q_rnn_and_grad

March 16, 2025

1 Introduction

In this notebook, we'll implement simple RNNs and LSTMs, then explore how gradients flow through these different networks.

This notebook does not require a Colab GPU. If it's enabled, you can turn it off through Runtime -> Change runtime type. (This will make it more likely for you to get Colab GPU access later in the REAL_RNN_LSTM.ipynb problem.)

2 Imports

Note: the ipympl installation will require you to restart the colab runtime.

[11]:
```
! pip install ipympl --quiet
```

```
WARNING: Ignoring invalid distribution -atplotlib
(c:\users\22020\.conda\envs\cs182hw1\lib\site-packages)
WARNING: Ignoring invalid distribution -atplotlib
(c:\users\22020\.conda\envs\cs182hw1\lib\site-packages)
WARNING: Ignoring invalid distribution -atplotlib
(c:\users\22020\.conda\envs\cs182hw1\lib\site-packages)
```

[1]:
```
import copy

import torch as th
from torch import nn
import torch.nn.functional as F
import torch.optim as optim
import numpy as np
import matplotlib.pyplot as plt
from ipywidgets import interactive, widgets, Layout
```

[2]:
```
%matplotlib ipympl
```

3 1.A: implementing a RNN layer

Consider using Pytorch's nn.Linear. You can implement this with either one Linear layer or two. If you use two, remember that you only need to include a bias term for one of the linear layers.

[3]:

```
class RNNLayer(nn.Module):
  def __init__(self, input_size, hidden_size, nonlinearity=th.tanh):
    """
    Initialize a single RNN layer.

    Inputs:
    - input_size: Data input feature dimension
    - hidden_size: RNN hidden state size (also the output feature dimension)
    - nonlinearity: Nonlinearity applied to the rnn output
    """
    super().__init__()
    self.input_size = input_size
    self.hidden_size = hidden_size
    self.nonlinearity = nonlinearity
   ␣
↪##############################################################################
    # TODO: Initialize any parameters your class needs.                          ␣
↪ #
   ␣
↪##############################################################################

    self.input_linear = nn.Linear(input_size, hidden_size, bias=True)
    self.hidden_linear = nn.Linear(hidden_size, hidden_size, bias=False)

   ␣
↪##############################################################################
    #                               END OF YOUR CODE                              ␣
↪ #
   ␣
↪##############################################################################

  def forward(self, x):
    """
    RNN forward pass

    Inputs:
    - x: input tensor (B, seq_len, input_size)

    Returns:
    - all_h: tensor of size (B, seq_len, hidden_size) containing hidden states
            produced for each timestep
    - last_h: hidden state from the last timestep (B, hidden_size)
    """
    h_list = []  # List to store the hidden states [h_1, ... h_T]
   ␣
↪##############################################################################
```

```
    # TODO: Implement the RNN forward step                                    ␣
↪ #
    # 1. Initialize h0 with zeros                                             ␣
↪ #
    # 2. Roll out the RNN over the sequence, storing hidden states in h_list  ␣
↪ #
    # 3. Return the appropriate outputs                                       ␣
↪ #
  ␣
↪###############################################################################

    B, seq_len, _ = x.size()
    h = th.zeros(B, self.hidden_size, device=x.device)
    for t in range(seq_len):
      h = self.nonlinearity(self.input_linear(x[:, t, :]) + self.
↪hidden_linear(h))
      h_list.append(h)

    last_h = h

        ###############################################################################
    #                               END OF YOUR CODE                          ␣
↪ #
  ␣
↪###############################################################################

    # h_list should now contain all hidden states, each of size (B, hidden_size)
    # We will store the hidden states so we can analyze their gradients later
    self.store_h_for_grad(h_list)
    all_h = th.stack(h_list, dim=1)
    return all_h, last_h

  def store_h_for_grad(self, h_list):
    """
    Store input list and allow gradient computation for all list elements
    """
    for h in h_list:
      h.retain_grad()
    self.h_list = h_list
```

3.0.1 Test Cases

If your implementation is correct, you should expect to see errors of less than 1e-4.

[4]:
```
rnn = RNNLayer(1, 1)
# Overwrite initial parameters with fixed values.
# Should give deterministic results even with different implementations.
```

```python
rnn.load_state_dict({k: v * 0 + .1 for k, v in rnn.state_dict().items()})
data = th.ones((1, 1, 1))
expected_out = th.FloatTensor([[[0.1973753273487091]]])
all_h, last_h = rnn(data)
assert all_h.shape == expected_out.shape
assert th.all(th.isclose(all_h, last_h))
print(f'Expected: {expected_out.item()}, got: {last_h.item()}, max error: {th.
↪max(th.abs(expected_out - last_h)).item()}')

rnn = RNNLayer(2, 3, nonlinearity=lambda x: x)  # no nonlinearity

num_params = sum(p.numel() for p in rnn.parameters())
assert num_params == 18, f'expected 18 parameters but found {num_params}'

rnn.load_state_dict({k: v * 0 - .1 for k, v in rnn.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_all_h = th.FloatTensor([[[-0.1250, -0.1250, -0.1250],
         [-0.1075, -0.1075, -0.1075],
         [-0.1328, -0.1328, -0.1328],
         [-0.1452, -0.1452, -0.1452]],

        [[ 0.0600,  0.0600,  0.0600],
         [ 0.1520,  0.1520,  0.1520],
         [ 0.2344,  0.2344,  0.2344],
         [-0.0853, -0.0853, -0.0853]]])
expected_last_h = th.FloatTensor([[-0.1452, -0.1452, -0.1452],
        [-0.0853, -0.0853, -0.0853]])
all_h, last_h = rnn(data)
assert all_h.shape == expected_all_h.shape
assert last_h.shape == expected_last_h.shape
print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_h - last_h)).item()}')
```

```
Expected: 0.1973753273487091, got: 0.1973753273487091, max error: 0.0
Max error all_h: 5.0008296966552734e-05
Max error last_h: 2.498924732208252e-05
```

4 1.B Implementing a RNN regression model.

[5]:
```python
class RecurrentRegressionModel(nn.Module):
  def __init__(self, recurrent_net, output_dim=1):
    """
    Initialize a simple RNN regression model

    Inputs:
    - recurrent_net: an RNN or LSTM (single or multi layer)
```

```
        - output_dim: feature dimension of the output
        """
        super().__init__()
        self.recurrent_net = recurrent_net
        self.output_dim = output_dim
 ␣
↪###########################################################################
        # TODO: Initialize any parameters you need                           ␣
↪ #
        # HINT: use recurrent_net.hidden_size to find the hidden state size  ␣
↪ #
 ␣
↪###########################################################################

        self.output_linear = nn.Linear(recurrent_net.hidden_size, output_dim,␣
↪bias=True)

 ␣
↪###########################################################################
        #                              END OF YOUR CODE                      ␣
↪ #
 ␣
↪###########################################################################

    def forward(self, x):
        """
        Forward pass

        Inputs:
        - x: input tensor (B, seq_len, input_size)

        Returns:
        - out: predictions of shape (B, seq_len, self.output_dim).
        - all_h: tensor of size (B, seq_len, hidden_size) containing hidden states
                 produced for each timestep.
        """
 ␣
↪###########################################################################
        # TODO: Implement the forward step.                                  ␣
↪ #
 ␣
↪###########################################################################

        all_h, last_h = self.recurrent_net(x)
        out = self.output_linear(all_h)
```

```
    ↪#########################################################################
    #                               END OF YOUR CODE                          
    ↪ #
    
    ↪#########################################################################
    return out, all_h
```

4.1 Tests

[6]:
```
rnn = RecurrentRegressionModel(RNNLayer(2, 3), 4)

num_params = sum(p.numel() for p in rnn.parameters())
assert num_params == 34, f'expected 34 parameters but found {num_params}'

rnn.load_state_dict({k: v * 0 - .1 for k, v in rnn.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
 ↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_preds = th.FloatTensor([[[-0.0627, -0.0627, -0.0627, -0.0627],
         [-0.0678, -0.0678, -0.0678, -0.0678],
         [-0.0604, -0.0604, -0.0604, -0.0604],
         [-0.0567, -0.0567, -0.0567, -0.0567]],

        [[-0.1180, -0.1180, -0.1180, -0.1180],
         [-0.1453, -0.1453, -0.1453, -0.1453],
         [-0.1692, -0.1692, -0.1692, -0.1692],
         [-0.0748, -0.0748, -0.0748, -0.0748]]])
expected_all_h = th.FloatTensor([[[-0.1244, -0.1244, -0.1244],
         [-0.1073, -0.1073, -0.1073],
         [-0.1320, -0.1320, -0.1320],
         [-0.1444, -0.1444, -0.1444]],

        [[ 0.0599,  0.0599,  0.0599],
         [ 0.1509,  0.1509,  0.1509],
         [ 0.2305,  0.2305,  0.2305],
         [-0.0840, -0.0840, -0.0840]]])
preds, all_h = rnn(data)
assert all_h.shape == expected_all_h.shape
assert preds.shape == expected_preds.shape
print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_preds - preds)).item()}')
```

```
Max error all_h: 4.699826240539551e-05
Max error last_h: 4.312396049499512e-05
```

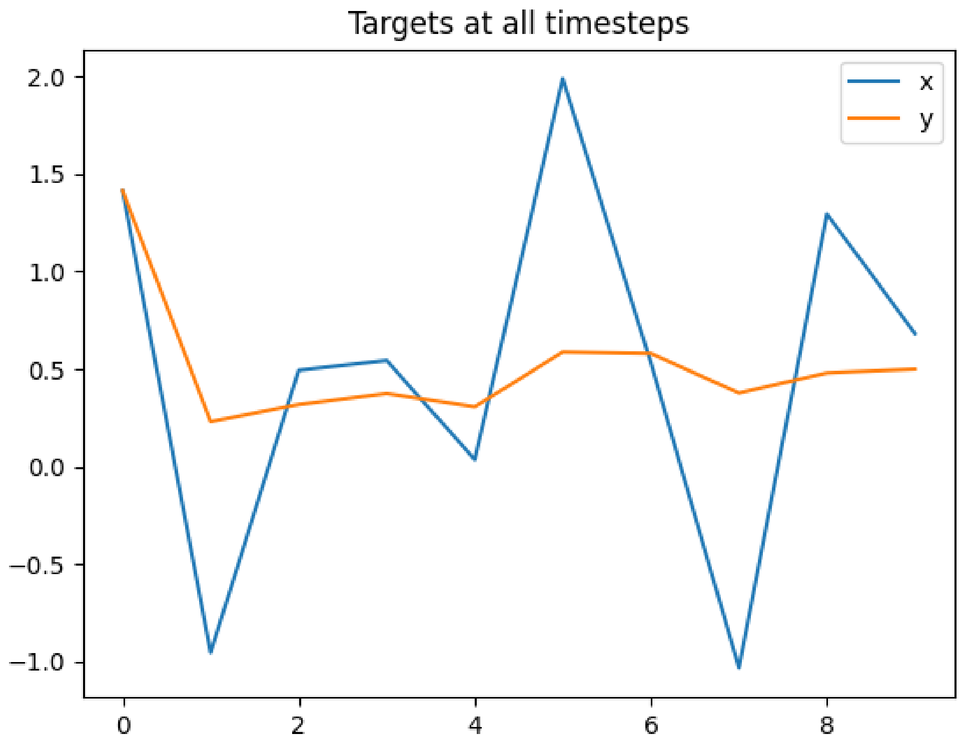
5 Problem 1.C: Dataset and loss function

