Algorithm Library

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Contents

1 String

1.1 AhoCorasickAutomaton

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct AhoCorasickAutomaton {
    // 子节点记录数组
    int Son[maxn][26];
    int Val[maxn];
    // 失配指针 Fail 数组
    int Fail[maxn];
    // 节点数量
    int Tot;

    // Trie Tree 初始化
    void TrieInit() {
        Tot = 0;
        memset(Son, 0, sizeof(Son));
        memset(Val, 0, sizeof(Val));
        memset(Fail, 0, sizeof(Fail));
    }

    // 计算字母下标
    int Pos(char X) {
        return X - 'a';
    }

    // 向 Trie Tree 中插入 Str 模式字符串
    void Insert(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                Son[Cur][Index] = ++Tot;
            }
            Cur = Son[Cur][Index];
        }
        Val[Cur]++;
    }

    // Bfs 求得 Trie Tree 上失配指针
```

```cpp
    void GetFail() {
        std::queue<int> Que;
        for (int i = 0; i < 26; ++i) {
            if (Son[0][i]) {
                Fail[Son[0][1]] = 0;
                Que.push(Son[0][i]);
            }
        }
        while (!Que.empty()) {
            int Cur = Que.front(); Que.pop();
            for (int i = 0; i < 26; ++i) {
                if (Son[Cur][i]) {
                    Fail[Son[Cur][i]] = Son[Fail[Cur]][i];
                    Que.push(Son[Cur][i]);
                }
                else {
                    Son[Cur][i] = Son[Fail[Cur]][i];
                }
            }
        }
    }

    // 询问 Str 中出现的模式串数量
    int Query(string Str) {
        int Len = int(Str.length());
        int Cur = 0, Ans = 0;
        for (int i = 0; i < Len; ++i) {
            Cur = Son[Cur][Pos(Str[i])];
            for (int j = Cur; j && ~Val[j]; j = Fail[j]) {
                Ans += Val[j];
                Val[j] = -1;
            }
        }
        return Ans;
    }
};
```

1.2 KMP

```cpp
#include <bits/stdc++.h>

// 对模式串 Pattern 计算 Next 数组
void KMPPre(string Pattern, vector<int> &Next) {
    int i = 0, j = -1;
```

```
    Next[0] = -1;
    int Len = int(Pattern.length());
    while (i != Len) {
        if (j == -1 || Pattern[i] == Pattern[j]) {
            Next[++i] = ++j;
        }
        else {
            j = Next[j];
        }
    }
}

// 优化对模式串 Pattern 计算 Next 数组
void PreKMP(string Pattern, vector<int> &Next) {
    int i, j;
    i = 0;
    j = Next[0] = -1;
    int Len = int(Pattern.length());
    while (i < Len) {
        while (j != -1 && Pattern[i] != Pattern[j]) {
            j = Next[j];
        }
        if (Pattern[++i] == Pattern[++j]) {
            Next[i] = Next[j];
        }
        else {
            Next[i] = j;
        }
    }
}

// 利用预处理 Next 数组计数模式串 Pattern 在主串 Main 中出现次数
int KMPCount(string Pattern, string Main) {
    int PatternLen = int(Pattern.length()), MainLen =
    ↪  int(Main.length());
    vector<int> Next(PatternLen + 1, 0);
    //PreKMP(Pattern, Next);
    KMPPre(Pattern, Next);
    int i = 0, j = 0;
    int Ans = 0;
    while (i < MainLen) {
        while (j != -1 && Main[i] != Pattern[j]) {
            j = Next[j];
```

```
        }
        i++; j++;
        if (j >= PatternLen) {
            Ans++;
            j = Next[j];
        }
    }
    return Ans;
}
```

1.3 Manacher

```
#include <bits/stdc++.h>

const int maxn = "Edit";

char ConvertStr[maxn << 1];
int Len[maxn << 1];

// Manacher 算法求 Str 字符串最长回文子串长度
int Manacher(char Str[]) {
    int L = 0, StrLen = int(strlen(Str));
    ConvertStr[L++] = '$'; ConvertStr[L++] = '#';
    for (int i = 0; i < StrLen; ++i) {
        ConvertStr[L++] = Str[i];
        ConvertStr[L++] = '#';
    }
    int MX = 0, ID = 0, Ans = 0;
    for (int i = 0; i < L; ++i) {
        Len[i] = MX > i ? min(Len[2 * ID - i], MX - i) : 1;
        while (ConvertStr[i + Len[i]] == ConvertStr[i -
        ↪   Len[i]]) {
            Len[i]++;
        }
        if (i + Len[i] > MX) {
            MX = i + Len[i];
            ID = i;
        }
        Ans = max(Ans, Len[i] - 1);
    }
    return Ans;
}
```

1.4 PalindromicTree

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct PalindromicTree {
    // 子节点记录数组
    long long Son[maxn][26];
    // 失配指针 Fail 数组
    long long Fail[maxn];
    // Len[i]: 节点 i 表示的回文串长度 (一个节点表示一个回文串)
    long long Len[maxn];
    // Cnt[i]: 节点 i 表示的本质不同的串的个数 (最后需要运行
    ↪  Count() 函数才可求出正确结果)
    long long Cnt[maxn];
    // Num[i]: 以节点 i 表示的最长回文串的最右端为回文串结尾的回
    ↪  文串个数
    long long Num[maxn];
    // 字符
    long long Str[maxn];
    // 新添加字符后最长回文串表示的节点
    long long Last;
    // 字符数量
    long long StrLen;
    // 节点数量
    long long Tot;

    // 新建节点
    long long NewNode(long long X) {
        for (long long i = 0; i < 26; ++i) {
            Son[Tot][i] = 0;
        }
        Cnt[Tot] = 0;
        Num[Tot] = 0;
        Len[Tot] = X;
        return Tot++;
    }

    // 初始化
    void Init() {
        Tot = 0;
        NewNode(0); NewNode(-1);
        Last = 0;
```

```cpp
        StrLen = 0;
        // 开头存字符集中没有的字符, 减少特判
        Str[0] = -1;
        Fail[0] = 1;
    }

    long long GetFail(long long X) {
        while (Str[StrLen - Len[X] - 1] != Str[StrLen]) {
            X = Fail[X];
        }
        return X;
    }

    void Add(long long Char) {
        Char -= 'a';
        Str[++StrLen] = Char;
        long long Cur = GetFail(Last);
        if (!Son[Cur][Char]) {
            long long New = NewNode(Len[Cur] + 2);
            Fail[New] = Son[GetFail(Fail[Cur])][Char];
            Son[Cur][Char] = New;
            Num[New] = Num[Fail[New]] + 1;
        }
        Last = Son[Cur][Char];
        Cnt[Last]++;
    }

    void Count() {
        // 若 Fail[V]=U, 则 U 一定是 V 回文子串, 所以双亲累加孩
        ↪ 子的 Cnt
        for (long long i = Tot - 1; i >= 0; --i) {
            Cnt[Fail[i]] += Cnt[i];
        }
    }
};
```

2 Math

2.1 Catalan

```
#include <bits/stdc++.h>

const int maxn = "Edit";

long long Catalan[maxn];

// 递推求卡特兰数
void CalalanInit() {
    memset(Catalan, 0, sizeof(Catalan));
    Catalan[0] = Catalan[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Catalan[i] = Catalan[i - 1] * (4 * i - 2) / (i + 1);
    }
}
```

2.2 Derangement

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const int mod = 1e9 + 7;

// Staggered: 错排数
long long Staggered[maxn];

// 求错排数
void StaggeredInit() {
    Staggered[1] = 0;
    Staggered[2] = 1;
    // 递推求错排数
    for (int i = 3; i < maxn; ++i) {
        Staggered[i] = (i - 1) * (Staggered[i - 1] +
        ↪  Staggered[i - 2]) % mod;
    }
}
```

2.3 Euler

2.3.1 Euler

```
#include <bits/stdc++.h>

// 单独求解欧拉函数
int Phi(int X) {
    int Ans = X;
    for (int i = 2; i * i <= X; ++i) {
        if (!(X % i)) {
            Ans = Ans / i * (i - 1);
            while (!(X % i)) {
                X /= i;
            }
        }
    }
    if (X > 1) {
        Ans = Ans / X * (X - 1);
    }
    return Ans;
}
```

2.3.2 PrimeEuler

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 素数标记
bool IsPrime[maxn];
// 欧拉函数
int Phi[maxn];
// 素数
int Prime[maxn];
// 素数个数
int Tot;

// 同时求得欧拉函数和素数表
void PhiPrime() {
    memset(IsPrime, false, sizeof(IsPrime));
    Phi[1] = 1;
    Tot = 0;
    for (int i = 2; i < maxn; ++i) {
        if (!IsPrime[i]) {
```

```
                Prime[Tot++] = i;
                Phi[i] = i - 1;
            }
            for (int j = 0; j < Tot; ++j) {
                if (i * Prime[j] > maxn) {
                    break;
                }
                IsPrime[i * Prime[j]] = true;
                if (!(i % Prime[j])) {
                    Phi[i * Prime[j]] = Phi[i] * Prime[j];
                break;
                }
                else {
                    Phi[i * Prime[j]] = Phi[i] * (Prime[j] - 1);
                }
            }
        }
}
```

2.3.3 Sieve

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 欧拉函数
int Phi[maxn];

// 筛法求欧拉函数
void Euler() {
    for (int i = 1; i < maxn; ++i) {
        Phi[i] = i;
    }
    for (int i = 2; i < maxn; i += 2) {
        Phi[i] /= 2;
    }
    for (int i = 3; i < maxn; i += 2) {
        if (Phi[i] == i) {
            for (int j = i; j < maxn; j += i) {
                Phi[j] = Phi[j] / i * (i - 1);
            }
        }
    }
}
```

2.4 FFT

```cpp
#include <bits/stdc++.h>

const int maxn = "Edit";
const double pi = acos(-1.0);

// 复数
struct Complex {
    double X, Y;

    Complex operator + (const Complex &B) const {
        return Complex {X + B.X, Y + B.Y};
    }

    Complex operator - (const Complex &B) const {
        return Complex {X - B.X, Y - B.Y};
    }

    Complex operator * (const Complex &B) const {
        return Complex {X * B.X - Y * B.Y, X * B.Y + Y * B.X};
    }

    Complex operator / (const Complex &B) const {
        double Temp = B.X * B.X + B.Y * B.Y;
        return Complex {(X * B.X + Y * B.Y) / Temp, (Y * B.X -
        ↪  X * B.Y) / Temp};
    }
};

