

Algorithm Library

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Contents

1	String	4
1.1	AhoCorasickAutomaton	4
1.2	KMP	5
1.3	Manacher	6
1.4	PalindromicTree	7
1.5	TrieTree	9
2	Math	11
2.1	Catalan	11
2.2	Derangement	11
2.3	Euler	11
2.3.1	Euler	11
2.3.2	Screen	12
2.3.3	Sieve	12
2.4	FFT	13
2.5	Fibonacci	15
2.6	GeneratingFunction	16
2.7	InverseElement	16
2.7.1	ExtendGcd	16
2.7.2	Factorial	17
2.7.3	FermatLittleTheorem	18
2.7.4	Recursive	18
2.8	Mobius	18
2.9	NimGame	19
2.10	Prime	19
2.10.1	PrimeFactor	19
2.10.2	SieveOfEratosthenes	20
2.11	QuickPow	20
2.12	Stirling	21
3	DataStructure	22
3.1	BinaryIndexedTree	22
3.2	DfsOrder	22
3.3	FunctionalSegmentTree	22
3.4	Hash	23
3.5	MultipleTree	24
3.6	SegmentTree	30
3.7	SplayTree	31
4	GraphTheory	35
4.1	LCA	35
4.1.1	DFS+ST	35
4.1.2	Tarjan	37
4.2	MinimumSpanningTree	39
4.2.1	Kruskal	39
4.2.2	Prim	40
4.3	NetworkFlow	41
4.3.1	Dinic	41

4.3.2	FordFulkerson	42
4.3.3	MinCostMaxFlow	43
4.4	ShortestPath	45
4.4.1	BellmanFord	45
4.4.2	Dijkstra	46
4.4.3	Floyd	48
4.4.4	SPFA	48
5	DynamicProgramming	50
5.1	Contour	50
5.2	Digit	50
5.3	LCS	51
5.4	LIS	52
5.5	Pack	52
6	ComputationalGeometry	54
6.1	JlsGeo	54
6.2	Plane	66
6.3	Simpson	71
6.4	Stereoscopic	71
7	Others	75
7.1	Factorial	75
7.2	FastIO	75
7.3	LeapYear	79
7.4	vim	80

1 String

1.1 AhoCorasickAutomaton

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct AhoCorasickAutomaton {
    // 子节点记录数组
    int Son[maxn][26];
    int Val[maxn];
    // 失配指针 Fail 数组
    int Fail[maxn];
    // 节点数量
    int Tot;

    // Trie Tree 初始化
    void TrieInit() {
        Tot = 0;
        memset(Son, 0, sizeof(Son));
        memset(Val, 0, sizeof(Val));
        memset(Fail, 0, sizeof(Fail));
    }

    // 计算字母下标
    int Pos(char X) {
        return X - 'a';
    }

    // 向 Trie Tree 中插入 Str 模式字符串
    void Insert(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                Son[Cur][Index] = ++Tot;
            }
            Cur = Son[Cur][Index];
        }
        Val[Cur]++;
    }

    // Bfs 求得 Trie Tree 上失配指针
    void GetFail() {
        std::queue<int> Que;
        for (int i = 0; i < 26; ++i) {
            if (Son[0][i]) {
                Fail[Son[0][i]] = 0;
                Que.push(Son[0][i]);
            }
        }
    }
};
```

```
    }
}
while (!Que.empty()) {
    int Cur = Que.front(); Que.pop();
    for (int i = 0; i < 26; ++i) {
        if (Son[Cur][i]) {
            Fail[Son[Cur][i]] = Son[Fail[Cur]][i];
            Que.push(Son[Cur][i]);
        }
        else {
            Son[Cur][i] = Son[Fail[Cur]][i];
        }
    }
}
}

// 询问 Str 中出现的模式串数量
int Query(string Str) {
    int Len = int(Str.length());
    int Cur = 0, Ans = 0;
    for (int i = 0; i < Len; ++i) {
        Cur = Son[Cur][Pos(Str[i])];
        for (int j = Cur; j && ~Val[j]; j = Fail[j]) {
            Ans += Val[j];
            Val[j] = -1;
        }
    }
    return Ans;
}
};
```

1.2 KMP

```
#include <bits/stdc++.h>

// 对模式串 Pattern 计算 Next 数组
void KMPPre(string Pattern, vector<int> &Next) {
    int i = 0, j = -1;
    Next[0] = -1;
    int Len = int(Pattern.length());
    while (i != Len) {
        if (j == -1 || Pattern[i] == Pattern[j]) {
            Next[++i] = ++j;
        }
        else {
            j = Next[j];
        }
    }
}
```

// 优化对模式串 *Pattern* 计算 *Next* 数组

```
void PreKMP(string Pattern, vector<int> &Next) {
    int i, j;
    i = 0;
    j = Next[0] = -1;
    int Len = int(Pattern.length());
    while (i < Len) {
        while (j != -1 && Pattern[i] != Pattern[j]) {
            j = Next[j];
        }
        if (Pattern[++i] == Pattern[++j]) {
            Next[i] = Next[j];
        }
        else {
            Next[i] = j;
        }
    }
}
```

// 利用预处理 *Next* 数组计数模式串 *Pattern* 在主串 *Main* 中出现次数

```
int KMPCount(string Pattern, string Main) {
    int PatternLen = int(Pattern.length()), MainLen = int(Main.length());
    vector<int> Next(PatternLen + 1, 0);
    //PreKMP(Pattern, Next);
    KMPPre(Pattern, Next);
    int i = 0, j = 0;
    int Ans = 0;
    while (i < MainLen) {
        while (j != -1 && Main[i] != Pattern[j]) {
            j = Next[j];
        }
        i++; j++;
        if (j >= PatternLen) {
            Ans++;
            j = Next[j];
        }
    }
    return Ans;
}
```

1.3 Manacher

```
#include <bits/stdc++.h>
```

```
const int maxn = "Edit";
```

```
char ConvertStr[maxn << 1];
```

```
int Len[maxn << 1];
```

// Manacher 算法求 *Str* 字符串最长回文子串长度

```
int Manacher(char Str[]) {
    int L = 0, StrLen = int(strlen(Str));
    ConvertStr[L++] = '$'; ConvertStr[L++] = '#';
    for (int i = 0; i < StrLen; ++i) {
        ConvertStr[L++] = Str[i];
        ConvertStr[L++] = '#';
    }
    int MX = 0, ID = 0, Ans = 0;
    for (int i = 0; i < L; ++i) {
        Len[i] = MX > i ? min(Len[2 * ID - i], MX - i) : 1;
        while (ConvertStr[i + Len[i]] == ConvertStr[i - Len[i]]) {
            Len[i]++;
        }
        if (i + Len[i] > MX) {
            MX = i + Len[i];
            ID = i;
        }
        Ans = max(Ans, Len[i] - 1);
    }
    return Ans;
}
```

1.4 PalindromicTree

```
#include <bits/stdc++.h>
```

```
const int maxn = "Edit";
```

```
struct PalindromicTree {
    // 子节点记录数组
    long long Son[maxn][26];
    // 失配指针 Fail 数组
    long long Fail[maxn];
    // Len[i]: 节点 i 表示的回文串长度 (一个节点表示一个回文串)
    long long Len[maxn];
    // Cnt[i]: 节点 i 表示的本质不同的串的个数 (最后需要运行 Count() 函数才可求出正确
    // 结果)
    long long Cnt[maxn];
    // Num[i]: 以节点 i 表示的最长回文串的最右端为回文串结尾的回文串个数
    long long Num[maxn];
    // 字符
    long long Str[maxn];
    // 新添加字符后最长回文串表示的节点
    long long Last;
    // 字符数量
    long long StrLen;
    // 节点数量
    long long Tot;

    // 新建节点
```

```
long long NewNode(long long X) {
    for (long long i = 0; i < 26; ++i) {
        Son[Tot][i] = 0;
    }
    Cnt[Tot] = 0;
    Num[Tot] = 0;
    Len[Tot] = X;
    return Tot++;
}

// 初始化
void Init() {
    Tot = 0;
    NewNode(0); NewNode(-1);
    Last = 0;
    StrLen = 0;
    // 开头存字符集中没有的字符，减少特判
    Str[0] = -1;
    Fail[0] = 1;
}

long long GetFail(long long X) {
    while (Str[StrLen - Len[X] - 1] != Str[StrLen]) {
        X = Fail[X];
    }
    return X;
}

void Add(long long Char) {
    Char -= 'a';
    Str[++StrLen] = Char;
    long long Cur = GetFail(Last);
    if (!Son[Cur][Char]) {
        long long New = NewNode(Len[Cur] + 2);
        Fail[New] = Son[GetFail(Fail[Cur])][Char];
        Son[Cur][Char] = New;
        Num[New] = Num[Fail[New]] + 1;
    }
    Last = Son[Cur][Char];
    Cnt[Last]++;
}

void Count() {
    // 若 Fail[V]=U, 则 U 一定是 V 回文子串, 所以双亲累加孩子的 Cnt
    for (long long i = Tot - 1; i >= 0; --i) {
        Cnt[Fail[i]] += Cnt[i];
    }
}

};
```


1.5 TrieTree

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Trie {
    // Trie Tree 节点
    int Son[maxn][26];
    // Trie Tree 节点数量
    int Tot;

    // 字符串数量统计数组
    int Cnt[maxn];

    // Trie Tree 初始化
    void TrieInit() {
        Tot = 0;
        memset(Cnt, 0, sizeof(Cnt));
        memset(Son, 0, sizeof(Son));
    }

    // 计算字母下标
    int Pos(char X) {
        return X - 'a';
    }

    // 向 Trie Tree 中插入字符串 Str
    void Insert(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                Son[Cur][Index] = ++Tot;
            }
            Cur = Son[Cur][Index];

            Cnt[Cur]++;
        }
    }

    // 查找字符串 Str, 存在返回 true, 不存在返回 false
    bool Find(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                return false;
            }
            Cur = Son[Cur][Index];
        }
    }
};
```

```
    }  
    return true;  
}  
  
// 查询字典树中以 Str 为前缀的字符串数量  
int PathCnt(string Str) {  
    int Cur = 0, Len = int(Str.length());  
    for (int i = 0; i < Len; ++i) {  
        int Index = Pos(Str[i]);  
        if (!Son[Cur][Index]) {  
            return 0;  
        }  
        Cur = Son[Cur][Index];  
    }  
    return Cnt[Cur];  
}  
};
```

2 Math

2.1 Catalan

```
#include <bits/stdc++.h>

const int maxn = "Edit";

long long Catalan[maxn];

// 递推求卡特兰数
void CalalanInit() {
    memset(Catalan, 0, sizeof(Catalan));
    Catalan[0] = Catalan[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Catalan[i] = Catalan[i - 1] * (4 * i - 2) / (i + 1);
    }
}
```

2.2 Derangement

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const int mod = 1e9 + 7;

// Staggered: 错排数
long long Staggered[maxn];

// 求错排数
void StaggeredInit() {
    Staggered[1] = 0;
    Staggered[2] = 1;
    // 递推求错排数
    for (int i = 3; i < maxn; ++i) {
        Staggered[i] = (i - 1) * (Staggered[i - 1] + Staggered[i - 2]) % mod;
    }
}
```

2.3 Euler

2.3.1 Euler

```
#include <bits/stdc++.h>

// 单独求解欧拉函数
int Phi(int X) {
    int Ans = X;
    for (int i = 2; i * i <= X; ++i) {
        if (!(X % i)) {
            Ans = Ans / i * (i - 1);
        }
    }
}
```

```
        while (!(X % i)) {
            X /= i;
        }
    }
}
if (X > 1) {
    Ans = Ans / X * (X - 1);
}
return Ans;
}
```

2.3.2 Screen

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 欧拉函数
int Phi[maxn];

// 筛法求欧拉函数
void Euler() {
    for (int i = 1; i < maxn; ++i) {
        Phi[i] = i;
    }
    for (int i = 2; i < maxn; i += 2) {
        Phi[i] /= 2;
    }
    for (int i = 3; i < maxn; i += 2) {
        if (Phi[i] == i) {
            for (int j = i; j < maxn; j += i) {
                Phi[j] = Phi[j] / i * (i - 1);
            }
        }
    }
}
```

2.3.3 Sieve

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 素数标记
bool IsPrime[maxn];
// 欧拉函数
int Phi[maxn];
// 素数
int Prime[maxn];
// 素数个数
```

```
int Tot;

// 同时求得欧拉函数和素数表
void PhiPrime() {
    memset(IsPrime, false, sizeof(IsPrime));
    Phi[1] = 1;
    Tot = 0;
    for (int i = 2; i < maxn; ++i) {
        if (!IsPrime[i]) {
            Prime[Tot++] = i;
            Phi[i] = i - 1;
        }
        for (int j = 0; j < Tot && i * Prime[j] < maxn; ++j) {
            IsPrime[i * Prime[j]] = true;
            if (!(i % Prime[j])) {
                Phi[i * Prime[j]] = Phi[i] * Prime[j];
                break;
            }
            else {
                Phi[i * Prime[j]] = Phi[i] * (Prime[j] - 1);
            }
        }
    }
}
```

2.4 FFT

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const double pi = acos(-1.0);

// 复数
struct Complex {
    double X, Y;

    Complex operator + (const Complex &B) const {
        return Complex {X + B.X, Y + B.Y};
    }

    Complex operator - (const Complex &B) const {
        return Complex {X - B.X, Y - B.Y};
    }

    Complex operator * (const Complex &B) const {
        return Complex {X * B.X - Y * B.Y, X * B.Y + Y * B.X};
    }

    Complex operator / (const Complex &B) const {
        double Temp = B.X * B.X + B.Y * B.Y;
```

```
        return Complex {(X * B.X + Y * B.Y) / Temp, (Y * B.X - X * B.Y) / Temp};
    }
};

// 多项式系数数量
int N, M;
int L;
int Limit;
int R[maxn << 2];

// 快速傅里叶变换 (FFT)
void FFT(Complex F[], int Op) {
    for (int i = 0; i < Limit; ++i) {
        if (i < R[i]) {
            std::swap(F[i], F[R[i]]);
        }
    }
    for (int j = 1; j < Limit; j <= 1) {
        Complex Temp = Complex {cos(pi / j), Op * sin(pi / j)};
        for (int k = 0; k < Limit; k += (j < 1)) {
            Complex Buffer = Complex {1.0, 0.0};
            for (int l = 0; l < j; ++l) {
                Complex Tx = F[k + l], Ty = Buffer * F[k + j + l];
                F[k + l] = Tx + Ty;
                F[k + j + l] = Tx - Ty;
                Buffer = Buffer * Temp;
            }
        }
    }
}

// 多项式系数
Complex A[maxn], B[maxn];

// 多项式卷积计算
void Cal() {
    Limit = 1; L = 0;
    while (Limit <= N + M) {
        Limit <= 1;
        L++;
    }
    for (int i = 0; i < Limit; ++i) {
        R[i] = (R[i >> 1] >> 1) | ((i & 1) << (L - 1));
    }
    FFT(A, 1);
    FFT(B, 1);
    for (int i = 0; i <= Limit; ++i) {
        A[i] = A[i] * B[i];
    }
}
```

```
    FFT(A, -1);  
}
```

