

Algorithm Library

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April 12, 2019

Contents

1	String	4
1.1	AhoCorasickAutomaton	4
1.2	KMP	5
1.3	Manacher	6
1.4	PalindromicTree	6
1.5	TrieTree	8
2	Math	10
2.1	Catalan	10
2.2	CombinatorialNumber	10
2.3	Derangement	10
2.4	Euler	11
2.4.1	Euler	11
2.4.2	Screen	11
2.4.3	Sieve	11
2.5	FFT	12
2.6	Fibonacci	13
2.7	GeneratingFunction	14
2.8	InverseElement	14
2.8.1	ExtendGcd	14
2.8.2	Factorial	15
2.8.3	FermatLittleTheorem	15
2.8.4	Recursive	15
2.9	Mobius	15
2.10	NimGame	16
2.11	Prime	16
2.11.1	PrimeFactor	16
2.11.2	SieveOfEratosthenes	17
2.12	QuickPow	17
2.13	Stirling	18
3	DataStructure	19
3.1	BinaryIndexedTree	19
3.2	DfsOrder	19
3.3	FunctionalSegmentTree	19
3.4	Hash	20
3.5	LCA	21
3.5.1	DFS+ST	21
3.5.2	Multiplication	22
3.5.3	Tarjan	23
3.6	MultipleTree	25
3.7	STTable	31
3.8	SegmentTree	31
3.8.1	AreaCombination	31
3.8.2	AreaXorCombination	33
3.8.3	SegmentTree	36
3.9	SplayTree	37
3.10	SplayTreeArray	40

4	GraphTheory	44
4.1	AStar	44
4.2	MinimumSpanningTree	45
4.2.1	Kruskal	45
4.2.2	Prim	46
4.3	NetFlow	47
4.3.1	Dinic	47
4.3.2	FordFulkerson	48
4.3.3	MaxFlow	49
4.3.4	MinCostMaxFlow	50
4.4	ShortestPath	51
4.4.1	BellmanFord	51
4.4.2	Dijkstra	52
4.4.3	Floyd	53
4.4.4	SPFA	53
5	DynamicProgramming	55
5.1	Contour	55
5.2	Digit	55
5.3	LCS	56
5.4	LIS	56
5.5	Pack	57
6	Geometry	58
6.1	DynamicConvexhull	58
6.2	JlsGeo	59
6.3	Plane	71
6.4	Simpson	77
6.5	Stereoscopic	77
7	Others	80
7.1	FastIO	80
7.2	LeapYear	83
7.3	MoAlgorithm	83
7.3.1	Dynamic	83
7.3.2	Static	85
7.4	vim	86

1 String

1.1 AhoCorasickAutomaton

```
const int maxn = "Edit";

class AhoCorasickAutomaton {
public:
    int son[maxn][26];
    int val[maxn];
    int fail[maxn];
    int tot;

    // Trie Tree 初始化
    void TrieInit() {
        tot = 0;
        memset(son, 0, sizeof(son));
        memset(val, 0, sizeof(val));
        memset(fail, 0, sizeof(fail));
    }

    // 计算字母下标
    int Pos(char x) {
        return x - 'a';
    }

    // 向 Trie Tree 中插入 Str 模式字符串
    void Insert(string str) {
        int cur = 0, Len = int(str.length());
        for (int i = 0; i < Len; ++i) {
            int index = Pos(str[i]);
            if (!son[cur][index]) son[cur][index] = ++tot;
            cur = son[cur][index];
        }
        val[cur]++;
    }

    // Bfs 求得 Trie Tree 上失配指针
    void GetFail() {
        std::queue<int> que;
        for (int i = 0; i < 26; ++i) {
            if (son[0][i]) {
                fail[son[0][i]] = 0;
                que.push(son[0][i]);
            }
        }
        while (!que.empty()) {
            int cur = que.front(); que.pop();
            for (int i = 0; i < 26; ++i) {
                if (son[cur][i]) {

```

```
        fail[son[cur][i]] = son[fail[cur]][i];
        que.push(son[cur][i]);
    }
    else son[cur][i] = son[fail[cur]][i];
}
}
}

// 询问 Str 中出现的模式串数量
int Query(string str) {
    int len = int(str.length());
    int cur = 0, ret = 0;
    for (int i = 0; i < len; ++i) {
        cur = son[cur][Pos(str[i])];
        for (int j = cur; j && ~val[j]; j = fail[j]) {
            ret += val[j];
            val[j] = -1;
        }
    }
    return ret;
}
};
```

1.2 KMP

```
// 对模式串 pattern 计算 next 数组
void KMPPre(std::string pattern, vector<int> &next) {
    int i = 0, j = -1;
    next[0] = -1;
    int len = (int)pattern.length();
    while (i != len) {
        if (j == -1 || pattern[i] == pattern[j]) next[++i] = ++j;
        else j = next[j];
    }
}

// 优化对模式串 pattern 计算 next 数组
void PreKMP(std::string pattern, vector<int> &next) {
    int i, j;
    i = 0;
    j = next[0] = -1;
    int len = (int)pattern.length();
    while (i < len) {
        while (j != -1 && pattern[i] != pattern[j]) j = next[j];
        if (pattern[++i] == pattern[++j]) next[i] = next[j];
        else next[i] = j;
    }
}

// 利用预处理 next 数组计数模式串 pattern 在主串 main 中出现次数
```

```
int KMPCount(std::string pattern, std::string main) {
    int pattern_len = (int)pattern.length(), main_len = (int)main.length();
    vector<int> next(pattern_len + 1, 0);
    //PreKMP(pattern, next);
    KMPPre(pattern, next);
    int i = 0, j = 0;
    int ret = 0;
    while (i < main_len) {
        while (j != -1 && main[i] != pattern[j]) j = next[j];
        i++; j++;
        if (j >= pattern_len) {
            ret++;
            j = next[j];
        }
    }
    return ret;
}
```

1.3 Manacher

```
const int maxn = "Edit";

char convert_str[maxn << 1];
int len[maxn << 1];

// Manacher 算法求 Str 字符串最长回文子串长度
int Manacher(char Str[]) {
    int L = 0, str_len = (int)strlen(Str);
    convert_str[L++] = '$'; convert_str[L++] = '#';
    for (int i = 0; i < str_len; ++i) {
        convert_str[L++] = Str[i];
        convert_str[L++] = '#';
    }
    int mx = 0, id = 0, ret = 0;
    for (int i = 0; i < L; ++i) {
        len[i] = mx > i ? std::min(len[2 * id - i], mx - i) : 1;
        while (convert_str[i + len[i]] == convert_str[i - len[i]]) len[i]++;
        if (i + len[i] > mx) {
            mx = i + len[i];
            id = i;
        }
        ret = std::max(ret, len[i] - 1);
    }
    return ret;
}
```

1.4 PalindromicTree

```
const int maxn = "Edit";
```

```
class PalindromicTree {
public:
    // 子节点记录数组
    long long son[maxn][26];
    // 失配指针 Fail 数组
    long long fail[maxn];
    // len[i]: 节点 i 表示的回文串长度 (一个节点表示一个回文串)
    long long len[maxn];
    // cnt[i]: 节点 i 表示的本质不同的串的个数 (最后需要运行 Count() 函数才可求出正确
    // 结果)
    long long cnt[maxn];
    // num[i]: 以节点 i 表示的最长回文串的最右端为回文串结尾的回文串个数
    long long num[maxn];
    // 字符
    long long str[maxn];
    // 新添加字符后最长回文串表示的节点
    long long last;
    // 字符数量
    long long str_len;
    // 节点数量
    long long tot;

    // 新建节点
    long long NewNode(long long x) {
        for (long long i = 0; i < 26; ++i) son[tot][i] = 0;
        cnt[tot] = 0;
        num[tot] = 0;
        len[tot] = x;
        return tot++;
    }

    // 初始化
    void Init() {
        tot = 0;
        NewNode(0); NewNode(-1);
        last = 0;
        str_len = 0;
        // 开头存字符集中没有的字符, 减少特判
        str[0] = -1;
        fail[0] = 1;
    }

    long long GetFail(long long x) {
        while (str[str_len - len[x] - 1] != str[str_len]) x = fail[x];
        return x;
    }

    void Add(long long c) {
        c -= 'a';
```

```
Str[++str_len] = c;
long long Cur = GetFail(last);
if (!son[Cur][c]) {
    long long New = NewNode(len[Cur] + 2);
    fail[New] = son[GetFail(fail[Cur])][c];
    son[Cur][c] = New;
    num[New] = num[fail[New]] + 1;
}
last = son[Cur][c];
cnt[last]++;
}

void Count() {
    // 若 fail[V]=U, 则 U 一定是 V 回文子串, 所以双亲累加孩子的 cnt
    for (long long i = tot - 1; i >= 0; --i) cnt[fail[i]] += cnt[i];
}

};
```

1.5 TrieTree

```
const int maxn = "Edit";

class Trie {
public:
    int son[maxn][26];
    int tot;
    int cnt[maxn];

    void TrieInit() {
        tot = 0;
        memset(cnt, 0, sizeof(cnt));
        memset(son, 0, sizeof(son));
    }

    int Pos(char x) {
        return x - 'a';
    }

    // 向 Trie Tree 中插入字符串 Str
    void Insert(string str) {
        int cur = 0, len = int(str.length());
        for (int i = 0; i < len; ++i) {
            int index = Pos(str[i]);
            if (!son[cur][index]) son[cur][index] = ++tot;
            cur = son[cur][index];
            cnt[cur]++;
        }
    }

    // 查找字符串 str, 存在返回 true, 不存在返回 false
};
```



```
bool Find(string str) {
    int cur = 0, len = int(str.length());
    for (int i = 0; i < len; ++i) {
        int index = Pos(str[i]);
        if (!son[cur][index]) return false;
        cur = son[cur][index];
    }
    return true;
}

// 查询字典树中以 str 为前缀的字符串数量
int PathCnt(string str) {
    int cur = 0, len = int(str.length());
    for (int i = 0; i < len; ++i) {
        int index = Pos(Str[i]);
        if (!son[cur][index]) return 0;
        cur = son[cur][index];
    }
    return cnt[cur];
}

};
```

2 Math

2.1 Catalan

```
const int maxn = "Edit";

long long catalan[maxn];

void GetCalalan() {
    memset(catalan, 0, sizeof(catalan));
    catalan[0] = catalan[1] = 1;
    for (int i = 2; i < maxn; ++i) catalan[i] = Catalan[i - 1] * (4 * i - 2) / (i +
↪ 1);
}
```

2.2 CombinatorialNumber

```
const int mod = "Edit";
const int maxn = "Edit";

int c[maxn][maxn];

void GetC() {
    memset(c, 0, sizeof(c));
    c[0][0] = 1;
    for (int i = 1; i < maxn; ++i) {
        c[i][0] = 1;
        for (int j = 1; j <= i; ++j) {
            if (j == i) c[i][j] = 1;
            else c[i][j] = (c[i - 1][j - 1] + c[i - 1][j]) % mod;
        }
    }
}
```

2.3 Derangement

```
const int maxn = "Edit";
const int mod = "Edit";

long long staggered[maxn];

// 错排
void GetStaggered() {
    staggered[1] = 0; staggered[2] = 1;
    for (int i = 3; i < maxn; ++i) staggered[i] = (i - 1) * (staggered[i - 1] +
↪ staggered[i - 2]) % mod;
}
```

2.4 Euler

2.4.1 Euler

```
int GetPhi(int x) {
    int ret = x;
    for (int i = 2; i * i <= x; ++i) {
        if (!(x % i)) {
            ret = ret / i * (i - 1);
            while (!(x % i)) x /= i;
        }
    }
    if (x > 1) ret = ret / x * (x - 1);
    return ret;
}
```

2.4.2 Screen

```
const int maxn = "Edit";

int phi[maxn];

void Euler() {
    for (int i = 1; i < maxn; ++i) phi[i] = i;
    for (int i = 2; i < maxn; i += 2) phi[i] /= 2;
    for (int i = 3; i < maxn; i += 2)
        if (phi[i] == i)
            for (int j = i; j < maxn; j += i) phi[j] = phi[j] / i * (i - 1);
}
```

2.4.3 Sieve

```
const int maxn = "Edit";

bool is_prime[maxn];
int phi[maxn];
std::vector<int> prime;

void Sieve() {
    memset(is_prime, true, sizeof(is_prime));
    phi[1] = 1; is_prime[0] = is_prime[1] = false;
    for (long long i = 2; i < maxn; ++i) {
        if (is_prime[i]) {
            phi[i] = i - 1;
            prime.emplace_back(i);
        }
        for (auto &p : prime) {
            if (p * i >= maxn) break;
            is_prime[i * p] = false;
            if (i % p == 0) {
                phi[i * p] = phi[i] * p;
            }
        }
    }
}
```

```
        break;
    }
    phi[i * p] = phi[i] * phi[p];
}
}
}
```

2.5 FFT

```
const int maxn = "Edit";
const double pi = acos(-1.0);
```

// 复数

```
struct complex {
    double x, y;
    complex operator + (const complex &b) const {return complex {x + b.x, y + b.y};}
    complex operator - (const complex &b) const {return complex {x - b.x, y - b.y};}
    complex operator * (const complex &b) const {return complex {x * b.x - y * b.y, x
↵ * b.y + y * b.x};}
    complex operator / (const complex &b) const {
        double tmp = b.x * b.x + b.y * b.y;
        return complex {(x * b.x + y * b.y) / tmp, (y * b.x - x * b.y) / tmp};
    }
};
```

// 多项式系数数量

```
int n, m;
int l;
int limit;
int r[maxn << 2];
```

// 快速傅里叶变换 (FFT)

```
void FFT(complex f[], int op) {
    for (int i = 0; i < limit; ++i) {
        if (i < r[i]) std::swap(f[i], f[r[i]]);
    }
    for (int j = 1; j < limit; j <= 1) {
        complex tmp = complex {cos(pi / j), op * sin(pi / j)};
        for (int k = 0; k < limit; k += (j < 1)) {
            complex Buffer = complex {1.0, 0.0};
            for (int l = 0; l < j; ++l) {
                complex tx = f[k + l], ty = Buffer * f[k + j + l];
                f[k + l] = tx + ty;
                f[k + j + l] = tx - ty;
                Buffer = Buffer * tmp;
            }
        }
    }
}
```

```
complex a[maxn], b[maxn];
```

```
// 多项式卷积计算
```

```
void Cal() {  
    limit = 1; l = 0;  
    while (limit <= n + m) {  
        limit <<= 1;  
        l++;  
    }  
    for (int i = 0; i < limit; ++i) r[i] = (r[i >> 1] >> 1) | ((i & 1) << (l - 1));  
    FFT(a, 1);  
    FFT(b, 1);  
    for (int i = 0; i <= limit; ++i) a[i] = a[i] * b[i];  
    FFT(a, -1);  
}
```

2.6 Fibonacci

```
const int mod = "Edit";
```

```
struct matrix {long long mat[2][2];
```

```
matrix operator * (matrix &k1, matrix &k2) const {  
    matrix ret;  
    memset(ret.mat, 0, sizeof(ret.mat));  
    for (int i = 0; i < 2; ++i) {  
        for (int j = 0; j < 2; ++j) {  
            for (int k = 0; k < 2; ++k) {  
                ret.mat[i][j] = (ret.mat[i][j] + k1.mat[i][k] * k2.mat[k][j] % mod) %  
                    mod;  
            }  
        }  
    }  
    return ret;  
}
```

```
// 矩阵快速幂
```

```
matrix operator ^ (matrix base, long long k) {  
    matrix ret;  
    memset(ret.mat, 0, sizeof(ret.mat));  
    ret.mat[0][0] = ret.mat[1][1] = 1;  
    while (k) {  
        if (k & 1) ret = ret * base;  
        base = base * base;  
        k >>= 1;  
    }  
    return ret;  
}
```

```
// 斐波那契数列中第  $x$  项
```

```
long long Fib(long long x) {
    matrix base;
    base.mat[0][0] = base.mat[1][0] = base.mat[0][1] = 1;
    base.mat[1][1] = 0;
    return (base ^ x).mat[0][1];
}
```

2.7 GeneratingFunction

```
const int maxn = "Edit";

void GetGeneratingFunction() {
    int n;
    int c1[maxn], c2[maxn];
    scanf("%d", &n);
    for (int i = 0; i < maxn; ++i) {
        c1[i] = 1;
        c2[i] = 0;
    }
    // c1[i] 为  $x^i$  的系数
    // c2 为中间变量
    for (int i = 2; i <= n; ++i) {
        for (int j = 0; j <= n; ++j) {
            for (int k = 0; k + j <= n; k += i) {
                c2[j + k] += c1[i];
            }
        }
        for (int j = 0; j <= n; ++j) {
            c1[j] = c2[j];
            c2[j] = 0;
        }
    }
}
```

2.8 InverseElement

2.8.1 ExtendGcd

```
const int mod = "Edit";

// 扩展欧几里得,  $ax+by=d$ 
long long ExtendGcd(long long a, long long b, long long &x, long long &y) {
    if (a == 0 && b == 0) return -1;
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    long long d = ExtendGcd(b, a % b, y, x);
    y -= a / b * x;
    return d;
}
```

```
}

// 逆元,  $ax = 1 \pmod{\text{mod}}$ 
long long GetInv(long long a) {
    long long x, y;
    long long d = ExtendGcd(a, mod, x, y);
    if (d == 1) return (x % mod + mod) % mod;
    else return -1;
}
```