5.1 1.C.i: Understanding the dataset (no implementation needed)

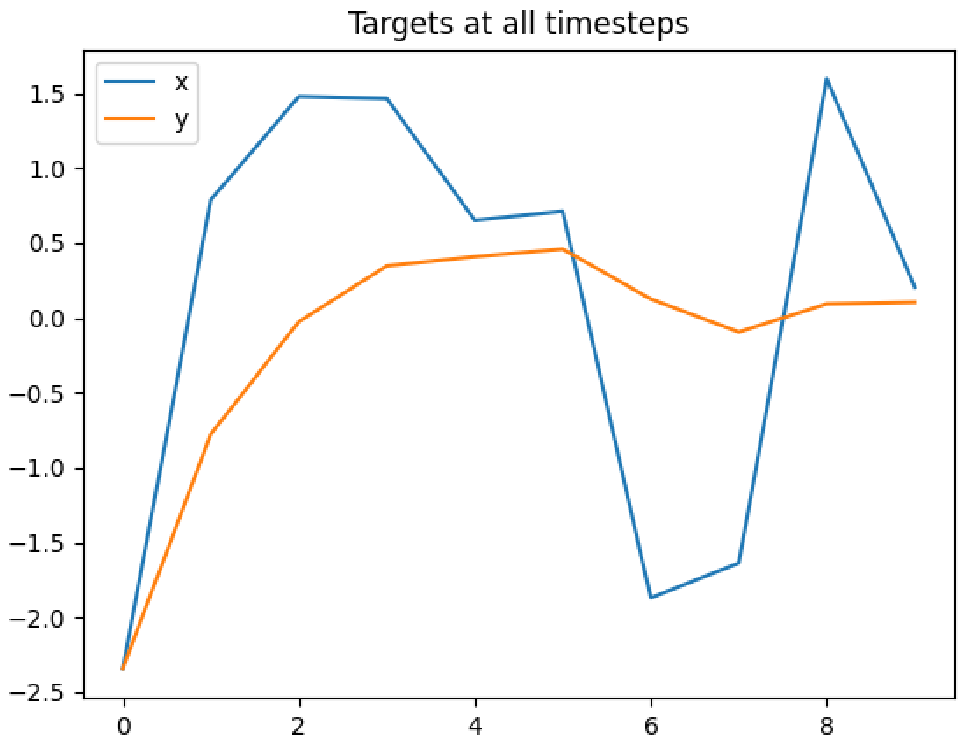
Inspect the code and plots below to visualize the dataset

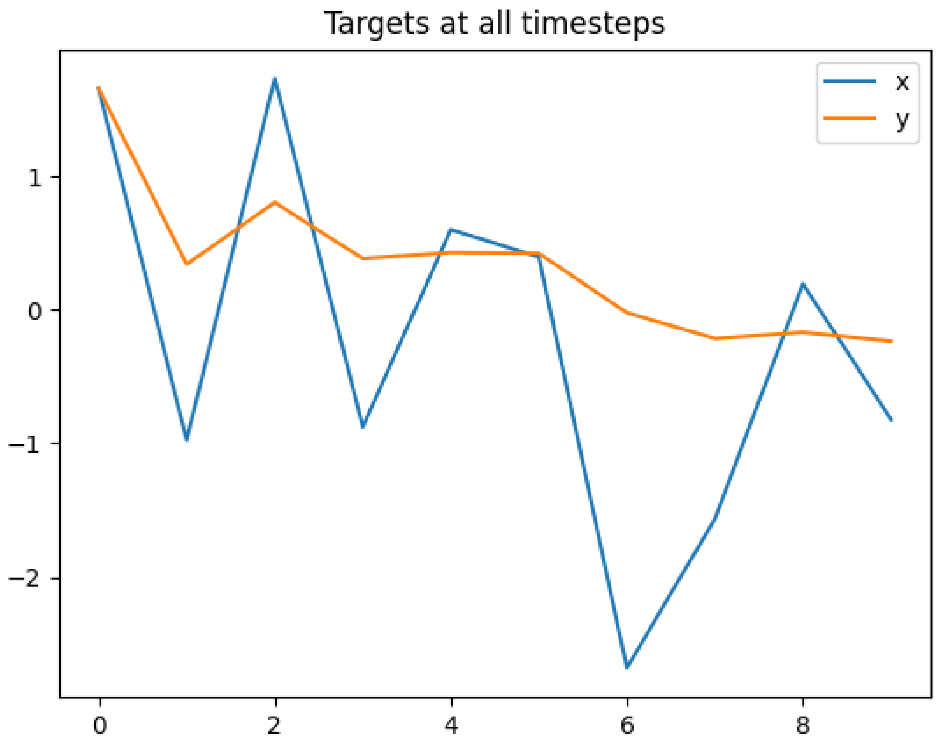
[7]:
```
def generate_batch(seq_len=10, batch_size=1):
  data = th.randn(size=(batch_size, seq_len, 1))
  sums = th.cumsum(data, dim=1)
  div = (th.arange(seq_len) + 1).unsqueeze(0).unsqueeze(2)
  target = sums / div
  return data, target
```

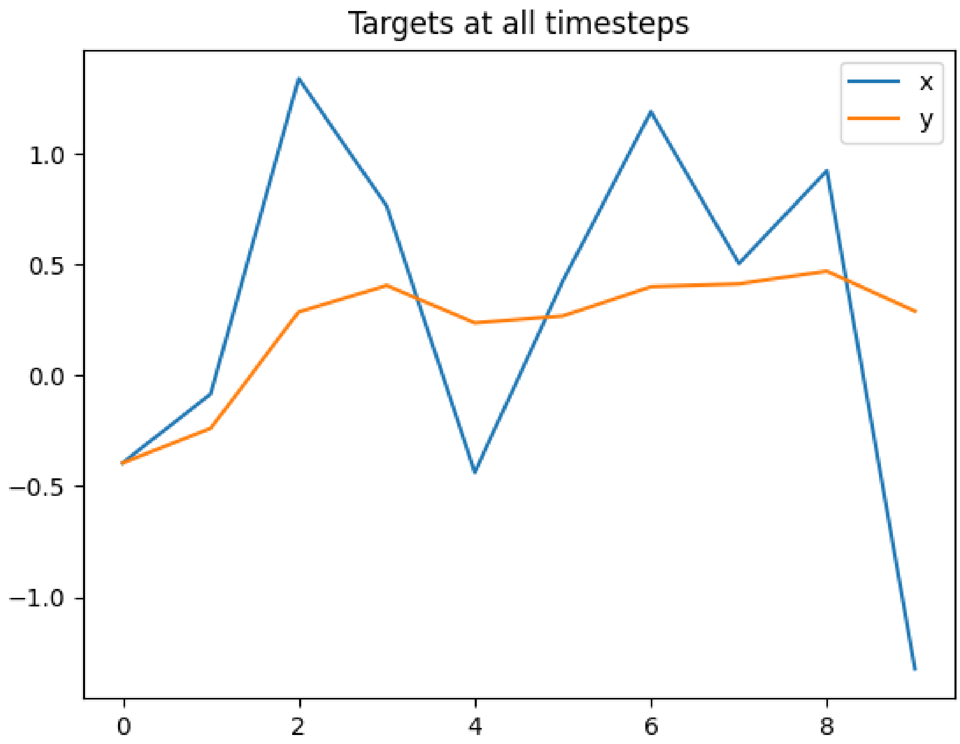
[8]:
```
x, y = generate_batch(seq_len=10, batch_size=4)
for i in range(4):
  fig, ax1 = plt.subplots(1)
  ax1.plot(x[i, :, 0])
  ax1.plot(y[i, :, 0])
  ax1.legend(['x', 'y'])
  plt.title('Targets at all timesteps')
  plt.show()

for i in range(4):
  fig, ax1 = plt.subplots(1)
  ax1.plot(x[i, :, 0])
  ax1.plot(np.arange(10), [y[i, -1].item()] * 10)
  ax1.legend(['x', 'y'])
  plt.title('Predict only at the last timestep')
  plt.show()
```

