Contents

1 Basic

1.1 compile

```
# preset before coding
echo "cd ~/Desktop" >> ~/.bashrc
gedit -> preference -> tab width: 4

# Editor
gedit a.cpp

# Compile
g++ a.cpp -std=c++14 -Wall -fsanitize=address

g++ -Wall -Wextra -Wshadow -Wconversion
    -g -fsanitize=address,undefined
    main.cpp -o main
// -Wmisleading-indentation  # 檢測縮排不對 for Loop 沒
    括號卻寫兩行
// -Wfatal-errors #讓編譯器只跳一個錯

ASAN_OPTIONS=detect_leaks=0 ./main  #叫他不要叫mem leak
// -fsanitize=address 檢測記憶體違規存取

**All file will be compiled to a.out unless you use -o(
    not recommanded, just use a.out)**
# Run
./a.out

# Run with file input
./a.out < input.txt

# Run with file input and output
./a.out < input.txt > output.txt

# Python Run
python3 a.py < input.txt > output.txt

# Copy Paste In Ubuntu
* copy: ctrl+insert
* paste: shift+insert

# 比對文件相同
sdiff a.txt b.txt
```

1.2 default code

```cpp
#include <bits/stdc++.h>
using namespace std;
#define int long long
typedef pair<int,int> pii;

// #define _GLIBCXX_DEBUG

#ifdef ONLINE_JUDGE
#define cerr if(false) cerr
#endif

int32_t main(){
#ifndef ONLINE_JUDGE
  //freopen("input.txt","r",stdin);
  freopen("output.txt","w",stdout);
  freopen("debug.txt","w",stderr);
#else
  ios_base::sync_with_stdio(0);
  cin.tie(false);
#endif
}
```

1.3 debug list

```
記得測試 python 的內建函數庫有哪些
bits/std++.h 跟 global variable y1 衝突，不能用
模板要記得 init
priority_queue 要清空
事先將把邊界測資加入測試
邊界條件 (過程溢位, 題目數據範圍), 會不會爆 long long
是否讀錯題目, 想不到時可以自己讀一次題目
比較容易有問題的地方換人寫
注意公式有沒有推錯或抄錯
精度誤差 sqrt(大大的東西) + EPS
喇分 random_suffle 隨機演算法
```

1.4 時間複雜度

時間複雜度	可處理的最大 N 數量級 (約)
$O(1)$	幾乎沒限制
$O(\log N)$	10^{18} 級別 (如快速冪)
$O(\sqrt{N})$	10^{10}
$O(N)$	10^8
$O(N \log N)$	$2 \times 10^7 \sim 5 \times 10^7$
$O(N\sqrt{N})$	$1 \times 10^5 \sim 2 \times 10^5$
$O(N^2)$	$10^4 \sim 1.5 \times 10^4$
$O(N^2 \log N)$	約 3×10^3
$O(N^3)$	$500 \sim 1000$
$O(2^N)$	$N \leq 20$
$O(N!)$	$N \leq 10$

2 Dark Code

2.1 IO optimization

```
*if output to much, consider put all output in array
    first, then output the array.
getchar() -> getchar_unlocked()
fread() -> fread_unlocked()
--------------------------
inline char readchar() {
  const int S = 1<<20; // buffer size
  static char buf[S], *p = buf, *q = buf;
  if(p == q && (q = (p=buf)+fread(buf,1,S,stdin)) ==
      buf) return EOF;
  return *p++;
}

inline int nxtint() {
  // if readchar can't use, change readchar() to
      getchar()
  int x = 0;
  int c = readchar(), neg = false;
  if (c == EOF) return -1;
  while (('0' > c || c > '9') && c != '-' && c != EOF)
      c = readchar();
  if (c == '-')neg = true, c = readchar();
  while ('0' <= c && c <= '9') x = x * 10 + (c ^ '0'),
      c = readchar();
  if (neg) x = -x;
  return x;
}
```

3 Geometry

3.1 2D point

```
typedef double Double;
struct Point {
  Double x,y;

  bool operator < (const Point &b)const{
    //return tie(x,y) < tie(b.x,b.y);
    return atan2(y,x) < atan2(b.y,b.x);
  }
  Point operator + (const Point &b)const{
    return (Point){x+b.x,y+b.y};
  }
  Point operator - (const Point &b)const{
    return (Point){x-b.x,y-b.y};
  }
  Point operator * (const Double &d)const{
    return Point(d*x,d*y);
  }
  Double operator * (const Point &b)const{
    return x*b.x + y*b.y;
  }
  Double operator % (const Point &b)const{
    return x*b.y - y*b.x;
  }
  friend Double abs2(const Point &p){
    return p.x*p.x + p.y*p.y;
  }
  friend Double abs(const Point &p){
    return sqrt( abs2(p) );
  }
};
typedef Point Vector;

struct Line{
  Point P; Vector v;
  bool operator < (const Line &b)const{
    return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
  }
};
```

3.2 兩線段交點

```
using type = long long;
const type EPS = 0 /*1e-9*/;
struct Point { type x, y; };

inline type cross(const Point &a, const Point &b, const
     Point &c) {
    return (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c
        .x - a.x);
}

inline bool overlap(type a, type b, type c, type d) {
    if(a > b) swap(a,b); if(c > d) swap(c,d);
    return max(a,c) <= min(b,d) + EPS;
}

bool equal_zero(type x) {
  return abs(x) <= EPS;
}
bool sgn(type x) {
  return (x > EPS) - (x < -EPS);
}

#define CROSS(i,j,k) cross(p[i],p[j],p[k])

#define CHECK_COLLINEAR(i,j,k) (equal_zero(CROSS(i,j,k)
    ) && overlap(p[i].x,p[j].x,p[k].x) &&
    overlap(p[i].y,p[j].y,p[k].y))

bool intersect(const vector<Point> &p){
    type d[4];
    for(int i=0;i<4;i++){
        if(i<2) d[i] = CROSS(0,1,i+2);
        else    d[i] = CROSS(2,3,i-2);
    }
    for(int i=0;i<4;i++)
  /**/if(CHECK_COLLINEAR(i<2?0:2,i<2?1:3,i<2?i+2:i-2))
    /**/return true;
  return sgn(d[0]) != sgn(d[1]) && sgn(d[2]) != sgn(d
      [3]);
}
```

```
// 求交點 不處理共線重疊
pair<long double,long double> intersection(const vector
    <Point> &p){
    long double A1 = p[1].y - p[0].y, B1 = p[0].x - p
        [1].x, C1 = A1*p[0].x+B1*p[0].y;
    long double A2 = p[3].y - p[2].y, B2 = p[2].x - p
        [3].x, C2 = A2*p[2].x+B2*p[2].y;
    long double det = A1*B2 - A2*B1;
    return {(C1*B2-C2*B1)/det,(A1*C2-A2*C1)/det};
}
```

3.3 兩圓交點

```
vector<Point> interCircle(Point o1, type r1, Point o2,
    type r2) {
    type d2 = abs2(o1 - o2);
    type d = sqrt(d2);
    if (d < fabs(r1 - r2) || d > r1 + r2) return {};
    Point u = (o1 + o2) * 0.5 + ((r2*r2 - r1*r1) /
        (2.0*d2)) * (o1 - o2);
    type A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
```

```cpp
    Point v = Point{o1.y - o2.y, -(o1.x - o2.x)} * (A /
         (2.0*d2));
    return { u + v, u - v };
}
```

3.4 Convex Hull

```cpp
#include "2Dpoint.cpp"

// return H, The first will occured TWICE in vector H!
void ConvexHull(vector<Point> &P, vector<Point> &H){
    int n = P.size(), m=0;
    sort(P.begin(),P.end());
    H.clear();

    for (int i=0; i<n; i++){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }

    for (int i=n-2; i>=0; i--){
        while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
            <0)H.pop_back(), m--;
        H.push_back(P[i]), m++;
    }
}
```

