

Contents

1 Basic	1
1.1 compile	1
1.2 default code	1
1.3 debug list	1
1.4 時間複雜度	1
2 Dark Code	1
2.1 IO optimization	1
3 Geometry	1
3.1 2D point	1
3.2 兩線段交點	2
3.3 兩圓交點	2
3.4 Convex Hull	2
4 Flow	2
4.1 Dinic	2
4.2 min cost flow	3
5 Mathematics	4
5.1 ax+by=gcd(a,b)	4
5.2 Gauss Elimination	4
5.3 Inverse	4
5.4 LinearPrime 歐拉篩	4
5.5 Miller Rabin	4
5.6 Pollard's rho	4
5.7 NTT	5
5.8 數論基本工具	5
5.9 Mobius	5
5.10SG	5
5.11Theorem	5
6 Graph	6
6.1 BCC	6
6.2 Prim	6
6.3 Bellman Ford	6
6.4 Kruskal	7
6.5 Dijkstra	7
6.6 Strongly Connected Component(SCC)	7
6.7 Hungarian	8
6.8 KM	8
6.9 最小平均環	8
6.10偵測負環	9
6.11Tarjan	9
6.12Topological Sort	10
7 Data Structure	10
7.1 2D Range Tree	10
7.2 Segment Tree	10
7.3 ZKW 線段樹	11
7.4 Sparse Table	11
7.5 Lazy Tag	11
7.6 BIT 樹狀樹組	12
7.7 並查集 union find	12
8 String	12
8.1 KMP	12
8.2 smallest rotation	12
8.3 Suffix Array	12
8.4 Z-value	13
8.5 旋轉哈希	13
8.6 後綴自動機	13
9 Others	14
9.1 矩陣樹定理	14
9.2 1D/1D dp 優化	14
9.3 Theorm - DP optimization	15
9.4 Stable Marriage	15
9.5 莫隊	15
9.6 矩陣乘法	16
9.7 c++ 小抄	16
9.8 python 小抄	17
9.9 萬年曆	17

```

**All file will be compiled to a.out unless you use -o(
  not recommended, just use a.out)**
# Run
./a.out

# Run with file input
./a.out < input.txt

# Run with file input and output
./a.out < input.txt > output.txt

# Python Run
python3 a.py < input.txt > output.txt

# Copy Paste In Ubuntu
* copy: ctrl+insert
* paste: shift+insert

# 比對文件相同
sdiff a.txt b.txt

```

1.2 default code

```

#include <bits/stdc++.h>
using namespace std;
#define int long long
typedef pair<int,int> pii;

#ifdef ONLINE_JUDGE
#define cerr if(false) cerr
#endif

int32_t main(){
#ifdef ONLINE_JUDGE
    freopen("input.txt","r",stdin);
    freopen("output.txt","w",stdout);
    freopen("debug.txt","w",stderr);
#else
    ios_base::sync_with_stdio(0);
    cin.tie(false);
#endif
}

```

1.3 debug list

記得測試 python 的內建函數庫有哪些
bits/std++.h 跟 global variable y1 衝突，不能用
模板要記得 init
priority_queue 要清空
事先將把邊界測資加入測試
邊界條件（過程溢位，題目數據範圍），會不會爆 long long
是否讀錯題目，想不到時可以自己讀一次題目
比較容易有問題的地方換人寫
注意公式有沒有推錯或抄錯
精度誤差 sqrt(大大的東西) + EPS
喇分 random_shuffle 隨機演算法

1 Basic

1.1 compile

```

# preset before coding
echo "cd ~/Desktop" >> ~/.bashrc
gedit -> preference -> tab width: 4

# Editor
gedit a.cpp

# Compile
g++ a.cpp -std=c++14 -Wall -fsanitize=address
// -fsanitize=address 檢測記憶體違規存取

```

1.4 時間複雜度

時間複雜度	可處理的最大 N 數量級 (約)
$O(1)$	幾乎沒限制
$O(\log N)$	10^{18} 級別 (如快速冪)
$O(\sqrt{N})$	10^{10}
$O(N)$	10^8
$O(N \log N)$	$2 \times 10^7 \sim 5 \times 10^7$
$O(N\sqrt{N})$	$1 \times 10^5 \sim 2 \times 10^5$
$O(N^2)$	$10^4 \sim 1.5 \times 10^4$
$O(N^2 \log N)$	約 3×10^3
$O(N^3)$	500 ~ 1000
$O(2^N)$	$N \leq 20$
$O(N!)$	$N \leq 10$

2 Dark Code

2.1 IO optimization

```
*if output to much, consider put all output in array
  first, then output the array.
getchar() -> getchar_unlocked()
fread() -> fread_unlocked()
-----
inline char readchar() {
    const int S = 1<<20; // buffer size
    static char buf[S], *p = buf, *q = buf;
    if(p == q && (q = (p=buf)+fread(buf,1,S,stdin)) ==
        buf) return EOF;
    return *p++;
}

inline int nxtint() {
    // if readchar can't use, change readchar() to
    // getchar()
    int x = 0;
    int c = readchar(), neg = false;
    if (c == EOF) return -1;
    while (('0' > c || c > '9') && c != '-' && c != EOF)
        c = readchar();
    if (c == '-') neg = true, c = readchar();
    while ('0' <= c && c <= '9') x = x * 10 + (c ^ '0'),
        c = readchar();
    if (neg) x = -x;
    return x;
}
```

3 Geometry

3.1 2D point

```
typedef double Double;
struct Point {
    Double x,y;

    bool operator < (const Point &b)const{
        //return tie(x,y) < tie(b.x,b.y);
        return atan2(y,x) < atan2(b.y,b.x);
    }
    Point operator + (const Point &b)const{
        return (Point){x+b.x,y+b.y};
    }
    Point operator - (const Point &b)const{
        return (Point){x-b.x,y-b.y};
    }
    Point operator * (const Double &d)const{
        return Point(d*x,d*y);
    }
    Double operator * (const Point &b)const{
        return x*b.x + y*b.y;
    }
    Double operator % (const Point &b)const{
        return x*b.y - y*b.x;
    }
    friend Double abs2(const Point &p){
        return p.x*p.x + p.y*p.y;
    }
    friend Double abs(const Point &p){
        return sqrt( abs2(p) );
    }
};
typedef Point Vector;

struct Line{
    Point P; Vector v;
    bool operator < (const Line &b)const{
        return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
    }
};
```

3.2 兩線段交點

```
using type = long long;
const type EPS = 0 /*1e-9*/;
struct Point { type x, y; };

inline type cross(const Point &a, const Point &b, const
    Point &c) {
    return (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c
        .x - a.x);
}

inline bool overlap(type a, type b, type c, type d) {
    if(a > b) swap(a,b); if(c > d) swap(c,d);
    return max(a,c) <= min(b,d) + EPS;
}

bool equal_zero(type x) {
    return abs(x) <= EPS;
}
bool sgn(type x) {
    return (x > EPS) - (x < -EPS);
}

#define CROSS(i,j,k) cross(p[i],p[j],p[k])

#define CHECK_COLLINEAR(i,j,k) (equal_zero(CROSS(i,j,k)
    ) && overlap(p[i].x,p[j].x,p[k].x,p[k].x) &&
    overlap(p[i].y,p[j].y,p[k].y,p[k].y))

bool intersect(const vector<Point> &p){
    type d[4];
    for(int i=0;i<4;i++){
        if(i<2) d[i] = CROSS(0,1,i+2);
        else d[i] = CROSS(2,3,i-2);
    }
    for(int i=0;i<4;i++)
        /**/if(CHECK_COLLINEAR(i<2?0:2,i<2?1:3,i<2?i+2:i-2))
        /**/return true;
    return sgn(d[0]) != sgn(d[1]) && sgn(d[2]) != sgn(d
        [3]);
}

// 求交點 不處理共線重疊
pair<long double,long double> intersection(const vector
    <Point> &p){
    long double A1 = p[1].y - p[0].y, B1 = p[0].x - p
        [1].x, C1 = A1*p[0].x+B1*p[0].y;
    long double A2 = p[3].y - p[2].y, B2 = p[2].x - p
        [3].x, C2 = A2*p[2].x+B2*p[2].y;
    long double det = A1*B2 - A2*B1;
    return {(C1*B2-C2*B1)/det,(A1*C2-A2*C1)/det};
}
```