2.8.2 Factorial

```
const int mod = "Edit";
const int maxn = "Edit";

// fac: 阶乘, facinv: 阶乘逆元
long long fac[maxn], facinv[maxn];

void GetFacInv() {
    fac[0] = 1; fac[1] = 1;
    for (int i = 2; i < maxn; ++i) fac[i] = (fac[i - 1] * i) % mod;
    facinv[maxn - 1] = Pow(fac[maxn - 1], mod - 2);
    for (int i = maxn - 2; i >= 0; --i) facinv[i] = (facinv[i + 1] * (i + 1)) % mod;
}
```

2.8.3 FermatLittleTheorem

```
const int mod = "Edit";

long long Inv(long long x) {
    return Pow(x, mod - 2);
}
```

2.8.4 Recursive

```
const int mod = "Edit";
const int maxn = "Edit";

long long inv[maxn];

// 递推求逆元
void GetInv() {
    inv[1] = 1;
    for (int i = 2; i < maxn; ++i) inv[i] = (mod - mod / i) * inv[mod % i] % mod;
}
```

2.9 Mobius

```
#include <bits/stdc++.h>

const int maxn = "Edit";
```

```
bool is_prime[maxn];
std::vector<int> prime;
int mu[maxn];

void Sieve() {
    memset(is_prime, true, sizeof(is_prime));
    mu[1] = 1; is_prime[0] = is_prime[1] = false;
    for (int i = 2; i < maxn; ++i) {
        if (is_prime[i]) {
            prime.emplace_back(i);
            mu[i] = -1;
        }
        for (auto &it : prime) {
            if (it * i >= maxn) break;
            is_prime[i * it] = false;
            if (i % it == 0) {
                mu[i * it] = 0;
                break;
            }
            mu[i * it] = -mu[i];
        }
    }
}
```

2.10 NimGame

```
// ret 不为零则先手赢，否则为后手赢
bool GetNim(std::vector<int> arr) {
    int ret = 0;
    for (auto &v : arr) ret ^= v;
    return ret != 0;
}
```

2.11 Prime

2.11.1 PrimeFactor

```
const int maxn = "Edit"

bool is_prime[maxn];
vector<int> prime_fac[maxn];

void GetPrimeFac() {
    memset(is_prime, true, sizeof(is_prime));
    for (long long i = 2; i < maxn; ++i) {
        if (is_prime[i]) {
            prime_fac[i].push_back(i);
            for (long long j = i + i; j < maxn; ++j) {
                is_prime[j] = false;
                prime_fac[j].push_back(i);
            }
        }
    }
}
```



```
    }  
  }  
}  
is_prime[1] = false;  
}
```

2.11.2 SieveOfEratosthenes

```
const int maxn = "Edit";  
  
bool is_prime[maxn];  
std::vector<int> prime  
  
void Sieve() {  
    memset(is_prime, true, sizeof(is_prime));  
    is_prime[0] = is_prime[1] = false;  
    for (long long i = 2; i < maxn; ++i) {  
        if (is_prime[i]) prime.emplace_back(i);  
        for (auto &p : prime) {  
            if (p * i >= maxn) break;  
            is_prime[i * p] = false;  
        }  
    }  
}
```

2.12 QuickPow

```
const int mod = "edit";  
  
int Mul(int x, int y) {  
    int ret = 0;  
    while (y) {  
        if (y & 1) ret = (ret + x) % mod;  
        x = (x + x) % mod;  
        y >>= 1;  
    }  
    return ret;  
}  
  
int Pow(int x, int n) {  
    int ret = 1;  
    while (n) {  
        if (n & 1) ret = (ret * x) % mod;  
        x = 1ll * x * x % mod;  
        n >>= 1;  
    }  
    return ret;  
}
```

2.13 Stirling

```
const double pi = acos(-1.0);
const double e = 2.718281828459;

int GetStirling(int x) {
    if (x <= 1) return 1;
    return int(ceil(log10(2 * pi * x) / 2 + x * log10(x / e)));
}
```

3 DataStructure

3.1 BinaryIndexedTree

```
#define lowbit(x) (x&(-x))
const int maxn = "Edit";

class binary_indexed_tree {
public:
    int arr[maxn];

    void Update(int x, int v) {
        while (x < maxn) {
            arr[x] += v;
            x += lowbit(x);
        }
    }

    int GetSum(int x) {
        int ans = 0;
        while (x > 0) {
            ans += arr[x];
            x -= lowbit(x);
        }
    }
};
```

3.2 DfsOrder

```
std::vector<std::vector<int>>> g;
int dfs_clock;
std::vector<int> in, out;

// Dfs 序
void DfsOrder(int cur, int pre) {
    in[cur] = ++dfs_clock;
    for (auto &it : g) {
        if (it == pre) continue;
        DfsOrder(it, cur);
    }
    out[cur] = dfs_clock;
}
```

3.3 FunctionalSegmentTree

```
// 主席树, 静态区间第 k 小
const int maxn = "Edit";

class FuncSegTree {
public:
    int tot;
```

```
int rt[maxn];
int lson[maxn << 5], rson[maxn << 5];
int cnt[maxn << 5];

int Build(int l, int r) {
    int t = ++tot;
    int m = (l + r) >> 1;
    if (l != r) {
        lson[t] = Build(l, m);
        rson[t] = Build(m + 1, r);
    }
    return t;
}

int Modify(int prev, int l, int r, int x) {
    int t = ++tot;
    lson[t] = lson[prev]; rson[t] = rson[prev];
    cnt[t] = cnt[prev] + 1;
    int m = (l + r) >> 1;
    if (l != r) {
        if (x <= m) lson[t] = Modify(lson[t], l, m, x);
        else rson[t] = Modify(rson[t], m + 1, r, x);
    }
    return t;
}

int Query(int u, int v, int l, int r, int k) {
    if (l == r) return l;
    int m = (l + r) >> 1;
    int num = cnt[lson[v]] - cnt[lson[u]];
    if (num >= k) return Query(lson[u], lson[v], l, m, k);
    return Query(rson[u], rson[v], m + 1, r, k - num);
}
};
```

3.4 Hash

```
template <typename type>
class Hash {
public:
    int size;
    vector<int> arr;

    Hash(const vector<type> &v) {
        arr.assign(v.begin(), v.end());
        sort(arr.begin(), arr.end());
        arr.erase(unique(arr.begin(), arr.end()), arr.end());
        size = arr.size();
    }
};
```

```
    int Get(type k) {
        return lower_bound(arr.begin(), arr.end(), k) - arr.begin();
    }
};
```

3.5 LCA

3.5.1 DFS+ST

```
const int maxn = "Edit";

struct edge {int v, c, next;};

edge edges[maxn << 1];
int head[maxn];
int tot;

void AddEdge(int u, int v, int c) {
    edges[tot] = edge {v, c, head[u]};
    head[u] = tot++;
}

namespace LCA {
    // 节点深度
    int rmq[maxn << 1];
    // 深搜遍历顺序
    int vertex[maxn << 1];
    // 节点在深搜中第一次出现的位置
    int first[maxn];
    int fa[maxn];
    int dis[maxn];
    int lca_tot;

    // 最小值对应下标
    int dp[maxn << 1][20];

    // rmq 初始化
    void Work(int n) {
        for (int i = 1; i <= n; ++i) dp[i][0] = i;
        for (int j = 1; (1 << j) <= n; ++j) {
            for (int i = 1; i + (1 << j) - 1 <= n; ++i) {
                dp[i][j] = rmq[dp[i][j - 1]] < rmq[dp[i + (1 << (j - 1))][j - 1]] ? dp[i][j - 1] : dp[i + (1 << (j - 1))][j - 1];
            }
        }
    }

    // 深搜
    void Dfs(int cur, int pre, int dep) {
        vertex[++lca_tot] = cur;
```

```
first[cur] = lca_tot;
rmq[lca_tot] = dep;
fa[cur] = pre;
for (int i = head[cur]; ~i; i = edges[i].next) {
    if (edges[i].v == pre) continue;
    dis[edges[i].v] = dis[cur] + edges[i].c;
    Dfs(edges[i].v, cur, dep + 1);
    vertex[++lca_tot] = cur;
    rmq[lca_tot] = dep;
}
}

// rmq 查询
int Query(int l, int r) {
    if (l > r) swap(l, r);
    int len = int(log2(r - l + 1));
    return rmq[dp[l][len]] <= rmq[dp[r - (1 << len) + 1][len]] ? dp[l][len] : dp[r - (1 << len) + 1][len];
}

// LCA 初始化
void Init(int rt, int num) {
    memset(dis, 0, sizeof(dis));
    lca_tot = 0;
    Dfs(rt, 0, 0);
    fa[1] = 0;
    Work(2 * num - 1);
}

// 查询节点 u、v 的距离
int GetDis(int u, int v) {
    return dis[u] + dis[v] - 2 * dis[LCA(u, v)];
}

// 查询节点 u、v 的最近公共祖先 (LCA)
int GetLCA(int u, int v) {
    return vertex[Query(first[u], first[v])];
}
}
```

3.5.2 Multiplication

```
const int maxn = "Edit";

namespace LCA {
    int n, k;
    std::vector<int> g[maxn];

    void AddEdge(int u, int v) {
        g[u].push_back(v);
    }
}
```

```
    g[v].push_back(u);
}

int anc[maxn][25];
int dep[maxn];

void Dfs(int u, int prev, int depth) {
    anc[u][0] = prev; dep[u] = depth;
    for (auto &v : g[u]) {
        if (v == prev) continue;
        Dfs(v, u, depth + 1);
    }
}

void Init(int rt) {
    Dfs(rt, 0, 1);
    for (int j = 1; j < k; ++j) {
        for (int i = 1; i <= n; ++i) {
            anc[i][j] = anc[anc[i][j - 1]][j - 1];
        }
    }
}

void Swim(int &u, int h) {
    for (int i = 0; h > 0; ++i) {
        if (h & 1) u = anc[u][i];
        h >>= 1;
    }
}

int Query(int u, int v) {
    if (dep[u] < dep[v]) std::swap(u, v);
    Swim(u, dep[u] - dep[v]);
    if (u == v) return u;
    for (int i = k - 1; i >= 0; --i) {
        if (anc[u][i] != anc[v][i]) {
            u = anc[u][i];
            v = anc[v][i];
        }
    }
    return anc[u][0];
}
```

3.5.3 Tarjan

```
const int maxn = "Edit";

struct edge {int v, next;};
struct query {int q, next, index;};
```

```
int pre[maxn << 2];
edge g[maxn << 2];
int head[maxn];
int tot;
query qg[maxn << 2];
int qhead[maxn];
int qtot;
int vis[maxn];
int anc[maxn];
int ans[maxn];

int Find(int x) {return pre[x] == x ? x : pre[x] = Find(pre[x]);}
void Union(int x, int y) {pre[Find(x)] = Find(y);}

void AddEdge(int u, int v) {
    g[tot] = edge {v, head[u]};
    head[u] = tot++;
}

// 添加询问
void AddQuery(int u, int v, int index) {
    qg[qtot] = query {v, qhead[u], index};
    qhead[u] = qtot++;
    qg[qtot] = query {u, qhead[v], index};
    qhead[v] = qtot++;
}

// 初始化
void Init() {
    tot = 0;
    memset(head, -1, sizeof(head));
    qtot = 0;
    memset(qhead, -1, sizeof(qhead));
    memset(vis, false, sizeof(vis));
    memset(pre, -1, sizeof(pre));
    memset(anc, 0, sizeof(anc));
    for (int i = 0; i <= n; ++i) pre[i] = i;
}

// LCA 离线 Tarjan 算法
void Tarjan(int u) {
    anc[u] = u;
    vis[u] = true;
    for (int i = head[u]; ~i; i = g[i].next) {
        if (vis[g[i].v]) continue;
        Tarjan(g[i].v);
        Join(u, g[i].v);
        anc[Find(u)] = u;
    }
}
```



```
    }  
    for (int i = qhead[u]; ~i; i = qg[i].next) {  
        if (vis[qg[i].q]) ans[qg[i].index] = anc[Find(qg[i].q)];  
    }  
}
```

3.6 MultipleTree

```
/*  
    BZOJ 3196 (线段树套伸展树)  
    1. 查询  $k$  在区间内的排名  
    2. 查询区间内排名为  $k$  的值  
    3. 修改某一位值上的数值  
    4. 查询  $k$  在区间内的前驱 (前驱定义为小于  $x$ , 且最大的数)  
    5. 查询  $k$  在区间内的后继 (后继定义为大于  $x$ , 且最小的数)  
*/  
#include <bits/stdc++.h>  
using namespace std;  
const int inf = 2147483647;  
const int maxn = 5e4 + 5;  
const int maxm = maxn * 25;  
  
int n;  
int arr[maxn];  
  
namespace SplayTree {  
    int rt[maxm], tot;  
    int fa[maxm], son[maxm][2];  
    int val[maxm], cnt[maxm];  
    int sz[maxm];  
  
    void Push(int o) {  
        sz[o] = sz[son[o][0]] + sz[son[o][1]] + cnt[o];  
    }  
  
    bool Get(int o) {  
        return o == son[fa[o]][1];  
    }  
  
    void Clear(int o) {  
        son[o][0] = son[o][1] = fa[o] = val[o] = sz[o] = cnt[o] = 0;  
    }  
  
    void Rotate(int o) {  
        int p = fa[o], q = fa[p], ck = Get(o);  
        son[p][ck] = son[o][ck ^ 1];  
        fa[son[o][ck ^ 1]] = p;  
        son[o][ck ^ 1] = p;  
        fa[p] = o; fa[o] = q;  
        if (q) son[q][p == son[q][1]] = o;  
    }  
}
```

```
    Push(p); Push(o);
}

void Splay(int &root, int o) {
    for (int f = fa[o]; (f = fa[o]); Rotate(o))
        if (fa[f]) Rotate(Get(o) == Get(f) ? f : o);
    root = o;
}

void Insert(int &root, int x) {
    if (!root) {
        val[++tot] = x;
        cnt[tot]++;
        root = tot;
        Push(root);
        return;
    }
    int cur = root, f = 0;
    while (true) {
        if (val[cur] == x) {
            cnt[cur]++;
            Push(cur); Push(f);
            Splay(root, cur);
            break;
        }
        f = cur;
        cur = son[cur][val[cur] < x];
        if (!cur) {
            val[++tot] = x;
            cnt[tot]++;
            fa[tot] = f;
            son[f][val[f] < x] = tot;
            Push(tot); Push(f);
            Splay(root, tot);
            break;
        }
    }
}

int GetRank(int &root, int x) {
    int ans = 0, cur = root;
    while (cur) {
        if (x < val[cur]) {
            cur = son[cur][0];
            continue;
        }
        ans += sz[son[cur][0]];
        if (x == val[cur]) {
            Splay(root, cur);
        }
    }
}
```

```
        return ans;
    }
    if (x > val[cur]) {
        ans += cnt[cur];
        cur = son[cur][1];
    }
}
return ans;
}

int GetKth(int &root, int k) {
    int cur = root;
    while (true) {
        if (son[cur][0] && k <= sz[son[cur][0]]) cur = son[cur][0];
        else {
            k -= cnt[cur] + sz[son[cur][0]];
            if (k <= 0) return cur;
            cur = son[cur][1];
        }
    }
}

int Find(int &root, int x) {
    int ans = 0, cur = root;
    while (cur) {
        if (x < val[cur]) {
            cur = son[cur][0];
            continue;
        }
        ans += sz[son[cur][0]];
        if (x == val[cur]) {
            Splay(root, cur);
            return ans + 1;
        }
        ans += cnt[cur];
        cur = son[cur][1];
    }
}

int GetPrev(int &root) {
    int cur = son[root][0];
    while (son[cur][1]) cur = son[cur][1];
    return cur;
}

int GetPrevVal(int &root, int x) {
    int ans = -inf, cur = root;
    while (cur) {
        if (x > val[cur]) {
            ans = max(ans, val[cur]);
        }
    }
}
```

```
        cur = son[cur][1];
        continue;
    }
    cur = son[cur][0];
}
return ans;
}

int GetNext(int &root) {
    int cur = son[root][1];
    while (son[cur][0]) cur = son[cur][0];
    return cur;
}

int GetNextVal(int &root, int x) {
    int ans = inf, cur = root;
    while (cur) {
        if (x < val[cur]) {
            ans = min(ans, val[cur]);
            cur = son[cur][0];
            continue;
        }
        cur = son[cur][1];
    }
    return ans;
}

void Delete(int &root, int x) {
    Find(root, x);
    if (cnt[root] > 1) {
        cnt[root]--;
        Push(root);
        return;
    }
    if (!son[root][0] && !son[root][1]) {
        Clear(root);
        root = 0;
        return;
    }
    if (!son[root][0]) {
        int cur = root;
        root = son[root][1];
        fa[root] = 0;
        Clear(cur);
        return;
    }
    if (!son[root][1]) {
        int cur = root;
        root = son[root][0];
        fa[root] = 0;
    }
}
```

```
    Clear(cur);
    return;
}
int p = GetPrev(root), cur = root;
Splay(root, p);
fa[son[cur][1]] = p;
son[p][1] = son[cur][1];
Clear(cur);
Push(root);
}
};

namespace SegTree {
    int tree[maxn << 2];

    void Build(int o, int l, int r) {
        for (int i = l; i <= r; ++i) SplayTree::Insert(tree[o], arr[i - 1]);
        if (l == r) return;
        int m = (l + r) >> 1;
        Build(o << 1, l, m);
        Build(o << 1 | 1, m + 1, r);
    }

    void Modify(int o, int l, int r, int ll, int rr, int u, int v) {
        SplayTree::Delete(tree[o], u); SplayTree::Insert(tree[o], v);
        if (l == r) return;
        int m = (l + r) >> 1;
        if (ll <= m) Modify(o << 1, l, m, ll, rr, u, v);
        if (rr > m) Modify(o << 1 | 1, m + 1, r, ll, rr, u, v);
    }

    int QueryRank(int o, int l, int r, int ll, int rr, int v) {
        if (ll <= l && rr >= r) return SplayTree::GetRank(tree[o], v);
        int m = (l + r) >> 1, ans = 0;
        if (ll <= m) ans += QueryRank(o << 1, l, m, ll, rr, v);
        if (rr > m) ans += QueryRank(o << 1 | 1, m + 1, r, ll, rr, v);
        return ans;
    }

    int QueryPrev(int o, int l, int r, int ll, int rr, int v) {
        if (ll <= l && rr >= r) return SplayTree::GetPrevVal(tree[o], v);
        int m = (l + r) >> 1, ans = -inf;
        if (ll <= m) ans = max(ans, QueryPrev(o << 1, l, m, ll, rr, v));
        if (rr > m) ans = max(ans, QueryPrev(o << 1 | 1, m + 1, r, ll, rr, v));
        return ans;
    }

    int QueryNext(int o, int l, int r, int ll, int rr, int v) {
        if (ll <= l && rr >= r) return SplayTree::GetNextVal(tree[o], v);
```

```
    int m = (l + r) >> 1, ans = inf;
    if (ll <= m) ans = min(ans, QueryNext(o << 1, l, m, ll, rr, v));
    if (rr > m) ans = min(ans, QueryNext(o << 1 | 1, m + 1, r, ll, rr, v));
    return ans;
}

int QueryKth(int ll, int rr, int v) {
    int l = 0, r = 1e8 + 10;
    while (l < r) {
        int m = ((l + r) >> 1) + 1;
        if (QueryRank(1, 1, n, ll, rr, m) < v) l = m;
        else r = m - 1;
    }
    return l;
}

};

int main() {
    ios::sync_with_stdio(false); cin.tie(0); cout.tie(0);
    int m; cin >> n >> m;
    for (int i = 0; i < n; ++i) cin >> arr[i];
    SplayTree::tot = 0;
    SegTree::Build(1, 1, n);
    for (int i = 0, op, l, r, pos, k; i < m; ++i) {
        cin >> op;
        if (op == 1) {
            cin >> l >> r >> k;
            cout << SegTree::QueryRank(1, 1, n, l, r, k) + 1 << endl;
        }
        else if (op == 2) {
            cin >> l >> r >> k;
            cout << SegTree::QueryKth(l, r, k) << endl;
        }
        else if (op == 3) {
            cin >> pos >> k;
            SegTree::Modify(1, 1, n, pos, pos, arr[pos - 1], k);
            arr[pos - 1] = k;
        }
        else if (op == 4) {
            cin >> l >> r >> k;
            cout << SegTree::QueryPrev(1, 1, n, l, r, k) << endl;
        }
        else if (op == 5) {
            cin >> l >> r >> k;
            cout << SegTree::QueryNext(1, 1, n, l, r, k) << endl;
        }
    }
    return 0;
}
```

