# Quantitative Data Analysis II

SOC 781

**GLM** basics

### Today we're going to...

- More info on descriptive and reg tables
- Discuss the basics of GLM
  - compare LRM and GLM
- Briefly introduce unique models for different outcomes
  - briefly discuss MLE
- Briefly cover techniques for assessing
  - and interpreting GLM output

### Descriptive tables

- Descriptive tables
  - single table for ALL basics
    - additional for more than basics
  - only report on analytic sample
    - in this case, stratified
  - Only report (1) for binary measure
    - (0) deduced
  - Means and percentages can be reported in the same column
    - SD in (SD)

Source: 12 Hua

|                         | Non-Hispanic<br>blacks (n = 393; 24.90%) | Non-Hispanic<br>whites (n = 1185; 75.10%) | Black vs White $\chi^2/t$ |
|-------------------------|--|---|---------------------------|
| Predisposing factors    |  |   |                           |
| Female                  | 0.69                                     | 0.61                                      | -2.77**                   |
| Age                     | 59.53 (12.83)                            | 61.60 (14.64)                             | 2.50**                    |
| Less than High School   | 0.34                                     | 0.16                                      | -7.88***                  |
| Need factors            |  |   |                           |
| Self-Rated Health       | 3.22 (1.03)                              | 3.53 (1.01)                               | 5.31***                   |
| Functional Limitations  | 1.56 (1.03)                              | 1.41 (0.87)                               | -2.74**                   |
| Health Conditions Index | 1.46 (1.30)                              | 1.35 (1.24)                               | -1.58†                    |
| Enabling factors        |  |   |                           |
| Employed                | 0.73                                     | 0.69                                      | -1.18                     |
| Income                  | 4.19 (2.60)                              | 5.73 (2.40)                               | 10.84***                  |
| Insured                 | 0.94                                     | 0.96                                      | 1.21                      |
| Married                 | 0.46                                     | 0.70                                      | 8.79***                   |
| Rural                   | 0.22                                     | 0.35                                      | 5.04***                   |
| ED Used                 | 0.32                                     | 0.24                                      | -2.98***                  |
| PC Used                 | 0.88                                     | 0.91                                      | 1.55                      |
| Trust in physicians     |  |   |                           |
| Mistrust                | 1.44                                     | 1.49                                      | 1.29                      |
| Life course factors     |  |   | 211.72***                 |
| South-Never             | 0.28                                     | 0.64                                      | 12.79***                  |
| South-After 16          | 0.03                                     | 0.08                                      | 3.14***                   |
| South-Left After 16     | 0.17                                     | 0.04                                      | -8.84***                  |
| Health care utilization |  |   |                           |
| PC Utilization          | 5.65 (8.99)                              | 5.41 (8.89)                               | -0.46                     |
| ED Utilization          | 0.59 (1.13)                              | 0.40 (1.08)                               | -2.90**                   |

### Descriptive tables

- Descriptive tables
  - have some logical organization
    - in this case, theory
  - informative names
    - female, not gender
    - <HS, not education</li>
    - married, not marital status
  - adequate spacing
    - make sure easy to read
    - I don't like lines

Source: 12 Hua

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- Descriptive tables
  - no single best approach
  - include most pertinent information
    - here, missing data was a major concern
  - measures section provides details

Source: 15\_Zang\_OSS

Table 1 Variable mean before and after imputation by country

|                                    | China                     |                        |                          | Japan                     |                        |                          | South Korea               |                        |                       |
|------------------------------------|---------------------------|------------------------|--------------------------|---------------------------|------------------------|--------------------------|---------------------------|------------------------|-----------------------|
|                                    | Percentage<br>missing (%) | Mean before imputation | Mean after<br>imputation | Percentage<br>missing (%) | Mean before imputation | Mean after<br>imputation | Percentage<br>missing (%) | Mean before imputation | Mean after imputation |
| Health outcomes                    |                           |                        |                          |                           |                        |                          |                           |                        |                       |
| Self-rated health                  | 0.12                      | 0.79                   | 0.79                     | 0.13                      | 0.70                   | 0.70                     | 0.08                      | 0.73                   | 0.73                  |
| Depression                         | 0.39                      | 0.37                   | 0.37                     | 0.66                      | 0.29                   | 0.29                     | 0.16                      | 0.34                   | 0.34                  |
| Chronic diseases                   | 0.00                      | 0.39                   | 0.39                     | 0.00                      | 0.49                   | 0.49                     | 0.08                      | 0.35                   | 0.35                  |
| Objective social status            | 37.71                     |                        |                          | 42.43                     |                        |                          | 40.12                     |                        |                       |
| Lower status                       |                           | 0.50                   | 0.39                     |                           | 0.33                   | 0.28                     |                           | 0.30                   | 0.31                  |
| Lower-middle status                |                           | 0.06                   | 0.14                     |                           | 0.27                   | 0.25                     |                           | 0.30                   | 0.23                  |
| Upper-middle status                |                           | 0.24                   | 0.24                     |                           | 0.21                   | 0.24                     |                           | 0.16                   | 0.23                  |
| Upper status                       |                           | 0.20                   | 0.23                     |                           | 0.18                   | 0.23                     |                           | 0.24                   | 0.23                  |
| Subjective social status           | 0.45                      |                        |                          | 1.19                      |                        |                          | 1.01                      |                        |                       |
| Lower status                       |                           | 0.36                   | 0.36                     |                           | 0.17                   | 0.17                     |                           | 0.29                   | 0.29                  |
| Lower-middle status                |                           | 0.19                   | 0.19                     |                           | 0.14                   | 0.14                     |                           | 0.15                   | 0.16                  |
| Upper-middle status                |                           | 0.30                   | 0.30                     |                           | 0.15                   | 0.15                     |                           | 0.29                   | 0.29                  |
| Upper status                       |                           | 0.14                   | 0.14                     |                           | 0.54                   | 0.54                     |                           | 0.26                   | 0.26                  |
| Age                                | 0.03                      | 51.06                  | 51.06                    | 0.00                      | 56.73                  | 56.73                    |                           | 50.07                  | 50.01                 |
| Marital status                     | 0.54                      |                        |                          | 0.04                      |                        |                          | 0.31                      |                        |                       |
| Married or cohabiting              |                           | 0.86                   | 0.86                     |                           | 0.78                   | 0.78                     |                           | 0.76                   | 0.76                  |
| Widowed, separated or divorced     |                           | 0.12                   | 0.12                     |                           | 0.13                   | 0.13                     |                           | 0.16                   | 0.15                  |
| Never married                      |                           | 0.02                   | 0.02                     |                           | 0.09                   | 0.09                     |                           | 0.09                   | 0.09                  |
| Community type                     | 0.00                      |                        |                          | 0.22                      |                        |                          | 0.62                      |                        |                       |
| A big city                         |                           | 0.18                   | 0.18                     |                           | 0.05                   | 0.05                     |                           | 0.28                   | 0.28                  |
| Suburbs or outskirts of a big city |                           | 0.07                   | 0.07                     |                           | 0.16                   | 0.16                     |                           | 0.27                   | 0.27                  |
| A town or a small city             |                           | 0.35                   | 0.36                     |                           | 0.45                   | 0.45                     |                           | 0.29                   | 0.30                  |
| Country                            |                           | 0.39                   | 0.39                     |                           | 0.35                   | 0.35                     |                           | 0.15                   | 0.15                  |
| Female                             | 0.00                      | 0.51                   | 0.51                     | 0.00                      | 0.54                   | 0.54                     | 0.00                      | 0.54                   | 0.53                  |