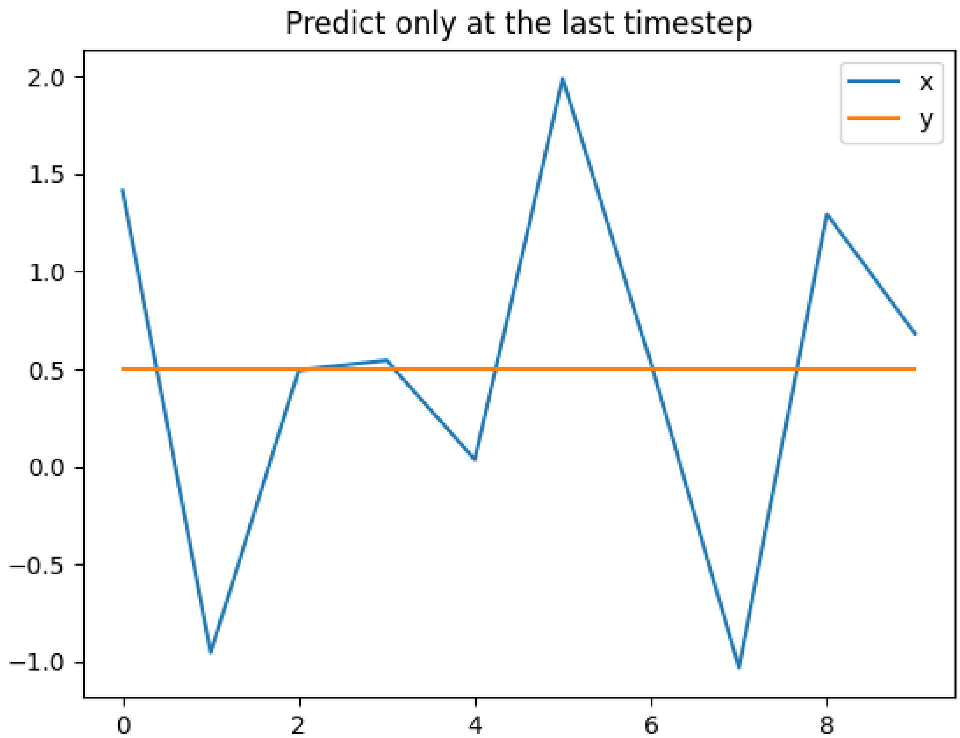


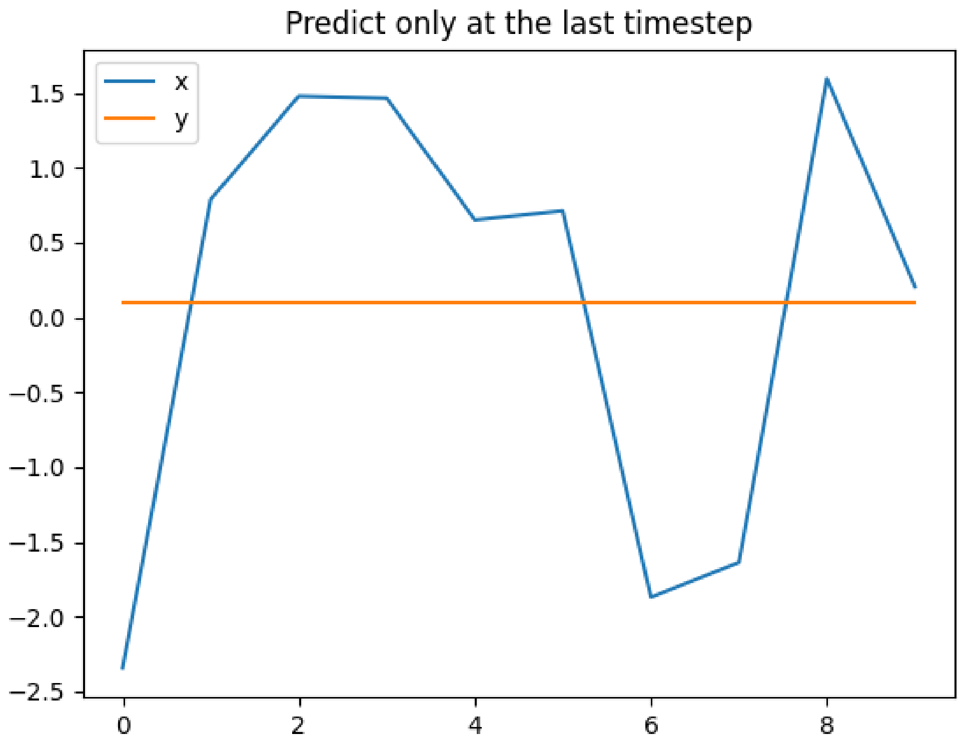
Targets at all timesteps
x
y
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8

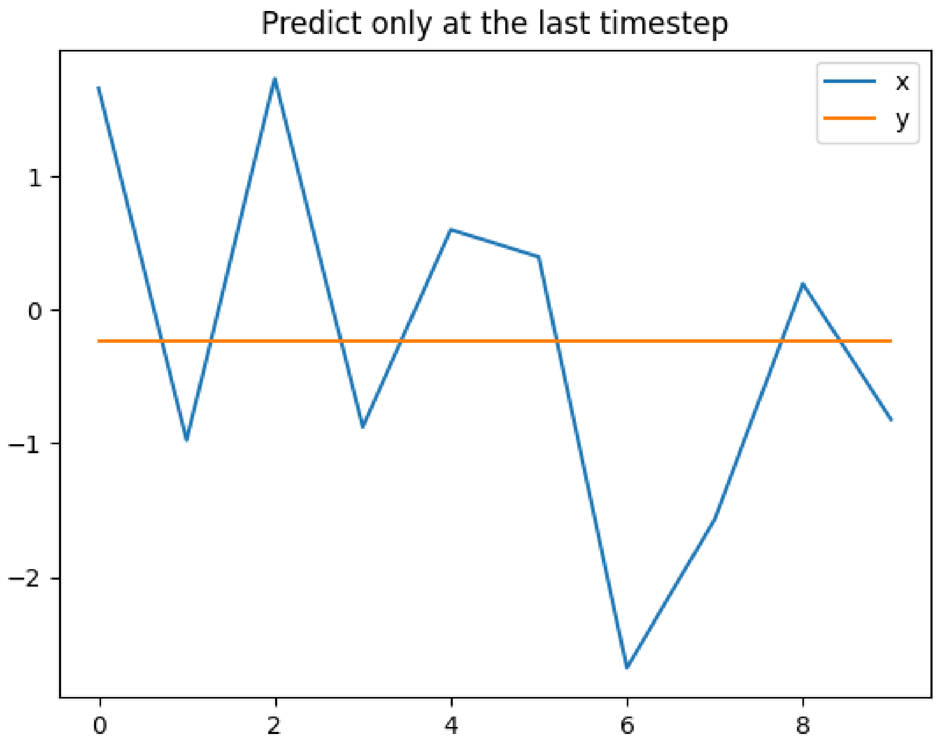

Targets at all timesteps
x
y
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
2
4
6
8

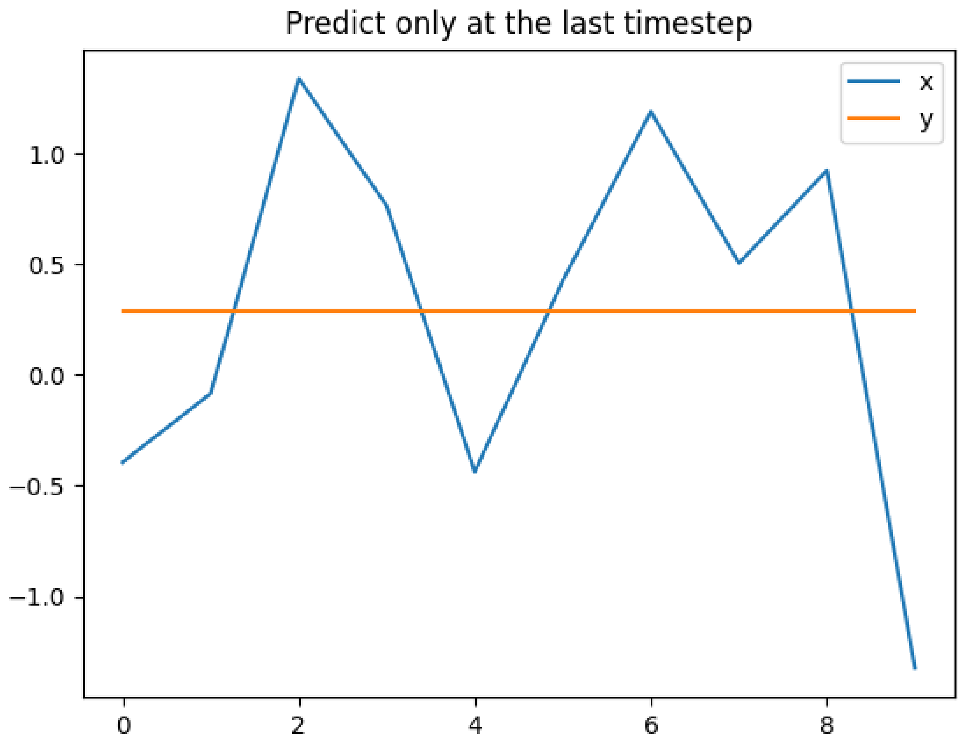

Targets at all timesteps
x
y
1
0
−1
−2
0
2
4
6
8


Targets at all timesteps
x
y
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8


Predict only at the last timestep
x
y
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8


Predict only at the last timestep
x
y
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
2
4
6
8


Predict only at the last timestep
x
y
1
0
−1
−2
0
2
4
6
8



5.2 1.C.ii Implement the loss function

[9]:
```
def loss_fn(pred, y, last_timestep_only=False):
  """
  Inputs:
  - pred: model predictions of size (batch, seq_len, 1)
  - y: targets of size (batch, seq_len, 1)
  - last_timestep_only: boolean indicating whether to compute loss for all
    timesteps or only the lat

  Returns:
  - loss: scalar MSE loss between pred and true labels
  """
  ##############################################################################
  # TODO: implement the loss (HINT: look for pytorch's MSELoss function)       #
  ##############################################################################

  if last_timestep_only:
    pred = pred[:, -1]
    y = y[:, -1]
```

```
    loss = F.mse_loss(pred, y)

    ##############################################################################
    #                               END OF YOUR CODE                             #
    ##############################################################################
    return loss
```

5.2.1 Tests

You should see errors < 1e-4

[10]:
```
pred = th.FloatTensor([[.1, .2, .3], [.4, .5, .6]])
y = th.FloatTensor([[-1.1, -1.2, -1.3], [-1.4, -1.5, -1.6]])
loss_all = loss_fn(pred, y, last_timestep_only=False)
loss_last = loss_fn(pred, y, last_timestep_only=True)
assert loss_all.shape == loss_last.shape == th.Size([])
print(f'Max error loss_all: {th.abs(loss_all - th.tensor(3.0067)).item()}')
print(f'Max error loss_last: {th.abs(loss_last - th.tensor(3.7)).item()}')
```

```
Max error loss_all: 3.314018249511719e-05
Max error loss_last: 2.384185791015625e-07
```

6 1.D: Analyzing RNN Gradients

You do not need to understand the details of the GradientVisualizer class in order to complete this problem.

