二.必做题（共1 题,33.3分)

1.

Oppenheim 课本，第二章课后习题，2.17，2.33，2.36，2.38，2.45，2.46

Oppenheim 课本，第三章课后习题，3.3，3.6，3.9，3.13，3.21，3.25

答案:

答案解析:

难度: 易

知识点:

三.选做题（共1 题,33.4分)

1.

下列题目中任选至少 3 道完成

Oppenheim 课本，第二章课后习题，2.56，2.60，2.65，2.84，2.85，2.86

下列题目中任选至少 3 道完成

Oppenheim 课本，第三章课后习题，3.22，3.42，3.49，3.52，3.56，3.57

2.17. **(a)** Determine the Fourier transform of the sequence

$$r[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right], & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

Sketch $w[n]$ and express $W(e^{j\omega})$, the Fourier transform of $w[n]$, in terms of $R(e^{j\omega})$, the Fourier transform of $r[n]$. (*Hint*: First express $w[n]$ in terms of $r[n]$ and the complex exponentials $e^{j(2\pi n/M)}$ and $e^{-j(2\pi n/M)}$.)

(c) Sketch the magnitude of $R(e^{j\omega})$ and $W(e^{j\omega})$ for the case when $M = 4$.

[解]

(a).

$$r[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & otherwise \end{cases}$$

$$DTFT\{r[n]\} = \sum_{n=-\infty}^{+\infty} r[n]e^{-j\omega n} = \sum_{n=0}^{M} e^{-j\omega n} = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$$

$$= \frac{\left(e^{+j\frac{\omega}{2}(M+1)} - e^{-j\frac{\omega}{2}(M+1)}\right)e^{-j\frac{\omega}{2}(M+1)}}{\left(e^{+j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)e^{-j\frac{\omega}{2}}} = e^{-j\frac{\omega}{2}M}\frac{\sin\left(\frac{\omega}{2}(M+1)\right)}{\sin\left(\frac{\omega}{2}\right)}$$

(b).

根据题目给出的 Hint，我们了解需要从(a)中找寻思路：

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right] & 0 \le n \le M \\ 0 & otherwise \end{cases}$$

$$DTFT\{w[n]\} = \frac{1}{2}DTFT\{r[n]\} - \frac{1}{2}\sum_{n=-\infty}^{+\infty}\cos\left(\frac{2\pi n}{M}\right)e^{-j\omega n}$$

$$\cos\left(\frac{2\pi n}{M}\right) = \frac{1}{2}\left(e^{+j\frac{2\pi}{M}n} + e^{-j\frac{2\pi}{M}n}\right)$$

这里记$R(e^{j\omega}) = DTFT\{r[n]\}$　　$DTFT\{w[n]\} = W(e^{j\omega})$

$$\frac{1}{2}\sum_{n=-\infty}^{+\infty}\cos\left(\frac{2\pi n}{M}\right)e^{-j\omega n} = \frac{1}{4}R\left(e^{j\left(\omega-\frac{2\pi}{M}\right)}\right) + \frac{1}{4}R\left(e^{j\left(\omega+\frac{2\pi}{M}\right)}\right)$$

$$W(e^{j\omega}) = \frac{1}{2}R(e^{j\omega}) - \frac{1}{4}R\left(e^{j\left(\omega-\frac{2\pi}{M}\right)}\right) - \frac{1}{4}R\left(e^{j\left(\omega+\frac{2\pi}{M}\right)}\right)$$

其中$R(e^{j\omega}) = e^{-j\frac{\omega}{2}M}\frac{\sin\left(\frac{\omega}{2}(M+1)\right)}{\sin\left(\frac{\omega}{2}\right)}$

无论是考试还是作业，表达到这个形式就很够了，不需要化简。

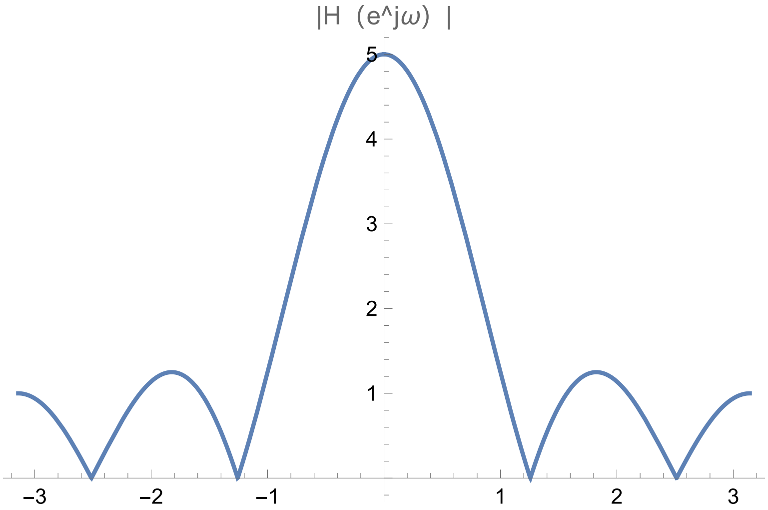
(c).

$$M = 4$$

我们只需要做幅频特性的图即可(Magnitude)

$$R(e^{j\omega}) = e^{-j\frac{\omega}{2}M}\frac{\sin\left(\frac{\omega}{2}(M+1)\right)}{\sin\left(\frac{\omega}{2}\right)} = \left|R(e^{j\omega})\right|e^{+j\angle R(e^{j\omega})} = \left|\frac{\sin\left(\frac{\omega}{2}(M+1)\right)}{\sin\left(\frac{\omega}{2}\right)}\right|e^{+j\left(-\frac{\omega}{2}M\right)}$$

$$R(e^{j\omega}) = \left|\frac{\sin\left(\frac{5\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right|e^{+j(-2\omega)}$$



$$W(e^{j\omega}) = W(e^{j\omega}) = \frac{1}{2}R(e^{j\omega}) - \frac{1}{4}R\left(e^{j\left(\omega-\frac{\pi}{2}\right)}\right) - \frac{1}{4}R\left(e^{j\left(\omega+\frac{\pi}{2}\right)}\right)$$

频域左右平移$\frac{\pi}{2}$后取幅频特性即可，图省略。

2.33. Consider an LTI system defined by the difference equation

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2].$$

(a) Determine the impulse response of this system.

(b) Determine the frequency response of this system. Express your answer in the form

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega n_d},$$

where $A(e^{j\omega})$ is a real function of ω. Explicitly specify $A(e^{j\omega})$ and the delay n_d of this system.

(c) Sketch a plot of the magnitude $|H(e^{j\omega})|$ and a plot of the phase $\angle H(e^{j\omega})$.

(d) Suppose that the input to the system is

$$x_1[n] = 1 + e^{j0.5\pi n} \qquad -\infty < n < \infty.$$

Use the frequency response function to determine the corresponding output $y_1[n]$.

(e) Now suppose that the input to the system is

$$x_2[n] = (1 + e^{j0.5\pi n})u[n] \qquad -\infty < n < \infty.$$

Use the defining difference equation or discrete convolution to determine the corresponding output $y_2[n]$ for $-\infty < n < \infty$. Compare $y_1[n]$ and $y_2[n]$. They should be equal for certain values of n. Over what range of values of n are they equal?