Table 1 (continued)

|                | China Jap                 |                        | Japan                 | South Korea               |                        |                          |                           |                        |                          |
|----------------|---------------------------|------------------------|-----------------------|---------------------------|------------------------|--------------------------|---------------------------|------------------------|--------------------------|
|                | Percentage<br>missing (%) | Mean before imputation | Mean after imputation | Percentage<br>missing (%) | Mean before imputation | Mean after<br>imputation | Percentage<br>missing (%) | Mean before imputation | Mean after<br>imputation |
| Have insurance | 3.08                      | 0.90                   | 0.90                  | 3.19                      | 0.97                   | 0.97                     | 0.78                      | 0.94                   | 0.94                     |
| N              |                           | 3307                   |                       |                           | 2260                   |                          |                           | 1281                   |                          |

### Regression Tables

- coefficients and p-values
  - are minimum
- often include SE or 95% CI
  - focus on readability
    - additional info in other, or supplementary, tables
- N should match descriptive table
  - naming convention too
- informative title
  - include notes to aid interpretation

| Variables                   | M1      | M2      | M3      | M4      | M5      | M6      | M7      | M8      | M9      | M1   |
|-----------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|------|
| Predisposing factors        |         |         |         |         |         |         |         |         |         |      |
| Black                       | 1.04    | 1.28*   | 0.92    | 0.62**  | 0.76    | 0.68*   | 0.70*   | 0.67*   | 0.69    | 0.60 |
| Female                      | 1.28*** | 1.27*** | 1.11*** | 1.11*** | 1.16*** | 1.17*** | 1.17*** | 1.17*** | 1.17*** | 1.16 |
| Age                         | 1.01*** | 1.01*** | 0.99*** | 0.99*   | 0.99**  | 0.99*   | 0.99**  | 0.99**  | 0.99**  | 0.99 |
| Age × Black                 |         | 0.99    | 1.00    | 0.99    | 0.99*   | 0.99    | 0.99    | 0.99    | 0.99    | 1.00 |
| Less than HS                | 1.05    | 1.05    | 0.83*** | 0.83*** | 0.93**  | 0.92**  | 0.92**  | 0.92**  | 0.93*   | 0.94 |
| Need factors                |         |         |         |         |         |         |         |         |         |      |
| SRH                         |         |         | 0.77*** | 0.75*** | 0.74*** | 0.75*** | 0.74*** | 0.74*** | 0.74*** | 0.74 |
| Functional Limit.           |         |         | 1.19*** | 1.18*** | 1.18*** | 1.17*** | 1.17*** | 1.17*** | 1.18*** | 1.18 |
| Health Conditions           |         |         | 1.18*** | 1.15*** | 1.14*** | 1.13*** | 1.13*** | 1.13*** | 1.13*** | 1.13 |
| SRH × Black                 |         |         |         | 1.10*** | 1.07*   | 1.07*   | 1.06*   | 1.06*   | 1.06*   | 1.04 |
| Func. Lim. × Black          |         |         |         | 1.02    | 1.03    | 1.04    | 1.03    | 1.03    | 1.02    | 1.03 |
| Health Con. x Black         |         |         |         | 1.10*** | 1.10*** | 1.10*** | 1.10*** | 1.10*** | 1.10*** | 1.10 |
| Enabling factors            |         |         |         |         |         |         |         |         |         |      |
| Employed                    |         |         |         |         | 1.01    | 1.02    | 1.02    | 1.02    | 1.03    | 1.02 |
| Income                      |         |         |         |         | 1.04*** | 1.04*** | 1.04*** | 1.04*** | 1.04*** | 1.04 |
| Insured                     |         |         |         |         | 2.08*** | 2.09*** | 2.05*** | 2.05*** | 2.02*** | 1.95 |
| Married                     |         |         |         |         | 0.94*   | 0.95*   | 0.94*   | 0.94*   | 0.96    | 0.96 |
| Rural                       |         |         |         |         | 0.89*** | 0.89*** | 0.89*** | 0.89*** | 0.91*** | 0.91 |
| ED Use                      |         |         |         |         |         | 1.21*** | 1.21*** | 1.21*** | 1.21*** | 1.21 |
| Trust in physicians         |         |         |         |         |         |         |         |         |         |      |
| Mistrust                    |         |         |         |         |         |         | 0.93*** | 0.93*** | 0.93*** | 0.93 |
| Mistrust × Black            |         |         |         |         |         |         |         | 1.02    | 1.00    | 0.97 |
| Life course factors         |         |         |         |         |         |         |         |         |         |      |
| South-Never                 |         |         |         |         |         |         |         |         | 1.15*** | 1.07 |
| South-After 16              |         |         |         |         |         |         |         |         | 1.39*** | 1.17 |
| South-Left After 16         |         |         |         |         |         |         |         |         | 0.97    | 0.87 |
| South-Never × Black         |         |         |         |         |         |         |         |         |         | 1.25 |
| South-After 16 × Black      |         |         |         |         |         |         |         |         |         | 2.25 |
| South-Left After 16 × Black | k       |         |         |         |         |         |         |         |         | 1.26 |

