

SECURITY

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Exercise 1

- A. Since we know the Alice's private key we may compute the private key as follows:

$K_{AB} = B^a$ where B is Bob's public key which is g^b

This means that K_{AB} can be computed as follows:

$$\begin{aligned} K_{AB} &= (g^b)^a \bmod p \\ &= (491)^{317} \bmod 1021 \\ &= 1,18 \times 10^{853} \bmod 1021 \\ &= 71 \end{aligned}$$

- B. Bob's secret key may be derived using the value of g^b . We know that $g = 10$ and that $g^b = 491$, so we should find the value for b for which $10^b \bmod 1021 = 491$. The mod 1021 we can derive from the p that have been sent. Using Wolfram Alpha we find that $b = 12$.

- C. In Bob's case the shared will equal A^b . Using $b = 12$ from the computation in the previous exercise we came to the following computation:

$$\begin{aligned} K_{AB} &= A^b \bmod p \\ &= (g^a)^b \bmod p \\ &= 93^{12} \bmod 1021 \\ &= 71 \end{aligned}$$

We see that this indeed equals the computed shared key from Alice.

- D. i). The messages are as follows:

$A \rightarrow E(B): p = 1021, g = 10, g^a = 93$

$E(A) \rightarrow B: p = 1021, g = 10, g^{404} = \dots$

$B \rightarrow E(A): g^b = 491$

$E(B) \rightarrow A: g^{37}$

- ii). First, we will compute the key between Alice and Eve:

$$\begin{aligned} K_{AE} &= (g^{rB})^a \bmod p \\ &= (10^{404})^{317} \bmod 1021 \\ &= (1 * 10^{404})^{317} \bmod 1021 \\ &= 622 \end{aligned}$$

We may compute the shared key between Eve and Bob using Bob's computed private key.

$$\begin{aligned}
 K_{BE} &= (g^{rA})^b \bmod p \\
 &= (10^{37})^{12} \bmod 1021 \\
 &= (1 * 10^{37})^{12} \bmod 1021 \\
 &= 73
 \end{aligned}$$

Exercise 2

A. $A = g^a \bmod p$
 $= 3^{17} \bmod 31$
 $= 129.140.163 \bmod 31$
 $= 22$

B.

Encryption	r	e	m	e	m	b	e	r
Mapping m	18	5	13	5	13	2	5	18
r	3	6	9	12	15	18	21	24
A^r	15	8	27	2	30	16	23	4
$c_1 = g^r$	27	16	29	8	30	4	15	2
$c_2 = m * A^r$	270	40	351	10	390	32	115	72

C. The decryption table of the encrypted message in B is as follows:

Decryption of ciphertext (c_1, c_2)								
$(A^r)^{-1} = c_1^{-a}$	29	4	23	16	30	2	27	8
$m = c_2 * A^{-r}$	18	5	13	5	13	2	5	18

Exercise 3

The first number 595581987651106688365284842778515858399666547859870373300567 can be factored by the following two integers since the first factor is a common factor with the last number:

- 521192137187180935658403029827
- 1142730185580695709964614614621

The second number has no GCD higher than 1 with any of the other numbers.

The third number

697998237255232517803133139640937207091669333334886072165381 can be factored in the following two numbers:

- 701397335649456892851007539749
- 995153819067366445814664502369

The fourth number has no GCD higher than 1 with any of the other numbers.

The fifth number 176294427788887166758409622538881387638478405478915857712513 can be factored in the following numbers:

- 701397335649456892851007539749
- 251347444349282901859447208237

The last number 592339248856319601455928821705423109007342115448431777433343 can be factored in the following numbers:

- 521192137187180935658403029827
- 1136508413294782483013101218709

Common factors

1. GCD with all is 1 except for the last one it's: 521192137187180935658403029827
2. All GCD are 1
3. GCD with 5th: 701397335649456892851007539749 and all others are 1
4. All gcd are 1
5. All gcd are 1 except with third
6. All gcd are 1 except with the first