[解]:

(a).

这里我们需要求取所谓的"impulse response"，根据定义我们只需要令:

$$x[n] = \delta[n]$$

$$y[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$$

$$h[n] = y[n]|_{x[n]=\delta[n]} = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$$

(b).

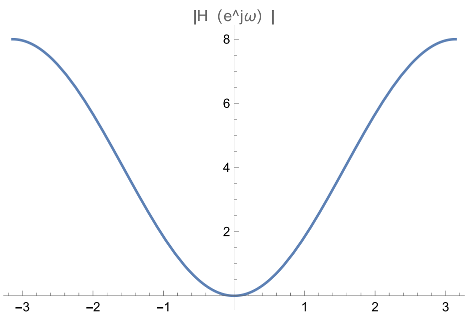
$$H(e^{j\omega}) = DTFT\{-2\delta[n] + 4\delta[n-1] - 2\delta[n-2]\} = -2 + 4e^{-j\omega} - 2e^{-j2\omega}$$

$$= 2e^{-j\omega}[2 - (e^{+j\omega} + e^{-j\omega})] = 2e^{-j\omega}(2 - 2cos\omega) = 4e^{-j\omega}(1 - cos\omega)$$

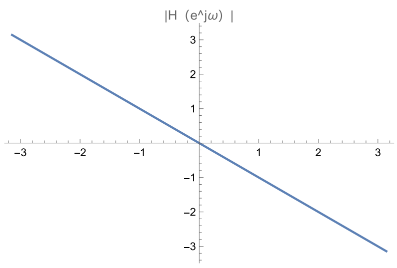
$$H(e^{j\omega}) = 4(1 - cos\omega)e^{-j\omega n_d} = 4(1 - cos\omega)e^{-j\omega}$$

(c).

$$|H(e^{j\omega})| = |4(1 - cos\omega)| = 4(1 - cos\omega)$$



$$\angle H(e^{j\omega}) = -\omega$$



(d).

$$x_1[n] = 1 + e^{+j\frac{\pi}{2}n}$$

$$DTFT\{x_1[n]\} = X_1(e^{j\omega}) = 2\pi\delta(\omega) + 2\pi\delta\left(\omega - \frac{\pi}{2}\right)$$

$$Y_1\left(e^{j\omega}\right) = X_1\left(e^{j\omega}\right)H\left(e^{j\omega}\right) = 4(1 - cos\omega)e^{-j\omega}2\pi\delta(\omega) + 4(1 - cos\omega)e^{-j\omega}2\pi\delta\left(\omega - \frac{\pi}{2}\right)$$

$$Y_1(j\omega) = 0 + 2\pi * 4\left(1 - \cos\left(\frac{\pi}{2}\right)\right)e^{-j\frac{\pi}{2}}\delta\left(\omega - \frac{\pi}{2}\right) = 2\pi * \left(4e^{-j\frac{\pi}{2}}\right)\delta\left(\omega - \frac{\pi}{2}\right)$$

$$y_1[n] = -4j\ e^{+j\frac{\pi}{2}n}$$

(e).

$$x_2[n] = \left(1 + e^{+j\frac{\pi}{2}n}\right)u[n]$$

$$y_2[n] = -2x_2[n] + 4x_2[n-1] - 2x_2[n-2]$$

$$x_2[n-1] = \left(1 + e^{+j\frac{\pi}{2}(n-1)}\right)u[n-1] = \left(1 - je^{+j\frac{\pi}{2}n}\right)u[n-1]$$

$$x_@[n-2] = \left(1 + e^{+j\frac{\pi}{2}(n-2)}\right)u[n-2] = \left(1 - e^{+j\frac{\pi}{2}n}\right)u[n-2]$$

$$y_2[n] = -2\left(1 + e^{+j\frac{\pi}{2}n}\right)u[n] + 4\left(1 - je^{+j\frac{\pi}{2}n}\right)u[n-1] - 2\left(1 - e^{+j\frac{\pi}{2}n}\right)u[n-2]$$

$n \geq 2$时有

$$y_2[n] = y_1[n]$$

2.36. An LTI discrete-time system has frequency response given by

$$H(e^{j\omega}) = \frac{(1-je^{-j\omega})(1+je^{-j\omega})}{1-0.8e^{-j\omega}} = \frac{1+e^{-j2\omega}}{1-0.8e^{-j\omega}} = \frac{1}{1-0.8e^{-j\omega}} + \frac{e^{-j2\omega}}{1-0.8e^{-j\omega}}.$$

(a) Use one of the above forms of the frequency response to obtain an equation for the impulse response $h[n]$ of the system.

(b) From the frequency response, determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$ of the system.

(c) If the input to this system is

$$x[n] = 4 + 2\cos(\omega_0 n) \quad \text{for } -\infty < n < \infty,$$

for what value of ω_0 will the output be of the form

$$y[n] = A = \text{constant}$$

for $-\infty < n < \infty$? What is the constant A?

[解]:

我们考虑经典变换，形如$x[n] = a^n u[n] \quad |a| < 1$

$$DTFT\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n} = \sum_{n=0}^{+\infty} a^n e^{-j\omega n} = \sum_{n=0}^{+\infty} \left(ae^{-j\omega}\right)^n = \lim_{N\to+\infty} \frac{1-\left(ae^{-j\omega}\right)^{N+1}}{1-ae^{-j\omega}}$$

$$= \frac{1}{1-ae^{-j\omega}}$$

(a).

显然，这里$a = 0.8$

$$IDTFT\left\{\frac{1}{1-0.8e^{-j\omega}}\right\} = h_1[n] = (0.8)^n u[n]$$

$$IDTFT\left\{\frac{e^{-j2\omega}}{1-0.8e^{-j\omega}}\right\} = h_1[n-2] = h_2[n] = (0.8)^{n-2}u[n-2]$$

$$h[n] = h_1[n] + h_2[n] = (0.8)^n u[n] + (0.8)^{n-2}u[n-2]$$

(b).

$$h[n] = (0.8)^n u[n] + (0.8)^{n-2}u[n-2]$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1+e^{-j2\omega}}{1-0.8e^{-j\omega}}$$

$$X(e^{j\omega})(1+e^{-j2\omega}) = Y(e^{j\omega})(1-0.8e^{-j\omega})$$

形如$X(e^{j\omega})e^{-j2\omega}$,我们可以作$IDTFT$:

$$IDTFT\{X(e^{j\omega})e^{-j2\omega}\} = x[n-2]$$

所以有差分方程为:

$$x[n] + x[n-2] = y[n] - 0.8y[n-1]$$

保险起见，最后移项:

$$y[n] = 0.8y[n-1] + x[n] + x[n-2]$$

(c).