3.7 STTable

```
template <typename type>
class STTable {
public:
    std::vector<std::vector<type>> max, min;

    STTable(const std::vector<type> &arr) {
        int n = (int)arr.size(), m = log2(n) + 1;
        max = min = std::vector<std::vector<type>>(n, std::vector<type>(m, 0));
        for (int i = 0; i < n; ++i) max[i][0] = min[i][0] = arr[i];
        for (int j = 1; j < m; ++j) {
            for (int i = 0; i + (1 << j) - 1 < n; ++i) {
                max[i][j] = std::max(max[i][j - 1], max[i + (1 << (j - 1))][j - 1]);
                min[i][j] = std::min(min[i][j - 1], min[i + (1 << (j - 1))][j - 1]);
            }
        }
    }

    type QueryMax(int l, int r) {
        int k = log2(r - l + 1);
        return std::max(max[l][k], max[r - (1 << k) + 1][k]);
    }

    type QueryMin(int l, int r) {
        int k = log2(r - l + 1);
        return std::min(min[l][k], min[r - (1 << k) + 1][k]);
    }
};
```

3.8 SegmentTree

3.8.1 AreaCombination

```
// HDU 1542 矩形面积并
#include <bits/stdc++.h>
typedef double db;
const int maxn = 1e2 + 5;
const db eps = 1e-9;

int Sgn(db k) {return fabs(k) < eps ? 0 : (k < 0 ? -1 : 1);}
int Cmp(db k1, db k2) {return Sgn(k1 - k2);}

struct seg {
    db l, r, h;
    int flag;
};

bool operator < (seg &k1, seg &k2) {return Cmp(k1.h, k2.h) < 0;}
std::vector<seg> segs;

std::vector<db> pos;
```

```
int BinarySearch(db k) {
    int ret = (int)pos.size() - 1, l = 0, r = (int)pos.size() - 1;
    while (l <= r) {
        int m = (l + r) >> 1;
        if (Cmp(pos[m], k) >= 0) {
            ret = m;
            r = m - 1;
        }
        else l = m + 1;
    }
    return ret;
}

struct node {
    int l, r, cnt;
    db len;
};
node seg_tree[maxn << 4];

void PushUp(int o) {
    if (seg_tree[o].cnt) seg_tree[o].len = pos[seg_tree[o].r + 1] -
        pos[seg_tree[o].l];
    else if (seg_tree[o].l == seg_tree[o].r) seg_tree[o].len = 0.0;
    else seg_tree[o].len = seg_tree[o << 1].len + seg_tree[o << 1 | 1].len;
}

void Build(int l, int r, int o) {
    seg_tree[o].l = l; seg_tree[o].r = r;
    seg_tree[o].cnt = 0; seg_tree[o].len = 0.0;
    if (l == r) return;
    int Mid = (l + r) >> 1;
    Build(l, Mid, o << 1);
    Build(Mid + 1, r, o << 1 | 1);
    PushUp(o);
}

void Update(int l, int r, int v, int o) {
    if (l <= seg_tree[o].l && r >= seg_tree[o].r) {
        seg_tree[o].cnt += v;
        PushUp(o);
        return;
    }
    int Mid = (seg_tree[o].l + seg_tree[o].r) >> 1;
    if (r <= Mid) Update(l, r, v, o << 1);
    else if (l > Mid) Update(l, r, v, o << 1 | 1);
    else {
        Update(l, Mid, v, o << 1);
        Update(Mid + 1, r, v, o << 1 | 1);
    }
}
```



```
    }
    PushUp(o);
}

int cas;
int n;
db x1, y1, x2, y2;
db ans;

int main() {
    while (~scanf("%d", &n) && n) {
        segs.clear(); pos.clear();
        for (int i = 0; i < n; ++i) {
            scanf("%lf%lf%lf%lf", &x1, &y1, &x2, &y2);
            segs.push_back((seg){x1, x2, y1, 1});
            segs.push_back((seg){x1, x2, y2, -1});
            pos.push_back(x1); pos.push_back(x2);
        }
        sort(segs.begin(), segs.end());
        sort(pos.begin(), pos.end(), [&](db k1, db k2) {return Cmp(k1, k2) < 0;});
        int Cur = 1;
        for (int i = 1; i < (int)pos.size(); ++i)
            if (Cmp(pos[i], pos[i - 1]) != 0)
                pos[Cur++] = pos[i];
        pos.erase(pos.begin() + Cur, pos.end());
        Build(0, (int)pos.size(), 1);
        ans = 0.0;
        for (int i = 0; i < (int)segs.size() - 1; ++i) {
            int l = BinarySearch(segs[i].l), r = BinarySearch(segs[i].r);
            Update(l, r - 1, segs[i].flag, 1);
            ans += (segs[i + 1].h - segs[i].h) * seg_tree[1].len;
        }
        printf("Test case #%d\n", ++cas);
        printf("Total explored area: %.2lf\n\n", ans);
    }
    return 0;
}
```

3.8.2 AreaXorCombination

```
// CodeForces GYM 101982 F 矩形面积异或并
#include <bits/stdc++.h>
typedef long long ll;

std::vector<int> x;
int Get(int k) {
    return std::lower_bound(x.begin(), x.end(), k) - x.begin();
}

class SegTree {
```

```
public:
    struct node {
        int v, lazy;
        node(int _v = 0, int _lazy = 0): v(_v), lazy(_lazy) {}
    };

    node Unite(const node &k1, const node &k2) {
        node ans;
        ans.v = k1.v + k2.v;
        return ans;
    }

    void Pull(int o) {
        tree[o] = Unite(tree[o << 1], tree[o << 1 | 1]);
    }

    void Push(int o, int l, int r) {
        int m = (l + r) >> 1;
        if (tree[o].lazy != 0) {
            tree[o << 1].v = x[m] - x[l - 1] - tree[o << 1].v;
            tree[o << 1 | 1].v = x[r] - x[m] - tree[o << 1 | 1].v;
            tree[o << 1].lazy ^= 1;
            tree[o << 1 | 1].lazy ^= 1;
            tree[o].lazy = 0;
        }
    }

    int n;
    std::vector<node> tree;

    void Build(int o, int l, int r) {
        if (l == r) return;
        int m = (l + r) >> 1;
        Build(o << 1, l, m);
        Build(o << 1 | 1, m + 1, r);
        Pull(o);
    }

    SegTree(int _n): n(_n) {
        tree.resize(n << 2);
        Build(1, 1, n);
    }

    void Modify(int o, int l, int r, int ll, int rr) {
        if (ll <= l && rr >= r) {
            tree[o].v = x[r] - x[l - 1] - tree[o].v;
            tree[o].lazy ^= 1;
            return;
        }
    }
```

```
    Push(o, l, r);
    int m = (l + r) >> 1;
    if (ll <= m) Modify(o << 1, l, m, ll, rr);
    if (rr > m) Modify(o << 1 | 1, m + 1, r, ll, rr);
    Pull(o);
}

void Modify(int ll, int rr) {
    Modify(1, 1, n, ll, rr);
}

node Query(int o, int l, int r, int ll, int rr) {
    if (ll <= l && rr >= r) return tree[o];
    Push(o, l, r);
    int m = (l + r) >> 1;
    node ans;
    if (ll <= m) ans = Unite(ans, Query(o << 1, l, m, ll, rr));
    if (rr > m) ans = Unite(ans, Query(o << 1 | 1, m + 1, r, ll, rr));
    Pull(o);
    return ans;
}

node Query() {
    return Query(1, 1, n, 1, n);
}

};

struct seg {int l, r, h, flag;};
bool operator < (seg k1, seg k2) {return k1.h < k2.h;}
std::vector<seg> s;

int main() {
    std::ios::sync_with_stdio(false);
    std::cin.tie(nullptr); std::cout.tie(nullptr);
    int n; std::cin >> n;
    for (int i = 0, x1, y1, x2, y2; i < n; ++i) {
        std::cin >> x1 >> y1 >> x2 >> y2;
        if (x1 > x2) std::swap(x1, x2);
        if (y1 > y2) std::swap(y1, y2);
        x.emplace_back(x1); x.emplace_back(x2);
        s.emplace_back((seg){x1, x2, y1, 1});
        s.emplace_back((seg){x1, x2, y2, -1});
    }
    sort(s.begin(), s.end());
    sort(x.begin(), x.end());
    x.erase(unique(x.begin(), x.end()), x.end());
    SegTree tree((int)x.size());
    ll ans = 0;
    for (int i = 0, l, r; i < (int)s.size() - 1; ++i) {
        l = Get(s[i].l), r = Get(s[i].r);
        tree.Modify(l + 1, r);
    }
}
```

```
    ans += (ll)tree.Query().v * (s[i + 1].h - s[i].h);
}
std::cout << ans << '\n';
return 0;
}
```

3.8.3 SegmentTree

// 求和线段树

```
template <typename type>
class segtree {
public:
    struct node {
        type v, lazy;
        node() {v = 0; lazy = 0;}
    };

    int n;
    std::vector<node> tree;

    node Unite(const node &k1, const node &k2) {
        node ret;
        ret.v = k1.v + k2.v;
        return ret;
    }

    void Pull(int o) {
        tree[o] = Unite(tree[o << 1], tree[o << 1 | 1]);
    }

    void Push(int o, int l, int r) {
        int m = (l + r) >> 1;
        if (tree[o].lazy != 0) {
            tree[o << 1].v += (m - l + 1) * tree[o].lazy;
            tree[o << 1 | 1].v += (r - m) * tree[o].lazy;
            tree[o << 1].lazy += tree[o].lazy;
            tree[o << 1 | 1].lazy += tree[o].lazy;
            tree[o].lazy = 0;
        }
    }

    template <typename t>
    void Build(int o, int l, int r, const std::vector<t> &v) {
        if (l == r) {
            tree[o].v = v[l - 1];
            return;
        }
        int m = (l + r) >> 1;
        Build(o << 1, l, m, v);
        Build(o << 1 | 1, m + 1, r, v);
    }
};
```

```
    Pull(o);
}

template <typename t>
segtree(const std::vector<t> &v) {
    n = v.size();
    tree.resize((n << 2) + 1);
    Build(1, 1, n, v);
}

template <typename t>
void Modify(int o, int l, int r, int ll, int rr, t v) {
    if (ll <= l && rr >= r) {
        tree[o].v += (r - l + 1) * v;
        tree[o].lazy += v;
        return;
    }
    Push(o, l, r);
    int m = (l + r) >> 1;
    if (ll <= m) Modify(o << 1, l, m, ll, rr, v);
    if (rr > m) Modify(o << 1 | 1, m + 1, r, ll, rr, v);
    Pull(o);
}

template <typename t>
void Modify(int ll, int rr, t v) {
    Modify(1, 1, n, ll, rr, v);
}

node Query(int o, int l, int r, int ll, int rr) {
    if (ll <= l && rr >= r) return tree[o];
    Push(o, l, r);
    int m = (l + r) >> 1;
    node ret;
    if (ll <= m) ret = Unite(ret, Query(o << 1, l, m, ll, rr));
    if (rr > m) ret = Unite(ret, Query(o << 1 | 1, m + 1, r, ll, rr));
    return ret;
}

node Query(int ll, int rr) {
    return Query(1, 1, n, ll, rr);
}
};
```

3.9 SplayTree

```
const int inf = "Edit";
const int maxn = "Edit";

class SplayTree {
public:
    int rt, tot;
```

```
int fa[maxn], son[maxn][2];
int val[maxn], cnt[maxn];
int sz[maxn];

void Push(int o) {
    sz[o] = sz[son[o][0]] + sz[son[o][1]] + cnt[o];
}

bool Get(int o) {
    return o == son[fa[o]][1];
}

void Clear(int o) {
    son[o][0] = son[o][1] = fa[o] = val[o] = sz[o] = cnt[o] = 0;
}

void Rotate(int o) {
    int p = fa[o], q = fa[p], ck = Get(o);
    son[p][ck] = son[o][ck ^ 1];
    fa[son[o][ck ^ 1]] = p;
    son[o][ck ^ 1] = p;
    fa[p] = o; fa[o] = q;
    if (q) son[q][p == son[q][1]] = o;
    Push(p); Push(o);
}

void Splay(int o) {
    for (int f = fa[o]; f = fa[o], f; Rotate(o))
        if (fa[f]) Rotate(Get(o) == Get(f) ? f : o);
    rt = o;
}

void Insert(int x) {
    if (!rt) {
        val[++tot] = x;
        cnt[tot]++;
        rt = tot;
        Push(rt);
        return;
    }
    int cur = rt, f = 0;
    while (true) {
        if (val[cur] == x) {
            cnt[cur]++;
            Push(cur); Push(f);
            Splay(cur);
            break;
        }
        f = cur;
    }
}
```

```
    cur = son[cur][val[cur] < x];
    if (!cur) {
        val[++tot] = x;
        cnt[tot]++;
        fa[tot] = f;
        son[f][val[f] < x] = tot;
        Push(tot); Push(f);
        Splay(tot);
        break;
    }
}
}

int GetRank(int x) {
    int ans = 0, cur = rt;
    while (true) {
        if (x < val[cur]) cur = son[cur][0];
        else {
            ans += sz[son[cur][0]];
            if (x == val[cur]) {
                Splay(cur);
                return ans + 1;
            }
            ans += cnt[cur];
            cur = son[cur][1];
        }
    }
}

int GetKth(int k) {
    int cur = rt;
    while (true) {
        if (son[cur][0] && k <= sz[son[cur][0]]) cur = son[cur][0];
        else {
            k -= cnt[cur] + sz[son[cur][0]];
            if (k <= 0) return cur;
            cur = son[cur][1];
        }
    }
}

// after insert, before delete
int GetPrev() {
    int cur = son[rt][0];
    while (son[cur][1]) cur = son[cur][1];
    return cur;
}

int GetNext() {
```

```
    int cur = son[rt][1];
    while (son[cur][0]) cur = son[cur][0];
    return cur;
}

void Delete(int x) {
    GetRank(x);
    if (cnt[rt] > 1) {
        cnt[rt]--;
        Push(rt);
        return;
    }
    if (!son[rt][0] && !son[rt][1]) {
        Clear(rt);
        rt = 0;
        return;
    }
    if (!son[rt][0]) {
        int cur = rt;
        rt = son[rt][1];
        fa[rt] = 0;
        Clear(cur);
        return;
    }
    if (!son[rt][1]) {
        int cur = rt;
        rt = son[rt][0];
        fa[rt] = 0;
        Clear(cur);
        return;
    }
    int p = GetPrev(), cur = rt;
    Splay(p);
    fa[son[cur][1]] = p;
    son[p][1] = son[cur][1];
    Clear(cur);
    Push(rt);
}
};
```

