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// -fsanitize=address 檢測記憶體違規存取

```

**All file will be compiled to a.out unless you use -o(
    not recommended, just use a.out)**
# Run
./a.out

# Run with file input
./a.out < input.txt

# Run with file input and output
./a.out < input.txt > output.txt

# Python Run
python3 a.py < input.txt > output.txt

# Copy Paste In Ubuntu
* copy: ctrl+insert
* paste: shift+insert

# 比對文件相同
sdiff a.txt b.txt

```

1.2 default code

```

#include <bits/stdc++.h>
using namespace std;
#define int long long
typedef pair<int,int> pii;

#ifdef ONLINE_JUDGE
#define cerr if(false) cerr
#endif

int32_t main(){
#ifdef ONLINE_JUDGE
    freopen("input.txt", "r", stdin);
    freopen("output.txt", "w", stdout);
    freopen("debug.txt", "w", stderr);
#else
    ios_base::sync_with_stdio(0);
    cin.tie(false);
#endif
}

```

1.3 debug list

記得測試 python 的內建函數庫有哪些
bits/stdc++.h 跟 global variable y1 衝突，不能用
模板要記得 init
priority_queue 要清空
事先將把邊界測資加入測試
邊界條件（過程溢位，題目數據範圍），會不會爆 long long
是否讀錯題目，想不到時可以自己讀一次題目
比較容易有問題的地方換人寫
注意公式有沒有推錯或抄錯
精度誤差 sqrt(大大的東西) + EPS
喇分 random_shuffle 隨機演算法

1 Basic

1.1 compile

```

# preset before coding
echo "cd ~/Desktop" >> ~/.bashrc
gedit -> preference -> tab width: 4

# Editor
gedit a.cpp

# Compile
g++ a.cpp -std=c++14 -Wall -fsanitize=address

```

1.4 時間複雜度

時間複雜度	可處理的最大 N 數量級 (約)
$O(1)$	幾乎沒限制
$O(\log N)$	10^{18} 級別 (如快速幂)
$O(\sqrt{N})$	10^{10}
$O(N)$	10^8
$O(N \log N)$	$2 \times 10^7 \sim 5 \times 10^7$
$O(N\sqrt{N})$	$1 \times 10^5 \sim 2 \times 10^5$
$O(N^2)$	$10^4 \sim 1.5 \times 10^4$
$O(N^2 \log N)$	約 3×10^3
$O(N^3)$	$500 \sim 1000$
$O(2^N)$	$N \leq 20$
$O(N!)$	$N \leq 10$

2 Dark Code

2.1 IO optimization

```
*if output to much, consider put all output in array
  first, then output the array.
getchar() -> getchar_unlocked()
fread() -> fread_unlocked()
-----
inline char readchar() {
    const int S = 1<<20; // buffer size
    static char buf[S], *p = buf, *q = buf;
    if(p == q && (q = (p=buf)+fread(buf,1,S,stdin)) ==
        buf) return EOF;
    return *p++;
}

inline int nxtint() {
    // if readchar can't use, change readchar() to
    // getchar()
    int x = 0;
    int c = readchar(), neg = false;
    if (c == EOF) return -1;
    while (('0' > c || c > '9') && c != '-' && c != EOF)
        c = readchar();
    if (c == '-') neg = true, c = readchar();
    while ('0' <= c && c <= '9') x = x * 10 + (c ^ '0'),
        c = readchar();
    if (neg) x = -x;
    return x;
}
```

3 Geometry

3.1 2D point

```
typedef double Double;
struct Point {
    Double x,y;

    bool operator < (const Point &b)const{
        //return tie(x,y) < tie(b.x,b.y);
        return atan2(y,x) < atan2(b.y,b.x);
    }
    Point operator + (const Point &b)const{
        return (Point){x+b.x,y+b.y};
    }
    Point operator - (const Point &b)const{
        return (Point){x-b.x,y-b.y};
    }
    Point operator * (const Double &d)const{
        return Point(d*x,d*y);
    }
    Double operator * (const Point &b)const{
        return x*b.x + y*b.y;
    }
    Double operator % (const Point &b)const{
        return x*b.y - y*b.x;
    }
    friend Double abs2(const Point &p){
        return p.x*p.x + p.y*p.y;
    }
    friend Double abs(const Point &p){
        return sqrt( abs2(p) );
    }
};
typedef Point Vector;

struct Line{
    Point P; Vector v;
    bool operator < (const Line &b)const{
        return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
    }
};
```

3.2 兩線段交點

```
using type = long long;
const type EPS = 0 /*1e-9*/;
struct Point { type x, y; };

inline type cross(const Point &a, const Point &b, const
    Point &c) {
    return (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c
        .x - a.x);
}

inline bool overlap(type a, type b, type c, type d) {
    if(a > b) swap(a,b); if(c > d) swap(c,d);
    return max(a,c) <= min(b,d) + EPS;
}

bool equal_zero(type x) {
    return abs(x) <= EPS;
}
bool sgn(type x) {
    return (x > EPS) - (x < -EPS);
}

#define CROSS(i,j,k) cross(p[i],p[j],p[k])

#define CHECK_COLLINEAR(i,j,k) (equal_zero(CROSS(i,j,k)
    ) && overlap(p[i].x,p[j].x,p[k].x,p[k].x) &&
    overlap(p[i].y,p[j].y,p[k].y,p[k].y))

bool intersect(const vector<Point> &p){
    type d[4];
    for(int i=0;i<4;i++){
        if(i<2) d[i] = CROSS(0,1,i+2);
        else d[i] = CROSS(2,3,i-2);
    }
    for(int i=0;i<4;i++)
        /**/if(CHECK_COLLINEAR(i<2?0:2,i<2?1:3,i<2?i+2:i-2))
        /**/return true;
    return sgn(d[0]) != sgn(d[1]) && sgn(d[2]) != sgn(d
        [3]);
}

// 求交點 不處理共線重疊
pair<long double,long double> intersection(const vector
    <Point> &p){
    long double A1 = p[1].y - p[0].y, B1 = p[0].x - p
        [1].x, C1 = A1*p[0].x+B1*p[0].y;
    long double A2 = p[3].y - p[2].y, B2 = p[2].x - p
        [3].x, C2 = A2*p[2].x+B2*p[2].y;
    long double det = A1*B2 - A2*B1;
    return {(C1*B2-C2*B1)/det,(A1*C2-A2*C1)/det};
}
```

3.3 兩圓交點

```
vector<Point> interCircle(Point o1, type r1, Point o2,
    type r2) {
    type d2 = abs2(o1 - o2);
    type d = sqrt(d2);
    if (d < fabs(r1 - r2) || d > r1 + r2) return {};
    Point u = (o1 + o2) * 0.5 + ((r2*r2 - r1*r1) /
        (2.0*d2)) * (o1 - o2);
    type A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    Point v = Point{o1.y - o2.y, -(o1.x - o2.x)} * (A /
        (2.0*d2));
    return { u + v, u - v };
}
```

3.4 Convex Hull

```
#include "2Dpoint.cpp"

// return H, The first will occurred TWICE in vector H!
void ConvexHull(vector<Point> &P, vector<Point> &H){
    int n = P.size(), m=0;
```

```

sort(P.begin(), P.end());
H.clear();

for (int i=0; i<n; i++){
    while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
           <0) H.pop_back(), m--;
    H.push_back(P[i]), m++;
}

for (int i=n-2; i>=0; i--){
    while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
           <0) H.pop_back(), m--;
    H.push_back(P[i]), m++;
}
}

```

4 Flow

4.1 Dinic

(a) 有源匯上下界最大流 (Bounded Maxflow)

目標：在滿足所有邊的流量上下界限制的前提下，從源點 s 到匯點 t 的最大流量。

先依照 (b) 的方法建立可行流模型。

檢查是否存在可行流 (即 $\text{max_flow}(ss, tt)$ 是否等於所有流量下界 l 的總和)。如果不可行，則此問題無解。

重要：如果可行，不要重新初始化圖。直接在當前的殘留網絡上繼續計算 $\text{dinic.max_flow}(s, t)$ 。

最終的答案就是步驟 3 中計算出的從 s 到 t 的附加流量。

(b) 有上下界可行流 (Bounded Possible Flow)

目標：檢查是否存在一種流量分配，使得每條邊的流量 f 都滿足其下界 l 和上界 r 的限制 ($l \leq f \leq r$)。

新增兩個節點：超級源點 ss 和超級匯點 tt 。

準備一個變數 total_lower_bound 來累加所有下界 l 。

對於每一條原始邊 $u \rightarrow v$ ，其容量為 $[l, r]$ ：

$\text{dinic.add_edge}(u, v, r - l)$; (邊的彈性容量)

$\text{dinic.add_edge}(ss, v, l)$; (節點 v 需要 l 的流入)

$\text{dinic.add_edge}(u, tt, l)$; (節點 u 提供 l 的流出)