3.3 兩圓交點

```
vector<Point> interCircle(Point o1, type r1, Point o2,
    type r2) {
    type d2 = abs2(o1 - o2);
    type d = sqrt(d2);
    if (d < fabs(r1 - r2) || d > r1 + r2) return {};
    Point u = (o1 + o2) * 0.5 + ((r2*r2 - r1*r1) /
        (2.0*d2)) * (o1 - o2);
    type A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    Point v = Point{o1.y - o2.y, -(o1.x - o2.x)} * (A /
        (2.0*d2));
    return { u + v, u - v };
}
```

3.4 Convex Hull

```
#include "2Dpoint.cpp"

// return H, The first will occurred TWICE in vector H!
void ConvexHull(vector<Point> &P, vector<Point> &H){
    int n = P.size(), m=0;
```

```

sort(P.begin(), P.end());
H.clear();

for (int i=0; i<n; i++){
    while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
           <0) H.pop_back(), m--;
    H.push_back(P[i]), m++;
}

for (int i=n-2; i>=0; i--){
    while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
           <0) H.pop_back(), m--;
    H.push_back(P[i]), m++;
}
}

```

4 Flow

4.1 Dinic

(a) 有源匯上下界最大流 (Bounded Maxflow)

目標：在滿足所有邊的流量上下界限制的前提下，從源點 s 到匯點 t 的最大流量。

先依照 (b) 的方法建立可行流模型。

檢查是否存在可行流 (即 $\text{max_flow}(ss, tt)$ 是否等於所有流量下界 l 的總和)。如果不可行，則此問題無解。

重要：如果可行，不要重新初始化圖。直接在當前的殘留網絡上繼續計算 $\text{dinic.max_flow}(s, t)$ 。

最終的答案就是步驟 3 中計算出的從 s 到 t 的附加流量。

(b) 有上下界可行流 (Bounded Possible Flow)

目標：檢查是否存在一種流量分配，使得每條邊的流量 f 都滿足其下界 l 和上界 r 的限制 ($l \leq f \leq r$)。

新增兩個節點：超級源點 ss 和超級匯點 tt 。

準備一個變數 total_lower_bound 來累加所有下界 l 。

對於每一條原始邊 $u \rightarrow v$ ，其容量為 $[l, r]$ ：

$\text{dinic.add_edge}(u, v, r - l)$; (邊的彈性容量)

$\text{dinic.add_edge}(ss, v, l)$; (節點 v 需要 l 的流入)

$\text{dinic.add_edge}(u, tt, l)$; (節點 u 提供 l 的流出)

$\text{total_lower_bound} += l$;

計算 $\text{flow} = \text{dinic.max_flow}(ss, tt)$ 。

如果 $\text{flow} == \text{total_lower_bound}$ ，則表示所有下界需求都被滿足，存在可行流；否則不存在。

(c) 有源匯上下界最小流 (Bounded Minimum Flow)

目標：在滿足所有邊的流量上下界限制的前提下，從源點 s 到匯點 t 的最小流量。

注意：這個問題通常需要透過二分搜尋答案來解決，無法直接用一次最大流求出。

二分搜尋一個流量值 F 。

對於每個猜測的 F ，建立一個無源匯可行流模型來檢查其可行性：

使用 (b) 的方法建構基本圖。

額外加入一條邊 $t \rightarrow s$ ，容量為 $[F, \text{INF}]$ 。這條邊強制要求從 s 到 t 的淨流量至少為 F 。

檢查這個新的循環圖是否存在可行流。如果存在，表示流量 F 是可達成的，可以嘗試更小的 F ；反之， F 太小了，需要增加。

(e) 最小割 (Minimum Cut)

目標：找出一個邊集，其總容量最小，且移除這些邊後 s 和 t 不再連通。

根據最大流-最小割定理，最小割的值等於最大流的值。先執行 $\text{ll min_cut_value} = \text{dinic.max_flow}(s, t)$ ；

呼叫 $\text{vector<bool> side} = \text{dinic.get_min_cut_nodes}(s)$ ；來取得節點的劃分。

$\text{side}[i] == \text{true}$ 表示節點 i 屬於源點 s 所在的集合 (S 集合)。

$\text{side}[i] == \text{false}$ 表示節點 i 屬於匯點 t 所在的集合 (T 集合)。

最小割的邊集就是所有從 S 集合指向 T 集合的原始邊

using ll = longlong;

```

const ll INF = 1e18;

struct Dinic {
    struct Edge {
        int to;
        ll cap;
        int rev; // 反向邊的索引
    };
    vector<vector<Edge>> adj;
    vector<int> level, iter;
    vector<bool> side;
    int n;
    Dinic(int v) : n(v), adj(v), level(v), iter(v) {}
    void add_edge(int u, int v, ll cap) {
        adj[u].push_back({v, cap, (int)adj[v].size()});
        adj[v].push_back({u, 0, (int)adj[u].size() - 1});
    }
    bool bfs(int s, int t) {
        fill(level.begin(), level.end(), -1);
        queue<int> q;
        level[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (const auto& edge : adj[u]) {
                if (edge.cap > 0 && level[edge.to] < 0) {
                    level[edge.to] = level[u] + 1;
                    q.push(edge.to);
                }
            }
        }
        return level[t] != -1;
    }
    ll dfs(int u, int t, ll f) {
        if (u == t) return f;
        for (int& i = iter[u]; i < (int)adj[u].size(); ++i) {
            Edge& e = adj[u][i];
            if (e.cap > 0 && level[u] < level[e.to]) {
                ll d = dfs(e.to, t, min(f, e.cap));
                if (d > 0) {
                    e.cap -= d;
                    adj[e.to][e.rev].cap += d;
                    return d;
                }
            }
        }
        return 0;
    }
    ll max_flow(int s, int t) {
        ll flow = 0;
        while (bfs(s, t)) {
            fill(iter.begin(), iter.end(), 0);
            ll f;
            while ((f = dfs(s, t, INF)) > 0) {
                flow += f;
            }
        }
        return flow;
    }
    void _find_cut(int u) {
        side[u] = true;
        for (const auto& e : adj[u]) {
            if (e.cap > 0 && !side[e.to]) {
                _find_cut(e.to);
            }
        }
    }
    vector<bool> get_min_cut_nodes(int s) {
        fill(side.begin(), side.end(), false);
        _find_cut(s);
        return side;
    }
};

```

4.2 min cost flow

```
// Long Long version
typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MAXN = 350;
    static const long long INF = 1LL<<60;
    struct Edge {
        int to, r;
        long long rest, c;
    };
    int n, pre[MAXN], preL[MAXN]; bool inq[MAXN];
    long long dis[MAXN], fl, cost;
    vector<Edge> G[MAXN];
    void init() {
        for (int i = 0; i < MAXN; i++) G[i].clear();
    }
    void add_edge(int u, int v, long long rest, long long c) {
        G[u].push_back({v, (int)G[v].size(), rest, c});
        G[v].push_back({u, (int)G[u].size()-1, 0, -c});
    }
    pll flow(int s, int t) {
        fl = cost = 0;
        while (true) {
            fill(dis, dis+MAXN, INF);
            fill(inq, inq+MAXN, 0);
            dis[s] = 0;
            queue<int> que;
            que.push(s);
            while (!que.empty()) {
                int u = que.front(); que.pop();
                inq[u] = 0;
                for (int i = 0; i < (int)G[u].size(); i++) {
                    int v = G[u][i].to;
                    long long w = G[u][i].c;
                    if (G[u][i].rest > 0 && dis[v] > dis[u] + w) {
                        pre[v] = u; preL[v] = i;
                        dis[v] = dis[u] + w;
                        if (!inq[v]) {
                            inq[v] = 1;
                            que.push(v);
                        }
                    }
                }
            }
            if (dis[t] == INF) break;
            long long tf = INF;
            for (int v = t, u, l; v != s; v = u) {
                u = pre[v]; l = preL[v];
                tf = min(tf, G[u][l].rest);
            }
            for (int v = t, u, l; v != s; v = u) {
                u = pre[v]; l = preL[v];
                G[u][l].rest -= tf;
                G[v][G[u][l].r].rest += tf;
            }
            cost += tf * dis[t];
            fl += tf;
        }
        return {fl, cost};
    }
} flow;
```

5 Mathematics

5.1 $ax+by=\gcd(a,b)$

```
typedef pair<int, int> pii;

pii exgcd(int a, int b) {
    if (b == 0) return make_pair(1, 0);
    else {
        int p = a / b;
        pii q = exgcd(b, a % b);
        int aa = q.second, bb = q.first - q.second * p;
```

```
    if (aa < 0) aa += b, bb -= a;
    return make_pair(aa, bb);
}
```