3.10 SplayTreeArray

```
const int inf = "Edit"
const int maxn = "Edit";

class SplayTree {
public:
    // rt:Splay Tree 根节点
    int rt;
    // son[i][0]:i 节点的左孩子, son[i][1]:i 节点的右孩子
```



```
int son[maxn][2];
// fa[i]: i 节点的父节点
int fa[maxn];
// val[i]: i 节点的权值
int val[maxn];
// sz[i]: 以 i 节点为根的 Splay Tree 的节点数 (包含自身)
int sz[maxn];
// 惰性标记数组
bool lazy[maxn];

void Push(int o) {
    sz[o] = sz[son[o][0]] + sz[son[o][1]] + 1;
}

void Pull(int o) {
    if (lazy[o]) {
        std::swap(son[o][0], son[o][1]);
        if (son[o][0]) lazy[son[o][0]] ^= 1;
        if (son[o][1]) lazy[son[o][1]] ^= 1;
        lazy[o] = 0;
    }
}

// 判断 o 节点是其父节点的左孩子还是右孩子
bool Get(int o) {
    return son[fa[o]][1] == o;
}

// 旋转节点 o
void Rotate(int o) {
    int p = fa[o], q = fa[p], ck = Get(o);
    Pull(p); Pull(o);
    son[p][ck] = son[o][ck ^ 1];
    fa[son[p][ck]] = fa[o];
    son[o][ck ^ 1] = fa[o];
    fa[p] = o;
    fa[o] = q;
    if (q) son[q][p == son[q][1]] = o;
    Push(p); Push(o);
}

// 旋转 o 节点到节点 tar
void Splay(int o, int tar = 0) {
    for (int cur = fa[o]; (cur = fa[o]) != tar; Rotate(o)) {
        Pull(fa[cur]); Pull(cur); Pull(o);
        if (fa[cur] != tar) {
            if (Get(o) == Get(cur)) Rotate(cur);
            else Rotate(o);
        }
    }
}
```

```
    }
    if (!tar) rt = o;
}

// 获取以 r 为根节点 Splay Tree 中的第 k 大个元素在 Splay Tree 中的位置
int Kth(int r, int k) {
    Pull(r);
    int tmp = sz[son[r][0]] + 1;
    if (tmp == k) return r;
    if (tmp > k) return Kth(son[r][0], k);
    else return Kth(son[r][1], k - tmp);
}

// 获取 Splay Tree 中以 o 为根节点子树的最小值位置
int GetMin(int o) {
    Pull(o);
    while (son[o][0]) {
        o = son[o][0];
        Pull(o);
    }
    return o;
}

// 获取 Splay Tree 中以 o 为根节点子树的最大值位置
int GetMax(int o) {
    Pull(o);
    while (son[o][1]) {
        o = son[o][1];
        Pull(o);
    }
    return o;
}

// 求节点 o 的前驱节点
int GetPath(int o) {
    Splay(o, rt);
    int cur = son[rt][0];
    while (son[cur][1]) cur = son[cur][1];
    return cur;
}

// 求节点 y 的后继节点
int GetNext(int o) {
    Splay(o, rt);
    int cur = son[rt][1];
    while (son[cur][0]) cur = son[cur][0];
    return cur;
}
```

```
// 翻转 Splay Tree 中 l~r 区间
void Reverse(int l, int r) {
    int o = Kth(rt, l), Y = Kth(rt, r);
    Splay(o, 0); Splay(Y, o);
    lazy[son[Y][0]] ^= 1;
}

// 建立 Splay Tree
void Build(int l, int r, int o) {
    if (l > r) return;
    int m = (l + r) >> 1;
    Build(l, m - 1, m);
    Build(m + 1, r, m);
    fa[m] = o;
    val[m] = m - 1;
    lazy[m] = 0;
    Push(m);
    if (m < o) son[o][0] = m;
    else son[o][1] = m;
}

// 输出 Splay Tree
void Print(int o) {
    Pull(o);
    if (son[o][0]) Print(son[o][0]);
    // 哨兵节点判断
    if (val[o] != -inf && val[o] != inf) printf("%d ", val[o]);
    if (son[o][1]) Print(son[o][1]);
}

};
```

4 GraphTheory

4.1 AStar

```
const int inf = "Edit";
const int maxn = "Edit";

struct edge {int v, c, next;};

edge g[maxn << 1];
int head[maxn];
int tot;
edge rev_g[maxn << 1];
int rev_head[maxn];
int rev_tot;

void Init() {
    tot = 0;
    memset(head, -1, sizeof(head));
    rev_tot = 0;
    memset(rev_head, -1, sizeof(rev_head));
}

void AddEdge(int u, int v, int c) {
    g[tot] = edge {v, c, head[u]};
    head[u] = tot++;
    rev_g[rev_tot] = edge {u, c, rev_head[v]};
    rev_head[v] = rev_tot++;
}

int dis[maxn];

struct Cmp {
    bool operator() (const int &k1, const int &k2) {
        return dis[k1] > dis[k2];
    }
};

// 利用反向边图求各点到终点的最短路
void Dijkstra(int s) {
    priority_queue<int, vector<int>, Cmp> que;
    memset(dis, inf, sizeof(dis));
    dis[s] = 0;
    que.push(s);
    while (!que.empty()) {
        int u = que.top(); que.pop();
        for (int i = rev_head[u]; i != -1; i = rev_g[i].next) {
            if (dis[rev_g[i].v] > dis[u] + rev_g[i].c) {
                dis[rev_g[i].v] = dis[u] + rev_g[i].c;
                que.push(rev_g[i].v);
            }
        }
    }
}
```

```
    }
  }
}

struct node {
  int f, g, p;
  // k* 核心:  $f=g+H(p)$ , 这里  $H(p)=dis[p]$ 
  bool operator < (const node &k) const {
    if (f == k.f) return g > k.g;
    return f > k.f;
  }
};

// A* 算法求起点 s 到终点 t 的第 k 短路
int AStar(int s, int t, int k) {
  int cnt = 0;
  priority_queue<node> que;
  // 注意特盘相同点是否算最短路
  if (s == t) k++;
  if (dis[s] == inf) return -1;
  que.push(node {dis[s], 0, s});
  while (!que.empty()) {
    node keep = que.top(); que.pop();
    if (keep.p == t) {
      cnt++;
      if (cnt == k) return keep.g;
    }
    for (int i = head[keep.p]; i != -1; i = g[i].next) {
      node tmp;
      tmp.p = g[i].v;
      tmp.g = keep.g + g[i].c;
      tmp.f = tmp.g + dis[tmp.p];
      que.push(tmp);
    }
  }
  return -1;
}
```

4.2 MinimumSpanningTree

4.2.1 Kruskal

```
const int maxn = "Edit";

struct edge {int u, v, c;};
bool operator < (edge k1 edge k2) {return k1.c < k2.c;}

int n, e, pre[maxn];
std::vector<edge> g;
```

```
void Init() {for (int i = 0; i <= n; ++i) pre[i] = i;}
int Find(int x) {return pre[x] == x ? x : pre[x] = Find(pre[x]);}
void Union(int x, int y) {pre[Find(x)] = Find(y);}

int Kruskal() {
    std::sort(g.begin(), g.end());
    Init();
    int ret = 0;
    for (auto &e : g) {
        if (Find(e.u) != Find(e.v)) {
            Union(e.u, e.v);
            ret += e.c;
        }
    }
    return ret;
}
```

4.2.2 Prim

```
const int inf = "Edit";
const int maxn = "Edit";

struct edge {int v, dis;};

int n;
int dis[maxn];
int vis[maxn];
std::vector<edge> g[maxn];

void AddEdge(int u, int v, int c) {
    g[u].push_back(edge (v, c));
    g[v].push_back(edge (u, c));
}

// Prim 算法
int Prim(int s) {
    memset(dis, inf, sizeof(dis));
    memset(vis, 0, sizeof(vis));
    dis[s] = 0;
    int ret = 0;
    for (int i = 1; i <= n; ++i) {
        int u = -1, min = inf;
        for (int j = 1; j <= n; ++j) {
            if (!vis[j] && dis[j] < min) {
                u = j;
                min = dis[j];
            }
        }
        vis[u] = 1;
    }
}
```

```
    ret += min;
    for (int j = 0; j < int(g[u].size()); ++j) {
        int v = g[u][j].v;
        if (!vis[v] && g[u][j].dis < dis[v]) dis[v] = g[u][j].dis;
    }
}
return ret;
}
```

4.3 NetFlow

4.3.1 Dinic

```
namespace NetFlow {
    const int maxn = 1e5 + 5;
    const int inf = 0x3f3f3f3f;
    struct edge {int v, flow, next;};
    edge g[maxn << 2];
    int tot;
    int head[maxn];
    int dep[maxn];
    int cur[maxn];

    void AddEdge(int u, int v, int flow, int rev = 0) {
        g[tot] = (edge){v, flow, head[u]};
        head[u] = tot++;
        g[tot] = (edge){u, rev, head[v]};
        head[v] = tot++;
    }

    bool Bfs(int s, int t) {
        memset(dep, -1, sizeof(dep));
        std::queue<int> que;
        dep[s] = 0;
        que.push(s);
        while (!que.empty()) {
            int u = que.front(); que.pop();
            for (int i = head[u]; ~i; i = g[i].next) {
                if (dep[g[i].v] == -1 && g[i].flow > 0) {
                    dep[g[i].v] = dep[u] + 1;
                    que.push(g[i].v);
                }
            }
        }
        return dep[t] != -1;
    }

    int Dfs(int u, int t, int flow) {
        if (u == t || flow == 0) return flow;
        int max = 0, find_flow;

```

```

    for (int &i = cur[u]; ~i; i = g[i].next) {
        if (g[i].flow > 0 && dep[g[i].v] == dep[u] + 1) {
            find_flow = Dfs(g[i].v, t, std::min(flow - max, g[i].flow));
            if (find_flow > 0) {
                g[i].flow -= find_flow;
                g[i ^ 1].flow += find_flow;
                max += find_flow;
                if (max == flow) return flow;
            }
        }
    }
    if (!max) dep[u] = -2;
    return max;
}

int Dinic(int s, int t) {
    int ans = 0;
    while (Bfs(s, t)) {
        for (int i = s; i <= t; ++i) cur[i] = head[i];
        ans += Dfs(s, t, inf);
    }
    return ans;
}
};

```

4.3.2 FordFulkerson

```

const int inf = "Edit";
const int maxn = "Edit";

int n, e;
bool vis[maxn];
int g[maxn][maxn];

// Dfs 搜索增广路径, u: 当前搜索顶点, t: 搜索终点, now_flow: 当前最大流量
int Dfs(int u, int t, int now_flow) {
    if (u == t) return now_flow;
    vis[u] = true;
    for (int i = 1; i <= n; ++i) {
        if (!vis[i] && g[u][i]) {
            int FindFlow = Dfs(i, t, now_flow < g[u][i] ? now_flow : g[u][i]);
            if (!FindFlow) continue;
            g[u][i] -= FindFlow;
            g[i][u] += FindFlow;
            return FindFlow;
        }
    }
    return false;
}

```



```
// Ford-Fulkerson 算法, s: 起点, t: 终点
int FordFulkerson(int s, int t) {
    int max_flow = 0, Flow = 0;
    memset(vis, false, sizeof(vis));
    while (Flow = Dfs(s, t, inf)) {
        max_flow += Flow;
        memset(vis, false, sizeof(vis));
    }
    return max_flow;
}
```

4.3.3 MaxFlow

```
namespace NetFlow {
    const int inf = 0x3f3f3f3f;
    int s, t;
    struct edge {int to, cap, rev;};
    std::vector<std::vector<edge>> g;
    std::vector<bool> vis;

    void Init(int n) {
        s = 0; t = n;
        g.resize(n + 1);
    }

    void AddEdge(int u, int v, int cap, int rev = 0) {
        g[u].push_back((edge){v, cap, (int)g[v].size()});
        g[v].push_back((edge){u, rev, (int)g[u].size() - 1});
    }

    int Dfs(int u, int t, int flow) {
        if (u == t) return flow;
        vis[u] = true;
        for (edge &e : g[u]) {
            if (!vis[e.to] && e.cap > 0) {
                int f = Dfs(e.to, t, std::min(e.cap, flow));
                if (f > 0) {
                    e.cap -= f;
                    g[e.to][e.rev].cap += f;
                    return f;
                }
            }
        }
        return 0;
    }

    int GetMaxFlow(int s, int t) {
        int ans = 0;
        while (true) {
            vis.assign(t + 1, false);

```

```
    int flow = Dfs(s, t, inf);
    if (flow == 0) return ans;
    ans += flow;
}
}
};
```

4.3.4 MinCostMaxFlow

```
const int inf = "Edit";
const int maxn = "Edit";

struct edge {int v, cap, cost, flow, next;};
int n, e;
int head[maxn];
int path[maxn];
int dis[maxn];
bool vis[maxn];
int tot;
edge g[maxn];

void AddEdge(int u, int v, int cap, int cost) {
    g[tot] = (edge){v, cap, cost, 0, head[u]};
    head[u] = tot++;
    g[tot] = (edge){u, 0, -cost, 0, head[v]};
    head[v] = tot++;
}

bool SPFA(int s, int t) {
    memset(dis, inf, sizeof(dis));
    memset(vis, false, sizeof(vis));
    memset(path, -1, sizeof(path));
    dis[s] = 0;
    vis[s] = true;
    std::queue<int> que;
    while (!que.empty()) que.pop();
    que.push(s);
    while (!que.empty()) {
        int U = que.front();
        que.pop();
        vis[U] = false;
        for (int i = head[U]; ~i; i = g[i].next) {
            int v = g[i].v;
            if (g[i].cap > g[i].flow && dis[v] > dis[U] + g[i].cost) {
                dis[v] = dis[U] + g[i].cost;
                path[v] = i;
                if (!vis[v]) {
                    vis[v] = true;
                    que.push(v);
                }
            }
        }
    }
}
```

```
    }
  }
}
return path[t] != -1;
}

int MinCostMaxflow(int s, int t, int &min_cost) {
  int max_flow = 0;
  min_cost = 0;
  while (SPFA(s, t)) {
    int min = inf;
    for (int i = path[t]; ~i; i = path[g[i ^ 1].v]) {
      if (g[i].cap - g[i].flow < min) min = g[i].cap - g[i].flow;
    }
    for (int i = path[t]; ~i; i = path[g[i ^ 1].v]) {
      g[i].flow += min;
      g[i ^ 1].flow -= min;
      min_cost += g[i].cost * min;
    }
    max_flow += min;
  }
  return max_flow;
}
```

4.4 ShortestPath

4.4.1 BellmanFord

```
const int inf = "Edit";
const int maxn = "Edit";

int n;
struct edge {int u, v, dis;};
int dis[maxn];
std::vector<edge> g;

// Bellman_Ford 算法判断是否存在负环回路
bool BellmanFord(int s) {
  memset(dis, inf, sizeof(dis));
  dis[s] = 0;
  // 最多做 N-1 次
  for (int i = 1; i < n; ++i) {
    bool flag = false;
    for (int j = 0; j < (int)g.size(); ++j) {
      if (dis[g[j].v] > dis[g[j].u] + g[j].dis) {
        dis[g[j].v] = dis[g[j].u] + g[j].dis;
        flag = true;
      }
    }
  }
  if (!flag) return true;
}
```

```
    }
    for (int j = 0; j < (int)g.size(); ++j) {
        if (dis[g[j].v] > dis[g[j].u] + g[j].dis) return false;
    }
    return true;
}
```

4.4.2 Dijkstra

```
const int maxn = "Edit";
const int inf = "Edit";

struct edge {int v, c, next;};

edge g[maxn << 1];
int head[maxn];
int tot;
int dis[maxn];

void AddEdge(int u, int v, int c) {
    g[tot] = (edge){v, c, head[u]};
    head[u] = tot++;
}

struct Cmp {
    bool operator() (const int &k1, const int &k2) {
        return dis[k1] > dis[k2];
    }
};

int n, e;

void Dijkstra(int s) {
    std::priority_queue<int, std::vector<int>, Cmp> que;
    memset(dis, inf, sizeof(dis));
    dis[s] = 0;
    que.push(s);
    while (!que.empty()) {
        int u = que.top(); que.pop();
        for (int i = head[u]; ~i; i = g[i].next) {
            if (dis[g[i].v] > dis[u] + g[i].c) {
                dis[g[i].v] = dis[u] + g[i].c;
                que.push(g[i].v);
            }
        }
    }
}
```

4.4.3 Floyd

```
const int maxn = "Edit";

int n;
int dis[maxn][maxn];

void Floyd() {
    for (int k = 1; k <= n; ++k) {
        for (int i = 1; i <= n; ++i) {
            for (int j = 1; j <= n; ++j) {
                dis[i][j] = std::min(dis[i][j], dis[i][k] + dis[k][j]);
            }
        }
    }
}
```