// 多项式系数数量
int N, M;
int L;
int Limit;
int R[maxn << 2];

// 快速傅里叶变换 (FFT)
void FFT(Complex F[], int Op) {
    for (int i = 0; i < Limit; ++i) {
        if (i < R[i]) {
            std::swap(F[i], F[R[i]]);
        }
    }
    for (int j = 1; j < Limit; j <<= 1) {
```

```
        Complex Temp = Complex {cos(pi / j), Op * sin(pi /
        ↪  j)};
        for (int k = 0; k < Limit; k += (j << 1)) {
            Complex Buffer = Complex {1.0, 0.0};
            for (int l = 0; l < j; ++l) {
                Complex Tx = F[k + l], Ty = Buffer * F[k + j +
                ↪  l];
                F[k + l] = Tx + Ty;
                F[k + j + l] = Tx - Ty;
                Buffer = Buffer * Temp;
            }
        }
    }
}

// 多项式系数
Complex A[maxn], B[maxn];

// 多项式卷积计算
void Cal() {
    Limit = 1; L = 0;
    while (Limit <= N + M) {
        Limit <<= 1;
        L++;
    }
    for (int i = 0; i < Limit; ++i) {
        R[i] = (R[i >> 1] >> 1) | ((i & 1) << (L - 1));
    }
    FFT(A, 1);
    FFT(B, 1);
    for (int i = 0; i <= Limit; ++i) {
        A[i] = A[i] * B[i];
    }
    FFT(A, -1);
}
```

2.5 Fibonacci

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;

// 矩阵结构体
struct Matrix {
```

```
    // 矩阵
    long long Mat[2][2];
    Matrix() {}
    // 重载矩阵乘法
    Matrix operator * (Matrix const &A) const {
        Matrix Res;
        memset(Res.Mat, 0, sizeof(Res.Mat));
        for (int i = 0; i < 2; ++i) {
            for (int j = 0; j < 2; ++j) {
                for (int k = 0; k < 2; ++k) {
                    Res.Mat[i][j] = (Res.Mat[i][j] + Mat[i][k]
                    ↪  * A.Mat[k][j] % mod) % mod;
                }
            }
        }
        return Res;
    }
};

// 重载矩阵快速幂
Matrix operator ^ (Matrix Base, long long K) {
    Matrix Res;
    memset(Res.Mat, 0, sizeof(Res.Mat));
    Res.Mat[0][0] = Res.Mat[1][1] = 1;
    while (K) {
        if (K & 1) {
            Res = Res * Base;
        }
        Base = Base * Base;
        K >>= 1;
    }
    return Res;
}

// 斐波那契数列中第 X 项
long long Fib(long long X) {
    Matrix Base;
    Base.Mat[0][0] = Base.Mat[1][0] = Base.Mat[0][1] = 1;
    Base.Mat[1][1] = 0;
    return (Base ^ X).Mat[0][1];
}
```

2.6 GeneratingFunction

```cpp
#include <bits/stdc++.h>

const int maxn = "Edit";

void GeneratingFunction() {
    int n;
    int c1[maxn], c2[maxn];
    scanf("%d", &n);
    for (int i = 0; i < maxn; ++i) {
        c1[i] = 1;
        c2[i] = 0;
    }
    // c1[i] 为 x^i 的系数
    // c2 为中间变量
    for (int i = 2; i <= n; ++i) {
        for (int j = 0; j <= n; ++j) {
            for (int k = 0; k + j <= n; k += i) {
                c2[j + k] += c1[i];
            }
        }
        for (int j = 0; j <= n; ++j) {
            c1[j] = c2[j];
            c2[j] = 0;
        }
    }
}
```

2.7 InverseElement

2.7.1 ExtendGcd

```cpp
#include <bits/stdc++.h>

// 扩展欧几里得, A*X+B*Y=D
long long ExtendGcd(long long A, long long B, long long &X,
↪   long long &Y) {
    // 无最大公约数
    if (A == 0 && B == 0) {
        return -1;
    }
    if (B == 0) {
        X = 1;
        Y = 0;
```

```
        return A;
    }
    long long D = ExtendGcd(B, A % B, Y, X);
    Y -= A / B * X;
    return D;
}

// 逆元, AX = 1(mod M)
long long Inv(long long A, long long N) {
    long long X, Y;
    long long D = ExtendGcd(A, N, X, Y);
    if (D == 1) {
        return (X % N + N) % N;
    }
    else {
        return -1;
    }
}
```

2.7.2 Factorial

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;
const int maxn = "Edit";

// 快速乘
long long QuickMul(long long A, long long B) {
    long long Ans = 0;
    while (B) {
        if (B & 1) {
            Ans = (Ans + A) % mod;
        }
        A = (A + A) % mod;
        B >>= 1;
    }
    return Ans;
}

// 快速幂
long long QuickPow(long long A, long long B) {
    long long Ans = 1;
    while (B) {
        if (B & 1) {
```

```cpp
            Ans = QuickMul(Ans, A) % mod;
        }
        A = QuickMul(A, A) % mod;
        B >>= 1;
    }
    return Ans;
}

// Factorial: 阶乘, FactorialInv: 阶乘逆元
long long Factorial[maxn], FactorialInv[maxn];

// 求阶乘逆元
void FactorialInvInit() {
    // 求阶乘
    Factorial[0] = 0;
    Factorial[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Factorial[i] = (Factorial[i - 1] * i) % mod;
    }
    // 飞马小定理求最大值阶乘逆元
    FactorialInv[maxn - 1] = QuickPow(Factorial[maxn - 1], mod
    ↪   - 2);
    // 递推求阶乘逆元
    for (int i = maxn - 2; i >= 0; --i) {
        FactorialInv[i] = (FactorialInv[i + 1] * (i + 1)) %
        ↪   mod;
    }
}
```

2.7.3 FermatLittleTheorem

```cpp
#include <bits/stdc++.h>

const int mod = 1e9 + 7;

// 快速幂、费马小定理求逆元
long long Inv(long long X) {
    return QuickPow(X, mod - 2);
}
```

2.7.4 Recursive

```cpp
#include <bits/stdc++.h>
```

```
const int mod = 1e9 + 7;
const int maxn = "Edit";

long long Inv[maxn];

// 递推求逆元
void InvInit() {
    Inv[1] = 1;
    for (int i = 2; i < maxn; ++i) {
        Inv[i] = (mod - mod / i) * Inv[mod % i] % mod;
    }
}
```

2.8 Prime

2.8.1 PrimeFactor

```
#include <bits/stdc++.h>

const int maxn = "Edit"

bool IsPrime[maxn];
vector<int> PrimeFactor[maxn];

void Init() {
    memset(IsPrime, true, sizeof(IsPrime));
    for (long long i = 2; i < maxn; ++i) {
        if (IsPrime[i]) {
            PrimeFactor[i].push_back(i);
            for (long long j = i + i; j < maxn; ++j) {
                IsPrime[j] = false;
                PrimeFactor[j].push_back(i);
            }
        }
    }
    IsPrime[1] = false;
}
```

2.8.2 SieveOfEratosthenes

```
#include <bits/stdc++.h>

const int maxn = "Edit";

bool IsPrime[maxn];
```

```
void Init() {
    memset(IsPrime, true, sizeof(IsPrime));
    IsPrime[0] = IsPrime[1] = false;
    for (long long i = 2; i < maxn; ++i) {
        if (IsPrime[i]) {
            for (long long j = i * i; j < maxn; j += i) {
                IsPrime[j] = false;
            }
        }
    }
}
```

2.9 QuickPow

```
#include <bits/stdc++.h>

const int mod = 1e9 + 7;

// 快速乘求 A*B%mod
long long QuickMul(long long A, long long B) {
    long long Ans = 0;
    while (B) {
        if (B & 1) {
            Ans = (Ans + A) % mod;
        }
        A = (A + A) % mod;
        B >>= 1;
    }
    return Ans;
}

// 快速幂求 A^B%mod
long long QuickPow(long long A, long long B) {
    long long Ans = 1;
    while (B) {
        if (B & 1) {
            // Ans = Ans * A % mod;
            Ans = QuickMul(Ans, A) % mod;
        }
        // Ans = A * A % mod;
        A = QuickMul(A, A) % mod;
        B >>= 1;
    }
```

```
    return Ans;
}
```

2.10 Stirling

```
#include <bits/stdc++.h>

const double pi = acos(-1.0);
const double e = 2.718281828459;

int Stirling(int x) {
    if (x <= 1) {
        return 1;
    }
    return int(ceil(log10(2 * pi * x) / 2 + x * log10(x /
    ↪   e)));
}
```

3 DataStructure

3.1 BinaryIndexedTree

```
#include <bits/stdc++.h>
#define lowbit(x) (x&(-x))

const int maxn = "Edit";

// 树状数组
int Array[maxn];

// 更新树状数组信息
void Update(int X, int Val) {
    while (X < maxn) {
        Array[X] += Val;
        X += lowbit(X);
    }
}

// 查询
int Query(int X) {
    int Ans = 0;
    while (X > 0) {
        Ans += Array[X];
        X -= lowbit(X);
    }
    return Ans;
}
```

3.2 DfsOrder

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 链式前向星建图
struct Link {
    int V, Next;
};

Link edges[maxn << 1];
int Head[maxn];
int Tot = 0;
```

```
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

void AddEdge(int U, int V) {
    edges[++Tot] = Link {V, Head[U]};
    Head[U] = Tot;
    edges[++Tot] = Link {U, Head[V]};
    Head[V] = Tot;
}

int Cnt;
int InIndex[maxn], OutIndex[maxn];

// Dfs 序
void DfsSequence(int Node, int Pre) {
    Cnt++;
    InIndex[Node] = Cnt;
    for (int i = Head[U]; i != -1; i = edges[i].Next) {
        if (edges[i].V != Pre) {
            DfsSequence(edges[i].V, Node);
        }
    }
    OutIndex[U] = Cnt;
}
```

3.3 SegmentTree

3.3.1 SegmentTree

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// Sum: 线段树信息 (此模板为求和), Lazy: 惰性标记
int Sum[maxn << 2], Lazy[maxn << 2];

// 更新节点信息, 这里是求和
void PushUp(int Root) {
    Sum[Root] = Sum[Root << 1] + Sum[Root << 1 | 1];
}