5.2 GaussElimination

```
// by bcw_codebook

const int MAXN = 300;
const double EPS = 1e-8;

int n;
double A[MAXN][MAXN];

void Gauss() {
    for (int i = 0; i < n; i++) {
        bool ok = 0;
        for (int j = i; j < n; j++) {
            if (fabs(A[j][i]) > EPS) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if (!ok) continue;

        double fs = A[i][i];
        for (int j = i+1; j < n; j++) {
            double r = A[j][i] / fs;
            for (int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}

template<class T>
void Gauss(vector<vector<T>> &A) {
    int n = A.size();
    for (int i = 0; i < n; i++) {
        bool ok = 0;
        for (int j = i; j < n; j++) {
            if (A[j][i] != 0) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if (!ok) continue;

        T fs = A[i][i];
        for (int j = i+1; j < n; j++) {
            T r = A[j][i] / fs;
            for (int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}
```

5.3 Inverse

```
int inverse[100000];
void invTable(int b, int p) {
    inverse[1] = 1;
    for (int i = 2; i <= b; i++) {
        inverse[i] = (long long)inverse[p%i] * (p-p/i) % p;
    }
}

int inv(int b, int p) {
    return b == 1 ? 1 : ((long long)inv(p % b, p) * (p-p/b) % p);
}
```

5.4 LinearPrime 歐拉篩

```
const int MAXP = 100; //max prime
vector<int> P; // primes
void build_prime(){
    static bitset<MAXP> ok;
    int np=0;
    for (int i=2; i<MAXP; i++){
        if (ok[i]==0)P.push_back(i), np++;
        for (int j=0; j<np && i*P[j]<MAXP; j++){
            ok[ i*P[j] ] = 1;
            if ( i%P[j]==0 )break;
        }
    }
}
```

5.5 Miller Rabin

```
typedef long long LL;

inline LL bin_mul(LL a, LL n, const LL& MOD){
    return __int128(a) * n % MOD;
}

inline LL bin_pow(LL a, LL n, const LL& MOD){
    LL re=1;
    while (n>0){
        if (n&1) re = bin_mul(re,a,MOD);
        a = bin_mul(a,a,MOD);
        n>>=1;
    }
    return re;
}

bool is_prime(LL n){
    //static LL sprp[3] = { 2LL, 7LL, 61LL};
    static LL sprp[7] = { 2LL, 325LL, 9375LL,
        28178LL, 450775LL, 9780504LL,
        1795265022LL };
    if (n==1 || (n&1)==0 ) return n==2;
    int u=n-1, t=0;
    while ( (u&1)==0 ) u>>=1, t++;
    for (int i=0; i<3; i++){
        LL x = bin_pow( sprp[i]%n, u, n);
        if (x==0 || x==1 || x==n-1)continue;

        for (int j=1; j<t; j++){
            x=x*x%n;
            if (x==1 || x==n-1)break;
        }
        if (x==n-1)continue;
        return 0;
    }
    return 1;
}
```

5.6 Pollard's rho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
    return add(mul(x, x, mod), 1, mod);
}

Int pollard_rho(Int n) {
    if ( !(n & 1) ) return 2;
    while (true) {
        Int y = 2, x = rand()%(n-1) + 1, res = 1;
        for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
            for ( int i = 0 ; i < sz && res <= 1 ; i++ ) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if ( res != 0 && res != n ) return res;
    }
}
```

5.7 NTT

```
constexpr int P = 998244353;
const int G = 3;
/*預處理 Lim*/
int lim = 1;
while (lim < (lenSum - 1)) lim <<= 1;
/*每個多項式都要resize(lim)*/
/*998244353 3 1004535809 3 469762049 3 167772161 3
754974721 11*/
void init_rev(vector<int> &rev, int lim) {
    int lg = __builtin_ctz(lim); // Lim 是 2^k
    rev.resize(lim);
    for (int i = 0; i < lim; ++i)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (lg - 1));
}

// a.size() == lim
void ntt(vector<int> &a, int opt) { // opt == -1 =>
    reverse ntt
    int n = a.size();
    static vector<int> rev;
    init_rev(rev, n);
    for (int i = 0; i < n; ++i)
        if (i < rev[i]) swap(a[i], a[rev[i]]);

    for (int m = 2; m <= n; m <<= 1) {
        int k = m >> 1;
        int gn = qpow(G, (P - 1) / m);
        if (opt == -1) gn = qpow(gn, P - 2);
        for (int i = 0; i < n; i += m) {
            int g = 1;
            for (int j = 0; j < k; ++j) {
                int t = 1ll * a[i + j + k] * g % P;
                a[i + j + k] = (a[i + j] - t + P) % P;
                a[i + j] = (a[i + j] + t) % P;
                g = 1ll * g * gn % P;
            }
        }

        if (opt == -1) {
            int inv_n = qpow(n, P - 2);
            for (int &x : a) x = 1ll * x * inv_n % P;
        }
    }
}
```

5.8 數論基本工具

```
Int POW(Int a, Int n, Int mod){
    Int re=1;
    while (n>0){
        if (n&1LL) re = re*a%mod;
        a = a*a%mod;
        n>>=1;
    }
    return re;
}

Int C(Int n, Int m){
    if (m<0 || m>n)return 0;
    return J[n] * inv(J[m]*J[n-m]%MOD) %MOD;
}
```

5.9 Mobius

```
void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
    mu[1] = 1, num = 0;
    for (int i = 2; i < MAXN; ++i) {
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
        static int d;
        for (int j = 0; j < num && (d = i * primes[j])
            < MAXN; ++j) {
            isPrime[d] = false;
            if (i % primes[j] == 0) {
                mu[d] = 0; break;
            }
        }
    }
}
```

```

    } else mu[d] = -mu[i];
  }
}
}

```

5.10 SG

Anti Nim (取走最後一個石子者敗)

先手必勝 **if and only if**

1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。

Anti-SG (決策集合為空的遊戲者贏)

定義 SG 值為 0 時，遊戲結束，

則先手必勝 **if and only if**

1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。

Sprague-Grundy

1. 雙人、回合制
2. 資訊完全公開
3. 無隨機因素
4. 可在有限步內結束
5. 沒有和局
6. 雙方可採取的行動相同

SG(S) 的值為 0：後手(P)必勝

不為 0：先手(N)必勝

```

int mex(set S) {
    // find the min number >= 0 that not in the S
    // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
    if (A not in state) {
        S = sub_states(A)
        if( len(S) > 1 ) state[A] = reduce(operator.xor, [
            SG(B) for B in S])
        else state[A] = mex(set(SG(B) for B in next_states(
            A)))
    }
    return state[A]
}

```

5.11 Theorem

```

/*
Lucas's Theorem
For non-negative integer n,m and prime P,
C(m,n) mod P = C(m/M,n/M) * C(m%M,n%M) mod P
= mult_i ( C(m_i,n_i) )
where m_i is the i-th digit of m in base P.

```

```

Pick's Theorem
A = i + b/2 - 1

```

```

Kirchhoff's theorem
A_{ii} = deg(i), A_{ij} = (i,j) \in E ? -1 : 0
Deleting any one row, one column, and cal the det(A)

```

```

Nth Catalan recursive function:
C_0 = 1, C_{n+1} = C_n * 2(2n + 1)/(n+2)

```

```

Mobius Formula
u(n) = 1, if n = 1
(-1)^m, 若 n 無平方數因數, 且 n = p1*p2*p3*...*pk

```

0, 若 n 有大於 1 的平方數因數

- Property

1. (積性函數) $u(a)u(b) = u(ab)$
2. $\sum_{d|n} u(d) = [n == 1]$

Mobius Inversion Formula

```

if f(n) = \sum_{d|n} g(d)
then g(n) = \sum_{d|n} u(n/d)f(d)
          = \sum_{d|n} u(d)f(n/d)

```

- Application

the number/power of gcd(i, j) = k

- Trick

分塊, $O(\sqrt{n})$

Chinese Remainder Theorem (m_i 兩兩互質)

```

x = a_1 (mod m_1)
x = a_2 (mod m_2)
...
x = a_i (mod m_i)

```

construct a solution:

```

Let M = m_1 * m_2 * m_3 * ... * m_n
Let M_i = M / m_i

```

```

t_i = 1 / M_i
t_i * M_i = 1 (mod m_i)

```

```

solution x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + ...
            + a_n * t_n * M_n + k * M
            = k*M + \sum a_i * t_i * M_i, k is positive integer.

```