2.5 Fibonacci

```
#include <bits/stdc++.h>
```

```
const int mod = 1e9 + 7;
```

```
// 矩阵结构体
```

```
struct Matrix {  
    // 矩阵  
    long long Mat[2][2];  
};
```

```
// 重载矩阵乘法
```

```
Matrix operator * (Matrix &Key1, Matrix &Key2) const {  
    Matrix Res;  
    memset(Res.Mat, 0, sizeof(Res.Mat));  
    for (int i = 0; i < 2; ++i) {  
        for (int j = 0; j < 2; ++j) {  
            for (int k = 0; k < 2; ++k) {  
                Res.Mat[i][j] = (Res.Mat[i][j] + Key1.Mat[i][k] * Key2.Mat[k][j] %  
                    ↪ mod) % mod;  
            }  
        }  
    }  
    return Res;  
}
```

```
// 重载矩阵快速幂
```

```
Matrix operator ^ (Matrix Base, long long K) {  
    Matrix Res;  
    memset(Res.Mat, 0, sizeof(Res.Mat));  
    Res.Mat[0][0] = Res.Mat[1][1] = 1;  
    while (K) {  
        if (K & 1) {  
            Res = Res * Base;  
        }  
        Base = Base * Base;  
        K >>= 1;  
    }  
    return Res;  
}
```

```
// 斐波那契数列中第 x 项
```

```
long long Fib(long long X) {  
    Matrix Base;  
    Base.Mat[0][0] = Base.Mat[1][0] = Base.Mat[0][1] = 1;  
    Base.Mat[1][1] = 0;  
}
```

```
    return (Base ^ X).Mat[0][1];  
}
```

2.6 GeneratingFunction

```
#include <bits/stdc++.h>  
  
const int maxn = "Edit";  
  
void GeneratingFunction() {  
    int n;  
    int c1[maxn], c2[maxn];  
    scanf("%d", &n);  
    for (int i = 0; i < maxn; ++i) {  
        c1[i] = 1;  
        c2[i] = 0;  
    }  
    // c1[i] 为  $x^i$  的系数  
    // c2 为中间变量  
    for (int i = 2; i <= n; ++i) {  
        for (int j = 0; j <= n; ++j) {  
            for (int k = 0; k + j <= n; k += i) {  
                c2[j + k] += c1[i];  
            }  
        }  
        for (int j = 0; j <= n; ++j) {  
            c1[j] = c2[j];  
            c2[j] = 0;  
        }  
    }  
}
```

2.7 InverseElement

2.7.1 ExtendGcd

```
#include <bits/stdc++.h>  
  
// 扩展欧几里得,  $A*X+B*Y=D$   
long long ExtendGcd(long long A, long long B, long long &X, long long &Y) {  
    // 无最大公约数  
    if (A == 0 && B == 0) {  
        return -1;  
    }  
    if (B == 0) {  
        X = 1;  
        Y = 0;  
        return A;  
    }  
    long long D = ExtendGcd(B, A % B, Y, X);  
    Y -= A / B * X;
```



```
    return D;
}

// 逆元,  $AX = 1 \pmod M$ 
long long Inv(long long A, long long N) {
    long long X, Y;
    long long D = ExtendGcd(A, N, X, Y);
    if (D == 1) {
        return (X % N + N) % N;
    }
    else {
        return -1;
    }
}
```

2.7.2 Factorial

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;
const int maxn = "Edit";

// 快速乘
long long QuickMul(long long A, long long B) {
    long long Ans = 0;
    while (B) {
        if (B & 1) {
            Ans = (Ans + A) % mod;
        }
        A = (A + A) % mod;
        B >>= 1;
    }
    return Ans;
}

// 快速幂
long long QuickPow(long long A, long long B) {
    long long Ans = 1;
    while (B) {
        if (B & 1) {
            Ans = QuickMul(Ans, A) % mod;
        }
        A = QuickMul(A, A) % mod;
        B >>= 1;
    }
    return Ans;
}

// Factorial: 阶乘, FactorialInv: 阶乘逆元
long long Factorial[maxn], FactorialInv[maxn];
```

```
// 求阶乘逆元
void FactorialInvInit() {
    // 求阶乘
    Factorial[0] = 0;
    Factorial[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Factorial[i] = (Factorial[i - 1] * i) % mod;
    }
    // 飞马小定理求最大值阶乘逆元
    FactorialInv[maxn - 1] = QuickPow(Factorial[maxn - 1], mod - 2);
    // 递推求阶乘逆元
    for (int i = maxn - 2; i >= 0; --i) {
        FactorialInv[i] = (FactorialInv[i + 1] * (i + 1)) % mod;
    }
}
```

2.7.3 FermatLittleTheorem

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;

// 快速幂、费马小定理求逆元
long long Inv(long long X) {
    return QuickPow(X, mod - 2);
}
```

2.7.4 Recursive

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;
const int maxn = "Edit";

long long Inv[maxn];

// 递推求逆元
void InvInit() {
    Inv[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Inv[i] = (mod - mod / i) * Inv[mod % i] % mod;
    }
}
```

2.8 Mobius

```
#include <bits/stdc++.h>

const int maxn = "Edit";
```

```
bool is_prime[maxn];
vector<int> prime;
int mu[maxn];

void Sieve() {
    memset(is_prime, true, sizeof(is_prime));
    mu[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        if (is_prime[i]) {
            prime.emplace_back(i);
            mu[i] = -1;
        }
        for (auto &it : prime) {
            if (it * i >= maxn) break;
            is_prime[i * it] = false;
            if (i % it == 0) {
                mu[i * it] = 0;
                break;
            }
            mu[i * it] = -mu[i];
        }
    }
}
```

2.9 NimGame

```
#include <bits/stdc++.h>

// 尼姆博弈
bool Nim(std::vector<int> Num) {
    int Ans = 0;
    for (int i = 0; i < int(Num.size()); ++i) {
        Ans ^= Num[i];
    }
    // ans 不为零则先手赢, 否则为后手赢
    return Ans != 0 ? true : false;
}
```

2.10 Prime

2.10.1 PrimeFactor

```
#include <bits/stdc++.h>

const int maxn = "Edit"

bool IsPrime[maxn];
vector<int> PrimeFactor[maxn];

void Init() {
    memset(IsPrime, true, sizeof(IsPrime));
```

```
for (long long i = 2; i < maxn; ++i) {
    if (IsPrime[i]) {
        PrimeFactor[i].push_back(i);
        for (long long j = i + i; j < maxn; ++j) {
            IsPrime[j] = false;
            PrimeFactor[j].push_back(i);
        }
    }
}
IsPrime[1] = false;
}
```

2.10.2 SieveOfEratosthenes

```
#include <bits/stdc++.h>

typedef long long ll;
const int maxn = "Edit";

bool is_prime[maxn];
vector<int> prime

void Sieve() {
    memset(is_prime, true, sizeof(is_prime));
    is_prime[0] = is_prime[1] = false;
    for (ll i = 2; i < maxn; ++i) {
        if (is_prime[i]) prime.emplace_back(i);
        for (auto &it : prime) {
            if (it * i >= maxn) break;
            is_prime[i * it] = false;
        }
    }
}
```

2.11 QuickPow

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;

// 快速乘求 A*B%mod
long long QuickMul(long long A, long long B) {
    long long Ans = 0;
    while (B) {
        if (B & 1) {
            Ans = (Ans + A) % mod;
        }
        A = (A + A) % mod;
        B >>= 1;
    }
}
```

```
    return Ans;
}

// 快速幂求  $A^{B \bmod}$ 
long long QuickPow(long long A, long long B) {
    long long Ans = 1;
    while (B) {
        if (B & 1) {
            //  $Ans = Ans * A \% mod$ ;
            Ans = QuickMul(Ans, A) % mod;
        }
        //  $Ans = A * A \% mod$ ;
        A = QuickMul(A, A) % mod;
        B >>= 1;
    }
    return Ans;
}
```

2.12 Stirling

```
#include <bits/stdc++.h>

const double pi = acos(-1.0);
const double e = 2.718281828459;

int Stirling(int x) {
    if (x <= 1) {
        return 1;
    }
    return int(ceil(log10(2 * pi * x) / 2 + x * log10(x / e)));
}
```

3 DataStructure

3.1 BinaryIndexedTree

```
#define lowbit(x) (x&(-x))
const int maxn = "Edit";

namespace binary_indexed_tree {
    int arr[maxn];

    void Update(int x, int v) {
        while (x < maxn) {
            arr[x] += v;
            x += lowbit(x);
        }
    }

    int GetSum(int x) {
        int ans = 0;
        while (x > 0) {
            ans += arr[x];
            x -= lowbit(x);
        }
    }
};
```

3.2 DfsOrder

```
const int maxn = "Edit";

vector<vector<int>> g;

int dfs_clock;
int in[maxn], out[maxn];

// Dfs 序
void DfsOrder(int cur, int pre) {
    dfs_clock++;
    in[cur] = dfs_clock;
    for (auto &it : g) {
        if (it == pre) continue;
        DfsOrder(it, cur);
    }
    out[cur] = dfs_clock;
}
```

3.3 FunctionalSegmentTree

```
// 主席树, 静态区间第 k 小
const int maxn = "Edit";
```

```
namespace FuncSegTree {
    int tot;
    int rt[maxn];
    int lson[maxn << 5], rson[maxn << 5];
    int cnt[maxn << 5];

    int Build(int l, int r) {
        int t = ++tot;
        int m = (l + r) >> 1;
        if (l != r) {
            lson[t] = Build(l, m);
            rson[t] = Build(m + 1, r);
        }
        return t;
    }

    int Modify(int prev, int l, int r, int x) {
        int t = ++tot;
        lson[t] = lson[prev]; rson[t] = rson[prev];
        cnt[t] = cnt[prev] + 1;
        int m = (l + r) >> 1;
        if (l != r) {
            if (x <= m) lson[t] = Modify(lson[t], l, m, x);
            else rson[t] = Modify(rson[t], m + 1, r, x);
        }
        return t;
    }

    int Query(int u, int v, int l, int r, int k) {
        if (l == r) return l;
        int m = (l + r) >> 1;
        int num = cnt[lson[v]] - cnt[lson[u]];
        if (num >= k) return Query(lson[u], lson[v], l, m, k);
        return Query(rson[u], rson[v], m + 1, r, k - num);
    }
};
```

3.4 Hash

// 离散化

```
namespace Hash {
    int size;
    vector<int> arr;

    Hash(const vector<int> &v) {
        arr.assign(v.begin(), v.end());
        sort(arr.begin(), arr.end());
        arr.erase(unique(arr.begin(), arr.end()), arr.end());
        size = arr.size();
    }
}
```

```
int Get(int k) {  
    return lower_bound(arr.begin(), arr.end(), k) - arr.begin();  
}  
};
```

3.5 MultipleTree

```
/*  
    BZOJ 3196 (线段树套伸展树)  
    1. 查询  $k$  在区间内的排名  
    2. 查询区间内排名为  $k$  的值  
    3. 修改某一位值上的数值  
    4. 查询  $k$  在区间内的前驱 (前驱定义为小于  $x$ , 且最大的数)  
    5. 查询  $k$  在区间内的后继 (后继定义为大于  $x$ , 且最小的数)  
*/  
#include <bits/stdc++.h>  
using namespace std;  
const int inf = 2147483647;  
const int maxn = 5e4 + 5;  
const int maxm = maxn * 25;  
  
int n;  
int arr[maxn];  
  
namespace SplayTree {  
    int rt[maxm], tot;  
    int fa[maxm], son[maxm][2];  
    int val[maxm], cnt[maxm];  
    int sz[maxm];  
  
    void Push(int o) {  
        sz[o] = sz[son[o][0]] + sz[son[o][1]] + cnt[o];  
    }  
  
    bool Get(int o) {  
        return o == son[fa[o]][1];  
    }  
  
    void Clear(int o) {  
        son[o][0] = son[o][1] = fa[o] = val[o] = sz[o] = cnt[o] = 0;  
    }  
  
    void Rotate(int o) {  
        int p = fa[o], q = fa[p], ck = Get(o);  
        son[p][ck] = son[o][ck ^ 1];  
        fa[son[o][ck ^ 1]] = p;  
        son[o][ck ^ 1] = p;  
        fa[p] = o; fa[o] = q;  
        if (q) son[q][p == son[q][1]] = o;  
    }  
}
```



```
    Push(p); Push(o);
}

void Splay(int &root, int o) {
    for (int f = fa[o]; (f = fa[o]); Rotate(o))
        if (fa[f]) Rotate(Get(o) == Get(f) ? f : o);
    root = o;
}

void Insert(int &root, int x) {
    if (!root) {
        val[++tot] = x;
        cnt[tot]++;
        root = tot;
        Push(root);
        return;
    }
    int cur = root, f = 0;
    while (true) {
        if (val[cur] == x) {
            cnt[cur]++;
            Push(cur); Push(f);
            Splay(root, cur);
            break;
        }
        f = cur;
        cur = son[cur][val[cur] < x];
        if (!cur) {
            val[++tot] = x;
            cnt[tot]++;
            fa[tot] = f;
            son[f][val[f] < x] = tot;
            Push(tot); Push(f);
            Splay(root, tot);
            break;
        }
    }
}

int GetRank(int &root, int x) {
    int ans = 0, cur = root;
    while (cur) {
        if (x < val[cur]) {
            cur = son[cur][0];
            continue;
        }
        ans += sz[son[cur][0]];
        if (x == val[cur]) {
            Splay(root, cur);
        }
    }
}
```

```
        return ans;
    }
    if (x > val[cur]) {
        ans += cnt[cur];
        cur = son[cur][1];
    }
}
return ans;
}

int GetKth(int &root, int k) {
    int cur = root;
    while (true) {
        if (son[cur][0] && k <= sz[son[cur][0]]) cur = son[cur][0];
        else {
            k -= cnt[cur] + sz[son[cur][0]];
            if (k <= 0) return cur;
            cur = son[cur][1];
        }
    }
}

int Find(int &root, int x) {
    int ans = 0, cur = root;
    while (cur) {
        if (x < val[cur]) {
            cur = son[cur][0];
            continue;
        }
        ans += sz[son[cur][0]];
        if (x == val[cur]) {
            Splay(root, cur);
            return ans + 1;
        }
        ans += cnt[cur];
        cur = son[cur][1];
    }
}

int GetPrev(int &root) {
    int cur = son[root][0];
    while (son[cur][1]) cur = son[cur][1];
    return cur;
}

int GetPrevVal(int &root, int x) {
    int ans = -inf, cur = root;
    while (cur) {
        if (x > val[cur]) {
            ans = max(ans, val[cur]);
        }
    }
}
```

```
        cur = son[cur][1];
        continue;
    }
    cur = son[cur][0];
}
return ans;
}

int GetNext(int &root) {
    int cur = son[root][1];
    while (son[cur][0]) cur = son[cur][0];
    return cur;
}

int GetNextVal(int &root, int x) {
    int ans = inf, cur = root;
    while (cur) {
        if (x < val[cur]) {
            ans = min(ans, val[cur]);
            cur = son[cur][0];
            continue;
        }
        cur = son[cur][1];
    }
    return ans;
}

void Delete(int &root, int x) {
    Find(root, x);
    if (cnt[root] > 1) {
        cnt[root]--;
        Push(root);
        return;
    }
    if (!son[root][0] && !son[root][1]) {
        Clear(root);
        root = 0;
        return;
    }
    if (!son[root][0]) {
        int cur = root;
        root = son[root][1];
        fa[root] = 0;
        Clear(cur);
        return;
    }
    if (!son[root][1]) {
        int cur = root;
        root = son[root][0];
        fa[root] = 0;
    }
}
```

```
        Clear(cur);
        return;
    }
    int p = GetPrev(root), cur = root;
    Splay(root, p);
    fa[son[cur][1]] = p;
    son[p][1] = son[cur][1];
    Clear(cur);
    Push(root);
}
};

namespace SegTree {
    int tree[maxn << 2];

    void Build(int o, int l, int r) {
        for (int i = l; i <= r; ++i) SplayTree::Insert(tree[o], arr[i - 1]);
        if (l == r) return;
        int m = (l + r) >> 1;
        Build(o << 1, l, m);
        Build(o << 1 | 1, m + 1, r);
    }

    void Modify(int o, int l, int r, int ll, int rr, int u, int v) {
        SplayTree::Delete(tree[o], u); SplayTree::Insert(tree[o], v);
        if (l == r) return;
        int m = (l + r) >> 1;
        if (ll <= m) Modify(o << 1, l, m, ll, rr, u, v);
        if (rr > m) Modify(o << 1 | 1, m + 1, r, ll, rr, u, v);
    }

    int QueryRank(int o, int l, int r, int ll, int rr, int v) {
        if (ll <= l && rr >= r) return SplayTree::GetRank(tree[o], v);
        int m = (l + r) >> 1, ans = 0;
        if (ll <= m) ans += QueryRank(o << 1, l, m, ll, rr, v);
        if (rr > m) ans += QueryRank(o << 1 | 1, m + 1, r, ll, rr, v);
        return ans;
    }

    int QueryPrev(int o, int l, int r, int ll, int rr, int v) {
        if (ll <= l && rr >= r) return SplayTree::GetPrevVal(tree[o], v);
        int m = (l + r) >> 1, ans = -inf;
        if (ll <= m) ans = max(ans, QueryPrev(o << 1, l, m, ll, rr, v));
        if (rr > m) ans = max(ans, QueryPrev(o << 1 | 1, m + 1, r, ll, rr, v));
        return ans;
    }

    int QueryNext(int o, int l, int r, int ll, int rr, int v) {
        if (ll <= l && rr >= r) return SplayTree::GetNextVal(tree[o], v);
```

```
    int m = (l + r) >> 1, ans = inf;
    if (ll <= m) ans = min(ans, QueryNext(o << 1, l, m, ll, rr, v));
    if (rr > m) ans = min(ans, QueryNext(o << 1 | 1, m + 1, r, ll, rr, v));
    return ans;
}

int QueryKth(int ll, int rr, int v) {
    int l = 0, r = 1e8 + 10;
    while (l < r) {
        int m = ((l + r) >> 1) + 1;
        if (QueryRank(1, 1, n, ll, rr, m) < v) l = m;
        else r = m - 1;
    }
    return l;
}

};

int main() {
    ios::sync_with_stdio(false); cin.tie(0); cout.tie(0);
    int m; cin >> n >> m;
    for (int i = 0; i < n; ++i) cin >> arr[i];
    SplayTree::tot = 0;
    SegTree::Build(1, 1, n);
    for (int i = 0, op, l, r, pos, k; i < m; ++i) {
        cin >> op;
        if (op == 1) {
            cin >> l >> r >> k;
            cout << SegTree::QueryRank(1, 1, n, l, r, k) + 1 << endl;
        }
        else if (op == 2) {
            cin >> l >> r >> k;
            cout << SegTree::QueryKth(l, r, k) << endl;
        }
        else if (op == 3) {
            cin >> pos >> k;
            SegTree::Modify(1, 1, n, pos, pos, arr[pos - 1], k);
            arr[pos - 1] = k;
        }
        else if (op == 4) {
            cin >> l >> r >> k;
            cout << SegTree::QueryPrev(1, 1, n, l, r, k) << endl;
        }
        else if (op == 5) {
            cin >> l >> r >> k;
            cout << SegTree::QueryNext(1, 1, n, l, r, k) << endl;
        }
    }
    return 0;
}
```