$$x[n] = 4 + 2\cos(\omega_0 n) = 4\cos(\omega n)|_{\omega=0} + 2\cos(\omega_0 n)$$

$$H(e^{j\omega}) = |H(e^{j\omega})|e^{+j\angle H(e^{j\omega})}$$

问题其实就是形如$\cos(\omega_0 n)$输入$LTI\ System$，输出会是如何

$$\cos(\omega_0 n)=\frac{1}{2}\left(e^{+j\omega_0 n}+e^{-j\omega_0 n}\right)$$

先给出我们之前推到过的结论:

考虑现在我们对于一个单位冲激响应为$h[n]$的LTI系统给入的输入为$x[n]=e^{j\omega_{in}n}$

$$y[n]=h[n]*x[n]=\sum_{k=-\infty}^{+\infty}h[k]e^{j\omega_i(n-k)}=e^{j\omega_i n}\sum_{k=-\infty}^{+\infty}h[k]e^{-j\omega_i k}=e^{j\omega_i n}H\left(e^{j\omega}\right)\Big|_{\omega=\omega_i}$$
$$=e^{j\omega_i n}H\left(e^{j\omega_i}\right)$$

$$x[n]=\cos(\omega_0 n)=\frac{1}{2}\left(e^{+j\omega_0 n}+e^{-j\omega_0 n}\right)$$

$$y[n]=\frac{1}{2}e^{+j\omega_0 n}H\left(e^{+j\omega_0}\right)+\frac{1}{2}e^{-j\omega_0 n}H\left(e^{-j\omega_0}\right)$$

考虑这是一个实系统:

$$H\left(e^{j\omega}\right)=\left|H\left(e^{j\omega}\right)\right|e^{+j\angle H\left(e^{j\omega}\right)}$$

那么会有（我们习题课讲过这个推论）:

$$\left|H\left(e^{j\omega}\right)\right|=\left|H\left(e^{j(-\omega)}\right)\right|$$
$$\angle H\left(e^{j\omega}\right)=-\angle H\left(e^{j(-\omega)}\right)$$

所以会有:

$$y[n]=\frac{1}{2}\left|H\left(e^{+j\omega_0}\right)\right|e^{+j[\omega_0 n\angle H(e^{j\omega_0})]}+\frac{1}{2}\left|H\left(e^{+j\omega_0}\right)\right|e^{-j[\omega_0 n+\angle H(e^{j\omega_0})]}$$
$$=\left|H\left(e^{+j\omega_0}\right)\right|\cos\left(\omega_0 n+\angle H\left(e^{j\omega}\right)\right)$$

所以本题结果为:

$$y[n]=4\left|H\left(e^{+j0}\right)\right|+2\left|H\left(e^{+j\omega_0}\right)\right|\cos\left(\omega_0 n+\angle H\left(e^{j\omega}\right)\right)=constant=A$$
$$\left|H\left(e^{+j\omega_0}\right)\right|=0$$
$$\left|H\left(e^{+j\omega_0}\right)\right|=\left|\frac{1+e^{-j2\omega_0}}{1-0.8e^{-j\omega_0}}\right|=0$$

$$1+e^{-j2\omega_0}=0$$

$$\boldsymbol{\omega_0=\frac{\pi}{2}+2k\pi\qquad k\in N}$$
$$\boldsymbol{A=40}$$

2.38. Consider the cascade of two LTI systems shown in Figure P2.38.

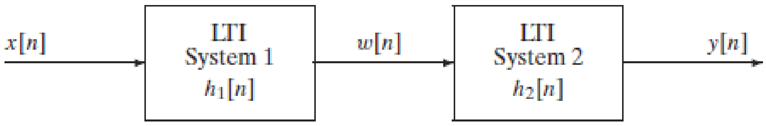


Figure P2.38

The impulse responses of the two systems are:

$$h_1[n] = u[n-5] \qquad \text{and} \qquad h_2[n] = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

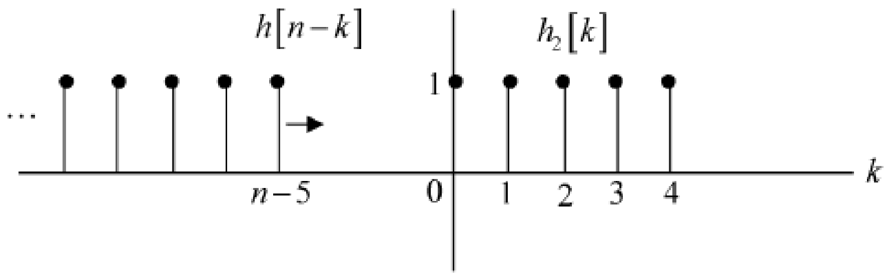
(a) Make a sketch showing both $h_2[k]$ and $h_1[n-k]$ (for some arbitrary $n < 0$) as functions of k.

(b) Determine $h[n] = h_1[n] * h_2[n]$, the impulse response of the overall system. Give your answer as an equation (or set of equations) that define $h[n]$ for $-\infty < n < \infty$ or as a carefully labelled plot of $h[n]$ over a range sufficient to define it completely.

[解]:

本题中两个系统产生级联($Cascade$)关系

A.

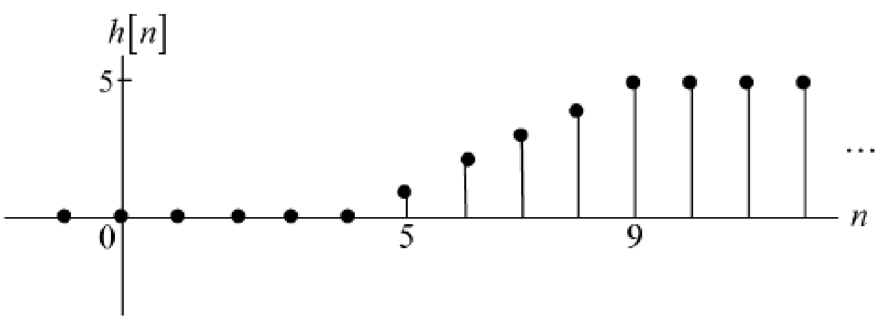


B. Clearly, $h[n]=0$ for $n-5<0$ or $n<5$.

Then $h[n]$ increases linearly until $n-5=4$ or $n=9$.

After $n=9$ the output is constant at $h[n]=5$.

$$h[n]=\begin{cases} 0, & n<5 \\ n-4, & 5\le n<9 \\ 5, & n>9. \end{cases}$$



2.45. Consider the cascade of LTI discrete-time systems shown in Figure P2.45.

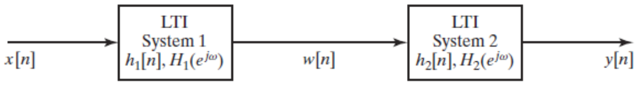


Figure P2.45

System 1 is described by the difference equation

$$w[n] = x[n] - x[n-1],$$

and System 2 is described by

$$h_2[n] = \frac{\sin(0.5\pi n)}{\pi n} \Longleftrightarrow H_2(e^{j\omega}) = \begin{cases} 1 & |\omega| < 0.5\pi \\ 0 & 0.5\pi < |\omega| < \pi. \end{cases}$$

The input $x[n]$ is

$$x[n] = \cos(0.4\pi n) + \sin(.6\pi n) + 5\delta[n-2] + 2u[n].$$

Determine the overall output $y[n]$.