4 Flow

4.1 Dinic

```
(a) 有源匯上下界最大流 (Bounded Maxflow)
目標：在滿足所有邊的流量上下界限制的前提下，從源點 s 到
    匯點 t 的最大流量。
先依照 (b) 的方法建立 可行流 模型。
檢查是否存在可行流 (即 max_flow(ss, tt) 是否等於所有流
    量下界 l 的總和) 。如果不可行，則此問題無解。
重要：如果可行，不要重新初始化圖。直接在當前的殘留網路
    上繼續計算 dinic.max_flow(s, t)。
最終的答案就是步驟 3 中計算出的從 s 到 t 的附加流量。

(b) 有上下界可行流 (Bounded Possible Flow)
目標：檢查是否存在一種流量分配，使得每條邊的流量 f 都滿
    足其下界 l 和上界 r 的限制 (l ≤ f ≤ r)。
新增兩個節點：超級源點 ss 和超級匯點 tt。
準備一個變數 total_lower_bound 來累加所有下界 l。
對於每一條原始邊 u -> v，其容量為 [l, r]：
dinic.add_edge(u, v, r - l);  (邊的彈性容量)
dinic.add_edge(ss, v, l);     (節點 v 需要 l 的流入)
dinic.add_edge(u, tt, l);     (節點 u 提供 l 的流出)
total_lower_bound += l;
計算 flow = dinic.max_flow(ss, tt)。
如果 flow == total_lower_bound，則表示所有下界需求都被
    滿足，存在可行流；否則不存在。

(c) 有源匯上下界最小流 (Bounded Minimum Flow)
目標：在滿足所有邊的流量上下界限制的前提下，從源點 s 到
    匯點 t 的最小流量。
注意：這個問題通常需要透過二分搜尋答案來解決，無法直接
    用一次最大流求出。
二分搜尋一個流量值 F。
對於每個猜測的 F，建立一個無源匯可行流模型來檢查其可行
    性：
使用 (b) 的方法建構基本圖。
額外加入一條邊 t -> s，容量為 [F, INF]。這條邊強制要求
    從 s 到 t 的淨流量至少為 F。
檢查這個新的循環圖是否存在可行流。如果存在，表示流量 F
    是可達成的，可以嘗試更小的 F；反之，F 太小了，需要
    增加。

(e) 最小割 (Minimum Cut)
目標：找出一個邊集，其總容量最小，且移除這些邊後 s 和 t
    不再連通。
根據最大流-最小割定理，最小割的值等於最大流的值。先執行
    ll min_cut_value = dinic.max_flow(s, t);。
呼叫 vector<bool> side = dinic.get_min_cut_nodes(s); 來
    取得節點的劃分。
side[i] == true 表示節點 i 屬於源點 s 所在的集合 (S 集
    合)。
side[i] == false 表示節點 i 屬於匯點 t 所在的集合 (T 集
    合)。
最小割的邊集就是所有從 S 集合指向 T 集合的原始邊
```

```cpp
using ll = long long;
const ll INF = 1e18;

struct Dinic {
    struct Edge {
        int to;
        ll cap;
        int rev; // 反向邊的索引
    };
    vector<vector<Edge>> adj;
    vector<int> level, iter;
    vector<bool> side;
    int n;
    Dinic(int v) : n(v), adj(v), level(v), iter(v),
        side(v) {}
    void add_edge(int u, int v, ll cap) {
        adj[u].push_back({v, cap, adj[v].size()});
        adj[v].push_back({u, 0, adj[u].size() - 1});
    }
    bool bfs(int s, int t) {
        fill(level.begin(), level.end(), -1);
        queue<int> q;
        level[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (auto& edge : adj[u]) {
                if (edge.cap > 0 && level[edge.to] < 0)
                    {
                    level[edge.to] = level[u] + 1;
                    q.push(edge.to);
                }
            }
        }
        return level[t] != -1;
    }
    ll dfs(int u, int t, ll f) {
        if (u == t) return f;
        for (int& i = iter[u]; i < adj[u].size(); ++i)
            {
            Edge& e = adj[u][i];
            if (e.cap > 0 && level[u] < level[e.to]) {
                ll d = dfs(e.to, t, min(f, e.cap));
                if (d > 0) {
                    e.cap -= d;
                    adj[e.to][e.rev].cap += d;
                    return d;
                }
            }
        }
        return 0;
    }
    ll max_flow(int s, int t) {
        ll flow = 0;
        while (bfs(s, t)) {
            fill(iter.begin(), iter.end(), 0);
            ll f;
            while ((f = dfs(s, t, INF)) > 0) {
                flow += f;
            }
        }
        return flow;
    }
    void _find_cut(int u) {
        side[u] = true;
        for(auto& e : adj[u]) {
            if(e.cap > 0 && !side[e.to]) {
                _find_cut(e.to);
            }
        }
    }
    vector<bool> get_min_cut_nodes(int s) { // 跟 S 同
        側 true ; 跟 T 同側 false
        fill(side.begin(), side.end(), false);
        _find_cut(s);
        return side;
    }
};
```

4.2 min cost flow

```
struct MinCostMaxFlow { // 0-base N-maximum edge
  struct Edge {
    ll from, to, cap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  int inq[N], n, s, t;
  ll dis[N], up[N], pot[N];
  bool BellmanFord() {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
      if (cap > 0 && dis[u] > d) {
        dis[u] = d, up[u] = cap, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax(e.to, d2, min(up[u], e.cap - e.flow), &e)
            ;
      }
    }
    return dis[t] != INF;
  }
  void solve(int _s, int _t, ll &flow, ll &cost, bool
      neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    for (; BellmanFord(); copy_n(dis, n, pot)) {
      for (int i = 0; i < n; ++i) dis[i] += pot[i] -
          pot[s];
      flow += up[t], cost += up[t] * dis[t];
      for (int i = t; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        e.flow += up[t], G[e.to][e.rev].flow -= up[t];
      }
    }
  }
  void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
    G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
  }
};
```

5 Mathmatics

5.1 ax+by=gcd(a,b)

```
typedef pair<int, int> pii;

pii exgcd(int a, int b){
  if(b == 0) return make_pair(1, 0);
  else{
    int p = a / b;
    pii q = exgcd(b, a % b);
    int aa = q.second, bb = q.first - q.second * p;
    if(aa < 0) aa += b, bb -= a;
    return make_pair(aa, bb);
  }
}
```

5.2 GaussElimination

```
// by bcw_codebook

const int MAXN = 300;
const double EPS = 1e-8;

int n;
double A[MAXN][MAXN];

void Gauss() {
  for(int i = 0; i < n; i++) {
    bool ok = 0;
    for(int j = i; j < n; j++) {
      if(fabs(A[j][i]) > EPS) {
        swap(A[j], A[i]);
        ok = 1;
        break;
      }
    }
    if(!ok) continue;

    double fs = A[i][i];
    for(int j = i+1; j < n; j++) {
      double r = A[j][i] / fs;
      for(int k = i; k < n; k++) {
        A[j][k] -= A[i][k] * r;
      }
    }
  }
}

template<class T>
void Gauss(vector<vector<T>> &A) {
  int n = A.size();
  for(int i = 0; i < n; i++) {
    bool ok = 0;
    for(int j = i; j < n; j++) {
      if(A[j][i] != 0) {
        swap(A[j], A[i]);
        ok = 1;
        break;
      }
    }
    if(!ok) continue;

    T fs = A[i][i];
    for(int j = i+1; j < n; j++) {
      T r = A[j][i] / fs;
      for(int k = i; k < n; k++) {
        A[j][k] -= A[i][k] * r;
      }
    }
  }
}
```

5.3 Inverse

```
int inverse[100000];
void invTable(int b, int p) {
  inverse[1] = 1;
  for( int i = 2; i <= b; i++ ) {
    inverse[i] = (long long)inverse[p%i] * (p-p/i) % p;
  }
}

int inv(int b, int p) {
  return b == 1 ? 1 : ((long long)inv(p % b, p) * (p-p/
      b) % p);
}
```

5.4 LinearPrime 歐拉篩

```
const int MAXP = 100; //max prime
vector<int> P;  // primes
void build_prime(){
  static bitset<MAXP> ok;
  int np=0;
  for (int i=2; i<MAXP; i++){
    if (ok[i]==0)P.push_back(i), np++;
    for (int j=0; j<np && i*P[j]<MAXP; j++){
      ok[ i*P[j] ] = 1;
      if ( i%P[j]==0 )break;
    }
  }
}
```