3.6 SegmentTree

// 求和线段树

```
const int maxn = "Edit"

namespace SegTree {
    int n;
    long long sum[maxn << 2], lazy[maxn << 2];

    void Pull(int o) {
        sum[o] = sum[o << 1] + sum[o << 1 | 1];
    }

    void Push(int o, int l, int r) {
        int m = (l + r) >> 1;
        if (lazy[o] != 0) {
            sum[o << 1] += (m - l + 1) * lazy[o];
            sum[o << 1 | 1] += (r - m) * lazy[o];
            lazy[o << 1] += lazy[o];
            lazy[o << 1 | 1] += lazy[o];
            lazy[o] = 0;
        }
    }

    template <typename t>
    void Build(int o, int l, int r, const vector<t> &v) {
        sum[o] = 0; lazy[o] = 0;
        if (l == r) {
            sum[o] = v[l - 1];
            return;
        }
        int m = (l + r) >> 1;
        Build(o << 1, l, m, v);
        Build(o << 1 | 1, m + 1, r, v);
        Pull(o);
    }

    template <typename t>
    void Init(const vector<t> &v) {
        n = v.size();
        Build(1, 1, n, v);
    }

    template <typename t>
    void Modify(int o, int l, int r, int ll, int rr, t v) {
        if (ll <= l && rr >= r) {
            sum[o] += (r - l + 1) * v;
            lazy[o] += v;
            return;
        }
    }
```

```
    Push(o, l, r);
    int m = (l + r) >> 1;
    if (ll <= m) Modify(o << 1, l, m, ll, rr, v);
    if (rr > m) Modify(o << 1 | 1, m + 1, r, ll, rr, v);
    Pull(o);
}
template <typename t>
void Modify(int ll, int rr, t v) {
    Modify(1, 1, n, ll, rr, v);
}

long long Query(int o, int l, int r, int ll, int rr) {
    if (ll <= l && rr >= r) return sum[o];
    Push(o, l, r);
    int m = (l + r) >> 1;
    long long ans = 0;
    if (ll <= m) ans += Query(o << 1, l, m, ll, rr);
    if (rr > m) ans += Query(o << 1 | 1, m + 1, r, ll, rr);
    return ans;
}
long long Query(int ll, int rr) {
    return Query(1, 1, n, ll, rr);
}
};
```

3.7 SplayTree

```
const int inf = "Edit"
const int maxn = "Edit";

namespace SplayTree {
    int rt, tot;
    int fa[maxn], son[maxn][2];
    int val[maxn], cnt[maxn];
    int sz[maxn];

    void Push(int o) {
        sz[o] = sz[son[o][0]] + sz[son[o][1]] + cnt[o];
    }

    bool Get(int o) {
        return o == son[fa[o]][1];
    }

    void Clear(int o) {
        son[o][0] = son[o][1] = fa[o] = val[o] = sz[o] = cnt[o] = 0;
    }

    void Rotate(int o) {
        int p = fa[o], q = fa[p], ck = Get(o);
```

```
son[p][ck] = son[o][ck ^ 1];
fa[son[o][ck ^ 1]] = p;
son[o][ck ^ 1] = p;
fa[p] = o; fa[o] = q;
if (q) son[q][p == son[q][1]] = o;
Push(p); Push(o);
}

void Splay(int o) {
    for (int f = fa[o]; (f = fa[o]); Rotate(o))
        if (fa[f]) Rotate(Get(o) == Get(f) ? f : o);
    rt = o;
}

void Insert(int x) {
    if (!rt) {
        val[++tot] = x;
        cnt[tot]++;
        rt = tot;
        Push(rt);
        return;
    }
    int cur = rt, f = 0;
    while (true) {
        if (val[cur] == x) {
            cnt[cur]++;
            Push(cur); Push(f);
            Splay(cur);
            break;
        }
        f = cur;
        cur = son[cur][val[cur] < x];
        if (!cur) {
            val[++tot] = x;
            cnt[tot]++;
            fa[tot] = f;
            son[f][val[f] < x] = tot;
            Push(tot); Push(f);
            Splay(tot);
            break;
        }
    }
}

int GetRank(int x) {
    int ans = 0, cur = rt;
    while (true) {
        if (x < val[cur]) cur = son[cur][0];
        else {
```



```
    ans += sz[son[cur][0]];
    if (x == val[cur]) {
        Splay(cur);
        return ans + 1;
    }
    ans += cnt[cur];
    cur = son[cur][1];
}
}
}

int GetKth(int k) {
    int cur = rt;
    while (true) {
        if (son[cur][0] && k <= sz[son[cur][0]]) cur = son[cur][0];
        else {
            k -= cnt[cur] + sz[son[cur][0]];
            if (k <= 0) return cur;
            cur = son[cur][1];
        }
    }
}

int GetPrev() {
    int cur = son[rt][0];
    while (son[cur][1]) cur = son[cur][1];
    return cur;
}

int GetPrevVal(int x) {
    int ans = -inf, cur = rt;
    while (cur) {
        if (x > val[cur]) {
            ans = max(ans, val[cur]);
            cur = son[cur][1];
            continue;
        }
        cur = son[cur][0];
    }
    return ans;
}

int GetNext() {
    int cur = son[rt][1];
    while (son[cur][0]) cur = son[cur][0];
    return cur;
}

int GetNextVal(int x) {
    int ans = inf, cur = rt;
    while (cur) {

```

```
        if (x < val[cur]) {
            ans = min(ans, val[cur]);
            cur = son[cur][0];
            continue;
        }
        cur = son[cur][1];
    }
    return ans;
}

void Delete(int x) {
    GetRank(x);
    if (cnt[rt] > 1) {
        cnt[rt]--;
        Push(rt);
        return;
    }
    if (!son[rt][0] && !son[rt][1]) {
        Clear(rt);
        rt = 0;
        return;
    }
    if (!son[rt][0]) {
        int cur = rt;
        rt = son[rt][1];
        fa[rt] = 0;
        Clear(cur);
        return;
    }
    if (!son[rt][1]) {
        int cur = rt;
        rt = son[rt][0];
        fa[rt] = 0;
        Clear(cur);
        return;
    }
    int p = GetPrev(), cur = rt;
    Splay(p);
    fa[son[cur][1]] = p;
    son[p][1] = son[cur][1];
    Clear(cur);
    Push(rt);
}

};
```

4 GraphTheory

4.1 LCA

4.1.1 DFS+ST

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 链式前向星存图
struct Edge {
    int V, Weight, Next;
};

Edge edges[maxn << 1];
int Head[maxn];
int Tot;

void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

void AddEdge(int U, int V, int Weight) {
    edges[Tot] = Edge {V, Weight, Head[U]};
    Head[U] = Tot++;
}

struct LCAOnline {
    // 节点深度
    int Rmq[maxn << 1];
    // 深搜遍历顺序
    int Vertex[maxn << 1];
    // 节点在深搜中第一次出现的位置
    int First[maxn];
    // 记录父节点
    int Parent[maxn];
    // 记录与根节点距离
    int Dis[maxn];
    // 遍历节点数量
    int LCATot;

    // 最小值对应下标
    int Dp[maxn << 1][20];

    // RMQ 初始化
    void Work(int N) {
        for (int i = 1; i <= N; ++i) {
            Dp[i][0] = i;
        }
    }
};
```

```

    for (int j = 1; (1 << j) <= N; ++j) {
        for (int i = 1; i + (1 << j) - 1 <= N; ++i) {
            Dp[i][j] = Rmq[Dp[i][j - 1]] < Rmq[Dp[i + (1 << (j - 1))][j - 1]] ?
                ↪ Dp[i][j - 1] : Dp[i + (1 << (j - 1))][j - 1];
        }
    }
}

// 深搜
void Dfs(int Cur, int Pre, int Depth) {
    Vertex[++LCATot] = Cur;
    First[Cur] = LCATot;
    Rmq[LCATot] = Depth;
    Parent[Cur] = Pre;
    for (int i = Head[Cur]; ~i; i = edges[i].Next) {
        if (edges[i].V == Pre) {
            continue;
        }
        Dis[edges[i].V] = Dis[Cur] + edges[i].Weight;
        Dfs(edges[i].V, Cur, Depth + 1);
        Vertex[++LCATot] = Cur;
        Rmq[LCATot] = Depth;
    }
}

// RMQ 查询
int Query(int Left, int Right) {
    if (Left > Right) {
        swap(Left, Right);
    }
    int Len = int(log2(Right - Left + 1));
    return Rmq[Dp[Left][Len]] <= Rmq[Dp[Right - (1 << Len) + 1][Len]] ?
        ↪ Dp[Left][Len] : Dp[Right - (1 << Len) + 1][Len];
}

// LCA 初始化
void Init(int Root, int NodeNum) {
    memset(Dis, 0, sizeof(Dis));
    LCATot = 0;
    Dfs(Root, 0, 0);
    Parent[1] = 0;
    Work(2 * NodeNum - 1);
}

// 查询节点 U、V 的距离
int GetDis(int U, int V) {
    return Dis[U] + Dis[V] - 2 * Dis[LCA(U, V)];
}

```

```
// 查询节点  $U$ 、 $V$  的最近公共祖先 (LCA)
int LCA(int U, int V) {
    return Vertex[Query(First[U], First[V])];
}
}LCA;
```

4.1.2 Tarjan

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 树边
struct Edge {
    int V, Next;
};

// 询问
struct Query {
    int Q, Next;
    int Index;
};

// 并查集数组
int Pre[maxn << 2];
// 树边
Edge edges[maxn << 2];
int Head[maxn];
int Tot;
// 询问
Query queries[maxn << 2];
int QHead[maxn];
int QTot;
// 访问标记
int Vis[maxn];
int Ancestor[maxn];
// 结果
int Answer[maxn];

// 并查集查找
int Find(int X) {
    int R = X;
    while (Pre[R] != -1) {
        R = Pre[R];
    }
    return R;
}

// 并查集合并
void Join(int U, int V) {
```

```
    int RU = Find(U);
    int RV = Find(V);
    if (RU != RV) {
        Pre[RU] = RV;
    }
}

// 添加树边
void AddEdge(int U, int V) {
    edges[Tot] = Edge {V, Head[U]};
    Head[U] = Tot++;
}

// 添加询问
void AddQuery(int U, int V, int Index) {
    queries[QTot] = Query {V, QHead[U], Index};
    QHead[U] = QTot++;
    queries[QTot] = Query {U, QHead[V], Index};
    QHead[V] = QTot++;
}

// 初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
    QTot = 0;
    memset(QHead, -1, sizeof(QHead));
    memset(Vis, false, sizeof(Vis));
    memset(Pre, -1, sizeof(Pre));
    memset(Ancestor, 0, sizeof(Ancestor));
}

// LCA 离线 Tarjan 算法
void Tarjan(int Node) {
    Ancestor[Node] = Node;
    Vis[Node] = true;
    for (int i = Head[Node]; i != -1; i = edges[i].Next) {
        if (Vis[edges[i].V]) {
            continue;
        }
        Tarjan(edges[i].V);
        Join(Node, edges[i].V);
        Ancestor[Find(Node)] = Node;
    }
    for (int i = QHead[Node]; i != -1; i = queries[i].Next) {
        if (Vis[queries[i].Q]) {
            Answer[queries[i].Index] = Ancestor[Find(queries[i].Q)];
        }
    }
}
```

```
}
```

4.2 MinimumSpanningTree

4.2.1 Kruskal

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Edge {
    int U, V, Dis;

    bool operator < (const Edge &B) const {
        return Dis < B.Dis;
    }
};

// N: 顶点数, E: 边数, Pre 并查集
int N, E, Pre[maxn];
// edges: 边
Edge edges[maxn];

void Init() {
    // 并查集初始化
    for (int i = 0; i <= N; ++i) {
        Pre[i] = i;
    }
}

