Quantitative Sociological Analysis

Inferential Statistics
Confidence Intervals and Hypothesis Testing

Part 6

March 27, 2025

Confidence intervals for sample statistics

- Why is a different equation needed for nominal and ordinal variables?
  - sample mean vs proportion

mean

MoE

$$CI = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{N}}\right)$$

when population sd (σ) is known

$$CI = \hat{\theta} \pm Z \times SE$$

when population sd (σ) is unknown

Recall

$$SE = \frac{sd}{\sqrt{N}}$$

proportion

MoE

$$CI = \hat{P} \pm Z\sqrt{\frac{\hat{P}(1-\hat{P})}{N}}$$

- Why would a CI get wider if alpha were set smaller?

- Why would a CI get narrower if the random sample size were larger?

Hypothesis testing: one-sample

- Through the confidence interval (CI) exercise you constructed a range of plausible values within which the true population parameter, in this case mean age, was likely to fall given a specified level of confidence

- However, this did not tell us whether your sample statistic was consistent with the population value if known, or hypothesized population value if unknown
  - Was your sample mean ($\bar{X}$) statistically different from the population mean (μ)?

- The probability of obtaining a sample statistic as extreme as the observed one, assuming the null hypothesis is true, is given by the p-value in a hypothesis test
  - Null hypothesis (H_0): assumes no difference
  - Alternative hypothesis (H_a): suggests a difference

Who has heard about p-values, statistical significance, before? What are some contexts?

Hypothesis testing: sample mean

$\mu_{age} = 19.82$

$\sigma_{age} = 4.09$

- Let's pretend that our Netflix survey results are the consumer population
  - Since we do not know the true Netflix consumer population parameters

- One team obtained a relatively large mean age ($\bar{X} = 23.8$) and
  - relatively wide 95% confidence interval (16.47,31.13)

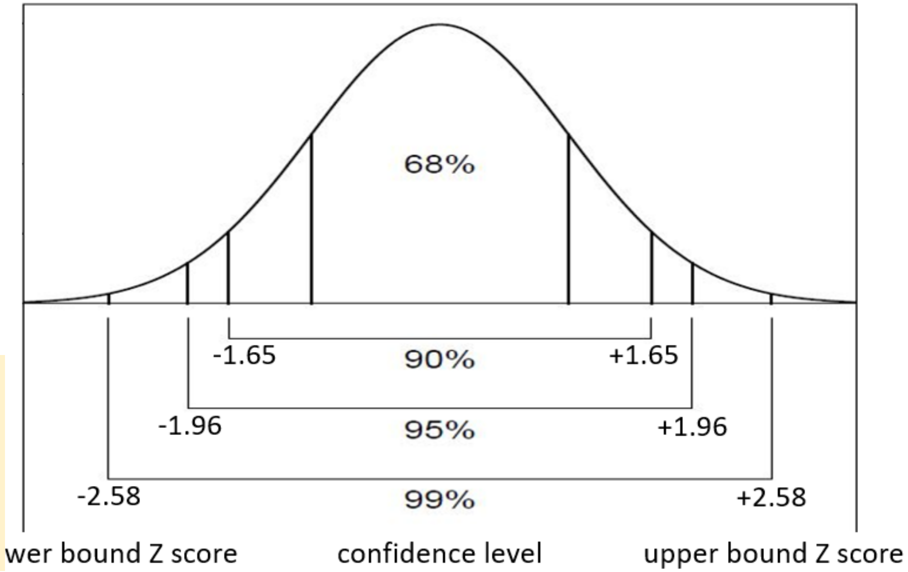
- What is the probability of obtaining a sample with a mean age of 23.8?
  - two-tailed hypothesis test with known parameters (μ, σ)
    - set alpha, significance level: in this case let's use 0.05

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{N}}, \text{ so } Z = \frac{23.8-19.82}{4.09/\sqrt{5}} = 2.18$$

Thus, should we reject or fail to reject the null hypothesis at that 0.05 level?

What about at the 0.01 level?

