PrivFL: Practical Privacy-preserving Federated Regressions on High-dimensional Data over Mobile Networks

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ABSTRACT

Federated Learning (FL) enables a large number of users to jointly learn a shared machine learning (ML) model, coordinated by a centralized server, where the data is distributed across multiple devices. This approach enables the server or users to train and learn an ML model using gradient descent, while keeping all the training data on users' devices. We consider training an ML model over a mobile network where *user dropout* is a common phenomenon. Although federated learning was aimed at reducing data privacy risks, the ML model privacy has not received much attention.

In this work, we present PrivFL, a privacy-preserving system for training (predictive) linear and logistic regression models and oblivious predictions in the federated setting, while guaranteeing data and model privacy as well as ensuring robustness to users dropping out in the network. We design two privacy-preserving protocols for training linear and logistic regression models based on an additive homomorphic encryption (HE) scheme and an aggregation protocol. Exploiting the training algorithm of federated learning, at the core of our training protocols is a secure multiparty global gradient computation on alive users' data. We analyze the security of our training protocols against semi-honest adversaries. As long as the aggregation protocol is secure under the aggregation privacy game and the additive HE scheme is semantically secure, PrivFL guarantees the users' data privacy against the server, and the server's regression model privacy against the users. We demonstrate the performance of PrivFL on real-world datasets and show its applicability in the federated learning system.

CCS CONCEPTS

• **Security and privacy** → Privacy-preserving protocols.

KEYWORDS

Privacy-preserving computation, Predictive analysis, Federated learning, Machine learning

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1 INTRODUCTION

Due to their powerful capabilities, machine learning (ML) algorithms are deployed in various applications, from mobile applications to massive-scale data centers for serving various tasks such as predictive analysis, classification, and clustering. Companies such as browser, telecom, web services, edge computing, and advertisement collect a huge amount of data from users to learn ML models for improving quality of services and user experiences, and for providing intelligent services. Predictive analytics is a fundamental task in machine learning, which has many applications ranging from advertisement analytics to financial modeling, supply chain analysis, to health analytics. A common approach to the data analytics is that users send their data to a centralized server where the server executes machine learning algorithms on collected data. A major drawback of this approach is that the transparency of data processing at the server is not clear, and such centralized servers (or Artificial Intelligence (AI) platforms) are easy targets for hackers (e.g., AI.type [1]). Recently the European Union's General Data Protection Regulation (GDPR) mandates companies to exhibit transparency in handling personal data, data processing, etc.

Federated learning [6, 45] is a promising distributed machine learning approach, and its goal is to collaboratively learn a shared model, coordinated by a (centralized) server, while the training data remains on user devices. In federated learning, an ML model on users' data is learnt in an iterative way, which has four steps: 1) a set of users is chosen by the server to compute an updated model; 2) each user computes an updated model on its local data; 3) the updated local models are sent to the server; and 4) the server aggregates these local models to construct a global model. With the advancement of 5G, the federated learning approach will be an attractive solution for machine leaning in edge computing.

With the growing concern of data privacy, federated learning aimed at reducing data privacy risks by avoiding storing data at a centralized server. In privacy-preserving machine learning, two major privacy concerns are the user input privacy and the ML model privacy [21, 53]. However, the model privacy in federated learning has not received much attention. Besides the model leaks sensitive information about training data, the model privacy must also be protected against unauthorized use or misuse. For instance, a *covert user* belonging to one organization participating in a federated training process may disclose the trained model to another organization, which may lead to obtaining a better model by further training it on their dataset or may use it as a prediction service to do business. Although training data remain on devices, we have

found certain scenarios in which the local gradient computation *leaks private information* about locally stored data (see Section 3.2).

In this work, we consider a federated machine learning setting in which a server that coordinates the machine learning process wishes to learn an ML model on a joint dataset belonging to a set of mobile users. We consider both the mobile users' data privacy against the server, and the model privacy against the users in the federated setting. A *user dropout* is an important consideration in mobile applications, recently considered in [7, 43]. Especially, during the training phase which is a computationally intensive and time consuming task, it can happen any time from the network. Our goal is to design secure and dropout-robust training protocols for predictive analytics for mobile applications where, in this work, we consider two fundamental regression models, namely linear/ridge regression and logistic regression.

In a recent work, the authors of [7] considered a general problem of secure aggregation for privacy-preserving machine learning in the federated setting. In a follow-up work, the authors of [43] presented an improved protocol for secure aggregation. No complete training protocol for any machine learning algorithm is developed in [7, 43]. Privacy-preserving training for linear and logistic regressions is not a new problem. Secure training protocols for linear regression have been proposed, e.g., in [22, 48] where, unlike ours, a set of users participating in the training process is connected on a stable network. Secure training protocols for Machine-Learning-as-a-Service (MLaaS) for logistic regression are proposed in [2, 8, 37, 60]. Although several solutions have been proposed for linear and logistic regressions using somewhat or fully homomorphic encryption (SWHE or FHE) [24], garbled circuit [58], or hybrid techniques e.g., in [46], until now, the regression model training over mobile networks has not been considered, under the scenario of users dropping out. Moreover, the suitability of expensive cryptographic primitives such as SWHE or FHE on mobile devices has not been studied. Our goal is to design protocols using lightweight crypto-primitives and schemes suitable for mobile devices in both training and prediction phases.

1.1 Our Contributions

We design, analyze, and evaluate PrivFL, a system for privacy-preserving training and oblivious prediction of regression models, namely linear, ridge and logistic regressions in the federated setting.

Dropout-robust regression training protocols. We design two privacy-preserving protocols – one is for a linear regression and another is for a logistic regression – for training a regression model over a mobile network while providing robustness in the event of users dropping out. In a nutshell, our privacy-preserving protocol for multiparty regression training consists of multiple (parallel) two-party shared local gradient computation protocols, followed by a global gradient share-reconstruction protocol (see Section 4.2). In our protocol, the users and the server execute the following three steps: 1) a shared local gradient computation protocol is run between the server and a user to securely compute two (additive) shares of the local gradient on the user's data to prevent the input leakage, even if the user has a single data point; 2) the server and all alive users execute an aggregation protocol to construct one share of the global gradient; and 3) the server computes the second share of the global gradient from its local gradient shares. This offers a great flexibility for computing the global gradients robustly. Our regression training protocols are developed using an additive homomorphic encryption scheme and a secure aggregation protocol built using practical crypto-primitives. We also show how to obliviously compute regression models for prediction services for the future use of the trained models by the users.

Security. We prove the security of the training protocol, in three different threat models, against semi-honest adversaries in the simulation paradigm. As our training protocol is built upon several subprotocols, we first prove the security of the shared local gradient computation protocol for linear and logistic regression models. We formally show that the security of the training protocol is based on the semantic security of the additive HE scheme that guarantees the server's model privacy and the aggregation privacy game, as defined in [10], which ensures users' data privacy.

Experimental evaluation. We implement and evaluate the efficiency of PrivFL for training linear and logistic regression models on eleven real-world datasets from the UCI ML repository [17]. In our experiment, we use the Joye-Libert (JL) cryptosystem [31] to realize the additive HE scheme, and implement an aggregation protocol that is a compilation of the protocols of [7] and [43]. As machine learning algorithms work on floating-point numbers, we show how to encode floating-point numbers for cryptographic operations, and how to decode results obtained after applying crypto-operations. We present the benchmark results on the execution time, data transfer, and storage overhead for cryptographic operations, and provide a comparison of our scheme with other approaches in Section 6.

2 PRELIMINARIES

Here, we briefly describe the regression algorithms, namely linear, and logistic regressions, federated learning, and the cryptographic schemes and protocols that we use to build our new protocols.

Basic notations. We denote the message space by $\mathbb{Z}_{2^k}$, a ring of 2^k elements, and E by an additive HE encryption scheme. $\mathbf{x} = (x_1, \cdots, x_n), x_i \in \mathbb{Z}_{2^k}$ denotes an n dimensional vector over $\mathbb{Z}_{2^k}$.

- For $\mathbf{x} = (x_1, \cdots, x_n)$, $\mathrm{E}(\mathbf{x}) = (\mathrm{E}(x_1), \cdots, \mathrm{E}(x_n))$.
- $\mathrm{E}(\mathbf{x}) \cdot \mathrm{E}(\mathbf{y}) = (\mathrm{E}(x_1) \cdot \mathrm{E}(y_1), \cdots, \mathrm{E}(x_n) \cdot \mathrm{E}(y_n))$
- $\mathrm{E}(\mathbf{x})^{\mathbf{y}} = (\mathrm{E}(x_1 y_1), \cdots, \mathrm{E}(x_n y_n))$
- For $\boldsymbol{\theta} = (\theta_0, \theta_1, \cdots, \theta_n)$, $\boldsymbol{\theta} \cdot \mathbf{x} = \theta_0 + \sum_{i=1}^{n} \theta_i x_i$.

2.1 Overview of Regression Algorithms

We now describe three regression algorithms and the gradient descent algorithm for the training process. Let $\mathcal{D} = \left\{(\mathbf{x}^{(i)}, y^{(i)})\right\}_{i=1}^{d}$ be a training dataset with labeled output $y^{(i)} = h(\boldsymbol{\theta}, \mathbf{x}^{(i)})$ and input $\mathbf{x}^{(i)} = (x_1^{(i)}, x_2^{(i)}, \cdots, x_n^{(i)})$, where h is the regression algorithm, $\boldsymbol{\theta}$ is the model, and n is the dimension of $\mathbf{x}$. The task in the training process is to learn the model $\boldsymbol{\theta}$, given $\mathcal{D}$ and h.