// 并查集查询
int Find(int X) {
    return Pre[X] == X ? X : Pre[X] = Find(Pre[X]);
}

// 并查集合并
void Join(int X, int Y) {
    int XX = Find(X);
    int YY = Find(Y);
    if (XX != YY) {
        Pre[XX] = YY;
    }
}

// Kruskal 算法
int Kruskal() {
    // 贪心排序
    std::sort(edges + 1, edges + E + 1);
    Init();
    int Res = 0;
```

```
// 选边计算
for (int i = 1; i <= E; ++i) {
    Edge Temp = edges[i];
    if (Find(Temp.U) != Find(Temp.V)) {
        Join(Temp.U, Temp.V);
        Res += Temp.Dis;
    }
}
return Res;
}
```

4.2.2 Prim

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Edge {
    // V: 连接点, Dis: 边权
    int V, Dis;
};

// N: 顶点数, E: 边数
int N, E;
// 松弛更新权值数组
int Dis[maxn];
// 访问标记数组
int Vis[maxn];
// 邻接表
std::vector<Edge> Adj[maxn];

// 建图加边, U, V: 顶点, Weight: 权值
void AddEdge(int U, int V, int Weight) {
    Adj[U].push_back(Edge (V, Weight));
    // 无向图反向建边
    Adj[V].push_back(Edge (U, Weight));
}

// Prim 算法
int Prim(int Start) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, 0, sizeof(Vis));
    Dis[Start] = 0;
    int Res = 0;
    for (int i = 1; i <= N; ++i) {
        // 选择距已生成树权值最小的顶点
        int U = -1, Min = INF;
        for (int j = 1; j <= N; ++j) {
            if (!Vis[j] && Dis[j] < Min) {
```



```
        U = j;
        Min = Dis[j];
    }
}
// 更新、标记
Vis[U] = 1;
Res += Min;
// 松弛
for (int j = 0; j < int(Adj[U].size()); ++j) {
    int V = Adj[U][j].V;
    if (!Vis[V] && Adj[U][j].Dis < Dis[V]) {
        Dis[V] = Adj[U][j].Dis;
    }
}
}
// 返回结果
return Res;
}
```

4.3 NetworkFlow

4.3.1 Dinic

```
const int maxn = "Edit";

namespace graph {
    struct edge {int v, flow, next;};

    edge g[maxn << 2];
    int head[maxn];
    int tot;
    int dep[maxn];
    int cur[maxn];

    void Init() {
        tot = 0;
        memset(head, -1, sizeof(head));
    }

    void AddEdge(int u, int v, int flow, int rev = 0) {
        g[tot] = (edge){v, flow, head[u]};
        head[u] = tot++;
        g[tot] = (edge){u, rev, head[v]};
        head[v] = tot++;
    }

    bool Bfs(int s, int t) {
        memset(dep, -1, sizeof(dep));
        queue<int> que;
        dep[s] = 0;
```

```
    que.push(s);
    while (!que.empty()) {
        int u = que.front(); que.pop();
        for (int i = head[u]; ~i; i = g[i].next) {
            if (dep[g[i].v] == -1 && g[i].flow > 0) {
                dep[g[i].v] = dep[u] + 1;
                que.push(g[i].v);
            }
        }
    }
    return dep[t] != -1;
}

int Dfs(int u, int t, int flow) {
    if (u == t || flow == 0) return flow;
    int max_flow = 0, find_flow;
    for (int &i = cur[u]; ~i; i = g[i].next) {
        if (g[i].flow > 0 && dep[g[i].v] == dep[u] + 1) {
            find_flow = Dfs(g[i].v, t, min(flow - max_flow, g[i].flow));
            if (find_flow > 0) {
                g[i].flow -= find_flow;
                g[i ^ 1].flow += find_flow;
                max_flow += find_flow;
                if (max_flow == flow) return flow;
            }
        }
    }
    if (!max_flow) dep[u] = -2;
    return max_flow;
}

int Dinic(int s, int t) {
    int ans = 0;
    while (Bfs(s, t)) {
        for (int i = s; i <= t; ++i) cur[i] = head[i];
        ans += Dfs(s, t, inf);
    }
    return ans;
}
};
```

4.3.2 FordFulkerson

```
#include <bits/stdc++.h>
// 正无穷
const int INF = "Edit";
const int maxn = "Edit";

// N: 顶点数, E: 边数
int N, E;
```

```
// 访问标记数组
bool Vis[maxn];
// 邻接矩阵
int Adj[maxn][maxn];

// Dfs 搜索增广路径, Vertex: 当前搜索顶点, End: 搜索终点, NowFlow: 当前最大流量
int Dfs(int Vertex, int End, int NowFlow) {
    // 搜索到终点结束
    if (Vertex == End) {
        return NowFlow;
    }
    // 标记访问过的顶点
    Vis[Vertex] = true;
    // 枚举寻找顶点
    for (int i = 1; i <= N; ++i) {
        if (!Vis[i] && Adj[Vertex][i]) {
            int FindFlow = Dfs(i, End, NowFlow < Adj[Vertex][i] ? NowFlow :
                               ↪ Adj[Vertex][i]);
            if (!FindFlow) {
                continue;
            }
            // 找到增广路径后更新邻接矩阵残留网
            Adj[Vertex][i] -= FindFlow;
            Adj[i][Vertex] += FindFlow;
            // 返回搜索结果
            return FindFlow;
        }
    }
    // 未找到增广路径, 搜索失败
    return false;
}

// Ford-Fulkerson 算法, Start: 起点, End: 终点
int FordFulkerson(int Start, int End) {
    // MaxFlow: 最大流, Flow: 搜索到的增广路径最大流
    int MaxFlow = 0, Flow = 0;
    memset(Vis, false, sizeof(Vis));
    // 搜索增广路径
    while (Flow = Dfs(Start, End, INF)) {
        MaxFlow += Flow;
        memset(Vis, false, sizeof(Vis));
    }
    // 返回结果
    return MaxFlow;
}
```

4.3.3 MinCostMaxFlow

```
#include <bits/stdc++.h>
```

```
const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Edge {
    // V: 连接点, Flow: 流量, Cost: 费用
    int V, Cap, Cost, Flow, Next;
};

// N: 顶点数, E: 边数
int N, E;
int Head[maxn];
// 前驱记录数组
int Path[maxn];
int Dis[maxn];
// 访问标记数组
bool Vis[maxn];
int Tot;
// 链式前向星
Edge edges[maxn];

// 链式前向星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 建图加边, U, V 之间建立一条费用为 Cost 的边
void AddEdge(int U, int V, int Cap, int Cost) {
    edges[Tot] = Edge {V, Cap, Cost, 0, Head[U]};
    Head[U] = Tot++;
    edges[Tot] = Edge {U, 0, -Cost, 0, Head[V]};
    Head[V] = Tot++;
}

// SPFA 算法, Start: 起点, End: 终点
bool SPFA(int Start, int End) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, false, sizeof(Vis));
    memset(Path, -1, sizeof(Path));
    Dis[Start] = 0;
    Vis[Start] = true;
    std::queue<int> Que;
    while (!Que.empty()) {
        Que.pop();
    }
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.front();
```

```

    Que.pop();
    Vis[U] = false;
    for (int i = Head[U]; ~i; i = edges[i].Next) {
        int V = edges[i].V;
        if (edges[i].Cap > edges[i].Flow && Dis[V] > Dis[U] + edges[i].Cost) {
            Dis[V] = Dis[U] + edges[i].Cost;
            Path[V] = i;
            if (!Vis[V]) {
                Vis[V] = true;
                Que.push(V);
            }
        }
    }
}
return Path[End] != -1;
}

```

// 最小费用最大流, *Start*: 起点, *End*: 终点, *Cost*: 最小费用

```

int MinCostMaxFlow(int Start, int End, int &MinCost) {
    int MaxFlow = 0;
    MinCost = 0;
    while (SPFA(Start, End)) {
        int Min = INF;
        for (int i = Path[End]; ~i; i = Path[edges[i ^ 1].V]) {
            if (edges[i].Cap - edges[i].Flow < Min) {
                Min = edges[i].Cap - edges[i].Flow;
            }
        }
        for (int i = Path[End]; ~i; i = Path[edges[i ^ 1].V]) {
            edges[i].Flow += Min;
            edges[i ^ 1].Flow -= Min;
            MinCost += edges[i].Cost * Min;
        }
        MaxFlow += Min;
    }
    // 返回最大流
    return MaxFlow;
}

```

4.4 ShortestPath

4.4.1 BellmanFord

```
#include <bits/stdc++.h>
```

```

const int INF = "Edit";
const int maxn = "Edit";

```

```

struct Edge {
    // U V: 顶点, Dis: 边权

```

```
    int U, V;
    int Dis;
};
// 松弛更新数组
int Dis[maxn];
// 边
std::vector<Edge> edges;

// Bellman_Ford 算法判断是否存在负环回路
bool BellmanFord(int Start, int N) {
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    // 最多做 N-1 次
    for (int i = 1; i < N; ++i) {
        bool flag = false;
        for (int j = 0; j < int(edges.size()); ++j) {
            if (Dis[edges[j].V] > Dis[edges[j].U] + edges[j].Dis) {
                Dis[edges[j].V] = Dis[edges[j].U] + edges[j].Dis;
                flag = true;
            }
        }
        // 没有负环回路
        if (!flag) {
            return true;
        }
    }
    // 有负环回路
    for (int j = 0; j < int(edges.size()); ++j) {
        if (Dis[edges[j].V] > Dis[edges[j].U] + edges[j].Dis) {
            return false;
        }
    }
    // 没有负环回路
    return true;
}
```

4.4.2 Dijkstra

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const int INF = "Edit";

// 边
struct Edge {
    // V: 连接点, Weight: 权值, Next: 上一条边的编号
    int V, Weight, Next;
};

// 边, 一定要开到足够大
```

```
Edge edges[maxn << 1];
// Head[i] 为点 i 上最后一条边的编号
int Head[maxn];
// 增加边时更新编号
int Tot;
// 松弛更新数组, 最短路
int Dis[maxn];

// 链式前向星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 添加一条 U 至 V 权值为 Weight 的边
void AddEdge(int U, int V, int Weight) {
    edges[Tot] = Edge(V, Weight, Head[U]);
    Head[U] = Tot++;
}

// 最短路优化堆排序规则
struct Cmp {
    bool operator() (const int &A, const int &B) {
        return Dis[A] > Dis[B];
    }
};

// N: 顶点数, E: 边数
int N, E;

// Dijkstra 算法, Start: 起点
void Dijkstra(int Start) {
    std::priority_queue<int, std::vector<int>, Cmp> Que;
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.top(); Que.pop();
        for (int i = Head[U]; ~i; i = edges[i].Next) {
            if (Dis[edges[i].V] > Dis[U] + edges[i].Weight) {
                Dis[edges[i].V] = Dis[U] + edges[i].Weight;
                Que.push(edges[i].V);
            }
        }
    }
}
```

4.4.3 Floyd

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// N: 顶点数
int N;
// Dis[i][j] 为 i 点到 j 点的最短路
int Dis[maxn][maxn];

// Floyd 算法
void Floyd() {
    for (int k = 1; k <= N; ++k) {
        for (int i = 1; i <= N; ++i) {
            for (int j = 1; j <= N; ++j) {
                Dis[i][j] = std::min(Dis[i][j], Dis[i][k] + Dis[k][j]);
            }
        }
    }
}
```

4.4.4 SPFA

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Edge {
    // V: 连接点, Dis: 边权
    int V, Dis;
};

// N: 顶点数, E: 边数
int N, E;
// 访问标记数组
bool Vis[maxn];
// 每个点的入队列次数
int Cnt[maxn];
// 最短路数组
int Dis[maxn];
// 邻接表
std::vector<Edge> Adj[maxn];

// 建图加边, U, V 之间权值为 Weight 的边
void AddEdge (int U, int V, int Weight) {
    Adj[U].push_back(Edge (V, Weight));
    // 无向图建立反向边
```



```
    Adj[V].push_back(Edge (U, Weight));
}

// SPFA 算法, Start: 起点
bool SPFA(int Start) {
    memset(Vis, false, sizeof(Vis));
    memset(Dis, INF, sizeof(Dis));
    memset(Cnt, 0, sizeof(Cnt));
    Vis[Start] = true;
    Dis[Start] = 0;
    Cnt[Start] = 1;
    std::queue<int> Que;
    while (!Que.empty()) {
        Que.pop();
    }
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.front();
        Que.pop();
        Vis[U] = false;
        for (int i = 0; i < int(Adj[U].size()); ++i) {
            int V = Adj[U][i].V;
            if (Dis[V] > Dis[U] + Adj[U][i].Dis) {
                Dis[V] = Dis[U] + Adj[U][i].Dis;
                if (!Vis[V]) {
                    Vis[V] = true;
                    Que.push(V);
                    // Cnt[i] 为 i 顶点入队列次数, 用来判定是否存在负环回路
                    if (++Cnt[V] > N) {
                        return false;
                    }
                }
            }
        }
    }
    return true;
}
```

5 DynamicProgramming

5.1 Contour

```
#include <bits/stdc++.h>

const int maxn = "Edit";

int Dp[2][1 << maxn];

void Update(int Cur, int A, int B) {
    if (B & (1 << M)) {
        Dp[Cur][B ^ (1 << M)] = Dp[Cur][B ^ (1 << M)] + Dp[Cur ^ 1][A];
    }
}

// 轮廓线 Dp(1*2 在 N*M 图上摆放数)
int Contour(int N, int M) {
    memset(Dp, 0, sizeof(Dp));
    int Cur = 0;
    Dp[Cur][(1 << M) - 1] = 1;
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < M; ++j) {
            Cur ^= 1;
            memset(Dp[Cur], 0, sizeof(Dp[Cur]));
            for (int k = 0; k < (1 << M); ++k) {
                Update(Cur, k, k << 1);
                if (i && !(k & (1 << (M - 1)))) {
                    Update(Cur, k, (k << 1) ^ (1 << M) ^ 1);
                }
                if (j && !(k & 1)) {
                    Update(Cur, k, (k << 1) ^ 3);
                }
            }
        }
    }
    return Dp[Cur][(1 << M) - 1];
}
```

5.2 Digit

```
#include <bits/stdc++.h>

const int maxn = "Edit";

long long Digit[25];
long long Dp[25][maxn];

// Site: 数位, Statu: 状态, Pre: 前导零, Limit: 数位上界
long long Dfs(long long Site, long long Statu, bool Pre, bool Limit) {
```

```
    if (Site == 0) {
        return ?;
    }
    if (!Limit && ~Dp[Site][Statu]) {
        return Dp[Site][Statu];
    }
    long long Max = Limit ? Digit[Site] : 9;
    long long Ans = 0;
    for (int i = 0; i <= Max; ++i) {
        long long NowStatu = /* 状态转移 */;
        if (NowStatu?) {
            Ans += Dfs(Site - 1, NowStatu, Pre && i == 0, Limit && i == Max);
        }
    }
    if (!Limit) {
        Dp[Site][Statu] = Ans;
    }
    return Ans;
}

long long Cal(long long X) {
    // 数位分解
    long long Len = 0;
    while (X) {
        Digit[++Len] = X % 10;
        X /= 10;
    }
    return Dfs(Len, 0, true, true);
}
```

5.3 LCS

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// Dp[i][j]:Str1[1]~Str1[i] 和 Str2[1]~Str2[j] 对应的公共子序列长度
int Dp[maxn][maxn];

// 最长公共子序列 (LCS)
void LCS(std::string Str1, std::string Str2) {
    for (int i = 0; i < int(Str1.length()); ++i) {
        for (int j = 0; j < int(Str2.length()); ++j) {
            if (Str1[i] == Str2[j]) {
                Dp[i + 1][j + 1] = Dp[i][j] + 1;
            }
            else {
                Dp[i + 1][j + 1] = std::max(Dp[i][j + 1], Dp[i + 1][j]);
            }
        }
    }
}
```

```
    }  
}
```