5.5 Miller Rabin

```
typedef long long LL;

inline LL bin_mul(LL a, LL n,const LL& MOD){
  return __int128(a) * n % MOD;
}

inline LL bin_pow(LL a, LL n,const LL& MOD){
  LL re=1;
  while (n>0){
    if (n&1) re = bin_mul(re,a,MOD);
    a = bin_mul(a,a,MOD);
    n>>=1;
  }
  return re;
}

bool is_prime(LL n){
  //static LL sprp[3] = { 2LL, 7LL, 61LL};
  static LL sprp[7] = { 2LL, 325LL, 9375LL,
    28178LL, 450775LL, 9780504LL,
    1795265022LL };
  if (n==1 || (n&1)==0 ) return n==2;
  int u=n-1, t=0;
  while ( (u&1)==0 ) u>>=1, t++;
  for (int i=0; i<3; i++){
    LL x = bin_pow( sprp[i]%n, u, n);
    if (x==0 || x==1 || x==n-1)continue;

    for (int j=1; j<t; j++){
      x=x*x%n;
      if (x==1 || x==n-1)break;
    }
    if (x==n-1)continue;
    return 0;
  }
  return 1;
}
```

5.6 Pollard’s rho

```
map<ll, int> cnt;
void PollardRho(ll n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2],
      void();
  ll x = 2, y = 2, d = 1, p = 1;
  #define f(x, n, p) ((mul(x, x, n) + p) % n)
  while (true) {
    if (d != n && d != 1) {
      PollardRho(n / d);
      PollardRho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p), y = f(f(y, n, p), n, p);
    d = gcd(abs(x - y), n);
  }
}
```

5.7 NTT

```
constexpr int P = 998244353;
const int G = 3;
/*預處理 lim*/
int lim = 1;
while (lim < (lenSum - 1)) lim <<= 1;
/*每個多項式都要resize(lim)*/
/*998244353 3 1004535809 3 469762049 3 167772161 3
    754974721 11*/
void init_rev(vector<int> &rev, int lim) {
    int lg = __builtin_ctz(lim); // lim 是 2^k
    rev.resize(lim);
    for (int i = 0; i < lim; ++i)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (lg -
             1));
}
// a.size() == lim
void ntt(vector<int> &a, int opt) { // opt == -1 =>
    reverse ntt
    int n = a.size();
    static vector<int> rev;
    init_rev(rev, n);
    for (int i = 0; i < n; ++i)
        if (i < rev[i]) swap(a[i], a[rev[i]]);

    for (int m = 2; m <= n; m <<= 1) {
        int k = m >> 1;
        int gn = qpow(G, (P - 1) / m);
        if (opt == -1) gn = qpow(gn, P - 2);
        for (int i = 0; i < n; i += m) {
            int g = 1;
            for (int j = 0; j < k; ++j) {
                int t = 1ll * a[i + j + k] * g % P;
                a[i + j + k] = (a[i + j] - t + P) % P;
                a[i + j] = (a[i + j] + t) % P;
                g = 1ll * g * gn % P;
            }
        }
    }

    if (opt == -1) {
        int inv_n = qpow(n, P - 2);
        for (int &x : a) x = 1ll * x * inv_n % P;
    }
}
```

5.8 數論基本工具

```
Int POW(Int a, Int n, Int mod){
  Int re=1;
  while (n>0){
    if (n&1LL) re = re*a%mod;
    a = a*a%mod;
    n>>=1;
  }
  return re;
}

Int C(Int n, Int m){
  if (m<0 || m>n)return 0;
  return J[n] * inv(J[m]*J[n-m]%MOD) %MOD;
}
```

5.9 Mobius

```
void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
    mu[1] = 1, num = 0;
    for (int i = 2; i < MAXN; ++i) {
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
        static int d;
        for (int j = 0; j < num && (d = i * primes[j])
            < MAXN; ++j) {
            isPrime[d] = false;
            if (i % primes[j] == 0) {
                mu[d] = 0; break;
            } else mu[d] = -mu[i];
        }
    }
}
```

5.10 SG

```
Anti Nim (取走最後一個石子者敗)

先手必勝 if and only if
1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。

-------------------------------------------------------
Anti-SG (決策集合為空的遊戲者贏)

定義 SG 值為 0 時，遊戲結束，
則先手必勝 if and only if
1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數
    為 0。
2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數
    不為 0。

-------------------------------------------------------
Sprague-Grundy

1. 雙人、回合制
2. 資訊完全公開
3. 無隨機因素
4. 可在有限步內結束
5. 沒有和局
6. 雙方可採取的行動相同

SG(S) 的值為 0：後手(P)必勝
不為 0：先手(N)必勝

int mex(set S) {
  // find the min number >= 0 that not in the S
  // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
  if (A not in state) {
    S = sub_states(A)
    if( len(S) > 1 ) state[A] = reduce(operator.xor, [
        SG(B) for B in S])
    else state[A] = mex(set(SG(B) for B in next_states(
        A)))
  }
  return state[A]
}
```

5.11 歐拉定理

歐拉降冪對於任何正整數 m、整數 a 和非負整數 k

$$a^k \equiv \begin{cases} a^{k \bmod \varphi(m)}, & \gcd(a,m)=1, \\ a^k, & \gcd(a,m)\neq 1,\ k<\varphi(m), \\ a^{(k \bmod \varphi(m))+\varphi(m)}, & \gcd(a,m)\neq 1,\ k\geq\varphi(m). \end{cases} \pmod m$$

5.12 Theorem

```
/*
Lucas's Theorem
  For non-negative integer n,m and prime P,
  C(m,n) mod P = C(m/M,n/M) * C(m%M,n%M) mod P
  = mult_i ( C(m_i,n_i) )
  where m_i is the i-th digit of m in base P.
-------------------------------------------------------
Pick's Theorem
  A = i + b/2 - 1
-------------------------------------------------------
Kirchhoff's theorem
  A_{ii} = deg(i), A_{ij} = (i,j) \in E ? -1 : 0
  Deleting any one row, one column, and cal the det(A)
-------------------------------------------------------
Nth Catalan recursive function:
C_0 = 1, C_{n+1} = C_n * 2(2n + 1)/(n+2)
-------------------------------------------------------
Mobius Formula
u(n) = 1          , if n = 1
       (-1)^m     , 若 n 無平方數因數，且 n = p1*p2*p3
           *...*pk
       0          , 若 n 有大於 1 的平方數因數
- Property
1. (積性函數) u(a)u(b) = u(ab)
2. ∑_{d|n} u(d) = [n == 1]
-------------------------------------------------------
Mobius Inversion Formula
if      f(n) = ∑_{d|n} g(d)
then    g(n) = ∑_{d|n} u(n/d)f(d)
             = ∑_{d|n} u(d)f(n/d)
- Application
the number/power of gcd(i, j) = k
- Trick
分塊, O(sqrt(n))
-------------------------------------------------------
Chinese Remainder Theorem (m_i 兩兩互質)

  x = a_1 (mod m_1)
  x = a_2 (mod m_2)
  ....
  x = a_i (mod m_i)

construct a solution:

  Let M = m_1 * m_2 * m_3 * ... * m_n
  Let M_i = M / m_i

  t_i = 1 / M_i
  t_i * M_i = 1 (mod m_i)

  solution x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + ...
      + a_n * t_n * M_n + k * M
  = k*M + ∑ a_i * t_i * M_i, k is positive integer.

  under mod M, there is one solution x = ∑ a_i * t_i *
      M_i
-------------------------------------------------------
Burnside's lemma
|G| * |X/G|  = sum( |X^g| ) where g in G
總方法數: 每一種旋轉下不動點的個數總和 除以 旋轉的方法
    數
*/
```

6 Graph

6.1 BCC

```
邊雙連通

任意兩點間至少有兩條不重疊的路徑連接，找法：
1. 標記出所有的橋
2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙
    連通

// from BCW

struct BccEdge {
  static const int MXN = 100005;
  struct Edge { int v,eid; };
  int n,m,step,par[MXN],dfn[MXN],low[MXN];
  vector<Edge> E[MXN];
  DisjointSet djs;
  void init(int _n) {
    n = _n; m = 0;
    for (int i=0; i<n; i++) E[i].clear();
    djs.init(n);
  }
  void add_edge(int u, int v) {
    E[u].PB({v, m});
    E[v].PB({u, m});
    m++;
  }
  void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
      if (it.eid == f_eid) continue;
      int v = it.v;
      if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], dfn[v]);
      }
    }
  }
  void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
      if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
  }
}graph;
```

