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## 1 Basic

### 1.1 compile

```
# preset before coding
echo "cd ~/Desktop" >> ~/.bashrc
gedit -> preference -> tab width: 4

# Editor
gedit a.cpp

# Compile
```

```
g++ a.cpp -std=c++11

**All file will be compiled to a.out unless you use -o(
    not recommended, just use a.out)**

# Run
./a.out

# Run with file input
./a.out < input.txt

# Run with file input and output
./a.out < input.txt > output.txt

# Python Run
python3 a.py < input.txt > output.txt

# Copy Paste In Ubuntu
* copy: ctrl+insert
* paste: shift+insert

# 比對文件相同
sdiff a.txt b.txt
```

### 1.2 default code

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
typedef pair<int,int> pii;

#ifdef ONLINE_JUDGE
#define cerr if(false) cerr
#endif

int32_t main(){
#ifdef ONLINE_JUDGE
    freopen("input.txt", "r", stdin);
    freopen("output.txt", "w", stdout);
    freopen("debug.txt", "w", stderr);
#else
    ios_base::sync_with_stdio(0);
    cin.tie(false);
#endif
}
```

### 1.3 debug list

記得測試 python 的內建函數庫有哪些  
bits/stdc++.h 跟 global variable y1 衝突，不能用  
模板要記得 init  
priority\_queue 要清空  
事先將把邊界測資加入測試  
邊界條件（過程溢位，題目數據範圍），會不會爆 long long  
是否讀錯題目，想不到時可以自己讀一次題目  
比較容易有問題的地方換人寫  
注意公式有沒有推錯或抄錯  
精度誤差 sqrt(大大的東西) + EPS  
喇分 random\_shuffle 隨機演算法

### 1.4 時間複雜度

時間複雜度	可處理的最大 $N$ 數量級 (約)
$O(1)$	幾乎沒限制
$O(\log N)$	$10^{18}$ 級別 (如快速幂)
$O(\sqrt{N})$	$10^{10}$
$O(N)$	$10^8$
$O(N \log N)$	$2 \times 10^7 \sim 5 \times 10^7$
$O(N\sqrt{N})$	$1 \times 10^5 \sim 2 \times 10^5$
$O(N^2)$	$10^4 \sim 1.5 \times 10^4$
$O(N^2 \log N)$	約 $3 \times 10^3$
$O(N^3)$	$500 \sim 1000$
$O(2^N)$	$N \leq 20$
$O(N!)$	$N \leq 10$

## 2 Dark Code

### 2.1 IO optimization

```
*if output to much, consider put all output in array
  first, then output the array.
getchar() -> getchar_unlocked()
fread() -> fread_unlocked()
-----
inline char readchar() {
    const int S = 1<<20; // buffer size
    static char buf[S], *p = buf, *q = buf;
    if(p == q && (q = (p=buf)+fread(buf,1,S,stdin)) ==
        buf) return EOF;
    return *p++;
}

inline int nxtint() {
    // if readchar can't use, change readchar() to
    // getchar()
    int x = 0;
    int c = readchar(), neg = false;
    if (c == EOF) return -1;
    while (('0' > c || c > '9') && c != '-' && c != EOF)
        c = readchar();
    if (c == '-') neg = true, c = readchar();
    while ('0' <= c && c <= '9') x = x * 10 + (c ^ '0'),
        c = readchar();
    if (neg) x = -x;
    return x;
}
```

## 3 Geometry

### 3.1 2D point

```
typedef double Double;
struct Point {
    Double x,y;

    bool operator < (const Point &b)const{
        //return tie(x,y) < tie(b.x,b.y);
        return atan2(y,x) < atan2(b.y,b.x);
    }
    Point operator + (const Point &b)const{
        return (Point){x+b.x,y+b.y};
    }
    Point operator - (const Point &b)const{
        return (Point){x-b.x,y-b.y};
    }
    Point operator * (const Double &d)const{
        return Point(d*x,d*y);
    }
    Double operator * (const Point &b)const{
        return x*b.x + y*b.y;
    }
    Double operator % (const Point &b)const{
        return x*b.y - y*b.x;
    }
    friend Double abs2(const Point &p){
        return p.x*p.x + p.y*p.y;
    }
    friend Double abs(const Point &p){
        return sqrt( abs2(p) );
    }
};
typedef Point Vector;

struct Line{
    Point P; Vector v;
    bool operator < (const Line &b)const{
        return atan2(v.y,v.x) < atan2(b.v.y,b.v.x);
    }
};
```

### 3.2 兩線段交點

```
struct Point {
    double x, y;
};