5.4 LIS

```
#include <bits/stdc++.h>  
  
// 最长不下降子序列 (LIS), Num: 序列  
int LIS(std::vector<int> &Num) {  
    int Ans = 1;  
    // Last[i] 为长度为 i 的不下降子序列末尾元素的最小值  
    std::vector<int> Last(int(Num.size()) + 1, 0);  
    Last[1] = Num[1];  
    for (int i = 2; i <= int(Num.size()); ++i) {  
        if (Num[i] >= Last[Ans]) {  
            Last[++Ans] = Num[i];  
        }  
        else {  
            int Index = std::upper_bound(Last.begin() + 1, Last.end(), Num[i]) -  
                Last.begin();  
            Last[Index] = Num[i];  
        }  
    }  
    // 返回结果  
    return Ans;  
}
```

5.5 Pack

```
#include <bits/stdc++.h>  
  
const int maxn = "Edit";  
  
int Dp[maxn];  
// NValue: 背包容量, NKind: 总物品数  
int NValue, NKind;  
  
// 01 背包, 代价为 Cost, 获得的价值为 Weight  
void ZeroOnePack(int Cost, int Weight) {  
    for (int i = NValue; i >= Cost; --i) {  
        Dp[i] = std::max(Dp[i], Dp[i - Cost] + Weight);  
    }  
}  
  
// 完全背包, 代价为 Cost, 获得的价值为 Weight  
void CompletePack(int Cost, int Weight) {  
    for (int i = Cost; i <= NValue; ++i) {  
        Dp[i] = std::max(Dp[i], Dp[i - Cost] + Weight);  
    }  
}
```

```
// 多重背包, 代价为 Cost, 获得的价值为 Weight, 数量为 Amount
void MultiplePack(int Cost, int Weight, int Amount) {
    if (Cost * Amount >= NValue) {
        CompletePack(Cost, Weight);
    }
    else {
        int k = 1;
        while (k < Amount) {
            ZeroOnePack(k * Cost, k * Weight);
            Amount -= k;
            k <<= 1;
        }
        ZeroOnePack(Amount * Cost, Amount * Weight);
    }
}
```

6 ComputationalGeometry

6.1 JlsGeo

```

#define mp make_pair
#define fi first
#define se second
#define pb push_back
typedef double db;
const db eps=1e-6;
const db pi=acos(-1);
int sign(db k){
    if (k>eps) return 1; else if (k<-eps) return -1; return 0;
}
int cmp(db k1,db k2){return sign(k1-k2);}
int inmid(db k1,db k2,db k3){return sign(k1-k3)*sign(k2-k3)<=0;}// k3 在 [k1,k2] 内
↪
struct point{
    db x,y;
    point operator + (const point &k1) const{return (point){k1.x+x,k1.y+y};}
    point operator - (const point &k1) const{return (point){x-k1.x,y-k1.y};}
    point operator * (db k1) const{return (point){x*k1,y*k1};}
    point operator / (db k1) const{return (point){x/k1,y/k1};}
    int operator == (const point &k1) const{return cmp(x,k1.x)==0&&cmp(y,k1.y)==0;}
    // 逆时针旋转
    point turn(db k1){return (point){x*cos(k1)-y*sin(k1),x*sin(k1)+y*cos(k1)};}
    point turn90(){return (point){-y,x};}
    bool operator < (const point k1) const{
        int a=cmp(x,k1.x);
        if (a==1) return 1; else if (a==1) return 0; else return cmp(y,k1.y)==-1;
    }
    db abs(){return sqrt(x*x+y*y);}
    db abs2(){return x*x+y*y;}
    db dis(point k1){return ((*this)-k1).abs();}
    point unit(){db w=abs(); return (point){x/w,y/w};}
    void scan(){double k1,k2; scanf("%lf%lf",&k1,&k2); x=k1; y=k2;}
    void print(){printf("%.11lf %.11lf\n",x,y);}
    db getw(){return atan2(y,x);}
    point getdel(){if (sign(x)==-1||(sign(x)==0&&sign(y)==-1)) return (*this)*(-1);
    ↪ else return (*this);}
    int getP() const{return sign(y)==1||(sign(y)==0&&sign(x)==-1);}
};
int inmid(point k1,point k2,point k3){return
    ↪ inmid(k1.x,k2.x,k3.x)&&inmid(k1.y,k2.y,k3.y);}
db cross(point k1,point k2){return k1.x*k2.y-k1.y*k2.x;}
db dot(point k1,point k2){return k1.x*k2.x+k1.y*k2.y;}
db rad(point k1,point k2){return atan2(cross(k1,k2),dot(k1,k2));}
// -pi -> pi
int compareangle (point k1,point k2){
    return k1.getP()<k2.getP()||(k1.getP()==k2.getP()&&sign(cross(k1,k2))>0);
}

```

```

}
point proj(point k1,point k2,point q){ // q 到直线 k1,k2 的投影
    point k=k2-k1; return k1+k*(dot(q-k1,k)/k.abs2());
}
point reflect(point k1,point k2,point q){return proj(k1,k2,q)*2-q;}
int clockwise(point k1,point k2,point k3){// k1 k2 k3 逆时针 1 顺时针 -1 否则 0
    return sign(cross(k2-k1,k3-k1));
}
int checkLL(point k1,point k2,point k3,point k4){// 求直线 (L) 线段 (S)k1,k2 和
    ↪ k3,k4 的交点
    return cmp(cross(k3-k1,k4-k1),cross(k3-k2,k4-k2))!=0;
}
point getLL(point k1,point k2,point k3,point k4){
    db w1=cross(k1-k3,k4-k3),w2=cross(k4-k3,k2-k3); return (k1*w2+k2*w1)/(w1+w2);
}
int intersect(db l1,db r1,db l2,db r2){
    if (l1>r1) swap(l1,r1); if (l2>r2) swap(l2,r2); return
    ↪ cmp(r1,l2)!=-1&&cmp(r2,l1)!=-1;
}
int checkSS(point k1,point k2,point k3,point k4){
    return intersect(k1.x,k2.x,k3.x,k4.x)&&intersect(k1.y,k2.y,k3.y,k4.y)&&
    sign(cross(k3-k1,k4-k1))*sign(cross(k3-k2,k4-k2))<=0&&
    sign(cross(k1-k3,k2-k3))*sign(cross(k1-k4,k2-k4))<=0;
}
db disSP(point k1,point k2,point q){
    point k3=proj(k1,k2,q);
    if (inmid(k1,k2,k3)) return q.dis(k3); else return min(q.dis(k1),q.dis(k2));
}
db disSS(point k1,point k2,point k3,point k4){
    if (checkSS(k1,k2,k3,k4)) return 0;
    else return
    ↪ min(min(disSP(k1,k2,k3),disSP(k1,k2,k4)),min(disSP(k3,k4,k1),disSP(k3,k4,k2)));
}
int onS(point k1,point k2,point q){return
    ↪ inmid(k1,k2,q)&&sign(cross(k1-q,k2-k1))==0;}
struct circle{
    point o; db r;
    void scan(){o.scan(); scanf("%lf",&r);}
    int inside(point k){return cmp(r,o.dis(k));}
};
struct line{
    // p[0]->p[1]
    point p[2];
    line(point k1,point k2){p[0]=k1; p[1]=k2;}
    point& operator [] (int k){return p[k];}
    int include(point k){return sign(cross(p[1]-p[0],k-p[0]))>0;}
    point dir(){return p[1]-p[0];}
    line push(){ // 向外 (左手边) 平移 eps
        const db eps = 1e-6;

```

```

        point delta=(p[1]-p[0]).turn90().unit()*eps;
        return {p[0]-delta,p[1]-delta};
    }
};

point getLL(line k1,line k2){return getLL(k1[0],k1[1],k2[0],k2[1]);}
int parallel(line k1,line k2){return sign(cross(k1.dir(),k2.dir()))==0;}
int sameDir(line k1,line k2){return
    ↪ parallel(k1,k2)&&sign(dot(k1.dir(),k2.dir()))==1;}
int operator < (line k1,line k2){
    if (sameDir(k1,k2)) return k2.include(k1[0]);
    return compareangle(k1.dir(),k2.dir());
}
int checkpos(line k1,line k2,line k3){return k3.include(getLL(k1,k2));}
vector<line> getHL(vector<line> &L){ // 求半平面交 , 半平面是逆时针方向 , 输出按照逆
    ↪ 时针
    sort(L.begin(),L.end()); deque<line> q;
    for (int i=0;i<(int)L.size();i++){
        if (i&&sameDir(L[i],L[i-1])) continue;
        while (q.size()>1&&!checkpos(q[q.size()-2],q[q.size()-1],L[i]))
            ↪ q.pop_back();
        while (q.size()>1&&!checkpos(q[1],q[0],L[i])) q.pop_front();
        q.push_back(L[i]);
    }
    while (q.size()>2&&!checkpos(q[q.size()-2],q[q.size()-1],q[0])) q.pop_back();
    while (q.size()>2&&!checkpos(q[1],q[0],q[q.size()-1])) q.pop_front();
    vector<line>ans; for (int i=0;i<q.size();i++) ans.push_back(q[i]);
    return ans;
}
db closepoint(vector<point>&A,int l,int r){ // 最近点对 , 先要按照 x 坐标排序
    if (r-l<=5){
        db ans=1e20;
        for (int i=l;i<=r;i++) for (int j=i+1;j<=r;j++)
            ↪ ans=min(ans,A[i].dis(A[j]));
        return ans;
    }
    int mid=l+r>>1; db ans=min(closepoint(A,l,mid),closepoint(A,mid+1,r));
    vector<point>B; for (int i=l;i<=r;i++) if (abs(A[i].x-A[mid].x)<=ans)
        ↪ B.push_back(A[i]);
    sort(B.begin(),B.end(),[](point k1,point k2){return k1.y<k2.y;});
    for (int i=0;i<B.size();i++) for (int j=i+1;j<B.size()&&B[j].y-B[i].y<ans;j++)
        ↪ ans=min(ans,B[i].dis(B[j]));
    return ans;
}
int checkposCC(circle k1,circle k2){// 返回两个圆的公切线数量
    if (cmp(k1.r,k2.r)==-1) swap(k1,k2);
    db dis=k1.o.dis(k2.o); int w1=cmp(dis,k1.r+k2.r),w2=cmp(dis,k1.r-k2.r);
    if (w1>0) return 4; else if (w1==0) return 3; else if (w2>0) return 2;
    else if (w2==0) return 1; else return 0;
}

```



```

vector<point> getCL(circle k1,point k2,point k3){ // 沿着 k2->k3 方向给出 , 相切给出
    ↪ 两个
    point k=proj(k2,k3,k1.o); db d=k1.r*k1.r-(k-k1.o).abs2();
    if (sign(d)==-1) return {};
    point del=(k3-k2).unit()*sqrt(max((db)0.0,d)); return {k-del,k+del};
}

vector<point> getCC(circle k1,circle k2){// 沿圆 k1 逆时针给出 , 相切给出两个
    int pd=checkposCC(k1,k2); if (pd==0||pd==4) return {};
    db
    ↪ a=(k2.o-k1.o).abs2(),cosA=(k1.r*k1.r+a-k2.r*k2.r)/(2*k1.r*sqrt(max(a,(db)0.0)));
    db b=k1.r*cosA,c=sqrt(max((db)0.0,k1.r*k1.r-b*b));
    point k=(k2.o-k1.o).unit(),m=k1.o+k*b,del=k.turn90()*c;
    return {m-del,m+del};
}

vector<point> TangentCP(circle k1,point k2){// 沿圆 k1 逆时针给出
    db a=(k2-k1.o).abs(),b=k1.r*k1.r/a,c=sqrt(max((db)0.0,k1.r*k1.r-b*b));
    point k=(k2-k1.o).unit(),m=k1.o+k*b,del=k.turn90()*c;
    return {m-del,m+del};
}

vector<line> TangentoutCC(circle k1,circle k2){
    int pd=checkposCC(k1,k2); if (pd==0) return {};
    if (pd==1){point k=getCC(k1,k2)[0]; return {(line){k,k}};}
    if (cmp(k1.r,k2.r)==0){
        point del=(k2.o-k1.o).unit().turn90().getdel();
        return
        ↪ {(line){k1.o-del*k1.r,k2.o-del*k2.r},{k1.o+del*k1.r,k2.o+del*k2.r}};
    } else {
        point p=(k2.o*k1.r-k1.o*k2.r)/(k1.r-k2.r);
        vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);
        vector<line>ans; for (int i=0;i<A.size();i++)
            ↪ ans.push_back((line){A[i],B[i]});
        return ans;
    }
}

vector<line> TangentinCC(circle k1,circle k2){
    int pd=checkposCC(k1,k2); if (pd<=2) return {};
    if (pd==3){point k=getCC(k1,k2)[0]; return {(line){k,k}};}
    point p=(k2.o*k1.r+k1.o*k2.r)/(k1.r+k2.r);
    vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);
    vector<line>ans; for (int i=0;i<A.size();i++) ans.push_back((line){A[i],B[i]});
    return ans;
}

vector<line> TangentCC(circle k1,circle k2){
    int flag=0; if (k1.r<k2.r) swap(k1,k2),flag=1;
    vector<line>A=TangentoutCC(k1,k2),B=TangentinCC(k1,k2);
    for (line k:B) A.push_back(k);
    if (flag) for (line &k:A) swap(k[0],k[1]);
    return A;
}

```

```

db getarea(circle k1,point k2,point k3){
    // 圆 k1 与三角形 k2 k3 k1.o 的有向面积交
    point k=k1.o; k1.o=k1.o-k; k2=k2-k; k3=k3-k;
    int pd1=k1.inside(k2),pd2=k1.inside(k3);
    vector<point>A=getCL(k1,k2,k3);
    if (pd1>=0){
        if (pd2>=0) return cross(k2,k3)/2;
        return k1.r*k1.r*rad(A[1],k3)/2+cross(k2,A[1])/2;
    } else if (pd2>=0){
        return k1.r*k1.r*rad(k2,A[0])/2+cross(A[0],k3)/2;
    } else {
        int pd=cmp(k1.r,disSP(k2,k3,k1.o));
        if (pd<=0) return k1.r*k1.r*rad(k2,k3)/2;
        return cross(A[0],A[1])/2+k1.r*k1.r*(rad(k2,A[0])+rad(A[1],k3))/2;
    }
}

circle getcircle(point k1,point k2,point k3){
    db a1=k2.x-k1.x,b1=k2.y-k1.y,c1=(a1*a1+b1*b1)/2;
    db a2=k3.x-k1.x,b2=k3.y-k1.y,c2=(a2*a2+b2*b2)/2;
    db d=a1*b2-a2*b1;
    point o=(point){k1.x+(c1*b2-c2*b1)/d,k1.y+(a1*c2-a2*c1)/d};
    return (circle){o,k1.dis(o)};
}

circle getScircle(vector<point> A){
    random_shuffle(A.begin(),A.end());
    circle ans=(circle){A[0],0};
    for (int i=1;i<A.size();i++){
        if (ans.inside(A[i])== -1){
            ans=(circle){A[i],0};
            for (int j=0;j<i;j++){
                if (ans.inside(A[j])== -1){
                    ans.o=(A[i]+A[j])/2; ans.r=ans.o.dis(A[i]);
                    for (int k=0;k<j;k++){
                        if (ans.inside(A[k])== -1)
                            ans=getcircle(A[i],A[j],A[k]);
                    }
                }
            }
        }
    }
    return ans;
}

db area(vector<point> A){ // 多边形用 vector<point> 表示 , 逆时针
    db ans=0;
    for (int i=0;i<A.size();i++) ans+=cross(A[i],A[(i+1)%A.size()]);
    return ans/2;
}

int checkconvex(vector<point>A){
    int n=A.size(); A.push_back(A[0]); A.push_back(A[1]);
    for (int i=0;i<n;i++) if (sign(cross(A[i+1]-A[i],A[i+2]-A[i]))== -1) return 0;
    return 1;
}