$\text{total_lower_bound} += l$;

計算 $\text{flow} = \text{dinic.max_flow}(ss, tt)$ 。

如果 $\text{flow} == \text{total_lower_bound}$ ，則表示所有下界需求都被滿足，存在可行流；否則不存在。

(c) 有源匯上下界最小流 (Bounded Minimum Flow)

目標：在滿足所有邊的流量上下界限制的前提下，從源點 s 到匯點 t 的最小流量。

注意：這個問題通常需要透過二分搜尋答案來解決，無法直接用一次最大流求出。

二分搜尋一個流量值 F 。

對於每個猜測的 F ，建立一個無源匯可行流模型來檢查其可行性：

使用 (b) 的方法建構基本圖。

額外加入一條邊 $t \rightarrow s$ ，容量為 $[F, \text{INF}]$ 。這條邊強制要求從 s 到 t 的淨流量至少為 F 。

檢查這個新的循環圖是否存在可行流。如果存在，表示流量 F 是可達成的，可以嘗試更小的 F ；反之， F 太小了，需要增加。

(e) 最小割 (Minimum Cut)

目標：找出一個邊集，其總容量最小，且移除這些邊後 s 和 t 不再連通。

根據最大流-最小割定理，最小割的值等於最大流的值。先執行 $\text{ll min_cut_value} = \text{dinic.max_flow}(s, t)$ ；

呼叫 $\text{vector<bool> side} = \text{dinic.get_min_cut_nodes}(s)$ ；來取得節點的劃分。

$\text{side}[i] == \text{true}$ 表示節點 i 屬於源點 s 所在的集合 (S 集合)。

$\text{side}[i] == \text{false}$ 表示節點 i 屬於匯點 t 所在的集合 (T 集合)。

最小割的邊集就是所有從 S 集合指向 T 集合的原始邊

`using ll = longlong;`

```

const ll INF = 1e18;

struct Dinic {
    struct Edge {
        int to;
        ll cap;
        int rev; // 反向邊的索引
    };
    vector<vector<Edge>> adj;
    vector<int> level, iter;
    vector<bool> side;
    int n;
    Dinic(int v) : n(v), adj(v), level(v), iter(v) {}
    void add_edge(int u, int v, ll cap) {
        adj[u].push_back({v, cap, (int)adj[v].size()});
        adj[v].push_back({u, 0, (int)adj[u].size() - 1});
    }
    bool bfs(int s, int t) {
        fill(level.begin(), level.end(), -1);
        queue<int> q;
        level[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (const auto& edge : adj[u]) {
                if (edge.cap > 0 && level[edge.to] < 0) {
                    level[edge.to] = level[u] + 1;
                    q.push(edge.to);
                }
            }
        }
        return level[t] != -1;
    }
    ll dfs(int u, int t, ll f) {
        if (u == t) return f;
        for (int& i = iter[u]; i < (int)adj[u].size(); ++i) {
            Edge& e = adj[u][i];
            if (e.cap > 0 && level[u] < level[e.to]) {
                ll d = dfs(e.to, t, min(f, e.cap));
                if (d > 0) {
                    e.cap -= d;
                    adj[e.to][e.rev].cap += d;
                    return d;
                }
            }
        }
        return 0;
    }
    ll max_flow(int s, int t) {
        ll flow = 0;
        while (bfs(s, t)) {
            fill(iter.begin(), iter.end(), 0);
            ll f;
            while ((f = dfs(s, t, INF)) > 0) {
                flow += f;
            }
        }
        return flow;
    }
    void _find_cut(int u) {
        side[u] = true;
        for (const auto& e : adj[u]) {
            if (e.cap > 0 && !side[e.to]) {
                _find_cut(e.to);
            }
        }
    }
    vector<bool> get_min_cut_nodes(int s) {
        fill(side.begin(), side.end(), false);
        _find_cut(s);
        return side;
    }
};

```

4.2 min cost flow

```
// Long Long version
typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MAXN = 350;
    static const long long INF = 1LL<<60;
    struct Edge {
        int to, r;
        long long rest, c;
    };
    int n, pre[MAXN], preL[MAXN]; bool inq[MAXN];
    long long dis[MAXN], fl, cost;
    vector<Edge> G[MAXN];
    void init() {
        for (int i = 0; i < MAXN; i++) G[i].clear();
    }
    void add_edge(int u, int v, long long rest, long long c) {
        G[u].push_back({v, (int)G[v].size(), rest, c});
        G[v].push_back({u, (int)G[u].size()-1, 0, -c});
    }
    pll flow(int s, int t) {
        fl = cost = 0;
        while (true) {
            fill(dis, dis+MAXN, INF);
            fill(inq, inq+MAXN, 0);
            dis[s] = 0;
            queue<int> que;
            que.push(s);
            while (!que.empty()) {
                int u = que.front(); que.pop();
                inq[u] = 0;
                for (int i = 0; i < (int)G[u].size(); i++) {
                    int v = G[u][i].to;
                    long long w = G[u][i].c;
                    if (G[u][i].rest > 0 && dis[v] > dis[u] + w) {
                        pre[v] = u; preL[v] = i;
                        dis[v] = dis[u] + w;
                        if (!inq[v]) {
                            inq[v] = 1;
                            que.push(v);
                        }
                    }
                }
            }
            if (dis[t] == INF) break;
            long long tf = INF;
            for (int v = t, u, l; v != s; v = u) {
                u = pre[v]; l = preL[v];
                tf = min(tf, G[u][l].rest);
            }
            for (int v = t, u, l; v != s; v = u) {
                u = pre[v]; l = preL[v];
                G[u][l].rest -= tf;
                G[v][G[u][l].r].rest += tf;
            }
            cost += tf * dis[t];
            fl += tf;
        }
        return {fl, cost};
    }
} flow;
```

5 Mathematics

5.1 $ax+by=\gcd(a,b)$

```
typedef pair<int, int> pii;
pii exgcd(int a, int b) {
    if (b == 0) return make_pair(1, 0);
    else {
        int p = a / b;
        pii q = exgcd(b, a % b);
        int aa = q.second, bb = q.first - q.second * p;
```

```
if (aa < 0) aa += b, bb -= a;
return make_pair(aa, bb);
}
```

5.2 BigInt

```
struct BigInt {
    static const int LEN = 60;
    static const int BIGMOD = 10000;
    int s;
    int vl, v[LEN];
    // vector<int> v;
    BigInt() : s(1) { vl = 0; }
    BigInt(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while (a) {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    BigInt(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-') {
            stPos = 1;
            s = -1;
        }
        for (int i = SZ(str)-1, q=1; i >= stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD) {
                push_back(num);
                num = 0; q = 1;
            }
        }
        if (num) push_back(num);
    }
    int len() const { return vl; /* return SZ(v); */ }
    bool empty() const { return len() == 0; }
    void push_back(int x) { v[vl++] = x; /* v.PB(x); */ }
    void pop_back() { vl--; /* v.pop_back(); */ }
    int back() const { return v[vl-1]; /* return v.back(); */ }
    void n() { while (!empty() && !back()) pop_back(); }
    void resize(int nl) {
        vl = nl; fill(v, v+vl, 0);
        /* v.resize(nl); // fill(ALL(v), 0); */
    }
    void print() const {
        if (empty()) { putchar('0'); return; }
        if (s == -1) putchar('-');
        printf("%d", back());
        for (int i = len()-2; i >= 0; i--) printf("%.4d", v[i]);
    }
    friend ostream& operator << (ostream& out, const BigInt &a) {
        if (a.empty()) { out << "0"; return out; }
        if (a.s == -1) out << "-";
        out << a.back();
        for (int i = a.len()-2; i >= 0; i--) {
            char str[10];
            snprintf(str, 5, "%.4d", a.v[i]);
            out << str;
        }
        return out;
    }
    int cp3(const BigInt &b) const {
        if (s != b.s) return s > b.s ? 1 : -1;
        if (s == -1) return -(*this).cp3(-b);
        if (len() != b.len()) return len() > b.len() ? 1 : -1;
        for (int i = len()-1; i >= 0; i--)
            if (v[i] != b.v[i]) return v[i] > b.v[i] ? 1 : -1;
        return 0;
    }
    bool operator < (const BigInt &b) const { return cp3(b) == -1; }
    bool operator <= (const BigInt &b) const { return cp3(b) <= 0; }
}
```

```

bool operator >= (const Bigint &b) const {return cp3(b)
    >=0;}
bool operator == (const Bigint &b) const {return cp3(b)
    ==0;}
bool operator != (const Bigint &b) const {return cp3(b)
    !=0;}
bool operator > (const Bigint &b) const {return cp3(b)
    ==1;}
Bigint operator - () const {
    Bigint r = (*this);
    r.s = -r.s;
    return r;
}
Bigint operator + (const Bigint &b) const {
    if (s == -1) return -(-(*this)+(-b));
    if (b.s == -1) return (*this)-(-b);
    Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
    for (int i=0; i<nl; i++) {
        if (i < len()) r.v[i] += v[i];
        if (i < b.len()) r.v[i] += b.v[i];
        if (r.v[i] >= BIGMOD) {
            r.v[i+1] += r.v[i] / BIGMOD;
            r.v[i] %= BIGMOD;
        }
    }
    r.n();
    return r;
}
Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(-(*this)-(-b));
    if (b.s == -1) return (*this)+(-b);
    if ((*this) < b) return -(b-(*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
        r.v[i] += v[i];
        if (i < b.len()) r.v[i] -= b.v[i];
        if (r.v[i] < 0) {
            r.v[i] += BIGMOD;
            r.v[i+1]--;
        }
    }
    r.n();
    return r;
}
Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
        for (int j=0; j<b.len(); j++) {
            r.v[i+j] += v[i] * b.v[j];
            if (r.v[i+j] >= BIGMOD) {
                r.v[i+j+1] += r.v[i+j] / BIGMOD;
                r.v[i+j] %= BIGMOD;
            }
        }
    }
    r.n();
    return r;
}
Bigint operator / (const Bigint &b) {
    Bigint r;
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
        int d=0, u=BIGMOD-1;
        while(d<u) {
            int m = (d+u+1)>>1;
            r.v[i] = m;
            if((r*b2) > (*this)) u = m-1;
            else d = m;
        }
        r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();