### Today's class...

- Focus on big picture
  - But stop me if have question: I may move on if in weeds
- Some probability background to justify
- GLM vs LRM: model choice depends on measurement of Y
- Different GLM models (Link functions)
  - Just need to know what model for what outcome
- MLE vs OLS: just different ways under hood to fit models
- GLM model fit
- GLM interpretation of results
- Play with my examples afterwards, then mess with your data.
  - More applied next week: logit

### Generalized linear models (GLM)

- accommodate many types of outcomes
  - binary, ordinal, nominal, count

- by computing most likely values of  $\beta$  given the observed data
  - less restrictive than linear regression model (LRM)
    - can fit nonlinear models w/o special transformations of Y
- usually by using Maximum Likelihood Estimation (MLE)
  - rather than minimizing the sum of square residuals (OLS)

### Probability: basics

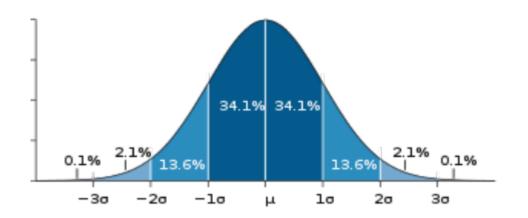
- Continuous outcomes (normal distribution)
  - closer to mean → greater likelihood to observe
- Peak of the function (mean of the distribution) most likely observed value

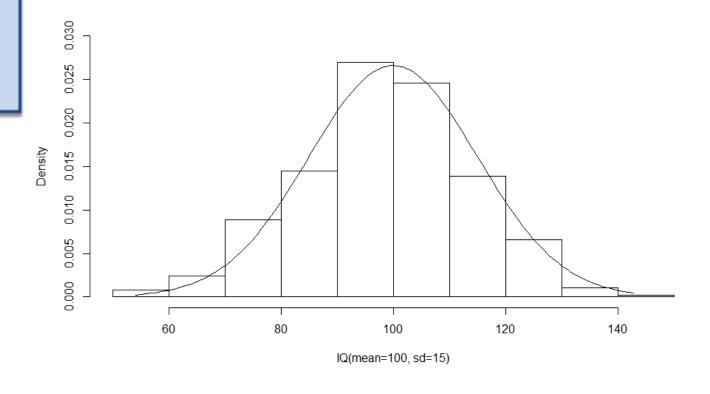
- IQ distributed normally, with mean 100 and SD 15
  - sample random individual
- What is more likely:
  - IQ = 100 or IQ = 80?

#### The 68-95-99.7 Rule

In the Normal distribution with mean  $\mu$  and standard deviation  $\sigma$ :

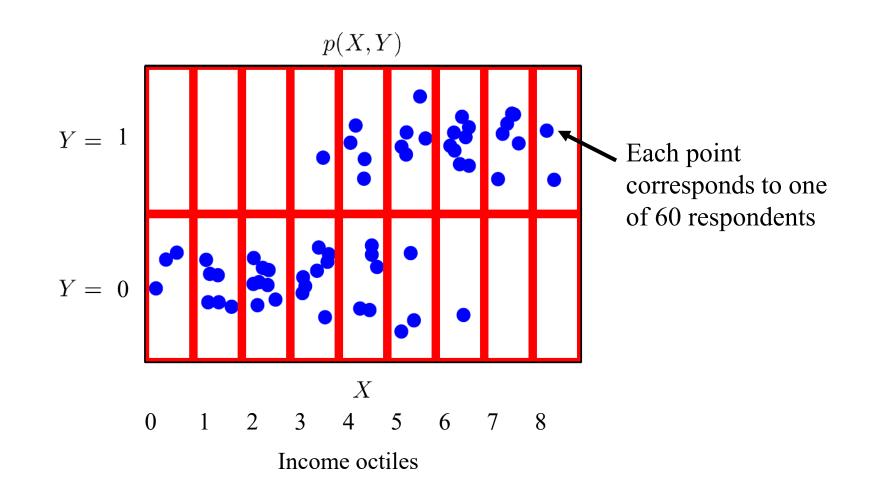
- Approximately **68%** of the observations fall within  $\sigma$  of  $\mu$ .
- Approximately **95%** of the observations fall within  $2\sigma$  of  $\mu$ .
- Approximately 99.7% of the observations fall within 3σ of μ.





- Let's suppose that:
  - We don't know the mean
  - We pick 2 random observations: IQ=[80,100]
  - We assume IQ is normally distributed
- What's best guess about value of the mean?
- MLE designed to address this issue
  - we know the data, we assume a distribution, and we estimate the parameters

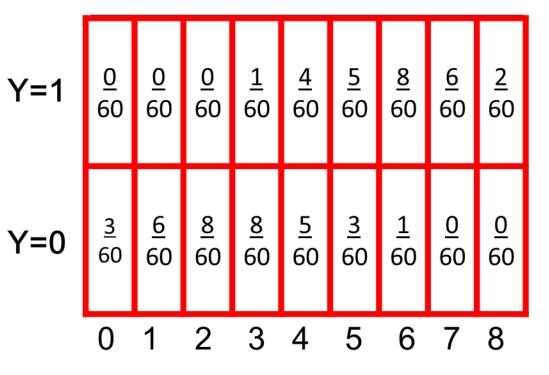
## Why GLM: ask 60 ppl. if happy (1) or not (0)



Happy (0=no; 1=yes)

## Probability: happy (0,1)

- P(X=i,Y=j): probability (relative frequency) of observing
   X = i and Y = j
- Properties:  $0 \le P \le 1$ ,  $\sum P = 1$