4.4.4 SPFA

```
const int inf = "Edit";
const int maxn = "Edit";

struct edge {int v, dis;};
int n, e;
bool vis[maxn];
int cnt[maxn];
int dis[maxn];
std::vector<edge> g[maxn];

void AddEdge (int u, int v, int c) {
    g[u].push_back((edge){v, c});
    g[v].push_back((edge){u, c});
}

bool SPFA(int s) {
    memset(vis, false, sizeof(vis));
    memset(dis, inf, sizeof(dis));
    memset(cnt, 0, sizeof(cnt));
    vis[s] = true;
    dis[s] = 0;
    cnt[s] = 1;
    std::queue<int> que;
    while (!que.empty()) que.pop();
    que.push(s);
    while (!que.empty()) {
        int U = que.front();
        que.pop();
        vis[U] = false;
        for (int i = 0; i < (int)g[U].size(); ++i) {
            int v = g[U][i].v;
```

```
    if (dis[v] > dis[U] + g[U][i].dis) {  
        dis[v] = dis[U] + g[U][i].dis;  
        if (!vis[v]) {  
            vis[v] = true;  
            que.push(v);  
            if (++cnt[v] > N) return false;  
        }  
    }  
}  
}  
}  
return true;  
}
```

5 DynamicProgramming

5.1 Contour

```
const int maxn = "Edit";

int dp[2][1 << maxn];

void Update(int cur, int a, int b) {
    if (b & (1 << M)) dp[cur][b ^ (1 << M)] = dp[cur][b ^ (1 << M)] + dp[cur ^ 1][a];
}

// 轮廓线 dp(1*2 在 N*M 图上摆放数)
int Contour(int N, int M) {
    memset(dp, 0, sizeof(dp));
    int cur = 0;
    dp[cur][(1 << M) - 1] = 1;
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < M; ++j) {
            cur ^= 1;
            memset(dp[cur], 0, sizeof(dp[cur]));
            for (int k = 0; k < (1 << M); ++k) {
                Update(cur, k, k << 1);
                if (i && !(k & (1 << (M - 1)))) Update(cur, k, (k << 1) ^ (1 << M) ^ 1);
                if (j && !(k & 1)) Update(cur, k, (k << 1) ^ 3);
            }
        }
    }
    return dp[cur][(1 << M) - 1];
}
```

5.2 Digit

```
const int maxn = "Edit";

long long digit[25];
long long dp[25][maxn];

// site: 数位, status: 状态, pre: 前导零, limit: 数位上界
long long Dfs(long long site, long long status, bool pre, bool limit) {
    if (site == 0) return ?;
    if (!limit && ~dp[site][status]) return dp[site][status];
    long long max = limit ? digit[site] : 9;
    long long ret = 0;
    for (int i = 0; i <= max; ++i) {
        long long new_status = /* 状态转移 */;
        if (new_status?) ret += Dfs(site - 1, new_status, pre && i == 0, limit && i ==
        ↪ max);
    }
    if (!limit) dp[site][status] = ret;
}
```

```
    return ret;
}

long long Get(long long x) {
    long long len = 0;
    while (x) {
        digit[++len] = x % 10;
        x /= 10;
    }
    return Dfs(len, 0, true, true);
}
```

5.3 LCS

```
const int maxn = "Edit";

// dp[i][j]: str1[1]~str1[i] 和 str2[1]~str2[j] 对应的公共子序列长度
int dp[maxn][maxn];

// 最长公共子序列 (LCS)
void GetLCS(std::string str1, std::string str2) {
    for (int i = 0; i < int(str1.length()); ++i) {
        for (int j = 0; j < int(str2.length()); ++j) {
            if (str1[i] == str2[j]) dp[i + 1][j + 1] = dp[i][j] + 1;
            else dp[i + 1][j + 1] = std::max(dp[i][j + 1], dp[i + 1][j]);
        }
    }
}
```

5.4 LIS

```
// 最长不上降子序列 (LIS), arr: 序列
int GetLIS(std::vector<int> &arr) {
    int ret = 1;
    // last[i]: 长度为 i 的不上降子序列末尾元素的最小值
    std::vector<int> last(int(arr.size()) + 1, 0);
    last[1] = arr[1];
    for (int i = 2; i <= int(arr.size()); ++i) {
        if (arr[i] >= last[ret]) last[++ret] = arr[i];
        else {
            int pos = std::upper_bound(last.begin() + 1, last.end(), arr[i]) -
                last.begin();
            last[pos] = arr[i];
        }
    }
    return ret;
}
```


5.5 Pack

```
const int maxn = "Edit";

int dp[maxn];
// cap: 背包容量, cnt: 总物品数
int cap, cnt;

// 01 背包, 代价为 cost, 获得的价值为 weight
void ZeroOnePack(int cost, int weight) {
    for (int i = cap; i >= cost; --i) dp[i] = std::max(dp[i], dp[i - cost] + weight);
}

// 完全背包, 代价为 cost, 获得的价值为 weight
void CompletePack(int cost, int weight) {
    for (int i = cost; i <= cap; ++i) dp[i] = std::max(dp[i], dp[i - cost] + weight);
}

// 多重背包, 代价为 cost, 获得的价值为 weight, 数量为 amount
void MultiplePack(int cost, int weight, int amount) {
    if (cost * amount >= cap) CompletePack(cost, weight);
    else {
        int k = 1;
        while (k < amount) {
            ZeroOnePack(k * cost, k * weight);
            amount -= k;
            k <<= 1;
        }
        ZeroOnePack(amount * cost, amount * weight);
    }
}
```

6 Geometry

6.1 DynamicConvexhull

```
// CodeForces 70D 动态凸包
#include <bits/stdc++.h>
typedef double db;
const int maxn = 1e5 + 5;
const db eps = 1e-9;
int Sgn(db k) {return fabs(k) < eps ? 0 : (k < 0 ? -1 : 1);}
int Cmp(db k1, db k2) {return Sgn(k1 - k2);}

struct point {db x, y;};
typedef point Vector;
Vector operator - (Vector k1, Vector k2) {return (Vector){k1.x - k2.x, k1.y -
↪ k2.y};}
Vector operator + (Vector k1, Vector k2) {return (Vector){k1.x + k2.x, k1.y +
↪ k2.y};}
db operator * (Vector k1, Vector k2) {return k1.x * k2.x + k1.y * k2.y;}
db operator ^ (Vector k1, Vector k2) {return k1.x * k2.y - k1.y * k2.x;}
db GetLen(Vector k) {return sqrt(k * k);}

int n;
point basic;
point p[maxn];
std::set<point> set;

bool operator < (point k1, point k2) {
    k1 = k1 - basic; k2 = k2 - basic;
    db Ang1 = atan2(k1.y, k1.x), Ang2 = atan2(k2.y, k2.x);
    db Len1 = GetLen(k1), Len2 = GetLen(k2);
    if (Cmp(Ang1, Ang2) != 0) return Cmp(Ang1, Ang2) < 0;
    return Cmp(Len1, Len2) < 0;
}

std::set<point>::iterator Prev(std::set<point>::iterator k) {
    if (k == set.begin()) k = set.end();
    return --k;
}

std::set<point>::iterator Next(std::set<point>::iterator k) {
    ++k;
    return k == set.end() ? set.begin() : k;
}

bool Query(point k) {
    std::set<point>::iterator it = set.lower_bound(k);
    if (it == set.end()) it = set.begin();
    return Sgn((k - *(Prev(it))) ^ (*(it) - *(Prev(it)))) <= 0;
}
```

```
void Insert(point k) {
    if (Query(k)) return;
    set.insert(k);
    std::set<point>::iterator cur = Next(set.find(k));
    while (set.size() > 3 && Sgn((k - *(Next(cur))) ^ (*(cur) - *(Next(cur)))) <= 0)
        ↪ {
        set.erase(cur);
        cur = Next(set.find(k));
    }
    cur = Prev(set.find(k));
    while (set.size() > 3 && Sgn((k - *(cur)) ^ (*(cur) - *(Prev(cur)))) >= 0) {
        set.erase(cur);
        cur = Prev(set.find(k));
    }
}

int main() {
    scanf("%d", &n);
    basic.x = basic.y = 0.0;
    for (int i = 1, T; i <= 3; ++i) {
        scanf("%d%lf%lf", &T, &p[i].x, &p[i].y);
        basic.x += p[i].x; basic.y += p[i].y;
    }
    basic.x /= 3.0; basic.y /= 3.0;
    for (int i = 1; i <= 3; ++i) set.insert(p[i]);
    for (int i = 4, T; i <= n; ++i) {
        scanf("%d%lf%lf", &T, &p[i].x, &p[i].y);
        if (T == 1) Insert(p[i]);
        else {
            if (Query(p[i])) printf("YES\n");
            else printf("NO\n");
        }
    }
    return 0;
}
```

6.2 JlsGeo

```
#define mp make_pair
#define fi first
#define se second
#define pb push_back
typedef double db;
const db eps=1e-6;
const db pi=acos(-1);
int sign(db k){
    if (k>eps) return 1; else if (k<eps) return -1; return 0;
}
int cmp(db k1,db k2){return sign(k1-k2);}
```

```

int inmid(db k1,db k2,db k3){return sign(k1-k3)*sign(k2-k3)<=0;}// k3 在 [k1,k2] 内
↪
struct point{
    db x,y;
    point operator + (const point &k1) const{return (point){k1.x+x,k1.y+y};}
    point operator - (const point &k1) const{return (point){x-k1.x,y-k1.y};}
    point operator * (db k1) const{return (point){x*k1,y*k1};}
    point operator / (db k1) const{return (point){x/k1,y/k1};}
    int operator == (const point &k1) const{return cmp(x,k1.x)==0&&cmp(y,k1.y)==0;}
    // 逆时针旋转
    point turn(db k1){return (point){x*cos(k1)-y*sin(k1),x*sin(k1)+y*cos(k1)};}
    point turn90(){return (point){-y,x};}
    bool operator < (const point k1) const{
        int a=cmp(x,k1.x);
        if (a==-1) return 1; else if (a==1) return 0; else return cmp(y,k1.y)==-1;
    }
    db abs(){return sqrt(x*x+y*y);}
    db abs2(){return x*x+y*y;}
    db dis(point k1){return ((*this)-k1).abs();}
    point unit(){db w=abs(); return (point){x/w,y/w};}
    void scan(){double k1,k2; scanf("%lf%lf",&k1,&k2); x=k1; y=k2;}
    void print(){printf("%.11lf %.11lf\n",x,y);}
    db getw(){return atan2(y,x);}
    point getdel(){if (sign(x)==-1||(sign(x)==0&&sign(y)==-1)) return (*this)*(-1);
        ↪ else return (*this);}
    int getP() const{return sign(y)==1||(sign(y)==0&&sign(x)==-1);}
};

int inmid(point k1,point k2,point k3){return
    ↪ inmid(k1.x,k2.x,k3.x)&&inmid(k1.y,k2.y,k3.y);}
db cross(point k1,point k2){return k1.x*k2.y-k1.y*k2.x;}
db dot(point k1,point k2){return k1.x*k2.x+k1.y*k2.y;}
db rad(point k1,point k2){return atan2(cross(k1,k2),dot(k1,k2));}
// -pi -> pi
int compareangle (point k1,point k2){
    return k1.getP()<k2.getP()||(k1.getP()==k2.getP()&&sign(cross(k1,k2))>0);
}
point proj(point k1,point k2,point q){ // q 到直线 k1,k2 的投影
    point k=k2-k1; return k1+k*(dot(q-k1,k)/k.abs2());
}
point reflect(point k1,point k2,point q){return proj(k1,k2,q)*2-q;}
int clockwise(point k1,point k2,point k3){// k1 k2 k3 逆时针 1 顺时针 -1 否则 0
    return sign(cross(k2-k1,k3-k1));
}
int checkLL(point k1,point k2,point k3,point k4){// 求直线 (L) 线段 (S)k1,k2 和
    ↪ k3,k4 的交点
    return cmp(cross(k3-k1,k4-k1),cross(k3-k2,k4-k2))!=0;
}
point getLL(point k1,point k2,point k3,point k4){
    db w1=cross(k1-k3,k4-k3),w2=cross(k4-k3,k2-k3); return (k1*w2+k2*w1)/(w1+w2);
}

```

```

}
int intersect(db l1,db r1,db l2,db r2){
    if (l1>r1) swap(l1,r1); if (l2>r2) swap(l2,r2); return
    ↪ cmp(r1,l2)!=-1&&cmp(r2,l1)!=-1;
}
int checkSS(point k1,point k2,point k3,point k4){
    return intersect(k1.x,k2.x,k3.x,k4.x)&&intersect(k1.y,k2.y,k3.y,k4.y)&&
    sign(cross(k3-k1,k4-k1))*sign(cross(k3-k2,k4-k2))<=0&&
    sign(cross(k1-k3,k2-k3))*sign(cross(k1-k4,k2-k4))<=0;
}
db disSP(point k1,point k2,point q){
    point k3=proj(k1,k2,q);
    if (inmid(k1,k2,k3)) return q.dis(k3); else return min(q.dis(k1),q.dis(k2));
}
db disSS(point k1,point k2,point k3,point k4){
    if (checkSS(k1,k2,k3,k4)) return 0;
    else return
    ↪ min(min(disSP(k1,k2,k3),disSP(k1,k2,k4)),min(disSP(k3,k4,k1),disSP(k3,k4,k2)));
}
int onS(point k1,point k2,point q){return
    ↪ inmid(k1,k2,q)&&sign(cross(k1-q,k2-k1))==0;}
struct circle{
    point o; db r;
    void scan(){o.scan(); scanf("%lf",&r);}
    int inside(point k){return cmp(r,o.dis(k));}
};
struct line{
    // p[0]->p[1]
    point p[2];
    line(point k1,point k2){p[0]=k1; p[1]=k2;}
    point& operator [] (int k){return p[k];}
    int include(point k){return sign(cross(p[1]-p[0],k-p[0]))>0;}
    point dir(){return p[1]-p[0];}
    line push(){ // 向外 ( 左手边 ) 平移 eps
        const db eps = 1e-6;
        point delta=(p[1]-p[0]).turn90().unit()*eps;
        return {p[0]-delta,p[1]-delta};
    }
};
point getLL(line k1,line k2){return getLL(k1[0],k1[1],k2[0],k2[1]);}
int parallel(line k1,line k2){return sign(cross(k1.dir(),k2.dir()))==0;}
int sameDir(line k1,line k2){return
    ↪ parallel(k1,k2)&&sign(dot(k1.dir(),k2.dir()))==1;}
int operator < (line k1,line k2){
    if (sameDir(k1,k2)) return k2.include(k1[0]);
    return compareangle(k1.dir(),k2.dir());
}
int checkpos(line k1,line k2,line k3){return k3.include(getLL(k1,k2));}
vector<line> getHL(vector<line> &L){ // 求半平面交 , 半平面是逆时针方向 , 输出按照逆
    ↪ 时针

```

```

sort(L.begin(),L.end()); deque<line> q;
for (int i=0;i<(int)L.size();i++){
    if (i&&sameDir(L[i],L[i-1])) continue;
    while (q.size()>1&&!checkpos(q[q.size()-2],q[q.size()-1],L[i])) q.pop_back();
    while (q.size()>1&&!checkpos(q[1],q[0],L[i])) q.pop_front();
    q.push_back(L[i]);
}
while (q.size()>2&&!checkpos(q[q.size()-2],q[q.size()-1],q[0])) q.pop_back();
while (q.size()>2&&!checkpos(q[1],q[0],q[q.size()-1])) q.pop_front();
vector<line>ans; for (int i=0;i<q.size();i++) ans.push_back(q[i]);
return ans;
}

db closepoint(vector<point>&A,int l,int r){ // 最近点对 , 先要按照 x 坐标排序
    if (r-l<=5){
        db ans=1e20;
        for (int i=l;i<=r;i++) for (int j=i+1;j<=r;j++) ans=min(ans,A[i].dis(A[j]));
        return ans;
    }
    int mid=l+r>>1; db ans=min(closepoint(A,l,mid),closepoint(A,mid+1,r));
    vector<point>B; for (int i=l;i<=r;i++) if (abs(A[i].x-A[mid].x)<=ans)
        ↪ B.push_back(A[i]);
    sort(B.begin(),B.end(),[](point k1,point k2){return k1.y<k2.y;});
    for (int i=0;i<B.size();i++) for (int j=i+1;j<B.size()&&B[j].y-B[i].y<ans;j++)
        ↪ ans=min(ans,B[i].dis(B[j]));
    return ans;
}

int checkposCC(circle k1,circle k2){// 返回两个圆的公切线数量
    if (cmp(k1.r,k2.r)==-1) swap(k1,k2);
    db dis=k1.o.dis(k2.o); int w1=cmp(dis,k1.r+k2.r),w2=cmp(dis,k1.r-k2.r);
    if (w1>0) return 4; else if (w1==0) return 3; else if (w2>0) return 2;
    else if (w2==0) return 1; else return 0;
}

vector<point> getCL(circle k1,point k2,point k3){ // 沿着 k2->k3 方向给出 , 相切给出
    ↪ 两个
    point k=proj(k2,k3,k1.o); db d=k1.r*k1.r-(k-k1.o).abs2();
    if (sign(d)==-1) return {};
    point del=(k3-k2).unit()*sqrt(max((db)0.0,d)); return {k-del,k+del};
}

vector<point> getCC(circle k1,circle k2){// 沿圆 k1 逆时针给出 , 相切给出两个
    int pd=checkposCC(k1,k2); if (pd==0||pd==4) return {};
    db
        ↪ a=(k2.o-k1.o).abs2(),cosA=(k1.r*k1.r+a-k2.r*k2.r)/(2*k1.r*sqrt(max(a,(db)0.0)));
    db b=k1.r*cosA,c=sqrt(max((db)0.0,k1.r*k1.r-b*b));
    point k=(k2.o-k1.o).unit(),m=k1.o+k*b,del=k.turn90()*c;
    return {m-del,m+del};
}

vector<point> TangentCP(circle k1,point k2){// 沿圆 k1 逆时针给出
    db a=(k2-k1.o).abs(),b=k1.r*k1.r/a,c=sqrt(max((db)0.0,k1.r*k1.r-b*b));
    point k=(k2-k1.o).unit(),m=k1.o+k*b,del=k.turn90()*c;