(*With careful thought, you will be able to use the properties of LTI systems to write down the answer by inspection.*)

[解]:

先对输入予以更正

$$x[n] = cos(0.4\pi n) + sin(0.6\pi n) + 5\delta[n-2] + 2u[n]$$

这个里面其实本质上只有$u[n]$非常不好处理，因为我们求过$u[n]$的$DTFT$，频域上直接变换不现实，那样会相当地复杂。

$$w[n] = \cos(0.4\pi n) - \cos[0.4\pi(n-1)] + \sin(0.6\pi n) - \sin[0.6\pi(n-1)] + 5\delta[n-2] - 5\delta[n-3] + 2u[n] - 2u[n-1]$$

第一个LTI系统这么设计是有用意的，这样就会消去$u[n]$

$$2u[n] - 2u[n-1] = 2\delta[n]$$

$$w[n] = \cos(0.4\pi n) - \cos[0.4\pi(n-1)] + \sin(0.6\pi n) - \sin[0.6\pi(n-1)] + 5\delta[n-2] - 5\delta[n-3] + 2\delta[n]$$

所以说，本质上我们只需要探究$\cos(\omega_0 n)$与$\sin(\omega_0 n)$通过LTI系统后结果如何

其实我们在 2.36 题就已经深入、细致讨论过这个事情，当时我们讨论的是$\cos(\omega_0 n)$的情况，所以这里我们有：

$$\sin(\omega_0 n) = \cos\left[\omega_0\left(n - \frac{\frac{\pi}{2}}{\omega_0}\right)\right] = \cos\left[\omega_0\left(n - \frac{\pi}{2\omega_0}\right)\right]$$

本质上sin相对于cos也只是做了一个延迟，他们信号的本质性质都没有区别

这里我们使用 2.36 的结论

所以会有：

$$y[n] = \frac{1}{2}\left|H\left(e^{+j\omega_0}\right)\right|e^{+j[\omega_0 n \angle H(e^{j\omega_0})]} + \frac{1}{2}\left|H\left(e^{+j\omega_0}\right)\right|e^{-j[\omega_0 n + \angle H(e^{j\omega_0})]}$$

$$= \left|H\left(e^{+j\omega_0}\right)\right|\cos\left(\omega_0 n + \angle H\left(e^{j\omega}\right)\right)$$

$$H\left(e^{j\omega}\right) = \begin{cases} 1 & |\omega| < \frac{\pi}{2} \\ 0 & otherwise \end{cases}$$

对于$x_1[n]=\cos(0.4\pi n)-\cos[0.4\pi(n-1)]$有：

$$\left|H\left(e^{j(0.4\pi)}\right)\right|=1 \qquad \angle H\left(e^{j(0.4\pi)}\right)=0$$

输出为：

$$y_1[n]=\cos(0.4\pi n)-\cos\left(0.4\pi(n-1)\right)$$

有同学大概会对这个延迟有疑惑，这里其实本质上是这个样子的：

$$DTFT\{x[n-1]\}=e^{-j\omega}X\left(e^{j\omega}\right)$$
$$Y\left(e^{j\omega}\right)=X\left(e^{j\omega}\right)H\left(e^{j\omega}\right)=DTFT\{x[n]*h[n]\}$$
$$Y'\left(e^{j\omega}\right)=e^{-j\omega}X\left(e^{j\omega}\right)H\left(e^{j\omega}\right)=e^{-j\omega}Y\left(e^{j\omega}\right)=DTFT\{x[n-1]*h[n]\}$$
$$y'[n]=IDTFT\left\{Y'\left(e^{j\omega}\right)\right\}=y[n-1]$$

其实就是卷积中有一个有延迟，结果也延迟，这个是需要烂熟于心的

对于$x_2[n]=\sin(0.6\pi n)-\sin[0.6\pi(n-1)]$有：

$$\left|H\left(e^{j(0.4\pi)}\right)\right|=0 \qquad \angle H\left(e^{j(0.4\pi)}\right)=0$$

$$y_2[n]=0$$

对于$x_3[n]=5\delta[n-2]-5\delta[n-3]+2\delta[n]$有：

$$y_3[n]=x_3[n]*h[n]=5\frac{\sin\left(0.5\pi(n-2)\right)}{\pi(n-2)}-5\frac{\sin\left(0.5\pi(n-3)\right)}{\pi(n-3)}+5\frac{\sin(0.5\pi n)}{\pi n}$$

最后总结果$y[n]=y_1[n]+y_2[n]+y_3[n]$

$$y[n]=\cos(0.4\pi n)-\cos\left(0.4\pi(n-1)\right)+5\frac{\sin\left(0.5\pi(n-2)\right)}{\pi(n-2)}-5\frac{\sin\left(0.5\pi(n-3)\right)}{\pi(n-3)}+5\frac{\sin(0.5\pi n)}{\pi n}$$

2.46. The DTFT pair

$$a^n u[n] \Longleftrightarrow \frac{1}{1-ae^{-j\omega}} \qquad |a|<1 \qquad \text{(P2.46-1)}$$

is given.

(a) Using Eq. (P2.46-1), determine the DTFT, $X(e^{j\omega})$, of the sequence

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n & n \le -1 \\ 0 & n \ge 0. \end{cases}$$

What restriction on b is necessary for the DTFT of $x[n]$ to exist?

(b) Determine the sequence $y[n]$ whose DTFT is

$$Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1+2e^{-j\omega}}.$$

[解]:

(a).

$$DTFT\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n} = \sum_{n=-\infty}^{-1} (-b^n)e^{-j\omega n} = -\sum_{n=-\infty}^{-1} b^n e^{-j\omega n} = -\sum_{n'=1}^{+\infty} \left(b^{-1}e^{+j\omega}\right)^{n'}$$

上式中我们做了一步换元:

$$n' = -n$$

$$DTFT\{x[n]\} == -\sum_{n'=1}^{+\infty} \left(b^{-1}e^{+j\omega}\right)^{n'} = -\left\{\lim_{N\to+\infty} \frac{1-\left(b^{-1}e^{+j\omega}\right)^{N+1}}{1-b^{-1}e^{+j\omega}} - 1\right\} = 1 - \frac{1}{1-b^{-1}e^{+j\omega}}$$

上面式子的收敛需要:

$$\left|b^{-1}e^{+j\omega}\right| < 1$$
$$|b| > 1$$

(b)

$$DTFT\{y[n]\} = Y\left(e^{j\omega}\right) = \frac{2e^{-j\omega}}{1+2e^{-j\omega}} = \frac{2}{e^{j\omega}+2} = \frac{1}{1+\frac{1}{2}e^{j\omega}}$$

带入(a)即可

$$y[n] = \begin{cases} -2^n & n \le 0 \\ 0 & otherwise \end{cases}$$

这里因为少了一个 1，所以需要凑一下上下限，没有难度，大家仔细想一下即可。

选做部分

2.60. Consider a discrete-time LTI system with frequency response $H(e^{j\omega})$ and corresponding impulse response $h[n]$.