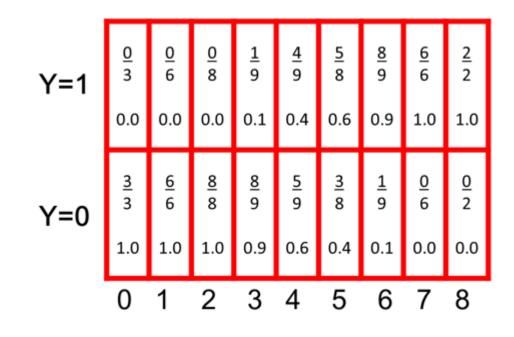


### Conditional probability: happy by income

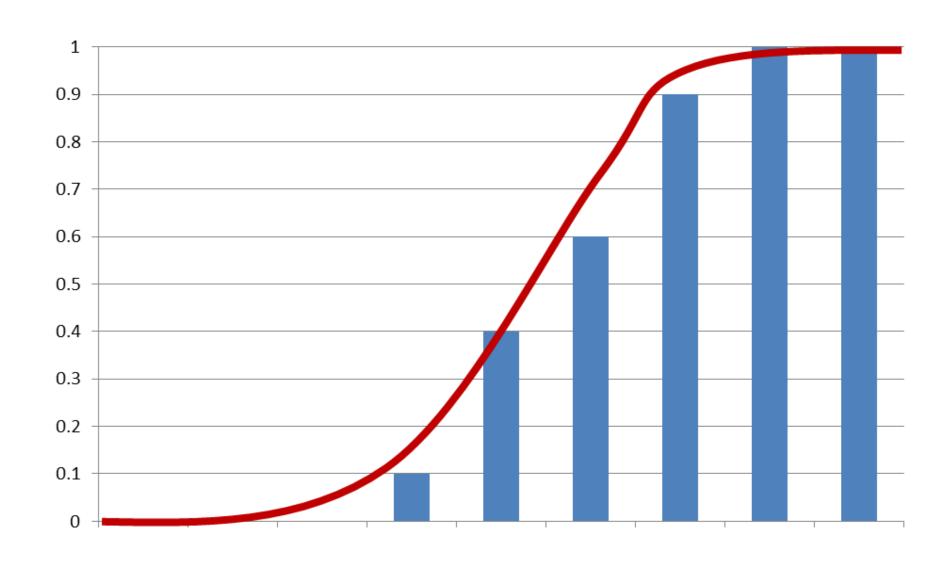
• Conditional probability P(Y=j|X=i)

• What is the probability of being happy if income is 4?

 General pattern: Probability of being happy increases as income increases



### P of being happy conditional on income



### Conditional probability

- Becomes extremely complex with multiple covariates (just like LRM),
   but still trying to describe effects of X on Y
  - Now, it's the probability of Y given X expressed in terms of odds

- Interpretation is relative, so odds are somewhat meaningless
  - > rely heavily on postestimation techniques
- Unique models for certain outcomes

| Outcome | Model                                    |                 |
|---------|--|-----------------|
| Binary  | Logit and probit                         | → Next week     |
| Ordinal | Ordered logit and probit                 | <b>←</b> FEB 26 |
| Nominal | Multinomial logit                        | <b>←</b> MAR 18 |
| Count   | Poisson and negative binomial regression | <b>←</b> MAR 25 |

#### LRM vs GLM

- LRM: the expected value of Y: or E(Y)
- can also be expressed as the conditional mean: or  $\mu$
- which equals the sum of the intercept  $\beta_0$ , the reg. coefs.  $\beta_{1...i}$ , and the respective independent variables  $X_{1...i}$
- Condensed using linear predictor eta: η

$$\eta = \sum_{k=1}^{K} \beta_k X_K$$

- When conditional mean makes no sense
  - Y is not a continuous normally distributed variable
- use GLM to "link"  $\eta$  and  $\mu$

### LRM vs GLM: example

- Now we are going to work with categorical outcomes
  - binary, ordinal, nominal, count
- The first outcomes we will work with are binary
  - only two categories; dichotomous (0, 1)
- We can operationalize ordinal happiness measure as dichotomy by collapsing the three response categories into two
  - happy (1) vs. unhappy (0)
- Let's first examine this binary measure using LRM with OLS

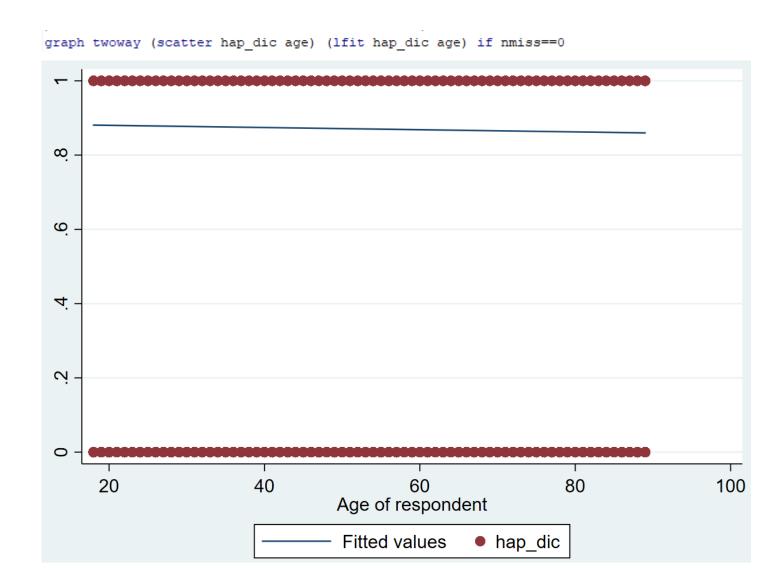
- Stata will give you results
- Hey, look! They're even significant!!!
- But what do they mean...
  - Why NOT use binary in Sobel mediation

reg hap dic age if nmiss==0

|   | Source            | SS                       | df          | MS              |                  | er of obs          | =   | 59,725<br>14.08     |
|---|-------------------|--------------------------|-------------|-----------------|------------------|--------------------|-----|---------------------|
|   | Model<br>Residual | 1.56594153<br>6641.76858 | 1<br>59,723 | 1.5659415       | 3 Prol<br>6 R-sc | > F<br>quared      | =   | 0.0002              |
| - | Total             | 6643.33452               | 59,724      | .11123391       | _                | R-squared<br>: MSE | =   | 0.0002<br>.33348    |
|   | hap_dic           | Coef.                    | Std. Err.   | t               | P> t             | [95% Co            | nf. | Interval]           |
| _ | age<br>_cons      | 0002914<br>.8859318      | .0000776    | -3.75<br>231.51 | 0.000<br>0.000   | 000443<br>.878431  |     | 0001392<br>.8934324 |