```

```

    return r;
}
Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
}
};

```

5.3 GaussElimination

// by bcw_codebook

```

const int MAXN = 300;
const double EPS = 1e-8;

int n;
double A[MAXN][MAXN];

void Gauss() {
    for(int i = 0; i < n; i++) {
        bool ok = 0;
        for(int j = i; j < n; j++) {
            if(fabs(A[j][i]) > EPS) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if(!ok) continue;

        double fs = A[i][i];
        for(int j = i+1; j < n; j++) {
            double r = A[j][i] / fs;
            for(int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}

template<class T>
void Gauss(vector<vector<T>> &A) {
    int n = A.size();
    for(int i = 0; i < n; i++) {
        bool ok = 0;
        for(int j = i; j < n; j++) {
            if(A[j][i] != 0) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if(!ok) continue;

        T fs = A[i][i];
        for(int j = i+1; j < n; j++) {
            T r = A[j][i] / fs;
            for(int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}

```

5.4 Inverse

```

int inverse[100000];
void invTable(int b, int p) {
    inverse[1] = 1;
    for( int i = 2; i <= b; i++ ) {
        inverse[i] = (long long)inverse[p%i] * (p-p/i) % p;
    }
}

int inv(int b, int p) {
    return b == 1 ? 1 : ((long long)inv(p % b, p) * (p-p/
        b) % p);
}

```

5.5 LinearPrime 歐拉篩

```
const int MAXP = 100; //max prime
vector<int> P; // primes
void build_prime(){
    static bitset<MAXP> ok;
    int np=0;
    for (int i=2; i<MAXP; i++){
        if (ok[i]==0)P.push_back(i), np++;
        for (int j=0; j<np && i*P[j]<MAXP; j++){
            ok[ i*P[j] ] = 1;
            if ( i%P[j]==0 )break;
        }
    }
}
```

5.6 Miller Rabin

```
typedef long long LL;

inline LL bin_mul(LL a, LL n, const LL& MOD){
    LL re=0;
    while (n>0){
        if (n&1) re += a;
        a += a; if (a>=MOD) a-=MOD;
        n>>=1;
    }
    return re%MOD;
}

inline LL bin_pow(LL a, LL n, const LL& MOD){
    LL re=1;
    while (n>0){
        if (n&1) re = bin_mul(re,a,MOD);
        a = bin_mul(a,a,MOD);
        n>>=1;
    }
    return re;
}

bool is_prime(LL n){
    //static LL sprp[3] = { 2LL, 7LL, 61LL};
    static LL sprp[7] = { 2LL, 325LL, 9375LL,
        28178LL, 450775LL, 9780504LL,
        1795265022LL };
    if (n==1 || (n&1)==0 ) return n==2;
    int u=n-1, t=0;
    while ( (u&1)==0 ) u>>=1, t++;
    for (int i=0; i<3; i++){
        LL x = bin_pow( sprp[i]%n, u, n);
        if (x==0 || x==1 || x==n-1)continue;

        for (int j=1; j<t; j++){
            x=x*x%n;
            if (x==1 || x==n-1)break;
        }
        if (x==n-1)continue;
        return 0;
    }
    return 1;
}
```

5.7 Pollard's rho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
    return add(mul(x, x, mod), 1, mod);
}

Int pollard_rho(Int n) {
    if ( !(n & 1) ) return 2;
    while (true) {
        Int y = 2, x = rand()%(n-1) + 1, res = 1;
        for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
            for ( int i = 0 ; i < sz && res <= 1 ; i++ ) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
        }
    }
}
```

```
    }
    y = x;
}
if ( res != 0 && res != n ) return res;
}
```

5.8 NTT

```
constexpr int P = 998244353;
const int G = 3;
/*預處理 Lim*/
int lim = 1;
while (lim < (lenSum - 1)) lim <= 1;
/*每個多項式都要resize(lim)*/
/*998244353 3 1004535809 3 469762049 3 167772161 3
754974721 11*/
void init_rev(vector<int> &rev, int lim) {
    int lg = __builtin_ctz(lim); // Lim 是 2^k
    rev.resize(lim);
    for (int i = 0; i < lim; ++i)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (lg - 1));
}
// a.size() == Lim
void ntt(vector<int> &a, int opt) { // opt == -1 =>
    reverse ntt
    int n = a.size();
    static vector<int> rev;
    init_rev(rev, n);
    for (int i = 0; i < n; ++i)
        if (i < rev[i]) swap(a[i], a[rev[i]]);

    for (int m = 2; m <= n; m <= 1) {
        int k = m >> 1;
        int gn = qpow(G, (P - 1) / m);
        if (opt == -1) gn = qpow(gn, P - 2);
        for (int i = 0; i < n; i += m) {
            int g = 1;
            for (int j = 0; j < k; ++j) {
                int t = 1ll * a[i + j + k] * g % P;
                a[i + j + k] = (a[i + j] - t + P) % P;
                a[i + j] = (a[i + j] + t) % P;
                g = 1ll * g * gn % P;
            }
        }

        if (opt == -1) {
            int inv_n = qpow(n, P - 2);
            for (int &x : a) x = 1ll * x * inv_n % P;
        }
    }
}
```

5.9 數論基本工具

```
Int POW(Int a, Int n, Int mod){
    Int re=1;
    while (n>0){
        if (n&1LL) re = re*a%mod;
        a = a*a%mod;
        n>>=1;
    }
    return re;
}

Int C(Int n, Int m){
    if (m<0 || m>n)return 0;
    return J[n] * inv(J[m]*J[n-m]%MOD) %MOD;
}
```

5.10 Mobius

```
void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
}
```



```

mu[1] = 1, num = 0;
for (int i = 2; i < MAXN; ++i) {
    if (isPrime[i]) primes[num++] = i, mu[i] = -1;
    static int d;
    for (int j = 0; j < num && (d = i * primes[j])
        < MAXN; ++j) {
        isPrime[d] = false;
        if (i % primes[j] == 0) {
            mu[d] = 0; break;
        } else mu[d] = -mu[i];
    }
}
}
}

```

5.11 SG

Anti Nim (取走最後一個石子者敗)

先手必勝 **if and only if**

1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。

Anti-SG (決策集合為空的遊戲者贏)

定義 SG 值為 0 時，遊戲結束，

則先手必勝 **if and only if**

1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。

Sprague-Grundy

1. 雙人、回合制
2. 資訊完全公開
3. 無隨機因素
4. 可在有限步內結束
5. 沒有和局
6. 雙方可採取的行動相同

SG(S) 的值為 0：後手(P)必勝

不為 0：先手(N)必勝