```

```

    return {m-del,m+del};
}
vector<line> TangentoutCC(circle k1,circle k2){
    int pd=checkposCC(k1,k2); if (pd==0) return {};
    if (pd==1){point k=getCC(k1,k2)[0]; return {(line){k,k}};}
    if (cmp(k1.r,k2.r)==0){
        point del=(k2.o-k1.o).unit().turn90().getdel();
        return
            ↪ {(line){k1.o-del*k1.r,k2.o-del*k2.r},{k1.o+del*k1.r,k2.o+del*k2.r}};
    } else {
        point p=(k2.o*k1.r-k1.o*k2.r)/(k1.r-k2.r);
        vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);
        vector<line>ans; for (int i=0;i<A.size();i++) ans.push_back((line){A[i],B[i]});
        return ans;
    }
}
vector<line> TangentinCC(circle k1,circle k2){
    int pd=checkposCC(k1,k2); if (pd<=2) return {};
    if (pd==3){point k=getCC(k1,k2)[0]; return {(line){k,k}};}
    point p=(k2.o*k1.r+k1.o*k2.r)/(k1.r+k2.r);
    vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);
    vector<line>ans; for (int i=0;i<A.size();i++) ans.push_back((line){A[i],B[i]});
    return ans;
}
vector<line> TangentCC(circle k1,circle k2){
    int flag=0; if (k1.r<k2.r) swap(k1,k2),flag=1;
    vector<line>A=TangentoutCC(k1,k2),B=TangentinCC(k1,k2);
    for (line k:B) A.push_back(k);
    if (flag) for (line &k:A) swap(k[0],k[1]);
    return A;
}
db getarea(circle k1,point k2,point k3){
    // 圆 k1 与三角形 k2 k3 k1.o 的有向面积交
    point k=k1.o; k1.o=k1.o-k; k2=k2-k; k3=k3-k;
    int pd1=k1.inside(k2),pd2=k1.inside(k3);
    vector<point>A=getCL(k1,k2,k3);
    if (pd1>=0){
        if (pd2>=0) return cross(k2,k3)/2;
        return k1.r*k1.r*rad(A[1],k3)/2+cross(k2,A[1])/2;
    } else if (pd2>=0){
        return k1.r*k1.r*rad(k2,A[0])/2+cross(A[0],k3)/2;
    } else {
        int pd=cmp(k1.r,disSP(k2,k3,k1.o));
        if (pd<=0) return k1.r*k1.r*rad(k2,k3)/2;
        return cross(A[0],A[1])/2+k1.r*k1.r*(rad(k2,A[0])+rad(A[1],k3))/2;
    }
}
circle getcircle(point k1,point k2,point k3){
    db a1=k2.x-k1.x,b1=k2.y-k1.y,c1=(a1*a1+b1*b1)/2;

```

```

    db a2=k3.x-k1.x,b2=k3.y-k1.y,c2=(a2*a2+b2*b2)/2;
    db d=a1*b2-a2*b1;
    point o=(point){k1.x+(c1*b2-c2*b1)/d,k1.y+(a1*c2-a2*c1)/d};
    return (circle){o,k1.dis(o)};
}

circle getScircle(vector<point> A){
    random_shuffle(A.begin(),A.end());
    circle ans=(circle){A[0],0};
    for (int i=1;i<A.size();i++){
        if (ans.inside(A[i])==1){
            ans=(circle){A[i],0};
            for (int j=0;j<i;j++){
                if (ans.inside(A[j])==1){
                    ans.o=(A[i]+A[j])/2; ans.r=ans.o.dis(A[i]);
                    for (int k=0;k<j;k++){
                        if (ans.inside(A[k])==1)
                            ans=getcircle(A[i],A[j],A[k]);
                    }
                }
            }
        }
    }
    return ans;
}

db area(vector<point> A){ // 多边形用 vector<point> 表示 , 逆时针
    db ans=0;
    for (int i=0;i<A.size();i++) ans+=cross(A[i],A[(i+1)%A.size()]);
    return ans/2;
}

int checkconvex(vector<point>A){
    int n=A.size(); A.push_back(A[0]); A.push_back(A[1]);
    for (int i=0;i<n;i++) if (sign(cross(A[i+1]-A[i],A[i+2]-A[i]))==1) return 0;
    return 1;
}

int contain(vector<point>A,point q){ // 2 内部 1 边界 0 外部
    int pd=0; A.push_back(A[0]);
    for (int i=1;i<A.size();i++){
        point u=A[i-1],v=A[i];
        if (onS(u,v,q)) return 1; if (cmp(u.y,v.y)>0) swap(u,v);
        if (cmp(u.y,q.y)>0||cmp(v.y,q.y)<0) continue;
        if (sign(cross(u-v,q-v))<0) pd^=1;
    }
    return pd<<1;
}

vector<point> ConvexHull(vector<point>A,int flag=1){ // flag=0 不严格 flag=1 严格
    int n=A.size(); vector<point>ans(n*2);
    sort(A.begin(),A.end()); int now=-1;
    for (int i=0;i<A.size();i++){
        while (now>0&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;
        ans[++now]=A[i];
    } int pre=now;
    for (int i=n-2;i>=0;i--){

```



```

    while (now>pre&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;
    ans[++now]=A[i];
} ans.resize(now); return ans;
}

db convexDiameter(vector<point>A){
    int now=0,n=A.size(); db ans=0;
    for (int i=0;i<A.size();i++){
        now=max(now,i);
        while (1){
            db k1=A[i].dis(A[now%n]),k2=A[i].dis(A[(now+1)%n]);
            ans=max(ans,max(k1,k2)); if (k2>k1) now++; else break;
        }
    }
    return ans;
}

vector<point> convexcut(vector<point>A,point k1,point k2){
    // 保留 k1,k2,p 逆时针的所有点
    int n=A.size(); A.push_back(A[0]); vector<point>ans;
    for (int i=0;i<n;i++){
        int w1=clockwise(k1,k2,A[i]),w2=clockwise(k1,k2,A[i+1]);
        if (w1>=0) ans.push_back(A[i]);
        if (w1*w2<0) ans.push_back(getLL(k1,k2,A[i],A[i+1]));
    }
    return ans;
}

int checkPoS(vector<point>A,point k1,point k2){
    // 多边形 A 和直线 ( 线段 )k1->k2 严格相交 , 注释部分为线段
    struct ins{
        point m,u,v;
        int operator < (const ins& k) const {return m<k.m;}
    }; vector<ins>B;
    //if (contain(A,k1)==2||contain(A,k2)==2) return 1;
    vector<point>poly=A; A.push_back(A[0]);
    for (int i=1;i<A.size();i++) if (checkLL(A[i-1],A[i],k1,k2)){
        point m=getLL(A[i-1],A[i],k1,k2);
        if (inmid(A[i-1],A[i],m)/*&&inmid(k1,k2,m)*/)
            B.push_back((ins){m,A[i-1],A[i]});
    }
    if (B.size()==0) return 0; sort(B.begin(),B.end());
    int now=1; while (now<B.size()&&B[now].m==B[0].m) now++;
    if (now==B.size()) return 0;
    int flag=contain(poly,(B[0].m+B[now].m)/2);
    if (flag==2) return 1;
    point d=B[now].m-B[0].m;
    for (int i=now;i<B.size();i++){
        if (!(B[i].m==B[i-1].m)&&flag==2) return 1;
        int tag=sign(cross(B[i].v-B[i].u,B[i].m+d-B[i].u));
        if (B[i].m==B[i].u||B[i].m==B[i].v) flag+=tag; else flag+=tag*2;
    }
}

```

```

    //return 0;
    return flag==2;
}
int checkinp(point r,point l,point m){
    if (compareangle(l,r)){return compareangle(l,m)&&compareangle(m,r);}
    return compareangle(l,m)||compareangle(m,r);
}
int checkPosFast(vector<point>A,point k1,point k2){ // 快速检查线段是否和多边形严格
    ↪ 相交
    if (contain(A,k1)==2||contain(A,k2)==2) return 1; if (k1==k2) return 0;
    A.push_back(A[0]); A.push_back(A[1]);
    for (int i=1;i+1<A.size();i++){
        if (checkLL(A[i-1],A[i],k1,k2)){
            point now=getLL(A[i-1],A[i],k1,k2);
            if (inmid(A[i-1],A[i],now)==0||inmid(k1,k2,now)==0) continue;
            if (now==A[i]){
                if (A[i]==k2) continue;
                point pre=A[i-1],ne=A[i+1];
                if (checkinp(pre-now,ne-now,k2-now)) return 1;
            } else if (now==k1){
                if (k1==A[i-1]||k1==A[i]) continue;
                if (checkinp(A[i-1]-k1,A[i]-k1,k2-k1)) return 1;
            } else if (now==k2||now==A[i-1]) continue;
            else return 1;
        }
    }
    return 0;
}
// 拆分凸包成上下凸壳 凸包尽量都随机旋转一个角度来避免出现相同横坐标
// 尽量特判只有一个点的情况 凸包逆时针
void getUDP(vector<point>A,vector<point>&U,vector<point>&D){
    db l=1e100,r=-1e100;
    for (int i=0;i<A.size();i++) l=min(l,A[i].x),r=max(r,A[i].x);
    int wherel,wherer;
    for (int i=0;i<A.size();i++) if (cmp(A[i].x,l)==0) wherel=i;
    for (int i=A.size();i;i--) if (cmp(A[i-1].x,r)==0) wherer=i-1;
    U.clear(); D.clear(); int now=wherel;
    while (1){D.push_back(A[now]); if (now==wherer) break; now++; if (now>=A.size())
        ↪ now=0;}
    now=wherel;
    while (1){U.push_back(A[now]); if (now==wherer) break; now--; if (now<0)
        ↪ now=A.size()-1;}
}
// 需要保证凸包点数大于等于 3,2 内部 ,1 边界 ,0 外部
int containCoP(const vector<point>&U,const vector<point>&D,point k){
    db lx=U[0].x,rx=U[U.size()-1].x;
    if (k==U[0]||k==U[U.size()-1]) return 1;
    if (cmp(k.x,lx)==-1||cmp(k.x,rx)==1) return 0;
    int wherel=lower_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();
    int where2=lower_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();

```

```

    int w1=clockwise(U[where1-1],U[where1],k),w2=clockwise(D[where2-1],D[where2],k);
    if (w1==1||w2==-1) return 0; else if (w1==0||w2==0) return 1; return 2;
}
// d 是方向 , 输出上方切点和下方切点
pair<point,point> getTangentCow(const vector<point> &U,const vector<point> &D,point
↪ d){
    if (sign(d.x)<0||(sign(d.x)==0&&sign(d.y)<0)) d=d*(-1);
    point whereU,whereD;
    if (sign(d.x)==0) return mp(U[0],U[U.size()-1]);
    int l=0,r=U.size()-1,ans=0;
    while (l<r){int mid=l+r>>1; if (sign(cross(U[mid+1]-U[mid],d))<=0)
↪ l=mid+1,ans=mid+1; else r=mid;}
    whereU=U[ans]; l=0,r=D.size()-1,ans=0;
    while (l<r){int mid=l+r>>1; if (sign(cross(D[mid+1]-D[mid],d))>=0)
↪ l=mid+1,ans=mid+1; else r=mid;}
    whereD=D[ans]; return mp(whereU,whereD);
}
// 先检查 contain, 逆时针给出
pair<point,point> getTangentCoP(const vector<point>&U,const vector<point>&D,point
↪ k){
    db lx=U[0].x,rx=U[U.size()-1].x;
    if (k.x<lx){
        int l=0,r=U.size()-1,ans=U.size()-1;
        while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==1) l=mid+1; else
↪ ans=mid,r=mid;}
        point w1=U[ans]; l=0,r=D.size()-1,ans=D.size()-1;
        while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1) l=mid+1; else
↪ ans=mid,r=mid;}
        point w2=D[ans]; return mp(w1,w2);
    } else if (k.x>rx){
        int l=1,r=U.size(),ans=0;
        while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==-1) r=mid; else
↪ ans=mid,l=mid+1;}
        point w1=U[ans]; l=1,r=D.size(),ans=0;
        while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==-1) r=mid; else
↪ ans=mid,l=mid+1;}
        point w2=D[ans]; return mp(w2,w1);
    } else {
        int where1=lower_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();
        int where2=lower_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();
        if ((k.x==lx&&k.y>U[0].y)|| (where1&&clockwise(U[where1-1],U[where1],k)==1)){
            int l=1,r=where1+1,ans=0;
            while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==-1)
↪ ans=mid,l=mid+1; else r=mid;}
            point w1=U[ans]; l=where1,r=U.size()-1,ans=U.size()-1;
            while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==-1) l=mid+1;
↪ else ans=mid,r=mid;}
            point w2=U[ans]; return mp(w2,w1);
        } else {

```

```

    int l=1,r=where2+1,ans=0;
    while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==-1)
        ↪ ans=mid,l=mid+1; else r=mid;}
    point w1=D[ans]; l=where2,r=D.size()-1,ans=D.size()-1;
    while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1) l=mid+1;
        ↪ else ans=mid,r=mid;}
    point w2=D[ans]; return mp(w1,w2);
}
}
}
struct P3{
    db x,y,z;
    P3 operator + (P3 k1){return (P3){x+k1.x,y+k1.y,z+k1.z};}
    P3 operator - (P3 k1){return (P3){x-k1.x,y-k1.y,z-k1.z};}
    P3 operator * (db k1){return (P3){x*k1,y*k1,z*k1};}
    P3 operator / (db k1){return (P3){x/k1,y/k1,z/k1};}
    db abs2(){return x*x+y*y+z*z;}
    db abs(){return sqrt(x*x+y*y+z*z);}
    P3 unit(){return (*this)/abs();}
    int operator < (const P3 k1) const{
        if (cmp(x,k1.x)!=0) return x<k1.x;
        if (cmp(y,k1.y)!=0) return y<k1.y;
        return cmp(z,k1.z)==-1;
    }
    int operator == (const P3 k1){
        return cmp(x,k1.x)==0&&cmp(y,k1.y)==0&&cmp(z,k1.z)==0;
    }
    void scan(){
        double k1,k2,k3; scanf("%lf%lf%lf",&k1,&k2,&k3);
        x=k1; y=k2; z=k3;
    }
};
P3 cross(P3 k1,P3 k2){return
    ↪ (P3){k1.y*k2.z-k1.z*k2.y,k1.z*k2.x-k1.x*k2.z,k1.x*k2.y-k1.y*k2.x};}
db dot(P3 k1,P3 k2){return k1.x*k2.x+k1.y*k2.y+k1.z*k2.z;}
//p=(3,4,5),l=(13,19,21),theta=85 ans=(2.83,4.62,1.77)
P3 turn3D(db k1,P3 l,P3 p){
    l=l.unit(); P3 ans; db c=cos(k1),s=sin(k1);
    ans.x=p.x*(l.x*l.x*(1-c)+c)+p.y*(l.x*l.y*(1-c)-l.z*s)+p.z*(l.x*l.z*(1-c)+l.y*s);
    ans.y=p.x*(l.x*l.y*(1-c)+l.z*s)+p.y*(l.y*l.y*(1-c)+c)+p.z*(l.y*l.z*(1-c)-l.x*s);
    ans.z=p.x*(l.x*l.z*(1-c)-l.y*s)+p.y*(l.y*l.z*(1-c)+l.x*s)+p.z*(l.x*l.x*(1-c)+c);
    return ans;
}
typedef vector<P3> VP;
typedef vector<VP> VVP;
db Acos(db x){return acos(max(-(db)1,min(x,(db)1)));}
// 球面距离 , 圆心原点 , 半径 1
db Odist(P3 a,P3 b){db r=Acos(dot(a,b)); return r;}
db r; P3 rnd;