Gradient Descent Algorithm. Gradient descent (GD) is an iterative optimization algorithm for minimizing a cost function. Given a non-empty training dataset $\mathcal{D}$, the cost function is defined as $J(\boldsymbol{\theta}) = \frac{1}{|\mathcal{B}|} \sum_{(\mathbf{x},y)\in\mathcal{B}} C(\boldsymbol{\theta}, (\mathbf{x}, y))$ where $C(\cdot,\cdot)$ is a cost function and $\mathcal{B} \subseteq \mathcal{D}$. An optimal $\boldsymbol{\theta}$ is computed iteratively as $\boldsymbol{\theta}^{i+1} \leftarrow \boldsymbol{\theta}^i - \eta \nabla J(\boldsymbol{\theta}^i), i \geq 0$ where $\boldsymbol{\theta}^0$ is initialized with a random value or all-zero, η is the learning rate, and ∇J is the gradient of J over $\mathcal{B}$.

Federated Learning. Federated learning enables a large number of users to collaboratively learn a model while keeping all training

data on their devices where the model training is coordinated by a central server [39, 44]. We assume that there are m users, denoted by P_i, with a set of data points, denoted by $\mathcal{D}_i$. The cost function for the joint dataset $\mathcal{D} = \cup_{i=1}^{m} \mathcal{D}_i$ is given by $J(\boldsymbol{\theta}) = \sum_{i=1}^{m} \frac{d_i}{d} F_i(\boldsymbol{\theta})$ where $F_i(\boldsymbol{\theta}) = \frac{1}{d_i} \sum_{(\mathbf{x},y)\in\mathcal{D}_i} C(\boldsymbol{\theta}, (\mathbf{x}, y))$, $d = \sum_{i=1}^{m} d_i$, and $d_i = |\mathcal{D}_i|$. The model update on the total training set $\mathcal{D}$ is performed as [44]

$$\boldsymbol{\theta}^{i+1} \leftarrow \boldsymbol{\theta}^{i} - \eta \sum_{j=1}^{m} \frac{d_j}{d} \nabla F_j(\boldsymbol{\theta}^{i}), i \geq 0$$

where ∇F_j is the *local* gradient value on $\mathcal{D}_j$. In federated averaging [44], the users are chosen randomly.

Linear Regression. For a linear regression model, the function $h(\cdot,\cdot)$ is an affine function, which is defined as $h(\boldsymbol{\theta}, \mathbf{x}^{(i)}) = \boldsymbol{\theta} \cdot \mathbf{x}^{(i)} = \theta_0 + \sum_{j=1}^{n} \theta_j x_j^{(i)}$. The model parameter $\boldsymbol{\theta}$ is obtained by optimizing the following cost function

$$J(\boldsymbol{\theta}) = \frac{1}{d} \sum_{i=1}^{d} (h(\boldsymbol{\theta}, \mathbf{x}^{(i)}) - y^{(i)})^2.$$

Ridge Regression. For the ridge regression, the function h is the same as the function of linear regression, but the model parameter is obtained by optimizing the following cost function

$$J(\boldsymbol{\theta}) = \frac{1}{d} \sum_{i=1}^{d} (h(\boldsymbol{\theta}, \mathbf{x}^{(i)}) - y^{(i)})^2 + \lambda ||\boldsymbol{\theta}||^2$$

where λ is the regularization parameter.

Logistic Regression. The logistic regression is a binary linear classifier, which maps an input $\mathbf{x}^{(i)}$ from the feature space to a value in $[0, 1]$ as follows:

$$h(\boldsymbol{\theta}, \mathbf{x}^{(i)}) = \sigma(\boldsymbol{\theta} \cdot \mathbf{x}^{(i)}) = \frac{1}{1 + e^{-(\sum_{i=j}^{n} \theta_i x_j^{(i)} + \theta_0)}}$$

where $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function. The binary class is decided based on a threshold value. The model parameter $\boldsymbol{\theta}$ is obtained by optimizing the following log-likelihood cost function

$$J(\boldsymbol{\theta}) = -\frac{1}{d} \sum_{i=1}^{d} y^{(i)} \log(h(\boldsymbol{\theta}, \mathbf{x}^{(i)})) + (1 - y^{(i)}) \log(1 - h(\boldsymbol{\theta}, \mathbf{x}^{(i)})).$$

2.2 Multiparty Federated Regression Training and Oblivious Prediction

Multiparty Gradient Computation. Assume that there are m users, and each user has a dataset $\mathcal{D}_i, 1 \leq i \leq m$. A server wishes to learn a regression model $\boldsymbol{\theta}$ on the joint dataset $\mathcal{D} = \mathcal{D}_1 \cup \cdots \cup \mathcal{D}_m = \left\{(\mathbf{x}^{(i)}, y^{(i)})\right\}$ (horizontally distributed). To train a model based on the gradient descent algorithm, the server jointly computes the gradient $\boldsymbol{\omega}$ on $\mathcal{D}$ as

$$\boldsymbol{\omega} = (\omega_0, \omega_1, \cdots, \omega_n) = \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \mathtt{GD}(\boldsymbol{\theta}, \mathcal{D}, d)$$

where $d = |\mathcal{D}|$, $\omega_0 = \sum_{i=1}^{d} e^{(i)}$, $\omega_j = \sum_{i=1}^{d} e^{(i)} x_j^{(i)}, 1 \leq j \leq n$, $e^{(i)} = h(\boldsymbol{\theta}, \mathbf{x}^{(i)}) - y^{(i)}$ and h is a (linear, ridge, or logistic) regression function. We denote the gradient computation on a single data point $(\mathbf{x}^{(i)}, y^{(i)})$ by $\mathbf{err}^{(i)} = f(\boldsymbol{\theta}, (\mathbf{x}^{(i)}, y^{(i)})) = (e^{(i)}, e^{(i)} x_1^{(i)}, \cdots, e^{(i)} x_n^{(i)})$ with $e^{(i)} = h(\boldsymbol{\theta}, \mathbf{x}^{(i)}) - y^{(i)}$. Knowing the model $\boldsymbol{\theta}$, each user can compute the local gradient $\boldsymbol{\omega}_i$ on $\mathcal{D}_i$ as $\boldsymbol{\omega}_i = \sum_{(\mathbf{x},y)\in\mathcal{D}_i} \mathbf{err}^{(i)} = \sum_{(\mathbf{x},y)\in\mathcal{D}_i} f(\boldsymbol{\theta}, (\mathbf{x}, y))$. Thus, the global gradient can be written as $\boldsymbol{\omega} = \sum_{i=1}^{m} \boldsymbol{\omega}_i = \sum_{j=1}^{d} \mathbf{err}^{(j)}$. Using the gradient $\boldsymbol{\omega}$ over $\mathcal{D}$, the server can update the model as $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \frac{\eta}{d} \cdot \boldsymbol{\omega}$ for linear and logistic regressions, and $\boldsymbol{\theta} \leftarrow (1 - 2\lambda\eta)\boldsymbol{\theta} - 2\eta \cdot \boldsymbol{\omega}$ for ridge regression for some regularization parameter λ. Secure gradient computation enables the server to learn only $\boldsymbol{\omega}$, while ensuring users' input privacy against the server, and the server's model privacy against the users.

Oblivious Regression Prediction. Oblivious prediction for neural networks was introduced in [40]. An oblivious prediction for regression models is defined as follows. Assume that the server holds a (private) regression model $\boldsymbol{\theta}$, and a user has a private input $\mathbf{x}$ and wishes to learn the predicted value $h(\boldsymbol{\theta}, \mathbf{x})$ for a regression function h with model $\boldsymbol{\theta}$. In an oblivious prediction computation, the user will learn only the predicted value $h(\boldsymbol{\theta}, \mathbf{x})$ without leaking any information about $\mathbf{x}$ to the server, and the user should learn nothing about $\boldsymbol{\theta}$, except what can be learnt from the output.

2.3 Cryptographic Tools

We now provide a description of the cryptographic schemes that we will use to construct our protocols.

Additive Homomorphic Encryption. An additive homomorphic encryption (HE) scheme consists of a tuple of four algorithms, denoted as HE = (KeyGen, Enc, Eval, Dec) where

- KeyGen: Given a security parameter ν, it generates a pair of private and public keys $(pk, sk) \leftarrow \mathsf{KeyGen}(1^{\nu})$.
- Enc: It is a randomize algorithm, denoted by E, takes the public key pk, a random coin r and the message $m \in \mathbb{Z}_{2^k}$ as input, and outputs a ciphertext $c = \mathrm{E}(pk; m, r)$.
- Eval: It supports the following operations on ciphertexts: 1) Addition: $\mathsf{Add}(\mathrm{E}(x), \mathrm{E}(y)) = \mathrm{E}(x + y)$; and 2) Constant multiplication: $\mathsf{ConstMul}(\mathrm{E}(x), z) = \mathrm{E}(xz)$.
- Dec: The algorithm, denoted by D, takes the secret key sk and a ciphertext c as input, and outputs a plaintext message $m = \mathrm{D}(sk; c)$.

We use an additive HE scheme that is semantically secure, and $\mathsf{Add}(,)$ and $\mathsf{ConstMul}(,)$ are ciphertext multiplication and exponentiation operations, respectively.

Secure Aggregation Protocol. We will use a secure aggregation protocol that can handle the dropout scenario as well as no dropout in an aggregated-sum computation. We call this protocol a *dropout-enabled aggregation* protocol, denoted by π_{DEA}. A dropout-enabled aggregation protocol accepts as input a set of users $\mathcal{U}$, their private inputs $\{x_u\}_{u\in\mathcal{U}}$, the total number of users m, and the threshold security parameter t, and outputs x_{sum}, and a set of *alive* users $\mathcal{U}_a \subseteq \mathcal{U}$ participated in the sum computation, i.e.,

$$(\mathcal{U}_a, x_{sum}) \leftarrow \pi_{\mathrm{DEA}}\left(\mathcal{U}, \{x_u\}_{u\in\mathcal{U}}, m, t\right)$$

where $x_{sum} = \sum_{u\in\mathcal{U}_a} x_u$ if $|\mathcal{U}_a| \geq t$, and $\perp$ otherwise. Note that the aggregation scheme is secure under a coalition of up to $(t-1)$ users in the system. Such protocols with the dropout-enabled property have been investigate recently in [7, 43]. For more details, see [7, 43]. These aggregation protocols need to establish a pair of pairwise keys: one key is used for realizing an authenticated channel, and another key is used to encrypt private inputs. We omit the cryptographic background behind the aggregation protocol here, but provide in the full paper [42].