Point operator*(double a, Point p) {
    return {a * p.x, a * p.y};
}

Point operator+(Point a, Point b) {
    return {a.x + b.x, a.y + b.y};
}

const double EPS = 1e-9;

double cross(Point p1, Point p2, Point q1) {
    return (p2.x - p1.x) * (q1.y - p1.y) - (p2.y - p1.y)
        * (q1.x - p1.x);
}

template<class T>
bool inter(T a, T b, T c) {
    if(b < a) swap(a, b);
    return c >= a && c <= b;
}

bool is_in(Point p1, Point p2, Point q) { // 判斷 q 是
    否在 線段 p1 p2 內
    return inter(p1.x, p2.x, q.x) && inter(p1.y, p2.y, q.
        y);
}

Point interPnt(Point p1, Point p2, Point q1, Point q2,
    bool &res){
    double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
    double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {};
    }

    Point ip = (f2 / f) * q1 + (f1 / f) * q2; //
        intersection point
    if(is_in(p1, p2, ip) && is_in(q1, q2, ip)) { // 兩直
        線交點不用這段
        res = true;
        return ip;
    }
    res = false;
    return {};
}
```

### 3.3 兩圓交點

```
vector<Double> interCircle(Double o1, Double r1, Double
    o2, Double r2) {
    Double d2 = abs2(o1 - o2);
    Double d = sqrt(d2);
    if (d < fabs(r1-r2) || r1+r2 < d) return {};
    Double u = 0.5*(o1+o2) + ((r2*r2-r1*r1)/(2.0*d2))*(o1
        -o2);
    Double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    Double v = A / (2.0*d2) * Double(o1.S-o2.S, -o1.F+o2.
        F);
    return {u+v, u-v};
}
```

### 3.4 Convex Hull

```
#include "2Dpoint.cpp"

// return H, The first will occurred TWICE in vector H!
void ConvexHull(vector<Point> &P, vector<Point> &H){
```

```

int n = P.size(), m=0;
sort(P.begin(),P.end());
H.clear();

for (int i=0; i<n; i++){
    while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
           <0)H.pop_back(), m--;
    H.push_back(P[i]), m++;
}

for (int i=n-2; i>=0; i--){
    while (m>=2 && (P[i]-H[m-2]) % (H[m-1]-H[m-2])
           <0)H.pop_back(), m--;
    H.push_back(P[i]), m++;
}
}

```

## 4 Flow

### 4.1 Dinic

(a) Bounded Maxflow Construction:

1. add two node ss, tt
2. add\_edge(ss, tt, INF)
3. for each edge u -> v with capacity [l, r]:
  - add\_edge(u, tt, l)
  - add\_edge(ss, v, l)
  - add\_edge(u, v, r-l)
4. see (b), check if it is possible.
5. answer is maxflow(ss, tt) + maxflow(s, t)

(b) Bounded Possible Flow:

1. same construction method as (a)
2. run maxflow(ss, tt)
3. for every edge connected with ss or tt:
  - rule: check if their rest flow is exactly 0
4. answer is possible if every edge do satisfy the rule
5. otherwise, it is NOT possible.

(c) Bounded Minimum Flow:

1. same construction method as (a)
2. answer is maxflow(ss, tt)

(d) Bounded Minimum Cost Flow:

- \* the concept is somewhat like bounded possible flow.
- 1. same construction method as (a)
- 2. answer is maxflow(ss, tt) + ( $\sum l * cost$  for every edge)

(e) Minimum Cut:

1. run maxflow(s, t)
2. run cut(s)
3. ss[i] = 1: node i is at the same side with s.

```

const long long INF = 1LL<<60;
struct Dinic { //O(VVE), with minimum cut
    static const int MAXN = 5003;
    struct Edge{
        int u, v;
        long long cap, rest;
    };

    int n, m, s, t, d[MAXN], cur[MAXN];
    vector<Edge> edges;
    vector<int> G[MAXN];

    void init(){
        edges.clear();
        for (int i = 0 ; i < MAXN ; i++ ) G[i].clear()
            ;
    }

    // min cut start
    bool side[MAXN];
    void cut(int u) {
        side[u] = 1;

```

```

        for (int i : G[u] ) {
            if ( !side[ edges[i].v ] && edges[i].rest )
                cut(edges[i].v);
        }
    }
    // min cut end

    void add_edge(int u, int v, long long cap){
        edges.push_back( {u, v, cap, cap} );
        edges.push_back( {v, u, 0, 0LL} );
        m = edges.size();
        G[u].push_back(m-2);
        G[v].push_back(m-1);
    }

    bool bfs(){
        memset(d, -1, sizeof(d));
        queue<int> que;
        que.push(s); d[s]=0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (int ei : G[u]){
                Edge &e = edges[ei];
                if (d[e.v] < 0 && e.rest > 0){
                    d[e.v] = d[u] + 1;
                    que.push(e.v);
                }
            }
        }
        return d[t] >= 0;
    }

