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T3 ________________	Problem Chosen	F3 ________________
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2018

MCM/ICM

Summary Sheet

Summary

abstract

Keywords: keyword1; keyword2

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1 Introduction

1.1 Statement of the problem

Energy production and energy consumption, which can be regarded as an important economic index, not only reflect the industrial development of a country but also relate to the lifehood of a country. But on the other hand, with the continuous advancement and deepening of the industrialization of human civilization, the consumption of non-renewable energy sources such as coal, petroleum is also accelerating. Hence, the development of cleaner renewable energy is particularly important. After all, if humans depend too much on non-renewable energy sources, the day when fossil fuels are depleted is also a day for humankind to return to agrarian society.

As the world's superpower, the US is a big country of energy. Many of America's energy policy decentralizes to the state level. to ensure cooperative action between the states [1], many compacts are formed between states. In this context, along the border with Mexico four states, California, Arizona, new Mexico, Texas hope to form a new energy compact focused on more and more widely used, cleaner renewable energy sources.

- Create an energy profile for each of the four states.
- Make governors easier to understand the four states'usage of cleaner, renewable energy sources and the similarities and difference between the four states.
- Determine a state that is appeared to have the "best" profile for use of cleaner, renewable energy in 2009.
- Predict the energy profile of each state in 2025 and 2050.
- Determine renewable energy usage targets for 2025 and 2050 which are also for the new four-state energy compact.
- Provide at least three suggestions about how to meet the energy compact goals.

1.2 Overview of Our Work

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2 Notations and Assumptions

2.1 Notations

Symbol	Specification
T_s	The total energy transferred.
T_e	The renewable energy transferred.
S_p	The total energy produced.
E_p	The renewable energy producted.
S_c	The total energe consumed.
E_c	The renewable energy consumed.
C_i	The proximity of the i-th evaluation object with the optimal solution.
D_i^-	The distance between the i-th evaluation object and the worst case.
D_i^+	The distance between the i-th evaluation object and the best case.
E^*	The energy status indicator.
E	The impact of energy, economy or environment.
W_{si}	The subjective evaluation weight of indicator I.
W_{oi}	The objective evaluation weight of indicator I.
W_{ti}	The overall weight of indicator I.

2.2 Assumptions

- Assume that the technology of renewable energy will not be an obstacle to the development of renewable energy.
- Assume that a very significant technological breakthrough will not occur suddenly which may result in that renewable energy source can be widely and easily used.
- Assume that the storage of fossil energy can meet the current trend of fossil energy production.
- Assume that various energy price fluctuations will not change too much.
- Assume that there will be no loss in energy transfer in the current four states.
- Assume that the current energy transfer costs are the same between the four states.

3 Energy Profile

3.1 Overview

We make an energy profile for the four states from 1960 to 2009 in the past 50 years, which covers four aspects of energy consumption, energy prices, energy expenditures, energy production.As shown in the following figures.

(Note that the following figures labels are five-character series names.The first two letters represent state code. According to the guide [2], [3], [4], the third and fourth letters like 'TC' means total consumption and 'PR' means product; the fifth letter represents the type of data such as 'B' means consumption in British thermal units (Btu), 'D' means price in dollars per million Btu and 'V' means expenditure in million dollars.Furthemore, the curve represents the change in energy over time.)