6.2 Prim

```
// edge strucute
struct edge{
  int a, b;
  double data;
  bool operator <(const edge b)const{
    return data > b.data;
  }
};

// main prim algorithm
int n, m, root, aa, bb, cc;
while (cin >> n >> m){
  priority_queue<edge>yee;
  int visit[500] = {}, p[500] = {};
  double a[500][500] = {};
  //undirectional edge aa to bb is weighted cc
  for (int i = 0; i < m; i++){
    cin >> aa >> bb >> cc;
    a[aa][bb] = a[bb][aa] = cc;
  }
  cin >> root;
  yee.push({ 0, root, 0 });
  edge tmp;
  double total = 0;
  while (!yee.empty()){
    tmp = yee.top(); yee.pop();
    if (visit[tmp.b])continue;
    total += tmp.data; p[tmp.b] = tmp.a; visit[tmp.b] =
         1;
    for (int i = 1; i <= n; i++){
      if (a[tmp.b][i]!=.0&&(!visit[i])){
        yee.push({tmp.b,i,a[tmp.b][i]});
      }
    }
  }
  cout << total << endl;
}
```

6.3 Bellman Ford

```
int a[100][100], d[100], p[100];

void bellman_ford(int root, int n){
  for (int i = 1; i <= n; i++)d[i] = 1e9;
  d[root] = 0, p[root] = 0;
  for (int i = 0; i<n - 1; i++){
    for (int j = 1; j <= n; j++){
      for (int k = 1; k <= n; k++){
        if (d[j] != 1e9 && a[j][k] != 1e9){
          if (d[j] + a[j][k] < d[k]){
            d[k] = d[j] + a[j][k], p[k] = j;
          }
        }
      }
    }
  }
}

bool nega_cyc(int n){
  for (int i = 1; i <= n; i++){
    for (int j = 1; j <= n; j++){
      if (d[i] != 1e9 && a[i][j] != 1e9)
      if (d[i] + a[i][j] < d[j]){
        return 0;
      }
    }
  }
  return 1;
}

int main(){
  int n, m, aa, bb, dd;
  while (cin >> n >> m){
    for (int i = 0; i <= n; i++)for (int j = 0; j <= n;
         j++){
      a[i][j] = E9;
    }
    memset(p, 0, sizeof(p));
    for (int i = 0; i < m; i++){
      cin >> aa >> bb >> dd;
      a[aa][bb] = min(a[aa][bb], dd);
    }
    cin >> aa;
    bellman_ford(aa, n);
    int t = nega_cyc(n);
    if(t){
      for (int i = 1; i <= n; i++)cout << d[i] << " \n"
          [i==n];
      for (int i = 1; i <= n; i++)cout << p[i] << " \n"
          [i==n];
    }
    else cout << "There is a negative weight cycle in
        the graph\n";
  }
}
```

6.4 Kruskal

```
struct v {
  int a, b, c;
};

int p[200001];v a[200001];

bool sor(v a, v b) {
  return a.c < b.c;
}

int find(int x) {
  return(x != p[x] ? (p[x] = find(p[x])) : x);
}

int main() {
  int n, m, i, j, sum;
  while (cin >> n >> m) {
    sum = 0;
    for (i = 0; i < 200001; i++)p[i] = i;
    for (i = 0; i<m; i++)cin >> a[i].a >> a[i].b >> a[i
        ].c;
    sort(a, a + m, sor);
    for (i =0,j = 0;j<m; j++) {
      if(find(a[j].a) != find(a[j].b)){
        i++;
        p[find(a[j].a)] = find(a[j].b);
        sum += a[j].c;
      }
    }
    cout << ((i==n-1)?sum:-1) << endl;
  }
}
```

6.5 Dijkstra

```
struct node {
    int num{}, w{};
    bool operator < (const node& other)const {
        return w > other.w;
    }
};

vector<int> dijkstra(int root, const vector<vector<node
    >> &graph) {
    vector<int> d(graph.size(), INT_MAX >> 1), p(graph.
        size());
    priority_queue<node> pq;
    d[root] = p[root] = 0;
    pq.push({root, d[root]});
    while (!pq.empty()) {
        node tmp = pq.top(); pq.pop();
        for (const node &i : graph[tmp.num]) {
            if (d[i.num] > d[tmp.num] + i.w) {
                d[i.num] = d[tmp.num] + i.w;
                p[i.num] = tmp.num;
                pq.push({i.num, d[tmp.num]});
            }
        }
    }
    return d;
}
```

6.6 Strongly Connected Component(SCC)

```
#define MXN 100005
#define PB push_back
#define FZ(s) memset(s,0,sizeof(s))

struct Scc{
int n, nScc, vst[MXN], bln[MXN];
vector<int> E[MXN], rE[MXN], vec;
void init(int _n){
  n = _n;
  for (int i=0; i<MXN; i++){
    E[i].clear();
    rE[i].clear();
  }
}
void add_edge(int u, int v){
  E[u].PB(v);
  rE[v].PB(u);
}
void DFS(int u){
  vst[u]=1;
  for (auto v : E[u])
    if (!vst[v]) DFS(v);
  vec.PB(u);
}
void rDFS(int u){
  vst[u] = 1;
  bln[u] = nScc;
  for (auto v : rE[u])
    if (!vst[v]) rDFS(v);
}
void solve(){
  nScc = 0;
  vec.clear();
  FZ(vst);
  for (int i=0; i<n; i++)
    if (!vst[i]) DFS(i);
  reverse(vec.begin(),vec.end());
  FZ(vst);
  for (auto v : vec){
    if (!vst[v]){
      rDFS(v);
      nScc++;
    }
  }
}
};
```

6.7 Hungarian

```
// Maximum Cardinality Bipartite Matching

struct Graph {
    static const int MAXN = 5005;
    vector<int> G[MAXN];
    int n;
    int match[MAXN]; // Matching Result
    int vis[MAXN];

    void init(int _n) {
        n = _n;
        for ( int i = 0 ; i < n ; i++ ) G[i].clear();
    }

    bool dfs(int u) {
        for ( auto v:G[u] ) {
            if (!vis[v]) {
                vis[v] = true;
                if (match[v] == -1 || dfs(match[v])) {
                    match[v] = u;
                    match[u] = v;
                    return true;
                }
            }
        }
        return false;
    }

    int solve() {
        int res = 0;
        memset(match, -1, sizeof(match));
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                memset(vis, 0, sizeof(vis));
                if (dfs(i)) res += 1;
            }
        }
        return res;
    }
} graph;
```

6.8 KM

```
Detect non-perfect-matching:
1. set all edge[i][j] as INF
2. if solve() >= INF, it is not perfectmatching.
-------------------------------------------------------
// Maximum Weight Perfect Bipartite Matching
// allow negative weight!

typedef long long Int;
struct KM {
    static const int MAXN = 1050;
    static const int INF = 1LL<<60;
    int n, match[MAXN], vx[MAXN], vy[MAXN];
    Int edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[
        MAXN];
    void init(int _n){
        n = _n;
        for ( int i = 0 ; i < n ; i++ )
            for ( int j = 0; j < n ; j++ )
                edge[i][j] = 0;
    }
    void add_edge(int x, int y, Int w){
        edge[x][y] = w;
    }
    bool DFS(int x){
        vx[x] = 1;
        for ( int y = 0 ; y < n ; y++ ) {
            if ( vy[y] ) continue;
            if ( lx[x] + ly[y] > edge[x][y] ) {
                slack[y] = min(slack[y], lx[x] + ly[y]
                    - edge[x][y]);
            } else {
                vy[y] = 1;
                if ( match[y] == -1 || DFS(match[y]) ){
                    match[y] = x;
                    return true;
                }
            }
        }
        return false;
    }
    Int solve() {
        fill(match, match + n, -1);
        fill(lx, lx + n, -INF);
        fill(ly, ly + n, 0);
        for ( int i = 0; i < n; i++ )
            for ( int j = 0; j < n; j++ )
                lx[i] = max(lx[i], edge[i][j]);
        for ( int i = 0 ; i < n; i++ ) {
            fill(slack, slack + n, INF);
            while (true){
                fill(vx, vx + n, 0);
                fill(vy, vy + n, 0);
                if ( DFS(i) ) break;
                Int d = INF;
                for ( int j = 0 ; j < n ; j++ )
                    if ( !vy[j] ) d = min(d, slack[j]);
                for ( int j = 0 ; j < n ; j++ ) {
                    if (vx[j]) lx[j] -= d;
                    if (vy[j]) ly[j] += d;
                    else slack[j] -= d;
                }
            }
        }
        Int res = 0;
        for ( int i = 0 ; i < n ; i++ ) {
            res += edge[ match[i] ][i];
        }
        return res;
    }
} graph;
```