// 下推标记函数, LeftNum, RightNum: 分别为左右子树的数字数量
```

```
void PushDown(int Root, int LeftNum, int RightNum) {
    if (Lazy[Root]) {
        // 下推标记
        Lazy[Root << 1] += Lazy[Root];
        Lazy[Root << 1 | 1] += Lazy[Root];
        // 根据惰性标修改子节点的值
        Sum[Root << 1] += Lazy[Root] * LeftNum;
        Sum[Root << 1 | 1] += Lazy[Root] * RightNum;
        // 清除本节点惰性标记
        Lazy[Root] = 0;
    }
}

// 建树, Left、Right: 当前节点区间, Root: 当前节点编号
void Build(int Left, int Right, int Root) {
    Lazy[Root] = 0;
    // 到达叶子节点
    if (Left == Right) {
        scanf("%d", &Sum[Root]);
        return;
    }
    int Mid = (Left + Right) >> 1;
    // 左子树
    Build(Left, Mid, Root << 1);
    // 右子树
    Build(Mid + 1, Right, Root << 1 | 1);
    // 更新信息
    PushUp(Root);
}

// 单点修改, Pos: 修改点位置, Value: 修改值, Left、Right: 当前区
↪  间, Root: 当前节点编号
void PointUpdate(int Pos, int Value, int Left, int Right, int
↪  Root) {
    // 修改叶子节点
    if (Left == Right) {
        Sum[Root] += Value;
        return;
    }
    int Mid = (Left + Right) >> 1;
    // 根据条件判断调用左子树还是右子树
    if (Pos <= Mid) {
        PointUpdate(Pos, Value, Left, Mid, Root << 1);
    }
```

```
    else {
        PointUpdate(Pos, Value, Mid + 1, Right, Root << 1 |
        ↪  1);
    }
    // 子节点更新后更新此节点
    PushUp(Root);
}

// 区间修改，OperateLeft、OperateRight: 操作区间，Left、Right:
↪  当前区间，Root: 当前节点编号
void IntervalUpdate(int OperateLeft, int OperateRight, int
↪  Value, int Left, int Right, int Root) {
    // 若本区间完全在操作区间内
    if (OperateLeft <= Left && OperateRight >= Right) {
        Sum[Root] += Value * (Right - Left + 1);
        // 增加惰性标记，表示本区间 Sum 正确，但子区间仍需要根据
        ↪  惰性标记调整更新
        Lazy[Root] += Value;
        return;
    }
    int Mid = (Left + Right) >> 1;
    // 下推标记
    PushDown(Root, Mid - Left + 1, Right - Mid);
    // 根据条件判断调用左子树还是右子树
    if (OperateLeft <= Mid) {
        IntervalUpdate(OperateLeft, OperateRight, Value, Left,
        ↪  Mid, Root << 1);
    }
    if (OperateRight > Mid) {
        IntervalUpdate(OperateLeft, OperateRight, Value, Mid +
        ↪  1, Right, Root << 1 | 1);
    }
    // 更新当前节点信息
    PushUp(Root);
}

// 区间查询，OperateLeft、OperateRight: 操作区间，Left、Right:
↪  当前区间，Root: 当前节点编号
int Query(int OperateLeft, int OperateRight, int Left, int
↪  Right, int Root) {
    // 区间内直接返回
    if (OperateLeft <= Left && OperateRight >= Right) {
        return Sum[Root];
```

```
    }
    int Mid = (Left + Right) >> 1;
    // 下推标记
    PushDown(Root, Mid - Left + 1, Right - Mid);
    // 叠加结果
    int Ans = 0;
    if (OperateLeft <= Mid) {
        Ans += Query(OperateLeft, OperateRight, Left, Mid,
        ↪  Root << 1);
    }
    if (OperateRight > Mid) {
        Ans += Query(OperateLeft, OperateRight, Mid + 1,
        ↪  Right, Root << 1 | 1);
    }
    // 返回结果
    return Ans;
}
```

3.3.2 SegmentTreestruct

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 线段树节点
struct Node {
    int Left, Right;
    int Lazy, Tag;
    int Sum;
};

Node SegmentTree[maxn << 2];

// 更新节点信息
void PushUp(int Root) {
    SegmentTree[Root].Sum = SegmentTree[Root << 1].Sum +
    ↪  SegmentTree[Root << 1 | 1].Sum;
}

// 建树，Left、Right: 当前节点区间，Root: 当前节点编号
void Build(int Left, int Right, int Root) {
    SegmentTree[Root].Left = Left;
    SegmentTree[Root].Right = Right;
    SegmentTree[Root].Lazy = 0;
```

```
    SegmentTree[Root].Tag = 0;
    // 叶子节点
    if (Left == Right) {
        scanf("%d", &SegmentTree[Root].Sum);
        return;
    }
    // 左右子树
    int Mid = (Left + Right) >> 1;
    Build(Left, Mid, Root << 1);
    Build(Mid + 1, Right, Root << 1 | 1);
    // 更新
    PushUp(Root);
}

// 单点更新, Pos: 修改点位置, Value: 修改值, Root: 当前节点编号
void PointUpdate(int Pos, int Value, int Root) {
    SegmentTree[Root].Sum += Value;
    if (SegmentTree[Root].Left == Pos && SegmentTree[Root].Right == Pos) {
        return;
    }
    int Mid = (SegmentTree[Root].Left + SegmentTree[Root].Right) >> 1;
    if (Pos <= Mid) {
        PointUpdate(Pos, Value, Root << 1);
    }
    else {
        PointUpdate(Pos, Value, Root << 1 | 1);
    }
    PushUp(Root);
}

// 区间修改, Left、Right: 修改区间, Value: 修改值, Root: 当前节点编号
void IntervalUpdate(int Left, int Right, int Value, int Root) {
    if (SegmentTree[Root].Left == Left && SegmentTree[Root].Right == Right) {
        SegmentTree[Root].Lazy = 1;
        SegmentTree[Root].Tag = Value;
        SegmentTree[Root].Sum = (Right - Left + 1) * Value;
        return;
    }
```

```
    int Mid = (SegmentTree[Root].Left +
    ↪  SegmentTree[Root].Right) >> 1;
    // 下推更新
    if (SegmentTree[Root].Lazy == 1) {
        SegmentTree[Root].Lazy = 0;
        IntervalUpdate(SegmentTree[Root].Left, Mid,
        ↪  SegmentTree[Root].Tag, Root << 1);
        IntervalUpdate(Mid + 1, SegmentTree[Root].Right,
        ↪  SegmentTree[Root].Tag, Root << 1 | 1);
        SegmentTree[Root].Tag = 0;
    }
    if (Right <= Mid) {
        IntervalUpdate(Left, Right, Value, Root << 1);
    }
    else if (Left > Mid) {
        IntervalUpdate(Left, Right, Value, Root << 1 | 1);
    }
    else {
        IntervalUpdate(Left, Mid, Value, Root << 1);
        IntervalUpdate(Mid + 1, Right, Value, Root << 1 | 1);
    }
    PushUp(Root);
}

// 区间查询, Left、Right: 查询区间, Root: 当前节点编号
int Query(int Left, int Right, int Root) {
    if (Left == SegmentTree[Root].Left && Right ==
    ↪  SegmentTree[Root].Right) {
        return SegmentTree[Root].Sum;
    }
    int Mid = (SegmentTree[Root].Left +
    ↪  SegmentTree[Root].Right) >> 1;
    if (Right <= Mid) {
        return Query(Left, Right, Root << 1);
    }
    else if (Left > Mid) {
        return Query(Left, Right, Root << 1 | 1);
    }
    else {
        return Query(Left, Mid, Root << 1) + Query(Mid + 1,
        ↪  Right, Root << 1 | 1);
    }
}
```

3.4 Splay

3.4.1 SplayTree

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct SplayTree {
    // Root:Splay Tree 根节点
    int Root, Tot;
    // Son[i][0]:i 节点的左孩子, Son[i][0]:i 节点的右孩子
    int Son[maxn][2];
    // Pre[i]:i 节点的父节点
    int Pre[maxn];
    // Val[i]:i 节点的权值
    int Val[maxn];
    // Size[i]: 以 i 节点为根的 Splay Tree 的节点数 (包含自身)
    int Size[maxn];
    // Cnt[i]: 节点 i 的权值的出现次数
    int Cnt[maxn];

    void PushUp(int X) {
        Size[X] = Size[Son[X][0]] + Size[Son[X][1]] + Cnt[X];
    }

    // 判断 X 节点是其父节点的左孩子还是右孩子
    bool Self(int X) {
        return X == Son[Pre[X]][1];
    }

    void Clear(int X) {
        Son[X][0] = Son[X][1] = Pre[X] = Val[X] = Size[X] =
        ↪ Cnt[X] = 0;
    }

    // 旋转
    void Rotate(int X) {
        int Fa = Pre[X], FaFa = Pre[Fa], XJ = Self(X);
        Son[Fa][XJ] = Son[X][XJ ^ 1];
        Pre[Son[Fa][XJ]] = Pre[X];
        Son[X][XJ ^ 1] = Pre[X];
        Pre[Fa] = X;
        Pre[X] = FaFa;
```

```
        if (FaFa) {
            Son[FaFa][Fa == Son[FaFa][1]] = X;
        }
        PushUp(Fa);
        PushUp(X);
    }

    // 旋转 X 节点到根节点
    void Splay(int X) {
        for (int i = Pre[X]; i = Pre[X]; Rotate(X)) {
            if (Pre[i]) {
                Rotate(Self(X) == Self(i) ? i : X);
            }
        }
        Root = X;
    }

    // 插入数 X
    void Insert(int X) {
        if (!Root) {
            Val[++Tot] = X;
            Cnt[Tot]++;
            Root = Tot;
            PushUp(Root);
            return;
        }
        int Cur = Root, F = 0;
        while (true) {
            if (Val[Cur] == X) {
                Cnt[Cur]++;
                PushUp(Cur);
                PushUp(F);
                Splay(Cur);
                break;
            }
            F = Cur;
            Cur = Son[Cur][Val[Cur] < X];
            if (!Cur) {
                Val[++Tot] = X;
                Cnt[Tot]++;
                Pre[Tot] = F;
                Son[F][Val[F] < X] = Tot;
                PushUp(Tot);
                PushUp(F);
```

```
                Splay(Tot);
                break;
            }
        }
    }

