror){c=-1;return;}
}
//fwrite->write
struct Ostream_fwrite{
    char *buf,*p1,*pend;
    Ostream_fwrite(){buf=new
        ↪ char[BUF_SIZE];p1=buf;pend=buf+BUF_SIZE;}
    void out(char ch){
        if (p1==pend){
            fwrite(buf,1,BUF_SIZE,stdout);p1=buf;
        }
        *p1++=ch;
    }
}
void print(int x){
    static char s[15],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1);
}
void println(int x){
    static char s[15],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1); out('\n');
}
void print(ll x){
    static char s[25],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1);
}
void println(ll x){

```

```

static char s[25],*s1;s1=s;
if (!x)*s1++='0';if (x<0)out('-'),x=-x;
while(x)*s1++=x%10+'0',x/=10;
while(s1--!=s)out(*s1); out('\n');
}
void print(double x,int y){
static ll
↪ mul[]={1,10,100,1000,10000,100000,1000000,10000000,100000000,
↪ 1000000000,10000000000LL,100000000000LL,1000000000000LL,10000000000000LL,
↪ 100000000000000LL,1000000000000000LL,10000000000000000LL,100000000000000000LL,1000000000000000000LL,10000000000000000000LL};
if (x<-1e-12)out('-'),x=-x;x*=mul[y];
ll x1=(ll)floor(x); if (x-floor(x)>=0.5)++x1;
ll x2=x1/mul[y],x3=x1-x2*mul[y]; print(x2);
if (y>0){out('.'); for (size_t
↪ i=1;i<y&& x3*mul[i]<mul[y];out('0'),++i);
↪ print(x3);}
}
void println(double x,int y){print(x,y);out('\n');}
void print(char *s){while (*s)out(*s++);}
void println(char *s){while (*s)out(*s++);out('\n');}
void flush(){if
↪ (p1!=buf){fwrite(buf,1,p1-buf,stdout);p1=buf;}}
~Ostream_fwrite(){flush();}
}Ostream;
inline void print(int x){Ostream.print(x);}
inline void println(int x){Ostream.println(x);}
inline void print(char x){Ostream.out(x);}
inline void println(char
↪ x){Ostream.out(x);Ostream.out('\n');}
inline void print(ll x){Ostream.print(x);}
inline void println(ll x){Ostream.println(x);}
inline void print(double x,int y){Ostream.print(x,y);}
inline void println(double x,int y){Ostream.println(x,y);}
inline void print(char *s){Ostream.print(s);}
inline void println(char *s){Ostream.println(s);}
inline void println(){Ostream.out('\n');}
inline void flush(){Ostream.flush();}
#undef ll
#undef OUT_SIZE
#undef BUF_SIZE
};
using namespace IO;

```

7.5 vim

```
syntax on
set nu
set tabstop=4
set shiftwidth=4
set cindent
set mouse=a

map <F9> :call Run()<CR>

func! Run()
    exec "w"
    exec "!g++ -Wall % -o %<"
    exec "!. /%<"
endfunc
```


8 Yiguang Li

8.1 匈牙利算法

```
// 匈牙利算法
// 复杂度  $O(nm)$ 
// 求最大匹配
bool find(int u)
{
    for (int v = 1; v <= n; v++)
    {
        if (e[u][v] && !vis[v])
        {
            vis[v] = true;
            if (cy[v] == -1 || find(cy[v]))
            {
                cy[v] = u;
                cx[u] = v; // 如果不用两个数组记录匹配，那么下
                ↪ 面的函数 里面不加 if 判断
                return true;
            }
        }
    }
    return false;
}

int maxmatch()
{
    int ans = 0;
    memset(cx, -1, sizeof cx);
    memset(cy, -1, sizeof cy);
    for (int i = 1; i <= n; i++)
    {
        if (cx[i] == -1)
        {
            memset(vis, false, sizeof vis);
            ans += find(i);
        }
    }
    return ans;
}
```

8.2 KM 算法求最佳匹配