```

int mex(set S) {
    // find the min number >= 0 that not in the S
    // e.g. S = {0, 1, 3, 4} mex(S) = 2
}

state = []
int SG(A) {
    if (A not in state) {
        S = sub_states(A)
        if( len(S) > 1 ) state[A] = reduce(operator.xor, [
            SG(B) for B in S])
        else state[A] = mex(set(SG(B) for B in next_states(
            A)))
    }
    return state[A]
}

```

5.12 Theorem

/*
 Lucas's Theorem
 For non-negative integer n, m and prime P ,
 $C(m, n) \bmod P = C(m/M, n/M) * C(m \% M, n \% M) \bmod P$
 $= \text{mult}_i (C(m_i, n_i))$
 where m_i is the i -th digit of m in base P .

Pick's Theorem
 $A = i + b/2 - 1$

Kirchhoff's theorem
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$

Deleting any one row, one column, and cal the det(A)

Nth Catalan recursive function:

$C_0 = 1, C_{n+1} = C_n * 2(2n + 1)/(n+2)$

Mobius Formula

$u(n) = 1$, if $n = 1$
 $(-1)^m$, 若 n 無平方數因數, 且 $n = p_1 * p_2 * p_3 * \dots * p_k$
 0 , 若 n 有大於 1 的平方數因數

- Property

1. (積性函數) $u(a)u(b) = u(ab)$
2. $\sum_{d|n} u(d) = [n == 1]$

Mobius Inversion Formula

if $f(n) = \sum_{d|n} g(d)$
 then $g(n) = \sum_{d|n} u(n/d)f(d)$
 $= \sum_{d|n} u(d)f(n/d)$

- Application

the number/power of $\gcd(i, j) = k$

- Trick

分塊, $O(\sqrt{n})$

Chinese Remainder Theorem (m_i 兩兩互質)

$x = a_1 \pmod{m_1}$
 $x = a_2 \pmod{m_2}$
 \dots
 $x = a_i \pmod{m_i}$

construct a solution:

Let $M = m_1 * m_2 * m_3 * \dots * m_n$
 Let $M_i = M / m_i$

$t_i = 1 / M_i$
 $t_i * M_i = 1 \pmod{m_i}$

solution $x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + \dots$
 $+ a_n * t_n * M_n + k * M$
 $= k * M + \sum a_i * t_i * M_i, k \text{ is positive integer.}$

under mod M , there is one solution $x = \sum a_i * t_i * M_i$

Burnside's Lemma

$|G| * |X/G| = \sum (|X^g|) \text{ where } g \text{ in } G$

總方法數：每一種旋轉下不動點的個數總和 除以 旋轉的方法數
 */

6 Graph

6.1 BCC

邊雙連通

任意兩點間至少有兩條不重疊的路徑連接，找法：

1. 標記出所有的橋
2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通

/* from BCW

```

struct BccEdge {
    static const int MXN = 100005;
    struct Edge { int v, eid; };
    int n, m, step, par[MXN], dfn[MXN], low[MXN];
    vector<Edge> E[MXN];
    DisjointSet djs;
    void init(int _n) {
        n = _n; m = 0;
        for (int i=0; i<n; i++) E[i].clear();
        djs.init(n);
    }
    void add_edge(int u, int v) {
        E[u].PB({v, m});
    }
}

```

```

    E[v].PB({u, m});
    m++;
}
void DFS(int u, int f, int f_eid) {
    par[u] = f;
    dfn[u] = low[u] = step++;
    for (auto it:E[u]) {
        if (it.eid == f_eid) continue;
        int v = it.v;
        if (dfn[v] == -1) {
            DFS(v, u, it.eid);
            low[u] = min(low[u], low[v]);
        } else {
            low[u] = min(low[u], dfn[v]);
        }
    }
}
void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
        if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
        if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
}
}graph;

```

6.2 Prim

```

// edge struct
struct edge{
    int a, b;
    double data;
    bool operator <(const edge b)const{
        return data > b.data;
    }
};

// main prim algorithm
int n, m, root, aa, bb, cc;
while (cin >> n >> m){
    priority_queue<edge> yee;
    int visit[500] = {}, p[500] = {};
    double a[500][500] = {};
    //undirectional edge aa to bb is weighted cc
    for (int i = 0; i < m; i++){
        cin >> aa >> bb >> cc;
        a[aa][bb] = a[bb][aa] = cc;
    }
    cin >> root;
    yee.push({ 0, root, 0 });
    edge tmp;
    double total = 0;
    while (!yee.empty()){
        tmp = yee.top(); yee.pop();
        if (visit[tmp.b]) continue;
        total += tmp.data; p[tmp.b] = tmp.a; visit[tmp.b] = 1;
        for (int i = 1; i <= n; i++){
            if (a[tmp.b][i] != 0 && (!visit[i])){
                yee.push({tmp.b, i, a[tmp.b][i]});
            }
        }
    }
    cout << total << endl;
}

```

6.3 Bellman Ford

```

int a[100][100], d[100], p[100];
void bellman_ford(int root, int n){
    for (int i = 1; i <= n; i++) d[i] = 1e9;
    d[root] = 0, p[root] = 0;
    for (int i = 0; i < n - 1; i++){

```

```

        for (int j = 1; j <= n; j++){
            for (int k = 1; k <= n; k++){
                if (d[j] != 1e9 && a[j][k] != 1e9){
                    if (d[j] + a[j][k] < d[k]){
                        d[k] = d[j] + a[j][k], p[k] = j;
                    }
                }
            }
        }
    }
}

bool nega_cyc(int n){
    for (int i = 1; i <= n; i++){
        for (int j = 1; j <= n; j++){
            if (d[i] != 1e9 && a[i][j] != 1e9)
                if (d[i] + a[i][j] < d[j]){
                    return 0;
                }
        }
    }
    return 1;
}

int main(){
    int n, m, aa, bb, dd;
    while (cin >> n >> m){
        for (int i = 0; i <= n; i++) for (int j = 0; j <= n; j++){
            a[i][j] = E9;
        }
        memset(p, 0, sizeof(p));
        for (int i = 0; i < m; i++){
            cin >> aa >> bb >> dd;
            a[aa][bb] = min(a[aa][bb], dd);
        }
        cin >> aa;
        bellman_ford(aa, n);
        int t = nega_cyc(n);
        if (t){
            for (int i = 1; i <= n; i++) cout << d[i] << " \n"
                [i==n];
            for (int i = 1; i <= n; i++) cout << p[i] << " \n"
                [i==n];
        }
        else cout << "There is a negative weight cycle in the graph\n";
    }
}

```

6.4 Kruskal

```

struct v {
    int a, b, c;
};

int p[200001]; v a[200001];

bool sor(v a, v b) {
    return a.c < b.c;
}

int find(int x) {
    return (x != p[x] ? (p[x] = find(p[x])) : x);
}

int main() {
    int n, m, i, j, sum;
    while (cin >> n >> m) {
        sum = 0;
        for (i = 0; i < 200001; i++) p[i] = i;
        for (i = 0; i < m; i++) cin >> a[i].a >> a[i].b >> a[i].c;
        sort(a, a + m, sor);
        for (i = 0, j = 0; j < m; j++) {
            if (find(a[j].a) != find(a[j].b)){
                i++;
                p[find(a[j].a)] = find(a[j].b);
                sum += a[j].c;
            }
        }
    }
}

```



```

    }
    cout << ((i==n-1)?sum:-1) << endl;
}
}

```

6.5 Dijkstra

```

struct node {
    int num{}, w{};
    bool operator < (const node& other) const {
        return w > other.w;
    }
};

vector<int> dijkstra(int root, const vector<vector<node>>& graph) {
    vector<int> d(graph.size(), INT_MAX >> 1), p(graph.size());
    priority_queue<node> pq;
    d[root] = p[root] = 0;
    pq.push({root, d[root]});
    while (!pq.empty()) {
        node tmp = pq.top(); pq.pop();
        for (const node &i : graph[tmp.num]) {
            if (d[i.num] > d[tmp.num] + i.w) {
                d[i.num] = d[tmp.num] + i.w;
                p[i.num] = tmp.num;
                pq.push({i.num, d[i.num]});
            }
        }
    }
    return d;
}

```

6.6 Strongly Connected Component(SCC)

```

#define MXN 100005
#define PB push_back
#define FZ(s) memset(s,0,sizeof(s))

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++){
            if (!vst[i]) DFS(i);
            reverse(vec.begin(),vec.end());
            FZ(vst);
            for (auto v : vec){
                if (!vst[v]){
                    rDFS(v);
                    nScc++;
                }
            }
        }
    }
}

```

```

    }
}
};

```

6.7 Hungarian

```

// Maximum Cardinality Bipartite Matching

struct Graph {
    static const int MAXN = 5005;
    vector<int> G[MAXN];
    int n;
    int match[MAXN]; // Matching Result
    int vis[MAXN];

    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) G[i].clear();
    }

    bool dfs(int u) {
        for (auto v:G[u]) {
            if (!vis[v]) {
                vis[v] = true;
                if (match[v] == -1 || dfs(match[v])) {
                    match[v] = u;
                    match[u] = v;
                    return true;
                }
            }
        }
        return false;
    }

    int solve() {
        int res = 0;
        memset(match, -1, sizeof(match));
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                memset(vis, 0, sizeof(vis));
                if (dfs(i)) res += 1;
            }
        }
        return res;
    }
} graph;

```

6.8 KM

Detect non-perfect-matching:
 1. set all edge[i][j] as INF
 2. if solve() >= INF, it is **not** perfectmatching.

 // Maximum Weight Perfect Bipartite Matching
 // allow negative weight!