3 OUR SYSTEM MODEL AND GOALS

In this section, we describe the system model, its goals, and possible privacy leakage in the existing multiparty gradient computation.

3.1 System Model and Trust

System model. We consider a system in the federated learning setting introduced in [6, 39]. In this model, the system consists of two types of parties: a server and a set of mobile users or parties connected in mobile network, where the server conducts the regression training process on mobile users' data. The users may *dropout* any time from the system, as recently considered in [7, 43].

Assume that there are m mobile users in the system, and each user has a unique identity in $[m] = \{1, 2, \cdots, m\}$. We denote the server by S, and a user by $P_i, i \in [m]$. Each user P_i holds a dataset with high-dimensional points, denoted by $\mathcal{D}_i = \{(\mathbf{x}^{(j)}, y^{(j)})\}$ where $|\mathcal{D}_i| \geq 1$. We have the following system goals:

- **Dropout-robust secure regression training:** The first goal of our system is to enable the server training a regression model $\boldsymbol{\theta}$ (e.g., linear, ridge or logistic) over the combined dataset $\mathcal{D} = \mathcal{D}_1 \cup \cdots \cup \mathcal{D}_m$, i.e., $\boldsymbol{\theta} \leftarrow \mathsf{Traning}(h, \mathcal{D}, m)$, while allowing user dropout any time in the system, and ensuring mobile users' input privacy and the servers' model privacy where h is the regression algorithm (see Section 2.2).
- **Oblivious prediction:** The second goal is to enable a user to learn the predicted output $y = h(\boldsymbol{\theta}, \mathbf{x})$ for a regression model $\boldsymbol{\theta}$ on its private input $\mathbf{x}$, without leaking any information about $\mathbf{x}$ to the server and about $\boldsymbol{\theta}$ to the user.

Figure 1 gives an overview of our system and approach to achieve system goals. The main challenges in the training phase are:

- *Correctness:* For correct inputs of the users, the protocol for the regression training should output the correct model. We claim no correctness of the regression model if any user uses an incorrect input or the server manipulates the model in the training phase.
- *Privacy:* Our system has aimed at protecting users' inputs privacy and the server's model privacy. The server should learn no information about mobile users' private inputs in $\mathcal{D}_i$. Similarly, the users should not learn anything about the model $\boldsymbol{\theta}$, except what they can infer from the output.
- *Efficiency:* As mobile users do not send the private inputs out of the devices, they should perform a minimal work, and the server should perform the majority of work in the training phase. The computational and communication costs of the training protocol should be minimal.

Threat model. In our system, we consider semi-honest adversaries (inside adversary) where a group of mobile users and/or the server compromised by an adversary follow and observe the prescribed actions of the protocol and aim at learning unintended information about $\boldsymbol{\theta}$ or honest users' $\mathcal{D}_i$ from the execution of the protocol. We consider three different threat models: 1) users-only adversary; 2) server-only adversary; and 3) users-server adversary. We assume the training is conducted in a synchronous way (each iteration within a time interval of length Δ), meaning all users use the correct model to compute the local gradient and the server updates the model consistently at every round of the model update.

3.2 Privacy Leakage and Model Privacy

Input privacy leakage from local gradient. When a user holds a single data point $(\mathbf{x}, y)$, i.e., $\mathcal{D}_j = \{(\mathbf{x}, y)\}$, the gradient computation on $\mathcal{D}_j$, denoted by $\boldsymbol{\omega} = (\omega_0, \omega_1, \cdots, \omega_n)$, leaks information about the private input $\mathbf{x}$ as $\omega_0 = (h(\boldsymbol{\theta}, \mathbf{x}) - y)$ and $\omega_j = (h(\mathbf{x}, \boldsymbol{\theta}) - y)x_j = \omega_0 \cdot x_j, 1 \leq j \leq n$, where h is a regression function. In the cases of stochastic gradient descent and federated averaging algorithm with $|\mathcal{D}_i| = 1$ [44], this directly allows the server to recover x_j from $\boldsymbol{\omega}$ for $\boldsymbol{\omega} \neq \mathbf{0}$.

In the application of smart grid data aggregation, an analysis on aggregation is performed in [10], which experimentally finds privacy leakage in the aggregate-sum when it has a small number of inputs. According to the analysis of [10], $\omega_j = \sum_{l=1}^{d_i}(h(\boldsymbol{\theta}, \mathbf{x}^{(l)}) - y^{(i)})x_j^{(l)}$ and $\omega_0 = \sum_{l=1}^{d_i}(h(\boldsymbol{\theta}, \mathbf{x}^{(l)}) - y^{(l)})$ may leak information about $\mathbf{x}^{(i)}$ when d_i is small. In mobile applications, when mobile devices do not have enough data, it may compromise the privacy of the input dataset $\mathcal{D}_i$. In this work, PrivFL enables mobile users' to train a regression model without leaking their privacy, even when $|\mathcal{D}_i| = 1$ and $d = \sum d_i$ is large enough to protect input privacy.

Model privacy of federated learning. In federated learning, a substantial focus has been put on users' data privacy. The authors of [44] mentioned about achieving the model privacy using differential privacy techniques [18]. However, there is no such concrete proposal, and it is also not known the accuracy of the training process after applying such techniques. PrivFL takes the secure MPC approach to provide the ML model privacy as well as data privacy.

4 OUR REGRESSION PROTOCOLS

The key objective of this work is to develop secure training protocols for regression models based on cryptographic primitives suitable for mobile devices as well as robust against the user dropout scenario. As the training process involves a pre-processing of the training data, called *scaling or normalization*, we start by describing how to securely perform this using an aggregation protocol while ensuring the dropout scenario.

4.1 Dropout-robust Secure Scaling Operation

Scaling a dataset is performed by computing the mean and standard deviation of the dataset. For a high-dimensional dataset $\mathcal{D} = \mathcal{D}_1 \cup \cdots \cup \mathcal{D}_m$ from users $\mathcal{U}$, the scaling operation is performed by scaling individual components of each data point. Note that scaling is performed only on $\mathbf{x}^{(i)}$ in $\mathcal{D}$. Let $\mathbf{x}^{(i)} = \left(x_1^{(i)}, x_2^{(i)}, \cdots, x_n^{(i)}\right)$ be an n-dimensional data point. The mean of the $\mathbf{x}^{(i)}$ component of $\mathcal{D}$ of size d is given by $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)$ where $\mu_j = \frac{1}{d}\sum_{i=1}^{d} x_j^{(i)}$, and the standard deviation is given by $\boldsymbol{\sigma} = (\sigma_1, \cdots, \sigma_n)$ where $\sigma_j = \sqrt{\frac{1}{d-1}(\sum_{i=1}^{d} {x_j^{(i)}}^2 - (2d-1)\mu_j^2)}, j \in [n]$. Secure computation of basis statistics such as mean and standard deviation has been widely investigated using secret sharing, (labeled) homomorphic encryption, and aggregation protocols, e.g., [4, 5, 12, 35].

As we have considered a scenario of users dropping out, we perform the scaling process using a dropout-enabled aggregation protocol (π_{DEA}), coordinated by the server. Instead of running π_{DEA} twice (once for computing μ and another for σ), our idea is to run π_{DEA} only once on $2n$ dimensional inputs $X^{(i)}$, which we construct

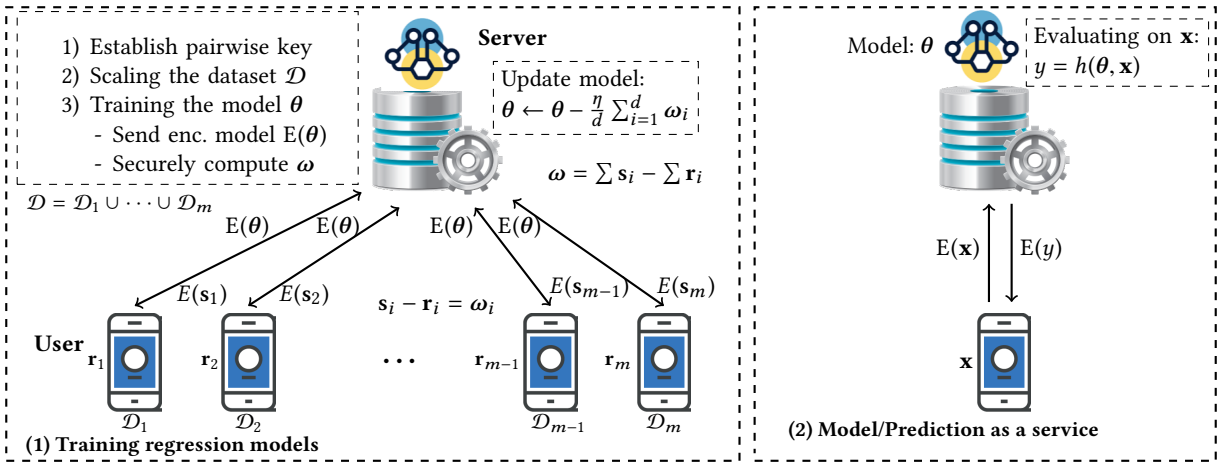


Figure 1: An overview of the regression training process and prediction as a service over a mobile network in the federated setting. For the training phase, each user holds a dataset $\mathcal{D}_i$, and the server holds an ML model θ. Users and the server jointly compute the global gradient ω. For the prediction service, the user holds a data point and the server holds a trained model.