*Null hypothesis: the sample mean ($\bar{X}$) is not statistically different from the population mean (μ)



Hypothesis testing

- designed to determine whether an observed difference is statistically significant
  - Unlikely to have occurred by random chance alone

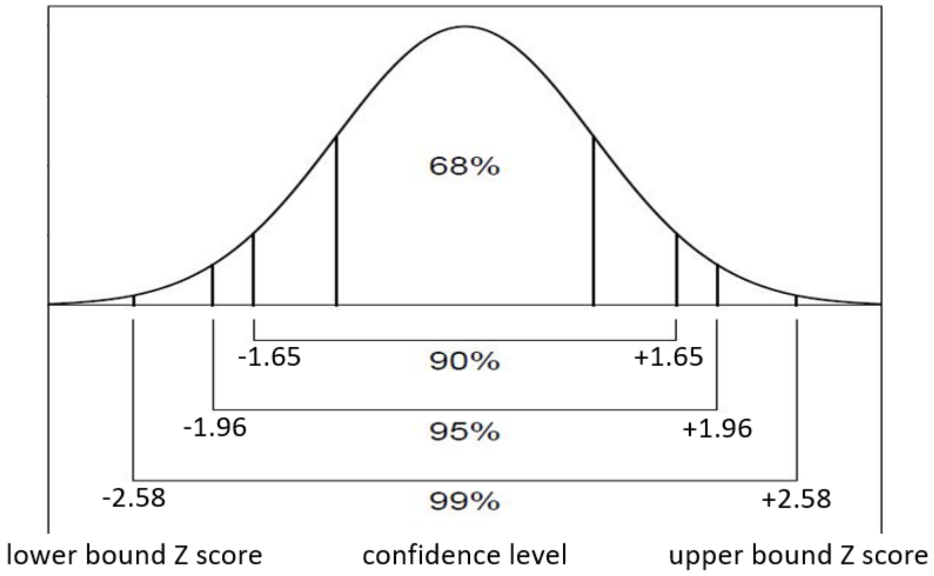
- Z-test: used when comparing a sample statistic to a known population parameter
  - given the population standard deviation (σ) is known

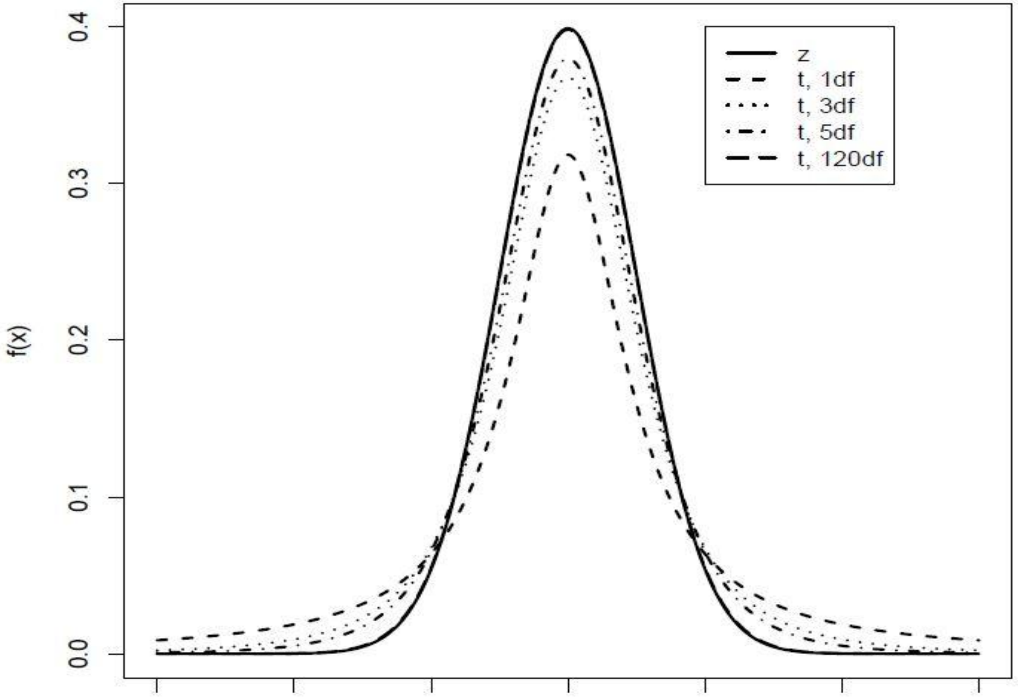
- t-test: used when the population standard deviation is unknown, either
  - to compare a sample statistic to a hypothesized value (one-sample), or
  - to compare a sample statistic between groups in the same sample (paired t-test) or
    - a different sample (two-sample t-test)