```

typedef long long Int;
struct KM {
    static const int MAXN = 1050;
    static const int INF = 1LL<<60;
    int n, match[MAXN], vx[MAXN], vy[MAXN];
    Int edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[
        MAXN];
    void init(int _n){
        n = _n;
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                edge[i][j] = 0;
    }
    void add_edge(int x, int y, Int w){
        edge[x][y] = w;
    }
    bool DFS(int x){
        vx[x] = 1;
        for (int y = 0; y < n; y++) {
            if (vy[y]) continue;
            if (lx[x] + ly[y] > edge[x][y]) {

```

```

        slack[y] = min(slack[y], lx[x] + ly[y]
            - edge[x][y]);
    } else {
        vy[y] = 1;
        if ( match[y] == -1 || DFS(match[y]) ){
            match[y] = x;
            return true;
        }
    }
}
return false;
}
Int solve() {
    fill(match, match + n, -1);
    fill(lx, lx + n, -INF);
    fill(ly, ly + n, 0);
    for ( int i = 0; i < n; i++ )
        for ( int j = 0; j < n; j++ )
            lx[i] = max(lx[i], edge[i][j]);
    for ( int i = 0; i < n; i++ ) {
        fill(slack, slack + n, INF);
        while (true){
            fill(vx, vx + n, 0);
            fill(vy, vy + n, 0);
            if ( DFS(i) ) break;
            Int d = INF;
            for ( int j = 0; j < n; j++ )
                if ( !vy[j] ) d = min(d, slack[j]);
            for ( int j = 0; j < n; j++ ) {
                if (vx[j]) lx[j] -= d;
                if (vy[j]) ly[j] += d;
                else slack[j] -= d;
            }
        }
    }
    Int res = 0;
    for ( int i = 0; i < n; i++ ) {
        res += edge[ match[i] ][i];
    }
    return res;
}
} graph;

```

6.9 最小平均環

```

// from BCW
/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
    int v,u;
    double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {

```

```

        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])
                /(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    for(int i=0; i<n; i++) vst[i] = 0;
    edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}

```

6.10 偵測負環

```

#include <bits/stdc++.h>
using namespace std;

const int INF = 1000000;
const int MAXN = 200;
int n, m, q;
int d[MAXN][MAXN];

int main () {
    while ( cin >> n >> m >> q && n ) {

        for ( int i = 0; i <= n; i++ ) {
            for ( int j = 0; j <= n; j++ ) d[i][j] =
                (i==j ? 0 : INF);
        }

        for ( int i = 0; i < m; i++ ) {
            int a, b, c;
            cin >> a >> b >> c;
            d[a][b] = min(d[a][b], c);
        }

        for ( int k = 0; k < n; k++ ) {
            for ( int i = 0; i < n; i++ ) {
                for ( int j = 0; j < n; j++ ) {
                    if ( d[i][j] > d[i][k] + d[k][j] &&
                        d[i][k] < INF && d[k][j] < INF ) {
                        //printf("%d > %d + %d\n", d[i]
                        ][j], d[i][k], d[k][j]);
                        //if ( d[i][k] >= INF || d[k][j]
                        j >= INF ) cout << "NO : "
                        << i << " " << j << " " <<
                        k << "--";
                        d[i][j] = min(d[i][j], d[i][k]
                            + d[k][j]);
                    }
                }
            }
        }

        for ( int i = 0; i < n; i++ ) {
            for ( int j = 0; j < n; j++ ) {
                for ( int k = 0; k < n && d[i][j] != -
                    INF; k++ ) {
                    if ( d[k][k] < 0 && d[i][k] != INF
                        && d[k][j] != INF )
                        d[i][j] = -INF;
                }
            }
        }
        int u, v;

```

```

    for (int i=0;i<q;i++){
        scanf("%d%d",&u,&v);

        if (d[u][v] == INF) printf("Impossible\n");
        else if (d[u][v] == -INF) printf("-Infinity\n");
        else printf("%d\n",d[u][v]);
    }
    puts("");
}
return 0;
}

```

6.11 Tarjan

割點

點 u 為割點 **if and only if** 滿足 1. **or** 2.

1. u 為樹根，且 u 有多於一個子樹。
2. u 不為樹根，且滿足存在 (u,v) 為樹枝邊（或稱父子邊，即 u 為 v 在搜索樹中的父親），使得 $DFN(u) \leq Low(v)$ 。

橋

一條無向邊 (u,v) 是橋 **if and only if** (u,v) 為樹枝邊，且滿足 $DFN(u) < Low(v)$ 。

// 0 base

```

struct TarjanSCC{
    static const int MAXN = 1000006;
    int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
    vector<int> G[MAXN];
    stack<int> stk;
    bool ins[MAXN];

    void tarjan(int u){
        dfn[u] = low[u] = ++count;
        stk.push(u);
        ins[u] = true;

        for(auto v:G[u]){
            if(!dfn[v]){
                tarjan(v);
                low[u] = min(low[u], low[v]);
            }else if(ins[v]){
                low[u] = min(low[u], dfn[v]);
            }
        }

        if(dfn[u] == low[u]){
            int v;
            do {
                v = stk.top();
                stk.pop();
                scc[v] = scn;
                ins[v] = false;
            } while(v != u);
            scn++;
        }
    }

    void getSCC(){
        memset(dfn,0,sizeof(dfn));
        memset(low,0,sizeof(low));
        memset(ins,0,sizeof(ins));
        memset(scc,0,sizeof(scc));
        count = scn = 0;
        for(int i = 0 ; i < n ; i++ ){
            if(!dfn[i]) tarjan(i);
        }
    }
}SCC;

```

6.12 Topological Sort

```

#define N 87

bool adj[N][N]; // adjacency matrix
int visit[N]; // record visited coordinations in DFS
int order[N], n; // save the order

bool cycle; // detect the cycle

void DFS(int s)
{
    // back edge occured, detected the cycle
    if (visit[s] == 1) {cycle = true; return;}
    // forward edge and cross edge
    if (visit[s] == 2) return;

    visit[s] = 1;
    for (int t=0; t<N; ++t){
        if (adj[s][t]) DFS(t);
    }
    visit[s] = 2;
    order[n--] = s; // record the order
}

void topological_ordering()
{
    memset(visit, 0, sizeof(visit));
    cycle = false;
    n = N - 1;

    for (int s=0; s<9; ++s)
        if (!v[s])
            DFS(s);

    if (cycle) cout << "The graph has the cycle!";
    else{
        for (int i=0; i<N; ++i)
            cout << order[i];
    }
}

```

7 Data Structure

7.1 2D Range Tree

```

// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x,a.y) < tie(b.x,b.y);
    }
};

struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];
struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];

vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
    }
}

```

```

    return;
}
int mid = (l+r)>>1;
build(l,mid,o+o,deep+1);
build(mid+1,r,o+o+1,deep+1);

TREE *ptr = &tree[deep][l];
TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][mid+1];
TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep+1][r+1];

int cnt = 0;
while(pl != nl && pr != nr) {
    *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++);
    *(pr++);
    ptr -> toleft = cnt; ptr++;
}
while(pl != nl) *(ptr) = *(pl++), ptr -> toleft = ++cnt, ptr++;
while(pr != nr) *(ptr) = *(pr++), ptr -> toleft = cnt, ptr++;
}
int main(){
    int n; cin >> n;
    for(int i = 0; i < n; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}

```

7.2 Segment Tree

```

struct Node{
    int mx; // 區間最大值
    int tag; // 子樹裡所有人的'值'都要加上 tag
};

vector<Node> seg;

// 節點 id 的整個區間要加上 tag
void addtag(int tag, int id){
    seg[id].mx += tag; // 最大值會加上 tag
    seg[id].tag += tag; // 注意可能本來就有標記了，所以是 +=
}

// 更新子節點資訊並把標記移到子節點身上
void push(int id){
    addtag(seg[id].tag, lc);
    addtag(seg[id].tag, rc);
    seg[id].tag = 0; // 標記被移到子節點上所以要改成 0
}

// 區間 [L,R] 加上 v
void modify(int l, int r, int v, int L, int R, int id){
    if(l <= L && R <= r){
        addtag(v, id);
        return;
    }
    push(id);
    if(r <= M) modify(l, r, v, L, M, lc);
    else if(l > M) modify(l, r, v, M + 1, R, rc);
    else{
        modify(l, r, v, L, M, lc);
        modify(l, r, v, M + 1, R, rc);
    }
    seg[id].mx = max(seg[lc].mx, seg[rc].mx);
}

int query(int l, int r, int L, int R, int id){
    if(l <= L && R <= r) return seg[id].mx;
    push(id);
    int M = (L + R) / 2;
    if(r <= M) return query(l, r, L, M, lc);
    else if(l > M) return query(l, r, M + 1, R, rc);
}

```

```

    else return max(query(l, r, L, M, lc),
                    query(l, r, M + 1, R, rc));
}

```

7.3 ZKW 線段樹