6.9 最小平均環

```
// from BCW

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
  for(int i=0; i<n; i++) d[0][i]=0;
  for(int i=0; i<n; i++) {
    fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {
      int v = e[j].v, u = e[j].u;
      if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
      }
    }
  }
}
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {
    double avg=-inf;
    for(int k=0; k<n; k++) {
      if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])
          /(n-k));
      else avg=max(avg,inf);
    }
    if (avg < mmc) tie(mmc, st) = tie(avg, i);
  }
  for(int i=0; i<n; i++) vst[i] = 0;
  edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  }
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
}
```

6.10 偵測負環

```
#include <bits/stdc++.h>
using namespace std;

const int INF = 1000000;
const int MAXN = 200;
int n, m, q;
int d[MAXN][MAXN];

int main () {
    while ( cin >> n >> m >> q && n) {

        for ( int i = 0 ; i <= n ; i++ ) {
            for ( int j = 0 ; j <= n ; j++ ) d[i][j] =
                (i==j ? 0 : INF);
        }

        for ( int i = 0 ; i < m ; i++ ) {
            int a, b, c;
            cin >> a >> b >> c;
            d[a][b] = min(d[a][b], c);
        }

        for ( int k = 0 ; k < n ; k++ ) {
            for ( int i = 0 ; i < n ; i++ ) {
                for ( int j = 0 ; j < n ; j++ ) {

                    if ( d[i][j] > d[i][k] + d[k][j] &&
                         d[i][k] < INF && d[k][j] < INF
                         ) {
                        //printf("%d > %d + %d\n", d[i
                            ][j], d[i][k], d[k][j]);
                        //if ( d[i][k] >= INF || d[k][j
                            ] >= INF ) cout << "NO : "
                            << i << " " << j << " " <<
                            k << "--";
                        d[i][j] = min(d[i][j], d[i][k]
                            + d[k][j]);
                    }
                }
            }
        }

        for ( int i = 0 ; i < n ; i++ ) {
            for ( int j = 0 ; j < n ; j++ ) {
                for ( int k = 0 ; k < n && d[i][j] != -
                    INF ; k++ ) {
                    if ( d[k][k] < 0 && d[i][k] != INF
                        && d[k][j] != INF   )
                        d[i][j] = -INF;
                }
            }
        }
        int u, v;
        for (int i=0;i<q;i++){
            scanf("%d%d",&u,&v);

            if (d[u][v] == INF) printf("Impossible\n");
            else if (d[u][v] == -INF) printf("-Infinity
                \n");
            else printf("%d\n",d[u][v]);
        }
        puts("");
    }
    return 0;
}
```

6.11 Tarjan

```
割點
點 u 為割點 if and only if 滿足 1. or 2.
1. u 爲樹根，且 u 有多於一個子樹。
2. u 不爲樹根，且滿足存在 (u,v) 爲樹枝邊 (或稱父子邊，
    即 u 爲 v 在搜索樹中的父親)，使得 DFN(u) <= Low(v)
    。

-------------------------------------------------------
橋
一條無向邊 (u,v) 是橋 if and only if (u,v) 爲樹枝邊，且
    滿足 DFN(u) < Low(v)。

// 0 base
struct TarjanSCC{
  static const int MAXN = 1000006;
  int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
  vector<int> G[MAXN];
  stack<int> stk;
  bool ins[MAXN];

  void tarjan(int u){
    dfn[u] = low[u] = ++count;
    stk.push(u);
    ins[u] = true;

    for(auto v:G[u]){
      if(!dfn[v]){
        tarjan(v);
        low[u] = min(low[u], low[v]);
      }else if(ins[v]){
        low[u] = min(low[u], dfn[v]);
      }
    }

    if(dfn[u] == low[u]){
      int v;
      do {
      v = stk.top();
      stk.pop();
      scc[v] = scn;
      ins[v] = false;
      } while(v != u);
      scn++;
    }
  }

  void getSCC(){
    memset(dfn,0,sizeof(dfn));
    memset(low,0,sizeof(low));
    memset(ins,0,sizeof(ins));
    memset(scc,0,sizeof(scc));
    count = scn = 0;
    for(int i = 0 ; i < n ; i++ ){
      if(!dfn[i]) tarjan(i);
    }
  }

}SCC;
```

6.12 Topological Sort

```
#define N 87

bool adj[N][N];     // adjacency matrix
int visit[N];       // record visited coordinations in
    DFS
int order[N], n;    // save the order

bool cycle;         // detect the cycle

void DFS(int s)
{
    // back edge occured, detected the cycle
    if (visit[s] == 1) {cycle = true; return;}
    // forward edge and cross edge¡C
    if (visit[s] == 2) return;

    visit[s] = 1;
    for (int t=0; t<N; ++t){
        if (adj[s][t]) DFS(t);
    }
    visit[s] = 2;
    order[n--] = s;     // record the order
}

void topological_ordering()
{
    memset(visit, 0, sizeof(visit));
    cycle = false;
    n = N - 1;

    for (int s=0; s<9; ++s)
        if (!v[s])
            DFS(s);

    if (cycle) cout << "The graph has the cycle!";
    else{
        for (int i=0; i<N; ++i)
            cout << order[i];
        }
  }
}
```

7 Data Structure

7.1 2D Range Tree

```
// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x,a.y) < tie(b.x,b.y);
    }
};
struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];
struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];

vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return;
    }
    int mid = (l+r)>>1;
    build(l,mid,o+o,deep+1);
    build(mid+1,r,o+o+1,deep+1);

    TREE *ptr = &tree[deep][l];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][
        mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep
        +1][r+1];

    int cnt = 0;
    while(pl != nl && pr != nr) {
        *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++):
            *(pr++);
        ptr -> toleft = cnt; ptr++;
    }
    while(pl != nl) *(ptr) = *(pl++), ptr -> toleft =
        ++cnt, ptr++;
    while(pr != nr) *(ptr) = *(pr++), ptr -> toleft =
        cnt, ptr++;

}
int main(){
    int n; cin >> n;
    for(int i = 0 ;i < n; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}
```

7.2 Segment Tree

```
struct Node{
    int mx;  // 區間最大值
    int tag; // 子樹裡所有人的'值'都要加上 tag
};

vector<Node> seg;

// 節點 id 的整個區間要加上 tag
void addtag(int tag, int id){
    seg[id].mx += tag; // 最大值會加上 tag
    seg[id].tag += tag; // 注意可能本來就有標記了，所以
        是 +=
}

// 更新子節點資訊並把標記移到子節點身上
void push(int id){
    addtag(seg[id].tag, lc);
    addtag(seg[id].tag, rc);
    seg[id].tag = 0; // 標記被移到子節點上所以要改成 0
}

// 區間 [l,r] 加上 v
void modify(int l, int r, int v, int L, int R, int id){
    if(l <= L && R <= r){
        addtag(v, id);
        return;
    }
    push(id);
    if(r <= M) modify(l, r, v, L, M, lc);
    else if(l > M) modify(l, r, v, M + 1, R, rc);
    else{
        modify(l, r, v, L, M, lc);
        modify(l, r, v, M + 1, R, rc);
    }
    seg[id].mx = max(seg[lc].mx, seg[rc].mx);
}

int query(int l, int r, int L, int R, int id){
    if(l <= L && R <= r) return seg[id].mx;
    push(id);
    int M = (L + R) / 2;
    if(r <= M) return query(l, r, L, M, lc);
    else if(l > M) return query(l, r, M + 1, R, rc);
    else return max(query(l, r, L, M, lc),
                    query(l, r, M + 1, R, rc));
}
```