(a) We are first given the following three clues about the system:

(i) The system is causal.

(ii) $H(e^{j\omega}) = H^*(e^{-j\omega})$.

(iii) The DTFT of the sequence $h[n+1]$ is real.

Using these three clues, show that the system has an impulse response of finite duration.

(b) In addition to the preceding three clues, we are now given two more clues:

(iv) $\frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})d\omega = 2.$

(v) $H(e^{j\pi}) = 0.$

Is there enough information to identify the system uniquely? If so, determine the impulse response $h[n]$. If not, specify as much as you can about the sequence $h[n]$.

[解]:

(a).

这里需要我们根据三个线索，证明符合这个线索的LTI系统会是有限冲激响应的，也就是到一定点数之后就全是 0 了，有截断($Truncation$)

(i)系统是因果的(Causal):

$$h[n] = h[n]u[n]$$

这个条件阐释了时域负半轴部分

(ii)系统正负频率所对应的频域函数是共轭关系的:

$$H\left(e^{j\omega}\right) = H^*\left(e^{j(-\omega)}\right)$$

我们在习题课里面其实讲过，这个系统很显然就是一个实系统，也就是$h[n] \in R$

(iii)系统时移后的$DTFT$是实的:

$$DTFT\{h[n+1]\} = \sum_{n=-\infty}^{+\infty} h[n+1]e^{-j\omega n} = \sum_{n=-1}^{+\infty} h[n+1]e^{-j\omega n}$$

第三个条件说明了，系统只有三点，并且 $h[0] = h[2]$，从而会有:

$$DTFT\{h[n+1]\} = h[0]e^{+j\omega} + h[1] + h[2]e^{-j\omega} = h[0]\{2\cos(\omega)\} + h[1]$$

所以确实是有限长度的。

(b).

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H\left(e^{j\omega}\right)e^{+j\omega n}d\omega$$

$$h[0] = \left\{\frac{1}{2\pi}\int_{-\pi}^{+\pi} H\left(e^{j\omega}\right)e^{+j\omega n}d\omega\right\}\Big|_{n=0} = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H\left(e^{j\omega}\right)d\omega = 2$$

$$h[2] = h[0] = 2$$

$$H\left(e^{j\omega}\right) = h[0]\{2\cos(\omega)\} + h[1]$$

$\omega = \pi$情况下有:

$$H\left(e^{j\pi}\right) = -2h[0] + h[1] = 0$$

$$h[1] = 4$$

所以这五个条件可以明确地确定这个系统的唯一性$h[0] = 2 \quad h[1] = 4 \quad h[2] = 2$

2.65. The LTI system

$$H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0, \end{cases}$$

is referred to as a 90° phase shifter and is used to generate what is referred to as an analytic signal $w[n]$ as shown in Figure P2.65-1. Specifically, the analytic signal $w[n]$ is a complex-valued signal for which

$$\mathcal{R}e\{w[n]\} = x[n],$$

$$\mathcal{I}m\{w[n]\} = y[n].$$

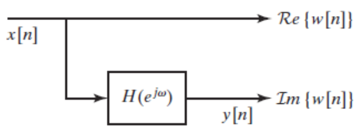


Figure P2.65–1

If $\mathcal{R}e\{X(e^{j\omega})\}$ is as shown in Figure P2.65-2 and $\mathcal{I}m\{X(e^{j\omega})\} = 0$, determine and sketch $W(e^{j\omega})$, the Fourier transform of the analytic signal $w[n] = x[n] + jy[n]$.

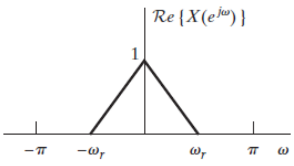


Figure P2.65–2

[解]:

这个题目的背景本质上就是$Hilbert$变换，我们在后续会学到。

$$Re\{w[n]\} = x[n]$$

$$Im\{w[n]\} = y[n]$$

$$w[n] = x[n] + jy[n]$$

$$DTFT\{w[n]\} = DTFT\{x[n] + jy[n]\}$$

由于$DTFT$变换是线性变换，自然有：

$$DTFT\{w[n]\} = W(e^{j\omega}) = DTFT\{x[n]\} + jDTFT\{y[n]\} = X(e^{j\omega}) + jY(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \begin{cases} -jX(e^{j\omega}) & 0 < \omega < \pi \\ +jX(e^{j\omega}) & -\pi < \omega < 0 \end{cases}$$

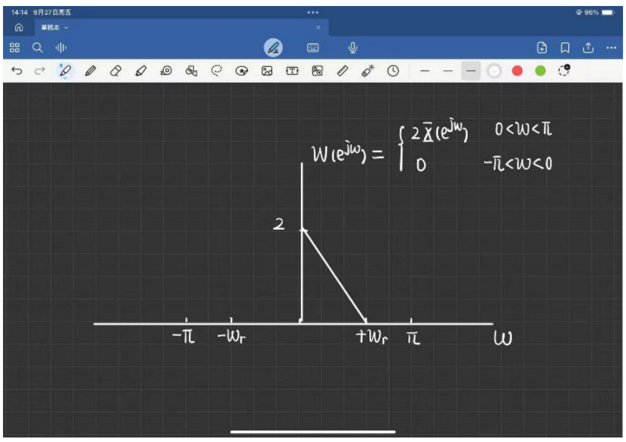
$$W(e^{j\omega}) = X(e^{j\omega}) + jY(e^{j\omega}) = \begin{cases} 2X(e^{j\omega}) & 0 < \omega < \pi \\ 0 & -\pi < \omega < 0 \end{cases}$$

已知$Re\{X(e^{j\omega})\}$如图，并且会有$Im\{X(e^{j\omega})\} = 0$

所以会有：

$$X(e^{j\omega}) = Re\{X(e^{j\omega})\} + jIm\{X(e^{j\omega})\} = Re\{X(e^{j\omega})\}$$

从而会有$W(e^{j\omega})$的图：



2.56. For the system in Figure P2.56, determine the output $y[n]$ when the input $x[n]$ is $\delta[n]$ and $H(e^{j\omega})$ is an ideal lowpass filter as indicated, i.e.,

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/2, \\ 0, & \pi/2 < |\omega| \le \pi. \end{cases}$$

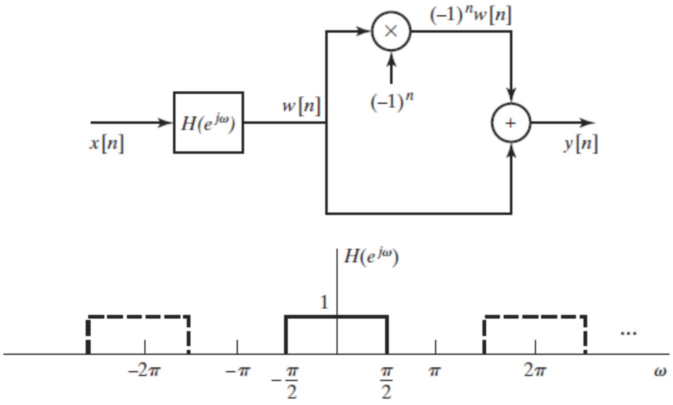


Figure P2.56

[解]:

$$x[n] = \delta[n]$$

$$W(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) = \begin{cases} 1 & |\omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\omega| < \pi \end{cases}$$