[12]:
```
def biggest_eig_magnitude(matrix):
    """
    Inputs: a square matrix
    Returns: the scalar magnitude of the largest eigenvalue
    """
    h, w = matrix.shape
    assert h == w, f'Matrix has shape {matrix.shape}, but eigenvalues can only be␣
 ↪computed for square matrices'
    eigs = th.linalg.eigvals(matrix)
    eig_magnitude = eigs.abs()
    eigs_sorted = sorted([i.item() for i in eig_magnitude], reverse=True)
    first_eig_magnitude = eigs_sorted[0]
    return first_eig_magnitude

class GradientVisualizer:

    def __init__(self, rnn, last_timestep_only):
        """
        Inputs:
        - rnn: rnn module
```

```
    - last_timestep_only: boolean indicating whether to compute loss for all
      timesteps or only the lat

    Returns:
    - loss: scalar MSE loss between pred and true labels
    """

    self.rnn = rnn
    self.last_timestep_only = last_timestep_only
    self.model = RecurrentRegressionModel(rnn)
    self.original_weights = copy.deepcopy(rnn.state_dict())

    # Generate a single batch to be used repeatedly
    self.x, self.y = generate_batch(seq_len=10)
    print(f'Data point: x={np.round(self.x[0, :, 0].detach().cpu().numpy(),␣
↪2)}, y={np.round(self.y.squeeze().detach().cpu().numpy(), 2)}')

  def plot_visuals(self):
    """ Generate plots which will be updated in realtime."""
    fig, (ax1, ax2) = plt.subplots(1, 2)
    ax1.set_title('RNN Outputs')
    ax1.set_xlabel('Unroll Timestep')
    ax1.set_ylabel('Hidden State Norm')
    ax1.set_ylim(-1, 5)
    plt_1 = ax1.plot(np.arange(1, 11), np.zeros(10) + 1)  # placeholder vals
    plt_1 = plt_1[0]

    ax2.set_title('Gradients')
    ax2.set_xlabel('Unroll Timestep')
    ax2.set_ylabel('RNN dLoss/d a_t Gradient Magitude')
    ax2.set_ylim( (10**-6,1e5) )
    ax2.set_yscale('log')
    # X-axis labels are reversed since the gradient flow is from later layers␣
↪to earlier layers
    ax2.set_xticks(np.arange(10), np.arange(10, 0, -1))
    plt_2 = ax2.plot(np.arange(10), np.arange(10) + 1)  # placeholder vals
    plt_2 = plt_2[0]
    self.fig = fig
    self.plots = [plt_1, plt_2]
    return plt_1, plt_2, fig

  # Main update function for interactive plot
  def update_plots(self, weight_val=0, bias_val=0):
    # Scale the original RNN weights by a constant
    w_dict = copy.deepcopy(self.original_weights)
   ␣
↪###############################################################################
```

```
    # TODO: Scale all W matrixes by weight_val, and all bias matrices by␣
↪bias_val#
    # If you're using PyTorch nn.Linear layers, you don't need to modify the␣
↪code#
    # provided, but if you're using custom layers, modify this block.           ␣
↪ #
    ␣
↪################################################################################

    for key in w_dict:
      if "weight" in key:
        w_dict[key] *= weight_val
      elif "bias" in key:
        w_dict[key] *= bias_val

    ␣
↪################################################################################
    #                              END OF YOUR CODE                              ␣
↪ #
    ␣
↪################################################################################
    self.rnn.load_state_dict(w_dict)

    # Don't compute for LSTMs, which don't have behavior dependent on a single␣
↪eigenvalue
    if isinstance(self.rnn, RNNLayer):
      ␣
↪################################################################################
      # TODO: Set W = the weight which most affects exploding/vanishing␣
↪gradients  #
      # Hint: Call module.weight or module.bias on the module you want to use  ␣
↪   #
      # If you used a single Linear layer, slice a square matrix from it.       ␣
↪   #
      ␣
↪################################################################################

      for name, param in self.rnn.named_parameters():
        if 'hidden_linear.weight' in name:
          W = param.detach().cpu()
          break

      ␣
↪################################################################################
      #                              END OF YOUR CODE                              ␣
↪   #
```

```
    ␣
↪#########################################################################
     biggest_eig = biggest_eig_magnitude(W)
     print(f' Biggest eigenvalue magnitude: {biggest_eig:.3}')

   # Run model
   pred, h = self.model(self.x)
   loss = loss_fn(pred, self.y, self.last_timestep_only)
   n_steps = len(h[0])

   plt_1, plt_2 = self.plots

   # Plot the hidden state magnitude
   max_h = th.linalg.norm(h[0], dim=-1).detach().cpu().numpy()
   print('Max H', ' '.join([f'{num:.3}' for num in max_h]))
   plt_1.set_data(np.arange(1, n_steps + 1), np.array(max_h))
   # Compute the gradient for the loss wrt the stored hidden states
   # Gradients are plotted backward since we go from later layers to earlier
   grads = [th.linalg.norm(num).item() for num in th.autograd.grad(loss, self.
↪rnn.h_list)][::-1]
   print('gradients d Loss/d h_t', ' '.join([f'{num:.3}' for num in grads]))
   # Add 1e-6 since it throws an error for gradients near 0
   plt_2.set_data(np.arange(n_steps), np.array(grads) + 1e-6)
   self.fig.canvas.draw_idle()

 def create_visualization(self):
   # Include sliders for relevant quantities
   self.plot_visuals()
   ip = interactive(self.update_plots,
                    weight_val=widgets.FloatSlider(value=0, min=-5, max=5,␣
↪step=.05, description="weight_scale", layout=Layout(width='100%')),
                    bias_val=widgets.FloatSlider(value=0, min=-5, max=5, step=.
↪05, description="bias_scale", layout=Layout(width='100%')),
                    )
   return ip
```

Adjust the sliders rescale the weight and bias parameters in the RNN. Observe the effect on exploding and vanishing gradients.

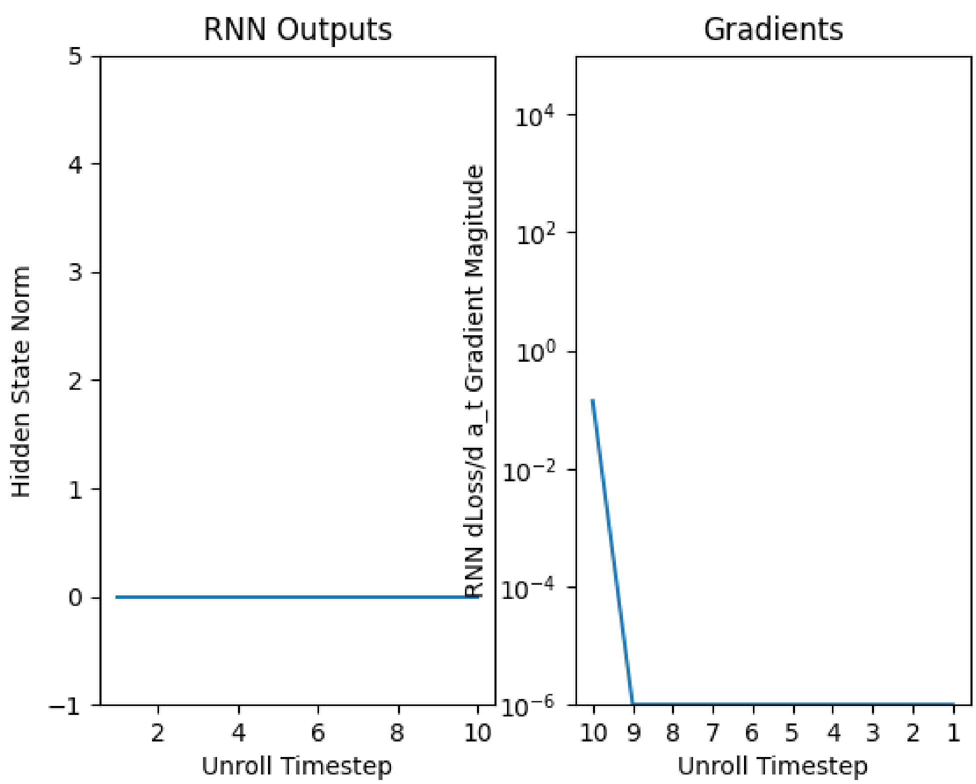
Parameters to try varying: * nonlinearity * last_target_only

[13]:
```
hidden_size = 16
nonlinearity = lambda x: x  # options include lambda x: x (no nonlinearity), nn.
↪functional.relu, and th.tanh
last_target_only = True
rnn = RNNLayer(1, hidden_size, nonlinearity=nonlinearity)
gv = GradientVisualizer(rnn, last_target_only)
gv.create_visualization()
```

```
# If for some reason the slider doesn't work for you, try calling gv.
↪update_plots
# with various values for weight and bias
```

```
Data point: x=[ 0.51 -0.08 -0.27 -0.03 -1.71 -0.09  1.24 -0.25 -0.33 -0.33], y=[
0.51  0.22  0.05  0.03 -0.32 -0.28 -0.06 -0.09 -0.11 -0.14]
```

[13]:
```
interactive(children=(FloatSlider(value=0.0, description='weight_scale',
layout=Layout(width='100%'), max=5.0,…
```



7 Problem 1.H: Implementing a single-layer LSTM

Hint: consider creating parameters using Pytorch's nn.Linear. You can implement this with either one Linear layer or two for each equation. If you use two, remember that you only need to include a bias term for one of the linear layers.