7.3 ZKW 線段樹

```
const int M=1e5+111;
int n,m,q;
int sum[M<<2],mn[M<<2],mx[M<<2],add[M<<2];

int read() {
    int x;
    cin >> x;
    return x;
}
void build(){
    for(m=1;m<=n;m<<=1);
    for(int i=m+1;i<=m+n;++i)
        sum[i]=mn[i]=mx[i]=read();
    for(int i=m-1;i;--i){
        sum[i]=sum[i<<1]+sum[i<<1|1];
        mn[i]=min(mn[i<<1],mn[i<<1|1]),
        mn[i<<1]-=mn[i],mn[i<<1|1]-=mn[i];
        mx[i]=max(mx[i<<1],mx[i<<1|1]),
        mx[i<<1]-=mx[i],mx[i<<1|1]-=mx[i];
    }
}
void update_node(int x,int v,int A=0){
    x+=m,mx[x]+=v,mn[x]+=v,sum[x]+=v;
    for(;x>1;x>>=1){
        sum[x]+=v;
        A=min(mn[x],mn[x^1]);
        mn[x]-=A,mn[x^1]-=A,mn[x>>1]+=A;
        A=max(mx[x],mx[x^1]),
        mx[x]-=A,mx[x^1]-=A,mx[x>>1]+=A;
    }
}
void update_part(int s,int t,int v){
    int A=0,lc=0,rc=0,len=1;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<<=1){
        if(s&1^1) add[s^1]+=v,lc+=len, mn[s^1]+=v,mx[s
            ^1]+=v;
        if(t&1)    add[t^1]+=v,rc+=len, mn[t^1]+=v,mx[t
            ^1]+=v;
        sum[s>>1]+=v*lc, sum[t>>1]+=v*rc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s
            >>1]+=A,
        A=min(mn[t],mn[t^1]),mn[t]-=A,mn[t^1]-=A,mn[t
            >>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s
            >>1]+=A,
        A=max(mx[t],mx[t^1]),mx[t]-=A,mx[t^1]-=A,mx[t
            >>1]+=A;
    }
    for(lc+=rc;s;s>>=1){
        sum[s>>1]+=v*lc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s
            >>1]+=A,
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s
            >>1]+=A;
    }
}
int query_node(int x,int ans=0){
    for(x+=m;x;x>>=1) ans+=mn[x]; return ans;
}
int query_sum(int s,int t){
    int lc=0,rc=0,len=1,ans=0;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<<=1){
        if(s&1^1) ans+=sum[s^1]+len*add[s^1],lc+=len;
        if(t&1) ans+=sum[t^1]+len*add[t^1],rc+=len;
        if(add[s>>1]) ans+=add[s>>1]*lc;
        if(add[t>>1]) ans+=add[t>>1]*rc;
    }
    for(lc+=rc,s>>=1;s;s>>=1) if(add[s]) ans+=add[s]*lc
        ;
    return ans;
}
int query_min(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mn[s],R+=mn[t];
        if(s&1^1) L=min(L,mn[s^1]);
        if(t&1) R=min(R,mn[t^1]);
    }
    for(ans=min(L,R),s>>=1;s;s>>=1) ans+=mn[s];
    return ans;
}
int query_max(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mx[s],R+=mx[t];
        if(s&1^1) L=max(L,mx[s^1]);
        if(t&1) R=max(R,mx[t^1]);
    }
    for(ans=max(L,R),s>>=1;s;s>>=1) ans+=mx[s];
    return ans;
}
```

7.4 Sparse Table

```
const int MAXN = 200005;
const int lgN = 20;
/* Sp[i][j] 為 區間 [i, i + 2^j - 1] 的值 */
/* 從 i 開始 長度為 2 ^ j */
/* 解決可重複貢獻問題 */
struct SP{ //sparse table
  int Sp[MAXN][lgN];
  function<int(int,int)> opt;
  void build(vector<int> &nums){ // 0 base
    for (int i = 0; i < nums.size(); i++) Sp[i][0]=nums
        [i];

    for (int h = 1; h < lgN; h++) {
      int len = 1 << (h - 1), i=0;
      for (; i + len < nums.size(); i++)
        Sp[i][h] = opt(Sp[i][h-1], Sp[i+len][h-1]);
      for (; i < nums.size(); i++)
        Sp[i][h] = Sp[i][h-1];
    }
  }
  int query(int l, int r){
    int h = __lg(r-l+1);
    int len = 1<<h;
    return opt(Sp[l][h], Sp[r-len+1][h] );
  }
};
```

7.5 Lazy Tag

```
void modify(type value, int l, int r, int L, int R,
    vertex v){
    if(l == L && r == R){
        //打懶標在v上;
        return;
    }
    int M = (L + R) / 2;
    if(r <= M) modify(value, l, r, L, M, //v的左子節點)
        ;
    else if(l > M) modify(value, l, r, M + 1, R, //v的
        右子節點);
    else{
        modify(value, l, M, L, M, v的左子節點);
        modify(value, M + 1, r, M + 1, R, //v的右子節點
            );
    }
    //用兩個子節點的答案更新v的答案;
}
```

7.6 BIT 樹狀樹組

```
class Bitree {
public:
    /* bit 一定是 1 indexed */
    vector<int> data;
    Bitree(const vector<int> &nums) {
        data.resize(nums.size() + 1, 0);
        for(int i = 0; i < nums.size(); i++ ) {
            update(i, nums[i]);
        }
    }
    void update(int x, int val) {
        x++; /*變成 1 indexed*/
        for(; x < data.size(); x += lowbit(x)) {
            data[x] += val;
        }
    }
    int query(int x) {
        x++; /*變成 1 indexed*/
        int result = 0;
        for(; x > 0; x -= lowbit(x)) {
            result += data[x];
        }
        return result;
    }
    static int lowbit(int x) {
        return x & (-x);
    }
};
```

7.7 並查集 union find

```
struct DisjointSet {
    vector<int> parent, sz;  // parent[i] = 父節點, sz[
        i] = 集合大小
    void init(int n) {
        parent.resize(n + 1);
        sz.assign(n + 1, 1);
        for (int i = 0; i <= n; i++) {
            parent[i] = i;
        }
    }
    int find(int x) {
        if (parent[x] != x) {
            parent[x] = find(parent[x]);  // 路徑壓縮
        }
        return parent[x];
    }
    bool unite(int x, int y) {
        x = find(x);
        y = find(y);
        if (x == y) return false;  // 已在同一集合
        // 啟發式合併：小的掛到大的下面
        if (sz[x] < sz[y]) swap(x, y);
        parent[y] = x;
        sz[x] += sz[y];
        return true;
    }

    bool same(int x, int y) {
        return find(x) == find(y);
    }
};
```

8 String

8.1 KMP

```
template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
  f[0]=-1, f[1]=0;
  for (int i=2; i<=n; i++){
    int w = f[i-1];
    while (w>=0 && s[w+1]!=s[i])w = f[w];
    f[i]=w+1;
  }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){
  build_KMP(m,b,f);
  int ans=0;

  for (int i=1, w=0; i<=n; i++){
    while ( w>=0 && b[w+1]!=a[i] )w = f[w];
    w++;
    if (w==m){
      ans++;
      w=f[w];
    }
  }
  return ans;
}
```

8.2 smallest rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}
/*
Booth 演算法
用於尋找一個字串的字典序最小的循環旋轉
*/
Contact GitHub API Training Shop Blog About
```

8.3 Suffix Array

```

/*he[i]保存了在後綴數組中相鄰兩個後綴的最長公共前綴長度
 *sa[i]表示的是字典序排名為i的後綴是誰（字典序越小的排
     名越靠前）
 *rk[i]表示的是後綴我所對應的排名是多少  */

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
  int len = strlen(ip);
  int alp = 256;
  memset(ct, 0, sizeof(ct));
  for(int i=0;i<len;i++) ct[ip[i]+1]++;
  for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
  for(int i=0;i<len;i++) rk[i]=ct[ip[i]];
  for(int i=1;i<len;i*=2){
    for(int j=0;j<len;j++){
      if(j+i>=len) tp[j][1]=0;
      else tp[j][1]=rk[j+i]+1;
      tp[j][0]=rk[j];
    }
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
    for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;
    memset(ct, 0, sizeof(ct));
    for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
    for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
    for(int j=0;j<len;j++)
      sa[ct[tp[tsa[j]][0]]++]=tsa[j];
    rk[sa[0]]=0;
    for(int j=1;j<len;j++){
      if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
      else
        rk[sa[j]] = j;
    }
  }
  for(int i=0,h=0;i<len;i++){
    if(rk[i]==0) h=0;
    else{
      int j=sa[rk[i]-1];
      h=max(0,h-1);
      for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]]=h;
  }
}
```

8.4 Z-value

```
z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
  if ( z[bst] + bst <= i ) z[i] = 0;
  else z[i] = min(z[i - bst], z[bst] + bst - i);
  while ( str[i + z[i]] == str[z[i]] ) z[i]++;
  if ( i + z[i] > bst + z[bst] ) bst = i;
}
```

```
// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
        centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-
            i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
               s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}
```

8.5 旋轉哈希

