308 notes 2.1-2.4

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1 Air resistance

The force of resistance is summed up in the equation

$$\mathbf{f} = -f(v)\hat{\mathbf{v}} \tag{1}$$

The function $f(v)$ gives the magnitude of the air resistance varies with v in a complicated way.

$$f(v) = bv + cv^2 = f_{\text{lin}} + f_{\text{quad}} \tag{2}$$

The linear term arises from the viscous drag of the medium and is generally proportional to the viscosity of the medium and the linear size of the projectile.

The quadratic term arises from the projectile's having to accelerate the mass of air with which it is continually colliding; f_{quad} is proportional to the density of the medium and the cross-section area of the projectile.

In particular, for a spherical projectile, the coefficients b and c have the form

$$b = \beta D \text{ and } c = \gamma D^2 \tag{3}$$

where D denotes the diameter of the sphere and the coefficients β and γ depend on the nature of the medium.

2 Linear air resistance

Supposing, for a minute, that the quadratic term doesn't exist, then the equation of motion is easy to solve, simply

$$m\ddot{\mathbf{r}} = m\mathbf{g} - b\mathbf{v} \tag{4}$$

Since neither of the forces depends on $\mathbf{r}$, we can rewrite $\ddot{\mathbf{r}}$ as $\dot{\mathbf{v}}$, so that we have

$$m\dot{\mathbf{v}} = m\mathbf{g} - b\mathbf{v} \tag{5}$$

a first order differential equation for $\mathbf{v}$.

Solve for the x component and y component separately, we can get an x position which our mass will infinitely close to, and get a terminal speed in the y direction.

One thing I found interesting is that a heavier object will have larger terminal speed than a lighter object. Initially, I thought that since the acceleration g is the same for both objects, they should touch the ground at the same time, since the distances traveled are also same, we would expect that the terminal speed is the same. The, I realized that that's only in the friction-less environment. So, pretty fun.

3 Trajectory and range in a linear medium

I found the way the textbook approximated the solution to R really beautiful, showing how scientist do math before computers, which is what I admire the most, as a person who don't like computers for research.

4 Quadratic air resistance

For most examples of projectile, a quadratic air resistance is a more reasonable approximation.

One interesting behavior of the quadratic air resistance is that as the velocity goes small, the air resistance goes small as well, which would fail to bring the speed of the object down when the object is at really low speeds. This insight tells us that at some point, the linear resistance is more significant than the quadratic resistance.