```

under mod M, there is one solution x = \sum a_i * t_i * M_i

```

Burnside's Lemma

$|G| * |X/G| = \sum (|X^g|)$ where g in G

總方法數：每一種旋轉下不動點的個數總和 除以 旋轉的方法數

*/

6 Graph

6.1 BCC

邊雙連通

任意兩點間至少有兩條不重疊的路徑連接，找法：

1. 標記出所有的橋
2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通

// from BCW

```

struct BccEdge {
    static const int MXN = 100005;
    struct Edge { int v,eid; };
    int n,m,step,par[MXN],dfn[MXN],low[MXN];
    vector<Edge> E[MXN];
    DisjointSet djs;
    void init(int _n) {
        n = _n; m = 0;
        for (int i=0; i<n; i++) E[i].clear();
        djs.init(n);
    }
    void add_edge(int u, int v) {
        E[u].PB({v, m});
        E[v].PB({u, m});
        m++;
    }
    void DFS(int u, int f, int f_eid) {
        par[u] = f;
        dfn[u] = low[u] = step++;
        for (auto it:E[u]) {
            if (it.eid == f_eid) continue;
            int v = it.v;

```



```

    if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
    } else {
        low[u] = min(low[u], dfn[v]);
    }
}
}
void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
        if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
        if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
}
}graph;

```

6.2 Prim

```

// edge strucute
struct edge{
    int a, b;
    double data;
    bool operator <(const edge b)const{
        return data > b.data;
    }
};

// main prim algorithm
int n, m, root, aa, bb, cc;
while (cin >> n >> m){
    priority_queue<edge>yee;
    int visit[500] = {}, p[500] = {};
    double a[500][500] = {};
    //undirectional edge aa to bb is weighted cc
    for (int i = 0; i < m; i++){
        cin >> aa >> bb >> cc;
        a[aa][bb] = a[bb][aa] = cc;
    }
    cin >> root;
    yee.push({ 0, root, 0 });
    edge tmp;
    double total = 0;
    while (!yee.empty()){
        tmp = yee.top(); yee.pop();
        if (visit[tmp.b])continue;
        total += tmp.data; p[tmp.b] = tmp.a; visit[tmp.b] = 1;
        for (int i = 1; i <= n; i++){
            if (a[tmp.b][i] != 0.0 && (!visit[i])){
                yee.push({tmp.b, i, a[tmp.b][i]});
            }
        }
    }
    cout << total << endl;
}

```

6.3 Bellman Ford

```

int a[100][100], d[100], p[100];
void bellman_ford(int root, int n){
    for (int i = 1; i <= n; i++)d[i] = 1e9;
    d[root] = 0, p[root] = 0;
    for (int i = 0; i<n - 1; i++){
        for (int j = 1; j <= n; j++){
            for (int k = 1; k <= n; k++){
                if (d[j] != 1e9 && a[j][k] != 1e9){
                    if (d[j] + a[j][k] < d[k]){
                        d[k] = d[j] + a[j][k], p[k] = j;
                    }
                }
            }
        }
    }
}

```

```

}
}
bool nega_cyc(int n){
    for (int i = 1; i <= n; i++){
        for (int j = 1; j <= n; j++){
            if (d[i] != 1e9 && a[i][j] != 1e9)
                if (d[i] + a[i][j] < d[j]){
                    return 0;
                }
        }
    }
    return 1;
}
int main(){
    int n, m, aa, bb, dd;
    while (cin >> n >> m){
        for (int i = 0; i <= n; i++)for (int j = 0; j <= n; j++){
            a[i][j] = E9;
        }
        memset(p, 0, sizeof(p));
        for (int i = 0; i < m; i++){
            cin >> aa >> bb >> dd;
            a[aa][bb] = min(a[aa][bb], dd);
        }
        cin >> aa;
        bellman_ford(aa, n);
        int t = nega_cyc(n);
        if(t){
            for (int i = 1; i <= n; i++)cout << d[i] << " \n"
                [i==n];
            for (int i = 1; i <= n; i++)cout << p[i] << " \n"
                [i==n];
        }
        else cout << "There is a negative weight cycle in the graph\n";
    }
}

```

6.4 Kruskal

```

struct v {
    int a, b, c;
};
int p[200001];v a[200001];
bool sor(v a, v b) {
    return a.c < b.c;
}
int find(int x) {
    return(x != p[x] ? (p[x] = find(p[x])) : x);
}
int main() {
    int n, m, i, j, sum;
    while (cin >> n >> m) {
        sum = 0;
        for (i = 0; i < 200001; i++)p[i] = i;
        for (i = 0; i<m; i++)cin >> a[i].a >> a[i].b >> a[i].c;
        sort(a, a + m, sor);
        for (i = 0; j = 0; j<m; j++) {
            if(find(a[j].a) != find(a[j].b)){
                i++;
                p[find(a[j].a)] = find(a[j].b);
                sum += a[j].c;
            }
        }
        cout << ((i==n-1)?sum:-1) << endl;
    }
}

```

6.5 Dijkstra

```

struct node {
    int num{}, w{};
    bool operator < (const node& other) const {
        return w > other.w;
    }
};

vector<int> dijkstra(int root, const vector<vector<node>
>> &graph) {
    vector<int> d(graph.size(), INT_MAX >> 1), p(graph.
        size());
    priority_queue<node> pq;
    d[root] = p[root] = 0;
    pq.push({root, d[root]});
    while (!pq.empty()) {
        node tmp = pq.top(); pq.pop();
        for (const node &i : graph[tmp.num]) {
            if (d[i.num] > d[tmp.num] + i.w) {
                d[i.num] = d[tmp.num] + i.w;
                p[i.num] = tmp.num;
                pq.push({i.num, d[tmp.num]});
            }
        }
    }
    return d;
}

```

6.6 Strongly Connected Component(SCC)

```

#define MXN 100005
#define PB push_back
#define FZ(s) memset(s,0,sizeof(s))

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec){
            if (!vst[v]){
                rDFS(v);
                nScc++;
            }
        }
    }
};

```

6.7 Hungarian

// Maximum Cardinality Bipartite Matching

```

struct Graph {
    static const int MAXN = 5005;
    vector<int> G[MAXN];
    int n;
    int match[MAXN]; // Matching Result
    int vis[MAXN];

    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) G[i].clear();
    }

    bool dfs(int u) {
        for (auto v:G[u]) {
            if (!vis[v]) {
                vis[v] = true;
                if (match[v] == -1 || dfs(match[v])) {
                    match[v] = u;
                    match[u] = v;
                    return true;
                }
            }
        }
        return false;
    }

    int solve() {
        int res = 0;
        memset(match, -1, sizeof(match));
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                memset(vis, 0, sizeof(vis));
                if (dfs(i)) res += 1;
            }
        }
        return res;
    }
} graph;

```

6.8 KM

Detect non-perfect-matching:
 1. set all edge[i][j] as INF
 2. if solve() >= INF, it is **not** perfectmatching.

 // Maximum Weight Perfect Bipartite Matching
 // allow negative weight!

```

typedef long long Int;
struct KM {
    static const int MAXN = 1050;
    static const int INF = 1LL<<60;
    int n, match[MAXN], vx[MAXN], vy[MAXN];
    Int edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[
        MAXN];
    void init(int _n){
        n = _n;
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                edge[i][j] = 0;
    }
    void add_edge(int x, int y, Int w){
        edge[x][y] = w;
    }
    bool DFS(int x){
        vx[x] = 1;
        for (int y = 0; y < n; y++) {
            if (vy[y]) continue;
            if (lx[x] + ly[y] > edge[x][y]) {
                slack[y] = min(slack[y], lx[x] + ly[y]
                    - edge[x][y]);
            } else {
                vy[y] = 1;
                if (match[y] == -1 || DFS(match[y])) {
                    match[y] = x;
                    return true;
                }
            }
        }
    }
}

```



```

    }
    return false;
}
Int solve() {
    fill(match, match + n, -1);
    fill(lx, lx + n, -INF);
    fill(ly, ly + n, 0);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            lx[i] = max(lx[i], edge[i][j]);
    for (int i = 0; i < n; i++) {
        fill(slack, slack + n, INF);
        while (true) {
            fill(vx, vx + n, 0);
            fill(vy, vy + n, 0);
            if (DFS(i)) break;
            Int d = INF;
            for (int j = 0; j < n; j++)
                if (!vy[j]) d = min(d, slack[j]);
            for (int j = 0; j < n; j++) {
                if (vx[j]) lx[j] -= d;
                if (vy[j]) ly[j] += d;
                else slack[j] -= d;
            }
        }
    }
    Int res = 0;
    for (int i = 0; i < n; i++) {
        res += edge[match[i]][i];
    }
    return res;
}
} graph;

```

6.9 最小平均環