```
typedef unsigned __int128 ulll;

ulll power(ulll a, ulll n, ulll m) {
    ulll re = 1;
    while (n > 0) {
        if (n & 1) re = re * a % m;
        a = a * a % m;
        n >>= 1;
    }
    return re;
}

ulll inv(ulll a, ulll m) {
    return power(a, m - 2, m);
}

struct Rh {
    const ulll p, mod;
    vector<ulll> ps{1};
    Rh(ulll p, ulll mod) : p(p), mod(mod) {}
    vector<ulll> build(const string &s) {
        vector<ulll> h(s.size() + 1);
        h[0] = 0;
        ps.resize(s.size() + 1);
        for (int i = 0; i < s.size(); i++) {
            ps[i + 1] = ps[i] * p % mod;
            h[i + 1] = (h[i] + s[i] * ps[i + 1] % mod)
                % mod;
        }
        return h;
    }
    ulll subhash(const vector<ulll> &h, int l, int r) {
        // [l, r] 指原字串
        return ((h[r + 1] - h[l]) * inv(ps[l], mod)) %
            mod;
    }
};

constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
  uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t
      )b, h2 = b>>32;
  uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
  uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >>
      29) + (m << 35 >> 3) + 1;
  ret = (ret & mod) + (ret>>61);
  ret = (ret & mod) + (ret>>61);
  return ret-1;
}
```

8.6 後綴自動機

```
struct state {
    int len{}, link{};
    array<int, 26> next{};
};

struct SAM {
    int sz{}, last{};
    vector<state> st;
    SAM(int maxlen) : st(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
    }

    void insert(char c) {
        insert_impl(c - 'a');
    }

    void insert_impl(char c) {
        int cur = sz++;
        st[cur].len = st[last].len + 1;
        int p = last;
        while(p != -1 && !st[p].next[c]) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if(p == -1) {
            st[cur].link = 0;
        }
        else {
            int q = st[p].next[c];
            if(st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            }
            else {
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                st[clone].next = st[q].next;
                st[clone].link = st[q].link;
                while(p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
    }
};
```

9 Others

9.1 矩陣樹定理

```
新的方法介紹
下面我們介紹一個新的方法－Matrix-Tree定理(Kirchhoff矩陣
    -樹定理)。
Matrix-Tree定理是解決生成樹數問題最有力的武器之一。它首
    先於1847年被Kirchhoff證明。在介紹定理之前，我們先先
    明確幾個概念：
1.G的度數矩陣D[G]是一個n*n的矩陣，並且滿足：當i≠j時,dij
    =0；當i=j時，dij等於vi的度數。
2.G的鄰接矩陣A[G]也是一個n*n的矩陣， 且滿足：若vi、vj之
    間有邊直接相連，則aij=1，否則為0。
我們定義G的Kirchhoff矩陣(也稱為拉普拉斯算子)C[G]為C[G]=
    D[G]-A[G]，
則Matrix-Tree定理可以描述為：G的所有不同的生成樹的個數
    等於其Kirchhoff矩陣C[G]任何一個n-1階主子式的行列式
    的絕對值。
所謂n-1階主子式，就是對於r(1≤r≤n)，將C[G]的第r行、第r列
    同時去掉後所得到的新矩陣，以Cr[G]表示。


生成樹計數
演算法步驟：
1、 建構拉普拉斯矩陣
 Matrix[i][j] =
degree(i) , i==j
 -1，i-j有邊
 0，其他情況
2、 去掉第r行，第r列 (r任意)
3、 計算矩陣的行列式

#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
{
    if(fabs(x) < eps)return 0;
    if(x < 0)return -1;
    else return 1;
}
double b[MAXN][MAXN];
double det(double a[][MAXN],int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0;i < n;i++)
    for(j = 0;j < n;j++) b[i][j] = a[i][j];
    for(i = 0;i < n;i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n;j++)
            if(sgn(b[j][i]) != 0) break;
            if(j == n)return 0;
            for(k = i;k < n;k++) swap(b[i][k],b[j][k]);
            sign++;
        }
        ret *= b[i][i];
        for(k = i + 1;k < n;k++) b[i][k]/=b[i][i];
        for(j = i+1;j < n;j++)
        for(k = i+1;k < n;k++) b[j][k] -= b[j][i]*b[i][
            k];
    }
    if(sign & 1)ret = -ret;
    return ret;
}
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
    int T;
    int n,m;
    int u,v;
    scanf("%d",&T);
    while(T--)
    {
        scanf("%d%d",&n,&m);
        memset(g,0,sizeof(g));
        while(m--)
        {
            scanf("%d%d",&u,&v);
            u--;v--;
            g[u][v] = g[v][u] = 1;
        }
        memset(a,0,sizeof(a));
        for(int i = 0;i < n;i++)
        for(int j = 0;j < n;j++)
        if(i != j && g[i][j])
        {
            a[i][i]++;
            a[i][j] = -1;
        }
        double ans = det(a,n-1);
        printf("%.0lf\n",ans);
    }
    return 0;
}
```

9.2 1D/1D dp 優化

```
#include<bits/stdc++.h>

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}
long double f(int i, int j) {
//    cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}
struct INV {
    int L, R, pos;
};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L
        , i) < f(stk[top].L, stk[top].pos) ) {
        stk[top - 1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos =
        stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top + 1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
            cin >> s[i];
            sum[i] = sum[i-1] + strlen(s[i]);
            dp[i] = numeric_limits<long double>::max();
        }
        stk[top] = (INV) {1, n + 1, 0};
        for ( int i = 1 ; i <= n ; i++ ) {
            if ( i >= stk[bot].R ) bot++;
            dp[i] = f(i, stk[bot].pos);
            update(i);
//              cout << (ll) f(i, stk[bot].pos) << endl;
        }
        if ( dp[n] > 1e18 ) {
            cout << "Too hard to arrange" << endl;
        } else {
            vector<PI> as;
            cout << (ll)dp[n] << endl;
        }
    }
    return 0;
}
```

9.3 Theorm - DP optimization

```
Monotonicity & 1D/1D DP & 2D/1D DP
-------------------------------------------------------
Definition xD/yD
1D/1D DP[j] = min(0≤i<j) { DP[i] + w(i, j) }; DP[0] = k
2D/1D DP[i][j] = min(i<k≤j) { DP[i][k - 1] + DP[k][j] }
     + w(i, j); DP[i][i] = 0
-------------------------------------------------------
Monotonicity
       c       d
  -----------------
a | w(a, c) w(a, d)
b | w(b, c) w(b, d)

Monge Condition
Concave(凹四邊形不等式): w(a, c) + w(b, d) >= w(a, d) +
     w(b, c)
Convex (凸四邊形不等式): w(a, c) + w(b, d) <= w(a, d) +
     w(b, c)

Totally Monotone
Concave(凹單調): w(a, c) <= w(b, d) -----> w(a, d) <= w
    (b, c)
Convex (凸單調): w(a, c) >= w(b, d) -----> w(a, d) >= w
    (b, c)
-------------------------------------------------------
1D/1D DP O(n^2) -> O(nlgn)
**CONSIDER THE TRANSITION POINT**
Solve 1D/1D Concave by Stack
Solve 1D/1D Convex by Deque
-------------------------------------------------------
2D/1D Convex DP (Totally Monotone) O(n^3) -> O(n^2)
h(i, j − 1) ≤ h(i, j) ≤ h(i + 1, j)
```

9.4 Stable Marriage

```
// normal stable marriage problem
// input:
//3
//Albert Laura Nancy Marcy
//Brad Marcy Nancy Laura
//Chuck Laura Marcy Nancy
//Laura Chuck Albert Brad
//Marcy Albert Chuck Brad
//Nancy Brad Albert Chuck

#include<bits/stdc++.h>
using namespace std;
const int MAXN = 505;

int n;
int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id
    ;
int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank
    ;
int current[MAXN]; // current[boy_id] = rank; boy_id
    will pursue current[boy_id] girl.
int girl_current[MAXN]; // girl[girl_id] = boy_id;

void initialize() {
  for ( int i = 0 ; i < n ; i++ ) {
    current[i] = 0;
    girl_current[i] = n;
    order[i][n] = n;
  }
}

map<string, int> male, female;
string bname[MAXN], gname[MAXN];
int fit = 0;

void stable_marriage() {

  queue<int> que;
  for ( int i = 0 ; i < n ; i++ ) que.push(i);
  while ( !que.empty() ) {
    int boy_id = que.front();
    que.pop();

    int girl_id = favor[boy_id][current[boy_id]];
    current[boy_id] ++;

    if ( order[girl_id][boy_id] < order[girl_id][
        girl_current[girl_id]] ) {
      if ( girl_current[girl_id] < n ) que.push(
          girl_current[girl_id]); // if not the first
          time
      girl_current[girl_id] = boy_id;
    } else {
      que.push(boy_id);
    }
  }