    // 查询 X 的排名
    int Rank(int X) {
        int Ans = 0, Cur = Root;
        while (true) {
            if (X < Val[Cur]) {
                Cur = Son[Cur][0];
            }
            else {
                Ans += Size[Son[Cur][0]];
                if (X == Val[Cur]) {
                    Splay(Cur);
                    return Ans + 1;
                }
                Ans += Cnt[Cur];
                Cur = Son[Cur][1];
            }
        }
    }

    // 查询排名为 X 的数
    int Kth(int X) {
        int Cur = Root;
        while (true) {
            if (Son[Cur][0] && X <= Size[Son[Cur][0]]) {
                Cur = Son[Cur][0];
            }
            else {
                X -= Cnt[Cur] + Size[Son[Cur][0]];
                if (X <= 0) {
                    return Val[Cur];
                }
                Cur = Son[Cur][1];
            }
        }
    }

    /*
```

```
 * 在 Insert 操作时 X 已经 Splay 到根了
 * 所以 X 的前驱就是 X 的左子树的最右边的节点
 * 后继就是 X 的右子树的最左边的节点
 */

// 求前驱
int GetPath() {
    int Cur = Son[Root][0];
    while (Son[Cur][1]) {
        Cur = Son[Cur][1];
    }
    return Cur;
}

// 求后继
int GetNext() {
    int Cur = Son[Root][1];
    while (Son[Cur][0]) {
        Cur = Son[Cur][0];
    }
    return Cur;
}

// 删除值为 X 的节点
void Delete(int X) {
    // 将 X 旋转到根
    Rank(X);
    if (Cnt[Root] > 1) {
        Cnt[Root]--;
        PushUp(Root);
        return;
    }
    if (!Son[Root][0] && !Son[Root][1]) {
        Clear(Root);
        Root = 0;
        return;
    }
    if (!Son[Root][0]) {
        int Temp = Root;
        Root = Son[Root][1];
        Pre[Root] = 0;
        Clear(Temp);
        return;
    }
```

```
        if (!Son[Root][1]) {
            int Temp = Root;
            Root = Son[Root][0];
            Pre[Root] = 0;
            Clear(Temp);
            return;
        }
        int Temp = GetPath(), Old = Root;
        Splay(Temp);
        Pre[Son[Old][1]] = Temp;
        Son[Temp][1] = Son[Old][1];
        Clear(Old);
        PushUp(Root);
    }
};
```

3.4.2 SplayTreeArray

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// Root:Splay Tree 根节点
int Root, Tot;
// Son[i][0]:i 节点的左孩子, Son[i][0]:i 节点的右孩子
int Son[maxn][2];
// Pre[i]:i 节点的父节点
int Pre[maxn];
// Val[i]:i 节点的权值
int Val[maxn];
// Size[i]: 以 i 节点为根的 Splay Tree 的节点数 (包含自身)
int Size[maxn];
// 惰性标记数组
bool Lazy[maxn];

void PushUp(int X) {
    Size[X] = Size[Son[X][0]] + Size[Son[X][1]] + 1;
}

void PushDown(int X) {
    if (Lazy[X]) {
        std::swap(Son[X][0], Son[X][1]);
        if (Son[X][0]) {
            Lazy[Son[X][0]] ^= 1;
```

```
        }
        if (Son[X][1]) {
            Lazy[Son[X][1]] ^= 1;
        }
        Lazy[X] = 0;
    }
}

// 判断 X 节点是其父节点的左孩子还是右孩子
bool Self(int X) {
    return Son[Pre[X]][1] == X;
}

// 旋转节点 X
void Rotate(int X) {
    int Fa = Pre[X], FaFa = Pre[Fa], XJ = Self(X);
    PushDown(Fa); PushDown(X);
    Son[Fa][XJ] = Son[X][XJ ^ 1];
    Pre[Son[Fa][XJ]] = Pre[X];
    Son[X][XJ ^ 1] = Pre[X];
    Pre[Fa] = X;
    Pre[X] = FaFa;
    if (FaFa) {
        Son[FaFa][Fa == Son[FaFa][1]] = X;
    }
    PushUp(Fa); PushUp(X);
}

// 旋转 X 节点到节点 Goal
void Splay(int X, int Goal = 0) {
    for (int Cur = Pre[X]; (Cur = Pre[X]) != Goal; Rotate(X))
    ↪  {
        PushDown(Pre[Cur]); PushDown(Cur); PushDown(X);
        if (Pre[Cur] != Goal) {
            if (Self(X) == Self(Cur)) {
                Rotate(Cur);
            }
            else {
                Rotate(X);
            }
        }
    }
    if (!Goal) {
        Root = X;
```

```
    }
}

// 获取以 R 为根节点 Splay Tree 中的第 K 大个元素在 Splay Tree
↪ 中的位置
int Kth(int R, int K) {
    PushDown(R);
    int Temp = Size[Son[R][0]] + 1;
    if (Temp == K) {
        return R;
    }
    if (Temp > K) {
        return Kth(Son[R][0], K);
    }
    else {
        return Kth(Son[R][1], K - Temp);
    }
}

// 获取 Splay Tree 中以 X 为根节点子树的最小值位置
int GetMin(int X) {
    PushDown(X);
    while (Son[X][0]) {
        X = Son[X][0];
        PushDown(X);
    }
    return X;
}

// 获取 Splay Tree 中以 X 为根节点子树的最大值位置
int GetMax(int X) {
    PushDown(X);
    while (Son[X][1]) {
        X = Son[X][1];
        PushDown(X);
    }
    return X;
}

// 求节点 X 的前驱节点
int GetPath(int X) {
    Splay(X, Root);
    int Cur = Son[Root][0];
```

```
    while (Son[Cur][1]) {
        Cur = Son[Cur][1];
    }
    return Cur;
}

// 求节点 Y 的后继节点
int GetNext(int X) {
    Splay(X, Root);
    int Cur = Son[Root][1];
    while (Son[Cur][0]) {
        Cur = Son[Cur][0];
    }
    return Cur;
}

// 翻转 Splay Tree 中 Left~Right 区间
void Reverse(int Left, int Right) {
    int X = Kth(Root, Left), Y = Kth(Root, Right);
    Splay(X, 0);
    Splay(Y, X);
    Lazy[Son[Y][0]] ^= 1;
}

// 建立 Splay Tree
void Build(int Left, int Right, int Cur) {
    if (Left > Right) {
        return;
    }
    int Mid = (Left + Right) >> 1;
    Build(Left, Mid - 1, Mid);
    Build(Mid + 1, Right, Mid);
    Pre[Mid] = Cur;
    Val[Mid] = Mid - 1;
    Lazy[Mid] = 0;
    PushUp(Mid);
    if (Mid < Cur) {
        Son[Cur][0] = Mid;
    }
    else {
        Son[Cur][1] = Mid;
    }
}
```

```cpp
// 输出 Splay Tree
void Print(int Cur) {
    PushDown(Cur);
    if (Son[Cur][0]) {
        Print(Son[Cur][0]);
    }
    // 哨兵节点判断
    if (Val[Cur] != -INF && Val[Cur] != INF) {
        printf("%d ", Val[Cur]);
    }
    if (Val[Son[Cur][1]]) {
        Print(Son[Cur][1]);
    }
}
```

3.5 TrieTree

```cpp
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Trie {
    // Trie Tree 节点
    int Son[maxn][26];
    // Trie Tree 节点数量
    int Tot;

    // 字符串数量统计数组
    int Cnt[maxn];

    // Trie Tree 初始化
    void TrieInit() {
        Tot = 0;
        memset(Cnt, 0, sizeof(Cnt));
        memset(Son, 0, sizeof(Son));
    }

    // 计算字母下标
    int Pos(char X) {
        return X - 'a';
    }

    // 向 Trie Tree 中插入字符串 Str
```

```
    void Insert(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                Son[Cur][Index] = ++Tot;
            }
            Cur = Son[Cur][Index];

            Cnt[Cur]++;
        }
    }

    // 查找字符串 Str, 存在返回 true, 不存在返回 false
    bool Find(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                return false;
            }
            Cur = Son[Cur][Index];
        }
        return true;
    }

    // 查询字典树中以 Str 为前缀的字符串数量
    int PathCnt(string Str) {
        int Cur = 0, Len = int(Str.length());
        for (int i = 0; i < Len; ++i) {
            int Index = Pos(Str[i]);
            if (!Son[Cur][Index]) {
                return 0;
            }
            Cur = Son[Cur][Index];
        }
        return Cnt[Cur];
    }
};
```

4 GraphTheory

4.1 LCA

4.1.1 DFS+ST

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 链式前向星存图
struct Edge {
    int V, Weight, Next;
};

Edge edges[maxn << 1];
int Head[maxn];
int Tot;

void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

void AddEdge(int U, int V, int Weight) {
    edges[Tot] = Edge {V, Weight, Head[U]};
    Head[U] = Tot++;
}

struct LCAOnline {
    // 节点深度
    int Rmq[maxn << 1];
    // 深搜遍历顺序
    int Vertex[maxn << 1];
    // 节点在深搜中第一次出现的位置
    int First[maxn];
    // 记录父节点
    int Parent[maxn];
    // 记录与根节点距离
    int Dis[maxn];
    // 遍历节点数量
    int LCATot;

    // 最小值对应下标
    int Dp[maxn << 1][20];
```

```
// RMQ 初始化
void Work(int N) {
    for (int i = 1; i <= N; ++i) {
        Dp[i][0] = i;
    }
    for (int j = 1; (1 << j) <= N; ++j) {
        for (int i = 1; i + (1 << j) - 1 <= N; ++i) {
            Dp[i][j] = Rmq[Dp[i][j - 1]] < Rmq[Dp[i + (1 << (j - 1))][j - 1]] ? Dp[i][j - 1] : Dp[i + (1 << (j - 1))][j - 1];
        }
    }
}

// 深搜
void Dfs(int Cur, int Pre, int Depth) {
    Vertex[++LCATot] = Cur;
    First[Cur] = LCATot;
    Rmq[LCATot] = Depth;
    Parent[Cur] = Pre;
    for (int i = Head[Cur]; ~i; i = edges[i].Next) {
        if (edges[i].V == Pre) {
            continue;
        }
        Dis[edges[i].V] = Dis[Cur] + edges[i].Weight;
        Dfs(edges[i].V, Cur, Depth + 1);
        Vertex[++LCATot] = Cur;
        Rmq[LCATot] = Depth;
    }
}