3.2 Each State

3.2.1 Arizona

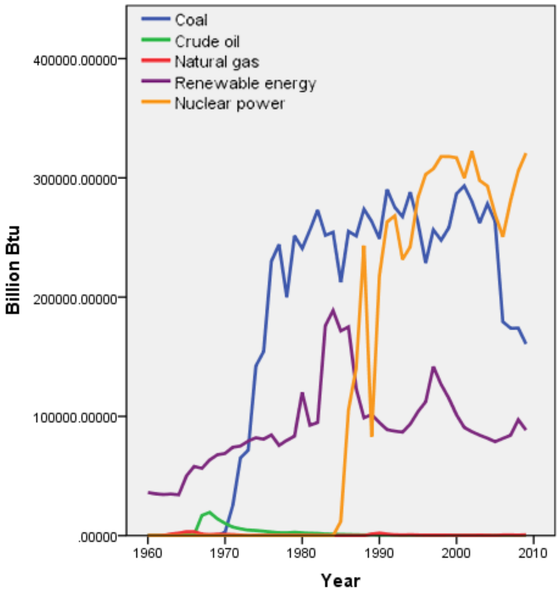

Figure 1: AZPRB

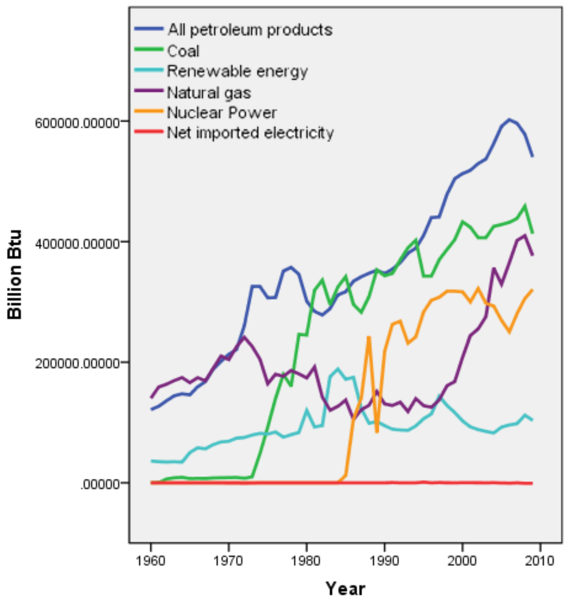

Figure 2: AZTCB

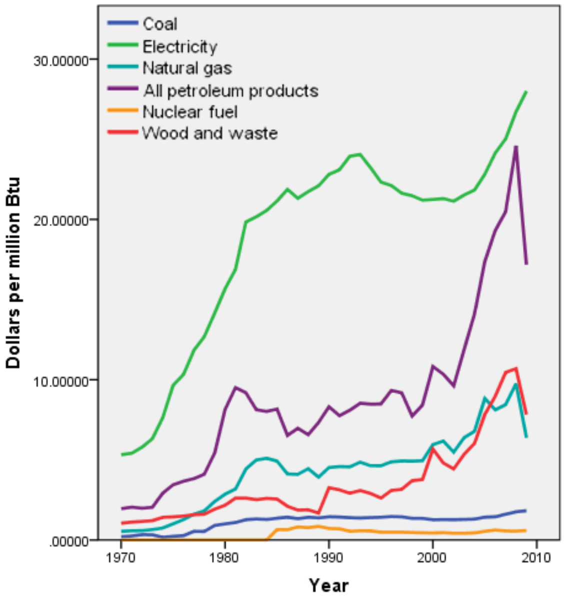

Figure 3: AZTCD

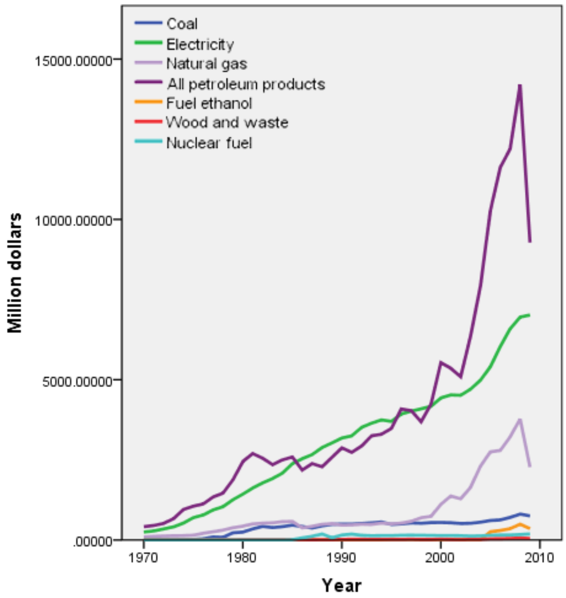

Figure 4: AZTCV

3.2.2 California

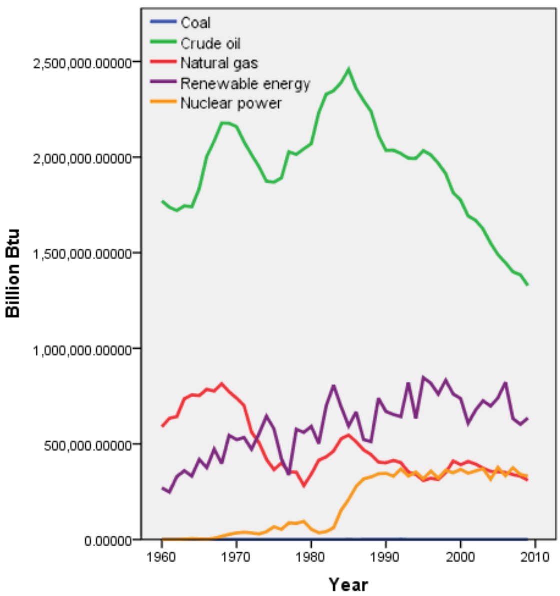

Figure 5: CAPRB

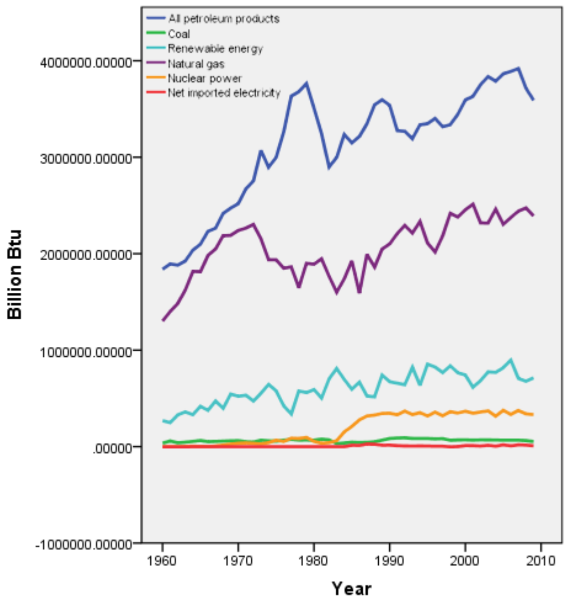

Figure 6: CATCB

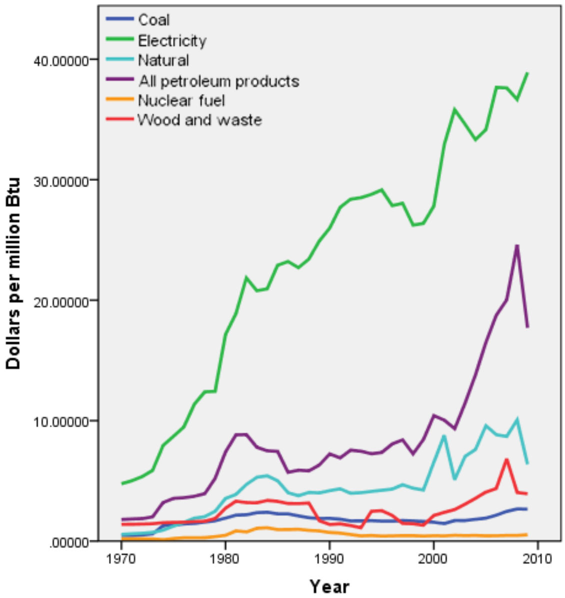

Figure 7: CATCD

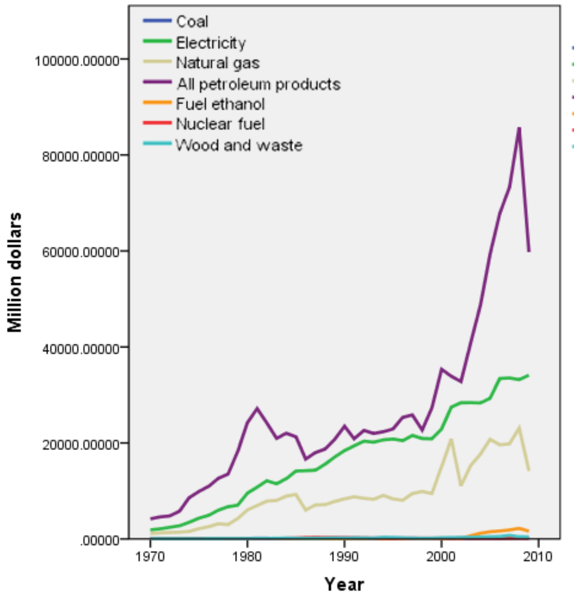

Figure 8: CATCV

3.2.3 New Mexico

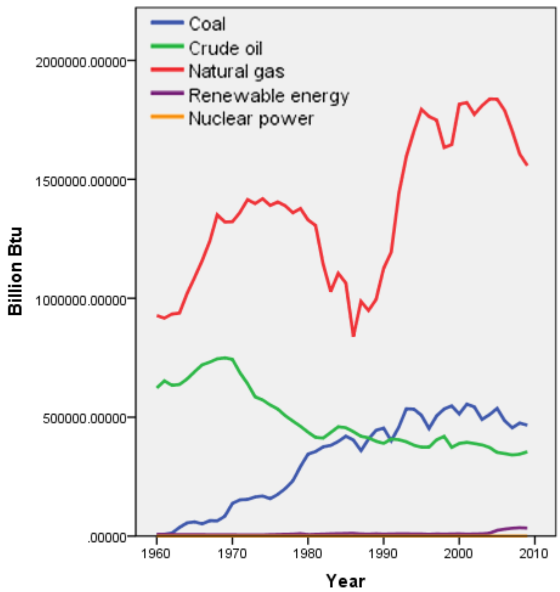

Figure 9: NMPRB

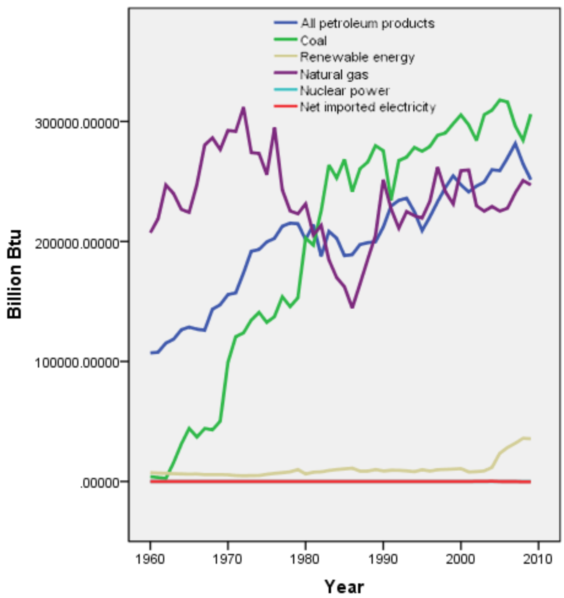

Figure 10: NMTCB

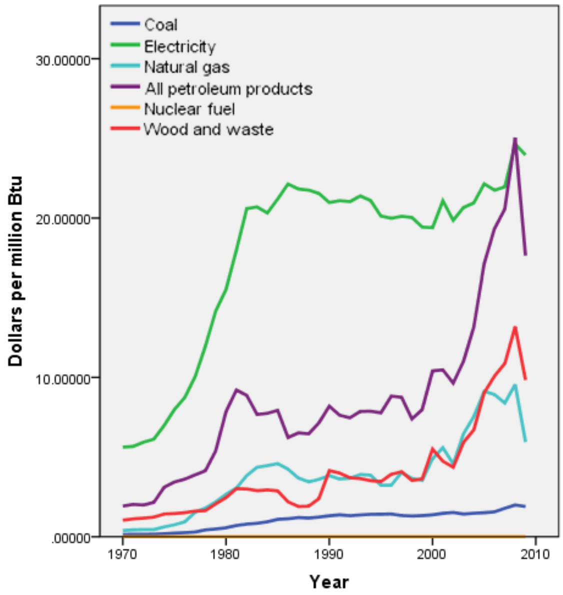

Figure 11: NMTCD

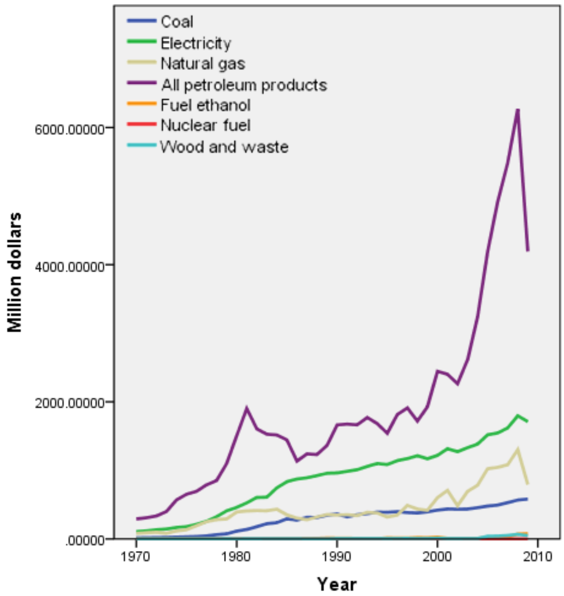

Figure 12: NMTCV

3.2.4 Texas

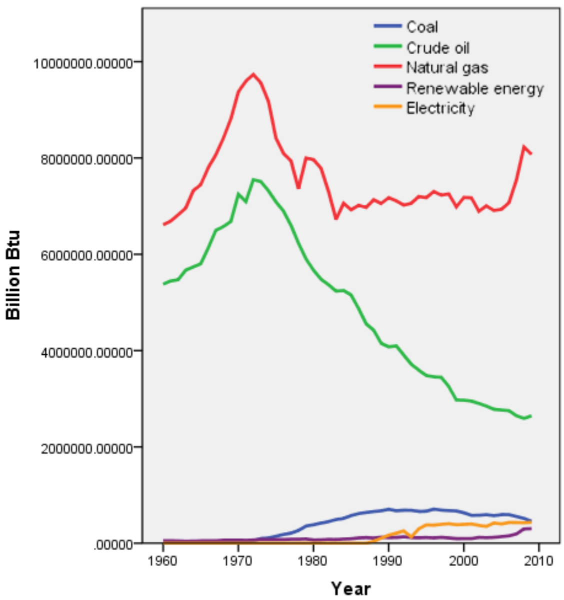

Figure 13: TXPRB

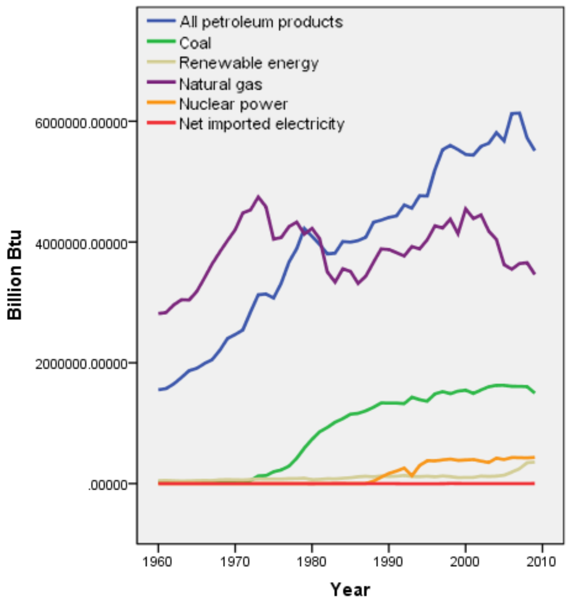

Figure 14: TXTCB

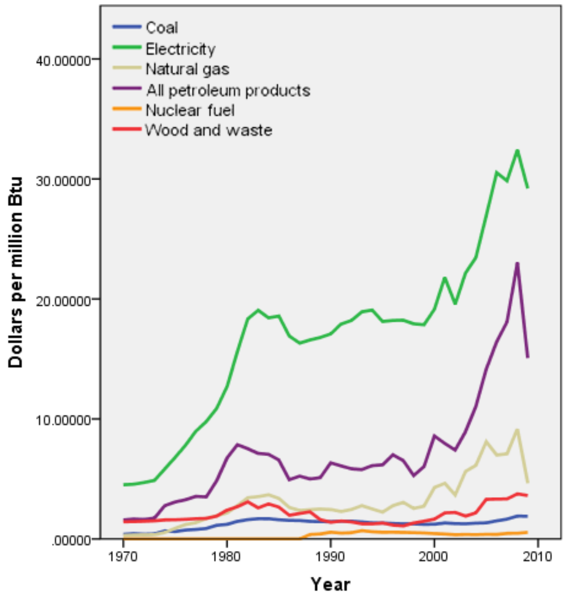

Figure 15: TXTCD

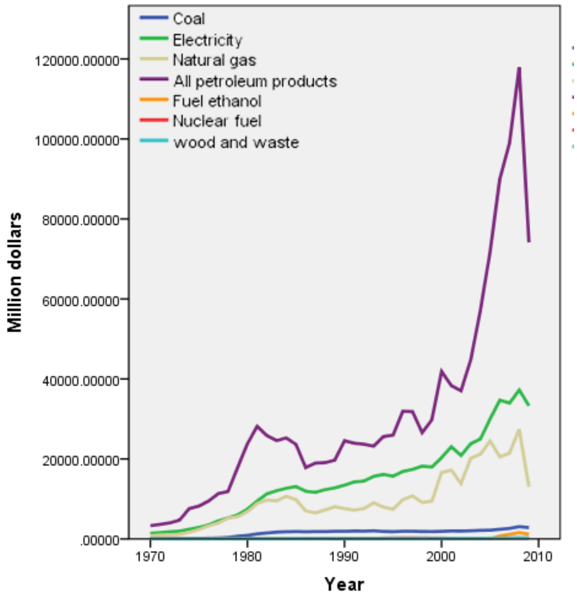

Figure 16: TXTCV

4 Sub-Model.I : 3E Evaluation Model

4.1 Result and Analysis

To describe the energy situation of the four states, first, we need to define an indicator that describes the overall condition of energy.Energy is closely related to the economy and the environment. therefore, the impact of economic and environment can not be ignored as we consider the condition of energy.

