Activity3:
Signals, Fourier, and Convolution

Goal

In this assignment, you will apply convolutions and Fourier transforms and their different rules to signals.

Instructions

You don't need to hand in anything for this Activity.

Solution key to the questions will be released one week after.

Q1. Convolving signals

You are given the following four 1D signals.
The underline indicates the value at the origin (e.g. at time zero).

S1 = [0 0 0 $\underline{1}$ 0 0 0]; % the origin is the 4th entry
S2 = [0 1 $\underline{0}$ 0 0 1 0]; % the origin is the 3rd entry
S3 = [0 4 0 $\underline{1}$ 0 1 0]; % the origin is the 4th entry
S4 = [10 50 $\underline{60}$ 10 20 40 30]; % the origin is the 3rd entry.

You have seen in class different ways to compute the convolution operation between two discrete signals. In this part, you will use the 3 following approaches to convolve signals S1 to S4:
* Method A: Using the discrete convolution equation
* Method B: Flip-Shift-Multiply-Add
* Method C: Convolving with impulses

Convolve each possible pair of signals S1 to S4 using the indicated method below. Make sure to highlight the value at the origin of the output signal with an underline. Show clearly the different steps leading to your answer.

a) S1 with S1 using Method A
b) S1 with S2 using Method B
c) S1 with S3 using Method C
d) S1 with S4 using Method A

e) S2 with S1 using whatever method or justification
f) S2 with S2 using Method B
g) S2 with S3 using Method C
h) S2 with S4 using Method A

i) S3 with S1 using whatever method or justification
j) S3 with S2 using whatever method or justification
k) S3 with S3 using Method B
l) S3 with S4 using Method C

m) S4 with S1 using Method A
n) S4 with S2 using Method B
o) S4 with S3 using Methods A, B, and C (all three methods).
p) S4 with S4 using Method C

Q2. Fourier Transform

Sketch (by hand) each of the following temporal signals at the range of "t" indicated after each signal and provide a justification. The $\otimes$ sign corresponds to the convolution operation.

a) $s_1(t) = \sin(2\pi t)$ (show for $t \in [-5,5]$)

b) $s_2(t) = \sin(2\pi t) + 4\sin(2\pi t)$ (show for $t \in [-5,5]$)

c) $s_3(t) = \begin{cases} 1 & if \quad -3 \le t \le 3 \\ 0 & otherwise \end{cases}$ (show for $t \in [-5,5]$)

d) $s_4(t) = \frac{\sin(\pi t)}{\pi t}$ (show for $t \in [-2,2]$)

e) $s_5(t) = \begin{cases} 1 \ if \ \frac{t}{5} = round\left(\frac{t}{5}\right) \\ 0 \quad otherwise \end{cases}$ (show for $t \in [-10,10]$)

f) $s_6(t) = s_2(t) + s_4(t)$ (show for $t \in [-5,5]$)

g) $s_7(t) = s_3(t) \otimes s_1(t)$ (show for $t \in [-5,5]$)

h) $s_8(t) = s_2(t) \otimes s_4(t)$ (show for $t \in [-2,2]$)

i) $s_9(t) = s_2(3t)$ (show for $t \in [-5,5]$)

j) $s_{10}(t) = s_2(0.5t)$ (show for $t \in [-5,5]$)

k) $s_{11}(t) = s_4(t) \times s_5(t)$ (show for $t \in [-10,10]$)

The following are Fourier Transforms of different signal (f: frequency; w: radian frequency; FT: Fourier transform; and IFT: inverse FT). Sketch (by hand) each of the following spectra at the range of "f" or "w" indicated after each signal:

a) $X_1(f) = \begin{cases} -f^2 + 16 & if\ |f| < 4 \\ 0 & otherwise \end{cases}$ (show for $f \in [-10,10]$)

b) $X_2(w) = \begin{cases} 4 & if\ -3\pi < w < 3\pi \\ 0 & otherwise \end{cases}$ (show for $w \in [-5\pi, 5\pi]$)

c) $X_3(f) = \begin{cases} -4|f| + 2 & if\ -0.5 < f < 0.5 \\ 0 & otherwise \end{cases}$ (show for $f \in [-2,2]$)

d) $X_4(f) = \begin{cases} 1 & if\ \frac{f}{2} = round\left(\frac{f}{2}\right) \\ 0 & otherwise \end{cases}$ (show for $f \in [-9,9]$)

e) $X_5(w) = X_2(3w)$ (show for $w \in [-5\pi, 5\pi]$)

f) $X_6(f) = FT\big(\ IFT\big(X_1(f)\big) \otimes\ IFT\big(\ X_2(f)\big)\ \big)$ (show for $f \in [-5,5]$)

g) $X_7(f) = FT\big(\ IFT\big(X_2(f)\big) \otimes\ IFT\big(\ X_3(f)\big)\ \big)$ (show for $f \in [-4,4]$)

h) $X_8(f) = FT\big(\ IFT\big(X_3(f)\big) \otimes\ s_5(t)\ \big)$ % $s_5(t)$ is the temporal signal defined in Q1. (show for $f \in [-5,5]$)

i) $X_9(f) = FT\big(\ IFT\big(X_3(f)\big) \otimes s_5(0.1t)\ \big)$ (show for $f \in [-5,5]$)