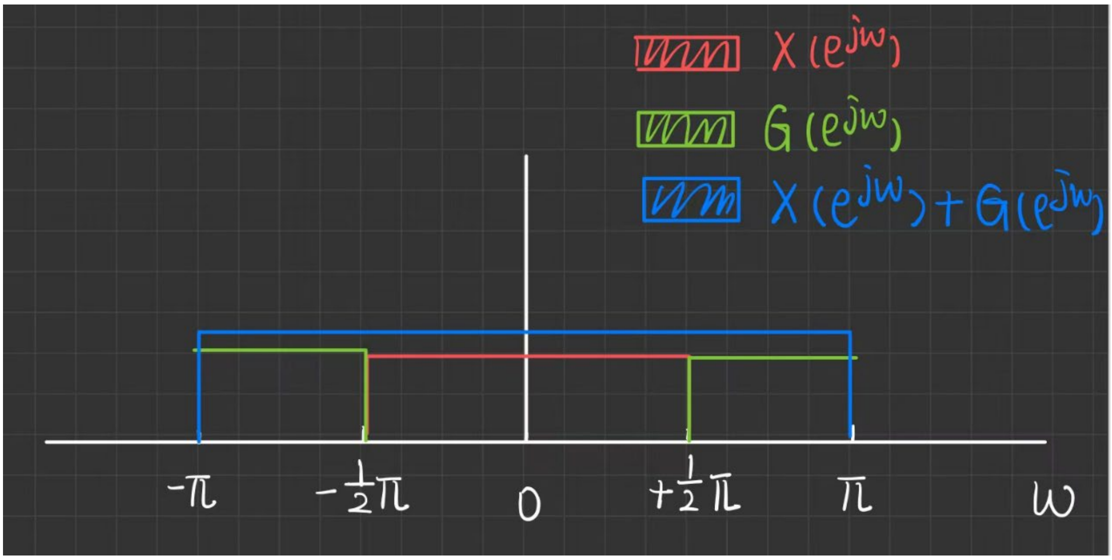
$$y[n] = x[n] + (-1)^n x[n]$$

$$g[n] = (-1)^n x[n]$$

$$G(e^{j\omega}) = DTFT\{g[n]\} = \sum_{n=-\infty}^{+\infty} g[n]e^{-j\omega n} = \sum_{n=-\infty}^{+\infty} (-1)^n x[n]e^{-j\omega n} = \sum_{n=-\infty}^{+\infty} x[n]e^{-j(\omega+\pi)n}$$

$$= X(e^{j(\omega+\pi)})$$

如下图:



红绿蓝三色信号其实是一样高，为了明显才画的不一样高

$$y[n] = \delta[n]$$

3.3. Determine the z-transform of each of the following sequences. Include with your answer the ROC in the z-plane and a sketch of the pole–zero plot. Express all sums in closed form; α can be complex.

(a) $x_a[n] = \alpha^{|n|}, \qquad 0 < |\alpha| < 1.$

(b) $x_b[n] = \begin{cases} 1, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$

(c) $x_c[n] = \begin{cases} n+1, & 0 \le n \le N-1, \\ 2N-1-n, & N \le n \le 2(N-1), \\ 0, & \text{otherwise.} \end{cases}$

Hint: Note that $x_b[n]$ is a rectangular sequence and $x_c[n]$ is a triangular sequence. First, express $x_c[n]$ in terms of $x_b[n]$.

[解]:

(a)

$$x_a[n] = \alpha^{|n|} \qquad 0 < |\alpha| < 1$$

$$\begin{aligned} X_a(z) &= \sum_{n=-\infty}^{-1} \alpha^{-n} z^{-n} + \sum_{n=0}^{\infty} \alpha^n z^{-n} \\ &= \sum_{n=1}^{\infty} \alpha^n z^n + \sum_{n=0}^{\infty} \alpha^n z^{-n} \\ &= \frac{\alpha z}{1-\alpha z} + \frac{1}{1-\alpha z^{-1}} = \frac{z(1-\alpha^2)}{(1-\alpha z)(z-\alpha)}, \qquad |\alpha| < |z| < \frac{1}{|\alpha|} \end{aligned}$$

(b)

$$x_b = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & N \le n \\ 0, & n < 0 \end{cases} \Rightarrow X_b(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1-z^{-N}}{1-z^{-1}} = \frac{z^N - 1}{z^{N-1}(z-1)} \qquad z \ne 0$$

(c)

$$x_c[n] = x_b[n-1] * x_b[n] \Leftrightarrow X_c(z) = z^{-1} X_b(z) \cdot X_b(z)$$

$$X_c(z) = z^{-1} \left(\frac{z^N - 1}{z^{N-1}(z-1)} \right)^2 = \frac{1}{z^{2N-1}} \left(\frac{z^N - 1}{z - 1} \right)^2 \qquad z \ne 0, 1$$

图略

3.6. Following are several z-transforms. For each, determine the inverse z-transform using both methods—partial fraction expansion and power series expansion—discussed in Section 3.3. In addition, indicate in each case whether the Fourier transform exists.

(a) $X(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}, \qquad |z| > \dfrac{1}{2}$

(b) $X(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}, \qquad |z| < \dfrac{1}{2}$

(c) $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}, \qquad |z| > \dfrac{1}{2}$

(d) $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}}, \qquad |z| > \dfrac{1}{2}$

(e) $X(z) = \dfrac{1-az^{-1}}{z^{-1}-a}, \qquad |z| > |1/a|$

[解]:

先给出两个最经典的Z变换，所有的Z变换都是在它们的基础上进行变换：

$$x[n] = a^n u[n]$$

$$Z\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n} = \sum_{n=0}^{+\infty} a^n z^{-n} = \frac{1}{1-az^{-1}}$$

$$ROC: |az^{-1}| < 1 \qquad |a| < |z|$$

$$x[n] = -a^n u[-n-1]$$

$$Z\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n} = -\sum_{n=-\infty}^{-1} a^n z^{-n} = -\left(\sum_{n=-\infty}^{0} a^n z^{-n} - 1\right) = 1 - \sum_{n=-\infty}^{0} a^n z^{-n}$$

$$= 1 - \sum_{n'=0}^{+\infty} (a^{-1}z)^{n'} = 1 - \frac{1}{1-a^{-1}z} = \frac{-a^{-1}z}{1-a^{-1}z} = \frac{1}{1-az^{-1}}$$

$$ROC: |a^{-1}z| < 1 \qquad |z| < |a|$$

(a).

