Math 335: Topology

Fall 2021

1. Technical Information

Instructor: Tarik Aougab (`taougab@haverford.edu`)
Office Hours: TBD– we'll survey the class and figure out the best times. Please see Moodle for the most up-to-date schedule.
Texts: There are no texts for this course. In fact, for reasons outlined below, the use of any textbook or other outside resource in this course will be considered a violation of the Honor Code, with the exception of your analysis text and, in the last part of the course, your algebra text.

2. Goals of the Course

2.1. **Content.** The goal of this course is to introduce you to the fundamental examples, problems, and machinery of topology. The central concepts in topology are those of *nearness* of points in a set and *continuity* of functions between sets for which a concept of nearness has been defined. The goal of topology is then to study properties of sets that are preserved under continuous functions, as well as properties of the continuous functions themselves. The result is a general and powerful theory that can make deep qualitative statements in a variety of situations, from the classic "rubber sheet geometry" formulation of topology, to vector calculus, dynamical systems, and even to robot motion planning or data analysis!

2.2. **Growth in Mathematical Thinking.** This course is structured to immerse you in the creative process of making new (to you) mathematics. You will prove theorems, construct (counter-) examples, and perhaps make conjectures. You will hone your mathematical thought processes in writing, speaking, and critiquing the work of others. You will gain increased ownership of and accountability for the material and your own mathematical processes.

Note that this is a less linear (and more frustrating) growth process than listening to lectures, reading a textbook, and doing problem sets — but it is also more rewarding. There is a very prevalent belief that you are either "good" or "bad" at math, and if you are "bad" at it, then you will always be bad at it no matter how hard you try. This is extremely false and the mathematics community bears a lot of responsibility for perpetuating this myth. In reality, mathematics is just like any other discipline or skill: you can improve more and more with practice, and this class is designed to challenge your practice. You should measure your success in this class by how much your understanding of the concepts have improved over the course of the semester.

3. Course Structure

3.1. **The Moore Method.** This course will be taught using a version of the Moore Method (a.k.a. inquiry-based learning). This means that instead of the more standard lecture-homework-exam course structure, I will give you course notes that consist of definitions, theorems, examples, and exercises (though yes, there will still be a final). *You* and your classmates will prove the theorems and work out the examples, both in writing and in class, with some guidance from me and structured collaboration with your classmates.

To give you an idea of what our class periods will look like, here is a description of an *ideal* period: we all arrive and I announce who has volunteered to present the three theorems we will be focusing on for that day, and also which student will serve as "facilitator" for the day. The presenting students then take turns proving the theorems at the board, to the rest of the class. In between each presentation, guided by the facilitator, the class provides feedback, asks questions, and guides the presenter to a complete and logically sound solution. This is a highly collaborative and open-ended process, but the ideal end result is that *everyone* in the class feels some ownership and understanding of the mathematics presented. I sit in the back and say essentially nothing the entire time.

This ideal will perhaps never be actually realized, and one of my roles in this course is to serve as training wheels or bumpers in a bowling alley, depending on your preferred recreational analogy. When the dynamics in the room stray a little too far from the above, I'll step in as necessary. This might mean that I'll participate in the "feedback" portion after a presentation has been given, and it may mean me giving a very brief lecture in between presentations just to provide extra context about what we just saw.

We will *all* work to make the classroom a welcoming community for all in which to experiment, collaborate, and learn. In particular, remember that we are not all coming to this class with the same privileges, resources, time, and knowledge. As a community, mathematicians and scientists need to do a much better job of making our disciplines more accessible to people of all races, genders (including gender non-conforming folks), sexual identities, and class backgrounds. While this is a priority for us in the classroom, we do not claim to know how to best honor this commitment, and so we are very open to feedback from you when it comes to making the course more accessible and inclusive to all identities.

I highly suggest reading the advice of former Moore Method students, which is available on Moodle.

Disclaimers:

(1) I'm a topologist, so I know this material quite well. On the other hand, I've never tried the Moore Method before, so I'm a complete pedagogical novice in this area. So, I'm going to be learning a lot as we go. A lot of the materials for this course have been borrowed from– and informed by– Josh, a very seasoned Moore Method instructor.