```
// 带权二分图的权值最大的完备匹配称为最佳匹配。
// KM 算法求最佳匹配
int val[N][N],lx[N],ly[N];
int linky[N];
int pre[N];
bool vis[N],visx[N],visy[N];
int slack[N];
int n;
void bfs(int k)
{
    int px, py = 0,yy = 0, d;
    memset(pre, 0, sizeof(pre));
    memset(slack, INF, sizeof(slack));
    linky[py]=k;
    do{
        px = linky[py],d = INF, vis[py] = 1;
        for(int i = 1; i <= n; i++)
            if(!vis[i])
            {
                if(slack[i] > lx[px] + ly[i] - val[px][i])
                    slack[i] = lx[px] + ly[i] -val[px][i],
↪ pre[i]=py;
                if(slack[i]<d) d=slack[i],yy=i;
            }
        for(int i = 0; i <= n; i++)
            if(vis[i]) lx[linky[i]] -= d, ly[i] += d;
            else slack[i] -= d;
        py = yy;
    }while(linky[py]);
    while(py) linky[py] = linky[pre[py]] , py=pre[py];
}
int KM()
{
    memset(lx, 0, sizeof(lx));
    memset(ly, 0, sizeof(ly));
    memset(linky, 0, sizeof(linky));
    for(int i = 1; i <= n; i++)
        memset(vis, false, sizeof(vis)), bfs(i);
    int ans = 0;
    for(int i = 1; i <= n; ++i)
        ans += lx[i] + ly[i];
    return ans;
}
```

```
}
```

8.3 HK 算法求最大匹配

```
// HK 算法求最大匹配
// 复杂度  $O(mn^{0.5})$ 
// 若存双向边则匹配数 *2
int dx[N], dy[N], mx[N], my[N], dis, uN; // dx, dy 表示长度, mx, my
    ↪ 表示匹配
bool vis[N];
bool searchp()
{
    queue<int> q;
    dis = INF;
    memset(dx, -1, sizeof dx);
    memset(dy, -1, sizeof dy);
    for (int i = 1; i <= uN; i++)
    {
        if (mx[i] == -1)
        {
            q.push(i);
            dx[i] = 0;
        }
    }
    while (!q.empty())
    {
        int u = q.front(); q.pop();
        if (dx[u] > dis) break;
        for (int i = head[u]; ~i; i = e[i].next)
        {
            int v = e[i].v;
            {
                if (dy[v] == -1)
                {
                    dy[v] = dx[u] + 1;
                    if (my[v] == -1) dis = dy[v];
                    else
                    {
                        dx[my[v]] = dy[v] + 1;
                        q.push(my[v]);
                    }
                }
            }
        }
    }
}
```

```
    }
    return dis != INF;
}
bool dfs(int u)
{
    for (int i = head[u]; ~i; i = e[i].next)
    {
        int v = e[i].v;
        if (vis[v] || (dy[v] != dx[u] + 1)) continue;
        vis[v] = 1;
        if (my[v] != -1 && dy[v] == dis) continue;
        if (my[v] == -1 || dfs(my[v]))
        {
            my[v] = u;
            mx[u] = v;
            return true;
        }
    }
    return false;
}
int HK()
{
    int res = 0;
    memset(mx, -1, sizeof mx);
    memset(my, -1, sizeof my);
    while (searchp())
    {
        memset(vis, false, sizeof vis);
        for (int i = 1; i <= uN; i++)
            if (mx[i] == -1 && dfs(i))
                res++;
    }
    return res;
}
```

8.4 Tarjan 求连通分量

```
// Tarjan 求连通分量
/* 求连通分量 */
bool vis[maxn];
int dfn[maxn]; //搜索的时间戳
int low[maxn]; //树中最小子树的根, 结点父亲的时间戳
int Stack[maxn]; //栈
int belong[maxn];
```