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{2}$$

$$x[n] = \left(-\frac{1}{2}\right)^n u[n]$$

(b)

$$X(z) = \frac{1}{1+\frac{1}{2}z^{-1}} \qquad |z| < \frac{1}{2}$$

$$x[n] = -\left(-\frac{1}{2}\right)^n u[-n-1]$$

(c)

$$X(z)=\frac{1-\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}=\frac{4}{1+\frac{1}{2}z^{-1}}+\frac{-3}{1+\frac{1}{4}z^{-1}}\qquad |z|>\frac{1}{2}$$

$$x[n]=4\left(-\frac{1}{2}\right)^{n}u[n]-3\left(-\frac{1}{4}\right)^{n}u[n]$$

(d)

$$X(z)=\frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}}=\frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}\qquad |z|>\frac{1}{2}$$

$$X(z)=\frac{1}{1+\frac{1}{2}z^{-1}}\qquad |z|>\frac{1}{2}$$

$$x[n]=\left(-\frac{1}{2}\right)^{n}u[n]$$

(e).

$$X(z)=\frac{1-az^{-1}}{z^{-1}-a}=\frac{1-az^{-1}}{a(a^{-1}z^{-1}-1)}=-\frac{1}{a}\frac{1-az^{-1}}{1-a^{-1}z^{-1}}$$

$$=-\frac{1}{a}\left(\frac{1}{1-a^{-1}z^{-1}}-a\frac{z^{-1}}{1-a^{-1}z^{-1}}\right)\qquad |z|>|a^{-1}|$$

$$x[n]=-\frac{1}{a}\{(a^{-1})^{n}u[n]-a*(a^{-1})^{n-1}u[n-1]\}$$

$$x[n]=-a^{-n-1}u[n]+a^{-n+1}u[n-1]$$

3.9. A causal LTI system has impulse response $h[n]$, for which the z-transform is

$$H(z) = \frac{1+z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}.$$

(a) What is the ROC of $H(z)$?
(b) Is the system stable? Explain.
(c) Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\frac{1}{3}\left(-\frac{1}{4}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1].$$

(d) Find the impulse response $h[n]$ of the system.

[解]:

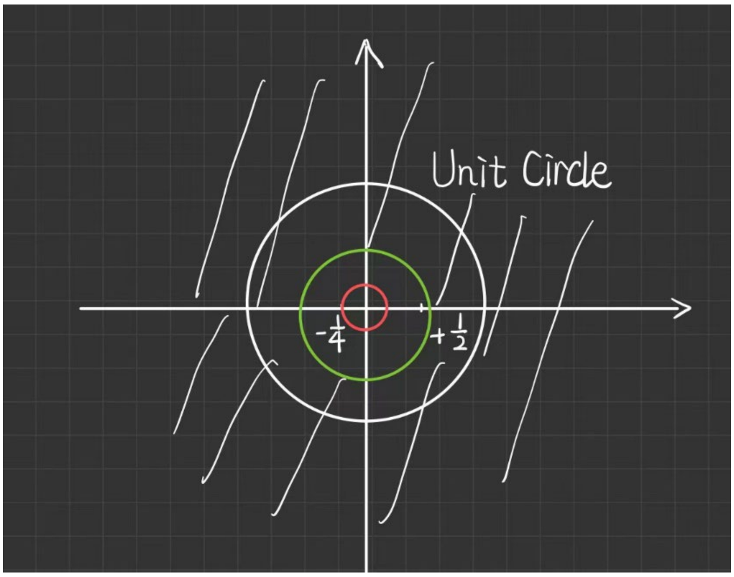
(a).

因为系统是因果的:

$$h[n] = h[n]u[n]$$

ROC形如$|z| > |a|$

如下图，必须有交



$$ROC: |z| > \frac{1}{2}$$

(b).

我们考虑一个系统如果是因果的，那么会有:

$$\exists M > 0 \quad such\ that \quad \sum_{n=-\infty}^{+\infty} |h[n]| < M$$

$$\left|\sum_{n=-\infty}^{+\infty} h[n]\right| < \sum_{n=-\infty}^{+\infty} |h[n]| < M$$

一个必要条件是$|z| = 1$所表征的$Unit\ Circle$需要在ROC之内，当然张老师上课降到了这个点，这只是个必要条件，是否一定如此判决还有待商榷。但是根据 Oppenheimer 书上所述，这里确实是属于$Stable$的范畴。

(c)

$$Y(z) = \sum_{n=-\infty}^{+\infty} y[n]z^{-n} = -\frac{1}{3}\frac{1}{1-\left(-\frac{1}{4}\right)z^{-1}} + (-1)\left(-\frac{4}{3}\right)\frac{1}{1-2z^{-1}}$$

$$Y(z) = -\frac{1}{3}\frac{1}{1+\frac{1}{4}z^{-1}} + \frac{4}{3}\frac{1}{1-2z^{-1}} \qquad ROC{:}\ \frac{1}{4} < |z| < 2$$

$$Y(z) = \frac{1+z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)} \qquad ROC{:}\ \frac{1}{4} < |z| < 2$$

$$X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}} \qquad |z| < 2$$

$$x[n] = -(2)^n u[-n-1] + \left(-\frac{1}{2}\right)(-1)(2)^{n-1}u[-(n-1)-1]$$

$$x[n] = -2^n u[-n-1] + 2^{n-2}u[-n]$$

(d)

$$h[n] = \left(\frac{1}{2}\right)^{n-1} u[n] - \left(-\frac{1}{4}\right)^n u[n]$$

3.13. A causal sequence $g[n]$ has the z-transform

$$G(z) = \sin(z^{-1})(1 + 3z^{-2} + 2z^{-4}).$$

Find $g[11]$.

[解]:

$$G(z) = \sin(z^{-1})\,(1 + 3z^{-2} + 2z^{-4})$$

我们对$\sin(z^{-1})$进行$Taylor$展开，或者更精确地来说，这里是关于 0 点的$Taylor$展开，也就是退化为了$Maclaurin$展开

$$\sin(z) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$$

这里我们替换z为z^{-1}

$$\sin(z^{-1}) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!} z^{-(2n+1)}$$

$$G(z) = \sum_{n=-\infty}^{+\infty} g[n] z^{-n}$$

令$n = 11$

$$g[11] \to z^{-11}$$

于是会有：

$$-(2n_0 + 1) + 0 = -11$$
$$n_0 = 5$$

$$-(2n_1 + 1) - 2 = -11$$
$$n_1 = 4$$

$$-(2n_2 + 1) - 4 = -11$$
$$n_2 = 3$$

对比系数可以有：

$$g[11] = \frac{(-1)^5}{11!} + \frac{(-1)^4}{9!} * 3 + \frac{(-1)^3}{7!} * 2 = -\frac{1}{11!} + \frac{3}{9!} - \frac{2}{7!}$$

3.21. A causal LTI system has the following system function:

$$H(z) = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})}$$

(a) What is the ROC for $H(z)$?

(b) Determine if the system is stable or not.
(c) Determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$.
(d) Use a partial fraction expansion to determine the impulse response $h[n]$.
(e) Find $Y(z)$, the z-transform of the output, when the input is $x[n] = u[-n-1]$. Be sure to specify the ROC for $Y(z)$.
(f) Find the output sequence $y[n]$ when the input is $x[n] = u[-n-1]$.