(2) Experienced Moore Method instructors make very clear that, *at its best*, this method is extremely free-flowing, organic, and disorganized! The point is

that we all learn more by putting ourselves in the position to work through ideas together, and this is an inherently messy process. By now, I'm sure you've experienced the difference between coming up with a proof on your own, and seeing a proof displayed for you by an instructor in a lecture. The latter experience feels *far* cleaner, more put together, and polished. The former experience is frustrating and time consuming, but it also has the potential to be infinitely more rewarding than just sitting in a room and listening to someone else's ideas. The Moore Method approach is about honoring this potential, and maximizing the level of ownership that each student feels over the material.

(3) The Moore Method is named after Robert Lee Moore, who was a topologist at the University of Pennsylvania from 1911-1920, and then at the University of Texas from 1920-1969. He was a rabid white supremacist, misogynist, and anti-semite, and was violently racist towards Black students and mathematicians. He developed this teaching method as a means of making mathematics more accessible to his students, but at the same time, he used white supremacy and misogyny as a way of determining who would be his students in the first place.

(4) I am making a point of saying the above because I think it's crucial we don't gloss over the less-than-palatable history of our discipline. Our worldviews impact our thought processes in an infinite variety of subtle ways, and so it's likely impossible to tease out exactly how, or if, Moore's Method reflects the creator's repugnant political beliefs. But being aware of his background and his beliefs at least puts us in a position to be ready to make correctives when we see the need. For one, we will be very careful to set classroom norms in a way that Moore likely never would have done, with an eye towards prioritizing mutual respect and care for one another above all else.

In any case, the whole point of this course is to work carefully through the content in a more conventionally-taught topology course, and therefore, most of the proofs students will be coming up with, presenting, and discussing with one another, can be easily found in other topology textbooks. Looking at those sources would spoil the entire exercise! So, I hope that you now understand the reason behind the following edict:

> *You are not allowed to use* any *outside sources — textbooks, the internet, notes from students who took the course before, etc. — in this class, with the exception of your analysis and algebra textbooks. I will consider any such use to be a violation of the Honor Code.*

3.2. **In-Class Presentations.** In-class presentations are the lifeblood of this course.

The evening before a class, students will email me,[1] volunteering to present one or more propositions from the class notes that I had suggested in the previous class. *Not all propositions in the class notes will be presented in class* — some will only

[1]You may send this email as late at night as you want, so long as I have it by 8 AM the next morning.

appear on homework, and some will simply be passed over unless you are interested in proving them on your own. You may volunteer a partial proof, or a proof of a special case, if that is where you are in the process. I will tell you if you have been chosen to present by about 10 AM. Please trust that I have my reasons for selecting who presents what.

At the beginning of the class, I will ask the volunteers to present proofs (or partial proofs) of propositions from the class notes. On a normal day, several students will present proofs to the class; I might also get up to make a brief presentation if the material warrants it. You may take notes to the board with you. If — *when* — you make a mistake, don't panic! You'll be given a chance to correct it on the fly, and if that doesn't pan out, I will either ask you to try again during the next class session or will let another student present, depending on the nature of the mistake and your preference.

If you are not presenting, that doesn't mean you get to kick back and relax: it's your responsibility to help the class (politely and respectfully) ensure that the presenter's proof is correct. You may ask questions of the presenter, but you may not give away answers and you may *certainly* not take over their presentation. You and the rest of the class are in this venture together — without everyone both presenting and critiquing at a high level, we won't get any math done!

A note on being in the audience: please remember that the entire vibe of this course is **collaboration**. The presenter is doing the extremely brave thing of stepping up and providing the backbone of what will hopefully and eventually become a complete solution or proof. It is the audience's job to respectfully guide them– and other audience members– towards full comprehension. This won't be accomplished by questioning that even has the *slightest tint* of aggression. Take it from me: you can know material extremely well, and it will still all fly out of your head when you feel like someone in the audience wants to catch you off guard and/or embarrass you. As a first rule of thumb: **question-asking is *not* the time for folks to show off how much more they think they understand than the presenter.** I've purposefully not factored in any sort of participation into final grades, so I'm not going to be keeping track of the clever things individual people say anyway. The point of a question should be to realign your own understanding with that of the presenter, always keeping in mind that you're both human beings with emotions and that you're both wrestling with very difficult mathematical content in real time.