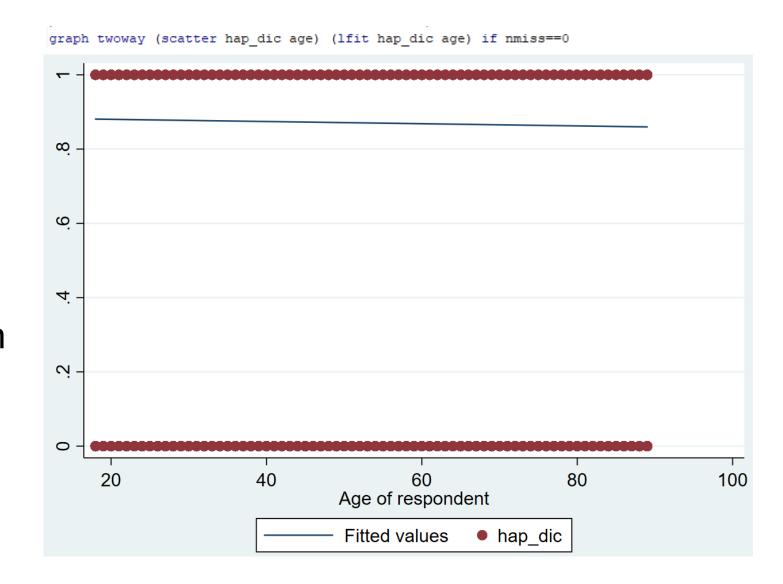
Interpret the slope

 What is the predicted value of happiness at age \_\_\_\_?

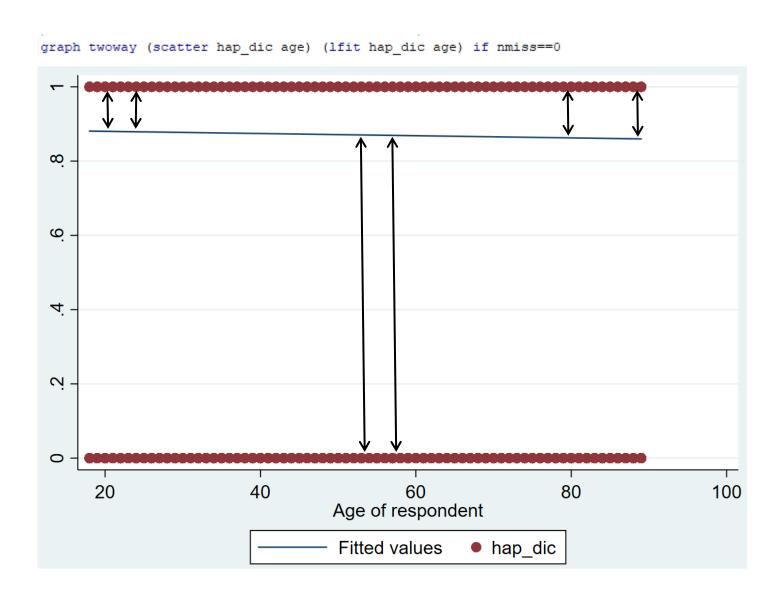


Linearity

 Is the linear functional form reasonable?

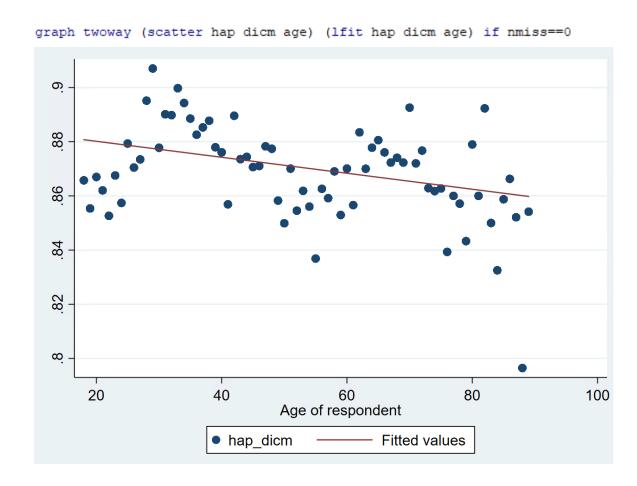


- Heteroscedasticity
- Is the error variance constant?
  - distance between observed and fitted values



- What do these results tell us?
  - linear slope of age between mean values of happiness

- What are mean values of hap dic?
  - percent happy at each year of age
- The identity function isn't appropriate for modeling dichotomous outcomes
  - need to use another function



### **GLM**

- Not required to understand how link functions work under the hood
- Know which model is appropriate for different types of outcomes
  - and why
- Be able to use these models, diagnose fit, and interpret outcomes

### Binary outcomes

- How can  $\eta$  link to  $\mu$ ?
- 0,1 implies a binomial distribution
- Thus, we can use a logit link:  $\eta = \log_e \left[ \frac{\mu}{1 \mu} \right]$ 
  - logistic model
- Or, an inverse normal link:  $\eta = \Phi^{-1}(\mu)$ 
  - probit model

#### Ordered outcomes

- The logit and inverse normal link also used for ordered outcomes
  - ordered logistic and ordered probit models
- Consider the original GSS happy measure
  - (0) not too happy, (1) pretty happy, (2) very happy
- As you'll see, these variables have "cut-points"
  - we have to make assumptions about

#### Nominal outcomes

- When there are more than two categories but NO meaningful order
  - need to account for unordered nature

- Thus, need to use another link function
- Generalized logit link:  $\eta_j = \log_e \left(\frac{\mu_j}{\mu_J}\right)$ 
  - Multinomial Logistic Regression

#### Count outcomes

- Count measures can not fall below zero
  - unlike continuous measures

- e.g., the number of children one expects to have
- Thus, need to use another link function
- Natural logarithm link:  $\eta = \log_e \mu$ 
  - Poisson and negative binomial models

### **GLM**

- Not required to understand how link functions work under the hood
- Know which model is appropriate for different types of outcomes
  - and why
- Be able to use these models, diagnose fit, and interpret outcomes

### Modeling technique

- Typically uses maximum likelihood estimation (MLE)
- Finds unknown parameters that make observed combination of  $X^s$  and  $Y^s$  most likely to occur
- Calculates how likely a set of outcome values are if a set of parameter estimates were true
- Keeps doing this (iterations) until the maximum of the likelihood function is found (convergence)
  - luckily, Stata does most of the work

#### MLE in Stata

The iteration process is shown at the top of the Stata log

```
logit hap_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0

Iteration 0: log likelihood = -22789.647

Iteration 1: log likelihood = -21556.708

Iteration 2: log likelihood = -21477.531

Iteration 3: log likelihood = -21477.296

Iteration 4: log likelihood = -21477.296
```