```

```

int contain(vector<point>A,point q){ // 2 内部 1 边界 0 外部
    int pd=0; A.push_back(A[0]);
    for (int i=1;i<A.size();i++){
        point u=A[i-1],v=A[i];
        if (onS(u,v,q)) return 1; if (cmp(u.y,v.y)>0) swap(u,v);
        if (cmp(u.y,q.y)>=0||cmp(v.y,q.y)<0) continue;
        if (sign(cross(u-v,q-v))<0) pd^=1;
    }
    return pd<<1;
}

vector<point> ConvexHull(vector<point>A,int flag=1){ // flag=0 不严格 flag=1 严格
    int n=A.size(); vector<point>ans(n*2);
    sort(A.begin(),A.end()); int now=-1;
    for (int i=0;i<A.size();i++){
        while (now>0&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;
        ans[++now]=A[i];
    } int pre=now;
    for (int i=n-2;i>=0;i--){
        while (now>pre&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag)
            now--;
        ans[++now]=A[i];
    } ans.resize(now); return ans;
}

db convexDiameter(vector<point>A){
    int now=0,n=A.size(); db ans=0;
    for (int i=0;i<A.size();i++){
        now=max(now,i);
        while (1){
            db k1=A[i].dis(A[now%n]),k2=A[i].dis(A[(now+1)%n]);
            ans=max(ans,max(k1,k2)); if (k2>k1) now++; else break;
        }
    }
    return ans;
}

vector<point> convexcut(vector<point>A,point k1,point k2){
    // 保留 k1,k2,p 逆时针的所有点
    int n=A.size(); A.push_back(A[0]); vector<point>ans;
    for (int i=0;i<n;i++){
        int w1=clockwise(k1,k2,A[i]),w2=clockwise(k1,k2,A[i+1]);
        if (w1>=0) ans.push_back(A[i]);
        if (w1*w2<0) ans.push_back(getLL(k1,k2,A[i],A[i+1]));
    }
    return ans;
}

int checkPoS(vector<point>A,point k1,point k2){
    // 多边形 A 和直线 ( 线段 )k1->k2 严格相交 , 注释部分为线段
    struct ins{
        point m,u,v;
        int operator < (const ins& k) const {return m<k.m;}
    }

```

```

}; vector<ins>B;
//if (contain(A,k1)==2||contain(A,k2)==2) return 1;
vector<point>poly=A; A.push_back(A[0]);
for (int i=1;i<A.size();i++) if (checkLL(A[i-1],A[i],k1,k2)){
    point m=getLL(A[i-1],A[i],k1,k2);
    if (inmid(A[i-1],A[i],m)/*inmid(k1,k2,m)*/)
        ↪ B.push_back((ins){m,A[i-1],A[i]});
}
if (B.size()==0) return 0; sort(B.begin(),B.end());
int now=1; while (now<B.size()&&B[now].m==B[0].m) now++;
if (now==B.size()) return 0;
int flag=contain(poly,(B[0].m+B[now].m)/2);
if (flag==2) return 1;
point d=B[now].m-B[0].m;
for (int i=now;i<B.size();i++){
    if (!(B[i].m==B[i-1].m)&&flag==2) return 1;
    int tag=sign(cross(B[i].v-B[i].u,B[i].m+d-B[i].u));
    if (B[i].m==B[i].u||B[i].m==B[i].v) flag+=tag; else flag+=tag*2;
}
//return 0;
return flag==2;
}
int checkinp(point r,point l,point m){
    if (compareangle(l,r)){return compareangle(l,m)&&compareangle(m,r);}
    return compareangle(l,m)||compareangle(m,r);
}
int checkPosFast(vector<point>A,point k1,point k2){ // 快速检查线段是否和多边形严格
    ↪ 相交
    if (contain(A,k1)==2||contain(A,k2)==2) return 1; if (k1==k2) return 0;
    A.push_back(A[0]); A.push_back(A[1]);
    for (int i=1;i+1<A.size();i++)
        if (checkLL(A[i-1],A[i],k1,k2)){
            point now=getLL(A[i-1],A[i],k1,k2);
            if (inmid(A[i-1],A[i],now)==0||inmid(k1,k2,now)==0)
                ↪ continue;
            if (now==A[i]){
                if (A[i]==k2) continue;
                point pre=A[i-1],ne=A[i+1];
                if (checkinp(pre-now,ne-now,k2-now)) return 1;
            } else if (now==k1){
                if (k1==A[i-1]||k1==A[i]) continue;
                if (checkinp(A[i-1]-k1,A[i]-k1,k2-k1)) return 1;
            } else if (now==k2||now==A[i-1]) continue;
            else return 1;
        }
    return 0;
}
// 拆分凸包成上下凸壳 凸包尽量都随机旋转一个角度来避免出现相同横坐标
// 尽量特判只有一个点的情况 凸包逆时针

```

```

void getUDP(vector<point>A,vector<point>&U,vector<point>&D){
    db l=1e100,r=-1e100;
    for (int i=0;i<A.size();i++) l=min(l,A[i].x),r=max(r,A[i].x);
    int wherel,wherer;
    for (int i=0;i<A.size();i++) if (cmp(A[i].x,l)==0) wherel=i;
    for (int i=A.size();i;i--) if (cmp(A[i-1].x,r)==0) wherer=i-1;
    U.clear(); D.clear(); int now=wherel;
    while (1){D.push_back(A[now]); if (now==wherer) break; now++; if
        ↪ (now>=A.size()) now=0;}
    now=wherel;
    while (1){U.push_back(A[now]); if (now==wherer) break; now--; if (now<0)
        ↪ now=A.size()-1;}
}
// 需要保证凸包点数大于等于 3,2 内部 ,1 边界 ,0 外部
int containCoP(const vector<point>&U,const vector<point>&D,point k){
    db lx=U[0].x,rx=U[U.size()-1].x;
    if (k==U[0]||k==U[U.size()-1]) return 1;
    if (cmp(k.x,lx)==-1||cmp(k.x,rx)==1) return 0;
    int wherel=lower_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();
    int where2=lower_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();
    int
        ↪ w1=clockwise(U[wherel-1],U[wherel],k),w2=clockwise(D[where2-1],D[where2],k);
    if (w1==1||w2==-1) return 0; else if (w1==0||w2==0) return 1; return 2;
}
// d 是方向 , 输出上方切点和下方切点
pair<point,point> getTangentCow(const vector<point> &U,const vector<point> &D,point
    ↪ d){
    if (sign(d.x)<0||(sign(d.x)==0&&sign(d.y)<0)) d=d*(-1);
    point whereU,whereD;
    if (sign(d.x)==0) return mp(U[0],U[U.size()-1]);
    int l=0,r=U.size()-1,ans=0;
    while (l<r){int mid=l+r>>1; if (sign(cross(U[mid+1]-U[mid],d))<=0)
        ↪ l=mid+1,ans=mid+1; else r=mid;}
    whereU=U[ans]; l=0,r=D.size()-1,ans=0;
    while (l<r){int mid=l+r>>1; if (sign(cross(D[mid+1]-D[mid],d))>=0)
        ↪ l=mid+1,ans=mid+1; else r=mid;}
    whereD=D[ans]; return mp(whereU,whereD);
}
// 先检查 contain, 逆时针给出
pair<point,point> getTangentCoP(const vector<point>&U,const vector<point>&D,point
    ↪ k){
    db lx=U[0].x,rx=U[U.size()-1].x;
    if (k.x<lx){
        int l=0,r=U.size()-1,ans=U.size()-1;
        while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==1) l=mid+1;
            ↪ else ans=mid,r=mid;}
        point w1=U[ans]; l=0,r=D.size()-1,ans=D.size()-1;
        while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==1) l=mid+1;
            ↪ else ans=mid,r=mid;}
    }
}

```

```

    point w2=D[ans]; return mp(w1,w2);
} else if (k.x>rx){
    int l=1,r=U.size(),ans=0;
    while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==-1) r=mid;
        ↪ else ans=mid,l=mid+1;}
    point w1=U[ans]; l=1,r=D.size(),ans=0;
    while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==1) r=mid;
        ↪ else ans=mid,l=mid+1;}
    point w2=D[ans]; return mp(w2,w1);
} else {
    int where1=lower_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();
    int where2=lower_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();
    if
        ↪ ((k.x==lx&&k.y>U[0].y)|| (where1&&clockwise(U[where1-1],U[where1],k)==1)){
        int l=1,r=where1+1,ans=0;
        while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==1)
            ↪ ans=mid,l=mid+1; else r=mid;}
        point w1=U[ans]; l=where1,r=U.size()-1,ans=U.size()-1;
        while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==1)
            ↪ l=mid+1; else ans=mid,r=mid;}
        point w2=U[ans]; return mp(w2,w1);
    } else {
        int l=1,r=where2+1,ans=0;
        while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==-1)
            ↪ ans=mid,l=mid+1; else r=mid;}
        point w1=D[ans]; l=where2,r=D.size()-1,ans=D.size()-1;
        while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1)
            ↪ l=mid+1; else ans=mid,r=mid;}
        point w2=D[ans]; return mp(w1,w2);
    }
}
}

struct P3{
    db x,y,z;
    P3 operator + (P3 k1){return (P3){x+k1.x,y+k1.y,z+k1.z};}
    P3 operator - (P3 k1){return (P3){x-k1.x,y-k1.y,z-k1.z};}
    P3 operator * (db k1){return (P3){x*k1,y*k1,z*k1};}
    P3 operator / (db k1){return (P3){x/k1,y/k1,z/k1};}
    db abs2(){return x*x+y*y+z*z;}
    db abs(){return sqrt(x*x+y*y+z*z);}
    P3 unit(){return (*this)/abs();}
    int operator < (const P3 k1) const{
        if (cmp(x,k1.x)!=0) return x<k1.x;
        if (cmp(y,k1.y)!=0) return y<k1.y;
        return cmp(z,k1.z)==-1;
    }
    int operator == (const P3 k1){
        return cmp(x,k1.x)==0&&cmp(y,k1.y)==0&&cmp(z,k1.z)==0;
    }
}

```

```

    void scan(){
        double k1,k2,k3; scanf("%lf%lf%lf",&k1,&k2,&k3);
        x=k1; y=k2; z=k3;
    }
};
P3 cross(P3 k1,P3 k2){return
↪ (P3){k1.y*k2.z-k1.z*k2.y,k1.z*k2.x-k1.x*k2.z,k1.x*k2.y-k1.y*k2.x};}
db dot(P3 k1,P3 k2){return k1.x*k2.x+k1.y*k2.y+k1.z*k2.z;}
//p=(3,4,5),l=(13,19,21),theta=85 ans=(2.83,4.62,1.77)
P3 turn3D(db k1,P3 l,P3 p){
    l=l.unit(); P3 ans; db c=cos(k1),s=sin(k1);

    ↪ ans.x=p.x*(l.x*l.x*(1-c)+c)+p.y*(l.x*l.y*(1-c)-l.z*s)+p.z*(l.x*l.z*(1-c)+l.y*s);

    ↪ ans.y=p.x*(l.x*l.y*(1-c)+l.z*s)+p.y*(l.y*l.y*(1-c)+c)+p.z*(l.y*l.z*(1-c)-l.x*s);

    ↪ ans.z=p.x*(l.x*l.z*(1-c)-l.y*s)+p.y*(l.y*l.z*(1-c)+l.x*s)+p.z*(l.x*l.x*(1-c)+c);
    return ans;
}
typedef vector<P3> VP;
typedef vector<VP> VVP;
db Acos(db x){return acos(max(-(db)1,min(x,(db)1)));}
// 球面距离 , 圆心原点 , 半径 1
db Odist(P3 a,P3 b){db r=Acos(dot(a,b)); return r;}
db r; P3 rnd;
vector<db> solve(db a,db b,db c){
    db r=sqrt(a*a+b*b),th=atan2(b,a);
    if (cmp(c,-r)==-1) return {0};
    else if (cmp(r,c)<=0) return {1};
    else {
        db tr=pi-Acos(c/r); return {th+pi-tr,th+pi+tr};
    }
}
vector<db> jiao(P3 a,P3 b){
    // dot(rd+x*cos(t)+y*sin(t),b) >= cos(r)
    if (cmp(Odist(a,b),2*r)>0) return {0};
    P3 rd=a*cos(r),z=a.unit(),y=cross(z,rnd).unit(),x=cross(y,z).unit();
    vector<db> ret =
    ↪ solve(-(dot(x,b)*sin(r)),-(dot(y,b)*sin(r)),-(cos(r)-dot(rd,b)));
    return ret;
}
db norm(db x,db l=0,db r=2*pi){ // change x into [l,r)
    while (cmp(x,l)==-1) x+=(r-l); while (cmp(x,r)>=0) x-=(r-l);
    return x;
}
db disLP(P3 k1,P3 k2,P3 q){
    return (cross(k2-k1,q-k1)).abs()/(k2-k1).abs();
}
db disLL(P3 k1,P3 k2,P3 k3,P3 k4){

```

```

    P3 dir=cross(k2-k1,k4-k3); if (sign(dir.abs())==0) return disLP(k1,k2,k3);
    return fabs(dot(dir.unit(),k1-k2));
}
VP getFL(P3 p,P3 dir,P3 k1,P3 k2){
    db a=dot(k2-p,dir),b=dot(k1-p,dir),d=a-b;
    if (sign(fabs(d))==0) return {};
    return {(k1*a-k2*b)/d};
}
VP getFF(P3 p1,P3 dir1,P3 p2,P3 dir2){// 返回一条线
    P3 e=cross(dir1,dir2),v=cross(dir1,e);
    db d=dot(dir2,v); if (sign(abs(d))==0) return {};
    P3 q=p1+v*dot(dir2,p2-p1)/d; return {q,q+e};
}
// 3D Convex Hull Template
db getV(P3 k1,P3 k2,P3 k3,P3 k4){ // get the Volume
    return dot(cross(k2-k1,k3-k1),k4-k1);
}
db rand_db(){return 1.0*rand()/RAND_MAX;}
VP convexHull2D(VP A,P3 dir){
    P3 x={(db)rand(),(db)rand(),(db)rand()}; x=x.unit();
    x=cross(x,dir).unit(); P3 y=cross(x,dir).unit();
    P3 vec=dir.unit()*dot(A[0],dir);
    vector<point>B;
    for (int i=0;i<A.size();i++) B.push_back((point){dot(A[i],x),dot(A[i],y)});
    B=ConvexHull(B); A.clear();
    for (int i=0;i<B.size();i++) A.push_back(x*B[i].x+y*B[i].y+vec);
    return A;
}
namespace CH3{
    VVP ret; set<pair<int,int> >e;
    int n; VP p,q;
    void wrap(int a,int b){
        if (e.find({a,b})==e.end()){
            int c=-1;
            for (int i=0;i<n;i++) if (i!=a&&i!=b){
                if (c==-1||sign(getV(q[c],q[a],q[b],q[i]))>0) c=i;
            }
            if (c!=-1){
                ret.push_back({p[a],p[b],p[c]});
                e.insert({a,b}); e.insert({b,c}); e.insert({c,a});
                wrap(c,b); wrap(a,c);
            }
        }
    }
}
VVP ConvexHull3D(VP _p){
    p=q=_p; n=p.size();
    ret.clear(); e.clear();
    for (auto &i:q) i=i+(P3){rand_db()*1e-4,rand_db()*1e-4,rand_db()*1e-4};
    for (int i=1;i<n;i++) if (q[i].x<q[0].x) swap(p[0],p[i]),swap(q[0],q[i]);
}