[解]:

(a)

此LTI系统是因果($Causal$)系统，所以会有ROC形如：$|z| > |a|$

$$ROC: |z| > 0.5$$

(b)

$|z| = 1$包含在ROC之中，所以是稳定的

(c)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{1 + 0.25z^{-1} - 0.125z^{-2}}$$

差分方程为：

$$y[n] + 0.25y[n-1] - 0.125y[n-2] = 4x[n] + 0.25x[n-1] - 0.5x[n-2]$$

保险起见，我建议大家都移项变成标准形式：

$$y[n] = 4x[n] + 0.25x[n-1] - 0.5x[n-2] + 0.125y[n-2] - 0.25y[n-1]$$

(e)

$$x = u[-n-1]$$

$$X(z) = \frac{-1}{1 - z^{-1}} \qquad |z| < 1$$

$$Y(z) = H(z)X(z) \qquad ROC: ROC_X \cap ROC_H$$

$$Y(z) = \frac{-4 - 0.25z^{-1} + 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})(1 - z^{-1})} \qquad ROC: 0.5 < |z| < 1$$

(f)

$$Y(z) = \frac{-4 - 0.25z^{-1} + 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})(1 - z^{-1})} = \frac{-\frac{1}{3}}{(1 - 0.25z^{-1})} + \frac{-\frac{1}{3}}{(1 + 0.5z^{-1})} + \frac{-\frac{10}{3}}{(1 - z^{-1})}$$

$$y[n] = -\frac{1}{3}(+0.25)^n u[n] - \frac{1}{3}(-0.5)^n u[n] + \frac{10}{3}u[-n-1]$$

3.25. Sketch each of the following sequences and determine their z-transforms, including the ROC:

(a) $\sum_{k=-\infty}^{\infty} \delta[n-4k]$

(b) $\frac{1}{2}\left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right)\right]u[n]$

[解]:

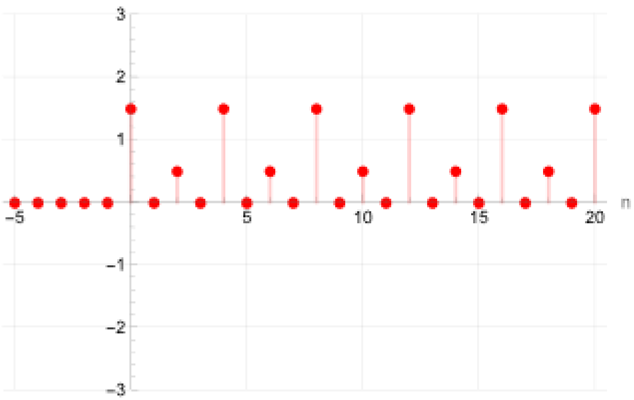
(a)

$$g[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]$$

$$Z\left\{\sum_{k=-\infty}^{+\infty} \delta[n-4k]\right\} = \sum_{n=-\infty}^{+\infty} \left\{\sum_{k=-\infty}^{+\infty} \delta[n-4k]\right\} z^{-n} = G(z)$$

$$G(z) = \sum_{k=-\infty}^{+\infty} z^{-4k}$$

(b)



我们画图可以看出:

$$m[n] = \frac{1}{2}u[n]\left\{e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2} + 2\pi n\right)\right\}$$

这个函数本质上是两个周期信号的叠加，幅值一个是$\frac{3}{2}$,一个是$\frac{1}{2}$，周期都是 4

$$M(z) = \sum_{k=0}^{+\infty} \frac{3}{2} z^{-4k} + \sum_{k=0}^{+\infty} \frac{1}{2} z^{-4k-2} = \frac{\frac{3}{2}}{1-z^{-4}} + \frac{\frac{1}{2}z^{-2}}{1-z^{-4}} \quad ROC: |z| > 1$$

选做部分

3.22. A causal LTI system has system function

$$H(z) = \frac{1 - 4z^{-2}}{1 + 0.5z^{-1}}.$$

The input to this system is

$$x[n] = u[n] + 2\cos\left(\frac{\pi}{2}n\right) \qquad -\infty < n < \infty,$$

Determine the output $y[n]$ for large positive n; i.e., find an expression for $y[n]$ that is asymptotically correct as n gets large. (*Of course, one approach is to find an expression for $y[n]$ that is valid for all n, but you should see an easier way.*)

[解]:

这个题本质上和 2.45 一个道理，无非是$DTFT$变为了Z变换，本质上也是一个道理。

这里只是$u[n]$会难以处理一些

$$x_{input}[n] = u[n]$$

$$y_{output}[n] = x_{input}[n] * h[n] = \sum_{k=-\infty}^{+\infty} u[k]h[n-k] = \sum_{k=0}^{+\infty} h[n-k] = \sum_{n'=-\infty}^{n} h[n'] \quad (n' = n-k)$$

n足够大的时候会有:

$$y_{output}[n] = \sum_{n'=-\infty}^{n} h[n'] = \sum_{n'=-\infty}^{+\infty} h[n'] = \sum_{n'=-\infty}^{+\infty} h[n']z^{-n'}|_{n'=0} = H(z)|_{z=1} = -2$$

3.42. In Figure P3.42, $H(z)$ is the system function of a causal LTI system.

(a) Using z-transforms of the signals shown in the figure, obtain an expression for $W(z)$ in the form

$$W(z) = H_1(z)X(z) + H_2(z)E(z),$$

where both $H_1(z)$ and $H_2(z)$ are expressed in terms of $H(z)$.

(b) For the special case $H(z) = z^{-1}/(1 - z^{-1})$, determine $H_1(z)$ and $H_2(z)$.

(c) Is the system $H(z)$ stable? Are the systems $H_1(z)$ and $H_2(z)$ stable?

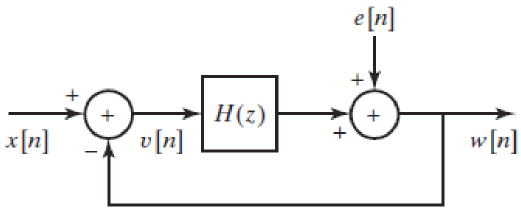


Figure P3.42

[解]:

(a)

$$v[n] = x[n] - w[n]$$
$$q[n] = v[n] * h[n]$$
$$w[n] = e[n] + q[n]$$
$$W(z) = E(z) + Q(z)$$
$$Q(z) = V(z)H(z) = \big(X(z) - W(z)\big)H(z) = X(z)H(z) - W(z)H(z)$$
$$W(z) = E(z) + X(z)H(z) - W(z)H(z)$$
$$H_1(z) = \frac{H(z)}{1 + H(z)} \qquad H_2(z) = \frac{1}{1 + H(z)}$$

(b)

$$H(z) = \frac{z^{-1}}{1 - z^{-1}}$$
$$H_1(z) = z^{-1} \qquad H_2(z) = 1 - z^{-1}$$

(c)

$H(z)$的极点是$z = 1$，所以是非稳定状态

$H(z)$要因果且结果收敛就有：

$$|z| > 1$$

完全不包含$Unit\ Circle$

对于$H_1(z)$　$H_2(z)$　$|z| = 1$是收敛的，所以稳定