308 PCS2

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1.1 a

First, we need to write down the positions of x, y in terms of t when there is no air resistance.

$$\begin{aligned} x(t) &= v_{xo}t, \\ y(t) &= v_{yo}t - \frac{1}{2}gt^2. \end{aligned} \tag{1}$$

Then, we need to express t in terms of x.

$$t = x/v_{xo}. \tag{2}$$

Now, we need to plug t into $y(t)$.

$$y(x) = v_{yo}x/v_{xo} - \frac{1}{2}g(x/v_{xo})^2. \tag{3}$$

1.2 b

We are given equation (2.37) from Taylor.

$$y = \frac{v_{yo} + v_{ter}}{v_{xo}}x + v_{ter}\tau \ln\left(1 - \frac{x}{v_{xo}\tau}\right). \tag{4}$$

Let $\epsilon = \frac{x}{v_{xo}\tau}$. From the Taylor expansion, equation (2.40) in Taylor, of the natural log, we can rewrite y with the second term being expanded. Since we are approximating it with really small ϵ values, it is reasonable to only keep the first two terms in the Taylor expansion.

$$-\ln(1-\epsilon) = -\left(\epsilon + \frac{1}{2}\epsilon^2\right). \tag{5}$$

$$y = \frac{v_{yo} + v_{ter}}{v_{xo}}x + v_{ter}\tau\left(-\left(\left(\frac{x}{v_{xo}\tau}\right) + \frac{1}{2}\left(\frac{x}{v_{xo}\tau}\right)^2\right)\right). \tag{6}$$

Since air resistance doesn't exist in this case, we take τ to infinity, and simplify equation (6).

$$\begin{aligned}
y &= \frac{v_{yo} + v_{ter}}{v_{xo}} x + v_{ter}\tau \left(- \left(\frac{x}{v_{xo}\tau} \right) - \frac{1}{2} \left(\frac{x}{v_{xo}\tau} \right)^2 \right) \\
&= \frac{v_{yo} + v_{ter}}{v_{xo}} x - v_{ter}\tau \left(\frac{x}{v_{xo}\tau} \right) - v_{ter}\tau \frac{1}{2} \left(\frac{x}{v_{xo}\tau} \right)^2 \\
&= \frac{v_{yo} + v_{ter}}{v_{xo}} x - v_{ter} \frac{x}{v_{xo}} - g \frac{1}{2} \left(\frac{x}{v_{xo}} \right)^2 \\
&= \frac{x}{v_{xo}} v_{yo} - g \frac{1}{2} \left(\frac{x}{v_{xo}} \right)^2 .
\end{aligned} \tag{7}$$

This expression matches our result found in (a).