```



```

    for (int i=2;i<n;i++) if
        ↪ ((q[i].x-q[0].x)*(q[1].y-q[0].y)>(q[i].y-q[0].y)*(q[1].x-q[0].x))
        ↪ swap(q[1],q[i]),swap(p[1],p[i]);
    wrap(0,1);
    return ret;
}
}
VVP reduceCH(VVP A){
    VVP ret; map<P3,VP> M;
    for (VP nowF:A){
        P3 dir=cross(nowF[1]-nowF[0],nowF[2]-nowF[0]).unit();
        for (P3 k1:nowF) M[dir].pb(k1);
    }
    for (pair<P3,VP> nowF:M) ret.pb(convexHull2D(nowF.se,nowF.fi));
    return ret;
}
// 把一个面变成 ( 点 , 法向量 ) 的形式
pair<P3,P3> getF(VP F){
    return mp(F[0],cross(F[1]-F[0],F[2]-F[0]).unit());
}
// 3D Cut 保留 dot(dir,x-p)>=0 的部分
VVP ConvexCut3D(VVP A,P3 p,P3 dir){
    VVP ret; VP sec;
    for (VP nowF: A){
        int n=nowF.size(); VP ans; int dif=0;
        for (int i=0;i<n;i++){
            int d1=sign(dot(dir,nowF[i]-p));
            int d2=sign(dot(dir,nowF[(i+1)%n]-p));
            if (d1>=0) ans.pb(nowF[i]);
            if (d1*d2<0){
                P3 q=getFL(p,dir,nowF[i],nowF[(i+1)%n])[0];
                ans.push_back(q); sec.push_back(q);
            }
            if (d1==0) sec.push_back(nowF[i]); else dif=1;
            ↪ dif|=(sign(dot(dir,cross(nowF[(i+1)%n]-nowF[i],nowF[(i+1)%n]-nowF[i])))==-1);
        }
        if (ans.size()>0&&dif) ret.push_back(ans);
    }
    if (sec.size()>0) ret.push_back(convexHull2D(sec,dir));
    return ret;
}
db vol(VVP A){
    if (A.size()==0) return 0; P3 p=A[0][0]; db ans=0;
    for (VP nowF:A)
        for (int i=2;i<nowF.size();i++)
            ans+=abs(getV(p,nowF[0],nowF[i-1],nowF[i]));
    return ans/6;
}

```

```

VVP init(db INF) {
    VVP pss(6,VP(4));
    pss[0][0] = pss[1][0] = pss[2][0] = {-INF, -INF, -INF};
    pss[0][3] = pss[1][1] = pss[5][2] = {-INF, -INF, INF};
    pss[0][1] = pss[2][3] = pss[4][2] = {-INF, INF, -INF};
    pss[0][2] = pss[5][3] = pss[4][1] = {-INF, INF, INF};
    pss[1][3] = pss[2][1] = pss[3][2] = {INF, -INF, -INF};
    pss[1][2] = pss[5][1] = pss[3][3] = {INF, -INF, INF};
    pss[2][2] = pss[4][3] = pss[3][1] = {INF, INF, -INF};
    pss[5][0] = pss[4][0] = pss[3][0] = {INF, INF, INF};
    return pss;
}

```

6.2 Plane

```
#include<bits/stdc++.h>
```

```

namespace Geometry {
    typedef double db;
    const db inf = 1e20;
    const int maxn = 1;
    const db eps = 1e-8;
    const db delta = 0.98;

    int Sgn(db k) { return fabs(k) < eps ? 0 : (k < 0 ? -1 : 1);}
    int Cmp(db k1, db k2) {return Sgn(k1 - k2);}

    /*-----点 (向量)-----*/
    struct point {db X, Y;};
    bool operator == (point k1, point k2) {return Cmp(k1.X, k2.X) == 0 && Cmp(k1.Y,
        ↪ k2.Y) == 0;}
    point operator + (point k1, point k2) {return (point){k1.X + k2.X, k1.Y +
        ↪ k2.Y};}
    point operator - (point k1, point k2) {return (point){k1.X - k2.X, k1.Y -
        ↪ k2.Y};}
    db operator * (point k1, point k2) {return k1.X * k2.X + k1.Y * k2.Y;}
    db operator ^ (point k1, point k2) {return k1.X * k2.Y - k1.Y * k2.X;}
    point operator * (point k1, db k2) {return (point){k1.X * k2, k1.Y * k2};}
    point operator / (point k1, db k2) {return (point){k1.X / k2, k1.Y / k2};}
    db GetLen(point k) {return sqrt(k * k);}
    db DisP2P(point k1, point k2) {return sqrt((k1 - k2) * (k1 - k2));}
    db DisP2P2(point k1, point k2) {return (k1 - k2) * (k1 - k2);}
    db GetAng(point k1, point k2) {return fabs(atan2(fabs(k1 ^ k2), k1 * k2));}
    point Rotate(point k, db ang) {return (point){k.X * cos(ang) - k.Y * sin(ang),
        ↪ k.X * sin(ang) + k.Y * cos(ang)};}
    point Rotate90(point k) {return (point){-k.Y, k.X};}
    bool IsConvexHull(vector<point> points) {
        int N = (int)points.size();
        for (int i = 0; i < N; ++i)

```

```

        if (Sgn((points[(i + 1) % N] - points[i]) ^ (points[(i + 2) % N] -
        ↪ points[(i + 1) % N])) < 0)
            return false;
    return true;
}

db ClosestP2P(point p[], int l, int r) {
    if (l + 1 == r) return GetDisP2P(p[l], p[r]);
    if (l + 2 == r) return min(GetDisP2P(p[l + 1], p[r]), min(GetDisP2P(p[l],
    ↪ p[l + 1]), GetDisP2P(p[l], p[r])));
    int mid = (l + r) >> 1;
    db ans = min(solve(l, mid), solve(mid + 1, r));
    vector<point> mid_p;
    for (int i = l; i <= r; ++i) {
        if (Cmp(fabs(p[i].x - p[mid].x), ans) <= 0) mid_p.push_back(p[i]);
    }
    sort(mid_p.begin(), mid_p.end(), [&](point k1, point k2) {return Cmp(k1.y,
    ↪ k2.y) < 0;});
    for (int i = 0; i < mid_p.size(); ++i) {
        for (int j = i + 1; j < mid_p.size(); ++j) {
            if (Cmp(mid_p[j].y - mid_p[i].y, ans) >= 0) break;
            ans = min(ans, GetDisP2P(mid_p[i], mid_p[j]));
        }
    }
    return ans;
}

/*-----多边形-----*/
typedef vector<point> poly;
void RotateCaliper() {
    ans = -1e20;
    if (ConvexHull.size() == 3) {
        if (Cmp(DisP2P(ConvexHull[0], ConvexHull[1]), ans) > 0) ans =
        ↪ DisP2P(ConvexHull[0], ConvexHull[1]);
        if (Cmp(DisP2P(ConvexHull[0], ConvexHull[2]), ans) > 0) ans =
        ↪ DisP2P(ConvexHull[0], ConvexHull[2]);
        if (Cmp(DisP2P(ConvexHull[1], ConvexHull[2]), ans) > 0) ans =
        ↪ DisP2P(ConvexHull[1], ConvexHull[2]);
        return;
    }
    int cur = 2, size = ConvexHull.size();
    for (int i = 0; i < size; ++i) {
        while (Cmp(fabs((ConvexHull[i] - ConvexHull[(i + 1) % size]) ^
        ↪ (ConvexHull[cur] - ConvexHull[(i + 1) % size])),
        ↪ fabs((ConvexHull[i] - ConvexHull[(i + 1) % size]) ^
        ↪ (ConvexHull[(cur + 1) % size] - ConvexHull[(i + 1) % size])))) < 0)
            cur = (cur + 1) % size;
        if (Cmp(DisP2P(ConvexHull[i], ConvexHull[cur]), ans) > 0) ans =
        ↪ DisP2P(ConvexHull[i], ConvexHull[cur]);
    }
}

```

```
    }
}

poly Grahamscan(point points[], int N) {
    poly ans;
    if (N < 3) {
        for (int i = 0; i < N; ++i) ans.push_back(points[i]);
        return ans;
    }
    int Basic = 0;
    for (int i = 0; i < N; ++i)
        if (Cmp(points[i].X, points[Basic].X) < 0 || (Cmp(points[i].X,
            ↪ points[Basic].X) == 0 && Cmp(points[i].Y, points[Basic].Y) < 0))
            Basic = i;
    std::swap(points[0], points[Basic]);
    std::sort(points + 1, points + N, [&](point k1, point k2) {
        double temp = (k1 - points[0]) ^ (k2 - points[0]);
        if (Sgn(temp) > 0) return true;
        else if (Sgn(temp) == 0 && Cmp(DisP2P(k2, points[0]), DisP2P(k1,
            ↪ points[0])) > 0) return true;
        return false;
    });
    ans.push_back(points[0]);
    for (int i = 1; i < N; ++i) {
        while ((int)ans.size() >= 2 && Sgn((ans.back() - ans[(ans.size()) - 2])
            ↪ ^ (points[i] - ans[(int)ans.size() - 2])) <= 0) {
            ans.pop_back();
        }
        ans.push_back(points[i]);
    }
    return ans;
}

db MinCircleCoverage(vector<point> points) {
    point cur = points[0];
    db Probability = 10000, ans = inf;
    while (Probability > eps) {
        int Book = 0;
        for (int i = 0; i < (int)points.size(); ++i)
            if (DisP2P(cur, points[i]) > DisP2P(cur, points[Book]))
                Book = i;
        db r = DisP2P(cur, points[Book]);
        if (Cmp(r, ans) < 0) ans = r;
        cur = cur + (points[Book] - cur) / r * Probability;
        Probability *= delta;
    }
    return ans;
}
```

```

/*-----线 (线段)-----*/
struct line {point s, t;};
typedef line seg;
db GetLen(seg k) {return Disp2p(k.s, k.t);}
db DisP2Line(point k1, line k2) {return fabs((k1 - k2.s) ^ (k2.t - k2.s)) /
↪ GetLen(k2);}
db DisP2Seg(point k1, seg k2) {
    if (Sgn((k1 - k2.s) * (k2.t - k2.s)) < 0 || Sgn((k1 - k2.t) * (k2.s -
↪ k2.t)) < 0) {
        return min(DisP2P(k1, k2.s), DisP2P(k1, k2.t));
    }
    return DisP2P(k1, k2);
}
bool IsParallel(line k1, line k2) {return Sgn((k1.s - k1.t) ^ (k2.s - k2.t)) ==
↪ 0;}
bool IsSegInterSeg(seg k1, seg k2) {
    return
        max(k1.s.X, k1.t.X) >= min(k2.s.X, k2.t.X) &&
        max(k2.s.X, k2.t.X) >= min(k1.s.X, k1.t.X) &&
        max(k1.s.Y, k1.t.Y) >= min(k2.s.Y, k2.t.Y) &&
        max(k2.s.Y, k2.t.Y) >= min(k1.s.Y, k1.t.Y) &&
        Sgn((k2.s - k1.t) ^ (k1.s - k1.t)) * Sgn((k2.t - k1.t) ^ (k1.s - k1.t))
↪ <= 0 &&
        Sgn((k1.s - k2.t) ^ (k2.s - k2.t)) * Sgn((k1.t - k2.t) ^ (k2.s - k2.t))
↪ <= 0;
}
bool IsLineInterSeg(line k1, seg k2) {
    return Sgn((k2.s - k1.t) ^ (k1.s - k1.t)) * Sgn((k2.t - k1.t) ^ (k1.s -
↪ k1.t)) <= 0;
}
bool IsLineInterLine(line k1, line k2) {
    return !IsParallel(k1, k2) || (IsParallel(k1, k2) && !(Sgn((k1.s - k2.s) ^
↪ (k2.t - k2.s)) == 0));
}
bool IsPointOnSeg(point k1, seg k2) {
    return Sgn((k1 - k2.s) ^ (k2.t - k2.s)) == 0 && Sgn((k1 - k2.s) * (k1 -
↪ k2.t)) <= 0;
}
point Cross(line k1, line k2) {
    db temp = ((k1.s - k2.s) ^ (k2.s - k2.t)) / ((k1.s - k1.t) ^ (k2.s -
↪ k2.t));
    return (point){k1.s.X + (k1.t.X - k1.s.X) * temp, k1.s.Y + (k1.t.Y -
↪ k1.s.Y) * temp};
}

/*-----半平面-----*/
// 表示 s->t 逆时针 (左侧) 的半平面
struct halfplane:public line {db ang;};
void GetAng(halfplane k) {k.ang = atan2(k.t.Y - k.s.Y, k.t.X - k.s.X);}

```

```
bool operator < (halfplane k1, halfplane k2) {
    if (Sgn(k1.ang - k2.ang) > 0) return k1.ang < k2.ang;
    return Sgn((k1.s - k2.s) ^ (k2.t - k2.s)) < 0;
}

struct HalfPlaneInsert {
    int tot;
    halfplane hp[maxn];
    halfplane deq[maxn];
    point points[maxn];
    point Res[maxn];
    int front, tail;

    void Push(halfplane k) {hp[tot++] = k;}

    void Unique() {
        int Cnt = 1;
        for (int i = 1; i < tot; ++i)
            if (fabs(hp[i].ang - hp[i - 1].ang) > eps)
                hp[Cnt++] = hp[i];
        tot = Cnt;
    }

    bool IsHalfPlaneInsert() {
        for (int i = 0; i < tot; ++i) GetAng(hp[i]);
        sort(hp, hp + tot);
        Unique();
        deq[front = 0] = hp[0];
        deq[tail = 1] = hp[1];
        for (int i = 2; i < tot; ++i) {
            if (fabs((deq[tail].t - deq[tail].s) ^ (deq[tail - 1].t - deq[tail - 1].s)) < eps || fabs((deq[front].t - deq[front].s) ^ (deq[front + 1].t - deq[front + 1].s)) < eps) return false;
            while (front < tail && ((Cross(deq[tail], deq[tail - 1]) - hp[i].s) ^ (hp[i].t - hp[i].s)) > eps) tail--;
            while (front < tail && ((Cross(deq[front], deq[front + 1]) - hp[i].s) ^ (hp[i].t - hp[i].s)) > eps) front++;
            deq[++tail] = hp[i];
        }
        while (front < tail && ((Cross(deq[tail], deq[tail - 1]) - deq[front].s) ^ (deq[front].t - deq[front].s)) > eps) tail--;
        while (front < tail && ((Cross(deq[front], deq[front + 1]) - deq[tail].s) ^ (deq[tail].t - deq[tail].t)) > eps) front++;
        if (tail <= front + 1) {
            return false;
        }
        return true;
    }

    void GetHalfPlaneInsertConvex() {
```

```
    int Cnt = 0;
    for (int i = front; i < tail; ++i) Res[Cnt++] = Cross(deq[i], deq[i +
        ↪ 1]);
    if (front < tail - 1) Res[Cnt++] = Cross(deq[front], deq[tail]);
}
};

/*-----圆-----*/
struct Circle {point o; db r;};
};
using namespace Geometry;
```