```

const int M=1e5+111;
int n,m,q;
int sum[M<<2],mn[M<<2],mx[M<<2],add[M<<2];

int read() {
    int x;
    cin >> x;
    return x;
}

void build(){
    for(m=1;m<=n;m<=1);
    for(int i=m+1;i<=m+n;++i)
        sum[i]=mn[i]=mx[i]=read();
    for(int i=m-1;i-->0){
        sum[i]=sum[i<<1]+sum[i<<1|1];
        mn[i]=min(mn[i<<1],mn[i<<1|1]);
        mx[i]=max(mx[i<<1],mx[i<<1|1]);
        mn[i<<1]-=mn[i],mn[i<<1|1]-=mn[i];
        mx[i<<1]-=mx[i],mx[i<<1|1]-=mx[i];
    }
}

void update_node(int x,int v,int A=0){
    x+=m,mx[x]+=v,mn[x]+=v,sum[x]+=v;
    for(;x>1;x>>=1){
        sum[x]+=v;
        A=min(mn[x],mn[x^1]);
        mn[x]-=A,mn[x^1]-=A,mn[x>>1]+=A;
        A=max(mx[x],mx[x^1]);
        mx[x]-=A,mx[x^1]-=A,mx[x>>1]+=A;
    }
}

void update_part(int s,int t,int v){
    int A=0,lc=0,rc=0,len=1;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<<=1){
        if(s&1^1) add[s^1]+=v,lc+=len, mn[s^1]+=v,mx[s^1]+=v;
        if(t&1) add[t^1]+=v,rc+=len, mn[t^1]+=v,mx[t^1]+=v;
        sum[s>>1]+=v*lc, sum[t>>1]+=v*rc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s>>1]+=A;
        A=min(mn[t],mn[t^1]),mn[t]-=A,mn[t^1]-=A,mn[t>>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s>>1]+=A;
        A=max(mx[t],mx[t^1]),mx[t]-=A,mx[t^1]-=A,mx[t>>1]+=A;
    }
    for(lc+=rc;s>>=1){
        sum[s>>1]+=v*lc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s>>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s>>1]+=A;
    }
}

int query_node(int x,int ans=0){
    for(x+=m;x>>=1) ans+=mn[x]; return ans;
}

int query_sum(int s,int t){
    int lc=0,rc=0,len=1,ans=0;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<<=1){
        if(s&1^1) ans+=sum[s^1]+len*add[s^1],lc+=len;
        if(t&1) ans+=sum[t^1]+len*add[t^1],rc+=len;
        if(add[s>>1]) ans+=add[s>>1]*lc;
        if(add[t>>1]) ans+=add[t>>1]*rc;
    }
    for(lc+=rc,s>>=1;s>>=1) if(add[s]) ans+=add[s]*lc;
    return ans;
}

int query_min(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){

```

```

        L+=mn[s],R+=mn[t];
        if(s&1^1) L=min(L,mn[s^1]);
        if(t&1) R=min(R,mn[t^1]);
    }
    for(ans=min(L,R),s>>=1;s>>=1) ans+=mn[s];
    return ans;
}
int query_max(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mx[s],R+=mx[t];
        if(s&1^1) L=max(L,mx[s^1]);
        if(t&1) R=max(R,mx[t^1]);
    }
    for(ans=max(L,R),s>>=1;s>>=1) ans+=mx[s];
    return ans;
}

```

7.4 Sparse Table

```

const int MAXN = 200005;
const int lgN = 20;
/* Sp[i][j] 為 區間 [i, i + 2^j - 1] 的值 */
/* 從 i 開始 長度為 2 ^ j */
/* 解決可重複貢獻問題 */
struct SP{ //sparse table
    int Sp[MAXN][lgN];
    function<int(int,int)> opt;
    void build(vector<int> &nums){ // 0 base
        for (int i = 0; i < nums.size(); i++) Sp[i][0]=nums[i];

        for (int h = 1; h < lgN; h++) {
            int len = 1 << (h - 1), i=0;
            for (; i + len < nums.size(); i++)
                Sp[i][h] = opt(Sp[i][h-1], Sp[i+len][h-1]);
            for (; i < nums.size(); i++)
                Sp[i][h] = Sp[i][h-1];
        }
    }
    int query(int l, int r){
        int h = __lg(r-l+1);
        int len = 1<<h;
        return opt(Sp[l][h], Sp[r-len+1][h] );
    }
};

```

7.5 Lazy Tag

```

void modify(type value, int l, int r, int L, int R,
vertex v){
    if(l == L && r == R){
        //打懶標在v上;
        return;
    }
    int M = (L + R) / 2;
    if(r <= M) modify(value, l, r, L, M, //v的左子節點)
    ;
    else if(l > M) modify(value, l, r, M + 1, R, //v的
    右子節點);
    else{
        modify(value, l, M, L, M, v的左子節點);
        modify(value, M + 1, r, M + 1, R, //v的右子節點
        );
    }
    //用兩個子節點的答案更新v的答案;
}

```

7.6 BIT 樹狀樹組

```

class Bitree {
public:
    /* bit 一定是 1 indexed */
    vector<int>data;

```

```

    Bitree(const vector<int> &nums) {
        data.resize(nums.size() + 1, 0);
        for(int i = 0; i < nums.size(); i++) {
            update(i, nums[i]);
        }
    }
    void update(int x, int val) {
        x++; /*變成 1 indexed*/
        for(; x < data.size(); x += lowbit(x)) {
            data[x] += val;
        }
    }
    int query(int x) {
        x++; /*變成 1 indexed*/
        int result = 0;
        for(; x > 0; x -= lowbit(x)) {
            result += data[x];
        }
        return result;
    }
    static int lowbit(int x) {
        return x & (-x);
    }
};

```

7.7 並查集 union find

```

struct DisjointSet {
    vector<int> parent, sz; // parent[i] = 父節點, sz[
    i] = 集合大小
    void init(int n) {
        parent.resize(n + 1);
        sz.assign(n + 1, 1);
        for (int i = 0; i <= n; i++) {
            parent[i] = i;
        }
    }
    int find(int x) {
        if (parent[x] != x) {
            parent[x] = find(parent[x]); // 路徑壓縮
        }
        return parent[x];
    }
    bool unite(int x, int y) {
        x = find(x);
        y = find(y);
        if (x == y) return false; // 已在同一集合
        // 啟發式合併: 小的掛到大的下面
        if (sz[x] < sz[y]) swap(x, y);
        parent[y] = x;
        sz[x] += sz[y];
        return true;
    }

    bool same(int x, int y) {
        return find(x) == find(y);
    }
};

```

8 String

8.1 KMP

```

template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
    f[0]=-1, f[1]=0;
    for (int i=2; i<=n; i++){
        int w = f[i-1];
        while (w>=0 && s[w+1]!=s[i])w = f[w];
        f[i]=w+1;
    }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){

```

```

build_KMP(m,b,f);
int ans=0;

for (int i=1, w=0; i<=n; i++){
    while ( w>=0 && b[w+1]!=a[i] ) w = f[w];
    w++;
    if (w==m){
        ans++;
        w=f[w];
    }
}
return ans;
}

```

8.2 smallest rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}
/*
Booth 演算法
用於尋找一個字串的字典序最小的循環旋轉
*/
Contact GitHub API Training Shop Blog About

```

8.3 Suffix Array

```

/*he[i]保存了後綴數組中相鄰兩個後綴的最長公共前綴長度
*sa[i]表示的是字典序排名為i的後綴是誰（字典序越小的排名越靠前）
*rk[i]表示的是後綴我所對應的排名是多少 */

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
    int len = strlen(ip);
    int alp = 256;
    memset(ct, 0, sizeof(ct));
    for(int i=0; i<len; i++) ct[ip[i]+1]++;
    for(int i=1; i<alp; i++) ct[i]+=ct[i-1];
    for(int i=0; i<len; i++) rk[i]=ct[ip[i]];
    for(int i=1; i<len; i*=2){
        for(int j=0; j<len; j++){
            if(j+i>len) tp[j][1]=0;
            else tp[j][1]=rk[j+i]+1;
            tp[j][0]=rk[j];
        }
        memset(ct, 0, sizeof(ct));
        for(int j=0; j<len; j++) ct[tp[j][1]+1]++;
        for(int j=1; j<len+2; j++) ct[j]+=ct[j-1];
        for(int j=0; j<len; j++) tsa[ct[tp[j][1]]++] = j;
        memset(ct, 0, sizeof(ct));
        for(int j=0; j<len; j++) ct[tp[j][0]+1]++;
        for(int j=1; j<len+1; j++) ct[j]+=ct[j-1];
        for(int j=0; j<len; j++)
            sa[ct[tp[tsa[j]][0]]++] = tsa[j];
        rk[sa[0]]=0;
        for(int j=1; j<len; j++){
            if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
               tp[sa[j]][1] == tp[sa[j-1]][1] )
                rk[sa[j]] = rk[sa[j-1]];
            else
                rk[sa[j]] = j;
        }
    }
}