from n-dimensional inputs $\mathbf{x}^{(i)}$ as follows. Each user P_i locally computes a single input $X^{(i)}$ on its dataset $\mathcal{D}_i$ as

$$X^{(i)} = \Big(\sum_{\mathbf{x}\in\mathcal{D}_i} x_1, \cdots, \sum_{\mathbf{x}\in\mathcal{D}_i} x_n, \sum_{\mathbf{x}\in\mathcal{D}_i} x_1^2, \cdots, \sum_{\mathbf{x}\in\mathcal{D}_i} x_n^2 \Big)$$

where $\mathbf{x} = (x_1, \cdots, x_n)$. This can be viewed as processing multiple-data using a single execution of the protocol. On inputs $\{X^{(i)}, d_i\}$ from users in $\mathcal{U}$ and a security parameter t, the server receives $(\mathcal{U}_a, X) \leftarrow \pi_{\text{DEA}}(\mathcal{U}, \{X^{(i)}, d_i\}, |\mathcal{U}|, t)$ with $|\mathcal{U}| \geq t$ where $X = (X_1, \cdots, X_n, X_{n+1}, \cdots, X_{2n}) = \sum_{u\in\mathcal{U}_a} X^{(u)}$ if $|\mathcal{U}_a| \geq t$, and $\mathcal{U}_a$ are the alive users who participated in the scaling process. If $|\mathcal{U}_a| < t$, abort the protocol. The server computes the mean $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)$ and standard deviation $\boldsymbol{\sigma} = (\sigma_1, \cdots, \sigma_n)$ as $\mu_j = \frac{X_j}{d_a}$, and $\sigma_j = \sqrt{\frac{1}{d_a - 1}(X_{n+j} - (2d_a - 1)\mu_j^2)}, j \in [n]$ and $d_a = \sum_{u\in\mathcal{U}_a} d_u$. The server then sends $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ to all users in $\mathcal{U}_a$ through an authenticated channel. Using $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$, the users scale their local datasets.

4.2 A High-level Overview of Our Regression Training Protocols

Basis idea. To train a regression model, the users and the server execute a multiparty global gradient computation protocol where the server gives the model and the users provide their local datasets as inputs, and the server obtains an updated model as an output. The novelty of PrivFL's multiparty gradient computation is that the global gradient computation is performed by executing multiple two-party shared local gradient protocols in parallel, followed by executing a global gradient share-reconstruction protocol realized using an aggregation protocol (see Figure 2). As shown in Section 3, sending the local gradient directly to the server may leak users' datasets $\mathcal{D}_i$. Our idea is to prevent such leakage by additively secret-sharing the local gradient $\boldsymbol{\omega}_i$ of a user P_i between the server and the user such that $\mathbf{s}_i - \mathbf{r}_i = \boldsymbol{\omega}_i$, where the server holds the share $\mathbf{s}_i$, and the user P_i holds the share $\mathbf{r}_i$. To prevent leaking the model to the users, the server encrypts the model θ using an additive homomorphic encryption scheme. In the first phase, each user computes an encrypted share E($\mathbf{s}_i$) of $\boldsymbol{\omega}_i$ from E(θ) and $\mathcal{D}_i$. This is computed by a protocol called, the *shared local gradient (SLG)* computation protocol. Instead of sending the individual users' shares to the server, the users send their shares in aggregate. For this, the server conducts a share reconstruction process to obtain an aggregate-sum of the shares. This phase is called the *share reconstruction phase*. After computing $\boldsymbol{\omega}$ on alive users' ($\mathcal{U}_a$) datasets from $\{\mathbf{s}_i\}$ and $\{\mathbf{r}_i\}$ as $\boldsymbol{\omega} = \sum_{u\in\mathcal{U}_a} \mathbf{s}_u - \sum_{u\in\mathcal{U}_a} \mathbf{r}_u$, the server updates the model as $\theta \leftarrow \theta - \frac{\eta}{d_a}\boldsymbol{\omega}$ where $d_a = \sum_{u\in\mathcal{U}_a} d_u$. Figure 2 depicts an overview of secure multiparty global gradient computation on the joint dataset $\mathcal{D}$.

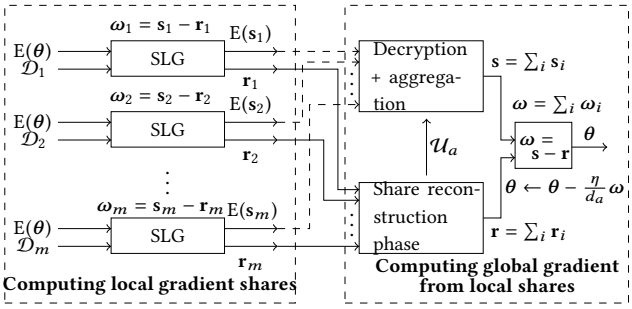


Figure 2: PrivFL's protocol flow of privacy-preserving global gradient computation for regression models.

Advantage. The advantages of the above flow of the protocol are as follows. At the i-th round of the training process, the users participating in the global gradient computation process do not need to know the other participating users ahead of the time. The information about the other participating users will be enough to know only in the share reconstruction phase. This provides a great flexibility in the global gradient computation phase for handling the dropout scenario in mobile applications. Moreover, the server can choose the participating users randomly based on their aliveness in the global gradient computation phase. Note that the server can choose the participating users and let all users know ahead of time, but this will be computationally expensive for both users and the server for handling the dropout scenario. Notice that any two-party protocol can be used to realize the SLG computation. However, we use an additive-HE based technique for regression models to make it communication efficient because of our application scenario.

4.3 Secure Shared Local Gradient Computation Protocols

A secure shared local gradient computation involves a user and the server. We present two protocols for computing the shared local gradient computation: one protocol is for the linear regression (see Figure 3), and another is for the logistic regression (see Figure 4).

Computing the SLG for linear regression. To minimize the number of rounds, the server encrypts the model using an additive HE scheme and sends it to the user P_i so that it can compute two shares of the local gradient $\boldsymbol{\omega}_i$ on its dataset $\mathcal{D}_i$ in one round of communication. Note that only the server holds the private key of the HE scheme. An inner product (IP) computation of two vectors is a common task in training and evaluating a linear or logistic regression model. Secure inner product computation has been studied in many settings, e.g., [25, 57]. We use a homomorphic IP computation technique from [25]. Figure 3 presents the shared local gradient computation protocol for a linear/ridge regression model.

Algorithm 1 Computing Shares of Local Gradient for Linear Regression

1: Input: $\mathcal{D} = \{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^{p}$ and $\mathrm{E}(\boldsymbol{\theta})$
2: Output: $(\mathrm{E}(\mathbf{s}), \mathbf{r})$ s.t. $\mathbf{s} - \mathbf{r} = \sum_{j=1}^{p} \mathbf{err}^{(j)}$
3: **procedure** Comp_LinSLG_Share
4: Randomly generate $\mathbf{r} = (r_0, r_1, \cdots, r_n)$
5: Set $\mathbf{t} \leftarrow (0, 0, \cdots, 0)$
6: **for** $i = 1$ to p **do**
7: $\mathrm{E}(e^{(i)}) = \prod_{j=1}^{n} \mathrm{E}(\theta_j)^{x_j^{(i)}} \cdot \mathrm{E}(\theta_0) \cdot \mathrm{E}(-y^{(i)})$
8: $\mathrm{E}(t_0) \leftarrow \mathrm{E}(t_0) \cdot \mathrm{E}(e^{(i)}) = \mathrm{E}(t_0 + e^{(i)})$
9: **for** $j = 1$ to n **do**
10: $\mathrm{E}(t_j) \leftarrow \mathrm{E}(t_j) \cdot \mathrm{E}(e^{(i)})^{x_j^{(i)}} = \mathrm{E}(t_j + e^{(i)} x_j^{(i)})$
11: **end for**
12: **end for**
13: Compute $\mathrm{E}(\mathbf{s}) = \mathrm{E}(\mathbf{t}) \cdot \mathrm{E}(\mathbf{r}) = \mathrm{E}(\mathbf{t} + \mathbf{r})$
14: **return** $(\mathrm{E}(\mathbf{s}), \mathbf{r})$
15: **end procedure**

Secure SLG Protocol: Linear regression (π_{LinSLG})

Input: Server provides $\boldsymbol{\theta}$, User (P_i) provides $\mathcal{D}_i$

Output: Server receives $\mathrm{E}(\mathbf{s}_i)$, P_i receives $\mathbf{r}_i$.

1. Server encrypts the model $\boldsymbol{\theta}$ and sends $\mathrm{E}(\boldsymbol{\theta})$ **to** the user P_i
2. User P_i runs Algorithm 1 on inputs $\mathrm{E}(\boldsymbol{\theta})$ and $\mathcal{D}_i$, and obtains $(\mathrm{E}(\mathbf{s}^i), \mathbf{r}^i)$ such that $\mathbf{s}_i - \mathbf{r}_i = \boldsymbol{\omega}_i = \mathtt{GD}(\boldsymbol{\theta}, \mathcal{D}_i, d_i)$.
3. P_i stores $\mathbf{r}_i$ and sends $\mathrm{E}(\mathbf{s}_i)$ **to** the server.

Figure 3: Secure shared local gradient computation protocol for a linear regression model.