[14]:
```
class LSTMLayer(nn.Module):
  def __init__(self, input_size, hidden_size):
    """
    Initialize a single LSTM layer.
```

```
        Inputs:
        - input_size: Data input feature dimension
        - hidden_size: RNN hidden state size (also the output feature dimension)
        """
        super().__init__()
        self.input_size = input_size
        self.hidden_size = hidden_size
       ␣
     ↪##############################################################################
        # TODO: Initialize any parameters your class needs.                          ␣
     ↪ #
       ␣
     ↪##############################################################################

        self.input_linear = nn.Linear(input_size, 4 * hidden_size, bias=True)
        self.hidden_linear = nn.Linear(hidden_size, 4 * hidden_size, bias=False)


       ␣
     ↪##############################################################################
        #                               END OF YOUR CODE                             ␣
     ↪ #
       ␣
     ↪##############################################################################

    def forward(self, x):
        """
        LSTM forward pass

        Inputs:
        - x: input tensor (B, seq_len, input_size)

        Returns:
        - all_h: tensor of size (B, seq_len, hidden_size) containing hidden states
                produced for each timestep
        - (h_last, c_last): hidden and cell states from the last timestep, each of
                size (B, hidden_size)
        """
        h_list = []
       ␣
     ↪##############################################################################
        # TODO: Implement the LSTM forward step                                      ␣
     ↪ #
        # 1. Initialize the hidden and cell states with zeros                        ␣
     ↪ #
```

```
    # 2. Roll out the LSTM over the sequence, populating h_list along the way  ␣
↪ #
    # 3. Return the appropriate outputs                                          ␣
↪ #
  ␣
↪###############################################################################

    B, seq_len, _ = x.shape
    h_t = th.zeros(B, self.hidden_size, device=x.device)
    c_t = th.zeros(B, self.hidden_size, device=x.device)

    for t in range(seq_len):
      x_t = x[:, t, :]

      gates = self.input_linear(x_t) + self.hidden_linear(h_t)
      i_t, f_t, o_t, c_hat_t = gates.chunk(4, dim=1)

      i_t = th.sigmoid(i_t)
      f_t = th.sigmoid(f_t)
      o_t = th.sigmoid(o_t)
      c_hat_t = th.tanh(c_hat_t)

      c_t = f_t * c_t + i_t * c_hat_t
      h_t = o_t * th.tanh(c_t)

      h_list.append(h_t)
      h_last, c_last = h_t, c_t

  ␣
↪###############################################################################
    #                               END OF YOUR CODE                             ␣
↪ #
  ␣
↪###############################################################################

    # h_list should now contain all hidden states, each of size (B, hidden_size)
    # We will store the hidden states so we can analyze their gradients later
    self.store_h_for_grad(h_list)
    all_h = th.stack(h_list, dim=1)

    return all_h, (h_last, c_last)

  def store_h_for_grad(self, h_list):
    """
    Store input list and allow gradient computation for all list elements
    """
```

```
    for h in h_list:
      h.retain_grad()
    self.h_list = h_list
```

7.0.1 Test Cases

A correct implementation should have errors < 1e-4.

[15]:
```
lstm = LSTMLayer(2, 3)
lstm.load_state_dict({k: v * 0 - .1 for k, v in lstm.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
 ↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_all_h = th.FloatTensor([[[-0.0273, -0.0273, -0.0273],
         [-0.0420, -0.0420, -0.0420],
         [-0.0514, -0.0514, -0.0514],
         [-0.0583, -0.0583, -0.0583]],

        [[ 0.0159,  0.0159,  0.0159],
         [ 0.0568,  0.0568,  0.0568],
         [ 0.1142,  0.1142,  0.1142],
         [ 0.0369,  0.0369,  0.0369]]])
expected_last_h = th.FloatTensor([[-0.0583, -0.0583, -0.0583],
        [ 0.0369,  0.0369,  0.0369]])
expected_last_c = th.FloatTensor([[-0.1280, -0.1280, -0.1280],
        [ 0.0759,  0.0759,  0.0759]])
all_h, (last_h, last_c) = lstm(data)
assert all_h.shape == (2, 4, 3)
assert last_h.shape == last_c.shape == (2, 3)
print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_h - last_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_c - last_c)).item()}')
```

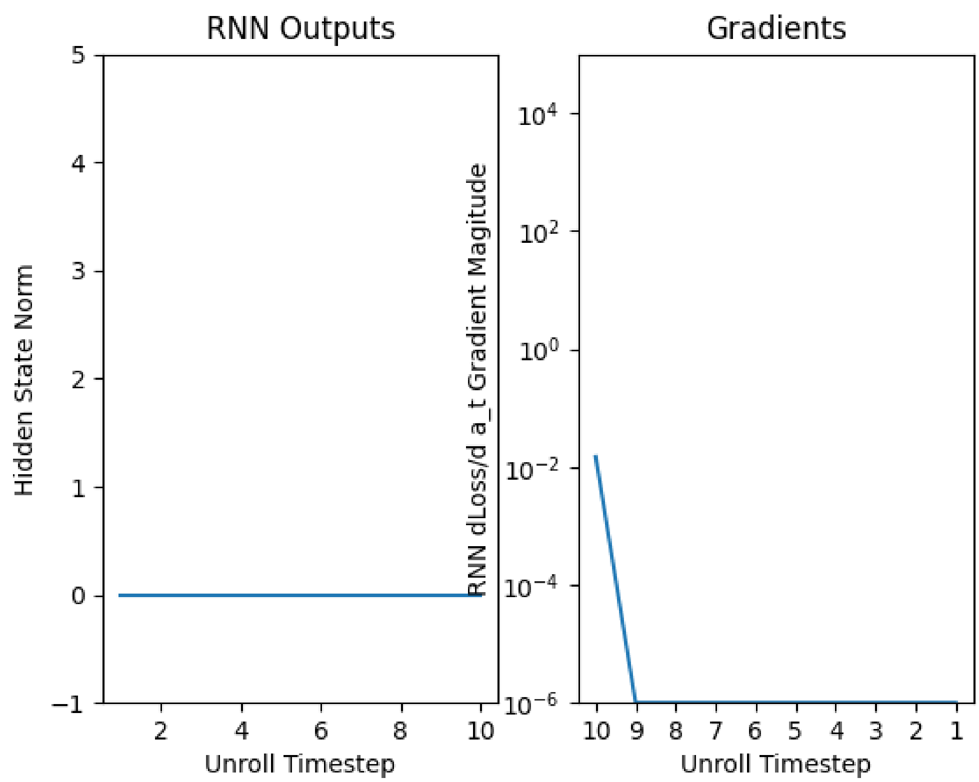
```
Max error all_h: 4.824250936508179e-05
Max error last_h: 4.824250936508179e-05
Max error last_h: 8.031725883483887e-06
```

7.1 Problem 1.8b: Analyzing gradient flow through a single-layer LSTM

[16]:
```
hidden_size = 3
last_target_only = True
rnn = LSTMLayer(1, hidden_size)
gv = GradientVisualizer(rnn, last_target_only)
gv.create_visualization()
```

```
Data point: x=[ 0.61 -1.25  0.2  -1.03  0.27  1.66  0.19  0.18 -1.67  1.64], y=[
0.61 -0.32 -0.15 -0.37 -0.24  0.08  0.09  0.1  -0.09  0.08]
```

[16]: interactive(children=(FloatSlider(value=0.0, description='weight_scale', layout=Layout(width='100%'), max=5.0,…



8 Problem 1.K: Making a multi-layer RNN and LSTM

8.1 1.K.i: Implementing multi-layer models

[17]:
```
class RNN(nn.Module):
  def __init__(self, input_size, hidden_size, num_layers):
    """
    Initialize a multilayer RNN

    Inputs:
    - input_size: Data input feature dimension
    - hidden_size: hidden state size (also the output feature dimension)
    - num_layers: number of layers
    """
    super().__init__()
    assert num_layers >= 1
    self.input_size = input_size
```

```
    self.hidden_size = hidden_size
    self.num_layers = num_layers
   ␣
↪###########################################################################
    # TODO: Initialize any parameters your class needs.                       ␣
↪ #
    # Consider using nn.ModuleList or nn.ModuleDict.                          ␣
↪ #
   ␣
↪###########################################################################

    self.layers = nn.ModuleList()
    self.layers.append(RNNLayer(input_size, hidden_size))
    for _ in range(num_layers - 1):
      self.layers.append(RNNLayer(hidden_size, hidden_size))


   ␣
↪###########################################################################
    #                              END OF YOUR CODE                           ␣
↪ #
   ␣
↪###########################################################################

  def forward(self, x):
    """
    Multilayer RNN forward pass

    Inputs:
    - x: input tensor (B, seq_len, input_size)

    Returns:
    - last_layer_h: tensor of size (B, seq_len, hidden_size) containing the
              outputs produced for each timestep from the last layer
    - last_step_h: all hidden states from the last step (num_layers, B,␣
↪hidden_size)
    """
   ␣
↪###########################################################################
    # TODO: Implement the RNN forward step                                    ␣
↪ #
   ␣
↪###########################################################################

    layer_input = x
    B, seq_len, _ = x.shape
    h_list = []
```

```
    for i, layer in enumerate(self.layers):
      all_h, last_h = layer(layer_input)
      h_list.append(last_h)
      layer_input = all_h

    last_layer_h = all_h
    last_step_h = th.stack(h_list, dim=0)

   ␣
 ↪###############################################################################
    #                                 END OF YOUR CODE                          ␣
 ↪ #
   ␣
 ↪###############################################################################
    return last_layer_h, last_step_h


class LSTM(nn.Module):
  def __init__(self, input_size, hidden_size, num_layers):
    """
    Initialize a multilayer LSTM

    Inputs:
    - input_size: Data input feature dimension
    - hidden_size: hidden state size (also the output feature dimension)
    - num_layers: number of layers
    """
    super().__init__()
    assert num_layers >= 1
    layers = [LSTMLayer(input_size, hidden_size)]
    for i in range(num_layers - 1):
      layers.append(LSTMLayer(hidden_size, hidden_size))
    self.layers = nn.ModuleList(layers)
    self.hidden_size = hidden_size
    self.num_layers = num_layers
   ␣
 ↪###############################################################################
    #                                 END OF YOUR CODE                          ␣
 ↪ #
   ␣
 ↪###############################################################################

  def forward(self, x, hc0=None):
    """
    Multilayer LSTM forward pass
```

```
    Inputs:
    - x: input tensor (B, seq_len, input_size)

    Returns:
    - last_layer_h: tensor of size (B, seq_len, hidden_size) containing the
              outputs produced for each timestep from the last layer
    - (last_step_h, last_step_c): all hidden and cell states from the last step
            size (num_layers, B, hidden_size)
    """
   ␣
↪##############################################################################
    # TODO: Implement the LSTM forward step                                    ␣
↪ #
   ␣
↪##############################################################################

    B, seq_len, _ = x.shape

    if hc0 is None:
        h_t = th.zeros(self.num_layers, B, self.hidden_size, device=x.device)
        c_t = th.zeros(self.num_layers, B, self.hidden_size, device=x.device)
    else:
        h_t, c_t = hc0

    h_list, c_list = [], []

    layer_input = x
    for i, layer in enumerate(self.layers):
      h_i = h_t[i]
      c_i = c_t[i]
      all_h, (last_h, last_c) = layer(layer_input)
      h_list.append(last_h)
      c_list.append(last_c)
      layer_input = all_h

    last_layer_h = all_h
    last_step_h = th.stack(h_list, dim=0)
    last_step_c = th.stack(c_list, dim=0)

   ␣
↪##############################################################################
    #                              END OF YOUR CODE                            ␣
↪ #
   ␣
↪##############################################################################
```

```
    return last_layer_h, (last_step_h, last_step_c)
```