```
int num[maxn]; // 各连通分量点个数 不一定需要
int id, tot, top, scc;
struct Edge
{
    int v, next;
}e[maxn*maxn];
int head[maxn];
void init()
{
    memset(head, -1, sizeof(head));
    memset(vis, false, sizeof(vis));
    memset(dfn, 0, sizeof(dfn));
    memset(low, 0, sizeof(low));
    memset(num, 0, sizeof(num));
    tot = 0;
    id = 0;
    top = 0;
    scc = 0; // 连通分量数
}
void addedge(int u, int v)
{
    e[tot].v = v;
    e[tot].next = head[u];
    head[u] = tot++;
}
void tarjan(int u)
{
    int v;
    dfn[u] = low[u] = ++id;
    Stack[top++] = u;
    vis[u] = true;
    for(int i = head[u]; i != -1; i = e[i].next)
    {
        v = e[i].v;
        if(!dfn[v])
        {
            tarjan(v);
            low[u] = min(low[u], low[v]);
        }
        else if(vis[v])
        {
            low[u] = min(low[u], dfn[v]);
        }
    }
}
```

```
    if(low[u] == dfn[u])
    {
        scc++;
        do
        {
            v = Stack[--top];
            vis[v] = false;
            belong[v] = scc;
            num[scc]++;
        }while(u != v);
    }
}
```

8.5 Tarjan 求割点和桥

```
// Tarjan 求割点 & 桥
/*
 * 求 无向图的割点和桥
 * 可以找出割点和桥，求删掉每个点后增加的连通块。
 */
bool vis[maxn];
int dfn[maxn]; //搜索的时间戳
int low[maxn]; //树中最小子树的根，结点父亲的时间戳
int num[maxn]; // 各连通分量点个数 不一定需要
int id, tot, top;
int bridge;
int add_block[maxn]; //删除一个点后增加的连通块
struct Edge
{
    int v, next;
    bool cut; // 是否为桥 (割边)
}e[maxn];
bool cut[maxn];
int head[maxn];
void init()
{
    memset(head, -1, sizeof(head));
    memset(vis, false, sizeof(vis));
    memset(dfn, 0, sizeof(dfn));
    memset(low, 0, sizeof(low));
    memset(cut, false, sizeof(cut));
    tot = 0;
    id = 0;
```



```

    top = 0;
    bridge = 0;
}
void addedge(int u, int v)
{
    e[tot].v = v;
    e[tot].next = head[u];
    e[tot].cut = false;
    head[u] = tot++;
}
void tarjan(int u, int pre)
{
    int v;
    low[u] = dfn[u] = ++id;
    vis[u] = true;
    int son = 0;
    for(int i = head[u]; i != -1; i = e[i].next)
    {
        v = e[i].v;
        if(v == pre) continue;
        if( !dfn[v] )
        {
            son++;
            tarjan(v, u);
            if(low[u] > low[v]) low[u] = low[v];
            //桥
            //一条无向边 (u,v) 是桥, 当且仅当 (u,v) 为树枝边, 且
            ↪ 满足  $DFS(u) < low(v)$ 。
            if(low[v] > dfn[u])
            {
                bridge++;
                e[i].cut = true;
                e[i^1].cut = true;
            }
            //割点
            //一个顶点 u 是割点, 当且仅当满足 (1) 或 (2) (1) u
            ↪ 为树根, 且 u 有多于一个子树。
            // (2) u 不为树根, 且满足存在 (u,v) 为树枝边 (或称父
            ↪ 子边,
            //即 u 为 v 在搜索树中的父亲), 使得  $DFS(u) \leq low(v)$ 
            if(u != pre && low[v] >= dfn[u]) //不是树根
            {
                cut[u] = true;
            }
        }
    }
}

```

```
        add_block[u]++;
    }
}
else if( low[u] > dfn[v])
    low[u] = dfn[v];
}
if(u == pre && son > 1) cut[u] = true;
if(u == pre) add_block[u] = son - 1;
vis[u] = false;
}
```

8.6 字符串最大最小表示法

// 字符串最大最小表示法