```

// from BCW
/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
    int v, u;
    double c;
};
int n, m, prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
    for (int i = 0; i < n; i++) d[0][i] = 0;
    for (int i = 0; i < n; i++) {
        fill(d[i+1], d[i+1] + n, inf);
        for (int j = 0; j < m; j++) {
            int v = e[j].v, u = e[j].u;
            if (d[i][v] < inf && d[i+1][u] > d[i][v] + e[j].c) {
                d[i+1][u] = d[i][v] + e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc = inf;
    int st = -1;
    bellman_ford();
    for (int i = 0; i < n; i++) {
        double avg = -inf;
        for (int k = 0; k < n; k++) {
            if (d[n][i] < inf - eps) avg = max(avg, (d[n][i] - d[k][i]) / (n - k));
            else avg = max(avg, inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    for (int i = 0; i < n; i++) vst[i] = 0;
}

```

```

edgeID.clear(); cycle.clear(); rho.clear();
for (int i = n; !vst[st]; st = prv[i-1][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
}
while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
}
reverse(ALL(edgeID));
edgeID.resize(SZ(cycle));
return mmc;
}

```

6.10 偵測負環

```

#include <bits/stdc++.h>
using namespace std;

const int INF = 1000000;
const int MAXN = 200;
int n, m, q;
int d[MAXN][MAXN];

int main () {
    while (cin >> n >> m >> q && n) {
        for (int i = 0; i <= n; i++) {
            for (int j = 0; j <= n; j++) d[i][j] =
                (i == j ? 0 : INF);
        }

        for (int i = 0; i < m; i++) {
            int a, b, c;
            cin >> a >> b >> c;
            d[a][b] = min(d[a][b], c);
        }

        for (int k = 0; k < n; k++) {
            for (int i = 0; i < n; i++) {
                for (int j = 0; j < n; j++) {
                    if (d[i][j] > d[i][k] + d[k][j] &&
                        d[i][k] < INF && d[k][j] < INF) {
                        //printf("%d > %d + %d\n", d[i][j], d[i][k], d[k][j]);
                        //if (d[i][k] >= INF || d[k][j] >= INF) cout << "NO : "
                        << i << " " << j << " " <<
                            k << "--";
                        d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
                    }
                }
            }
        }

        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                for (int k = 0; k < n && d[i][j] != -
                    INF; k++) {
                    if (d[k][k] < 0 && d[i][k] != INF
                        && d[k][j] != INF)
                        d[i][j] = -INF;
                }
            }
        }
        int u, v;
        for (int i = 0; i < q; i++) {
            scanf("%d%d", &u, &v);

            if (d[u][v] == INF) printf("Impossible\n");
            else if (d[u][v] == -INF) printf("-Infinity\n");
            else printf("%d\n", d[u][v]);
        }
        puts("");
    }
}

```

```

    }
    return 0;
}

```

6.11 Tarjan

割點

點 u 為割點 **if and only if** 滿足 1. **or** 2.

1. u 為樹根，且 u 有多於一個子樹。
2. u 不為樹根，且滿足存在 (u, v) 為樹枝邊（或稱父子邊，即 u 為 v 在搜索樹中的父親），使得 $DFN(u) \leq Low(v)$ 。

橋

一條無向邊 (u, v) 是橋 **if and only if** (u, v) 為樹枝邊，且滿足 $DFN(u) < Low(v)$ 。

```

// 0 base
struct TarjanSCC{
    static const int MAXN = 1000006;
    int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
    vector<int> G[MAXN];
    stack<int> stk;
    bool ins[MAXN];

    void tarjan(int u){
        dfn[u] = low[u] = ++count;
        stk.push(u);
        ins[u] = true;

        for(auto v:G[u]){
            if(!dfn[v]){
                tarjan(v);
                low[u] = min(low[u], low[v]);
            }else if(ins[v]){
                low[u] = min(low[u], dfn[v]);
            }
        }

        if(dfn[u] == low[u]){
            int v;
            do {
                v = stk.top();
                stk.pop();
                scc[v] = scn;
                ins[v] = false;
            } while(v != u);
            scn++;
        }
    }

    void getSCC(){
        memset(dfn, 0, sizeof(dfn));
        memset(low, 0, sizeof(low));
        memset(ins, 0, sizeof(ins));
        memset(scc, 0, sizeof(scc));
        count = scn = 0;
        for(int i = 0 ; i < n ; i++ ){
            if(!dfn[i]) tarjan(i);
        }
    }
}SCC;

```

6.12 Topological Sort

```

#define N 87

bool adj[N][N];          // adjacency matrix
int visit[N];             // record visited coordinations in DFS
int order[N], n;         // save the order

bool cycle;               // detect the cycle

void DFS(int s)

```

```

{
    // back edge occurred, detected the cycle
    if (visit[s] == 1) {cycle = true; return;}
    // forward edge and cross edge
    if (visit[s] == 2) return;

    visit[s] = 1;
    for (int t=0; t<N; ++t){
        if (adj[s][t]) DFS(t);
    }
    visit[s] = 2;
    order[n--] = s;      // record the order
}

void topological_ordering()
{
    memset(visit, 0, sizeof(visit));
    cycle = false;
    n = N - 1;

    for (int s=0; s<N; ++s)
        if (!visit[s])
            DFS(s);

    if (cycle) cout << "The graph has the cycle!";
    else{
        for (int i=0; i<N; ++i)
            cout << order[i];
    }
}

```

7 Data Structure

7.1 2D Range Tree

```

// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x, a.y) < tie(b.x, b.y);
    }
};

struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];

struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];

vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return;
    }
    int mid = (l+r)>>1;
    build(l, mid, o+o, deep+1);
    build(mid+1, r, o+o+1, deep+1);

    TREE *ptr = &tree[deep][l];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep+1][r+1];

```

```

int cnt = 0;
while(pl != n1 && pr != nr) {
    *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++):
        *(pr++);
    ptr -> toleft = cnt; ptr++;
}
while(pl != n1) *(ptr) = *(pl++), ptr -> toleft =
    ++cnt, ptr++;
while(pr != nr) *(ptr) = *(pr++), ptr -> toleft =
    cnt, ptr++;
}
int main(){
    int n; cin >> n;
    for(int i = 0 ; i < n; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}

```

7.2 Segment Tree

```

struct Node{
    int mx; // 區間最大值
    int tag; // 子樹裡所有人的'值'都要加上 tag
};

vector<Node> seg;

// 節點 id 的整個區間要加上 tag
void addtag(int tag, int id){
    seg[id].mx += tag; // 最大值會加上 tag
    seg[id].tag += tag; // 注意可能本來就有標記了，所以
        是 +=
}

// 更新子節點資訊並把標記移到子節點身上
void push(int id){
    addtag(seg[id].tag, lc);
    addtag(seg[id].tag, rc);
    seg[id].tag = 0; // 標記被移到子節點上所以要改成 0
}

// 區間 [l,r] 加上 v
void modify(int l, int r, int v, int L, int R, int id){
    if(l <= L && R <= r){
        addtag(v, id);
        return;
    }
    push(id);
    if(r <= M) modify(l, r, v, L, M, lc);
    else if(l > M) modify(l, r, v, M + 1, R, rc);
    else{
        modify(l, r, v, L, M, lc);
        modify(l, r, v, M + 1, R, rc);
    }
    seg[id].mx = max(seg[lc].mx, seg[rc].mx);
}

int query(int l, int r, int L, int R, int id){
    if(l <= L && R <= r) return seg[id].mx;
    push(id);
    int M = (L + R) / 2;
    if(r <= M) return query(l, r, L, M, lc);
    else if(l > M) return query(l, r, M + 1, R, rc);
    else return max(query(l, r, L, M, lc),
        query(l, r, M + 1, R, rc));
}

```

7.3 ZKW 線段樹