8.1.1 Test Cases

[18]:
```
rnn = RNN(2, 3, 1)
rnn.load_state_dict({k: v * 0 - .1 for k, v in rnn.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_all_h = th.FloatTensor([[[-0.1244, -0.1244, -0.1244],
         [-0.1073, -0.1073, -0.1073],
         [-0.1320, -0.1320, -0.1320],
         [-0.1444, -0.1444, -0.1444]],

        [[ 0.0599,  0.0599,  0.0599],
         [ 0.1509,  0.1509,  0.1509],
         [ 0.2305,  0.2305,  0.2305],
         [-0.0840, -0.0840, -0.0840]]])
expected_last_h = th.FloatTensor([[[-0.1444, -0.1444, -0.1444],
         [-0.0840, -0.0840, -0.0840]]])
all_h, last_h = rnn(data)
assert all_h.shape == expected_all_h.shape
assert last_h.shape == expected_last_h.shape
print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_h - last_h)).item()}')

rnn = RNN(2, 3, 2)
rnn.load_state_dict({k: v * 0 - .1 for k, v in rnn.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_all_h = th.FloatTensor([[[-0.0626, -0.0626, -0.0626],
         [-0.0490, -0.0490, -0.0490],
         [-0.0457, -0.0457, -0.0457],
         [-0.0430, -0.0430, -0.0430]],
        [[-0.1174, -0.1174, -0.1174],
         [-0.1096, -0.1096, -0.1096],
         [-0.1354, -0.1354, -0.1354],
         [-0.0342, -0.0342, -0.0342]]])
expected_last_h = th.FloatTensor([[[-0.1444, -0.1444, -0.1444],
         [-0.0840, -0.0840, -0.0840]],
        [[-0.0430, -0.0430, -0.0430],
         [-0.0342, -0.0342, -0.0342]]])
all_h, last_h = rnn(data)
assert all_h.shape == (2, 4, 3)
assert last_h.shape == (2, 2, 3)
print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_h - last_h)).item()}')
```

```
lstm = LSTM(2, 3, 1)
lstm.load_state_dict({k: v * 0 - .1 for k, v in lstm.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_all_h = th.FloatTensor([[[-0.0273, -0.0273, -0.0273],
         [-0.0420, -0.0420, -0.0420],
         [-0.0514, -0.0514, -0.0514],
         [-0.0583, -0.0583, -0.0583]],

        [[ 0.0159,  0.0159,  0.0159],
         [ 0.0568,  0.0568,  0.0568],
         [ 0.1142,  0.1142,  0.1142],
         [ 0.0369,  0.0369,  0.0369]]])
expected_last_h = th.FloatTensor([[[-0.0583, -0.0583, -0.0583],
         [ 0.0369,  0.0369,  0.0369]]])
expected_last_c = th.FloatTensor([[[-0.1280, -0.1280, -0.1280],
         [ 0.0759,  0.0759,  0.0759]]])
all_h, (last_h, last_c) = lstm(data)
assert all_h.shape == (2, 4, 3)
assert last_h.shape == last_c.shape == (1, 2, 3)
print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_h - last_h)).item()}')
print(f'Max error last_c: {th.max(th.abs(expected_last_c - last_c)).item()}')


lstm = LSTM(2, 3, 3)
lstm.load_state_dict({k: v * 0 - .1 for k, v in lstm.state_dict().items()})
data = th.FloatTensor([[[.1, .15], [.2, .25], [.3, .35], [.4, .45]], [[-.1, -1.
↪5], [-.2, -2.5], [-.3, -3.5], [-.4, -.45]]])
expected_all_h = th.FloatTensor([[[-0.0212, -0.0212, -0.0212],
         [-0.0296, -0.0296, -0.0296],
         [-0.0329, -0.0329, -0.0329],
         [-0.0343, -0.0343, -0.0343]],
        [[-0.0211, -0.0211, -0.0211],
         [-0.0291, -0.0291, -0.0291],
         [-0.0320, -0.0320, -0.0320],
         [-0.0332, -0.0332, -0.0332]]])
expected_last_h = th.FloatTensor([[[-0.0583, -0.0583, -0.0583],
         [ 0.0369,  0.0369,  0.0369]],
        [[-0.0320, -0.0320, -0.0320],
         [-0.0430, -0.0430, -0.0430]],
        [[-0.0343, -0.0343, -0.0343],
         [-0.0332, -0.0332, -0.0332]]])
expected_last_c = th.FloatTensor([[[-0.1280, -0.1280, -0.1280],
         [ 0.0759,  0.0759,  0.0759]],
        [[-0.0666, -0.0666, -0.0666],
```

```
         [-0.0907, -0.0907, -0.0907]],
        [[-0.0716, -0.0716, -0.0716],
         [-0.0693, -0.0693, -0.0693]]])
all_h, (last_h, last_c) = lstm(data)
assert all_h.shape == (2, 4, 3)
assert last_h.shape == last_c.shape == (3, 2, 3)

print(f'Max error all_h: {th.max(th.abs(expected_all_h - all_h)).item()}')
print(f'Max error last_h: {th.max(th.abs(expected_last_h - last_h)).item()}')
print(f'Max error last_c: {th.max(th.abs(expected_last_c - last_c)).item()}')
```

```
Max error all_h: 4.699826240539551e-05
Max error last_h: 4.124641418457031e-05
Max error all_h: 4.3526291847229004e-05
Max error last_h: 4.124641418457031e-05
Max error all_h: 4.824250936508179e-05
Max error last_h: 4.824250936508179e-05
Max error last_c: 8.031725883483887e-06
Max error all_h: 4.732981324195862e-05
Max error last_h: 4.824250936508179e-05
Max error last_c: 4.2885541915893555e-05
```

8.2 1.K.ii: Training your model

[19]:
```
def train(model, optimizer, num_batches, last_timestep_only, seq_len=10,␣
 ↪batch_size=32):
  model
  model.train()

  losses = []
  from tqdm import tqdm
  t = tqdm(range(0, num_batches))
  for i in t:
      data, labels = generate_batch(seq_len=seq_len, batch_size=batch_size)
      pred, h = model(data)
      loss = loss_fn(pred, labels, last_timestep_only)
      losses.append(loss.item())

      optimizer.zero_grad()
      loss.backward()
      optimizer.step()
      if i % 100 == 0:
          t.set_description(f"Batch: {i} Loss: {np.mean(losses[-10:])}")
  return losses
```

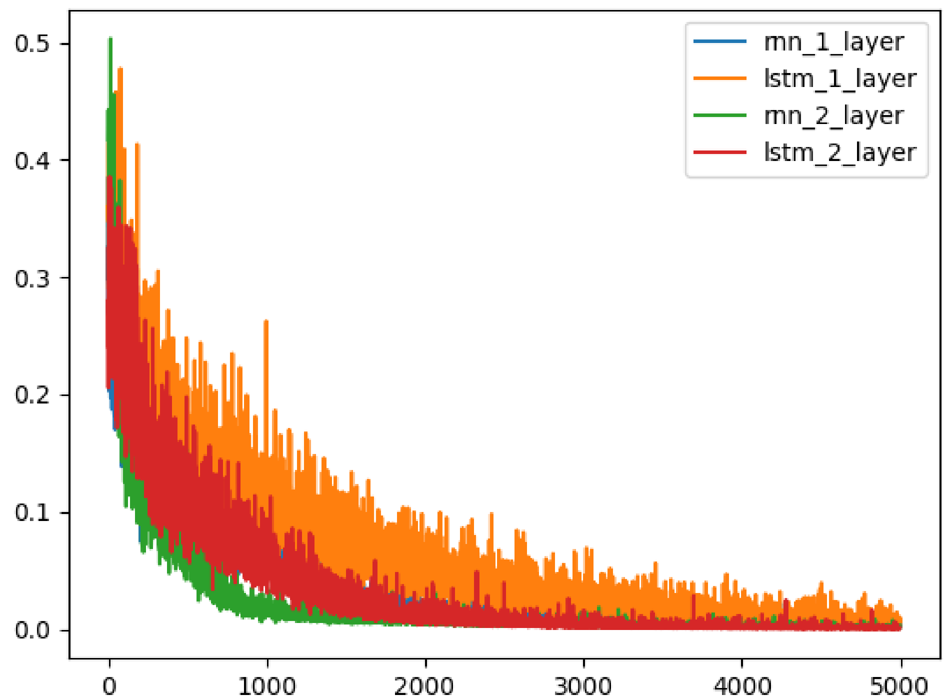
[20]:
```
def train_all(hidden_size, lr, num_batches, last_timestep_only):
  input_size = 1
```

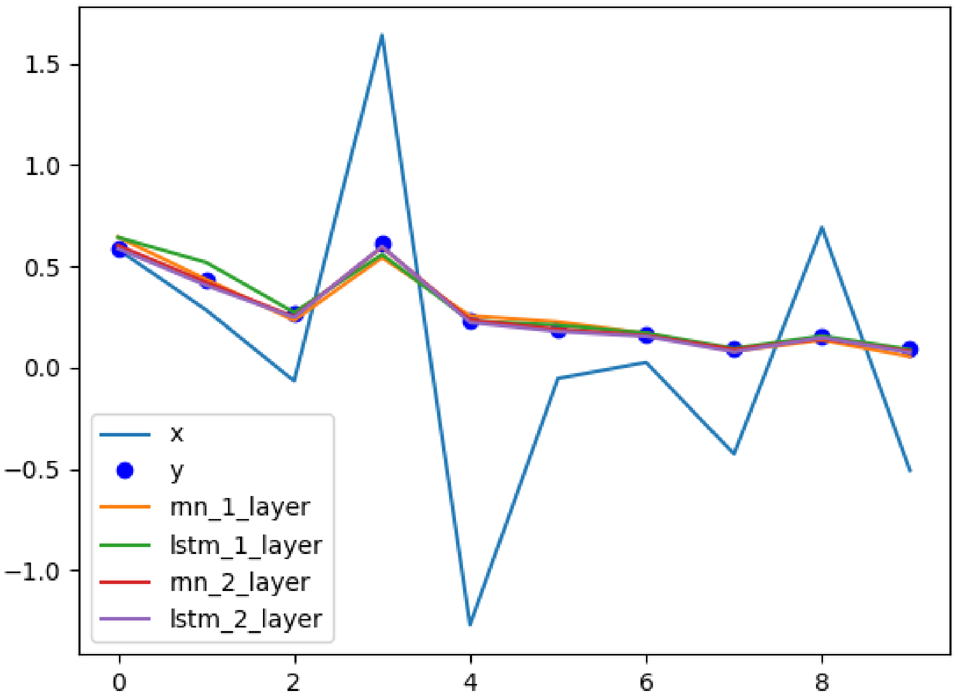
```
  rnn_1_layer = RecurrentRegressionModel(RNN(input_size, hidden_size, 1))
  lstm_1_layer = RecurrentRegressionModel(LSTM(input_size, hidden_size, 1))
  rnn_2_layer = RecurrentRegressionModel(RNN(input_size, hidden_size, 2))
  lstm_2_layer = RecurrentRegressionModel(LSTM(input_size, hidden_size, 2))
  models = [rnn_1_layer, lstm_1_layer, rnn_2_layer, lstm_2_layer]
  model_names = ['rnn_1_layer', 'lstm_1_layer', 'rnn_2_layer', 'lstm_2_layer']

  losses = []
  for model in models:
    optimizer = optim.Adam(model.parameters(), lr=lr)
    loss = train(model, optimizer, num_batches, last_timestep_only)
    losses.append(loss)

  # visualize the results
  fig, ax1 = plt.subplots(1)
  for loss in losses:
    ax1.plot(loss)
  ax1.legend(model_names)
  plt.show()

  batch_size = 4
  x, y = generate_batch(seq_len=10, batch_size=batch_size)
  preds_list = [model(x)[0] for model in models]
  for i in range(batch_size):
    fig, ax1 = plt.subplots(1)
    ax1.plot(x[i, :, 0])
    if last_timestep_only:
      ax1.plot(np.arange(10), [y[i, -1].item()] * 10, 'bo')
    else:
      ax1.plot(y[i, :, 0], 'bo')
    for pred in preds_list:
      if last_timestep_only:
        ax1.plot(np.arange(10), [pred[i, -1, 0].detach().cpu().numpy()] * 10)
      else:
        ax1.plot(pred[i, :, 0].detach().cpu().numpy())
    ax1.legend(['x', 'y'] + model_names)
    plt.show()
  return models, losses
```