```
int minstr()
{
    int i = 0, j = 1, k = 0;
    while (i < tlen && j < tlen && k < tlen)
    {
        int tem = t[(i + k) % tlen] - t[(j + k) % tlen];
        if (!tem) k++;
        else
        {
            if (tem > 0) i = i + k + 1;
            else j = j + k + 1;
            if (i == j) j++;
            k = 0;
        }
    }
    return i < j ? i : j;
}

int maxstr()
{
    int i = 0, j = 1, k = 0;
    while (i < tlen && j < tlen && k < tlen)
    {
        int tem = t[(i + k) % tlen] - t[(j + k) % tlen];
        if (!tem) k++;
        else
        {
            if (tem > 0) j = j + k + 1;
            else i = i + k + 1;

            if (i == j) j++;
        }
    }
}
```

```
        k = 0;
    }
}
return i < j ? i : j;
}
```

8.7 扩展 KMP

```
// Exkmp
void gettextNext()
{
    int p, a;
    int tlen = strlen(T);
    Next[0] = tlen;
    for (int i = 1, j = -1; i < tlen; i++, j--)
    {
        if (-1 == j || i + Next[i - a] >= p)
        {
            if (j == -1) p = i, j = 0;

            while (p < tlen && T[p] == T[j]) p++, j++;
            Next[i] = j;
            a = i;
        }
        else Next[i] = Next[i - a];
    }
}

void gettext()
{
    int p, a;
    gettextNext();
    int slen = strlen(S);
    int tlen = strlen(T);
    for (int i = 0, j = -1; i < slen; i++, j--)
    {
        if (-1 == j || i + Next[i - a] >= p)
        {
            if (j == -1) p = i, j = 0;

            while (p < slen && j < tlen && S[p] == T[j]) p++,
                j++;
            ext[i] = j;
            a = i;
        }
    }
}
```

```
    }
    else ext[i] = Next[i - a];
}
}
```

8.8 Manacher 求回文

// Manacher 求回文

```
int init()
{
    int i;
    memset(len, 0, sizeof len);
    int slen = strlen(S);
    T[0] = '$';

    for (int i = 1; i <= 2*slen; i += 2)
    {
        T[i] = '#';
        T[i+1] = S[i/2];
    }
    T[slen*2 + 1] = '#';
    T[slen*2 + 2] = '%';
    T[slen*2 + 3] = 0;
    return slen * 2 + 1;
}

int Manncher(int l)
{
    int mx = 0, ans = 0, p = 0;
    for (int i = 1; i <= l; i++)
    {
        if (mx > i) len[i] = min(mx - i, len[2 * p - i]);
        else len[i] = 1;
        while (T[i - len[i]] == T[i + len[i]]) len[i]++;
        if (i + len[i] > mx)
        {
            mx = i + len[i];
            p = i;
        }
        ans = ans < len[i] ? len[i] : ans;
    }
    return ans - 1;
}
```

```
}
```

8.9 树链剖分

```
// 树链剖分
```

```
struct edge {
    int v, next;
}e[N<<1];
int tot, head[N], id;
int pos[N], dep[N], top[N], fa[N], son[N], sz[N];
int val[N], num[N];
void add_edge(int u, int v)
{
    e[tot].v = v; e[tot].next = head[u]; head[u] = tot++;
}
void init()
{
    tot = 0; id = 0;
    memset(head, -1, sizeof head);
}
void dfs(int u, int pre, int d)
{
    dep[u] = d; sz[u] = 1; fa[u] = pre; son[u] = -1;
    for (int i = head[u]; ~i; i = e[i].next)
    {
        int v = e[i].v;
        if (v == pre) continue;
        dfs(v, u, d + 1);
        sz[u] += sz[v];
        if (son[u] == -1 || sz[son[u]] < sz[v])
            son[u] = v;
    }
}
void dfs1(int u, int tp)
{
    top[u] = tp; pos[u] = ++id;
    num[id] = u;
    if (~son[u]) dfs1(son[u], tp);
    for (int i = head[u]; ~i; i = e[i].next)
    {
        int v = e[i].v;
        if (v == fa[u] || v == son[u]) continue;
        dfs1(v, v);
    }
}
```

```
}
/*-----以上为剖分-----*/
int getSum(int u, int v) // 两点之间路径上的和等
{
    int f1 = top[u], f2 = top[v];
    int ans = 0;
    while (f1 != f2)
    {
        if (dep[f1] < dep[f2])
        {
            swap(u, v);
            swap(f1, f2);
        }
        ans += Query(pos[f1], pos[u], 1);
        u = fa[f1], f1 = top[u];
    }
    if (dep[u] > dep[v]) swap(u, v);
    ans += Query(pos[u], pos[v], 1);
    return ans;
}
void Change(int u, int v, int c) //更新两点之间的值等
{
    int f1 = top[u], f2 = top[v];
    while (f1 != f2)
    {
        if (dep[f1] < dep[f2])
        {
            swap(u, v);
            swap(f1, f2);
        }
        Update(pos[f1], pos[u], c, 1);
        u = fa[f1], f1 = top[u]; //爬到另一条链
    }
    if (dep[u] > dep[v]) swap(u, v);
    Update(pos[u], pos[v], c, 1);
}
```

8.10 字典树