So we study the paper [9] and take it into consideration. For energy, we use total energy consumption and clean energy consumption as a proportion of total energy consumption; For economy, we use the GDP of each state and real GDP per capita of each state; For the environment, we use carbon dioxide emissions and temperature data. Finally, these data are integrated to represent our indicators of clean energy. For the weight of the two kinds of data in all aspects, we use the method of principal component analysis and analytic hierarchy process to determine synthetically. This process can not only express the subjective will of policy makers, but also avoid the deviation of subjective wishes and the actual situation. In addtion, the original data can be fully utilized.Based on experience, we assume that subjective weights and objective weights are equally important, then the weight of each indicator is

$$W_{ti} = 0.5W_{si} + 0.5W_{oi} \tag{1}$$

First, we standardize on the six kinds of data. The way We use to standardize is the range-method . Formulas are as follows:

For positive indicators,

$$Y_i = \frac{X_i - X_{min}}{X_{max} - X_{min}} \tag{2}$$

For negative indicators,

$$Y_i = \frac{X_{max} - X_i}{X_{max} - X_{min}} \tag{3}$$

Therefore, the final calculation formula of energy, economy and environment is

$$E = \sum_{i=1}^{n} W_{ti} Y_i \tag{4}$$

We assume that energy, economy and environment are equally important to our indicators, so the formula is

$$E^* = \frac{E_e + E_c + E_v}{3} \tag{5}$$

The larger this indicator, the better the clean energy condition. Among them, as the reason of that carbon dioxide emissions is positive, the actual calculation need to take the opposite treatment.

After then, we make a statistical analysis of the energy status indicators of each state from 1980 to 2009, and make the figure of comprehensive energy status indicator time series in different states, as shown below.

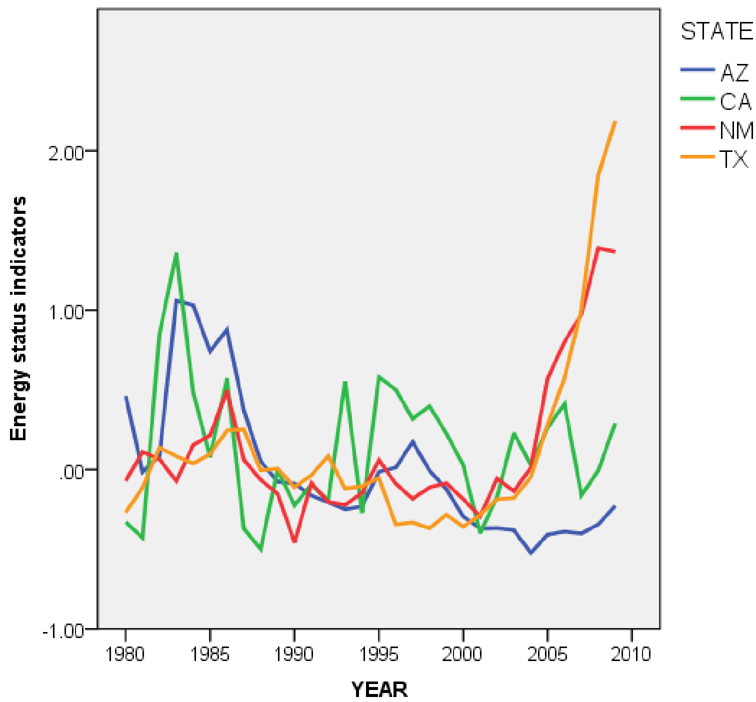

Figure 17: Comprehensive energy status indicator time series

As we can see, the beginning of the rapid economic growth leads to the indicator increasing. However, due to the impact of carbon dioxide emissions on environmental factors, the indicators of all states began to decrease around 1987. Then, with the concept of sustainable development strengthened, each state proposed its own new energy policy and the indicator began to increase. As we can see in Texas and New Mexico, because of their energetic efforts to develop new energy sources, their indicators were increasing at a relatively faster rate.