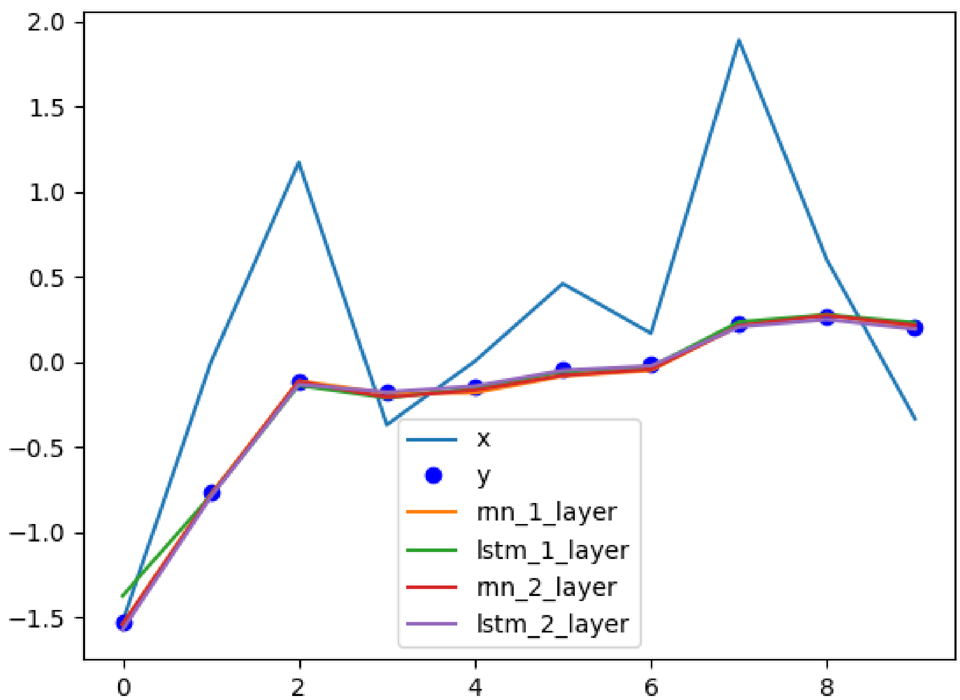
[21]:
```
hidden_size = 32
lr = 1e-4
num_batches = 5000
last_timestep_only = False

th.manual_seed(0)
predict_all_models, predict_all_losses = train_all(hidden_size, lr,␣
 ↪num_batches, last_timestep_only)
```

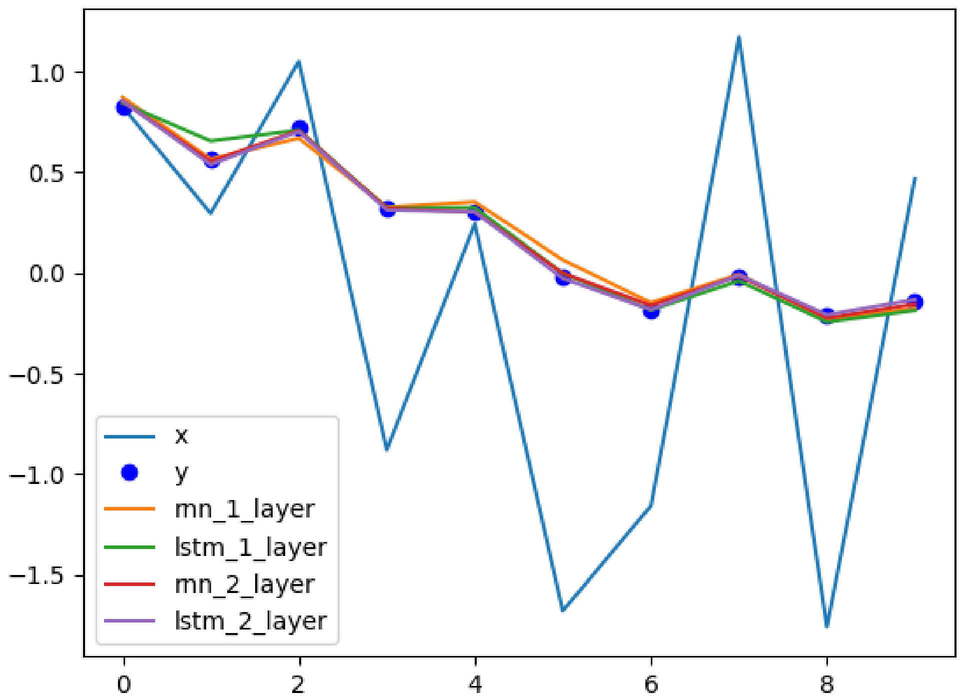
```
last_timestep_only = True
predict_one_models, predict_one_losses = train_all(hidden_size, lr,␣
↪num_batches, last_timestep_only)
```

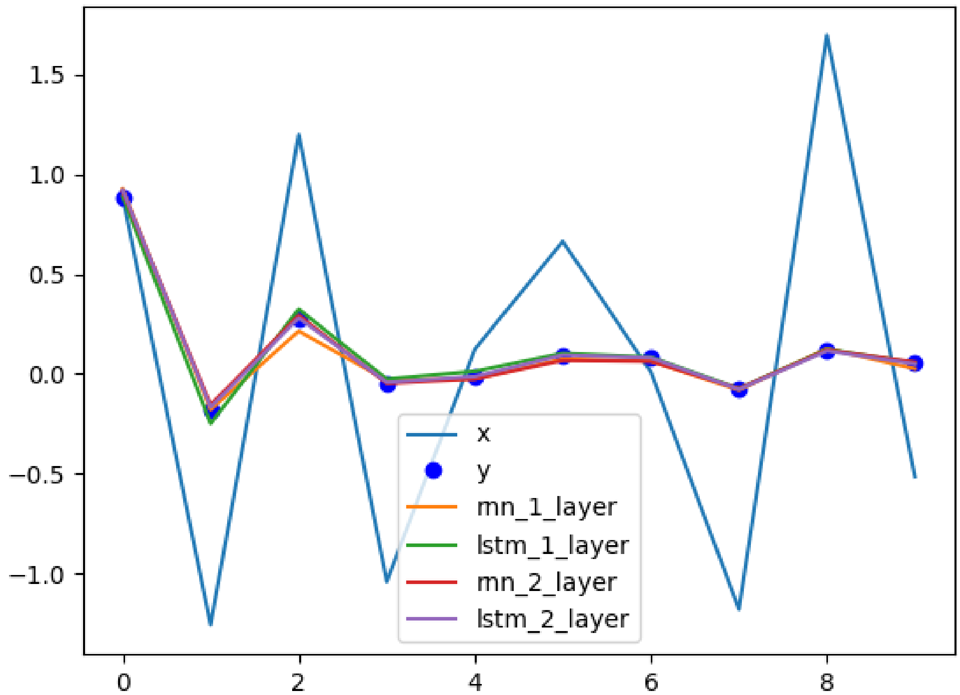
```
Batch: 4900 Loss: 0.00300768178422004: 100%|      | 5000/5000 [00:10<00:00,
478.01it/s]
Batch: 4900 Loss: 0.007475480530411005: 100%|      | 5000/5000 [00:28<00:00,
177.67it/s]
Batch: 4900 Loss: 0.0010111536423210055: 100%|      | 5000/5000
[00:17<00:00, 283.01it/s]
Batch: 4900 Loss: 0.0009569476067554205: 100%|      | 5000/5000
[01:13<00:00, 67.91it/s]
```