```

```

vector<db> solve(db a,db b,db c){
    db r=sqrt(a*a+b*b),th=atan2(b,a);
    if (cmp(c,-r)==-1) return {0};
    else if (cmp(r,c)<=0) return {1};
    else {
        db tr=pi-Acos(c/r); return {th+pi-tr,th+pi+tr};
    }
}

vector<db> jiao(P3 a,P3 b){
    // dot(rd+x*cos(t)+y*sin(t),b) >= cos(r)
    if (cmp(0dist(a,b),2*r)>0) return {0};
    P3 rd=a*cos(r),z=a.unit(),y=cross(z,rd).unit(),x=cross(y,z).unit();
    vector<db> ret =
        ↪ solve(-(dot(x,b)*sin(r)),-(dot(y,b)*sin(r)),-(cos(r)-dot(rd,b)));
    return ret;
}

db norm(db x,db l=0,db r=2*pi){ // change x into [l,r)
    while (cmp(x,l)==-1) x+=(r-l); while (cmp(x,r)>=0) x-=(r-l);
    return x;
}

db disLP(P3 k1,P3 k2,P3 q){
    return (cross(k2-k1,q-k1)).abs()/(k2-k1).abs();
}

db disLL(P3 k1,P3 k2,P3 k3,P3 k4){
    P3 dir=cross(k2-k1,k4-k3); if (sign(dir.abs())==0) return disLP(k1,k2,k3);
    return fabs(dot(dir.unit(),k1-k2));
}

VP getFL(P3 p,P3 dir,P3 k1,P3 k2){
    db a=dot(k2-p,dir),b=dot(k1-p,dir),d=a-b;
    if (sign(fabs(d))==0) return {};
    return {(k1*a-k2*b)/d};
}

VP getFF(P3 p1,P3 dir1,P3 p2,P3 dir2){// 返回一条线
    P3 e=cross(dir1,dir2),v=cross(dir1,e);
    db d=dot(dir2,v); if (sign(abs(d))==0) return {};
    P3 q=p1+v*dot(dir2,p2-p1)/d; return {q,q+e};
}

// 3D Convex Hull Template
db getV(P3 k1,P3 k2,P3 k3,P3 k4){ // get the Volume
    return dot(cross(k2-k1,k3-k1),k4-k1);
}

db rand_db(){return 1.0*rand()/RAND_MAX;}

VP convexHull2D(VP A,P3 dir){
    P3 x={(db)rand(),(db)rand(),(db)rand()}; x=x.unit();
    x=cross(x,dir).unit(); P3 y=cross(x,dir).unit();
    P3 vec=dir.unit()*dot(A[0],dir);
    vector<point>B;
    for (int i=0;i<A.size();i++) B.push_back((point){dot(A[i],x),dot(A[i],y)});
    B=ConvexHull(B); A.clear();
}

```

```

    for (int i=0;i<B.size();i++) A.push_back(x*B[i].x+y*B[i].y+vec);
    return A;
}
namespace CH3{
    VVP ret; set<pair<int,int> >e;
    int n; VP p,q;
    void wrap(int a,int b){
        if (e.find({a,b})==e.end()){
            int c=-1;
            for (int i=0;i<n;i++) if (i!=a&&i!=b){
                if (c==-1||sign(getV(q[c],q[a],q[b],q[i]))>0) c=i;
            }
            if (c!=-1){
                ret.push_back({p[a],p[b],p[c]});
                e.insert({a,b}); e.insert({b,c}); e.insert({c,a});
                wrap(c,b); wrap(a,c);
            }
        }
    }
    VVP ConvexHull3D(VP _p){
        p=q=_p; n=p.size();
        ret.clear(); e.clear();
        for (auto &i:q) i=i+(P3){rand_db()*1e-4,rand_db()*1e-4,rand_db()*1e-4};
        for (int i=1;i<n;i++) if (q[i].x<q[0].x) swap(p[0],p[i]),swap(q[0],q[i]);
        for (int i=2;i<n;i++) if
            ↪ ((q[i].x-q[0].x)*(q[1].y-q[0].y)>(q[i].y-q[0].y)*(q[1].x-q[0].x))
            ↪ swap(q[1],q[i]),swap(p[1],p[i]);
        wrap(0,1);
        return ret;
    }
}
VVP reduceCH(VVP A){
    VVP ret; map<P3,VP> M;
    for (VP nowF:A){
        P3 dir=cross(nowF[1]-nowF[0],nowF[2]-nowF[0]).unit();
        for (P3 k1:nowF) M[dir].pb(k1);
    }
    for (pair<P3,VP> nowF:M) ret.pb(convexHull2D(nowF.se,nowF.fi));
    return ret;
}
// 把一个面变成 ( 点 , 法向量 ) 的形式
pair<P3,P3> getF(VP F){
    return mp(F[0],cross(F[1]-F[0],F[2]-F[0]).unit());
}
// 3D Cut 保留 dot(dir,x-p)>=0 的部分
VVP ConvexCut3D(VVP A,P3 p,P3 dir){
    VVP ret; VP sec;
    for (VP nowF: A){
        int n=nowF.size(); VP ans; int dif=0;

```

```

    for (int i=0;i<n;i++){
        int d1=sign(dot(dir,nowF[i]-p));
        int d2=sign(dot(dir,nowF[(i+1)%n]-p));
        if (d1>=0) ans.pb(nowF[i]);
        if (d1*d2<0){
            P3 q=getFL(p,dir,nowF[i],nowF[(i+1)%n])[0];
            ans.push_back(q); sec.push_back(q);
        }
        if (d1==0) sec.push_back(nowF[i]); else dif=1;
        dif|=(sign(dot(dir,cross(nowF[(i+1)%n]-nowF[i],nowF[(i+1)%n]-nowF[i])))==-1);
    }
    if (ans.size()>0&&dif) ret.push_back(ans);
}
if (sec.size()>0) ret.push_back(convexHull2D(sec,dir));
return ret;
}
db vol(VVP A){
    if (A.size()==0) return 0; P3 p=A[0][0]; db ans=0;
    for (VP nowF:A)
        for (int i=2;i<nowF.size();i++)
            ans+=abs(getV(p,nowF[0],nowF[i-1],nowF[i]));
    return ans/6;
}
VVP init(db INF) {
    VVP pss(6,VP(4));
    pss[0][0] = pss[1][0] = pss[2][0] = {-INF, -INF, -INF};
    pss[0][3] = pss[1][1] = pss[5][2] = {-INF, -INF, INF};
    pss[0][1] = pss[2][3] = pss[4][2] = {-INF, INF, -INF};
    pss[0][2] = pss[5][3] = pss[4][1] = {-INF, INF, INF};
    pss[1][3] = pss[2][1] = pss[3][2] = {INF, -INF, -INF};
    pss[1][2] = pss[5][1] = pss[3][3] = {INF, -INF, INF};
    pss[2][2] = pss[4][3] = pss[3][1] = {INF, INF, -INF};
    pss[5][0] = pss[4][0] = pss[3][0] = {INF, INF, INF};
    return pss;
}

```

6.3 Plane

```

namespace Geometry {
    typedef double db;
    const db inf = 1e20;
    const int maxn = 1;
    const db eps = 1e-8;
    const db delta = 0.98;

    int Sgn(db k) { return fabs(k) < eps ? 0 : (k < 0 ? -1 : 1);}
    int Cmp(db k1, db k2) {return Sgn(k1 - k2);}
    db max(db k1, db k2) {return Cmp(k1, k2) > 0 ? k1 : k2;}
    db min(db k1, db k2) {return Cmp(k1, k2) < 0 ? k1 : k2;}
}

```

```

struct point {db x, y;};
bool operator == (point k1, point k2) {return Cmp(k1.x, k2.x) == 0 && Cmp(k1.y,
↪ k2.y) == 0;}
point operator + (point k1, point k2) {return (point){k1.x + k2.x, k1.y + k2.y};}
point operator - (point k1, point k2) {return (point){k1.x - k2.x, k1.y - k2.y};}
db operator * (point k1, point k2) {return k1.x * k2.x + k1.y * k2.y;}
db operator ^ (point k1, point k2) {return k1.x * k2.y - k1.y * k2.x;}
point operator * (point k1, db k2) {return (point){k1.x * k2, k1.y * k2};}
point operator / (point k1, db k2) {return (point){k1.x / k2, k1.y / k2};}
db GetLen(point k) {return sqrt(k * k);}
db GetDisP2P(point k1, point k2) {return sqrt((k1 - k2) * (k1 - k2));}
db GetDisP2P2(point k1, point k2) {return (k1 - k2) * (k1 - k2);}
db GetAng(point k1, point k2) {return fabs(atan2(fabs(k1 ^ k2), k1 * k2));}
point Rotate(point k, db ang) {return (point){k.x * cos(ang) - k.y * sin(ang),
↪ k.x * sin(ang) + k.y * cos(ang)};}
point Rotate90(point k) {return (point){-k.y, k.x};}
bool IsConvexhull(std::vector<point> &p) {
    int N = (int)p.size();
    for (int i = 0; i < N; ++i)
        if (Sgn((p[(i + 1) % N] - p[i]) ^ (p[(i + 2) % N] - p[(i + 1) % N])) < 0)
            return false;
    return true;
}

db ClosestP2P(point p[], int l, int r) {
    if (l + 1 == r) return GetDisP2P(p[l], p[r]);
    if (l + 2 == r) return min(GetDisP2P(p[l + 1], p[r]), min(GetDisP2P(p[l], p[l +
↪ 1]), GetDisP2P(p[l], p[r])));
    int mid = (l + r) >> 1;
    db ret = min(ClosestP2P(l, mid), ClosestP2P(mid + 1, r));
    std::vector<point> mid_p;
    for (int i = l; i <= r; ++i) {
        if (Cmp(fabs(p[i].x - p[mid].x), ret) <= 0) mid_p.push_back(p[i]);
    }
    std::sort(mid_p.begin(), mid_p.end(), [&](point k1, point k2) {return Cmp(k1.y,
↪ k2.y) < 0;});
    for (int i = 0; i < (int)mid_p.size(); ++i) {
        for (int j = i + 1; j < (int)mid_p.size(); ++j) {
            if (Cmp(mid_p[j].y - mid_p[i].y, ret) >= 0) break;
            ret = min(ret, GetDisP2P(mid_p[i], mid_p[j]));
        }
    }
    return ret;
}

typedef std::vector<point> poly;
void RotateCaliper() {
    ans = -1e20;
    if (ConvexHull.size() == 3) {

```



```
    if (Cmp(GetDisP2P(ConvexHull[0], ConvexHull[1]), ans) > 0) ans =
        ↪ GetDisP2P(ConvexHull[0], ConvexHull[1]);
    if (Cmp(GetDisP2P(ConvexHull[0], ConvexHull[2]), ans) > 0) ans =
        ↪ GetDisP2P(ConvexHull[0], ConvexHull[2]);
    if (Cmp(GetDisP2P(ConvexHull[1], ConvexHull[2]), ans) > 0) ans =
        ↪ GetDisP2P(ConvexHull[1], ConvexHull[2]);
    return;
}
int cur = 2, size = ConvexHull.size();
for (int i = 0; i < size; ++i) {
    while (Cmp(fabs((ConvexHull[i] - ConvexHull[(i + 1) % size]) ^
        ↪ (ConvexHull[cur] - ConvexHull[(i + 1) % size])), fabs((ConvexHull[i] -
        ↪ ConvexHull[(i + 1) % size]) ^ (ConvexHull[(cur + 1) % size] -
        ↪ ConvexHull[(i + 1) % size]))) < 0) cur = (cur + 1) % size;
    if (Cmp(GetDisP2P(ConvexHull[i], ConvexHull[cur]), ans) > 0) ans =
        ↪ GetDisP2P(ConvexHull[i], ConvexHull[cur]);
}
}

poly Grahamscan(std::vector<point> p) {
    poly ans;
    if ((int)p.size() < 3) {
        for (int i = 0; i < (int)p.size(); ++i) ans.push_back(p[i]);
        return ans;
    }
    int low = 0;
    for (int i = 0; i < (int)p.size(); ++i)
        if (Cmp(p[i].x, p[low].x) < 0 || (Cmp(p[i].x, p[low].x) == 0 && Cmp(p[i].y,
            ↪ p[low].y) < 0))
            low = i;
    std::swap(p[0], p[low]);
    std::sort(p.begin() + 1, p.end(), [&](point k1, point k2) {
        double tmp = (k1 - p[0]) ^ (k2 - p[0]);
        if (Sgn(tmp) > 0) return true;
        else if (Sgn(tmp) == 0 && Cmp(GetDisP2P(k2, p[0]), GetDisP2P(k1, p[0])) > 0)
            ↪ return true;
        return false;
    });
    ans.push_back(p[0]);
    for (int i = 1; i < (int)p.size(); ++i) {
        while ((int)ans.size() >= 2 && Sgn((ans.back() - ans[(ans.size()) - 2]) ^
            ↪ (p[i] - ans[(int)ans.size() - 2])) <= 0) {
            ans.pop_back();
        }
        ans.push_back(p[i]);
    }
    return ans;
}
```

```
db GetMinCircle(std::vector<point> p) {
    point cur = p[0];
    db pro = 10000, ans = inf;
    while (pro > eps) {
        int book = 0;
        for (int i = 0; i < (int)p.size(); ++i)
            if (GetDisP2P(cur, p[i]) > GetDisP2P(cur, p[book]))
                book = i;
        db r = GetDisP2P(cur, p[book]);
        if (Cmp(r, ans) < 0) ans = r;
        cur = cur + (p[book] - cur) / r * pro;
        pro *= delta;
    }
    return ans;
}

struct line {point s, t;};
typedef line seg;
db GetLen(seg k) {return GetDisP2P(k.s, k.t);}
db GetDisP2Line(point k1, line k2) {return fabs((k1 - k2.s) ^ (k2.t - k2.s)) /
    ↪ GetLen(k2);}
db GetDisP2Seg(point k1, seg k2) {
    if (Sgn((k1 - k2.s) * (k2.t - k2.s)) < 0 || Sgn((k1 - k2.t) * (k2.s - k2.t)) <
    ↪ 0) {
        return min(GetDisP2P(k1, k2.s), GetDisP2P(k1, k2.t));
    }
    return GetDisP2Line(k1, k2);
}
bool IsParallel(line k1, line k2) {return Sgn((k1.s - k1.t) ^ (k2.s - k2.t)) ==
    ↪ 0;}
bool IsSegInterSeg(seg k1, seg k2) {
    return
        max(k1.s.x, k1.t.x) >= min(k2.s.x, k2.t.x) &&
        max(k2.s.x, k2.t.x) >= min(k1.s.x, k1.t.x) &&
        max(k1.s.y, k1.t.y) >= min(k2.s.y, k2.t.y) &&
        max(k2.s.y, k2.t.y) >= min(k1.s.y, k1.t.y) &&
        Sgn((k2.s - k1.t) ^ (k1.s - k1.t)) * Sgn((k2.t - k1.t) ^ (k1.s - k1.t)) <= 0
        ↪ &&
        Sgn((k1.s - k2.t) ^ (k2.s - k2.t)) * Sgn((k1.t - k2.t) ^ (k2.s - k2.t)) <= 0;
}
bool IsLineInterSeg(line k1, seg k2) {
    return Sgn((k2.s - k1.t) ^ (k1.s - k1.t)) * Sgn((k2.t - k1.t) ^ (k1.s - k1.t))
    ↪ <= 0;
}
bool IsLineInterLine(line k1, line k2) {
    return !IsParallel(k1, k2) || (IsParallel(k1, k2) && !(Sgn((k1.s - k2.s) ^
    ↪ (k2.t - k2.s)) == 0));
}
bool IsPointOnSeg(point k1, seg k2) {
```

```

    return Sgn((k1 - k2.s) ^ (k2.t - k2.s)) == 0 && Sgn((k1 - k2.s) * (k1 - k2.t))
    ↪ ≤ 0;
}

point Cross(line k1, line k2) {
    db temp = ((k1.s - k2.s) ^ (k2.s - k2.t)) / ((k1.s - k1.t) ^ (k2.s - k2.t));
    return (point){k1.s.x + (k1.t.x - k1.s.x) * temp, k1.s.y + (k1.t.y - k1.s.y) *
    ↪ temp};
}

// 表示 s->t 逆时针 (左侧) 的半平面
struct halfplane:public line {db ang;};
void GetAng(halfplane k) {k.ang = atan2(k.t.y - k.s.y, k.t.x - k.s.x);}
bool operator < (halfplane k1, halfplane k2) {
    if (Sgn(k1.ang - k2.ang) > 0) return k1.ang < k2.ang;
    return Sgn((k1.s - k2.s) ^ (k2.t - k2.s)) < 0;
}

struct HalfPlaneInsert {
    int tot;
    halfplane hp[maxn];
    halfplane deq[maxn];
    point points[maxn];
    point res[maxn];
    int front, tail;

    void Push(halfplane k) {hp[tot++] = k;}

    void Unique() {
        int cnt = 1;
        for (int i = 1; i < tot; ++i)
            if (fabs(hp[i].ang - hp[i - 1].ang) > eps)
                hp[cnt++] = hp[i];
        tot = cnt;
    }

    bool IsHalfPlaneInsert() {
        for (int i = 0; i < tot; ++i) GetAng(hp[i]);
        sort(hp, hp + tot);
        Unique();
        deq[front = 0] = hp[0];
        deq[tail = 1] = hp[1];
        for (int i = 2; i < tot; ++i) {
            if (fabs((deq[tail].t - deq[tail].s) ^ (deq[tail - 1].t - deq[tail - 1].s))
            ↪ < eps || fabs((deq[front].t - deq[front].s) ^ (deq[front + 1].t -
            ↪ deq[front + 1].s)) < eps) return false;
            while (front < tail && ((Cross(deq[tail], deq[tail - 1]) - hp[i].s) ^
            ↪ (hp[i].t - hp[i].s)) > eps) tail--;
            while (front < tail && ((Cross(deq[front], deq[front + 1]) - hp[i].s) ^
            ↪ (hp[i].t - hp[i].s)) > eps) front++;
            deq[++tail] = hp[i];
        }
    }
}