```

const int M=1e5+111;
int n,m,q;
int sum[M<<2],mn[M<<2],mx[M<<2],add[M<<2];

```

```

int read() {
    int x;
    cin >> x;
    return x;
}

void build(){
    for(m=1;m<=n;m<=1);
    for(int i=m+1;i<=m+n;++i)
        sum[i]=mn[i]=mx[i]=read();
    for(int i=m-1;i-->0){
        sum[i]=sum[i<<1]+sum[i<<1|1];
        mn[i]=min(mn[i<<1],mn[i<<1|1]);
        mx[i]=max(mx[i<<1],mx[i<<1|1]);
        mx[i<<1]-=mx[i],mx[i<<1|1]-=mx[i];
    }
}

void update_node(int x,int v,int A=0){
    x+=m,mx[x]+=v,mn[x]+=v,sum[x]+=v;
    for(;x>1;x>>=1){
        sum[x]+=v;
        A=min(mn[x],mn[x^1]);
        mn[x]-=A,mn[x^1]-=A,mn[x>>1]+=A;
        A=max(mx[x],mx[x^1]);
        mx[x]-=A,mx[x^1]-=A,mx[x>>1]+=A;
    }
}

void update_part(int s,int t,int v){
    int A=0,lc=0,rc=0,len=1;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<<=1){
        if(s&1^1) add[s^1]+=v,lc+=len, mn[s^1]+=v,mx[s^1]+=v;
        if(t&1) add[t^1]+=v,rc+=len, mn[t^1]+=v,mx[t^1]+=v;
        sum[s>>1]+=v*lc, sum[t>>1]+=v*rc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s>>1]+=A;
        A=min(mn[t],mn[t^1]),mn[t]-=A,mn[t^1]-=A,mn[t>>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s>>1]+=A;
        A=max(mx[t],mx[t^1]),mx[t]-=A,mx[t^1]-=A,mx[t>>1]+=A;
    }
    for(lc+=rc;s>>=1;s>>=1){
        sum[s>>1]+=v*lc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s>>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s>>1]+=A;
    }
}

int query_node(int x,int ans=0){
    for(x+=m;x>>=1) ans+=mn[x]; return ans;
}

int query_sum(int s,int t){
    int lc=0,rc=0,len=1,ans=0;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<<=1){
        if(s&1^1) ans+=sum[s^1]+len*add[s^1],lc+=len;
        if(t&1) ans+=sum[t^1]+len*add[t^1],rc+=len;
        if(add[s>>1]) ans+=add[s>>1]*lc;
        if(add[t>>1]) ans+=add[t>>1]*rc;
    }
    for(lc+=rc,s>>=1;s>>=1) if(add[s]) ans+=add[s]*lc;
    return ans;
}

int query_min(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mn[s],R+=mn[t];
        if(s&1^1) L=min(L,mn[s^1]);
        if(t&1) R=min(R,mn[t^1]);
    }
    for(ans=min(L,R),s>>=1;s>>=1) ans+=mn[s];
    return ans;
}

int query_max(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mx[s],R+=mx[t];
    }
}

```

```

        if(s&1^1) L=max(L,mx[s^1]);
        if(t&1) R=max(R,mx[t^1]);
    }
    for(ans=max(L,R),s>>=1;s>>=1) ans+=mx[s];
    return ans;
}

```

7.4 Sparse Table

```

const int MAXN = 200005;
const int lgN = 20;
/* Sp[i][j] 為 區間 [i, i + 2^j - 1] 的值 */
/* 從 i 開始 長度為 2 ^ j */
/* 解決可重複貢獻問題 */
struct SP{ //sparse table
    int Sp[MAXN][lgN];
    function<int(int,int)> opt;
    void build(vector<int> &nums){ // 0 base
        for (int i = 0; i < nums.size(); i++) Sp[i][0]=nums[i];

        for (int h = 1; h < lgN; h++) {
            int len = 1 << (h - 1), i=0;
            for (; i + len < nums.size(); i++)
                Sp[i][h] = opt(Sp[i][h-1], Sp[i+len][h-1]);
            for (; i < nums.size(); i++)
                Sp[i][h] = Sp[i][h-1];
        }
    }
    int query(int l, int r){
        int h = __lg(r-l+1);
        int len = 1<<h;
        return opt(Sp[l][h], Sp[r-len+1][h] );
    }
};

```

7.5 Lazy Tag

```

void modify(type value, int l, int r, int L, int R,
vertex v){
    if(l == L && r == R){
        //打懶標在v上;
        return;
    }
    int M = (L + R) / 2;
    if(r <= M) modify(value, l, r, L, M, //v的左子節點)
    ;
    else if(l > M) modify(value, l, r, M + 1, R, //v的
        右子節點);
    else{
        modify(value, l, M, L, M, v的左子節點);
        modify(value, M + 1, r, M + 1, R, //v的右子節點
            );
    }
    //用兩個子節點的答案更新v的答案;
}

```

7.6 BIT 樹狀樹組

```

class Bitree {
public:
    /* bit 一定是 1 indexed */
    vector<int> data;
    Bitree(const vector<int> &nums) {
        data.resize(nums.size() + 1, 0);
        for(int i = 0; i < nums.size(); i++) {
            update(i, nums[i]);
        }
    }
    void update(int x, int val) {
        x++; /*變成 1 indexed*/
        for(; x < data.size(); x += lowbit(x)) {
            data[x] += val;
        }
    }
}

```

```

}
int query(int x) {
    x++; /*變成 1 indexed*/
    int result = 0;
    for(; x > 0; x -= lowbit(x)) {
        result += data[x];
    }
    return result;
}
static int lowbit(int x) {
    return x & (-x);
}
};

```

7.7 並查集 union find

```

struct DisjointSet {
    vector<int> parent, sz; // parent[i] = 父節點, sz[
        i] = 集合大小
    void init(int n) {
        parent.resize(n + 1);
        sz.assign(n + 1, 1);
        for (int i = 0; i <= n; i++) {
            parent[i] = i;
        }
    }
    int find(int x) {
        if (parent[x] != x) {
            parent[x] = find(parent[x]); // 路徑壓縮
        }
        return parent[x];
    }
    bool unite(int x, int y) {
        x = find(x);
        y = find(y);
        if (x == y) return false; // 已在同一集合
        // 啟發式合併: 小的掛到大的下面
        if (sz[x] < sz[y]) swap(x, y);
        parent[y] = x;
        sz[x] += sz[y];
        return true;
    }
    bool same(int x, int y) {
        return find(x) == find(y);
    }
};

```

8 String

8.1 KMP

```

template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
    f[0]=-1, f[1]=0;
    for (int i=2; i<=n; i++){
        int w = f[i-1];
        while (w>=0 && s[w+1]!=s[i])w = f[w];
        f[i]=w+1;
    }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){
    build_KMP(m,b,f);
    int ans=0;

    for (int i=1, w=0; i<=n; i++){
        while ( w>=0 && b[w+1]!=a[i] )w = f[w];
        w++;
        if (w==m){
            ans++;
            w=f[w];
        }
    }
    return ans;
}

```

}

8.2 smallest rotation

```
string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}
/*
Booth 演算法
用於尋找一個字串的字典序最小的循環旋轉
*/
Contact GitHub API Training Shop Blog About
```

8.3 Suffix Array

```
/*he[i]保存了在后綴數組中相鄰兩個後綴的最長公共前綴長度
*sa[i]表示的是字典序排名為i的後綴是誰（字典序越小的排名越靠前）
*rk[i]表示的是後綴我所對應的排名是多少 */

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
    int len = strlen(ip);
    int alp = 256;
    memset(ct, 0, sizeof(ct));
    for(int i=0;i<len;i++) ct[ip[i]+1]++;
    for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
    for(int i=0;i<len;i++) rk[i]=ct[ip[i]];
    for(int i=1;i<len;i*=2){
        for(int j=0;j<len;j++){
            if(j+i>len) tp[j][1]=0;
            else tp[j][1]=rk[j+i]+1;
            tp[j][0]=rk[j];
        }
        memset(ct, 0, sizeof(ct));
        for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
        for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
        for(int j=0;j<len;j++) tsa[ct[tp[j][1]]+=j];
        memset(ct, 0, sizeof(ct));
        for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
        for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
        for(int j=0;j<len;j++){
            sa[ct[tp[j][1]][0]]+=j;
            rk[sa[0]]=0;
            for(int j=1;j<len;j++){
                if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
                   tp[sa[j]][1] == tp[sa[j-1]][1] )
                    rk[sa[j]] = rk[sa[j-1]];
                else
                    rk[sa[j]] = j;
            }
        }
        for(int i=0,h=0;i<len;i++){
            if(rk[i]==0) h=0;
            else{
                int j=sa[rk[i]-1];
                h=max(0,h-1);
                for(;ip[i+h]==ip[j+h];h++);
            }
            he[rk[i]]=h;
        }
    }
}
```

8.4 Z-value