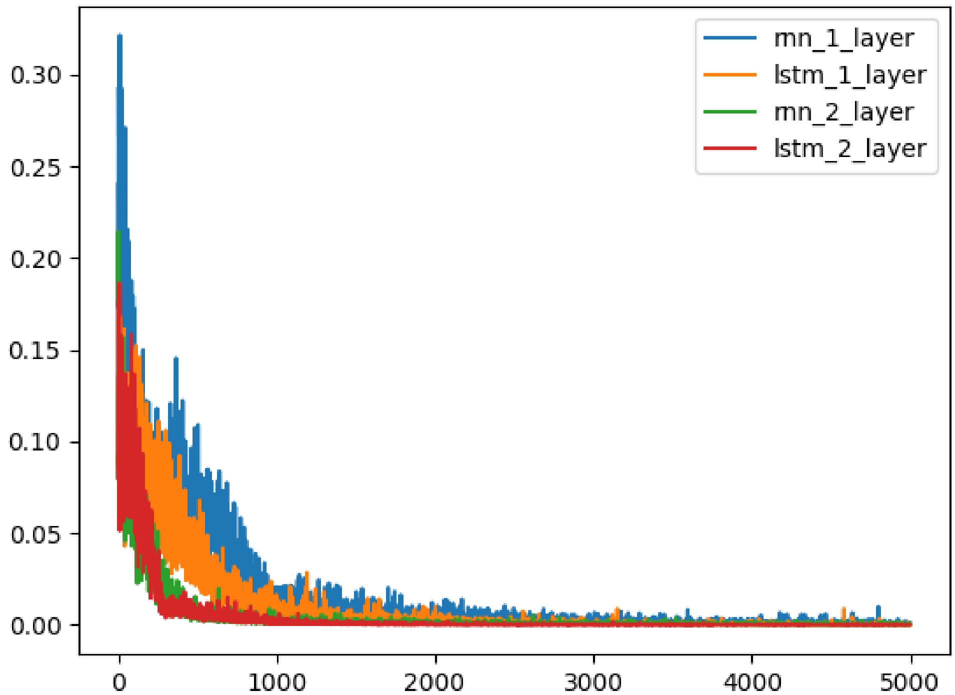

1.5
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer


x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8


1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer




```
Batch: 4900 Loss: 0.0006809785554651171: 100%|      | 5000/5000
[00:10<00:00, 482.97it/s]
Batch: 4900 Loss: 0.00023271610480151138: 100%|      | 5000/5000
[00:28<00:00, 174.80it/s]
Batch: 4900 Loss: 0.0003779473539907485: 100%|      | 5000/5000
[00:17<00:00, 278.81it/s]
Batch: 4900 Loss: 5.903654691792326e-05: 100%|      | 5000/5000
[01:20<00:00, 62.38it/s]
```


rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer
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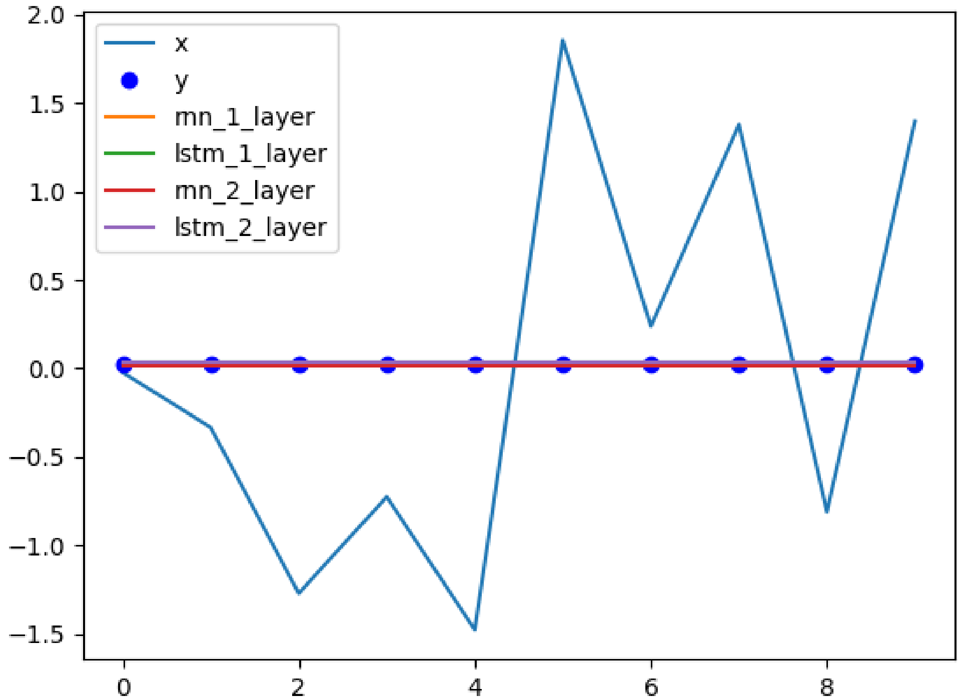
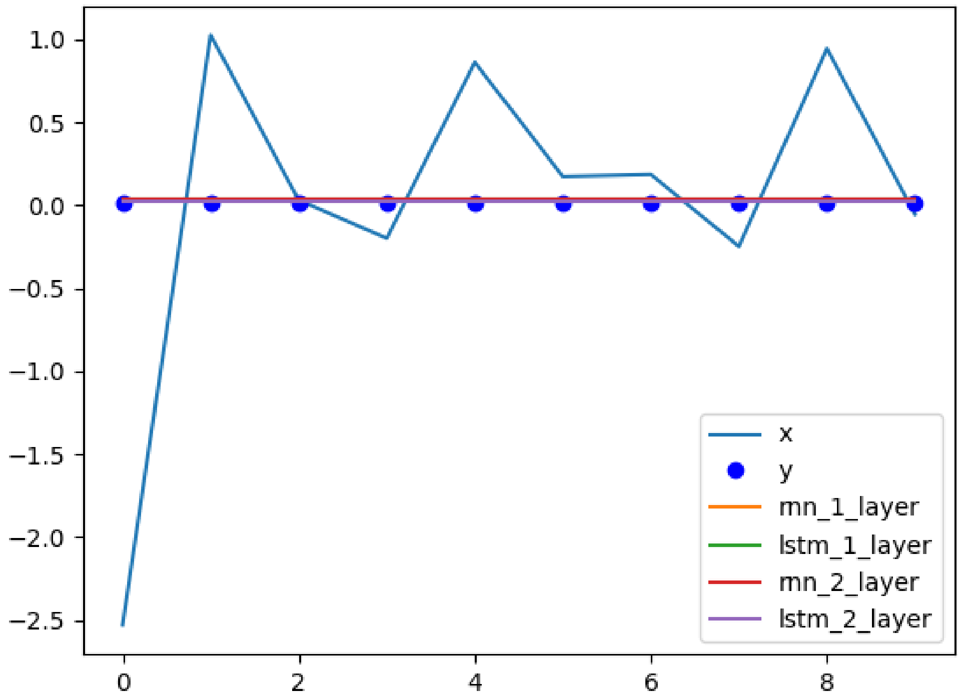
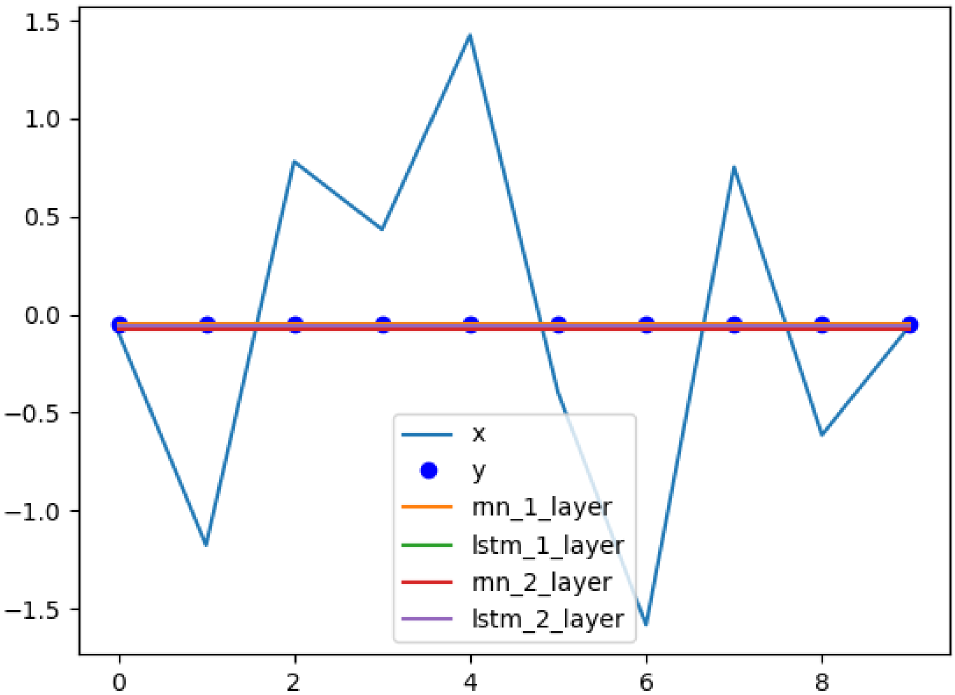
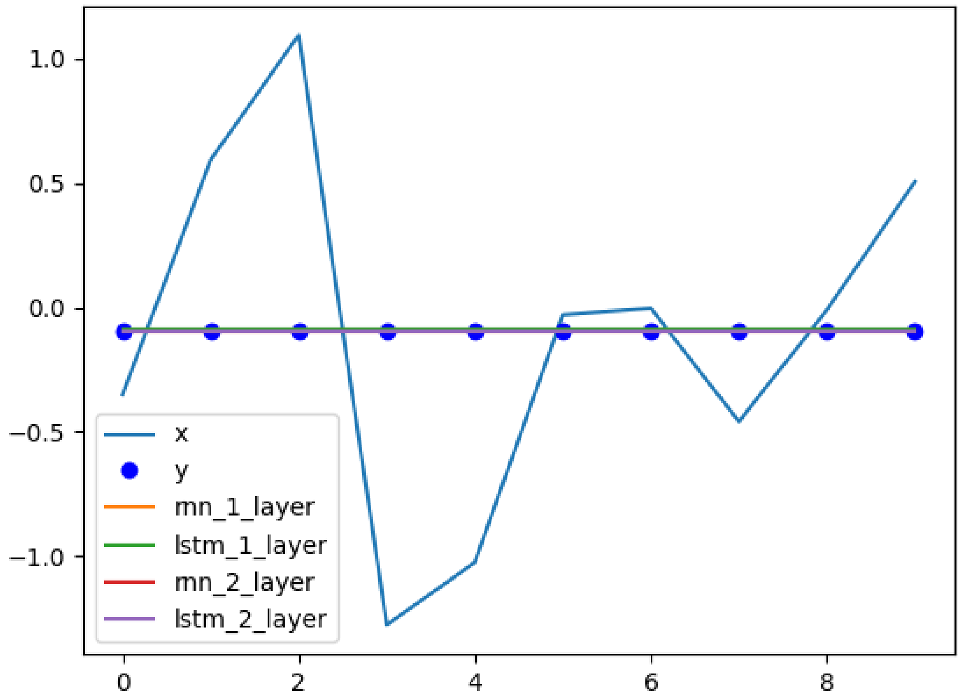
0.25
0.20
0.15
0.10
0.05
0.00
0
1000
2000
3000
4000
5000


x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8


1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
2
4
6
8
x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer


1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
x
y
rnn_1_layer
lstm_1_layer
rnn_2_layer
lstm_2_layer