Moreover, since there is a large amount of data in each state that affects the similarities and differences between the states, we analyze the renewable energy condition in each of the four states by making time series diagram of different states.

We plot the development trends of various renewable energy categories in Arizona over the past 50 years in figure.18.

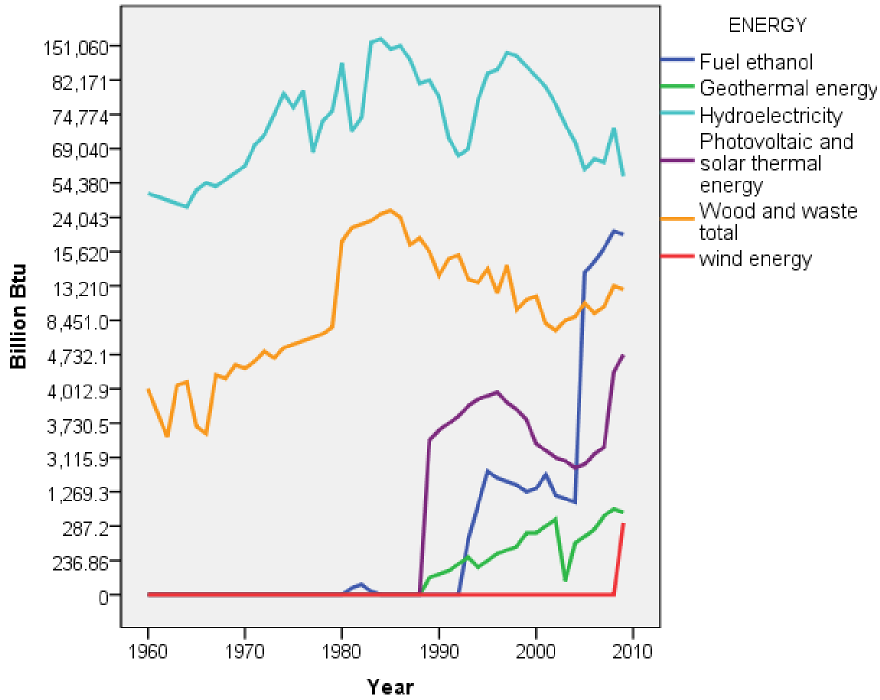

Figure 18: Arizona renewable energy time series

As we can see, hydropower in Arizona is growing earlier and occupies a significant proportion of all renewable energy sources, followed by wood and waste products. Other renewable energy sources began to develop in the 1980s and 1990s. It is is notable that fuel ethanol and solar power are developing rapidly. By 2009, fuel ethanol consumption exceeded that of wood and waste, making it the second largest consumption of renewable energy.

An analysis of Arizona's geography and climate shows that Arizona has abundant solar energy resources, averaging 300 sunny days a year. However, due to the long-term high cost of photovoltaic energy, the growth rate has gradually slowed down after the initial rapid growth. The current cost of photovoltaic energy is estimated at 0.15-0.25 dollars / kWh, so the development is not satisfactory. [5] Although the wind energy in Arizona is developing slowly, its development prospects are good. Located near the eastern edge of Mogollon Rim, eastern Arizona and eastern Arizona are geographically superior and resource-rich. There are also good wind resources on the edges and ridges across the state. [6]

We plot the development trends of various renewable energy categories in California over the past 50 years in figure.??.

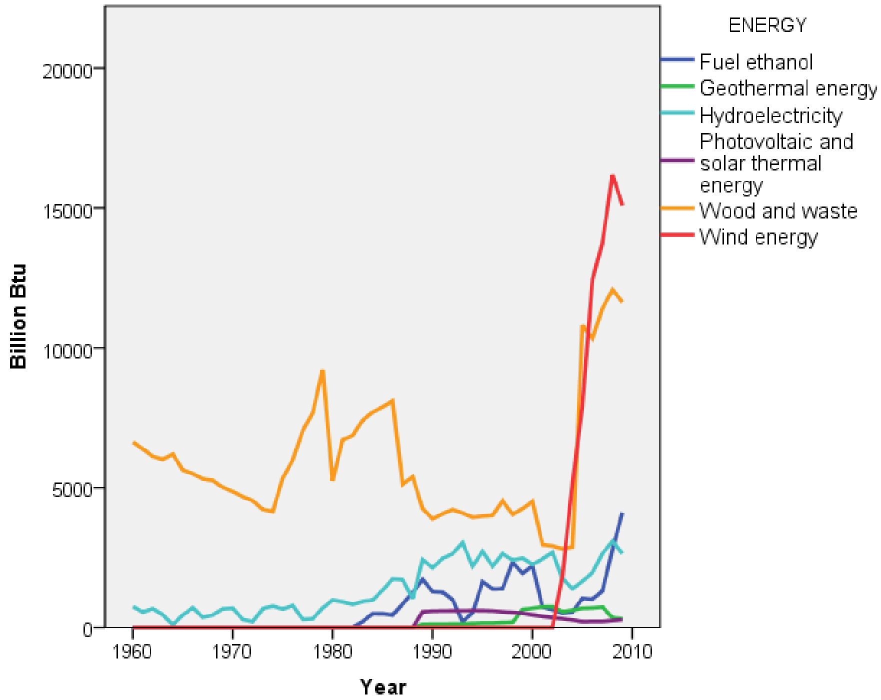

Figure 19: California renewable energy time series

The figure shows that California's total renewable energy consumption is huge. All types of renewable energy consumption is similar to that in Arizona, where hydropower accounts for the highest share of all renewable energy sources, followed by wood and waste. Other renewable energy sources began to develop in the 1980s and 1990s. It is is notable that fuel ethanol is developing fastest. Unlike other sources of energy, California's geothermal energy has grown earlier and is consumed much more.

After analyzing the state of California, we can see it clear that the eastern part of California, especially the southern tip and the northeast end is a desert, so there may be more geothermal energy. In addition, due to the large population, there are also many energy consumption.

We plot the development trends of various renewable energy categories in New Mexico over the past 50 years in figure.20.

As we can see, the state's total renewable energy consumption is less. By 2005, the largest share of renewable energy was wood and waste. Wind energy has grown rapidly since 2005, making it the biggest consumption of renewable energy.

We plot the development trends of various renewable energy categories in Texas over the past 50 years in figure.21.