Computing the SLG for logistic regression. The computation of the local gradient involves an evaluation of the sigmoid function on $\boldsymbol{\theta} \cdot \mathbf{x}^{(j)}$. As shown for linear regression, the server and the user can compute $\mathrm{E}(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)})$ in one round of communication. As we used an additive HE scheme, the user and the server need one more round of communication to compute $\mathrm{E}(\sigma_3(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)})), \mathbf{x}^{(j)} \in \mathcal{D}_i$ from $\mathrm{E}(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)})$ where the sigmoid function is approximated by a cubic polynomial, which provides a good tradeoff between the accuracy and the efficiency. Let $\sigma_3(x) = q_0 + q_1 x + q_2 x^2 + q_3 x^3$ be a cubic approximation of $\sigma(x)$ where the coefficients q_i are public. To protect an input $\mathbf{x}$ against the server, the user masks $y = \boldsymbol{\theta} \cdot \mathbf{x}$ as $z = y + r$ by choosing a random value r, and then sends it to the server. The computation of $\sigma_3(y)$ can be expressed as

$$\sigma_3(y) = \sigma_3(z) - \sigma_3(r) - (q_0 + 3q_3 r^3) - 3q_3 r z^2 - (2q_2 r - 6q_3 r^2) y.$$

To reduce computational cost for the users, the server computes $\mathrm{E}(z^2)$ and $\mathrm{E}(\sigma_3(z))$ from $\mathrm{E}(z)$ and sends $(\mathrm{E}(z^2), \mathrm{E}(\sigma_3(z)))$ to the user. Given $\mathrm{E}(y), \mathrm{E}(z^2), \mathrm{E}(\sigma_3(z))$ and r, the user can compute $\mathrm{E}(\sigma_3(y))$ using the homomorphic property of the encryption scheme as

$$\mathrm{E}(z^2)^{-3q_3 r} \mathrm{E}(y)^{-(2q_2 r - 6q_3 r^2)} \mathrm{E}(-(\sigma_3(r) + (q_0 + 3q_3 r^3))) \mathrm{E}(\sigma_3(z)).$$

Once the user has $\{\mathrm{E}(\sigma_3(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)}))\}$, it can compute the local gradient $\boldsymbol{\omega}_i$ on $\mathcal{D}_i$, using the steps described in Algorithm 2. Figure 4 summarizes the shared local gradient computation protocol for a logistic regression model.

Algorithm 2 Computing Shares of Local Gradient for Logistic Regression

1: Input: $\mathcal{D} = \{(\mathbf{x}^{(i)}, y_i)\}_{i=1}^{p}$ and $\{\mathrm{E}(\sigma_3(\boldsymbol{\theta} \cdot \mathbf{x}^{(i)}))\}$
2: Output: $(\mathrm{E}(\mathbf{s}), \mathbf{r})$ s.t. $\mathbf{s} - \mathbf{r} = \sum_{j=1}^{p} \mathbf{err}^{(j)}$
3: **procedure** Comp_LogSLG_Share
4: Randomly generate $\mathbf{r} = (r_0, r_1, \cdots, r_n)$
5: Set $\mathbf{t} \leftarrow (0, 0, \cdots, 0)$
6: **for** $i = 1$ to p **do**
7: $\mathrm{E}(e^{(i)}) = \mathrm{E}(\sigma_3(\boldsymbol{\theta} \cdot \mathbf{x}^{(i)})) \cdot \mathrm{E}(-y^{(i)})$
8: $\mathrm{E}(t_0) \leftarrow \mathrm{E}(t_0) \cdot \mathrm{E}(e^{(i)}) = \mathrm{E}(t_0 + e^{(i)})$
9: **for** $j = 1$ to n **do**
10: $\mathrm{E}(t_j) \leftarrow \mathrm{E}(t_j) \cdot \mathrm{E}(e^{(i)})^{x_j^{(i)}} = \mathrm{E}(t_j + e^{(i)} x_j^{(i)})$
11: **end for**
12: **end for**
13: Compute $\mathrm{E}(\mathbf{s}) = \mathrm{E}(\mathbf{t}) \cdot \mathrm{E}(\mathbf{r}) = \mathrm{E}(\mathbf{t} + \mathbf{r})$
14: **return** $(\mathrm{E}(\mathbf{s}), \mathbf{r})$
15: **end procedure**

Complexity. We measure the computational complexity of π_{LinSLG} and π_{LogSLG} for computing the local gradient in terms of the number of ciphertext multiplications, the number of encryptions, and the number of homomorphic constant multiplications. Table 1 summarizes their exact numbers. Asymptotically, the overall computational complexity for both π_{LinSLG} and π_{LogSLG} is $O(nd_i)$.

Let λ be the bit length of a ciphertext. Then, the communication cost involved in computing $\boldsymbol{\omega}_i$ for the linear regression is $2(n+1)\lambda$, which is for receiving the encrypted model and sending an encrypted share of $\boldsymbol{\omega}_i$ to the server. For the logistic regression, the communication cost is $(2(n+1) + 3d_i)\lambda$ where $d_i = |\mathcal{D}_i|$ is the size of the dataset, which is due to receiving the encrypted model, sending an encrypted share of $\boldsymbol{\omega}_i$ and information exchange for homomorphically computing the approximated sigmoid function.

Table 1: Number of unit operations for each user in the gradient computation protocols

Protocol	#CT MUL	#Const MUL	#Enc
Linear regression (π_{LinSLG})	$2(n+1)d_i - (n+1)$	$2nd_i$	$d_i + (n+1)$
Logistic regression (π_{LogSLG})	$(2n+5)d_i - (n+1)$	$2(n+2)d_i$	$3d_i + (n+1)$

Security. We prove the security of two SLG computation protocols in the simulation paradigm. Theorems 1 and 2 summarize the security of the SLG computation protocols for linear and logistic regressions. We show that an adversary $\mathcal{A}$ controlling a user P_i (resp. the server) does not learn anything about $\boldsymbol{\theta}$ (resp. $\boldsymbol{\omega}_i$) during the execution of π_{LinSLG}, and similarly for π_{LogSLG}. We present the security proofs in the full paper [42].

Secure SLG Protocol: Logistic regression (π_{LogSLG})

Input: Server provides $\boldsymbol{\theta}$, User (P_i) provides $\mathcal{D}_i$

Output: Server receives $\text{E}(\mathbf{s}_i)$, P_i receives $\mathbf{r}_i$.

1. Server encrypts the model $\boldsymbol{\theta}$ and sends $\text{E}(\boldsymbol{\theta})$ **to** the user P_i

2. P_i computes: **for** j = 1 to d_i **do**

Compute $\text{E}(z_j) \leftarrow \prod_{l=1}^{n} \text{E}(\theta_l)^{x_l^{(j)}} \cdot \text{E}(\theta_0) \cdot \text{E}(c_j)$, where $z_j = \boldsymbol{\theta} \cdot \mathbf{x}^{(j)} + c_j$ and c_j is chosen uniformly at random

endfor

3. P_i sends $\left\{\text{E}(z_j)\right\}_{j=1}^{d_i}$ **to** the server

4. Server decrypts $\left\{\text{E}(z_j)\right\}_{j=1}^{d_i}$ and computes $\left\{\text{E}(z_j^2), \text{E}(h_q(z_j))\right\}_{j=1}^{d_i}$ and sends to P_i

5. P_i computes: **for** j = 1 to d_i **do**

$t = \text{E}(-(h_q(c_j) + (q_0 + 3q_3c_j^3))) \cdot \text{E}(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)})^{-(2q_2c_j - 6q_3c_j^2)}$

$\text{E}(\sigma_3(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)})) = \text{E}(\sigma_3(z_j))\text{E}(z_j^2)^{-3q_3c_j} \cdot t$

endfor

6. P_i runs Algorithm 2 on inputs $\left\{\text{E}(\sigma_3(\boldsymbol{\theta} \cdot \mathbf{x}^{(j)}))\right\}$ and $\mathcal{D}_i$, and obtains $(\text{E}(\mathbf{s}_i), \mathbf{r}_i)$ such that $\mathbf{s}_i - \mathbf{r}_i = \boldsymbol{\omega}_i = \text{GD}(\boldsymbol{\theta}, \mathcal{D}_i, d_i)$.

7. P_i stores $\mathbf{r}_i$ and sends $\text{E}(\mathbf{s}_i)$ **to** the server.

Figure 4: Secure shared local gradient computation protocol for a logistic regression model.

Theorem 1. *Assume that the additive homomorphic encryption scheme E() is semantically secure. The protocol π_{LinSLG} between P_i and S securely computes two shares of the local gradient on the dataset $\mathcal{D}_i$ in the presence of semi-honest adversaries.*

Theorem 2. *Assume that the additive homomorphic encryption scheme E() is semantically secure. The protocol π_{LogSLG} between P_i and S securely computes two shares of the local gradient $\boldsymbol{\omega}_i$ on the dataset $\mathcal{D}_i$ in the presence of semi-honest adversaries.*

4.4 Privacy-preserving Regression Model Training Protocol

The training protocol is executed between the server, and a set of m users in a synchronous network and is divided into three phases: 1) a pairwise key establishment phase, 2) a dataset scaling phase and 3) computing the regression model by iteratively computing the global gradient over a subset of users' data. A pair of pairwise keys is established using a Diffie-Hellman (DH) key agreement protocol [15] to realize an authenticated channel between a pair of users and to use in the aggregation protocol. Before starting the training process, the users need to learn the mean and standard deviation on the entire dataset $\mathcal{D}$, which can be easily computed by running the aggregation protocol as shown in Section 4.1.

The process of the global gradient computation [1] consists of four main steps:

- the server randomly chooses a set of M users and broadcasts the current encrypted model to these users,
- the server and each user privately compute two shares of the local gradient on the chosen user's dataset in parallel,
- the server and alive users compute a single (aggregated) share from the alive users' shares of the local gradients,
- the server computes the second share of the global gradient from its local shares and then recovers the global gradient for updating the model.

The parameter M is chosen based on the security parameter of the aggregation protocol and the ϵ-privacy of the aggregate-sum. For simplicity, we assume that each user has an equal number of data points, i.e., $|\mathcal{D}_i| = d_i = \ell, \forall i$. For a coalition of c users and/or the server in the ϵ-private aggregation scheme, the number of dropouts, denoted by δ, must satisfy the following constraint: $\ell(M - \delta - c) \geq \epsilon$, which implies $\delta \leq (M - \rho - c)$ where $\rho = \lceil \frac{\epsilon}{\ell} \rceil$. To resist a coalition of up to t parties including users and the server, M must be at least $2t$ where $t = \lceil \frac{m}{3} \rceil$. For a coalition of size up to t and $M = 2t$, $\delta \leq (t - \rho)$. When $\ell = 1$, for such a coalition of size up to t and $\delta = t$ dropouts, the number of users must participate in the training process is at least $2t + \epsilon$. Thus, the threshold value of π_{DeA} in the scaling phase is at least $2t$. Figure 5 summarizes the complete protocol for training a linear or logistic regression model, given the parameters M, δ, t, ϵ, ℓ, and the number of iterations R.