// RMQ 查询
int Query(int Left, int Right) {
    if (Left > Right) {
        swap(Left, Right);
    }
    int Len = int(log2(Right - Left + 1));
    return Rmq[Dp[Left][Len]] <= Rmq[Dp[Right - (1 << Len) + 1][Len]] ? Dp[Left][Len] : Dp[Right - (1 << Len) + 1][Len];
}

// LCA 初始化
```

```
    void Init(int Root, int NodeNum) {
        memset(Dis, 0, sizeof(Dis));
        LCATot = 0;
        Dfs(Root, 0, 0);
        Parent[1] = 0;
        Work(2 * NodeNum - 1);
    }

    // 查询节点 U、 V 的距离
    int GetDis(int U, int V) {
        return Dis[U] + Dis[V] - 2 * Dis[LCA(U, V)];
    }

    // 查询节点 U、 V 的最近公共祖先 (LCA)
    int LCA(int U, int V) {
        return Vertex[Query(First[U], First[V])];
    }
}LCA;
```

4.1.2 Tarjan

```
#include <bits/stdc++.h>

const int maxn = "Edit";

// 树边
struct Edge {
    int V, Next;
};

// 询问
struct Query {
    int Q, Next;
    int Index;
};

// 并查集数组
int Pre[maxn << 2];
// 树边
Edge edges[maxn << 2];
int Head[maxn];
int Tot;
// 询问
Query querys[maxn << 2];
```

```
int QHead[maxn];
int QTot;
// 访问标记
int Vis[maxn];
int Ancestor[maxn];
// 结果
int Answer[maxn];

// 并查集查找
int Find(int X) {
    int R = X;
    while (Pre[R] != -1) {
        R = Pre[R];
    }
    return R;
}

// 并查集合并
void Join(int U, int V) {
    int RU = Find(U);
    int RV = Find(V);
    if (RU != RV) {
        Pre[RU] = RV;
    }
}

// 添加树边
void AddEdge(int U, int V) {
    edges[Tot] = Edge {V, Head[U]};
    Head[U] = Tot++;
}

// 添加询问
void AddQuery(int U, int V, int Index) {
    querys[QTot] = Query {V, QHead[U], Index};
    QHead[U] = QTot++;
    querys[QTot] = Query {U, QHead[V], Index};
    QHead[V] = QTot++;
}

// 初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
```

```
    QTot = 0;
    memset(QHead, -1, sizeof(QHead));
    memset(Vis, false, sizeof(Vis));
    memset(Pre, -1, sizeof(Pre));
    memset(Ancestor, 0, sizeof(Ancestor));
}

// LCA 离线 Tarjan 算法
void Tarjan(int Node) {
    Ancestor[Node] = Node;
    Vis[Node] = true;
    for (int i = Head[Node]; i != -1; i = edges[i].Next) {
        if (Vis[edges[i].V]) {
            continue;
        }
        Tarjan(edges[i].V);
        Join(Node, edges[i].V);
        Ancestor[Find(Node)] = Node;
    }
    for (int i = QHead[Node]; i != -1; i = querys[i].Next) {
        if (Vis[querys[i].Q]) {
            Answer[querys[i].Index] =
             ↪  Ancestor[Find(querys[i].Q)];
        }
    }
}
```

4.2 MinimumSpanningTree

4.2.1 Kruskal

```
#include <bits/stdc++.h>

const int maxn = "Edit";

struct Edge {
    int U, V, Dis;

    bool operator < (const Edge &B) const {
        return Dis < B.Dis;
    }
};

// N: 顶点数, E: 边数, Pre 并查集
```

```
int N, E, Pre[maxn];
// edges: 边
Edge edges[maxn];

void Init() {
    // 并查集初始化
    for (int i = 0; i <= N; ++i) {
        Pre[i] = i;
    }
}

// 并查集查询
int Find(int X) {
    int R = X;
    while (Pre[R] != R) {
        R = Pre[R];
    }
    int I = X, J;
    while (I != R) {
        J = Pre[I];
        Pre[I] = R;
        I = J;
    }
    return R;
}

// 并查集合并
void Join(int X, int Y) {
    int XX = Find(X);
    int YY = Find(Y);
    if (XX != YY) {
        Pre[XX] = YY;
    }
}

// Kruskal 算法
int Kruskal() {
    // 贪心排序
    std::sort(edges + 1, edges + E + 1);
    Init();
    int Res = 0;
    // 选边计算
    for (int i = 1; i <= E; ++i) {
        Edge Temp = edges[i];
```

```
        if (Find(Temp.U) != Find(Temp.V)) {
            Join(Temp.U, Temp.V);
            Res += Temp.Dis;
        }
    }
    return Res;
}
```

4.2.2 Prim

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Edge {
    // V: 连接点, Dis: 边权
    int V, Dis;
};

// N: 顶点数, E: 边数
int N, E;
// 松弛更新权值数组
int Dis[maxn];
// 访问标记数组
int Vis[maxn];
// 邻接表
std::vector<Edge> Adj[maxn];

// 建图加边, U、V: 顶点,Weight: 权值
void AddEdge(int U, int V, int Weight) {
    Adj[U].push_back(Edge (V, Weight));
    // 无向图反向建边
    Adj[V].push_back(Edge (U, Weight));
}

// Prim 算法
int Prim(int Start) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, 0, sizeof(Vis));
    Dis[Start] = 0;
    int Res = 0;
    for (int i = 1; i <= N; ++i) {
        // 选择距已生成树权值最小的顶点
```

```
        int U = -1, Min = INF;
        for (int j = 1; j <= N; ++j) {
            if (!Vis[j] && Dis[j] < Min) {
                U = j;
                Min = Dis[j];
            }
        }
        // 更新、标记
        Vis[U] = 1;
        Res += Min;
        // 松弛
        for (int j = 0; j < int(Adj[U].size()); ++j) {
            int V = Adj[U][j].V;
            if (!Vis[V] && Adj[U][j].Dis < Dis[V]) {
                Dis[V] = Adj[U][j].Dis;
            }
        }
    }
    // 返回结果
    return Res;
}
```

4.3 NetworkFlow

4.3.1 Dinic

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Edge {
    // V: 连接点, Weight: 权值, Next: 上一条边的编号
    int V, Weight, Next;
};

// 边, 一定要开到足够大
Edge edges[maxn << 1];
// Head[i] 为点 i 上最后一条边的编号
int Head[maxn];
// 增加边时更新编号
int Tot;
// N: 顶点数, E: 边数
```

```
int N, E;
// Bfs 分层深度
int Depth[maxn];
// 当前弧优化
int Current[maxn];

// 链式向前星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 添加一条由 U 至 V 权值为 Weight 的边
void AddEdge(int U, int V, int Weight, int ReverseWeight = 0)
↪  {
    edges[Tot] = Edge (V, Weight, Head[U]);
    Head[U] = Tot++;
    // 反向建边
    edges[Tot] = Edge (U, ReverseWeight, Head[V]);
    Head[V] = Tot++;
}

// Bfs 搜索分层图, Start: 起点, End: 终点
bool Bfs(int Start, int End) {
    memset(Depth, -1, sizeof(Depth));
    std::queue<int> Que;
    Depth[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int Cur = Que.front();
        Que.pop();
        for (int i = Head[Cur]; ~i; i = edges[i].Next) {
            if (Depth[edges[i].V] == -1 && edges[i].Weight >
            ↪  0) {
                Depth[edges[i].V] = Depth[Cur] + 1;
                Que.push(edges[i].V);
            }
        }
    }
    return Depth[End] != -1;
}

// Dfs 搜索增广路径, Cur: 当前搜索顶点, End: 终点, NowFlow: 当前
↪  最大流
```

```
int Dfs(int Cur, int End, int NowFlow) {
    // 搜索到终点或者可用当前最大流为 0 返回
    if (Cur == End || NowFlow == 0) {
        return NowFlow;
    }
    // UsableFlow: 可用流量, 当达到 NowFlow 时不可再增加,
    ↪  FindFlow: 递归深搜到的最大流
    int UsableFlow = 0, FindFlow;
    // &i=Current[Cur] 为当前弧优化, 每次更新 Current[Cur]
    for (int &i = Current[Cur]; ~i; i = edges[i].Next) {
        if (edges[i].Weight > 0 && Depth[edges[i].V] ==
        ↪  Depth[Cur] + 1) {
            FindFlow = Dfs(edges[i].V, End, std::min(NowFlow -
            ↪  UsableFlow, edges[i].Weight));
            if (FindFlow > 0) {
                edges[i].Weight -= FindFlow;
                // 反边
                edges[i ^ 1].Weight += FindFlow;
                UsableFlow += FindFlow;
                if (UsableFlow == NowFlow) {
                    return NowFlow;
                }
            }
        }
    }
    // 炸点优化
    if (!UsableFlow) {
        Depth[Cur] = -2;
    }
    return UsableFlow;
}

// Dinic 算法, Start: 起点, End: 终点
int Dinic(int Start, int End) {
    int MaxFlow = 0;
    while (Bfs(Start, End)) {
        // 当前弧优化
        for (int i = 1; i <= N; ++i) {
            Current[i] = Head[i];
        }
        MaxFlow += Dfs(Start, End, INF);
    }
    // 返回结果
    return MaxFlow;
```

```
}
```

4.3.2 FordFulkerson

```
#include <bits/stdc++.h>
// 正无穷
const int INF = "Edit";
const int maxn = "Edit";

// N: 顶点数, E: 边数
int N, E;
// 访问标记数组
bool Vis[maxn];
// 邻接矩阵
int Adj[maxn][maxn];

// Dfs 搜索增广路径, Vertex: 当前搜索顶点, End: 搜索终点,
↪ NowFlow: 当前最大流量
int Dfs(int Vertex, int End, int NowFlow) {
    // 搜索到终点结束
    if (Vertex == End) {
        return NowFlow;
    }
    // 标记访问过的顶点
    Vis[Vertex] = true;
    // 枚举寻找顶点
    for (int i = 1; i <= N; ++i) {
        if (!Vis[i] && Adj[Vertex][i]) {
            int FindFlow = Dfs(i, End, NowFlow <
            ↪ Adj[Vertex][i] ? NowFlow : Adj[Vertex][i]);
            if (!FindFlow) {
                continue;
            }
            // 找到增广路径后更新邻接矩阵残留网
            Adj[Vertex][i] -= FindFlow;
            Adj[i][Vertex] += FindFlow;
            // 返回搜索结果
            return FindFlow;
        }
    }
    // 未找到增广路径, 搜索失败
    return false;
}
```