```
// 字典树
void insert(char *s)
{
    int data, i;
    int u=0;
```

```
int len=strlen(s);
for(i=0;i<len;i++)
{
    data=s[i]-'0';
    if(!trie[u][data])
        trie[u][data]=zz++;
    u=trie[u][data];
    val[u]++;
}
}
int find(string s)
{
    int u=0,data;
    int len=s.length();
    for(int i=0;i<len;i++)
    {
        data=s[i]-'0';
        if(!trie[u][data])
            return 0;
        u=trie[u][data];
    }
    return val[u];
}
```

8.11 卢卡斯

```
LL PowMod(LL a,LL b,LL MOD){
    LL ret=1;
    while(b){
        if(b&1) ret=(ret*a)%MOD;
        a=(a*a)%MOD;
        b>>=1;
    }
    return ret;
}
LL fac[100005];
LL Get_Fact(LL p){
    fac[0]=1;
    for(LL i=1;i<=p;i++)
        fac[i]=(fac[i-1]*i)%p; //预处理阶乘
}
```

```
LL Lucas(LL n,LL m,LL p){
    LL ret=1;
    while(n&& m){
        LL a=n%p,b=m%p;
        if(a<b) return 0;
        ret=(ret*fac[a]*PowMod(fac[b]*fac[a-b]%p,p-2,p))%p;
        n/=p;
        m/=p;
    }
    return ret;
}
```


9 Hongliang Deng

9.1 树的重心

/*

树的重心也叫树的质心。对于一棵树 n 个节点的无根树，找到一个点，使
→ 得把树变成以该点为根的有根树时，最大子树的结点数最小。换句话
→ 说，删除这个点后最大连通块（一定是树）的结点数最小。

和树的最大独立问题类似，先任选一个结点作为根节点，把无根树变成有根
→ 树，然后设 $d(i)$ 表示以 i 为根的子树的结点的个数。不难发现
→ $d(i) = d(j) + 1, j \in s(i)$ 。 $s(i)$ 为 i 结点的所有儿子结点的编号的集
→ 合。程序也十分简单：只需要 *DFS* 一次，在无根树有根数的同时计算
→ 即可，连记忆化都不需要——因为本来就没有重复计算。

那么，删除结点 i 后，最大的连通块有多少个呢？结点 i 的子树中最大
→ 有 $\max\{d(j)\}$ 个结点， i 的“上方子树”中有 $n - d(i)$ 个结点，

如图这样，在动态规划的过程中就可以随便找出树的重心了。

*/

```
#include <iostream>
#include <stdio.h>
#include <algorithm>
#include <math.h>
#include <vector>
#include <set>
#include <map>
#include <string>
#include <string.h>
#include <queue>
#include <stack>
#include <deque>
#include <stdlib.h>
#include <bitset>
```

```
using namespace std;
```

```
#define ll long long
#define ull unsigned long long
#define INF 0x3f3f3f3f
#define maxn 10
#define eps 0.00000001
#define M 1e9 + 7
```

```
vector<int> tree[maxn]; //tree[i] 是与 i 相邻的点
```

```
int d[maxn + 5]; //以 i 为根的子树的节点个数
int minNode;
int minBalance;
int n;

void dfs(int node, int parent) { //自己, 父节点
    d[node] = 1; // 自身
    int maxSubTree = 0; //最大子树节点个数
    for (int i = 0; i < (int)tree[node].size(); i++) {
        int son = tree[node][i]; //子树
        if(son != parent) {
            dfs(son, node);
            d[node] += d[son]; //node 的节点数加上 son 的节点个
            ↪ 数
            maxSubTree = max(maxSubTree, d[son]); //比较更大的子
            ↪ 树节点个数
        }
    }
    maxSubTree = max(maxSubTree, n - d[node]);
    if(maxSubTree < minBalance) {
        minBalance = maxSubTree;
        minNode = node;
    }
}

int main(int argc, const char * argv[]) {
    scanf("%d", &n);
    for (int i = 1; i < n; i++) {
        int s, e;
        scanf("%d %d", &s, &e);
        tree[s].push_back(e);
        tree[e].push_back(s);
    }
    minNode = 0;
    minBalance = INF;
    dfs(1, 0);
    printf("%d %d\n", minNode, minBalance);
    return 0;
}