Efficiency. It is easy to verify that the training protocol is correct if the users provide their true inputs and the server does not alter the model. Since the aggregation protocol is executed $(R + 1)$ times, the users establish the pairwise keys using a Diffie-Hellman protocol, coordinated by the server, as shown in [7], and then in each execution of π_{DeA} the users derive a pair of one-time pairwise keys from the master pairwise keys using a hash-chain, as shown in [43]. Thus, the key establishment phase is executed only once. The computational complexity of π_{DeA} with m users and n dimensional inputs for a user is $O(m^2 + mn)$, and for the server is $O(m^2 n)$. The computational complexity for one execution of the global gradient computation involves the computational complexities of π_{LinSLG} or π_{LogSLG} and π_{DeA}, which is $O(4t^2 + 2tn + nd_i) = O(m^2 + mn + nd_i)$ when $t = \lceil \frac{m}{3} \rceil$. If the training phase takes R iterations to obtain a model, the overall computational complexity of a user is $O(R(m^2 + mn + nd_i))$, and the computational complexity of the server for the training phase is $O(R(m^2 n + mnd_i))$.

The communication complexity of each user includes the costs of the DH key agreement protocol coordinated by the server, the scaling phase and R iterations of π_{LinSLG} or π_{LogSLG} and π_{DeA}. A detailed analysis on the communication complexity and storage overhead can be found in the full paper [42].

Security. We consider the security of the training protocols against semi-honest adversaries where three different threat models, namely users-only, server-only, and users-server threat models are considered. The security is proved in the simulation paradigm using the hybrid arguments. In Theorem 3, we present that a semi-honest adversary corrupting at most t parties, including the server and a set of at most $(t - 1)$ users, can learn no information about the honest users' datasets. The security of the training protocols relies on that of the SLG protocols, the aggregation protocol, and the aggregation privacy game defined in [10].

[1] In the training phase, the batch size in the gradient computation varies in the range of $(t + \rho - 1)\ell$ and $2t\ell$, where $|\mathcal{D}_i| = \ell$.

Privacy-preserving Regression Training Protocol (π_{LinTrain} or π_{LogTrain})

//Pairwise key establishment phase

1. Mobile users and the server establish pairwise key among themselves using a DH key agreement protocol. Let $\mathcal{U}_0 \subseteq \mathcal{U}$ be set of users that established pairwise keys and agreed to participate in the training phase. If $|\mathcal{U}_0| < M$, abort the protocol.

//Data scaling phase

2. Each user **in** $\mathcal{U}_0$ locally computes the mean of its dataset $\mathcal{D}_i$ $\mathbf{X}_i = \Big(\sum_{j=1}^{|\mathcal{D}_i|} x_1^{(j)}, \cdots, \sum_{j=1}^{|\mathcal{D}_i|} x_n^{(j)}, \sum_{j=1}^{|\mathcal{D}_i|} x_1^{(j)^2}, \cdots, \sum_{j=1}^{|\mathcal{D}_i|} x_n^{(j)^2} \Big)$.

3. Server and the users in $\mathcal{U}_0$ execute the aggregation protocol π_{DeA} with inputs $\{\mathbf{X}_u\}_{u\in\mathcal{U}_0}$ and obtains $(\mathcal{U}_1, \mathbf{X}_{sum}) \leftarrow \pi_{\text{DeA}}\big(\mathcal{U}_0, \{\mathbf{X}_u\}_{u\in\mathcal{U}_0}, |\mathcal{U}_0|, 2t\big)$ where $\mathbf{X}_{sum} = \sum_{u\in\mathcal{U}_1} \mathbf{X}_u = (X_1, \cdots, X_n, X_{n+1}, \cdots, X_{2n})$. If $|\mathcal{U}_1| \geq 2t$, the server computes the mean $\boldsymbol{\mu}$ over $\mathcal{D} = \cup_{u\in\mathcal{U}_1} \mathcal{D}_u$ as $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)$ with $\mu_j = \frac{1}{d_1} X_j$ and $d_1 = |\mathcal{U}_1| \cdot \ell$, and the standard deviation $\boldsymbol{\sigma}$ vector as $\boldsymbol{\sigma} = (\sigma_1, \cdots \sigma_n)$ with $\sigma_j = \sqrt{\frac{1}{d_1} X_{n+j} - (2d_1 - 1)\mu_j^2}$, $1 \leq j \leq n$. Server sends $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ to all users in $\mathcal{U}_1$.

4. After receiving $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$, each user P_i **in** $\mathcal{U}_1$ scales its dataset as $\mathbf{x} \leftarrow \frac{\mathbf{x}-\boldsymbol{\mu}}{\boldsymbol{\sigma}}$, $\mathbf{x} \in \mathcal{D}_i$.

// Iterative training process

5. Server randomly initializes the model $\boldsymbol{\theta}$

6. **for** $j = 1 \ldots R$ **do**

- Let $\mathcal{V} \subseteq \mathcal{U}_1$ be the set of alive users at the current time. If $|\mathcal{V}| < M$, abort the protocol.
- Server chooses $\mathcal{V}_j \leftarrow$ (randomly choosing M users from $\mathcal{V}$) at time T_j. // `Gradient computation at time interval` $[T_j, T_j + \Delta]$
- Server encrypts the regression model $\boldsymbol{\theta}$ and then broadcasts the encrypted model $\text{E}(\boldsymbol{\theta})$ to the users in $\mathcal{V}_j$.
- **for** each user $P_i \in \mathcal{V}_j$ **in parallel do**

  Server runs the SLG computation protocol π with P_i. Server receives $\text{E}(\mathbf{s}_i)$ and P_i receives $\mathbf{r}_i$ as output where $(\text{E}(\mathbf{s}_i), \mathbf{r}_i) \leftarrow \pi(\boldsymbol{\theta}, \mathcal{D}_i)$ and $\pi \in \{\pi_{\text{LinSLG}}, \pi_{\text{LogSLG}}\}$.

  endfor
- Let $\mathcal{V}'_j \subseteq \mathcal{V}_j$ be the set of users who successfully completed the SLG computation protocol and sent $\text{E}(\mathbf{s}_i)$ to the server. If $|\mathcal{V}'_j| < (t + \rho)$, abort the protocol.
- Server and users **in** $\mathcal{V}'_j$ run the aggregation protocol π_{DeA} and complete the protocol execution in time $(T_j + \Delta)$. Server receives $(\mathcal{V}''_j, \mathbf{r}_{sum}) \leftarrow \pi_{\text{DeA}}\big(\mathcal{V}'_j, \{\mathbf{r}_u\}_{u\in\mathcal{V}'_j}, |\mathcal{V}'_j|, t\big)$. If $|\mathcal{V}''_j| < (t + \rho)$, abort the protocol.
- Server decrypts $\text{E}(\mathbf{s}_u)$ for each $u \in \mathcal{V}''_j$, and then computes $\mathbf{s}_{sum} = \sum_{u\in\mathcal{V}''_j} \mathbf{s}_u$.
- Server computes $\boldsymbol{\omega}$ as $\boldsymbol{\omega} = \mathbf{s}_{sum} - \mathbf{r}_{sum}$ and updates the model as $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \frac{\eta}{d_a} \cdot \boldsymbol{\omega}$ with $d_a = \sum_{v\in\mathcal{V}''_j} d_v = |\mathcal{V}''_j| \cdot \ell$.

endfor

7. Server stores the regression model $\boldsymbol{\theta}$.

Figure 5: Privacy-preserving training protocol for a linear or logistic regression model over a mobile network.

Theorem 3 (Privacy in Users-Server Threat Model). *The protocols π_{LinTrain} and π_{LogTrain} are secure in the presence of semi-honest adversaries, meaning they leak no information about the honest users' inputs $\mathcal{D}_i$ with $|\mathcal{D}_i| \geq 1$ to the adversary corrupting the server and a set of users of size up to $(t-1)$.*

In Theorem 4, we show that a semi-honest adversary corrupting at most $(t-1)$ users in the training phase learns no information about the honest server's model. Intuitively, since the model is encrypted using a semantically secure additive HE scheme, the model privacy is protected against semi-honest users. The proofs of Theorems 3 and 4 can be found in the full paper [42].

Theorem 4 (Privacy in Users-only Threat Model). *The protocols π_{LinTrain} and π_{LogTrain} are secure in the presence of semi-honest adversaries, meaning they leak no information about the honest server's model $\boldsymbol{\theta}$ to the adversary corrupting a set of users of size up to $(t-1)$.*

4.5 Oblivious Regression Prediction Protocols

We consider a scenario where, after training the model, users wish to use the trained regression model as predictions as a service, which is quite natural because the model was trained on their datasets, or the server wishes to offer regression predictions as a service to other clients. An oblivious regression prediction protocol is run between the server and a user. In this computation, the server holds a secret regression model $\boldsymbol{\theta}$, and a user has a private input $\mathbf{x}$. An oblivious regression prediction protocol allows the user only to learn $h(\boldsymbol{\theta}, \mathbf{x})$ without revealing $\mathbf{x}$ to the server and $\boldsymbol{\theta}$ to the users.

We assume that each user has a public and private key pair, denoted by (pk_{P_i}, sk_{P_i}), of an additive homomorphic encryption scheme and its public key is known to the server. For oblivious predictions, the user sends an encrypted input to the server. For linear regression, the server computes $\text{E}_{pk_{P_i}}(\boldsymbol{\theta} \cdot \mathbf{x})$ from $\text{E}_{pk_{P_i}}(\mathbf{x})$ and $\boldsymbol{\theta}$. For logistic regression, our oblivious prediction protocol uses the same idea on a single input as used in the SLG computation protocol. See the full paper [42] for the protocol descriptions, efficiency analysis, and their experimental evaluations.

