PROOFS FOR CHAPTER 0

A. STUDENT

Proposition 0.1. The real number $\sqrt{2}$ is irrational.

Version: 1
Comments / Collaborators: New

Proof. Suppose, for the sake of contradiction, that $\sqrt{2}$ is rational, i.e. it can be represented by the fraction $\frac{p}{q}$, where p and q have no common divisors. Squaring both sides of the identity $\sqrt{2} = \frac{p}{q}$ and multiplying both sides by q^2 yields the identity

$$p^2 = 2q^2. \tag{1}$$

Thus, 2 divides p^2, and hence 2 divides p. That is, we may write $p = 2r$, and hence Equation (1) implies that $4r^2 = 2q^2$. As before, this means that 2 divides q^2, and hence 2 divides q. We have arrived at a contradiction: on one hand, we assumed that p and q have no common divisors; on the other, 2 divides both p and q. □

Proposition 0.2. If $a_n \to A$ and $b_n \to B$, then $a_n + b_n \to A + B$.

Version: 2 (correcting small error)
Comments / Collaborators: E. Milicevic and R. Manning

Proof. Given $\epsilon > 0$, let N_a be such that if $n \geq N_a$, then $|a_n - A| < \frac{\epsilon}{2}$. Similarly, let N_b be such that if $n \geq N_b$, then $|b_n - B| < \frac{\epsilon}{2}$.

Choose $N = \sup\{N_a, N_b\}$. We then verify for a given $n \geq N$:

$$\begin{aligned} |a_n + b_n - (A + B)| &\leq |a_n - A| + |b_n - B| \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \end{aligned}$$

□

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