308 Q3

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1 In your own words (and in no more than a few sentences) explain why this result suggests that should two bodies emerge from an elastic collision traveling on perpendicular paths suggests they have equal mass.

Suppose that they have different masses, m_1 and m_2, and their velocities are non-zero and are still in perpendicular paths. Then we know that the dot product between the final velocity vectors is zero.

By conservation of momentum and energy, we got the following equations.

$$\begin{aligned} m_1\vec{\mathbf{v}}_1 &= m_1\vec{\mathbf{v}}_1' + m_2\vec{\mathbf{v}}_2', \\ 1/2m_1\|\vec{\mathbf{v}}_1\|^2 &= 1/2m_1\|\vec{\mathbf{v}}_1'\|^2 + 1/2m_2\|\vec{\mathbf{v}}_2'\|^2. \end{aligned} \tag{1}$$

Cancel the 1/2's in the second line, we have

$$m_1\|\vec{\mathbf{v}}_1\|^2 = m_1\|\vec{\mathbf{v}}_1'\|^2 + m_2\|\vec{\mathbf{v}}_2'\|^2. \tag{2}$$

Dot product $\vec{\mathbf{v}}_1'$ for the first line of equation (1), we get

$$m_1\vec{\mathbf{v}}_1 \cdot \vec{\mathbf{v}}_1' = m_1\|\vec{\mathbf{v}}_1'\|^2 + 0. \tag{3}$$

Cancel m_1 and $\vec{\mathbf{v}}_1'$ on both sides, we get

$$\vec{\mathbf{v}}_1 = \vec{\mathbf{v}}_1'. \tag{4}$$

This shows that $\vec{\mathbf{v}}_2'$ in equation (2) is 0, contradicts with our assumptions.