5 EXPERIMENTAL EVALUATION

In this section, we evaluate the performance of PrivFL under semi-honest adversaries, providing 128-bit security. We first provide implementation details and then show how to deal with floating-point numbers when conjunct the regression algorithms with cryptographic primitives. Finally, we present experimental results of PrivFL on real-world datasets from the UCI ML repository.

5.1 Implementation and Dataset Details

Implementation details. We have implemented PrivFL in C using GMP [27] for large number operations, the FLINT library [30] for secret sharing implementations and OpenSSL [49] for implementing an authenticated channel. We choose the Joye-Libert (JL) cryptosystem [31] to instantiate the additive HE scheme, over Paillier's cryptosystem [50] as its ciphertext size is 2× smaller. If the server is a cloud provider (e.g., Google, Microsoft, Apple), we can save the communication cost by a factor of two with the computation ability of the server, which is quite reasonable for mobile applications. We implement an aggregation protocol that is a compilation of Bonawitz et al.'s [7] and Mandal et al.'s [43] protocols that handle user dropouts. We omit the implement details of the aggregation protocol here, but can be found in the full paper [42]. We have chosen the message space $\mathbb{Z}_{2^k}$ with $k = 256$ where $q < 2^k$, which is enough for handling high-dimension data used in our experiment. Our experiments were conducted on a desktop with a `3.40GHz Intel i7` and 12 GB RAM. The codes were compiled using gcc 5.4.0 with -std=c99 -O1 -fomit-frame-pointer flag.

Datasets. We evaluate the performance of PrivFL on 11 different real-world datasets from the UCI repository [17]. We use 6 different datasets with various sizes and dimensions (see Table 2) for evaluating the linear/ridge regression training protocol, and 5 different datasets (see Table 3) for the logistic regression training protocol. For linear regression, we use the root mean squared error (RMSE) to measure the accuracy of the model. For logistic regression, we measure the accuracy of the model using the standard method by computing a confusion matrix. Note that no privacy-preserving mechanism is used to compute the accuracy of the model.

Table 2: Dataset used in our experiment for linear/ridge regression models.

Id	Name	n	d	Reference
1	Auto MPG	8	392	[3]
2	Boston Housing Dataset	14	506	[9]
3	Energy Efficiency	9	768	[56]
4	Wine Quality	12	1,599	[13]
5	Parkinsons Telemonitoring	22	5,875	[55]
6	Bike Sharing Dataset	12	17,379	[19]

Table 3: Dataset used in our experiment for logistic regression model. † 20 out of 90 features were chosen.

Id	Name	n	d	Reference
7	Breast Cancer Dataset	32	454	[17]
8	Credit Approval Dataset	14	652	[17]
9	Diabetes Dataset	9	768	[14]
10	Credit Card Clients	24	30,000	[59]
11	US Census Income Dataset	$20^{\dagger}$	48,842	[11, 38]

5.2 Dealing with Floating Point Numbers

In the regression algorithms, the datasets and model parameters are floating point numbers, both positive and negative, but the cryptographic techniques such as additive HE work over the finite ring of integers. We provide the details on encoding floating point numbers to elements of $\mathbb{Z}_{2^k}$ and vice-versa via decoding.

Encoding and decoding. The encoding operation is applied on floating point numbers before performing cryptographic operations.

Table 4: Number of users and their training dataset size. Accuracy of the trained regression models achieved by PrivFL.

Linear regression				Logistic regression			
ID	m	d_i	**RMSE**	ID	m	d_i	**Score** (%)
1	28	10	3.16	7	32	10	96.00
2	36	10	4.91	8	46	10	87.60
3	54	10	3.85	9	54	10	76.48
4	112	10	0.68	10	700	30	80.72
5	206	20	3.45	11	1140	30	89.60
6	609	20	147.80				

As the message space is $\mathbb{Z}_{2^k}$, we divide the message space into two halves: the positive numbers are in $[0, 2^{k-1} - 1]$, and the negative numbers are in $[2^{k-1}, 2^k - 1] \equiv [-2^{k-1}, -1]$. We convert each floating point number to an element of $\mathbb{Z}_{2^k}$ while maintaining its precision. Given an absolute floating point number x in $\mathbf{x}^{(i)}$, the corresponding ring element, denoted as $\tilde{x}$ in $\mathbb{Z}_{2^k}$, is computed as $\tilde{x} = \mathsf{FE}(x, \tau) = \mathsf{round}(x \cdot 2^{\tau})$, and each floating point number y in $\{y^{(i)}\}$ is converted to a ring element as $\tilde{y} = \mathsf{FE}(y, 2\tau)$. If x is negative, the corresponding ring element is $\tilde{x} = 2^k - \mathsf{FE}(x, \tau)$, and similarly for y in $\{y^{(i)}\}$. Given $\tilde{x} \in \mathbb{Z}_{2^k}$, the decoding of $\tilde{x}$ is given by $x = \mathsf{FD}(\tilde{x}, \tau) = -\frac{2^k - z}{2^{\tau}}$ if $\tilde{x} \geq 2^{k-1}$, otherwise $\frac{z}{2^{\tau}}$.

Evaluating inner product. Given $\boldsymbol{\theta} \in \mathbb{R}^{n+1}$ and $\mathbf{x} \in \mathbb{R}^n$ and the corresponding vectors in $\mathbb{Z}_{2^k}$ are $\tilde{\boldsymbol{\theta}} \in \mathbb{Z}_{2^k}^{n+1}$ and $\tilde{\mathbf{x}} \in \mathbb{Z}_{2^k}^{n}$ where $\tilde{\theta}_0 = \mathsf{FE}(\theta_0, 2\tau)$, $\tilde{\theta}_i = \mathsf{FE}(\theta_i, \tau)$ and $\tilde{x_i} = \mathsf{FE}(x_i, \tau)$. Then, $\tilde{\boldsymbol{\theta}} \cdot \tilde{\mathbf{x}} = 2^{2\tau} \boldsymbol{\theta} \cdot \mathbf{x}$. Thus, $\boldsymbol{\theta} \cdot \mathbf{x} = \mathsf{FD}(\tilde{\boldsymbol{\theta}} \cdot \tilde{\mathbf{x}})$. If $\tilde{y} = \mathsf{FE}(-y, 2\tau)$, then $\boldsymbol{\theta} \cdot \mathbf{x} - y = \mathsf{FD}(\tilde{\boldsymbol{\theta}} \cdot \tilde{\mathbf{x}} + \tilde{y})$.

Evaluating sigmoid. We approximate the sigmoid function $\sigma(x)$ over $[-l, l]$ for some l to a cubic polynomial as $\sigma_3(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$. Note that the coefficients of the polynomial are public. To evaluate $\sigma_3(z)$ over $\mathbb{Z}_{2^k}$, we convert the coefficients of $\sigma_3(x)$ as $q_0 = \mathsf{FE}(c_0, 7\tau)$, $q_1 = \mathsf{FE}(c_1, 5\tau)$, $q_2 = \mathsf{FE}(c_2, 3\tau)$, and $q_3 = \mathsf{FE}(c_3, \tau)$, i.e., $q_i = \mathsf{FE}\big(c_i, (7 - 2 * i)\tau\big)$ and $z \in \mathbb{R}$ as $\tilde{z} = \mathsf{FE}(x, 2\tau)$. Then $\widetilde{\sigma_3(z)} = 2^{7\tau}\sigma_3(z)$, this implies $\sigma_3(z) = \mathsf{FD}(\widetilde{\sigma_3(z)}, 7\tau)$, where τ is chosen so that there is no overflow in the message space $\mathbb{Z}_{2^k}$.

5.3 Experimental Results

This section presents the performance of PrivFL where we report the timings, communication costs, and storage overhead for training the linear and logistic regression algorithms for each user and the server. As the model privacy and data privacy while considering users dropping out in mobile applications have not been considered in previous work, e.g., [7, 43], we do not compare the performance of PrivFL with others in numerical values. However, we provide a system goalwise comparison in Section 6.

Micro-benchmarking. The additive homomorphic encryption and the aggregation protocol are two main operations that are frequently performed. We perform micro-benchmarks that measure the timings of the basic operations, namely vector encryption, decryption and constant multiplication for the JL cryptosystem including floating-point encoding and decoding, and the aggregation protocol for a user and the server to understand the deeper insight about the performance of the overall protocol. Table 5 presents the timings for HE operations and the aggregation protocol.

Timing for training regression models. For each dataset, we shuffle the dataset and use approximately 70% for training and 30% for testing to calculate the accuracy of the model. The training dataset is then distributed into a set of m users and each user holds

an equal number of data points, given in Table 4. We randomly choose $2t$ users out of the m users for their participations in computing the global gradient in the training phase where $t = \lceil \frac{m}{3} \rceil$ and m is the total number of users. In our experiment, we set the number of dropout users in the gradient computation to $\delta = \lceil \frac{t}{2} \rceil$, and choose the aggregation size large enough to protect users' inputs privacy. For each dataset, the experiments were repeated five times, except Credit Card Clients and US Census Income datasets.

Table 5: Time in milliseconds (ms) (using a single CPU) for vector encryption, decryption and const. multiplication of the JL cryptosystem and the aggregation protocol.