```

```

    }
    while (front < tail && ((Cross(deq[tail], deq[tail - 1]) - deq[front].s) ^
        ↪ (deq[front].t - deq[front].s)) > eps) tail--;
    while (front < tail && ((Cross(deq[front], deq[front - 1]) - deq[tail].s) ^
        ↪ (deq[tail].t - deq[tail].t)) > eps) front++;
    if (tail <= front + 1) return false;
    return true;
}

void GetHalfPlaneInsertConvex() {
    int cnt = 0;
    for (int i = front; i < tail; ++i) res[cnt++] = Cross(deq[i], deq[i + 1]);
    if (front < tail - 1) res[cnt++] = Cross(deq[front], deq[tail]);
}
};

struct circle {point o; db r;};
circle GetCircle(point k1, point k2, point k3) {
    db a1 = k2.x - k1.x, b1 = k2.y - k1.y, c1 = (a1 * a1 + b1 * b1) / 2;
    db a2 = k3.x - k1.x, b2 = k3.y - k1.y, c2 = (a2 * a2 + b2 * b2) / 2;
    db d = a1 * b2 - a2 * b1;
    point o = (point){k1.x + (c1 * b2 - c2 * b1) / d, k1.y + (a1 * c2 - a2 * c1) /
        ↪ d};
    return (circle){o, GetDisP2P(k1, o)};
}

circle GetMinCircle(std::vector<point> p) {
    std::random_shuffle(p.begin(), p.end());
    int n = (int)p.size();
    circle ret = (circle){p[0], 0.0};
    for (int i = 1; i < n; ++i) {
        if (Cmp(GetDisP2P(ret.o, p[i]), ret.r) <= 0) continue;
        ret = (circle){p[i], 0.0};
        for (int j = 0; j < i; ++j) {
            if (Cmp(GetDisP2P(ret.o, p[j]), ret.r) <= 0) continue;
            ret.o = (p[i] + p[j]) / 2; ret.r = GetDisP2P(ret.o, p[i]);
            for (int k = 0; k < j; ++k) {
                if (Cmp(GetDisP2P(ret.o, p[k]), ret.r) <= 0) continue;
                ret = GetCircle(p[i], p[j], p[k]);
            }
        }
    }
    return ret;
}
};
using namespace Geometry;

```

6.4 Simpson

```
typedef double db;

namespace Simpson {
    db a, b, c, d;

    db F(db x) {
        return (c * x + d) / (a * x + b);
    }

    db Simpson(db l, db r) {
        db m = (l + r) / 2.0;
        return (F(l) + 4 * F(m) + F(r)) * (r - l) / 6.0;
    }

    db Asr(db l, db r, db ans, db eps) {
        db m = (l + r) / 2.0;
        db l_ans = Simpson(l, m), r_ans = Simpson(m, r);
        if (fabs(l_ans + r_ans - ans) <= 15.0 * eps) return l_ans + r_ans + (l_ans +
            ↪ r_ans - ans) / 15.0;
        return Asr(l, m, l_ans, eps / 2.0) + Asr(m, r, r_ans, eps / 2.0);
    }
};
```

6.5 Stereoscopic

```
namespace Geometry3D {
    typedef double db;
    const db inf = 1e20;
    const int maxn = "Edit";
    const db eps = 1e-9;
    const db delta = 0.98;

    int Sgn(db k) {return fabs(k) < eps ? 0 : (k < 0 ? -1 : 1);}
    int Cmp(db k1, db k2) {return Sgn(k1 - k2);}

    struct point {db x, y, z;};
    bool operator == (point k1, point k2) {return Sgn(k1.x - k2.x) == 0 && Sgn(k1.y -
        ↪ k2.y) == 0 && Sgn(k1.z - k2.z) == 0;}
    point operator + (point k1, point k2) {return (point){k1.x + k2.x, k1.y + k2.y,
        ↪ k1.z + k2.z};}
    point operator - (point k1, point k2) {return (point){k1.x - k2.x, k1.y - k2.y,
        ↪ k1.z - k2.z};}
    db operator * (point k1, point k2) {return k1.x * k2.x + k1.y * k2.y + k1.z *
        ↪ k2.z;}
    db GetLen(point k) {return sqrt(k * k);}
    db GetLen2(point k) {return k * k;}
    db operator ^ (point k1, point k2) {return GetLen((point){k1.y * k2.z - k1.z *
        ↪ k2.y, k1.z * k2.x - k1.x * k2.z, k1.x * k2.y - k1.y * k2.x});}
```

```

point operator * (point k1, db k2) {return (point){k1.x * k2, k1.y * k2, k1.z *
↪ k2};}
point operator / (point k1, db k2) {return (point){k1.x / k2, k1.y / k2, k1.z /
↪ k2};}
db GetDisP2P(point k1, point k2) {return GetLen(k2 - k1);}
db GetDisP2P2(point k1, point k2) {return GetLen2(k2 - k1);}
db GetAngle(point k1, point k2) {return fabs(atan2(fabs(k1 ^ k2), k1 * k2));}

db GetMinSphere(vector<point> p) {
    Point cur = p[0];
    db pro = 10000, ret = inf;
    while (pro > eps) {
        int mark = 0;
        for (int i = 0; i < (int)p.size(); ++i) {
            if (Cmp(GetDisP2P(cur, p[i]), GetDisP2P(cur, p[mark])) > 0) mark = i;
        }
        db r = GetDisP2P(cur, p[mark]);
        ret = min(ret, r);
        cur = cur + (p[mark] - cur) / r * pro;
        pro *= delta;
    }
    return ret;
}

struct line {point s, t;};
typedef line seg;
db GetLen(seg k) {return GetDisP2P(k.s, k.t);}
db GetLe2(seg k) {return GetDisP2P2(k.s, k.t);}
db GetDisP2Line(point k1, line k2) {return fabs((k1 - k2.s) ^ (k2.t - k2.s)) /
↪ GetLen(k2);}
db GetDisP2Seg(point k1, seg k2) {
    if (Sgn((k1 - k2.s) * (k2.t - k2.s)) < 0 || Sgn((k1 - k2.t) * (k2.s - k2.t)) <
↪ 0) {
        return min(GetDisP2P(k1, k2.s), GetDisP2P(k1, k2.t));
    }
    return GetDisP2Line(k1, k2);
}

struct sphere {point o;db r;};
db GetV(sphere k) {return 4.0 / 3.0 * pi * k.r * k.r * k.r;}
db GetSphereInterV(sphere k1, sphere k2) {
    db ret = 0.0;
    db dis = GetDisP2P(k1.o, k2.o);
    if (Sgn(dis - k1.r - k2.r) >= 0) return ret;
    if (Sgn(k2.r - (dis + k1.r)) >= 0) return GetV(k1);
    else if (Sgn(k1.r - (dis + k2.r)) >= 0) return GetV(k2);
    db len1 = ((k1.r * k1.r - k2.r * k2.r) / dis + dis) / 2;
    db len2 = dis - len1;
    db x1 = k1.r - len1, x2 = k2.r - len2;

```

```
    db v1 = pi * x1 * x1 * (k1.r - x1 / 3.0);
    db v2 = pi * x2 * x2 * (k2.r - x2 / 3.0);
    return v1 + v2;
}

struct ray {point o, dir;};
bool IsRayInterSphere(ray k1, sphere k2, db &dis) {
    db a = k1.dir * k1.dir;
    db b = (k1.o - k2.o) * k1.dir * 2.0;
    db c = ((k1.o - k2.o) * (k1.o - k2.o)) - (k2.r * k2.r);
    db dlt = b * b - 4.0 * a * c;
    if (Sgn(dlt) < 0) return false;
    db x1 = (-b - sqrt(dlt)) / (2.0 * a), x2 = (-b + sqrt(dlt)) / (2.0 * a);
    if (Cmp(x1, x2) > 0) swap(x1, x2);
    if (Sgn(x1) <= 0) return false;
    dis = x1;
    return true;
}

void Reflect(ray &k1, sphere k2, db dis) {
    Point pos = k1.o + (k1.dir * dis);
    Vector temp = k2.o + (((pos - k2.o) * ((pos - k2.o) * (k1.o - k2.o))) /
        ↪ GetLen2(pos - k2.o));
    k1.dir = temp * 2.0 - k1.o - pos; k1.o = pos;
}
};
using namespace Geometry3D;
```

7 Others

7.1 FastIO

// 普通读入挂

```
template <typename t>
inline bool read(t &ret) {
    char c; int sgn;
    if (c = getchar(), c == EOF) return false;
    while (c != '-' && (c < '0' || c > '9')) c = getchar();
    sgn = (c == '-') ? -1 : 1;
    ret = (c == '-') ? 0 : (c - '0');
    while (c = getchar(), c >= '0' && c <= '9') ret = ret * 10 + (c - '0');
    ret *= sgn;
    return true;
}
```

// 普通输出挂

```
template <typename t>
inline void out(t x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) out(x / 10);
    putchar(x % 10 + '0');
}
```

// 牛逼读入挂

```
namespace fastIO {
    const int MX = 4e7;
    char buf[MX];
    int c, sz;
    void begin() {
        c = 0;
        sz = fread(buf, 1, MX, stdin);
    }
    template <class T>
    inline bool read(T &t) {
        while (c < sz && buf[c] != '-' && (buf[c] < '0' || buf[c] > '9')) c++;
        if (c >= sz) return false;
        bool flag = 0;
        if (buf[c] == '-') {
            flag = 1;
            c++;
        }
        for (t = 0; c < sz && '0' <= buf[c] && buf[c] <= '9'; ++c) t = t * 10 + buf[c]
            - '0';
        if (flag) t = -t;
        return true;
    }
}
```



```
    }
};

// 超级读写挂
namespace IO{
#define BUF_SIZE 100000
#define OUT_SIZE 100000
#define ll long long
//fread->read

bool IOerror=0;
inline char nc(){
    static char buf[BUF_SIZE],*p1=buf+BUF_SIZE,*pend=buf+BUF_SIZE;
    if (p1==pend){
        p1=buf; pend=buf+fread(buf,1,BUF_SIZE,stdin);
        if (pend==p1){IOerror=1;return -1;}
        //printf("IO error!\n");system("pause");for (;;);exit(0);}
    }
    return *p1++;
}
inline bool blank(char ch){return ch==' '||ch=='\n'||ch=='\r'||ch=='\t';}
inline void read(int &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}
inline void read(ll &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}
inline void read(double &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (ch=='.'){
        double tmp=1; ch=nc();
        for (;ch>='0'&&ch<='9';ch=nc())tmp/=10.0,x+=tmp*(ch-'0');
    }
    if (sign)x=-x;
}
```

```
inline void read(char *s){
    char ch=nc();
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    for (;!blank(ch)&&!IOerror;ch=nc())*s++=ch;
    *s=0;
}
inline void read(char &c){
    for (c=nc();blank(c);c=nc());
    if (IOerror){c=-1;return;}
}
//fwrite->write
struct Ostream_fwrite{
    char *buf,*p1,*pend;
    Ostream_fwrite(){buf=new char[BUF_SIZE];p1=buf;pend=buf+BUF_SIZE;}
    void out(char ch){
        if (p1==pend) fwrite(buf,1,BUF_SIZE,stdout);p1=buf;
        *p1++=ch;
    }
    void print(int x){
        static char s[15],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1);
    }
    void println(int x){
        static char s[15],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1); out('\n');
    }
    void print(ll x){
        static char s[25],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1);
    }
    void println(ll x){
        static char s[25],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1); out('\n');
    }
    void print(double x,int y){
        static ll mul[]={1,10,100,1000,10000,100000,1000000,10000000,100000000,
            1000000000,10000000000LL,100000000000LL,1000000000000LL,10000000000000LL,
            ↵ 100000000000000LL,1000000000000000LL,10000000000000000LL,100000000000000000LL};
        if (x<-1e-12)out('-'),x=-x;x*=mul[y];
```

```
ll x1=(ll)floor(x); if (x-floor(x)>=0.5)++x1;
ll x2=x1/mul[y],x3=x1-x2*mul[y]; print(x2);
if (y>0){out('.'); for (size_t i=1;i<y&& x3*mul[i]<mul[y];out('0'),++i);
    ↪ print(x3);}
}
void println(double x,int y){print(x,y);out('\n');}
void print(char *s){while (*s)out(*s++);}
void println(char *s){while (*s)out(*s++);out('\n');}
void flush(){if (p1!=buf){fwrite(buf,1,p1-buf,stdout);p1=buf;}}
~Ostream_fwrite(){flush();}
}Ostream;
inline void print(int x){Ostream.print(x);}
inline void println(int x){Ostream.println(x);}
inline void print(char x){Ostream.out(x);}
inline void println(char x){Ostream.out(x);Ostream.out('\n');}
inline void print(ll x){Ostream.print(x);}
inline void println(ll x){Ostream.println(x);}
inline void print(double x,int y){Ostream.print(x,y);}
inline void println(double x,int y){Ostream.println(x,y);}
inline void print(char *s){Ostream.print(s);}
inline void println(char *s){Ostream.println(s);}
inline void println(){Ostream.out('\n');}
inline void flush(){Ostream.flush();}
#undef ll
#undef OUT_SIZE
#undef BUF_SIZE
};
using namespace IO;
```

7.2 LeepYear

```
bool IsLeep(int year) {
    return (!(year % 4) && (year % 100)) || !(year % 400);
}
```

7.3 MoAlgorithm

7.3.1 Dynamic

```
const int maxn = "Edit";

// 动态莫队算法求区间不同数字数量（支持单点修改）
class MoCap {
public:
    int n, m;
    int block;
    int arr[maxn];
    struct query {int l, r, pre, id;};
    int q_tot;
    query q[maxn];
    struct change {int pos, val;};
```

```
int c_tot;
change c[maxn];
int cnt[maxn << 7];
int cur;
int ans[maxn];

void Add(int x) {
    cur += (++cnt[arr[x]] == 1);
}

void Del(int x) {
    cur -= (--cnt[arr[x]] == 0);
}

void Modify(int x, int i) {
    if (c[x].pos >= q[i].l && c[x].pos <= q[i].r) {
        cur -= (--cnt[arr[c[x].pos]] == 0);
        cur += (++cnt[c[x].val] == 1);
    }
    std::swap(c[x].val, arr[c[x].pos]);
}

void Solve() {
    scanf("%d%d", &n, &m);
    block = (int)sqrt(n);
    for (int i = 1; i <= n; ++i) scanf("%d", &arr[i]);
    for (int i = 1; i <= m; ++i) {
        char op; getchar();
        scanf("%c", &op);
        if (op == 'Q') {
            int l, r; scanf("%d%d", &l, &r);
            q[++q_tot] = (query){l, r, c_tot, q_tot};
        }
        else {
            int p, v; scanf("%d%d", &p, &v);
            c[++c_tot] = (change){p, v};
        }
    }
    std::sort(q + 1, q + q_tot + 1, [&](query k1, query k2) {
        if ((k1.l / block) == k2.l / block) {
            if ((k1.r / block) == (k2.r / block)) return k1.pre < k2.pre;
            return k1.r < k2.r;
        }
        return k1.l < k2.l;
    });

    int l = 1, r = 0, t = 0;
    for (int i = 1; i <= q_tot; ++i) {
        while (l < q[i].l) Del(l++);
```

```
        while (l > q[i].l) Add(--l);
        while (r < q[i].r) Add(++r);
        while (r > q[i].r) Del(r--);
        while (t < q[i].pre) Modify(++t, i);
        while (t > q[i].pre) Modify(t--, i);
        ans[q[i].id] = cur;
    }

    for (int i = 1; i <= q_tot; ++i) printf("%d\n", ans[i]);
}
}mo;
```

7.3.2 Static

```
const int maxn = "Edit";

// 静态莫队算法求区间不同数字数量
class MoCap {
public:
    int n, m;
    int block;
    int arr[maxn];
    struct query {int l, r, id;};
    query q[maxn];
    int cnt[maxn << 1];
    int cur;
    int ans[maxn];

    void Add(int x) {
        cur += (++cnt[arr[x]] == 1);
    }

    void Del(int x) {
        cur -= (--cnt[arr[x]] == 0);
    }

    void Solve() {
        scanf("%d%d", &n, &m);
        block = (int)sqrt(n);
        for (int i = 1; i <= n; ++i) scanf("%d", &arr[i]);
        for (int i = 1; i <= m; ++i) {
            scanf("%d%d", &q[i].l, &q[i].r);
            q[i].id = i;
        }
        std::sort(q + 1, q + m + 1, [&](query k1, query k2) {
            return (k1.l / block) == (k2.l / block) ? k1.r < k2.r : k1.l < k2.l;
        });

        int l = 0, r = 0;
        for (int i = 1; i <= m; ++i) {
```

```
        while (l < q[i].l) Del(l++);
        while (l > q[i].l) Add(--l);
        while (r < q[i].r) Add(++r);
        while (r > q[i].r) Del(r--);
        ans[q[i].id] = cur;
    }

    for (int i = 1; i <= m; ++i) printf("%d\n", ans[i]);
}
}mo;
```

7.4 vim

syntax on
set nu ts=2 sw=2 et mouse=a
set cindent

```
"map <F9> :call Run()<CR>
"func! Run()
"    exec "w"
"    exec "!g++ % -o %<"
"    exec "! %<"
"endfunc

"map <F2> :call SetTitle()<CR>
"func SetTitle()
"    let l = 0
"    let l = l + 1 | call setline(l, "#include <bits/stdc++.h>")
"    let l = l + 1 | call setline(l, "using namespace std;")
"    let l = l + 1 | call setline(l, "")
"    let l = l + 1 | call setline(l, "int main(int argc, char *argv[]) {")
"    let l = l + 1 | call setline(l, "    std::ios::sync_with_stdio(false);")
"    let l = l + 1 | call setline(l, "    std::cin.tie(nullptr);")
↵    std::cout.tie(nullptr);")
"    let l = l + 1 | call setline(l, "    return 0;")
"    let l = l + 1 | call setline(l, "}")
"    let l = l + 1 | call setline(l, "")
"endfunc
```