```
// Ford-Fulkersone 算法, Start: 起点, End: 终点
int FordFulkerson(int Start, int End) {
    // MaxFlow: 最大流, Flow: 搜索到的增广路径最大流
    int MaxFlow = 0, Flow = 0;
    memset(Vis, false, sizeof(Vis));
    // 搜索增广路径
    while (Flow = Dfs(Start, End, INF)) {
        MaxFlow += Flow;
        memset(Vis, false, sizeof(Vis));
    }
    // 返回结果
    return MaxFlow;
}
```

4.3.3 MinCostMaxFlow

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Edge {
    // V: 连接点, Flow: 流量, Cost: 费用
    int V, Cap, Cost, Flow, Next;
};

// N: 顶点数, E: 边数
int N, E;
int Head[maxn];
// 前驱记录数组
int Path[maxn];
int Dis[maxn];
// 访问标记数组
bool Vis[maxn];
int Tot;
// 链式前向星
Edge edges[maxn];

// 链式前向星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}
```

```cpp
// 建图加边, U、 V 之间建立一条费用为 Cost 的边
void AddEdge(int U, int V, int Cap, int Cost) {
    edges[Tot] = Edge {V, Cap, Cost, 0, Head[U]};
    Head[U] = Tot++;
    edges[Tot] = Edge {U, 0, -Cost, 0, Head[V]};
    Head[V] = Tot++;
}

//  SPFA 算法, Start: 起点, End: 终点
bool SPFA(int Start, int End) {
    memset(Dis, INF, sizeof(Dis));
    memset(Vis, false, sizeof(Vis));
    memset(Path, -1, sizeof(Path));
    Dis[Start] = 0;
    Vis[Start] = true;
    std::queue<int> Que;
    while (!Que.empty()) {
        Que.pop();
    }
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.front();
        Que.pop();
        Vis[U] = false;
        for (int i = Head[U]; ~i; i = edges[i].Next) {
            int V = edges[i].V;
            if (edges[i].Cap > edges[i].Flow && Dis[V] >
             ↪  Dis[U] + edges[i].Cost) {
                Dis[V] = Dis[U] + edges[i].Cost;
                Path[V] = i;
                if (!Vis[V]) {
                    Vis[V] = true;
                    Que.push(V);
                }
            }
        }
    }
    return Path[End] != -1;
}

// 最小费用最大流, Start: 起点, End: 终点, Cost: 最小费用
int MinCostMaxFlow(int Start, int End, int &MinCost) {
    int MaxFlow = 0;
```

```
    MinCost = 0;
    while (SPFA(Start, End)) {
        int Min = INF;
        for (int i = Path[End]; ~i; i = Path[edges[i ^ 1].V])
         ↪  {
            if (edges[i].Cap - edges[i].Flow < Min) {
                Min = edges[i].Cap - edges[i].Flow;
            }
        }
        for (int i = Path[End]; ~i; i = Path[edges[i ^ 1].V])
         ↪  {
            edges[i].Flow += Min;
            edges[i ^ 1].Flow -= Min;
            MinCost += edges[i].Cost * Min;
        }
        MaxFlow += Min;
    }
    // 返回最大流
    return MaxFlow;
}
```

4.4 ShortestPath

4.4.1 BellmanFord

```
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

struct Edge {
    // U、 V: 顶点, Dis: 边权
    int U, V;
    int Dis;
};
// 松弛更新数组
int Dis[maxn];
// 边
std::vector<Edge> edges;

// Bellman_Ford 算法判断是否存在负环回路
bool BellmanFord(int Start, int N) {
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
```

```
    // 最多做 N-1 次
    for (int i = 1; i < N; ++i) {
        bool flag = false;
        for (int j = 0; j < int(edges.size()); ++j) {
            if (Dis[edges[j].V] > Dis[edges[j].U] +
            ↪  edges[j].Dis) {
                Dis[edges[j].V] = Dis[edges[j].U] +
                ↪  edges[j].Dis;
                flag = true;
            }
        }
        // 没有负环回路
        if (!flag) {
            return true;
        }
    }
    // 有负环回路
    for (int j = 0; j < int(edges.size()); ++j) {
        if (Dis[edges[j].V] > Dis[edges[j].U] + edges[j].Dis)
        ↪  {
            return false;
        }
    }
    // 没有负环回路
    return true;
}
```

4.4.2 Dijkstra

```
#include <bits/stdc++.h>

const int maxn = "Edit";
const int INF = "Edit";

// 边
struct Edge {
    // V: 连接点, Weight: 权值, Next: 上一条边的编号
    int V, Weight, Next;
};

// 边, 一定要开到足够大
Edge edges[maxn << 1];
// Head[i] 为点 i 上最后一条边的编号
int Head[maxn];
```

```
// 增加边时更新编号
int Tot;
// 松弛更新数组，最短路
int Dis[maxn];

// 链式前向星初始化
void Init() {
    Tot = 0;
    memset(Head, -1, sizeof(Head));
}

// 添加一条 U 至 V 权值为 Weight 的边
void AddEdge(int U, int V, int Weight) {
    edges[Tot] = Edge (V, Weight, Head[U]);
    Head[U] = Tot++;
}

// 最短路优化堆排序规则
struct Cmp {
    bool operator() (const int &A, const int &B) {
        return Dis[A] > Dis[B];
    }
};

// N: 顶点数, E: 边数
int N, E;

// Dijkstra 算法, Start: 起点
void Dijkstra(int Start) {
    std::priority_queue<int, std::vector<int>, Cmp> Que;
    memset(Dis, INF, sizeof(Dis));
    Dis[Start] = 0;
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.top(); Que.pop();
        for (int i = Head[U]; ~i; i = edges[i].Next) {
            if (Dis[edges[i].V] > Dis[U] + edges[i].Weight) {
                Dis[edges[i].V] = Dis[U] + edges[i].Weight;
                Que.push(edges[i].V);
            }
        }
    }
}
```

4.4.3 Floyd

```cpp
#include <bits/stdc++.h>

const int maxn = "Edit";

// N: 顶点数
int N;
// Dis[i][j] 为 i 点到 j 点的最短路
int Dis[maxn][maxn];

// Floyd 算法
void Floyd() {
    for (int k = 1; k <= N; ++k) {
        for (int i = 1; i <= N; ++i) {
            for (int j = 1; j <= N; ++j) {
                Dis[i][j] = std::min(Dis[i][j], Dis[i][k] +
                ↪  Dis[k][j]);
            }
        }
    }
}
```

4.4.4 SPFA

```cpp
#include <bits/stdc++.h>

const int INF = "Edit";
const int maxn = "Edit";

// 边
struct Edge {
    // V: 连接点, Dis: 边权
    int V, Dis;
};

// N: 顶点数, E: 边数
int N, E;
// 访问标记数组
bool Vis[maxn];
// 每个点的入队列次数
int Cnt[maxn];
// 最短路数组
int Dis[maxn];
```

```
// 邻接表
std::vector<Edge> Adj[maxn];

// 建图加边, U、V 之间权值为 Weight 的边
void AddEdge (int U, int V, int Weight) {
    Adj[U].push_back(Edge (V, Weight));
    // 无向图建立反向边
    Adj[V].push_back(Edge (U, Weight));
}

// SPFA 算法, Start: 起点
bool SPFA(int Start) {
    memset(Vis, false, sizeof(Vis));
    memset(Dis, INF, sizeof(Dis));
    memset(Cnt, 0, sizeof(Cnt));
    Vis[Start] = true;
    Dis[Start] = 0;
    Cnt[Start] = 1;
    std::queue<int> Que;
    while (!Que.empty()) {
        Que.pop();
    }
    Que.push(Start);
    while (!Que.empty()) {
        int U = Que.front();
        Que.pop();
        Vis[U] = false;
        for (int i = 0; i < int(Adj[U].size()); ++i) {
            int V = Adj[U][i].V;
            if (Dis[V] > Dis[U] + Adj[U][i].Dis) {
                Dis[V] = Dis[U] + Adj[U][i].Dis;
                if (!Vis[V]) {
                    Vis[V] = true;
                    Que.push(V);
                    // Cnt[i] 为 i 顶点入队列次数, 用来判定是否
                    ↪ 存在负环回路
                    if (++Cnt[V] > N) {
                        return false;
                    }
                }
            }
        }
    }
```

```
    return true;
}
```

5 DynamicProgramming

5.1 Contour

```
#include <bits/stdc++.h>

const int maxn = "Edit";

int Dp[2][1 << maxn];

void Update(int Cur, int A, int B) {
    if (B & (1 << M)) {
        Dp[Cur][B ^ (1 << M)] = Dp[Cur][B ^ (1 << M)] + Dp[Cur
        ↪ ^ 1][A];
    }
}

// 轮廓线 Dp(1*2 在 N*M 图上摆放数)
int Contour(int N, int M) {
    memset(Dp, 0, sizeof(Dp));
    int Cur = 0;
    Dp[Cur][(1 << M) - 1] = 1;
    for (int i = 0; i < N; ++i) {
        for (int j = 0; j < M; ++j) {
            Cur ^= 1;
            memset(Dp[Cur], 0, sizeof(Dp[Cur]));
            for (int k = 0; k < (1 << M); ++k) {
                Update(Cur, k, k << 1);
                if (i && !(k & (1 << (M - 1)))) {
                    Update(Cur, k, (k << 1) ^ (1 << M) ^ 1);
                }
                if (j && (!(k & 1))) {
                    Update(Cur, k, (k << 1) ^ 3);
                }
            }
        }
    }
    return Dp[Cur][(1 << M) - 1];
}
```

5.2 Digit

```
#include <bits/stdc++.h>

const int maxn = "Edit";
```

```cpp
long long Digit[25];
long long Dp[25][maxn];

// Site: 数位,Statu: 状态,Pre: 前导零,Limit: 数位上界
long long Dfs(long long Site, long long Statu, bool Pre, bool
↪ Limit) {
    if (Site == 0) {
        return ?;
    }
    if (!Limit && ~Dp[Site][Statu]) {
        return Dp[Site][Statu];
    }
    long long Max = Limie ? Digit[Site] : 9;
    long long Ans = 0;
    for (int i = 0; i <= Max; ++i) {
        long long NowStatu = /* 状态转移 */;
        if (NowStatu?) {
            Ans += Dfs(Site - 1, NowStatu, Pre && i == 0,
            ↪ Limit && i == Max);
        }
    }
    if (!Limit) {
        Dp[Site][Statu] = Ans;
    }
    return Ans;
}

long long Cal(long long X) {
    // 数位分解
    long long Len = 0;
    while (X) {
        Digit[++Len] = X % 10;
        X /= 10;
    }
    return Dfs(Len, 0, true, true);
}
```