```

```

}
for(int i=0, h=0; i<len; i++){
    if(rk[i]==0) h=0;
    else{
        int j=sa[rk[i]-1];
        h=max(0, h-1);
        for(; ip[i+h]==ip[j+h]; h++);
    }
    he[rk[i]]=h;
}
}

```

8.4 Z-value

```

z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
    if ( z[bst] + bst <= i ) z[i] = 0;
    else z[i] = min(z[i - bst], z[bst] + bst - i);
    while ( str[i + z[i]] == str[z[i]] ) z[i]++;
    if ( i + z[i] > bst + z[bst] ) bst = i;
}

// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
    // centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
               s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}

```

8.5 旋轉哈希

```

typedef unsigned __int128 ulll;

ulll power(ulll a, ulll n, ulll m) {
    ulll re = 1;
    while (n > 0) {
        if (n & 1) re = re * a % m;
        a = a * a % m;
        n >>= 1;
    }
    return re;
}

ulll inv(ulll a, ulll m) {
    return power(a, m - 2, m);
}

struct Rh {
    const ulll p, mod;
    vector<ulll> ps{1};
    Rh(ulll p, ulll mod) : p(p), mod(mod) {}
    vector<ulll> build(const string &s) {
        vector<ulll> h(s.size() + 1);
        h[0] = 0;
        ps.resize(s.size() + 1);
        for (int i = 0; i < s.size(); i++) {
            ps[i + 1] = ps[i] * p % mod;
            h[i + 1] = (h[i] + s[i] * ps[i + 1] % mod) % mod;
        }
        return h;
    }
    ulll subhash(const vector<ulll> &h, int l, int r) {
        // [l, r] 指原字串
        return ((h[r + 1] - h[l]) * inv(ps[l], mod)) % mod;
    }
}

```



```

    }
};

constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)
        b, h2 = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >>
        29) + (m << 35 >> 3) + 1;
    ret = (ret & mod) + (ret>>61);
    ret = (ret & mod) + (ret>>61);
    return ret-1;
}

```

8.6 後綴自動機

```

struct state {
    int len{}, link{};
    array<int, 26> next{};
};

struct SAM {
    int sz{}, last{};
    vector<state> st;
    SAM(int maxlen) : st(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
    }

    void insert(char c) {
        insert_impl(c - 'a');
    }

    void insert_impl(char c) {
        int cur = sz++;
        st[cur].len = st[last].len + 1;
        int p = last;
        while(p != -1 && !st[p].next[c]) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if(p == -1) {
            st[cur].link = 0;
        }
        else {
            int q = st[p].next[c];
            if(st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            }
            else {
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                st[clone].next = st[q].next;
                st[clone].link = st[q].link;
                while(p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
    }
};

```

9 Others

9.1 矩陣樹定理

新的方法介紹

下面我們介紹一個新的方法—Matrix-Tree定理(Kirchhoff矩陣-樹定理)。

Matrix-Tree定理是解決生成樹數問題最有力的武器之一。它首先於1847年被Kirchhoff證明。在介紹定理之前，我們先先明確幾個概念：

1.G的度數矩陣 $D[G]$ 是一個 $n \times n$ 的矩陣，並且滿足：當 $i \neq j$ 時， $d_{ij} = 0$ ；當 $i = j$ 時， d_{ij} 等於 v_i 的度數。

2.G的鄰接矩陣 $A[G]$ 也是一個 $n \times n$ 的矩陣，且滿足：若 v_i 、 v_j 之間有邊直接相連，則 $a_{ij} = 1$ ，否則為0。

我們定義G的Kirchhoff矩陣(也稱為拉普拉斯算子) $C[G]$ 為 $C[G] = D[G] - A[G]$ ，

則Matrix-Tree定理可以描述為：G的所有不同的生成樹的個數等於其Kirchhoff矩陣 $C[G]$ 任何一個 $n-1$ 階主子式的行列式的絕對值。

所謂 $n-1$ 階主子式，就是對於 $r(1 \leq r \leq n)$ ，將 $C[G]$ 的第 r 行、第 r 列同時去掉後所得到的新矩陣，以 $C_r[G]$ 表示。

生成樹計數

演算法步驟：

1、建構拉普拉斯矩陣

$Matrix[i][j] = degree(i)$ ， $i=j$

-1， $i-j$ 有邊

0，其他情況

2、去掉第 r 行，第 r 列 (r 任意)

3、計算矩陣的行列式

```

#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
{
    if(fabs(x) < eps) return 0;
    if(x < 0) return -1;
    else return 1;
}

double b[MAXN][MAXN];
double det(double a[][MAXN], int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0; i < n; i++)
        for(j = 0; j < n; j++) b[i][j] = a[i][j];
    for(i = 0; i < n; i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n; j++)
                if(sgn(b[j][i]) != 0) break;
            if(j == n) return 0;
            for(k = i; k < n; k++) swap(b[i][k], b[j][k]);
            sign++;
        }
        ret *= b[i][i];
        for(k = i + 1; k < n; k++) b[i][k] /= b[i][i];
        for(j = i + 1; j < n; j++)
            for(k = i + 1; k < n; k++) b[j][k] -= b[j][i] * b[i][k];
    }
    if(sign & 1) ret = -ret;
    return ret;
}

double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
    int T;
    int n, m;
    int u, v;
    scanf("%d", &T);
    while(T--)
    {
        scanf("%d%d", &n, &m);
        memset(g, 0, sizeof(g));
        while(m--)

```

```

{
    scanf("%d%d",&u,&v);
    u--;v--;
    g[u][v] = g[v][u] = 1;
}
memset(a,0,sizeof(a));
for(int i = 0;i < n;i++)
for(int j = 0;j < n;j++)
if(i != j && g[i][j])
{
    a[i][i]++;
    a[i][j] = -1;
}
double ans = det(a,n-1);
printf("%.0Lf\n",ans);
}
return 0;
}

```

9.2 1D/1D dp 優化

```

#include<bits/stdc++.h>

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}

long double f(int i, int j) {
    // cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}

struct INV {
    int L, R, pos;
};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L, i) < f(stk[top].L, stk[top].pos) ) {
        stk[top-1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos = stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top+1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
            cin >> s[i];
            sum[i] = sum[i-1] + strlen(s[i]);
            dp[i] = numeric_limits<long double>::max();
        }
        stk[top] = (INV) {1, n + 1, 0};
        for ( int i = 1 ; i <= n ; i++ ) {
            if ( i >= stk[bot].R ) bot++;
            dp[i] = f(i, stk[bot].pos);
            update(i);
        }
    }
}

```

```

// cout << (ll) f(i, stk[bot].pos) << endl;
}
if ( dp[n] > 1e18 ) {
    cout << "Too hard to arrange" << endl;
} else {
    vector<PI> as;
    cout << (ll)dp[n] << endl;
}
}
return 0;
}

```

9.3 Theorm - DP optimization

Monotonicity & 1D/1D DP & 2D/1D DP

Definition xD/yD

1D/1D DP[j] = min(0≤i<j) { DP[i] + w(i, j) }; DP[0] = k
 2D/1D DP[i][j] = min(i<k≤j) { DP[i][k-1] + DP[k][j] }
 + w(i, j); DP[i][i] = 0

Monotonicity

	c	d
a	w(a, c)	w(a, d)
b	w(b, c)	w(b, d)

Monge Condition

Concave (凹四邊形不等式): $w(a, c) + w(b, d) \geq w(a, d) + w(b, c)$

Convex (凸四邊形不等式): $w(a, c) + w(b, d) \leq w(a, d) + w(b, c)$

Totally Monotone

Concave (凹單調): $w(a, c) \leq w(b, d) \rightarrow w(a, d) \leq w(b, c)$

Convex (凸單調): $w(a, c) \geq w(b, d) \rightarrow w(a, d) \geq w(b, c)$

1D/1D DP $O(n^2) \rightarrow O(n \lg n)$

CONSIDER THE TRANSITION POINT

Solve 1D/1D Concave by Stack

Solve 1D/1D Convex by Deque

2D/1D Convex DP (Totally Monotone) $O(n^3) \rightarrow O(n^2)$

$h(i, j-1) \leq h(i, j) \leq h(i+1, j)$

9.4 Stable Marriage

```

// normal stable marriage problem
// input:
//3
//Albert Laura Nancy Marcy
//Brad Marcy Nancy Laura
//Chuck Laura Marcy Nancy
//Laura Chuck Albert Brad
//Marcy Albert Chuck Brad
//Nancy Brad Albert Chuck

#include<bits/stdc++.h>
using namespace std;
const int MAXN = 505;

int n;
int favor[MAXN][MAXN]; // favor[boy_id][rank] = girl_id;
int order[MAXN][MAXN]; // order[girl_id][boy_id] = rank;
int current[MAXN]; // current[boy_id] = rank; boy_id will pursue current[boy_id] girl.
int girl_current[MAXN]; // girl[girl_id] = boy_id;

void initialize() {
    for ( int i = 0 ; i < n ; i++ ) {
        current[i] = 0;
        girl_current[i] = n;
        order[i][n] = n;
    }
}