After each presentation, we (you) will discuss the presenter's argument to clarify or correct specific points, offering praise where warranted and constructive criticism when necessary. The day's facilitator will guide this discussion. We may then proceed to analyze the statement and proof further, by, for instance, examining the effect of changing hypotheses, discussing whether a converse holds, or thinking about whether the proof is good enough to prove a stronger statement. Conjectures are most welcome at this point! At the end of class, I will suggest next steps in the notes or perhaps inform you that I will put up a new section of the notes.

Presentations will be graded for the correctness of the mathematics (2 points), the (logical) structure of the presentation (2 points), and mastery of the technicalities (one point). We will discuss exactly what goes into each of these facets on the first day. *The first presentation you make will only be graded for completeness. Further, there is no penalty for a presentation that you have to abandon because of a serious mathematical error; such a presentation will simply go unrecorded.*

You need to present at least four times during the semester to earn a passing grade. Do not panic: there are plenty of theorems or examples in the course notes, so there are plenty of presentations to go around.

3.3. **Written Work.** You may hand in proofs of propositions in the class notes in five different ways:

(1) At any time, you can turn in a written proof of a proposition that has yet to be presented in class or assigned for homework and is in the current subsection of the notes. Please label these proofs as "BEFORE PRESENTATION" at the very top of your writeup. Submissions will be made to a class dropbox.
(2) If you present a problem, you must turn in a written proof within 24 hours of the presentation. Please tag these proofs as "PRESENTER" in your writeup. Please alert me when you upload such a proof so I can include it in the notes as a "draft" after your presentation.
(3) You may turn in, at any time, a written proof of a proposition that *has* been presented in class and is in the current subsection of the notes. These proofs should *not* be labeled "BEFORE PRESENTATION" in your writeup.
(4) I will assign write-ups of proofs that have already been presented and/or a few new exercises, depending on the week. These will be due every Monday by 5pm. I will inform you of what you need to turn in by sometime after class on Wednesday. While you are welcome to take notes in class, you may not look at those notes while writing up your proofs (see the "Collaboration Guidelines" section below). If you are handing in enough "before presentation" proofs, or at least working ahead, you will probably never have to do a homework assignment from scratch. When you do have some "before presentation" proofs already completed, just copy/paste those proofs into your assignment, including the label.
(5) You may rewrite proofs that I have graded *for full credit* at any time and as many times as you would like until I "close" a section of the notes (typically about two weeks after we are done presenting proofs from that section).

The grading rubric for the written work is as follows. For proofs whose first version is handed in by the original due date:

- **4 points:** The proof is correct and clearly written, which almost always means that the author has explained not only the steps of the proof, but the strategy of the proof.
- **3 points:** The proof suffers from a minor defect in correctness or clarity, though the author seems to understand the proof.
- **0 points:** All other proofs. This grade will appear as a "RW" (meaning "rewrite") on your paper.

+1 point: Proofs submitted using the first criterion, above, so long as that proof is deserving of a 3 or 4 the first time around.

+2 points: Proofs of particularly difficult statements, as designated in the course notes, so long as the proof eventually deserves a 3 or 4.

Late work is subject to a 50% penalty. These are still very abnormal times and we are still in a pandemic. So I'm extremely willing to be flexible when it comes to late work! I just need you to communicate with me. There are many of you and only one of me, so I don't expect to have the time to track you down when I don't see an assignment from you. I will work with you if you're struggling, but it's your responsibility to clue me in. So If I don't hear anything from you by the time an assignment is due, I'll apply the penalty. Note that the absolute maximum score on a typical proof is 5, though *expected* maximum score is 4.

