

Shengyi Liang

HPC

Prof. Peherstorfer

Homework 3

Here is my github address for homework: https://github.com/TonyLiang0518/Shengyi_Liang_HPC.git

2.

Series converges when $x \in [-\frac{\pi}{4}, \frac{\pi}{4}]$

Here, we have $\sin(-x) = -\sin(x) \Rightarrow \sin(x) = \text{sign}(x) \sin(|x|)$

By periodicity, $\sin(|x|) = \sin(|x| + 2n\pi)$, where $n \in \mathbb{Z}$

So, $(|x| \bmod 2\pi) \in [0, 2\pi] \Rightarrow \frac{1}{8}(|x| \bmod 2\pi) \in [0, \frac{\pi}{4}]$

Let $\theta = \frac{1}{8}(|x| \bmod 2\pi)$, then we have

$\sin(x)$

$= \text{sign}(x) \sin(|x|)$

$= \text{sign}(x) \sin(|x| \bmod 2\pi)$

$= \text{sign}(x) \sin(8\theta)$

$= \text{sign}(x) \cdot 2\sin(4\theta) \cos(4\theta)$

$= \text{sign}(x) \cdot 4\sin(2\theta) \cos(2\theta) (1 - 2\sin^2(2\theta))$

$= \text{sign}(x) \cdot 8\sin(\theta) \cos(\theta) (1 - 2\sin^2(\theta)) (1 - 8\sin^2(\theta) \cos^2(\theta))$

Machine: Intel i9-9900K 3.6GHz with 8 cores\16 threads and 32GBs memory

Resulting error and time for $N = 1000000$:

Reference time: 0.2328

Taylor time: 14.3591 Error: 3.599898e-13

Intrin time: 0.0031 Error: 3.599343e-13

Vector time: 0.0025 Error: 3.181672e+02

3.

I use Intel i9-9900K 3.6GHz with 8 cores\16 threads and 32GBs memory.

| | $N = 10000$ | $N = 1000000$ | $N = 100000000$ |
|------------|-------------|---------------|-----------------|
| Sequential | 0.000030s | 0.002787s | 0.234269s |
| 2 Threads | 0.000103s | 0.002986s | 0.232200s |
| 4 Threads | 0.000157s | 0.002905s | 0.253004s |
| 8 Threads | 0.000270s | 0.004038s | 0.193124s |