5.3 LCS

```cpp
#include <bits/stdc++.h>

const int maxn = "Edit";
```

```
// Dp[i][j]:Str1[1]~Str1[i] 和 Str2[1]~Str2[j] 对应的公共子序列
↪  长度
int Dp[maxn][maxn];

// 最长公共子序列 (LCS)
void LCS(std::string Str1, std::string Str2) {
    for (int i = 0; i < int(Str1.length()); ++i) {
        for (int j = 0; j < int(Str2.length()); ++j) {
            if (Str1[i] == Str2[j]) {
                Dp[i + 1][j + 1] = Dp[i][j] + 1;
            }
            else {
                Dp[i + 1][j + 1] = std::max(Dp[i][j + 1], Dp[i
                ↪  + 1][j]);
            }
        }
    }
}
```

5.4 LIS

```
#include <bits/stdc++.h>

// 最长不下降子序列 (LIS), Num: 序列
int LIS(std::vector<int> &Num) {
    int Ans = 1;
    // Last[i] 为长度为 i 的不下降子序列末尾元素的最小值
    std::vector<int> Last(int(Num.size()) + 1, 0);
    Last[1] = Num[1];
    for (int i = 2; i <= int(Num.size()); ++i) {
        if (Num[i] >= Last[Ans]) {
            Last[++Ans] = Num[i];
        }
        else {
            int Index = std::upper_bound(Last.begin() + 1,
            ↪  Last.end(), Num[i]) - Last.begin();
            Last[Index] = Num[i];
        }
    }
    // 返回结果
    return Ans;
}
```

5.5 Pack

```cpp
#include <bits/stdc++.h>

const int maxn = "Edit";

int Dp[maxn];
// NValue: 背包容量, NKind: 总物品数
int NValue, NKind;

// 01 背包, 代价为 Cost, 获得的价值为 Weight
void ZeroOnePack(int Cost, int Weight) {
    for (int i = NValue; i >= Cost; --i) {
        Dp[i] = std::max(Dp[i], Dp[i - Cost] + Weight);
    }
}

// 完全背包, 代价为 Cost, 获得的价值为 Weight
void CompletePack(int Cost, int Weight) {
    for (int i = Cost; i <= NValue; ++i) {
        Dp[i] = std::max(Dp[i], Dp[i - Cost] + Weight);
    }
}

// 多重背包, 代价为 Cost, 获得的价值为 Weight, 数量为 Amount
void MultiplePack(int Cost, int Weight, int Amount) {
    if (Cost * Amount >= NValue) {
        CompletePack(Cost, Weight);
        }
    else {
        int k = 1;
        while (k < Amount) {
            ZeroOnePack(k * Cost, k * Weight);
            Amount -= k;
            k <<= 1;
        }
        ZeroOnePack(Amount * Cost, Amount * Weight);
    }
}
```

6 ComputationalGeometry

6.1 Plane

```
#include <bits/stdc++.h>

const double eps = "Edit";

int Sgn(double X) {
    if (fabs(X) < eps) {
        return 0;
    }
    return X < 0 ? -1 : 1;
}

// 点
struct Point {
    // X: 横坐标, Y: 纵坐标
    double X, Y;

    // 减法
    Point operator - (Point B) {
        return (Point){X - B.X, Y - B.Y};
    }

    // 加法
    Point operator + (Point B) {
        return (Point){X + B.X, Y + B.Y};
    }

    // 点积
    double operator * (Point B) {
        return X * B.X + Y * B.Y;
    }

    // 叉积
    double operator ^ (Point B) {
        return X * B.Y - Y * B.X;
    }

    // 两点间距离
    double Dis(Point B) {
        return sqrt((B - *this) * (B - *this));
    }
```

```cpp
};

// 线
struct Line {
    Point S, T;

    // 向量叉积
    double operator ^ (Line B) {
        return (T - S) ^ (B.T - B.S);
    }

    // 判断是否平行
    bool IsParallel(Line B) {
        return Sgn((S - T) ^ (B.S - B.T)) == 0;
    }

    // 求交点
    Point operator & (Line B) {
        double Temp = ((S - B.S) ^ (B.S - B.T)) / ((S - T) ^
        ↪  (B.S - B.T));
        return (Point){S.X + (T.X - S.X) * Temp, S.Y + (T.Y -
        ↪  S.Y) * Temp};
    }
};

// 判断线段 A、B 是否相交
bool IsIntersect(Line A, Line B) {
    return
        max(A.S.X, A.T.X) >= min(B.S.X, B.T.X) &&
        max(B.S.X, B.T.X) >= min(A.S.X, A.T.X) &&
        max(A.S.Y, A.T.Y) >= min(B.S.Y, B.T.Y) &&
        max(B.S.Y, B.T.Y) >= min(A.S.Y, A.T.Y) &&
        Sgn((B.S - A.T) ^ (A.S - A.T)) * Sgn((B.T - A.T) ^
        ↪  (A.S - A.T)) <= 0 &&
        Sgn((A.S - B.T) ^ (B.S - B.T)) * Sgn((A.T - B.T) ^
        ↪  (B.S - B.T)) <= 0;
}

// 判断线段 A 所在直线与线段 B 是否相交
bool IsIntersect(Line A, Line B) {
    return Sgn((B.S - A.T) ^ (A.S - A.T)) * Sgn((B.T - A.T) ^
    ↪  (A.S - A.T)) <= 0;
}
```

```
// 判断直线 A、B 是否相交
bool IsIntersect(Line A, Line B) {
    return !A.IsParallel(B) || (A.IsParallel(B) && !(Sgn((A.S
    ↪   - B.S) ^ (B.T - B.S)) == 0));
}

// 判断 N 个点 (下标 1~N-1) 能否组成凸包
bool IsConvexHull(Point points[], int N) {
    for (int i = 0; i < N; ++i) {
        if (Sgn((points[(i + 1) % N] - points[i]) ^ (points[(i
        ↪   + 2) % N] - points[(i + 1) % N])) < 0) {
            return false;
        }
    }
    return true;
}

// 凸包, points: 所有点, 返回凸包总长度
double ConvexHull(std::vector<Point> points) {
    int N = int(points.size());
    // 特判点数小于等于 2 的情况
    if (N == 1) {
        return 0;
    }
    else if (N == 2) {
        return Distance(points[0], points[1]);
    }
    // 查找最左下角的基准点
    int Basic = 0;
    for (int i = 0; i < N; ++i) {
        if (points[i].Y > points[Basic].Y ||
            (points[i].Y == points[Basic].Y && points[i].X <
            ↪   points[Basic].X)) {
                Basic = i;
        }
    }
    std::swap(points[0], points[Basic]);
    // 对其它点进行极角排序
    std::sort(points.begin() + 1, points.end(), [&] (const
    ↪   Point &A, const Point &B) {
        double Temp = (A - points[0]) ^ (B - points[0]);
        if (Temp > 0) {
```

```
            return true;
        }
        else if (!Temp && Distance(A, points[0]) < Distance(B,
        ↪   points[0])) {
            return true;
        }
        return false;
    });
    // 凸包选点
    std::vector<Point> Stack;
    Stack.push_back(points[0]);
    for (int i = 2; i < N; ++i) {
        while (Stack.size() >= 2 && ((Stack.back() -
        ↪   Stack[int(Stack.size()) - 2]) ^ (points[i] -
        ↪   Stack[int(Stack.size()) - 2])) <= 0) {
            Stack.pop_back();
        }
    }
    Stack.push_back(points[0]);
    // 计算总长
    double Ans = 0;
    for (int i = 1; i < int(Stack.size()); ++i) {
        Ans += Distance(Stack[i], Stack[i - 1]);
    }
    // 返回结果
    return Ans;
}

// 半平面，表示 S->T 逆时针（左侧）的半平面
struct HalfPlane:public Line {
    double Angle;

    void CalAngle() {
        Angle = atan2(T.Y - S.Y, T.X - S.X);
    }

    bool operator < (HalfPlane B) {
        if (fabs(Angle - B.Angle) > eps) {
            return Angle < B.Angle;
        }
        return ((S - B.S) ^ (B.T - B.S)) < 0;
    }
};
```

```cpp
// 半平面交
struct HPI {
    // 半平面数量
    int Tot;
    // 半平面
    HalfPlane halfplanes[maxn];
    // 半平面交双向队列
    HalfPlane Deque[maxn];
    // 点队列
    Point points[maxn];
    // 半平面交内核
    Point Res[maxn];
    // 双向队列首尾指针
    int Front, Tail;

    // 添加半平面
    void Push(HalfPlane X) {
        halfplanes[Tot++] = X;
    }

    // 半平面去重
    void Unique() {
        int Cnt = 1;
        for (int i = 1; i < Tot; ++i) {
            if (fabs(halfplanes[i].Angle - halfplanes[i -
            ↪ 1].Angle) > eps) {
                halfplanes[Cnt++] = halfplanes[i];
            }
        }
        Tot = Cnt;
    }

    // 判断半平面交是否有内核
    bool HalfPlaneInsert() {
        for (int i = 0; i < Tot; ++i) {
            halfplanes[i].CalAngle();
        }
        sort(halfplanes, halfplanes + Tot);
        Unique();
        Deque[Front = 0] = halfplanes[0];
        Deque[Tail = 1] = halfplanes[1];
        for (int i = 2; i < Tot; ++i) {
```

```
        if (fabs((Deque[Tail].T - Deque[Tail].S) ^
        ↪   (Deque[Tail - 1].T - Deque[Tail - 1].S)) < eps
        ↪   || fabs((Deque[Front].T - Deque[Front].S) ^
        ↪   (Deque[Front + 1].T - Deque[Front + 1].S)) <
        ↪   eps) {
            return false;
        }
        while (Front < Tail && (((Deque[Tail] & Deque[Tail
        ↪   - 1]) - halfplanes[i].S) ^ (halfplanes[i].T -
        ↪   halfplanes[i].S)) > eps) {
            Tail--;
        }
        while (Front < Tail && (((Deque[Front] &
        ↪   Deque[Front + 1]) - halfplanes[i].S) ^
        ↪   (halfplanes[i].T - halfplanes[i].S)) > eps) {
            Front++;
        }
        Deque[++Tail] = halfplanes[i];
    }
    while (Front < Tail && (((Deque[Tail] & Deque[Tail -
    ↪   1]) - Deque[Front].S) ^ (Deque[Front].T -
    ↪   Deque[Front].S)) > eps) {
        Tail--;
    }
    while (Front < Tail && (((Deque[Front] & Deque[Front -
    ↪   1]) - Deque[Tail].S) ^ (Deque[Tail].T -
    ↪   Deque[Tail].T)) > eps) {
        Front++;
    }
    if (Tail <= Front + 1) {
        return false;
    }
    return true;
}

// 获取半平面交内核点集 Res
void GetHalfPlaneInsertConvex() {
    int Cnt = 0;
    for (int i = Front; i < Tail; ++i) {
        Res[Cnt++] = Deque[i] & Deque[i + 1];
    }
    if (Front < Tail - 1) {
        Res[Cnt++] = Deque[Front] & Deque[Tail];
    }
```

```
    }
};
```