// 数组模拟邻接表版

#include <iostream>
#include <stdio.h>
```

```
#include <algorithm>
#include <math.h>
#include <vector>
#include <set>
#include <map>
#include <string>
#include <string.h>
#include <queue>
#include <stack>
#include <deque>
#include <stdlib.h>
#include <bitset>

using namespace std;

#define ll long long
#define ull unsigned long long
#define INF 0x3f3f3f3f
#define maxn 100005
#define eps 0.00000001
#define M 1e9 + 7

struct Edge{
    int s, e;
    Edge() {}
    Edge(int _s, int _e) {
        s = _s;
        e = _e;
    }
}edge[2 * maxn];

int Next[maxn * 2], pre[maxn * 2], tot, ans[maxn], d[maxn];
int n, minBalance, minNode;

void add(int s, int e) {
    tot ++;
    Next[tot] = pre[s]; pre[s] = tot;
    edge[tot].s = s;
    edge[tot].e = e;
}

void init() {
    for (int i = 0; i < 2 * maxn; i ++)
        Next[i] = pre[i] = -1;
```

```
}

void dfs(int node, int parent) {
    d[node] = 1;
    int maxSubTree = 0;
    for (int i = pre[node]; i != -1; i = Next[i]) {
        int son = edge[i].e;
        if(son == parent) continue;
        dfs(son, node);
        d[node] += d[son];
        maxSubTree = max(maxSubTree, d[son]);
    }
    maxSubTree = max(maxSubTree, n - d[node]);
    if(maxSubTree < minBalance) {
        minBalance = maxSubTree;
        minNode = node;
    }
}
```

```
int main(int argc, const char * argv[]) {

    scanf("%d", &n);
    tot = 0;
    init();
    for (int i = 1; i < n; i++) {
        int s, e;
        scanf("%d %d", &s, &e);
        add(s, e);
        add(e, s);
    }
    minBalance = INF;
    minNode = 0;
    tot = 0;
    dfs(1, 0);