```
z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
    if ( z[bst] + bst <= i ) z[i] = 0;
    else z[i] = min(z[i - bst], z[bst] + bst - i);
    while ( str[i + z[i]] == str[z[i]] ) z[i]++;
    if ( i + z[i] > bst + z[bst] ) bst = i;
}

// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
    // centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
                s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}
```

8.5 旋轉哈希

```
typedef unsigned __int128 ull1;

ull1 power(ull1 a, ull1 n, ull1 m) {
    ull1 re = 1;
    while (n > 0) {
        if (n & 1) re = re * a % m;
        a = a * a % m;
        n >>= 1;
    }
    return re;
}

ull1 inv(ull1 a, ull1 m) {
    return power(a, m - 2, m);
}

struct Rh {
    const ull1 p, mod;
    vector<ull1> ps{1};
    Rh(ull1 p, ull1 mod) : p(p), mod(mod) {}
    vector<ull1> build(const string &s) {
        vector<ull1> h(s.size() + 1);
        h[0] = 0;
        ps.resize(s.size() + 1);
        for (int i = 0; i < s.size(); i++) {
            ps[i + 1] = ps[i] * p % mod;
            h[i + 1] = (h[i] + s[i] * ps[i + 1] % mod) % mod;
        }
        return h;
    }
    ull1 subhash(const vector<ull1> &h, int l, int r) {
        // [l, r] 指原字串
        return ((h[r + 1] - h[l]) * inv(ps[l], mod)) % mod;
    }
};

constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >> 29) + (m << 35 >> 3) + 1;
    ret = (ret & mod) + (ret>>61);
    ret = (ret & mod) + (ret>>61);
    return ret-1;
}
```

8.6 後綴自動機

```

struct state {
    int len{}, link{};
    array<int, 26> next{};
};

struct SAM {
    int sz{}, last{};
    vector<state> st;
    SAM(int maxlen) : st(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
    }

    void insert(char c) {
        insert_impl(c - 'a');
    }

    void insert_impl(char c) {
        int cur = sz++;
        st[cur].len = st[last].len + 1;
        int p = last;
        while(p != -1 && !st[p].next[c]) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if(p == -1) {
            st[cur].link = 0;
        }
        else {
            int q = st[p].next[c];
            if(st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            }
            else {
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                st[clone].next = st[q].next;
                st[clone].link = st[q].link;
                while(p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
    }
};

```

9 Others

9.1 矩陣樹定理

新的方法介紹

下面我們介紹一個新的方法—Matrix-Tree定理(Kirchhoff矩陣-樹定理)。

Matrix-Tree定理是解決生成樹數問題最有力的武器之一。它首先於1847年被Kirchhoff證明。在介紹定理之前，我們先先明確幾個概念：

1. G 的度數矩陣 $D[G]$ 是一個 $n \times n$ 的矩陣，並且滿足：當 $i \neq j$ 時， $d_{ij} = 0$ ；當 $i = j$ 時， d_{ij} 等於 v_i 的度數。

2. G 的鄰接矩陣 $A[G]$ 也是一個 $n \times n$ 的矩陣，且滿足：若 v_i 、 v_j 之間有邊直接相連，則 $a_{ij} = 1$ ，否則為0。

我們定義 G 的Kirchhoff矩陣(也稱為拉普拉斯算子) $C[G]$ 為 $C[G] = D[G] - A[G]$ ，

則Matrix-Tree定理可以描述為： G 的所有不同的生成樹的個數等於其Kirchhoff矩陣 $C[G]$ 任何一個 $n-1$ 階主子式的行列式的絕對值。

所謂 $n-1$ 階主子式，就是對於 $r(1 \leq r \leq n)$ ，將 $C[G]$ 的第 r 行、第 r 列同時去掉後所得到的新矩陣，以 $C_r[G]$ 表示。

生成樹計數

演算法步驟：

1、建構拉普拉斯矩陣

$Matrix[i][j] = degree(i)$ ， $i=j$

-1， $i-j$ 有邊

0，其他情況

2、去掉第 r 行，第 r 列 (r 任意)

3、計算矩陣的行列式

```

#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
{
    if(fabs(x) < eps) return 0;
    if(x < 0) return -1;
    else return 1;
}

double b[MAXN][MAXN];
double det(double a[][MAXN], int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0; i < n; i++)
        for(j = 0; j < n; j++) b[i][j] = a[i][j];
    for(i = 0; i < n; i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n; j++)
                if(sgn(b[j][i]) != 0) break;
            if(j == n) return 0;
            for(k = i; k < n; k++) swap(b[i][k], b[j][k]);
            sign++;
        }
        ret *= b[i][i];
        for(k = i + 1; k < n; k++) b[i][k] /= b[i][i];
        for(j = i + 1; j < n; j++)
            for(k = i + 1; k < n; k++) b[j][k] -= b[j][i] * b[i][k];
    }
    if(sign & 1) ret = -ret;
    return ret;
}

double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
    int T;
    int n, m;
    int u, v;
    scanf("%d", &T);
    while(T--)
    {
        scanf("%d%d", &n, &m);
        memset(g, 0, sizeof(g));
        while(m--)
        {
            scanf("%d%d", &u, &v);
            u--; v--;
            g[u][v] = g[v][u] = 1;
        }
        memset(a, 0, sizeof(a));
        for(int i = 0; i < n; i++)
            for(int j = 0; j < n; j++)
                if(i != j && g[i][j])
                {
                    a[i][i]++;
                    a[i][j] = -1;
                }
        double ans = det(a, n-1);
        printf("%.0lf\n", ans);
    }
    return 0;
}

```


|}

9.2 1D/1D dp 優化

```
#include<bits/stdc++.h>

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}

long double f(int i, int j) {
    // cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}

struct INV {
    int L, R, pos;
};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L, i) < f(stk[top].L, stk[top].pos) ) {
        stk[top-1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos = stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top+1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
            cin >> s[i];
            sum[i] = sum[i-1] + strlen(s[i]);
            dp[i] = numeric_limits<long double>::max();
        }
        stk[top] = (INV) {1, n + 1, 0};
        for ( int i = 1 ; i <= n ; i++ ) {
            if ( i >= stk[bot].R ) bot++;
            dp[i] = f(i, stk[bot].pos);
            update(i);
        }
        // cout << (ll) f(i, stk[bot].pos) << endl;
        if ( dp[n] > 1e18 ) {
            cout << "Too hard to arrange" << endl;
        } else {
            vector<PI> as;
            cout << (ll)dp[n] << endl;
        }
    }
    return 0;
}
```

9.3 Theorm - DP optimization

Monotonicity & 1D/1D DP & 2D/1D DP

Definition xD/yD

1D/1D DP[j] = min(0≤i<j) { DP[i] + w(i, j) }; DP[0] = k
 2D/1D DP[i][j] = min(i<k≤j) { DP[i][k-1] + DP[k][j] }
 + w(i, j); DP[i][i] = 0

Monotonicity

	c	d
a	w(a, c)	w(a, d)
b	w(b, c)	w(b, d)

Monge Condition

Concave (凹四邊形不等式): $w(a, c) + w(b, d) \geq w(a, d) + w(b, c)$ Convex (凸四邊形不等式): $w(a, c) + w(b, d) \leq w(a, d) + w(b, c)$

Totally Monotone

Concave (凹單調): $w(a, c) \leq w(b, d) \rightarrow w(a, d) \leq w(b, c)$ Convex (凸單調): $w(a, c) \geq w(b, d) \rightarrow w(a, d) \geq w(b, c)$ 1D/1D DP $O(n^2) \rightarrow O(n \lg n)$

CONSIDER THE TRANSITION POINT

Solve 1D/1D Concave by Stack

Solve 1D/1D Convex by Deque

2D/1D Convex DP (Totally Monotone) $O(n^3) \rightarrow O(n^2)$ $h(i, j-1) \leq h(i, j) \leq h(i+1, j)$

9.4 Stable Marriage

// normal stable marriage problem

// input:

//3

//Albert Laura Nancy Marcy

//Brad Marcy Nancy Laura

//Chuck Laura Marcy Nancy

//Laura Chuck Albert Brad

//Marcy Albert Chuck Brad

//Nancy Brad Albert Chuck

#include<bits/stdc++.h>

using namespace std;

const int MAXN = 505;

int n;

int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id

int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank

int current[MAXN]; // current[boy_id] = rank; boy_id will pursue current[boy_id] girl.

int girl_current[MAXN]; // girl[girl_id] = boy_id;

void initialize() {

for (int i = 0 ; i < n ; i++) {

current[i] = 0;

girl_current[i] = n;

order[i][n] = n;

}

}

map<string, int> male, female;

string bname[MAXN], gname[MAXN];

int fit = 0;

void stable_marriage() {

queue<int> que;

for (int i = 0 ; i < n ; i++) que.push(i);

while (!que.empty()) {

int boy_id = que.front();

que.pop();

int girl_id = favor[boy_id][current[boy_id]];