It can be seen that Texas is very much like New Mexico in the use of wind energy and wood and waste.After analysis of Texas data, we find that at present Texas is the

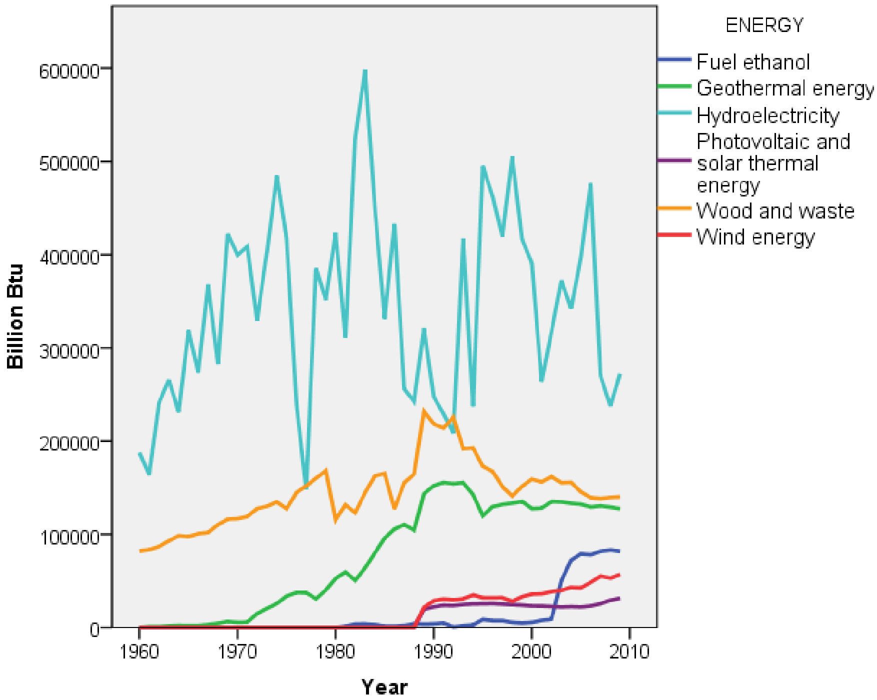

Figure 20: New Mexico renewable energy time series

largest state in the United States with the highest wind energy generation. What supers us is taht the wind energy consumptiion has been increased so fast in past ten years. The paper [8] shows that the reason is the state's renewables portfolio standard (RPS), which forces RPS Power suppliers proactively use large-scale wind and other renewable energy to generate electricity.

4.2 Conclution

A comparison of all the figures above demonstrates that Arizona and California are similar, while new Mexico and Texas are similar. Renewable energy sources in Arizona and California account for a large proportion of renewable energy. Wind energy in New Mexico and Texas accounts for a large proportion of renewable energy. As the result of the difference between different states in geography and climate, Arizona mainly develops solar energy while California, imainly developing geothermal energy, New Mexico and Texas mainly developing wind energy.In addition, each state's fuel ethanol energy has a certain development.

5 Determing "best" profile

In order to facilitate the further processing of the following problems, we use gray relational analysis.

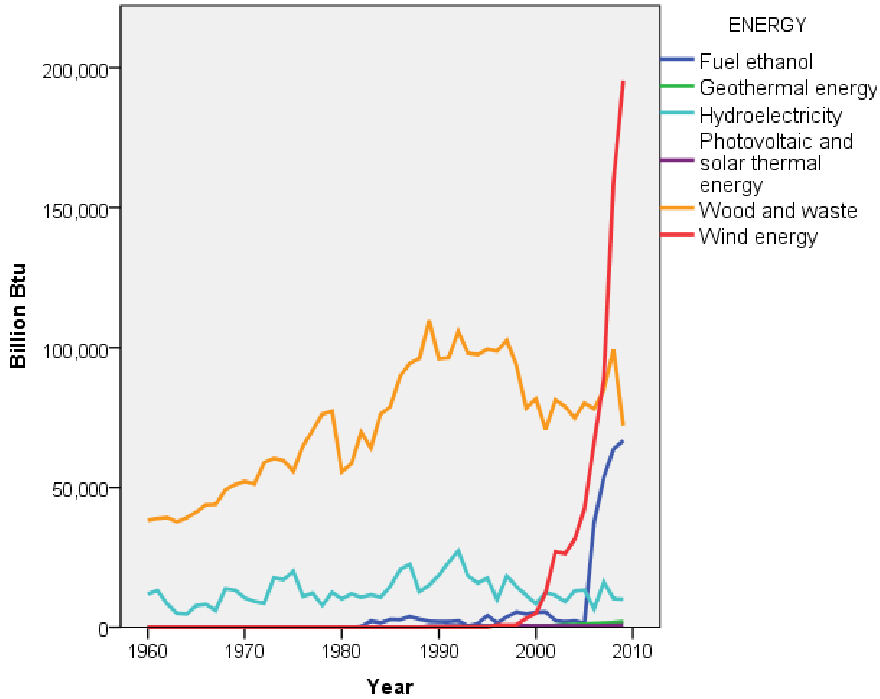

Figure 21: Texas renewable energy time series

5.1 gray relational analysis

Gray system theory puts forward the concept of analyzing the gray relational degree of each subsystem. It intends to seek the numerical relationship among the subsystems (or factors) in the system through certain methods. Therefore, gray relational analysis provides a quantitative measure of a system development trend and is very suitable for dynamic history analysis. [5]

5.2 Steps of Calculation

First, Identify reference sequences that reflect system behavior characteristics and comparison sequences that affect system behavior. The sequence of data that reflects the behavior of the system is called the reference sequence. Factors affecting the behavior of the system composed of data series, is called the comparison sequence.

After obtaining the reference sequence and the comparison sequence, the reference sequence and the comparison sequences should be treated dimensionless. Due to the different physical meaning of each factor in the system, the dimensionality of the data is not necessarily the same, which is not convenient for comparison, or it is difficult to get the correct conclusion in comparison. Therefore, in the analysis of grey relational degree, it is generally necessary to conduct dimensionless data processing.
Select reference sequence,

$$x_0 = \{x_0(k)|k = 1, 2, \cdots, n\} = (x_0(1), x_0(2), \cdots, x_0(n)) \tag{6}$$

Suppose there are m number of comparisons,

$$x_i = \{x_i(k)|k = 1, 2, \cdots, n\} = (x_i(1), x_i(2), \cdots, x_i(n)), i = 1, 2, \cdots, m \quad (7)$$

Where k represents the moment, a total of n moments.

Before calculating the correlation coefficient, the comparison sequence need to be treated dimensionless. The common dimensionless methods are average method and initialization method. What we use is the initialization method. Here $x_i'(k)$ indicates the comparison sequence just screened. $x_i(k)$ is the new processed comparison sequence.