Z and t are theoretical probability distributions used to compute p-values, which indicate the likelihood of obtaining a statistic as extreme as the observed one under the null hypothesis (H_0): assumes no difference

The p-value is the probability of obtaining a test statistic as extreme as or more extreme than the observed one, assuming the null hypothesis (H_0) is true

The Z and some t distributions





See how the shape of a t distribution differs based on degrees of freedom (df)

In this case, df is based on sample size $(df = n - 1)$

Why do t distributions with relatively smaller df have fatter tails compared to the Z distribution?

Note: area under each distribution, curve, respectively sums to 1

Hypothesis testing

1. Make assumptions and meet test requirements
    - random sample; level of measurement; sample size; are parameters un/known
2. State the null (H_0) and alternative (H_a) hypothesis
    - H_0 no difference; $H_a$there is a difference
3. Choose a significance level (critical value)
    - e.g., $\alpha = 0.05$
4. Compute the test statistic
5. Draw a conclusion and interpret the test results
    - If $p < \alpha$ then reject H_0

Hypothesis testing: step one

1. Make assumptions and meet test requirements
    - random sample; level of measurement; sample size; are parameters un/known

- Why would the data need to be from a random sample?
    - consider the CLT and its connection with probability theory
- Why does level of measurement matter?
    - consider how assumptions based on connections to probability distributions
- Why is it important to consider sample size?
    - consider how a t distribution is determined
- How would the test differ if parameters were known versus unknown?

Hypothesis testing: step two

2. State the null (H_0) and alternative (H_a) hypothesis
   - H_0 no difference; $H_a$there is a difference

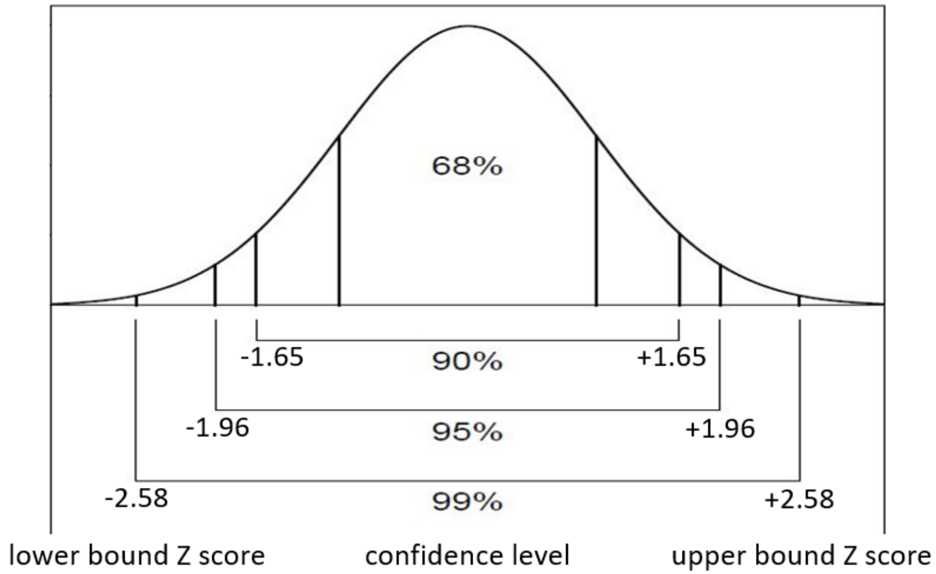
- H_0: $\theta = \theta_0$

- The alternative hypothesis can take on one of three forms
- H_a: $\theta \neq \theta_0$
- H_a: $\theta > \theta_0$
- H_a: $\theta < \theta_0$

two-tailed

one-tailed

What region(s) of the distribution would each be concerned with?