Some technicalities:

- You must type your written work in LaTeX. I will provide sample and template files for you to use. If you have never before used LaTeX, please see `http://www.haverford.edu/mathematics/resources/LaTeX.php` for instructions; you may also talk to me or contact David Lippel.
- You must write in complete sentences and complete paragraphs; as tempting as it is, you should mostly avoid using technical symbols like $\exists$, $\forall$, and $\Rightarrow$, as they make your prose all but unreadable except in rare circumstances. Remember: you are writing the textbook for this class!
- You should "hand in" your written work by placing a PDF file in the "New Submissions" subfolder of the Dropbox folder that you and I will be sharing *and* sending me an email when you have done so. Please use the following file naming convention:

  `your-last-name_section-subsection_date.pdf`

  For example, if your name is Judy Smith and you are handing in problems from Section 4.3 on September 15th, then the name of your file should be:

  `smith_4-3_9-15.pdf`

  In particular, if you are handing in proofs from different sections, they should go in different files.

4. Collaboration Guidelines

You may *not* collaborate with your classmates on propositions (etc.) that have yet to be correctly presented in class. This runs counter to my usual approach to collaboration, which is essentially: do it as much as you can! But the ultimate focus of this class is to maximize *your very own* feeling of familiarity and comfort level with the material. Plus, there will be ton of real-time collaboration in each of our class sessions when folks are giving feedback to presenters. And of course, you can always talk to me.

Once the class agrees that a proposition has been correctly presented (or I assign an exercise for you to write up that will not be presented), however, you are free to collaborate with your peers on any write-ups that you do, subject to the following guidelines:

- You must indicate on your written work who your collaborators were (you may note different collaborators on different problems!)
- The actual write-up of your homework assignment should be done out of sight of any material produced during the collaboration (notebooks, scraps of paper, chalkboards, etc.) and without looking at any notes you may have taken in class. If you cannot write up the solution without wanting to refer to your collaboration material, then you probably have not yet understood the problem. In that case, you should delete anything you have produced so far on the problem, go back to your notes (etc.) and start again.

5. Exams

There will be an open-notes un-timed take-home final exam.

6. Grading

Your grade for the course is determined by your written work, your class presentations, and your performance on the final exam.[2] It is difficult to translate individual scores into an overall grade in a course structured like this one, but roughly speaking, doing the minimum required work (i.e. completing all assigned homework problems and the minimum number of presentations with decent scores) will yield a 3.3, while completing over 1.5 times the required minimum work, with excellent scores, will earn a 4.0. I will also take your trajectory into account, especially for the quality of the presentations, as I expect there to be some growing pains at the beginning of the class.

The three inputs into your final grade are weighted as follows:

Written Work: 50%
Class Presentations: 30%
Final: 20%

7. Resources

7.1. Office Hours.

Please stop by to see me during office hours! Ask questions! Hang out with your classmates! Learn how to draw cartoon animals!

7.2. Math Question Center.

The MQC is place you can go to discuss homework with your classmates and ask questions of upperclass student tutors. The MQC is located in KINSC H011 (the same place as discussion sessions!) and is open Sunday through Thursday 7–9p.

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[2] I am deliberately *not* taking any qualitative measure of "class participation" into account, as my sense is that such grades are too easily subject to implicit bias.

7.3. **New Mathematicians' Study Center.** The New Mathematicians Study Center (NMSC) is a collaboration space for historically underrepresented or marginalized students in mathematics. The center is staffed by 1-2 undergraduates who are able to help students with some math courses. Students who identify as historically underrepresented or marginalized in mathematics are invited to stop by to work on homework, reinforce concepts learned previously in the course, or to simply meet with and get to know other math students. The NMSC meets 9–11p on Sundays in the OAR / STOKES 118K.

7.4. **Equal Access.** Haverford College is committed to providing equal access to students with a disability. If you have (or think you have) a learning difference or disability Ð including mental health, medical, or physical impairment, please contact the Office of Access and Disability Services (ADS) at `hc-ads@haverford.edu`. The Coordinator willconfidentially discuss the process to establish reasonable accommodations.

Students who have already been approved to receive academic accommodations and want to use their accommodations in this course should share their verification letter with me and also make arrangements to meet with me as soon as possible to discuss their the specific accommodations. Please note that accommodations are **not retroactive** and require advance notice to implement.

It is a state law in Pennsylvania that individuals must be given advance notice if they are to be recorded. Therefore, any student who has a disability-related need to audio record this class must first be approved for this accommodation from the Coordinator of Access and Disability Services and then must speak with me. Other class members will need to be aware that this class may be recorded.