```

    current[boy_id] ++;

    if ( order[girl_id][boy_id] < order[girl_id][
        girl_current[girl_id]] ) {
        if ( girl_current[girl_id] < n ) que.push(
            girl_current[girl_id]); // if not the first
            time
        girl_current[girl_id] = boy_id;
    } else {
        que.push(boy_id);
    }
}

int main() {
    cin >> n;

    for ( int i = 0 ; i < n ; i++ ) {
        string p, t;
        cin >> p;
        male[p] = i;
        bname[i] = p;
        for ( int j = 0 ; j < n ; j++ ) {
            cin >> t;
            if ( !female.count(t) ) {
                gname[fit] = t;
                female[t] = fit++;
            }
            favor[i][j] = female[t];
        }
    }

    for ( int i = 0 ; i < n ; i++ ) {
        string p, t;
        cin >> p;
        for ( int j = 0 ; j < n ; j++ ) {
            cin >> t;
            order[female[p]][male[t]] = j;
        }
    }

    initialize();
    stable_marriage();

    for ( int i = 0 ; i < n ; i++ ) {
        cout << bname[i] << " " << gname[favor[i][current[i]
            - 1]] << endl;
    }
}

```

9.5 莫隊

```

/* nums 長度 N ; query 長度為 M */
/* O(N * sqrt(M)) */

struct Query {
    int l, r, id;
};

void add(int pos) {
    /*更新狀態*/
    /*將pos所在的移入集合*/
}

void del(int pos) {
    /*更新狀態*/
    /*將pos所在的移出集合*/
}

int bsz = n / sqrt(m); /*分塊大小 block size*/
sort(query.begin(), query.end(), [bsz](const Query &a,
    const Query &b){
    if(a.l / bsz != b.l / bsz) {
        return a.l < b.l;
    }
    return (a.l / bsz) & 1 ? a.r < b.r : a.r > b.r;
});

```

```

int l = 1;
int r = 0;

vector<pair<int, int>> res(m);

for(int i = 0; i < query.size(); i++ ) {
    auto &q = query[i];
    /*順序不能換*/
    while (l > q.l) add(--l);
    while (r < q.r) add(++r);
    while (l < q.l) del(l++);
    while (r > q.r) del(r--);
    res[q.id] = /* 根據當前狀態求解 */
}

```

9.6 矩陣乘法

```

#define MOD INT_MAX
vector<vector<int>> operator *(const vector<vector<int>
    >> &a, const vector<vector<int>> &b) {
    vector<vector<int>> re(a.size(), vector<int>(b[0].
        size()));
    for (int i = 0; i < a.size(); i++) {
        for (int j = 0; j < b[0].size(); j++) {
            for (int k = 0; k < b.size(); k++) {
                re[i][j] += (a[i][k] * b[k][j]) % MOD;
            }
        }
    }
    return re;
}

```

9.7 c++ 小抄

```

//pbds tree
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> tr;

tr.find_by_order(k) // O(LogN) 取得第k大的元素
tr.order_of_key(ele) // O(LogN) 得到ele是tree中第幾大(
    有幾個元素小於ele)

//pbds pair priority_queue
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;

priority_queue<int, less<int>, pairing_heap_tag> pq;
auto it = pq.push(x);
// type of it = priority_queue<int, less<int>,
    pairing_heap_tag>::point_iterator
pq.pop();
pq.top();
pq.join(b);
pq.empty();
pq.size();
pq.modify(it,6); // O(LogN)
pq.erase(it);

//builtin functions
__builtin_popcount(x); // 1的個數
__builtin_popcountll(x); // for Long Long
__builtin_clz(x); // 前導0的個數
__builtin_ctz(x); // 後導0的個數
__builtin_parity(x); // 奇偶性

//溢位檢查
ret = __builtin_add_overflow(a, b, &res) // if ret = 1
    a+b 溢位
ret = __builtin_sub_overflow(a, b, &res) // if ret = 1
    a-b 溢位

```

```
ret = __builtin_mul_overflow(a, b, &res) // if ret = 1
    a*b 溢位
ret = __builtin_add_overflow_p(a, b, 0LL) // if ret = 1
    溢位 第三個參數是判斷的類型

//vector SIMD
typedef int v4si __attribute__((vector_size(4 * sizeof
(int))));

//大質數表
{1000000007, 1000000009, 1000000021, 1000000033,
 1000000087, 1000000093, 1000000097, 1000000123,
 1000000321};

//mt19937
#include <random>
#include <chrono>

int getRandom(int l, int r) {
    static auto seed = std::chrono::system_clock::now()
        .time_since_epoch().count();
    static std::mt19937 gen(seed);
    std::uniform_int_distribution<int> dis(l, r);
    return dis(gen);
}

//sorted vector 去重
vec.erase(unique(vec.begin(), vec.end()), vec.end());

//std::valarray
valarray<int> a(初始值, 數量); //就是那麼機八
valarray<int> a(10);
valarray<int> b(10);
valarray<int> c = a + b;
valarray<int> d = a * b;
valarray<int> e = a + 10;
valarray<int> f = a * 10;
valarray<int> g = a.cshift(1); //循環左移
valarray<bool> equal = a == b;
int sum = a.sum();
int max = a.max();
int min = a.min();
std::valarray<int> g = a.apply([](int x) { return x * x
; });

//regex ***very slow***
#include <regex>
using namespace std;
bool res = regex_match("abc", regex("a.c"));
bool res = regex_match("abc", regex("A.c", regex::icase
)); //忽略大小寫

//gp_hash_table
#include <ext/pb_ds/assoc_container.hpp>
__gnu_pbds::gp_hash_table<int, int/*, hashFunction */>
table;
```

9.8 python 小抄

```
#!/usr/bin/env python3

# 帕斯卡三角形
n = 10
dp = [ [1 for j in range(n)] for i in range(n) ]
for i in range(1,n):
    for j in range(1,n):
        dp[i][j] = dp[i][j-1] + dp[i-1][j]

for i in range(n):
    print( ''.join( '{:5d}'.format(x) for x in dp[i] )
)

# EOF1
while True:
    try:
        n, m = map(int, input().split())
    except:
        break
```

```
# EOF2
import sys
for s in sys.stdin:
    print(eval(s.replace("/", "///")))

# input a sequence of number
a = [ int(x) for x in input().split() ]
a.sort()
print( ''.join( str(x)+' ' for x in a ) )

# LCS
ncase = int( input() )
for _ in range(ncase):
    n, m = [int(x) for x in input().split()]
    a, b = "$"+input(), "$"+input()
    dp = [ [int(0) for j in range(m+1)] for i in range(
        n+1) ]
    for i in range(1,n+1):
        for j in range(1,m+1):
            dp[i][j] = max(dp[i-1][j], dp[i][j-1])
            if a[i]==b[j]:
                dp[i][j] = max(dp[i][j], dp[i-1][j-1]+1)

    for i in range(1,n+1):
        print(dp[i][1:])
    print( 'a={:s}, b={:s}, /LCS(a,b)/={:d}'.format(a
        [1:], b[1:], dp[n][m]) )

# list, dict, string
a = [1, 3, 4, 65, 65]
b = list.copy() # b = [1, 3, 4, 65], list a 跟 list b
互相獨立
cnt = list.count(65) # cnt == 2
loc = list.index(65) # loc == 3, find the leftmost
element, if not found then return ERROR
list.sort(reverse = True|False, key = None|lambda x:x
[1]) # list.sort has side effect but no return
value

# stack # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop() # pop()
stack[-1] # top()
len(stack) # size() 0(1)

# queue # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0] # front()
len(queue) # size() 0(1)
```

9.9 萬年曆

$$h = \left(q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor + 5J \right) \bmod 7$$

h : 星期 (0 = 星期六, 1 = 星期日, 2 = 星期一, ...)
q : 日期 (日)
m : 月份 (3= 三月, 4= 四月, ...; 1、2 月視為前一年的 13、14 月)
K : 年份的後兩位數 (year mod 100)
J : 年份的前兩位數 (year ÷ 100)