After that, the gray correlation coefficient $r_i(k)$ between the reference sequence and the comparison sequence can be calculated. The so-called correlation coefficient, is substantially between degree of difference curve geometry. Therefore, the size of the difference between the curves can be used as a measure of the degree of association For a reference sequence x_0, there are several comparison sequences $x_1, x_2,\ cdots, x_m$. The correlation coefficient $r_i(k)$ of each comparison sequence and the reference sequence at each moment (ie, points in the curve) can be calculated by following formula: Where ρ is the resolution coefficient, generally between 0 to 1, usually take 0.5.

$$r_i(k) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + \rho \times \min_i \min_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \rho \times \min_i \min_k |x_0(k) - x_i(k)|}, \quad k = 1, 2, \cdots, n \quad (8)$$

In Eq.8, $min\ limits_i\ min\ limits_k |x_0(k) - x_i(k)|, max\ limits_i\ max\ limits_k - x_i(k)|$ are the minimum difference between the two levels and the maximum difference between the two levels.

In the formula, $min\ limits_i\ min\ limits_k |x_0(k) - x_i(k)|, max\ limits_i\ max\ limits_k - x_i(k)|$ are the minimum difference between the two levels and the maximum difference between the two levels. Where rho is the resolution coefficient, which is usually taken as (0, 1). The smaller the rho, the greater the difference between the correlation coefficients and the stronger the ability to distinguish. Usually take 0.5.

The correlation coefficients defined in the formula describe an indicator of the correlation degree between the comparison sequence and the reference sequence at a certain time. Since there is an association number at each moment, the information is more scattered and it is not convenient to compare. Therefore, we give the formula to calculate the correlation between the comparison sequence and the reference sequence:

$$r_i = \frac{1}{n} \sum_{k=1}^{n} r_i(k) \quad (9)$$

According to Eq.9 to calculate the relevance of each comparison series and sort.

We select the same relevant indicators for each state here. First of all, we select five indicators that appear in all the states, and then assign the weights of the ten main influence indicators in each state, and sort them according to their relevance to the comprehensive evaluation indicators. We pick the largest of the three indicators other than the five mentioned above. So in the end we decide to adopt the following eight indicators: PAPRB NAMPB NNTCB FFTCB TETCB PATCB HTCB WWTCB

5.3 Use Topsis to Evaluate

Then we evaluated the indicators of the four states using the Topsis method, which is based on the principle [7].

There are n evaluation objects, m evaluation indicators. The original data can be written as a matrix:

$$\mathbf{X} = (X_{ij})_{nm} \tag{10}$$

The high-quality, low-quality indicators are normalized transformation, that is

$$Z_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{n} X_{ij}^2}} \tag{11}$$

After normalizing, we get the matrix

$$\mathbf{Z} = (Z_{ij})_{nm} \tag{12}$$

The maximum and minimum of each column constitute the best and worst vector respectively as:

$$\mathbf{Z}^+ = (Z_{max1}, Z_{max2}, \cdots, Z_{maxm}) \tag{13}$$
$$\mathbf{Z}^- = (Z_{min1}, Z_{min2}, \cdots, Z_{minm}) \tag{14}$$

The distances between the i-th evaluation object and the optimal scheme and the worst scheme are as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^{m}(Z_{maxj} - Z_{ij})^2} \tag{15}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m}(Z_{minj} - Z_{ij})^2} \tag{16}$$

The approximate degree of the i-th evaluation object and the optimal scheme is Ci:

$$C_i = D_i^-/(D_i^+ + D_i^-) \tag{17}$$

Here we use the DPS tool for processing, and the final result is as follows.

Table 1: The evaluation results of state by Topsis

Sample	D^+	$D-$	C_i	Rank
AZ	1.9694	0.7961	0.2879	4
CA	1.7148	1.4377	0.4561	2
NM	1.8401	1.4144	0.4346	3
TX	1.7362	1.574	0.4755	1

As the Tbl.1 shows, according to our standards, we conclude that Texas is the best state for clean energy.

6 Sub-Model.II : Energy Status Prediction Model

In terms of forecasting energy profile in 2025 and 2050, we mainly consider overall energy assessment, and the difference value between energy production and consumption. The difference between energy production and consumption is

$$S = PR - PC \tag{18}$$

where PR is energy production and PC is energy consumption. This value represents the level of the state's surplus of energy, the value greater than zero indicates that the state's energy production value is greater than the consumption value, which means the state can export energy to other states. Value less than zero indicates that the state energy consumption is greater than the production, which means the state need to import energy from other states.

For the prediction model, we use the gray prediction model.

6.1 GM(1,1)

We use the GM(1,1) model to forecast time-series. GM(1,1) is one of the models of gray theory, which has been utilized in many areas such as the power industry [].The process of developing the model is shown below.

The sequence of time-series data is assumed to be

$$X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), X^{(0)}(3), ..., X^{0}(n)\} \tag{19}$$

where n represents the total number of time-series data.

Use an accumulated generation operation(AGO) on the sequence to get a new squence

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i), \quad k = 1, 2, ..., n \tag{20}$$

Therefore, the new squence is

$$X^{(1)} = \{X^{(1)}(1), X^{(1)}(2), X^{(1)}(3), ..., X^{1}(n) \tag{21}$$

The grey differential equation is ued to develop the GM(1,1) model

$$X^{(0)}(k) + ay^{(1)}(k) = u \tag{22}$$

where a is the development coefficient; $y^{(0)}$ represents the mean generation with consecutive neighbors sequence of $X^{(1)}$ and its define as,

$$y^{(1)}(k) = \frac{1}{2} \times \left[y^{(1)}(k) + y^{(1)}(k-1)\right], \quad k = 1, 2, ..., n \tag{23}$$

Use the ordinary least squares method, first-order AGO sequence can be obtained as,

$$X^{(1)}(k) = \left(X^{(0)}(1) - \frac{\hat{u}}{\hat{a}}\right) \times e^{-\hat{a}(k-1)} + \frac{\hat{u}}{\hat{a}} \tag{24}$$

where $[\hat{a}, \hat{u}]^T$ is equal to $\left(C^T C\right)^{-1} C^T X$ and

$$C = \begin{bmatrix} -y^{(1)}(2) & 1 \\ -y^{(1)}(3) & 1 \\ ... & ... \\ -y^{(1)}(n) & 1 \end{bmatrix} \tag{25}$$

and

$$X = \left[X^{(0)}(2), X^{(0)}(2), ..., X^{(0)}(n)\right]^T \tag{26}$$

According to the prediction model above,

7 Target and Plan

7.1 Target

Based on our forecast results in Sub-Model.II, we find that the energy distribution is not balanced between states, while Arizona and California consume far more energy than production, New Mexico and Texas produce far more energy than they consume. Therefore, we consider that if four states can achieve energy complementarity, such as energy-rich states send energy to less energy-intensive states, and the energy distribution of the four states will be relatively balanced. At the same time, the state with large energy consumption will be paid to the energy export state, which will bring economic benefits to the energy-rich states and improve the overall economic benefits. This not only improves the utilization of resources, but also brings the average economic benefit. So our overall goal is to make the energy status of the four states more balanced and the economic benefits to be relatively optimal.