6.3 Simpson

```
typedef double db;

namespace Simpson {
    db a, b, c, d;

    db F(db x) {
        return (c * x + d) / (a * x + b);
    }

    db Simpson(db l, db r) {
        db m = (l + r) / 2.0;
        return (F(l) + 4 * F(m) + F(r)) * (r - l) / 6.0;
    }

    db Asr(db l, db r, db ans, db eps) {
        db m = (l + r) / 2.0;
        db l_ans = Simpson(l, m), r_ans = Simpson(m, r);
        if (fabs(l_ans + r_ans - ans) <= 15.0 * eps) return l_ans + r_ans + (l_ans +
            ↪ r_ans - ans) / 15.0;
        return Asr(l, m, l_ans, eps / 2.0) + Asr(m, r, r_ans, eps / 2.0);
    }
};
```

6.4 Stereoscopic

```
#include<bits/stdc++.h>

namespace Geometry3D {
    typedef double db;
    const db INF = 1e20;
    const int maxn = "Edit";
    const db eps = 1e-9;
    const db delta = 0.98;

    int Sgn(db Key) {return fabs(Key) < eps ? 0 : (Key < 0 ? -1 : 1);}
    int Cmp(db Key1, db Key2) {return Sgn(Key1 - Key2);}
}
```

```
/*-----点 (向量)-----*/
```

```
struct Point {db X, Y, Z;};
typedef Point Vector;
bool operator == (Point Key1, Point Key2) {return Sgn(Key1.X - Key2.X) == 0 &&
    ↪ Sgn(Key1.Y - Key2.Y) == 0 && Sgn(Key1.Z - Key2.Z) == 0;}
Vector operator + (Vector Key1, Vector Key2) {return (Vector){Key1.X + Key2.X,
    ↪ Key1.Y + Key2.Y, Key1.Z + Key2.Z};}
Vector operator - (Vector Key1, Vector Key2) {return (Vector){Key1.X - Key2.X,
    ↪ Key1.Y - Key2.Y, Key1.Z - Key2.Z};}
db operator * (Vector Key1, Vector Key2) {return Key1.X * Key2.X + Key1.Y *
    ↪ Key2.Y + Key1.Z * Key2.Z;}
db GetLen(Vector Key) {return sqrt(Key * Key);}
db GetLen2(Vector Key) {return Key * Key;}
db operator ^ (Vector Key1, Vector Key2) {return GetLen((Vector){Key1.Y *
    ↪ Key2.Z - Key1.Z * Key2.Y, Key1.Z * Key2.X - Key1.X * Key2.Z, Key1.X *
    ↪ Key2.Y - Key1.Y * Key2.X});}
Vector operator * (Vector Key1, db Key2) {return (Vector){Key1.X * Key2, Key1.Y
    ↪ * Key2, Key1.Z * Key2};}
Vector operator / (Vector Key1, db Key2) {return (Vector){Key1.X / Key2, Key1.Y
    ↪ / Key2, Key1.Z / Key2};}
db DisPointToPoint(Point Key1, Point Key2) {return GetLen(Key2 - Key1);}
db DisPointToPoint2(Point Key1, Point Key2) {return GetLen2(Key2 - Key1);}
db GetAngle(Vector Key1, Vector Key2) {return fabs(atan2(fabs(Key1 ^ Key2),
    ↪ Key1 * Key2));}

db MinimimSphereCoverage(vector<Point> points, int N) {
    Point Cur = points[0];
    db Probability = 10000, Ans = INF;
    while (Probability > eps) {
        int Book = 0;
        for (int i = 0; i < (int)points.size(); ++i) {
            if (Cmp(Distance(Cur, points[i]), Distance(Cur, points[Book])) > 0)
                ↪ {
                    Book = i;
                }
        }
        db Radius = Distance(Cur, points[Book]);
        Ans = min(Ans, Radius);
        Cur = Cur + (points[Book] - Cur) / Radius * Probability;
        Probability *= delta;
    }
    return Ans;
}
```

```
/*-----线 (线段)-----*/
```

```
struct Line {Point S, T;};
typedef Line Segment;
db Length(Segment Key) {return DisPointToPoint(Key.S, Key.T);}

```



```

db DisPointToLine(Point Key1, Line Key2) {return fabs((Key1 - Key2.S) ^ (Key2.T
↪ - Key2.S)) / Length(Key2);}
db DisPointToSeg(Point Key1, Segment Key2) {
    if (Sgn((Key1 - Key2.S) * (Key2.T - Key2.S)) < 0 || Sgn((Key1 - Key2.T) *
↪ (Key2.S - Key2.T)) < 0) {
        return min(DisPointToPoint(Key1, Key2.S), DisPointToPoint(Key1,
↪ Key2.T));
    }
    return DisPointToLine(Key1, Key2);
}

/*-----球-----*/
struct Sphere {Point Center;db Radius;};
db GetVolume(Sphere Key) {return 4.0 / 3.0 * pi * Key.Radius * Key.Radius *
↪ Key.Radius;}
db SphereIntersectVolume(Sphere Key1, Sphere Key2) {
    db Ans = 0.0;
    db Dis = DisPointToPoint(Key1.Center, Key2.Center);
    if (Sgn(Dis - Key1.Radius - Key2.Radius) >= 0) {
        return Ans;
    }
    if (Sgn(Key2.Radius - (Dis + Key1.Radius)) >= 0) {
        return CalVolume(Key1);
    }
    else if (Sgn(Key1.Radius - (Dis + Key2.Radius)) >= 0) {
        return CalVolume(Key2);
    }
    db Length1 = ((Key1.Radius * Key1.Radius - Key2.Radius * Key2.Radius) / Dis
↪ + Dis) / 2;
    db Length2 = Dis - Length1;
    db X1 = Key1.Radius - Length1, X2 = Key2.Radius - Length2;
    db V1 = pi * X1 * X1 * (Key1.Radius - X1 / 3.0);
    db V2 = pi * X2 * X2 * (Key2.Radius - X2 / 3.0);
    return V1 + V2;
}

bool IsRayInterSphere(Ray Key1, Sphere Key2, db &Dis) {
    db A = Key1.Dir * Key1.Dir;
    db B = (Key1.Origin - Key2.Center) * Key1.Dir * 2.0;
    db C = ((Key1.Origin - Key2.Center) * (Key1.Origin - Key2.Center)) -
↪ (Key2.Radius * Key2.Radius);
    db Delta = B * B - 4.0 * A * C;
    if (Sgn(Delta) < 0) return false;
    db X1 = (-B - sqrt(Delta)) / (2.0 * A), X2 = (-B + sqrt(Delta)) / (2.0 *
↪ A);
    if (Cmp(X1, X2) > 0) swap(X1, X2);
    if (Sgn(X1) <= 0) return false;
    Dis = X1;
    return true;
}

```

```
    }

    void Reflect(Ray &Key1, Sphere Key2, db Dis) {
        Point Pos = Key1.Origin + (Key1.Dir * Dis);
        Vector Temp = Key2.Center + (((Pos - Key2.Center) * ((Pos - Key2.Center) *
        ↪ (Key1.Origin - Key2.Center))) / GetLen2(Pos - Key2.Center));
        Key1.Dir = Temp * 2.0 - Key1.Origin - Pos; Key1.Origin = Pos;
    }
};
using namespace Geometry3D;
```

7 Others

7.1 Factorial

```
#include <bits/stdc++.h>

void Factorial() {
    int res[10010];
    int Book = 1;
    int BaoFour = 0;
    res[Book] = 1;
    int n;
    scanf("%d", &n);
    // 乘法计算
    for (int i = 1; i <= n; ++i) {
        BaoFour = 0;
        for (int j = 1; j <= Book; ++j) {
            res[j] = res[j] * i + BaoFour;
            BaoFour = res[j] / 10000;
            res[j] = res[j] % 10000;
        }
        if (BaoFour > 0) {
            res[++Book] += BaoFour;
        }
    }
    printf("%d", res[Book]);
    // 补零输出
    for (int i = Book - 1; i > 0; --i) {
        if (res[i] >= 1000) {
            printf("%d", res[i]);
        }
        else if (res[i] >= 100) {
            printf("0%d", res[i]);
        }
        else if (res[i] >= 10) {
            printf("00%d", res[i]);
        }
        else {
            printf("000%d", res[i]);
        }
    }
    putchar('\n');
}
```

7.2 FastIO

```
#include <bits/stdc++.h>

// 普通读入挂
template <class T>
```

```
inline bool read(T &ret) {
    char c;
    int sgn;
    if (c = getchar(), c == EOF) {
        return false;
    }
    while (c != '-' && (c < '0' || c > '9')) {
        c = getchar();
    }
    sgn = (c == '-') ? -1 : 1;
    ret = (c == '-') ? 0 : (c - '0');
    while (c = getchar(), c >= '0' && c <= '9') {
        ret = ret * 10 + (c - '0');
    }
    ret *= sgn;
    return true;
}

// 普通输出挂
template <class T>
inline void out(T x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) {
        out(x / 10);
    }
    putchar(x % 10 + '0');
}

// 牛逼读入挂
namespace fastIO {
    const int MX = 4e7;
    char buf[MX];
    int c, sz;
    void Begin() {
        c = 0;
        sz = fread(buf, 1, MX, stdin);
    }
    template <class T>
    inline bool Read(T &t) {
        while (c < sz && buf[c] != '-' && (buf[c] < '0' || buf[c] > '9')) {
            c++;
        }
        if (c >= sz) {
            return false;
        }
        bool flag = 0;

```

```

    if (buf[c] == '-') {
        flag = 1;
        c++;
    }
    for (t = 0; c < sz && '0' <= buf[c] && buf[c] <= '9'; ++c) {
        t = t * 10 + buf[c] - '0';
    }
    if (flag) {
        t = -t;
    }
    return true;
}
};

// 超级读写挂
namespace IO{
#define BUF_SIZE 100000
#define OUT_SIZE 100000
#define ll long long
//fread->read

bool IOerror=0;
inline char nc(){
    static char buf[BUF_SIZE],*p1=buf+BUF_SIZE,*pend=buf+BUF_SIZE;
    if (p1==pend){
        p1=buf; pend=buf+fread(buf,1,BUF_SIZE,stdin);
        if (pend==p1){IOerror=1;return -1;}
        //{printf("IO error!\n");system("pause");for (;;);exit(0);}
    }
    return *p1++;
}

inline bool blank(char ch){return ch==' '||ch=='\n'||ch=='\r'||ch=='\t';}
inline void read(int &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}

inline void read(ll &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}

inline void read(double &x){

```

```
bool sign=0; char ch=nc(); x=0;
for (;blank(ch);ch=nc());
if (IOerror)return;
if (ch=='-')sign=1,ch=nc();
for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
if (ch=='.'){
    double tmp=1; ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())tmp/=10.0,x+=tmp*(ch-'0');
}
if (sign)x=-x;
}
inline void read(char *s){
    char ch=nc();
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    for (;!blank(ch)&&!IOerror;ch=nc())*s++=ch;
    *s=0;
}
inline void read(char &c){
    for (c=nc();blank(c);c=nc());
    if (IOerror){c=-1;return;}
}
//fwrite->write
struct Ostream_fwrite{
    char *buf,*p1,*pend;
    Ostream_fwrite(){buf=new char[BUF_SIZE];p1=buf;pend=buf+BUF_SIZE;}
    void out(char ch){
        if (p1==pend){
            fwrite(buf,1,BUF_SIZE,stdout);p1=buf;
        }
        *p1++=ch;
    }
}
void print(int x){
    static char s[15],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1);
}
void println(int x){
    static char s[15],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1); out('\n');
}
void print(ll x){
    static char s[25],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1);
}
```

```

}
void println(ll x){
    static char s[25],*s1;s1=s;
    if (!x)*s1++='0';if (x<0)out('-'),x=-x;
    while(x)*s1++=x%10+'0',x/=10;
    while(s1--!=s)out(*s1); out('\n');
}
void print(double x,int y){
    static ll mul[]={1,10,100,1000,10000,100000,1000000,10000000,100000000,
        ↪ 1000000000,10000000000LL,100000000000LL,1000000000000LL,10000000000000LL,
        ↪ 100000000000000LL,1000000000000000LL,10000000000000000LL,100000000000000000LL,
    if (x<-1e-12)out('-'),x=-x;x*=mul[y];
    ll x1=(ll)floor(x); if (x-floor(x)>=0.5)++x1;
    ll x2=x1/mul[y],x3=x1-x2*mul[y]; print(x2);
    if (y>0){out('.'); for (size_t i=1;i<y&& x3*mul[i]<mul[y];out('0'),++i);
        ↪ print(x3);}
}
void println(double x,int y){print(x,y);out('\n');}
void print(char *s){while (*s)out(*s++);}
void println(char *s){while (*s)out(*s++);out('\n');}
void flush(){if (p1!=buf){fwrite(buf,1,p1-buf,stdout);p1=buf;}}
~Ostream_fwrite(){flush();}
}Ostream;
inline void print(int x){Ostream.print(x);}
inline void println(int x){Ostream.println(x);}
inline void print(char x){Ostream.out(x);}
inline void println(char x){Ostream.out(x);Ostream.out('\n');}
inline void print(ll x){Ostream.print(x);}
inline void println(ll x){Ostream.println(x);}
inline void print(double x,int y){Ostream.print(x,y);}
inline void println(double x,int y){Ostream.println(x,y);}
inline void print(char *s){Ostream.print(s);}
inline void println(char *s){Ostream.println(s);}
inline void println(){Ostream.out('\n');}
inline void flush(){Ostream.flush();}
#undef ll
#undef OUT_SIZE
#undef BUF_SIZE
};
using namespace IO;

```

7.3 LeapYear

```

#include <bits/stdc++.h>

inline bool Leap(int Year) {
    return (!(Year % 4) && (Year % 100)) || !(Year % 400);
}

```

7.4 vim

```
syntax on
set nu
set tabstop=2
set shiftwidth=2
set cindent
set mouse=a
set expandtab
set backspace=indent,eol,start
```

```
"map <F9> :call Run()<CR>
"func! Run()
"    exec "w"
"    exec "!g++ % -o %<"
"    exec "! %<"
"endfunc
```

```
"map <F2> :call SetTitle()<CR>
"func SetTitle()
"    let l = 0
"    let l = l + 1 | call setline(l, "#include <bits/stdc++.h>")
"    let l = l + 1 | call setline(l, "using namespace std;")
"    let l = l + 1 | call setline(l, "")
"    let l = l + 1 | call setline(l, "int main(int argc, char *argv[]) {")
"    let l = l + 1 | call setline(l, "    return 0;")
"    let l = l + 1 | call setline(l, "}")
"    let l = l + 1 | call setline(l, "")
"endfunc
```