Convergence was reached in fourth iteration

#### MLE in Stata: factor notation

- Thus far, not consistent with factor notation
  - just getting familiar with it
- Now, need to be more consistent
  - post estimation techniques rely on it
- As you'll see, we'll be using a lot of post estimation
- This can change variable name
  - use coeflegend

#### logit, coeflegend

| hap_dic                | Coef.                | Legend                       |
|------------------------|----------------------|------------------------------|
| age                    | 0413751              | _b[age]                      |
| c.age#c.age            | .0004016             | _b[c.age#c.age]              |
| 1.female<br>1.nonwhite |                      | _b[l.female] _b[l.nonwhite]  |
| educat<br>0<br>2       |                      | _b[0.educat]<br>_b[2.educat] |
| 1.married<br>_cons     | 1.026049<br>2.415824 | _b[l.married]<br>_b[_cons]   |

#### MLE model fit

- Likelihood Ratio (LR) test: constrained model [intercept only ( $Y = \alpha$ )] vs. unconstrained model [includes all independent variables]
  - Basically: do the independent variables "explain" Y better than nothing
    - more useful for comparing nested models
- McFadden (pseudo) adjusted R<sup>2</sup>
  - not a very interpretable, or agreed upon, statistic
- Information measures: comparing models
  - Akaike's Information Criterion (AIC): smaller is better
  - Bayesian Information Criterion (BIC): smaller is better

#### MLE in Stata: model fit

- Save fit statistics and compare to other models
  - Which is the better fitting model?

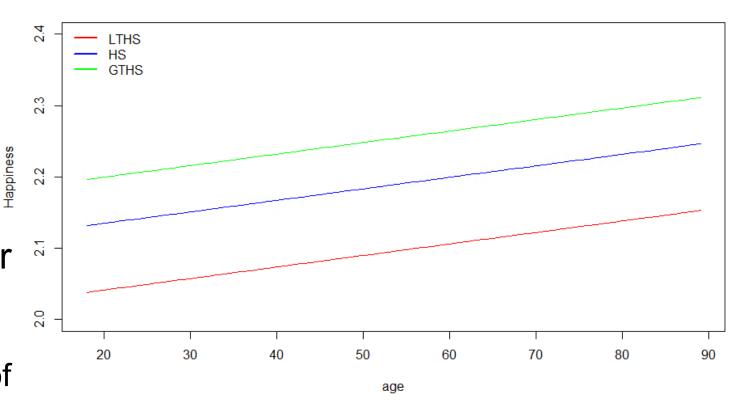
logit hap\_dic c.age##c.age i.female i.nonwhite i.married if nmiss==0
fitstat, saving(noedu)
logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
fitstat, using(noedu)

|                        | Current    | Saved      | Difference |
|------------------------|------------|------------|------------|
| Log-likelihood         |            |            |            |
| Model                  | -21477.296 | -21780.858 | 303.562    |
| Intercept-only         | -22789.647 | -22789.647 | 0.000      |
| Chi-square             |            |            |            |
| D(df=59717/59719/-2)   | 42954.591  | 43561.715  | -607.124   |
| LR(df=7/5/2)           | 2624.702   | 2017.578   | 607.124    |
| p-value                | 0.000      | 0.000      | 0.000      |
| R2                     |            |            |            |
| McFadden               | 0.058      | 0.044      | 0.013      |
| McFadden(adjusted)     | 0.057      | 0.044      | 0.013      |
| McKelvey & Zavoina     | 0.109      | 0.086      | 0.023      |
| Cox-Snell/ML           | 0.043      | 0.033      | 0.010      |
| Cragg-Uhler/Nagelkerke | 0.081      | 0.062      | 0.018      |
| Efron                  | 0.045      | 0.034      | 0.011      |
| Tjur's D               | 0.046      | 0.035      | 0.011      |
| Count                  | 0.873      | 0.873      | 0.000      |
| Count (adjusted)       | 0.000      | 0.000      | 0.000      |
| IC                     |            |            |            |
| AIC                    | 42970.591  | 43573.715  | -603.124   |
| AIC divided by N       | 0.719      | 0.730      | -0.010     |
| BIC(df=8/6/2)          | 43042.571  | 43627.700  | -585.129   |
| Variance of            |            |            |            |
| e                      | 3.290      | 3.290      | 0.000      |
| y-star                 | 3.692      | 3.599      | 0.092      |
|                        |            |            |            |

Note: Likelihood-ratio test assumes saved model nested in current model

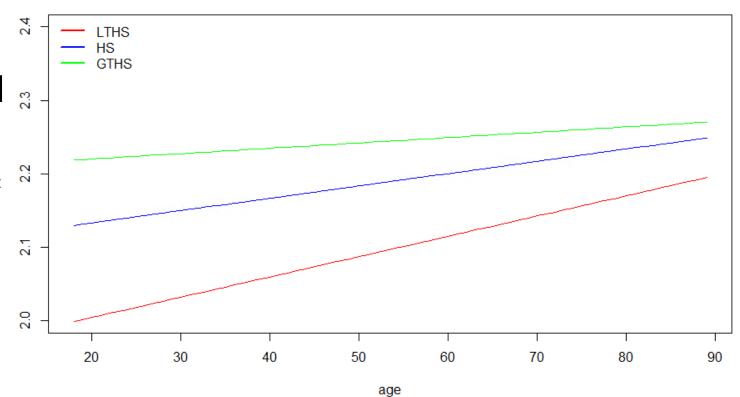
### LRM vs GLM interpretation

- Recall our happiness-age by education example
- $B_{age} = \Delta$  in happiness for every one-year  $\Delta$  in age
  - holding education constant
- Linear interpretation no longer applies with GLM
  - because the effect of the  $\Delta$  in one X depends on the values of all other  $X^s$



### OLS vs GLM interpretation

- This is sort of like what we did when we introduced age\*edu interaction terms
  - making model non-linear



- Estimated parameters typically don't make much sense
  - used to make predictions at meaningful values of the Xs via postestimation
- Certain techniques are more useful to address specific issues
  - that's why we'll learn many different techniques
- Some of this may not make sense right now that's okay
  - let's become familiar with the techniques and Stata programming first

- Predictions for each observation
  - a good starting point

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
predict prob if nmiss==0
summarize prob

| Variable | Obs    | Mean     | Std. Dev. | Min      | Max     |
|----------|--------|----------|-----------|----------|---------|
| prob     | 59,725 | .8725157 | .0722409  | .6214635 | .967612 |

- Mean probability of happy is 0.87
  - range: 0.62 to 0.97

- Try this with a different combination of IVs
  - Same or different? Why?