Based on this idea, we propose the following optimization model.Our optimization variable is set as energy transfer between states,such as $T_e(i,i)$, $T_s(i,j)$, where i represents a state. We set the goal of optimizing the proportion of renewable energy used in each of the four states as equal as possible, that is, the least variance in the use of renewable energy in the four states.

$$\left(\sigma^2\left(\frac{E_c(i)}{S_c(i)}\right)\right) \tag{27}$$

We set the state of energy transmission is 0, the same two states transfer each other the opposite number, so we set the transport constraints

$$T_e(i,i) = 0 \tag{28}$$
$$T_e(i,j) = -T_e(j,i) \tag{29}$$
$$T_S(i,i) = 0 \tag{30}$$
$$T_s(i,j) = -T_s(j,i) \tag{31}$$

At the same time, according to our forecasts, the total renewable energy produced by the four states is greater than the total renewable energy consumption of the four states, but the total energy consumption is slightly larger than that of production and needs to be imported from other places. Energy constraints are:

$$E_p(i) > E_c(i) \tag{32}$$

$$S_p(i) > S_c(i) \tag{33}$$

$$\sum_i S_c(i) < \sum_i S_p(i) \tag{34}$$

7.2 Plan

In response to the objective and the results of our model solution, and for short-term goals and long-term goals, the following four measures were proposed.

Short-term measures until 2025:

- Texas and New Mexico transfer excess renewable energy to Arizona and California.
- Increase the transmission of renewable energy and reduce the transmission of non-renewable energy such as fossil fuels.

Long-term measures until 2050:

- The states continue to vigorously develop renewable energy based on the current energy situation.
- The states can approprately expand the number of cooperating states in order to achieve better project.

8 The Evaluation of Model

we will discuss the gray prediction model in this section.

8.1 Strengths

- The model is simple and easy to implement.
- The model is calculated using a professional mathematical analysis software, a higher degree of feasibility.
- The indicators set up take into account a number of factors and are highly integrated.

8.2 Weaknesses

- The prediction model is too simple to predict well.

- For this problem, there are more external data introduced, and fewer things are created by ourself.
- Not devoting too much time to modify the part of the model, and the center is not prominent enough.

8.3 Improvements

- For 2009's and previous data, we can combine them with the government policies, the economic crisis and new energy technology breakthroughs to analyze and find the causes of the energy fluctuations in the time series curve, thus making the curve smoother.
- For the goal optimization problem, we can consider the transmission costs between states and make the model closer to the actual situation.

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Appendices

Table 2: txCO2

date	million metric tons of CO2
1980	496.5
1981	483.4
1982	461.1
1983	467.1
1984	492.7
1985	498.7
1986	496.2
1987	502.1
1988	534.9
1989	554.9
1990	565.1
1991	560.2
1992	559.9
1993	577.8
1994	576.9
1995	581.5
1996	624.9
1997	651.4
1998	656.6
1999	633.3
2000	657.6
2001	651.5
2002	661.6
2003	655.5
2004	649.6
2005	612.2
2006	623.4
2007	620
2008	585
2009	550.1
2010	582.5
2011	601.5
2012	596.3
2013	623
2014	625.3
2015	625.8

Table 3: nmCO2

date	million metric tons of CO2
1980	44.9
1981	44
1982	45.1
1983	48.6
1984	46.2
1985	46.5
1986	43
1987	46.2
1988	47.9
1989	50.4
1990	53.3
1991	49.2
1992	51.7
1993	52.6
1994	52.5
1995	51.1
1996	52.6
1997	56
1998	55.5
1999	56.4
2000	58.2
2001	58.3
2002	55.3
2003	57.6
2004	58.7
2005	59.3
2006	59.8
2007	59
2008	56.4
2009	57.3
2010	53.3
2011	55.7
2012	53.6
2013	53.2
2014	50.1
2015	50.2

Table 4: caCO2

date	million metric tons of CO2
1980	348.4
1981	337
1982	299.9
1983	293
1984	319.5
1985	324.2
1986	309.5
1987	340.1
1988	348.2
1989	363.5
1990	363.9
1991	351.7
1992	356.1
1993	345.5
1994	362.4
1995	351.4
1996	350.5
1997	353
1998	363.4
1999	367
2000	382.4
2001	386.9
2002	386.1
2003	373.8
2004	392.3
2005	389.3
2006	397.5
2007	402.5
2008	385.7
2009	372
2010	365.9
2011	352.2
2012	357.1
2013	359.8
2014	356.7
2015	363.5

Table 5: azCO2

date	million metric tons of CO2
1980	52.7
1981	59.6
1982	58.2
1983	53.9
1984	58.2
1985	60.7
1986	55.9
1987	56.1
1988	59.3
1989	65.2
1990	62.8
1991	63.7
1992	66.5
1993	69
1994	71.7
1995	66.7
1996	68.4
1997	71.6
1998	76.5
1999	80.4
2000	86.1
2001	88.4
2002	87.8
2003	89.6
2004	96.6
2005	96.7
2006	99.9
2007	101.9
2008	102.3
2009	93.4
2010	95.2
2011	93.3
2012	91.3
2013	95.1
2014	93.1
2015	90.9

Proof. x □

Lemma 1. *If $f \in C_L^{1,1}(\mathbb{R}^n)$, then $\forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ we have*

$$\left| f(\boldsymbol{y}) - f(\boldsymbol{x}) - \nabla f(\boldsymbol{x})^T (\boldsymbol{y} - \boldsymbol{x}) \right| \leq \frac{L}{2} \left\| \boldsymbol{y} - \boldsymbol{x} \right\|^2 . \quad (35)$$

Appendix A First appendix

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

Here are simulation programmes we used in our model as follow.

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
       aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===============================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===============================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
    {
        shuffle((int *)&table[0], 9);
```

```
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```