6.2 Stereoscopic

```
#include <bits/stdc++.h>

const double INF = 1e20;
const int maxn = "Edit";
const double eps = 1e-8;
const double delta = 0.98;

// 点
struct Point {
        double X, Y, Z;

    void Input() {
        scanf("%lf%lf%lf", &X, &Y, &Z);
    }
};

// 求点 A、B 间距离
double Distance(Poinn A, Point B) {
    return sqrt((A.X - B.X) * (A.X - B.X) + (A.Y - B.Y) * (A.Y
    ↪   - B.Y) + (A.Z - B.Z) * (A.Z - B.Z));
}

int N;
Point points[maxn];

// 模拟退火求 N 个点的最小球覆盖
double MinimimSphereCoverage() {
    Point Cur = points[0];
    double Probability = 10000, Ans = INF;
    while (Probability > eps) {
        int Book = 0;
        for (int i = 0; i < N; ++i) {
            if (Distance(Cur, points[i] > Distance(Cur,
            ↪  points[Book]))) {
                Book = i;
            }
        }
        double Redius = Distance(Cur, points[Book]);
        Ans = min(Ans, Redius);
```

```
        Cur.X += (points[Book].X - Cur.X) / Redius *
        ↪  Probability;
        Cur.Y += (points[Book].Y - Cur.Y) / Redius *
        ↪  Probability;
        Cur.Z += (points[Book].Z - Cur.Z) / Redius *
        ↪  Probability;
        Probability *= delta;
    }
    // 返回覆盖最小球半径
    return Ans;
}
```

7 Others

7.1 Factorial

```
#include <bits/stdc++.h>

void Factorial() {
    int res[10010];
    int Book = 1;
    int BaoFour = 0;
    res[Book] = 1;
    int n;
    scanf("%d", &n);
    // 乘法计算
    for (int i = 1;i <= n;++i) {
        BaoFour = 0;
        for (int j = 1;j <= Book;++j) {
            res[j] = res[j] * i + BaoFour;
            BaoFour = res[j] / 10000;
            res[j] = res[j] % 10000;
        }
        if (BaoFour > 0) {
            res[++Book] += BaoFour;
        }
    }
    printf("%d", res[Book]);
    // 补零输出
    for (int i = Book - 1;i > 0;--i) {
        if (res[i] >= 1000) {
            printf("%d", res[i]);
        }
        else if (res[i] >= 100) {
            printf("0%d",res[i]);
        }
        else if (res[i] >= 10) {
            printf("00%d",res[i]);
        }
        else {
            printf("000%d",res[i]);
        }
    }
    putchar('\n');
}
```

7.2 FastIO

```
#include <bits/stdc++.h>

// 普通读入挂
template <class T>
inline bool read(T &ret) {
    char c;
    int sgn;
    if (c = getchar(), c == EOF) {
        return false;
    }
    while (c != '-' && (c < '0' || c > '9')) {
        c = getchar();
    }
    sgn = (c == '-') ? -1 : 1;
    ret = (c == '-') ? 0 : (c - '0');
    while (c = getchar(), c >= '0' && c <= '9') {
        ret = ret * 10 + (c - '0');
    }
    ret *= sgn;
    return true;
}

// 普通输出挂
template <class T>
inline void out(T x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) {
        out(x / 10);
    }
    putchar(x % 10 + '0');
}

// 牛逼读入挂
namespace FastIO {
    const int MX = 4e7;
    char buf[MX];
    int c, sz;
    void begin() {
        c = 0;
```

```
        sz = fread(buf, 1, MX, stdin);
    }
    template <class T>
    inline bool read(T &t) {
        while (c < sz && buf[c] != '-' && (buf[c] < '0' ||
        ↪   buf[c] > '9')) {
            c++;
        }
        if (c >= sz) {
            return false;
        }
        bool flag = 0;
        if (buf[c] == '-') {
            flag = 1;
            c++;
        }
        for (t = 0; c < sz && '0' <= buf[c] && buf[c] <= '9';
        ↪   ++c) {
            t = t * 10 + buf[c] - '0';
        }
        if (flag) {
            t = -t;
        }
        return true;
    }
};

// 超级读写挂
namespace IO{
    #define BUF_SIZE 100000
    #define OUT_SIZE 100000
    #define ll long long
    //fread->read

    bool IOerror=0;
    inline char nc(){
        static char
        ↪   buf[BUF_SIZE],*p1=buf+BUF_SIZE,*pend=buf+BUF_SIZE;
        if (p1==pend){
            p1=buf; pend=buf+fread(buf,1,BUF_SIZE,stdin);
            if (pend==p1){IOerror=1;return -1;}
            //{printf("IO error!\n");system("pause");for
            ↪   (;;);exit(0);}
```

```
    }
    return *p1++;
}
inline bool blank(char ch){return ch=='
↪   '||ch=='\n'||ch=='\r'||ch=='\t';}
inline void read(int &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}
inline void read(ll &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (sign)x=-x;
}
inline void read(double &x){
    bool sign=0; char ch=nc(); x=0;
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    if (ch=='-')sign=1,ch=nc();
    for (;ch>='0'&&ch<='9';ch=nc())x=x*10+ch-'0';
    if (ch=='.'){
        double tmp=1; ch=nc();
        for
        ↪   (;ch>='0'&&ch<='9';ch=nc())tmp/=10.0,x+=tmp*(ch-'0');
    }
    if (sign)x=-x;
}
inline void read(char *s){
    char ch=nc();
    for (;blank(ch);ch=nc());
    if (IOerror)return;
    for (;!blank(ch)&&!IOerror;ch=nc())*s++=ch;
    *s=0;
}
inline void read(char &c){
    for (c=nc();blank(c);c=nc());
    if (IOerror){c=-1;return;}
```

```
}
//fwrite->write
struct Ostream_fwrite{
    char *buf,*p1,*pend;
    Ostream_fwrite(){buf=new
    ↪  char[BUF_SIZE];p1=buf;pend=buf+BUF_SIZE;}
    void out(char ch){
        if (p1==pend){
            fwrite(buf,1,BUF_SIZE,stdout);p1=buf;
        }
        *p1++=ch;
    }
    void print(int x){
        static char s[15],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1);
    }
    void println(int x){
        static char s[15],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1); out('\n');
    }
    void print(ll x){
        static char s[25],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1);
    }
    void println(ll x){
        static char s[25],*s1;s1=s;
        if (!x)*s1++='0';if (x<0)out('-'),x=-x;
        while(x)*s1++=x%10+'0',x/=10;
        while(s1--!=s)out(*s1); out('\n');
    }
    void print(double x,int y){
        static ll
        ↪  mul[]={1,10,100,1000,10000,100000,1000000,10000000,100000000,

            ↪  1000000000,10000000000LL,100000000000LL,1000000000000LL,10000000

            ↪  100000000000000LL,1000000000000000LL,10000000000000000LL,1000000
        if (x<-1e-12)out('-'),x=-x;x*=mul[y];
```

```
            ll x1=(ll)floor(x); if (x-floor(x)>=0.5)++x1;
            ll x2=x1/mul[y],x3=x1-x2*mul[y]; print(x2);
            if (y>0){out('.'); for (size_t
             ↪  i=1;i<y&&x3*mul[i]<mul[y];out('0'),++i);
             ↪  print(x3);}
        }
        void println(double x,int y){print(x,y);out('\n');}
        void print(char *s){while (*s)out(*s++);}
        void println(char *s){while (*s)out(*s++);out('\n');}
        void flush(){if
         ↪  (p1!=buf){fwrite(buf,1,p1-buf,stdout);p1=buf;}}
        ~Ostream_fwrite(){flush();}
    }Ostream;
    inline void print(int x){Ostream.print(x);}
    inline void println(int x){Ostream.println(x);}
    inline void print(char x){Ostream.out(x);}
    inline void println(char
     ↪  x){Ostream.out(x);Ostream.out('\n');}
    inline void print(ll x){Ostream.print(x);}
    inline void println(ll x){Ostream.println(x);}
    inline void print(double x,int y){Ostream.print(x,y);}
    inline void println(double x,int y){Ostream.println(x,y);}
    inline void print(char *s){Ostream.print(s);}
    inline void println(char *s){Ostream.println(s);}
    inline void println(){Ostream.out('\n');}
    inline void flush(){Ostream.flush();}
    #undef ll
    #undef OUT_SIZE
    #undef BUF_SIZE
};
using namespace IO;
```

7.3 LeepYear

```
#include <bits/stdc++.h>

inline bool Leep(int Year) {
    return (!(Year % 4) && (Year % 100)) || !(Year % 400);
}
```

7.4 NimGame

```
#include <bits/stdc++.h>
```

```cpp
// 尼姆博弈
bool Nim(std::vector<int> Num) {
    int Ans = 0;
    for (int i = 0; i < int(Num.size()); ++i) {
        Ans ^= Num[i];
    }
    // ans 不为零则先手赢，否则为后手赢
    return Ans != 0 ? true : false;
}
```

7.5 vim

```vim
syntax on
set nu
set tabstop=4
set shiftwidth=4
set cindent
set mouse=a

map <F9> :call Run()<CR>

func! Run()
        exec "w"
        if &filetype == "cpp"
                exec "!g++ -Wall % -o %<"
                exec "! ./%<"
        elseif &filetype == "python"
                exec "!python3 %"
        endif
endfunc
```