}

int main() {
  cin >> n;

  for ( int i = 0 ; i < n;  i++ ) {
    string p, t;
    cin >> p;
    male[p] = i;
    bname[i] = p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
      if ( !female.count(t) ) {
        gname[fit] = t;
        female[t] = fit++;
      }
      favor[i][j] = female[t];
    }
  }

  for ( int i = 0 ; i < n ; i++ ) {
    string p, t;
    cin >> p;
    for ( int j = 0 ; j < n ; j++ ) {
      cin >> t;
      order[female[p]][male[t]] = j;
    }
  }

  initialize();
  stable_marriage();

  for ( int i = 0 ; i < n ; i++ ) {
    cout << bname[i] << " " << gname[favor[i][current[i
        ] - 1]] << endl;
  }

}
```

9.5 莫隊

```
/* nums 長度 N ;; query 長度為 M */
/* O(N * sqrt(M))*/

struct Query {
    int l, r, id;
};

void add(int pos) {
    /*更新狀態*/
    /*將pos所在的移入集合*/
}

void del(int pos) {
    /*更新狀態*/
    /*將pos所在的移出集合*/
}

int bsz = n / sqrt(m); /*分塊大小 block size*/
sort(query.begin(), query.end(), [bsz](const Query &a,
    const Query &b){
    if(a.l / bsz != b.l / bsz) {
        return a.l < b.l;
    }
    return (a.l / bsz) & 1 ? a.r < b.r : a.r > b.r;
});

int l = 1;
int r = 0;

vector<pair<int, int>> res(m);

for(int i = 0; i < query.size(); i++ ) {
    auto &q = query[i];
    /*順序不能換*/
    while (l > q.l) add(--l);
    while (r < q.r) add(++r);
    while (l < q.l) del(l++);
    while (r > q.r) del(r--);
    res[q.id] = /* 根據當前狀態求解 */
}
```

9.6 莫隊帶修改

```
/*
Mo's Algorithm With modification
Block: N^{2/3}, Complexity: N^{5/3}
*/
struct Query {
  int L, R, LBid, RBid, T;
  Query(int l, int r, int t):
    L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    if (RBid != q.RBid) return RBid < q.RBid;
    return T < b.T;
  }
};
void solve(vector<Query> query) {
  sort(ALL(query));
  int L=0, R=0, T=-1;
  for (auto q : query) {
    while (T < q.T) addTime(L, R, ++T); // TODO
    while (T > q.T) subTime(L, R, T--); // TODO
    while (R < q.R) add(arr[++R]); // TODO
    while (L > q.L) add(arr[--L]); // TODO
    while (R > q.R) sub(arr[R--]); // TODO
    while (L < q.L) sub(arr[L++]); // TODO
    // answer query
  }
}
```

9.7 矩陣乘法

```
#define MOD INT_MAX
vector<vector<int>> operator *(const vector<vector<int
    >> &a, const vector<vector<int>> &b) {
    vector<vector<int>> re(a.size(), vector<int>(b[0].
        size()));
    for (int i = 0; i < a.size(); i++) {
        for (int j = 0; j < b[0].size(); j++) {
            for (int k = 0; k < b.size(); k++) {
                re[i][j] += (a[i][k] * b[k][j]) % MOD;
            }
        }
    }
    return re;
}
```

9.8 c++ 小抄

```cpp
//pbds tree
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> tr;

tr.find_by_order(k) // O(logN) 取得第k大的元素
tr.order_of_key(ele) // O(logN) 得到ele是tree中第幾大(
    有幾個元素小於ele)

//pbds pair priority_queue
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;

priority_queue<int, less<int>, pairing_heap_tag> pq;
auto it = pq.push(x);
// type of it = priority_queue<int, less<int>,
    pairing_heap_tag>::point_iterator
pq.pop();
pq.top();
pq.join(b);
pq.empty();
pq.size();
pq.modify(it,6);    // O(logN)
pq.erase(it);

//builtin functions
__builtin_popcount(x);  // 1的個數
__builtin_popcountll(x); // for long long
__builtin_clz(x);        // 前導0的個數
__builtin_ctz(x);        // 後導0的個數
__builtin_parity(x);     // 奇偶性

//溢位檢查
ret = __builtin_add_overflow(a, b, &res) // if ret = 1
    a+b 溢位
ret = __builtin_sub_overflow(a, b, &res) // if ret = 1
    a-b 溢位
ret = __builtin_mul_overflow(a, b, &res) // if ret = 1
    a*b 溢位
ret = __builtin_add_overflow_p(a, b, 0LL) // if ret = 1
     溢位 第三個參數是判斷的類型

//vector SIMD
typedef int v4si __attribute__ ((vector_size(4 * sizeof
    (int))));

//大質數表
{1000000007, 1000000009, 1000000021, 1000000033,
    1000000087, 1000000093, 1000000097, 1000000123,
    1000000321};


//mt19937
#include <random>
#include <chrono>

int getRendom(int l, int r) {
    static auto seed = std::chrono::system_clock::now()
        .time_since_epoch().count();
    static std::mt19937 gen(seed);
    std::uniform_int_distribution<int> dis(l, r);
    return dis(gen);
}

//sorted vector 去重
vec.erase(unique(vec.begin(), vec.end()), vec.end());

//std::valarray
valarray<int> a(初始值, 數量); //就是那麼機八
valarray<int> a(10);
valarray<int> b(10);
valarray<int> c = a + b;
valarray<int> d = a * b;
valarray<int> e = a + 10;
valarray<int> f = a * 10;
valarray<int> g = a.cshift(1); //循環左移
valarray<bool> equal = a == b;
int sum = a.sum();
int max = a.max();
int min = a.min();
std::valarray<int> g = a.apply([](int x) { return x * x
    ; });

//regex  ***very slow***
#include <regex>
using namespace std;
bool res = regex_match("abc", regex("a.c"));
bool res = regex_match("abc", regex("A.c", regex::icase
    )); //忽略大小寫

//gp_hash_table
#include <ext/pb_ds/assoc_container.hpp>
__gnu_pbds::gp_hash_table<int, int/*, hashFunctor */>
    table;
```

9.9 python 小抄

```python
#!/usr/bin/env python3

# 帕斯卡三角形
n = 10
dp = [ [1 for j in range(n)] for i in range(n) ]
for i in range(1,n):
    for j in range(1,n):
        dp[i][j] = dp[i][j-1] + dp[i-1][j]

for i in range(n):
    print( ' '.join( '{:5d}'.format(x) for x in dp[i] )
        )

# EOF1
while True:
    try:
        n, m = map(int, input().split())
    except:
        break
# EOF2
import sys
for s in sys.stdin:
    print(eval(s.replace("/", "//")))

# input a sequence of number
a = [ int(x) for x in input().split() ]
a.sort()
print( ''.join( str(x)+' ' for x in a ) )

# LCS
ncase = int( input() )
for _ in range(ncase):
    n, m = [int(x) for x in input().split()]
    a, b = "$"+input(), "$"+input()
    dp = [ [int(0) for j in range(m+1)] for i in range(
        n+1) ]
    for i in range(1,n+1):
        for j in range(1,m+1):
            dp[i][j] = max(dp[i-1][j],dp[i][j-1])
            if a[i]==b[j]:
                dp[i][j] = max(dp[i][j],dp[i-1][j-1]+1)

    for i in range(1,n+1):
        print(dp[i][1:])
    print('a={:s}, b={:s}, |LCS(a,b)|={:d}'.format(a
        [1:],b[1:],dp[n][m]))

# list, dict, string
a = [1, 3, 4, 65, 65]
b = list.copy() # b = [1, 3, 4, 65], list a 跟 llst b
    互相獨立
cnt = list.count(65) # cnt == 2
loc = list.index(65) # loc == 3, find the leftmost
    element, if not found then return ERROR
```

```
list.sort(reverse = True|False, key = none|lambda x:x
    [1]) # list.sort has side effect but no reture
    value

# stack         # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop()     # pop()
stack[-1]       # top()
len(stack)      # size() O(1)

# queue         # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0]        # front()
len(queue)      # size() O(1)
```

9.10 萬年曆

$$h = \left(q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor + 5J\right) \bmod 7$$

h : 星期（0 = 星期六, 1 = 星期日, 2 = 星期一, …）
q : 日期（日）
m : 月份（3= 三月, 4= 四月, …；1、2 月視為前一年的 13、14 月）
K : 年份的後兩位數（year mod 100）
J : 年份的前兩位數（year ÷ 100）