	Vector dimension (n)				
Operations	10	20	30	40	50
Encryption (E(**x**))	8.39	16.05	24.05	32.35	39.83
Decryption (D(**x**))	4,027.02	8,131.76	12,134.78	16,178.63	20,107.34
Const. multiplication (E(**x**)$^{\mathbf{z}}$)	60.56	128.78	181.83	248.58	301.96
Aggregation protocol (π_{DEA})	**Number of users (m)**				
Dropout = 25%, $n = 30$	50	100	150	200	250
User time	20.38	41.45	63.43	85.86	107.10
Server time	34.79	161.36	420.90	823.35	1437.02

We compare the accuracy of the trained model obtained using our privacy-preserving training protocol with the model obtained using the `sklearn` tool (in clear, no security) for each dataset. Our achieved accuracy is very closed to the no security one. The number of iterations required to achieve such accuracy is $R = 350$ for linear/ridge regression and $R = 300$ for logistic regression, respectively. Figures 6a and 6c present the timings for training the linear and logistic regression models per user. Figure 6b and 6b show the server's timings for training the linear and logistic regression models. All experiment results were obtained using a single CPU.

To train a linear regression model on the parkinsons telemonitoring data, a user elapses about 105 milliseconds (ms) to compute the global gradient, and in the worst case, it elapses about 170 seconds (sec) for the entire training process. The "worst case" is because of the fact that a user may be chosen a maximum of R times. On the other hand, the server's computation time is about 10 sec for each global gradient computation, and in total about 56.57 mins. To train a logistic regression model on the credit card clients data, a user elapses about 1066 ms to compute the global gradient, and in the worst case, a user elapses about 320 sec. for the entire training process. The server's computation time is about 99 mins for each global gradient computation, and in total about 495.3 hours, which is because of performing a total of 4, 219, 200 JL decryptions and 300 executions of π_{DEA} with $2t$ users and $t/2$ dropouts where $t = 234$. *By exploiting parallelism using 24 CPUs, the training time for the server can be reduced to approximately 20 hours (estimated using the timings of unit operations).*

Note that the communication latencies among users or the server interactions during the protocol execution are not included in our experiment. However, the total execution time of the training protocol will be the computation times plus the communication latencies.

For instance, according to our experimental settings, to train a linear model on the bike sharing dataset, PrivFL's computational overheads for the server and each user, compared to a normal system, are 2.7×10^5 and 1.2×10^6, resp., where the normal system (naive implementation) provides no model and data privacy. Similarly, for training a logistic regression model on the credit card clients dataset, PrivFL's computational overheads for the server and each user are 1.4×10^8 and 3.2×10^5, resp.. The server's high computational overhead is due to the JL decryption algorithm.

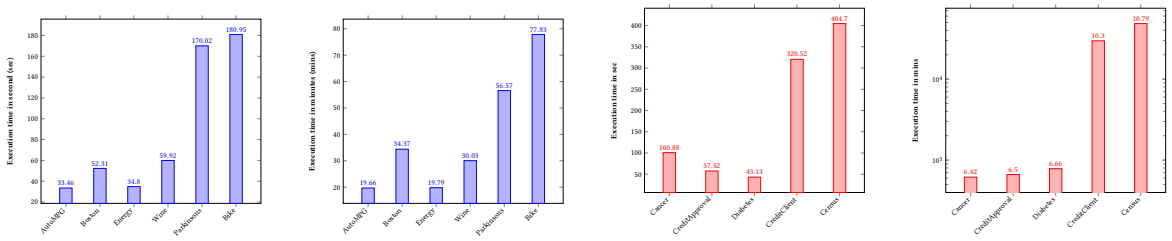
Communication and storage cost. To train a linear regression model, the communication cost involves receiving the encrypted model, transmitting the encrypted share of the local gradient and information exchange in the share reconstruction phase. For training a logistic regression model, in addition to the linear regression's communication cost, one more round of information exchange is required in the shared local gradient computation phase. Figure 7a presents the maximum amount of bits a user needs to transfer to accomplish the training phase for the choice of the parameters and the achieved accuracy in Table 4. Note that the communication cost for a user depends on how many times the user is chosen in the training phase. For instance, for training the parkinsons telemonitoring data, in the worst case, a user need to exchange 12.05 Megabyte (MB) of data to train the linear regression model. On the other hand, for the credit card clients data, in the worst case, a user need to exchange 36.10 Megabyte (MB) of data to train the logistic regression model. The communication cost for the server for each dataset is provided in Figure 7b. The storage overheads for each user and the server is presented in Figures 7c and 7d, respectively.

Discussions. In PrivFL, one can take a garbled circuit or purely secret-sharing based approach to implement the shared local gradient protocol. As the local gradient computation for linear regression can be expressed as a circuit of multiplicative depth two, a secret-sharing based approach will require at least two rounds of communications plus an additional cost of generating multiplication triplets in an offline phase. As mobile applications may have low bandwidth and slow connections, a garbled circuit or purely secret-sharing based approach will incur a higher communication overhead compared to an additive HE-based approach.

6 RELATED WORK

Privacy-preserving Linear Regression. Privacy-preserving computation of linear regression has received considerable attention. Early works [16, 32–34, 51] have considered learning linear regression model on either horizontally or vertically distributed datasets. Hall et al. [28] proposed protocols for linear regression based on homomorphic encryption techniques. Nikolaenko et al. [48] proposed a system for privacy-preserving computation of the ridge regression model by combining homomorphic encryption and Yao garbled circuits. In a follow-up work, Gascón et al. [23] proposed protocols for linear regression models based on hybrid-MPC techniques and Yao garbled circuits and using the conjugate gradient descent algorithm. Bogdanov et al. [5] developed tools for privacy-preserving linear regression based on secret sharing. Other approach for privacy-preserving linear regression is based on fully homomorphic encryption (FHE) scheme [24], which may not be suitable for mobile applications. The work of [26] can be applied to linear regression for the settings of MLaaS.

Privacy-preserving Logistic Regression. Logistic regression is an essential technique to classify data. Aono et al. [8] proposed a system for both training and predicting data using logistic regression relying on additive homomorphic encryption where they considered the computation outsourcing scenario in which a server computes the logistic regression model and sends an encrypted model to the user. They also showed how to make the system



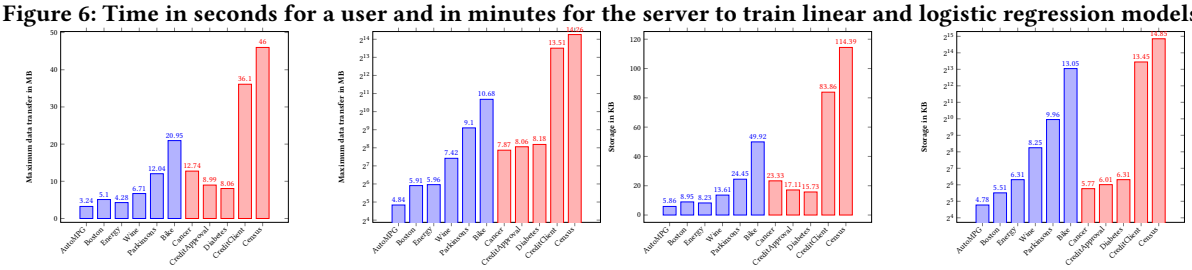
(a) Time per user to train linear regression models

(b) Time elapsed by the server to train linear regression models

(c) Time per user to train logistic regression models

(d) Time elapsed by the server to train logistic regression models

Figure 6: Time in seconds for a user and in minutes for the server to train linear and logistic regression models.



(a) Maximum data transfer per user

(b) Maximum data transfer for server

(c) Storage per user

(d) Storage for the server

Figure 7: Total data exchange in Megabyte (MB) and storage required in Kilobyte (KB) per user and the server to train linear and logistic regression models.

differential privacy enabled. Bonte and Vercauteren [2] explored secure training for logistic regression using somewhat homomorphic encryption in the computation outsourcing scenario where the training is done using Newton-Raphson method. Kim et al. [37] also investigated the training phase of the logistic regression model, using somewhat homomorphic encryption scheme, based on gradient descent algorithm and the Taylor series polynomial approximation. Zhu et al. [60] presented a secure outsourcing protocol for training and evaluating logistic regression classifier in cloud. Kim et al. in [36] proposed a method to train a logistic regression model based on the approximate homomorphic encryption. There is no satisfactory solution for training an ML model using SWHE or FHE when data come from multiple sources. All these techniques cannot handle the dropout scenario and are not suitable for mobile applications.

Privacy-preserving Federated Learning. Shokri and Shmatikov [52] presented a scheme for privacy-preserving deep learning. Hardy et al. [29] presented a three-party protocol for logistic regression using additive homomorphic encryption where the protocol consists of privacy-preserving entity resolution and federated logistic regression and the data is vertically partitioned. Note that their work have not considered the model privacy and the dropout scenario. Our solution is more general compared to theirs. Fioretto and Hentenryck in [20] proposed a protocol for federated data sharing to use under the framework of differential privacy, which allows the users to release a privacy-preserving version of the dataset to use in training an ML algorithm. Liu et al. [41] proposed a technique for privacy-preserving federated transfer learning. Truex et al. [54] presented an approach for private federated learning, decision tree, neural networks that provides data privacy guarantees using differential privacy and threshold HE schemes.

Generic Secure ML Systems. Systems, namely SecureML [47], Prio [12] and ABY^3 [46] can be used to perform privacy-preserving linear and logistic regression, but such systems need existence of addition servers (other than the server used for coordination) where users secret share their data among a set of servers. These systems provide stronger security compared to ours because in PrivFL, the server has access to the model in each iteration of the model update, which is quite suitable in the federated setting. However, these approaches are orthogonal to the federated learning setting where users send (using secret sharing) their private data to the servers, but in federated learning, users' data never go out of the devices.

7 CONCLUSIONS

This paper presented PrivFL, a privacy-preserving system for training and oblivious predictions of the predictive models such as linear and logistic regressions in the federated setting, while ensuring dropout robustness, data and model privacy. PrivFL enables a robust and secure training process by iteratively executing a secure multi-party global gradient protocol built using lightweight cryptographic primitives suitable for mobile applications. The security of PrivFL is analyzed against semi-honest adversaries. Our experimental results on several real-world datasets demonstrate the practicality of PrivFL to incorporate in the federated learning system.

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