```

```

    }
}

map<string, int> male, female;
string bname[MAXN], gname[MAXN];
int fit = 0;

void stable_marriage() {
    queue<int> que;
    for (int i = 0; i < n; i++) que.push(i);
    while (!que.empty()) {
        int boy_id = que.front();
        que.pop();

        int girl_id = favor[boy_id][current[boy_id]];
        current[boy_id]++;

        if (order[girl_id][boy_id] < order[girl_id][
            girl_current[girl_id]]) {
            if (girl_current[girl_id] < n) que.push(
                girl_current[girl_id]); // if not the first
                time
            girl_current[girl_id] = boy_id;
        } else {
            que.push(boy_id);
        }
    }
}

int main() {
    cin >> n;

    for (int i = 0; i < n; i++) {
        string p, t;
        cin >> p;
        male[p] = i;
        bname[i] = p;
        for (int j = 0; j < n; j++) {
            cin >> t;
            if (!female.count(t)) {
                gname[fit] = t;
                female[t] = fit++;
            }
            favor[i][j] = female[t];
        }
    }

    for (int i = 0; i < n; i++) {
        string p, t;
        cin >> p;
        for (int j = 0; j < n; j++) {
            cin >> t;
            order[female[p]][male[t]] = j;
        }
    }

    initialize();
    stable_marriage();

    for (int i = 0; i < n; i++) {
        cout << bname[i] << " " << gname[favor[i][current[i]
            ] - 1]] << endl;
    }
}

```

9.5 莫隊

```

/* nums 長度 N ; query 長度為 M */
/* O(N * sqrt(M)) */

```

```

struct Query {
    int l, r, id;
};

```

```

void add(int pos) {
    /*更新狀態*/
    /*將pos所在的移入集合*/
}

```

```

}

void del(int pos) {
    /*更新狀態*/
    /*將pos所在的移出集合*/
}

int bsz = n / sqrt(m); /*分塊大小 block size*/
sort(query.begin(), query.end(), [bsz](const Query &a,
    const Query &b){
    if(a.l / bsz != b.l / bsz) {
        return a.l < b.l;
    }
    return (a.l / bsz) & 1 ? a.r < b.r : a.r > b.r;
});

int l = 1;
int r = 0;

vector<pair<int, int>> res(m);

for(int i = 0; i < query.size(); i++) {
    auto &q = query[i];
    /*順序不能換*/
    while (l > q.l) add(--l);
    while (r < q.r) add(++r);
    while (l < q.l) del(l--);
    while (r > q.r) del(r--);
    res[q.id] = /* 根據當前狀態求解 */
}

```

9.6 矩陣乘法

```

#define MOD INT_MAX
vector<vector<int>> operator *(const vector<vector<int>
    >> &a, const vector<vector<int>> &b) {
    vector<vector<int>> re(a.size(), vector<int>(b[0].
        size()));
    for (int i = 0; i < a.size(); i++) {
        for (int j = 0; j < b[0].size(); j++) {
            for (int k = 0; k < b.size(); k++) {
                re[i][j] += (a[i][k] * b[k][j]) % MOD;
            }
        }
    }
    return re;
}

```

9.7 c++ 小抄

```

//pbds tree
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> tr;

tr.find_by_order(k) // O(LogN) 取得第k大的元素
tr.order_of_key(ele) // O(LogN) 得到ele是tree中第幾大(
    有幾個元素小於ele)

```

```

//pbds pair priority_queue
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;

priority_queue<int, less<int>, pairing_heap_tag> pq;
auto it = pq.push(x);
// type of it = priority_queue<int, less<int>,
    pairing_heap_tag>::point_iterator
pq.pop();
pq.top();
pq.join(b);
pq.empty();
pq.size();
pq.modify(it, 6); // O(LogN)

```

```

pq.erase(it);

//builtin functions
__builtin_popcount(x); // 1的個數
__builtin_popcountll(x); // for Long Long
__builtin_clz(x); // 前導0的個數
__builtin_ctz(x); // 後導0的個數
__builtin_parity(x); // 奇偶性

//溢位檢查
ret = __builtin_add_overflow(a, b, &res) // if ret = 1
    a+b 溢位
ret = __builtin_sub_overflow(a, b, &res) // if ret = 1
    a-b 溢位
ret = __builtin_mul_overflow(a, b, &res) // if ret = 1
    a*b 溢位
ret = __builtin_add_overflow_p(a, b, 0LL) // if ret = 1
    溢位 第三個參數是判斷的類型

//vector SIMD
typedef int v4si __attribute__((vector_size(4 * sizeof
    (int))));

//大質數表
{1000000007, 1000000009, 1000000021, 1000000033,
    1000000087, 1000000093, 1000000097, 1000000123,
    1000000321};

//mt19937
#include <random>
#include <chrono>

int getRandom(int l, int r) {
    static auto seed = std::chrono::system_clock::now()
        .time_since_epoch().count();
    static std::mt19937 gen(seed);
    std::uniform_int_distribution<int> dis(l, r);
    return dis(gen);
}

//sorted vector 去重
vec.erase(unique(vec.begin(), vec.end(), vec.end()));

//std::valarray
valarray<int> a(初始值, 數量); //就是那麼機八
valarray<int> a(10);
valarray<int> b(10);
valarray<int> c = a + b;
valarray<int> d = a * b;
valarray<int> e = a + 10;
valarray<int> f = a * 10;
valarray<int> g = a.cshift(1); //循環左移
valarray<bool> equal = a == b;
int sum = a.sum();
int max = a.max();
int min = a.min();
std::valarray<int> g = a.apply([](int x) { return x * x
    ; });

//regex ***very slow***
#include <regex>
using namespace std;
bool res = regex_match("abc", regex("a.c"));
bool res = regex_match("abc", regex("A.c", regex::icase
    )); //忽略大小寫

//gp_hash_table
#include <ext/pb_ds/assoc_container.hpp>
__gnu_pbds::gp_hash_table<int, int/*, hashFunction */>
    table;

```

9.8 python 小抄

```
#!/usr/bin/env python3
```

```
# 帕斯卡三角形
n = 10
```

```

dp = [ [1 for j in range(n)] for i in range(n) ]
for i in range(1,n):
    for j in range(1,n):
        dp[i][j] = dp[i][j-1] + dp[i-1][j]

for i in range(n):
    print( ' '.join( '{:5d}'.format(x) for x in dp[i] )
    )

# EOF1
while True:
    try:
        n, m = map(int, input().split())
    except:
        break

# EOF2
import sys
for s in sys.stdin:
    print(eval(s.replace("/", "///")))

# input a sequence of number
a = [ int(x) for x in input().split() ]
a.sort()
print( ''.join( str(x)+' ' for x in a ) )

# LCS
ncase = int( input() )
for _ in range(ncase):
    n, m = [int(x) for x in input().split()]
    a, b = "$"+input(), "$"+input()
    dp = [ [int(0) for j in range(m+1)] for i in range(
        n+1) ]
    for i in range(1,n+1):
        for j in range(1,m+1):
            dp[i][j] = max(dp[i-1][j], dp[i][j-1])
            if a[i]==b[j]:
                dp[i][j] = max(dp[i][j], dp[i-1][j-1]+1)

    for i in range(1,n+1):
        print(dp[i][1:])
    print( 'a={:s}, b={:s}, |LCS(a,b)|={:d}'.format(a
        [1:], b[1:], dp[n][m]) )

# list, dict, string
a = [1, 3, 4, 65, 65]
b = list.copy() # b = [1, 3, 4, 65], list a 跟 list b
    互相獨立
cnt = list.count(65) # cnt == 2
loc = list.index(65) # loc == 3, find the leftmost
    element, if not found then return ERROR
list.sort(reverse = True|False, key = none|lambda x:x
    [1]) # list.sort has side effect but no return
    value

```

```

# stack # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop() # pop()
stack[-1] # top()
len(stack) # size() 0(1)

```

```

# queue # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0] # front()
len(queue) # size() 0(1)

```

9.9 萬年曆

$$h = \left(q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor + 5J \right) \bmod 7$$

h : 星期 (0 = 星期六, 1 = 星期日, 2 = 星期一, ...)
 q : 日期 (日)
 m : 月份 (3= 三月, 4= 四月, ...; 1、2 月視為前一年的 13、14 月)
 K : 年份的後兩位數 (year mod 100)
 J : 年份的前兩位數 (year ÷ 100)