308 notes 7.1-7.5

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This is a wild note

1 Lagrange's Equations for Unconstrained Motion

The Lagrangian is

$$\mathcal{L} = T - U. \tag{1}$$

I like the sentences: "You are certainly entitled to ask why the quantity $T - U$ should be of any interest. There seems to be no simple answer to this question except that it is..."

Hamilton's Principle: The actual path which a particle follows between two points 1 and 2 in a given time interval, t_1 to t_2, is such that the action integral

$$S = \int_{t_1}^{t_2} \mathcal{L} \mathrm{d}t \tag{2}$$

is stationary when taken along the actual path.

The generalized force and generalized momentum sounded something physicists would name.

1.1 Several Unconstrained Particles

2 Constrained Systems: and Example

Not sure what this example is trying to present, that Lagrangian matches Newtonian definitions?

3 Constrained Systems in General

3.1 Generalized Coordinates

Since three dimensional world requires that the number n of generalized coordinates for N particles is certainly no more than $3N$, does that mean that two dimensional world requires that the number of generalized coordinates for N particles is no more than $2N$?

How do we know OUR number of the generalized coordinates is the smallest number that allows the system to be parametrized in this way?

The cart example definitely reminds me that we need an inertial frame.

3.2 Degrees of Freedom

Since the degrees of freedom is defined as the number of coordinates that can be independently varied in a small displacement, it should be plausible to assume that the nonholonomic systems would have degrees of freedom that is less than the number of generalized coordinates?

4 Proof of Lagrange's Equations with Constraints

"In general, the forces of constraint are not necessarily conservative, but this doesn't matter." WHY?????????????

AND then, Taylor says, "Notice, however, that if the constraining forces are nonconservative, Lagrange's equations in the simple unconstrained form of Section 7.1 certainly do not apply." So it actually matters???

Why is gravity a nonconstraint force?????????????

4.1 The Action Integral is Stationary at the Right Path

Why a sudden change of notation???

4.2 The Final Proof

5 Examples of Lagrange's Equations

Nobody cares about examples, good night.