    return 0;
}
```

9.2 一般的 SG 函数

// 一般的 SG 函数

```
#include<iostream>
```

```
#include<math.h>
#include<string.h>
using namespace std;
const int N = 1005;
int SG[N], f[N], S[N];
int m, n, p;
void ff() {
    for (int i = 0; i < N; i++) {
        f[i] = i;
    }
}
void GetSG(int n, int m) {
    memset(SG, 0, sizeof(SG));
    for (int i = 1; i <= n; i++) {
        memset(S, 0, sizeof(S));
        for (int j = 1; f[j] <= i && j <= m; j++) {
            S[SG[i - f[j]]] = 1;
        }
        for (int j = 0; ; j++) {
            if (!S[j]) {
                SG[i] = j;
                break;
            }
        }
    }
}
int main(int argc, char const *argv[]) {
    int n;
    ff();
    scanf("%d", &n);
    while (n--) {
        int a, b;
        scanf("%d %d", &a, &b);
        GetSG(a, b);
        //cout<<SG[a]<<endl;
        if (SG[a])
            printf("first\n");
        else
            printf("second\n");
    }
    return 0;
}
```

9.3 复杂的 SG 函数

/*

复杂的 SG 函数 Hdu-3797

题意：

有 n 个容量为 s 的盒子，每个盒子里原有的石子有 ci 个，问谁能赢
思路：

每个盒子中的 s 和 c 之间存在的输赢关系是一定的，把每个盒子的

→ SG 值先异或可以得最终这个游戏得 SG 值，根据 SG 来判断输赢

比如：一个盒子得 $s = 20$

如果 $c=20$ ，那么先手必败 $SG[20] = 0$;

如果 $c = 19$ $SG[19]$ 得后继为 $SG[20]$ ，即 $SG[19] = 1$

如果 $c = 18$ $SG[18]$ 得后继为 $SG[19]$ ， $SG[20]$ ，即 $SG[18] = 2$

.....

如果 $c = 4$ $SG[4]$ 得后继有。。。, $SG[4]$ 得后继为 $SG[4] = 16$

当 $c = 3$ 是 c 不能经过一步操作到达 20，所以他的后继中无 $SG[20]$ ，

→ 所以最小值为 0，所以 $SG[3] = 0$;

同理 $SG[2] = 1$;

$SG[1] = 0$;

$SG[0] = 0$;

$SG[0]$ 的后继就是他在他能一步到达的 SG 中已存在的数，从 0 开始不存

→ 在的数

比如 4 能到达 $SG[5] \rightarrow SG[20]$ ，而 $SG[5] \rightarrow SG[20]$ 包含的数从 0→15，

→ 所以 $SG[4]=16$ ，而 $SG[3]$ 只能到达 $SG[4] \rightarrow SG[12]$ ，而没有

→ $SG[20]=0$ ，所以 $SG[3] = 0$

从 4 后可以看出一个循环，3 和 20 都是必输点，而后面的 0 也是必输点

当一个人处在必胜点的时候，他可以通过操作来是到达离他最近的一个必

→ 输点，那么他就是必胜得，同理，当一个人先手处于必输点得时候，那

→ 么他就是必输的

*/

```
#include<iostream>
```

```
#include<math.h>
```

```
using namespace std;
```

```
int getsg(int s, int c) {
```

```
    int t = sqrt(s);
```

```
    while (t * t + t >= s) t --;
```

```
    if (c > t) return s - c;
```

```
    else return getsg(t, c);
```

```
}
```

```
int main() {
```

```
    int n;
```

```
    int cnt = 0;
```

```
    while(cin >> n && n) {
```

```
        cnt ++;
```



```
    int t = 0;
    while(n--) {
        int s, c;
        cin >> s >> c;
        t ^= gets(g(s, c));
    }
    cout << "Case " << cnt << ": " << endl;
    if (t) {
        cout << "Yes" << endl;
    } else {
        cout << "No" << endl;
    }
}
return 0;
}
```

9.4 多重集合的排列组合

/*
多重集合的排列组合

多重集合的排列

定理：设 S 是多重集合，他有 k 种不同类型的对象，每一种类型的有限

→ 重复数是 $n_1, n_2, n_3, \dots, n_k$ 。设 S 的大小为 $n = n_1 + n_2 + n_3 + \dots + n_k$ 。则

→ S 的 n 排列数目为 $n! / (n_1! n_2! n_3! \dots n_k!)$

多重集合的组合

定理：设 S 是有 k 种类型对象的多重集合，每种元素均具有无限重复数。

→ 那么 S 的 r 组合的个数

$= C(r+k-1, r) = C(r+k-1, k-1)$

*/

9.5 莫比乌斯反演

```
#include <bits/stdc++.h>
using namespace std;
#define maxn 10000005
#define LL long long
int minu[maxn], prim[maxn], vis[maxn], cnt;

void Init() {
    int N = maxn;
    memset(prim, 0, sizeof(prim));
    memset(minu, 0, sizeof(minu));
```

```
memset(vis, 0, sizeof(vis));
minu[1] = 1;
cnt = 0;
for (int i = 2; i < N; i++) {
    if(!vis[i]) {
        prim[cnt++] = i;
        minu[i] = -1;
    }
    for (int j = 0; j < cnt && i * prim[j] < N; j++) {
        vis[i * prim[j]] = 1;
        if(i % prim[j]) minu[i * prim[j]] = - minu[i];
        else {
            minu[i * prim[j]] = 0;
            break;
        }
    }
}

LL solve(int d) {
    LL ans = 0;
    for (int i = 1; i <= d; i++)
        ans += (LL)minu[i] * (d/i) * (d/i);
    return ans;
}

int main() {
    return 0;
}
```