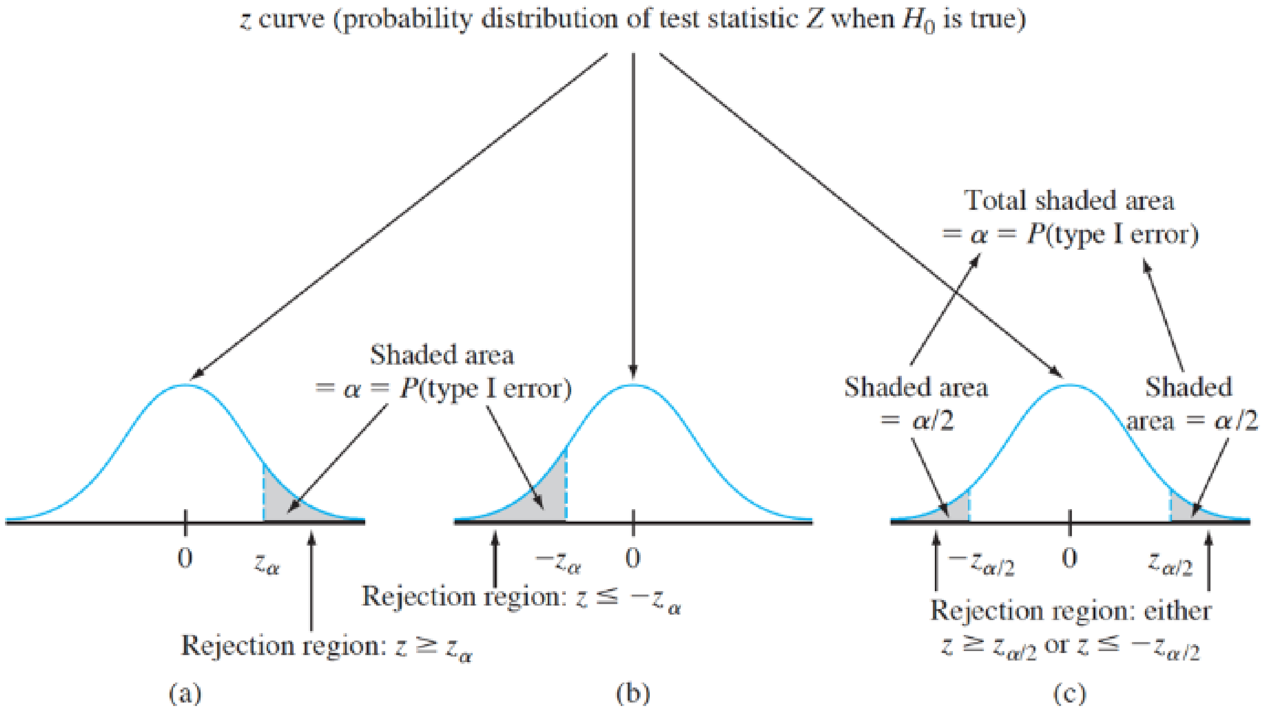


Hypothesis testing: step three

3. Choose a significance level (critical value)
   - e.g., $\alpha = 0.05$

- Threshold at which the null hypothesis will be reject
  - the probability of rejecting H_0 when it is actually true

Type I error



See how the region(s) of the curve differ based on one-tailed vs two-tailed test

Hypothesis testing: errors

- How improbable must a sample statistic be to reject the null hypothesis?

	Decision	
State of H_0	Reject H_0	Fail to reject H_0
TRUE	Type I Error	OK
FALSE	OK	Type II Error

false negative

false positive

Hypothesis testing: step four & five

4. Compute the test statistic
   - e.g., obtain a Z or t score

5. Draw a conclusion and interpret the test results

- A table of values that correspond with the test statistic can be used to determine if it falls beyond the critical value at which α was set
  - In which case, the null hypothesis H_0would be rejected

- Rather, critical values are associated with p-values, and we can use that information
  - If $p < \alpha$ then reject H_0, or If $p > \alpha$ then fail to reject H_0

Hypothesis testing: p-values

- The probability of obtaining a sample statistic as extreme, or more extreme, than the one observed, assuming the null hypothesis H_0 is true
  - p-values range between 0 and 1

- The smaller the p-value, the stronger the evidence to reject H_0
  - assuming H_0 is true

- NOT the probability that H_0 is true
- NOT the probability of an error

Let's use R to do some hypothesis testing, compute p-values, and make decisions about our sample data from Exercise 6…

Hypothesis testing: Exercise 6b

- See Rscript Ex_6b_CI_HypTest