- Predictions at specified values: substantively interesting combinations of X values → profiles or ideal types
  - e.g., the average respondent (ALL Xs set at mean)

```
logit hap dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0 margins, atmeans
```

```
: Pr(hap dic), predict()
: age
                         46.04745 (mean)
  0.female
                         .4422101 (mean)
  1.female
                         .5577899 (mean)
  0.nonwhite
                         .8068648 (mean)
  1.nonwhite
                         .1931352 (mean)
  0.educat
                         .2285308 (mean)
  1.educat
                         .3058853 (mean)
  2.educat
                         .4655839 (mean)
  0.married
                         .4702721 (mean)
  1.married
                         .5297279 (mean)
```

|       | 1        | Delta-method |        |       |            |                      |
|-------|----------|--------------|--------|-------|------------|----------------------|
|       | Margin   | Std. Err.    | z      | P> z  | [95% Conf. | <pre>Interval]</pre> |
| _cons | .8751479 | .0019592     | 446.69 | 0.000 | .871308    | .8789879             |

 Probability of being happy vs. not happy is 0.875 (CI 0.871-0.879), on average

- Often doesn't make too much sense
  - e.g., what's 0.55 female?

- Profiles or ideal types: set Xs at meaningful values
  - factor notation makes this easy when including interactions or polynomials

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
margins, at(age=30 female=1 nonwhite=0 educat=2 married=1)

|       | 1         | Delta-method |        |       |            |           |
|-------|-----------|--------------|--------|-------|------------|-----------|
|       | Margin    | Std. Err.    | z      | P> z  | [95% Conf. | Interval] |
| _cons | . 9534082 | .0015001     | 635.55 | 0.000 | .9504679   | . 9563484 |

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
margins, at(age=30 female=1 nonwhite=0 educat=0 married=1)

|       | Delta-method |           |        |       |            |           |  |
|-------|--------------|-----------|--------|-------|------------|-----------|--|
|       | Margin       | Std. Err. | z      | P> z  | [95% Conf. | Interval] |  |
| _cons | .9041268     | .0031965  | 282.85 | 0.000 | .8978617   | .9103918  |  |

- Probability of being happy vs. not happy is 0.953 (CI 0.950-0.956) for age 30, female, white, college educated, and married
- What if we change education to <HS?</li>
  - Whose more likely to be happy?

- Profiles or ideal types: set Xs at meaningful values
  - can simplify syntax for hypothetical comparisons

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
margins, at(age=30 female=1 nonwhite=0 educat=0 educat=1 educat=2 married=1)

|    | Delta-method |           |        |       |            |           |  |
|----|--------------|-----------|--------|-------|------------|-----------|--|
|    | Margin       | Std. Err. | z      | P> z  | [95% Conf. | Interval] |  |
| at |              |           |        |       |            |           |  |
| 1  | .9041268     | .0031965  | 282.85 | 0.000 | .8978617   | .9103918  |  |
| 2  | . 9343698    | .0020958  | 445.83 | 0.000 | .9302621   | .9384776  |  |
| 3  | .9534082     | .0015001  | 635.55 | 0.000 | .9504679   | .9563484  |  |

- Profiles or ideal types: set Xs at meaningful values
  - things can get messy quick, so make sure you know what you're looking at

mlistat

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
margins, at(age=(20(10)90) female=1 nonwhite=0 educat=0 educat=1 educat=2 ///
married=1) noatlegend

|     | 1         | Delta-method | l      |        |            |                      |
|-----|-----------|--------------|--------|--------|------------|----------------------|
|     | Margin    | Std. Err.    | z      | P>   z | [95% Conf. | <pre>Interval]</pre> |
|     |           |              |        |        |            |                      |
| _at |           |              |        |        |            |                      |
| 1   | .9210645  | .0034122     | 269.93 | 0.000  | .9143767   | .9277524             |
| 2   | .9462821  | .0022886     | 413.48 | 0.000  | .9417965   | .9507676             |
| 3   | .9620053  | .001652      | 582.33 | 0.000  | .9587675   | .9652432             |
| 4   | .9041268  | .0031965     | 282.85 | 0.000  | .8978617   | .9103918             |
| 5   | . 9343698 | .0020958     | 445.83 | 0.000  | .9302621   | .9384776             |
| 6   | . 9534082 | .0015001     | 635.55 | 0.000  | .9504679   | .9563484             |
| 7   | .891997   | .0033388     | 267.16 | 0.000  | .8854531   | .8985409             |
| 8   | . 9257522 | .0022075     | 419.36 | 0.000  | .9214255   | .9300789             |
| 9   | .9471489  | .0015711     | 602.87 | 0.000  | .9440697   | .9502282             |
| 10  | .8868522  | .0034209     | 259.24 | 0.000  | .8801473   | .893557              |
| 11  | .9220748  | .0023318     | 395.44 | 0.000  | .9175046   | .926645              |
| 12  | .9444678  | .0016687     | 566.00 | 0.000  | .9411972   | .9477383             |
| 13  | .8896312  | .0032111     | 277.05 | 0.000  | .8833376   | .8959249             |
| 14  | .9240628  | .0022839     | 404.60 | 0.000  | .9195865   | .9285391             |
| 15  | .945918   | .0016564     | 571.05 | 0.000  | .9426714   | .9491646             |
| 16  | .8998262  | .0030433     | 295.67 | 0.000  | .8938614   | .9057909             |
| 17  | .9313228  | .0022766     | 409.09 | 0.000  | .9268608   | .9357848             |
| 18  | .9511988  | .0016804     | 566.05 | 0.000  | .9479053   | .9544924             |
| 19  | .9155934  | .0034393     | 266.22 | 0.000  | .9088525   | .9223343             |
| 20  | .9424496  | .0025916     | 363.66 | 0.000  | .9373702   | .947529              |
| 21  | .9592463  | .0019174     | 500.29 | 0.000  | .9554884   | .9630043             |
| 22  | .9341881  | .0041649     | 224.30 | 0.000  | .926025    | .9423512             |
| 23  | .955416   | .0030437     | 313.90 | 0.000  | .9494505   | .9613814             |
| 24  | .9685546  | .0022238     | 435.53 | 0.000  | .9641959   | .9729133             |
|     |           |              |        |        |            |                      |

Interpretation?

- Margins is a base Stata command for postestimation
  - mtable is a powerful user-created package that simplifies margins

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
mtable, at(age=(20(10)90) female=1 nonwhite=0 educat=0 educat=1 educat=2 ///
married=1) statistics(ci)

|    | age | educat | Pr(y) | 11    | ul    |
|----|-----|--------|-------|-------|-------|
| 1  | 20  | 0      | 0.921 | 0.914 | 0.928 |
| 2  | 20  | 1      | 0.946 | 0.942 | 0.951 |
| 3  | 20  | 2      | 0.962 | 0.959 | 0.965 |
| 4  | 30  | 0      | 0.904 | 0.898 | 0.910 |
| 5  | 30  | 1      | 0.934 | 0.930 | 0.938 |
| 6  | 30  | 2      | 0.953 | 0.950 | 0.956 |
| 7  | 40  | 0      | 0.892 | 0.885 | 0.899 |
| 8  | 40  | 1      | 0.926 | 0.921 | 0.930 |
| 9  | 40  | 2      | 0.947 | 0.944 | 0.950 |
| 10 | 50  | 0      | 0.887 | 0.880 | 0.894 |
| 11 | 50  | 1      | 0.922 | 0.918 | 0.927 |
| 12 | 50  | 2      | 0.944 | 0.941 | 0.948 |
| 13 | 60  | 0      | 0.890 | 0.883 | 0.896 |
| 14 | 60  | 1      | 0.924 | 0.920 | 0.929 |
| 15 | 60  | 2      | 0.946 | 0.943 | 0.949 |
| 16 | 70  | 0      | 0.900 | 0.894 | 0.906 |
| 17 | 70  | 1      | 0.931 | 0.927 | 0.936 |
| 18 | 70  | 2      | 0.951 | 0.948 | 0.954 |
| 19 | 80  | 0      | 0.916 | 0.909 | 0.922 |
| 20 | 80  | 1      | 0.942 | 0.937 | 0.948 |
| 21 | 80  | 2      | 0.959 | 0.955 | 0.963 |
| 22 | 90  | 0      | 0.934 | 0.926 | 0.942 |
| 23 | 90  | 1      | 0.955 | 0.949 | 0.961 |
| 24 | 90  | 2      | 0.969 | 0.964 | 0.973 |

- Same results as previous slide
- A little more condensed
  - easier to interpret
- mtable is also useful when dealing with categorical and count outcomes
  - as you'll see things get more complex when Y is not dichotomous

 Marginal effects: how changes in one variable are associated with changes in the outcomes, holding all else constant

logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
margins, dydx(\*)

|            | dy/d <b>x</b> | Delta-method<br>Std. Err. | [95% Conf. | . Interval] |          |          |
|------------|---------------|---------------------------|------------|-------------|----------|----------|
|            |               |                           |            |             |          |          |
| age        | 000418        | .0000691                  | -6.05      | 0.000       | 0005533  | 0002826  |
| 1.female   | .0099826      | .0027294                  | 3.66       | 0.000       | .0046331 | .0153322 |
| 1.nonwhite | 0511805       | .0036188                  | -14.14     | 0.000       | 0582732  | 0440878  |
| educat     |               |                           |            |             |          |          |
| 0          | 0521551       | .0041551                  | -12.55     | 0.000       | 0602989  | 0440113  |
| 2          | .0352867      | .0030513                  | 11.56      | 0.000       | .0293062 | .0412671 |
| 1.married  | .1085001      | .0028608                  | 37.93      | 0.000       | .102893  | .1141071 |

 On average, the probability of happiness decreases by 0.052 moving from a HS education to <HS education, holding all else constant</li>

- Marginal effects: can get the same with mchange
  - will be more useful as we progress

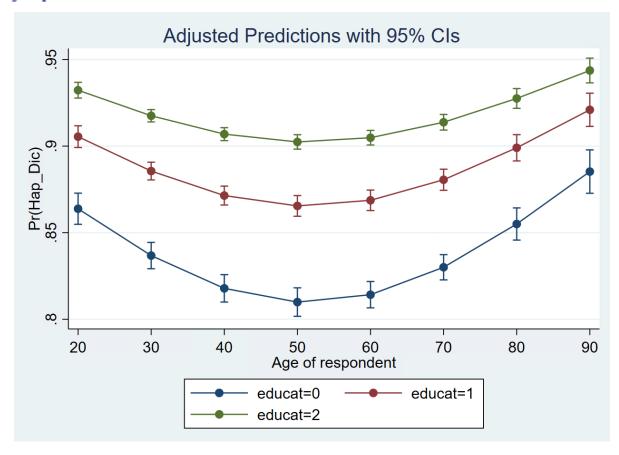
logit hap\_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
mchange, statistics(ci)

|          | Change | LL     | UL     |
|----------|--------|--------|--------|
| age      |        |        |        |
| +1       | -0.000 | -0.001 | -0.000 |
| +SD      | 0.003  | 0.001  | 0.006  |
| Marginal | -0.000 | -0.001 | -0.000 |
| female   |        |        |        |
| 1 vs 0   | 0.010  | 0.005  | 0.015  |
| nonwhite |        |        |        |
| 1 vs 0   | -0.051 | -0.058 | -0.044 |
| educat   |        |        |        |
| 1 vs 0   | 0.052  | 0.044  | 0.060  |
| 2 vs 0   | 0.087  | 0.080  | 0.095  |
| 2 vs 1   | 0.035  | 0.029  | 0.041  |
| married  |        |        |        |
| 1 vs 0   | 0.109  | 0.103  | 0.114  |

Can anyone figure out how to display the estimates for age?

Plotting probabilities useful for continuous Xs

```
logit hap_dic c.age##c.age i.female i.nonwhite ibl.educat i.married if nmiss==0
margins, at(age=(20(10)90) educat=(0 1 2)) atmeans
marginsplot
```



### Next Monday we will...

discuss logit and probit models (binary outcomes)

Read Hoffman CH3 and Long & Freese CH 5 & 6 before class