    long long dfs(int u, long long a){
        if ( u == t || a == 0 ) return a;
        long long flow = 0, f;
        for ( int &i=cur[u]; i < (int)G[u].size() ; i++ ) {
            Edge &e = edges[ G[u][i] ];
            if ( d[u] + 1 != d[e.v] ) continue;
            f = dfs(e.v, min(a, e.rest) );
            if ( f > 0 ) {
                e.rest -= f;
                edges[ G[u][i]^1 ].rest += f;
                flow += f;
                a -= f;
                if ( a == 0 ) break;
            }
        }
        return flow;
    }

    long long maxflow(int s, int t){
        this->s = s, this->t = t;
        long long flow = 0, mf;
        while ( bfs() ){
            memset(cur, 0, sizeof(cur));
            while ( (mf = dfs(s, INF)) ) flow += mf;
        }
        return flow;
    }
} dinic;

```

### 4.2 min cost flow

```

// Long Long version
typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MAXN = 350;
    static const long long INF = 1LL<<60;
    struct Edge {
        int to, r;
        long long rest, c;
    };

    int n, pre[MAXN], preL[MAXN]; bool inq[MAXN];
    long long dis[MAXN], fl, cost;
    vector<Edge> G[MAXN];
    void init() {
        for ( int i = 0 ; i < MAXN ; i++ ) G[i].clear();
    }
}

```

```

void add_edge(int u, int v, long long rest, long
long c) {
    G[u].push_back({v, (int)G[v].size(), rest, c});
    G[v].push_back({u, (int)G[u].size()-1, 0, -c});
}
pll flow(int s, int t) {
    fl = cost = 0;
    while (true) {
        fill(dis, dis+MAXN, INF);
        fill(inq, inq+MAXN, 0);
        dis[s] = 0;
        queue<int> que;
        que.push(s);
        while (!que.empty()) {
            int u = que.front(); que.pop();
            inq[u] = 0;
            for (int i = 0; i < (int)G[u].size(); i++) {
                int v = G[u][i].to;
                long long w = G[u][i].c;
                if (G[u][i].rest > 0 && dis[v] >
                    dis[u] + w) {
                    pre[v] = u; preL[v] = i;
                    dis[v] = dis[u] + w;
                    if (!inq[v]) {
                        inq[v] = 1;
                        que.push(v);
                    }
                }
            }
        }
        if (dis[t] == INF) break;
        long long tf = INF;
        for (int v = t, u, l; v != s; v = u) {
            u = pre[v]; l = preL[v];
            tf = min(tf, G[u][l].rest);
        }
        for (int v = t, u, l; v != s; v = u) {
            u = pre[v]; l = preL[v];
            G[u][l].rest -= tf;
            G[v][G[u][l].r].rest += tf;
        }
        cost += tf * dis[t];
        fl += tf;
    }
    return {fl, cost};
}
} flow;

```

## 5 Mathematics

### 5.1 $ax+by=\gcd(a,b)$

```

typedef pair<int, int> pii;
pii exgcd(int a, int b) {
    if (b == 0) return make_pair(1, 0);
    else {
        int p = a / b;
        pii q = exgcd(b, a % b);
        int aa = q.second, bb = q.first - q.second * p;
        if (aa < 0) aa += b, bb -= a;
        return make_pair(aa, bb);
    }
}

```

### 5.2 BigInt

```

struct BigInt {
    static const int LEN = 60;
    static const int BIGMOD = 100000;
    int s;
    int vl, v[LEN];
    // vector<int> v;
}

```

```

BigInt() : s(1) { vl = 0; }
BigInt(long long a) {
    s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
        push_back(a % BIGMOD);
        a /= BIGMOD;
    }
}
BigInt(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
        stPos = 1;
        s = -1;
    }
    for (int i = SZ(str)-1, q=1; i >= stPos; i--) {
        num += (str[i] - '0') * q;
        if ((q *= 10) >= BIGMOD) {
            push_back(num);
            num = 0; q = 1;
        }
    }
    if (num) push_back(num);
}
int len() const { return vl; /* return SZ(v); */ }
bool empty() const { return len() == 0; }
void push_back(int x) { v[vl++] = x; /* v.PB(x); */ }
void pop_back() { vl--; /* v.pop_back(); */ }
int back() const { return v[vl-1]; /* return v.back(); */ }
void n() { while (!empty() && !back()) pop_back(); }
void resize(int nl) {
    vl = nl; fill(v, v+vl, 0);
    // v.resize(nl); // fill(ALL(v), 0);
}
void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i = len()-2; i >= 0; i--) printf("%.4d", v[i]);
}
friend ostream& operator << (ostream& out,
    const BigInt &a) {
    if (a.empty()) { out << "0"; return out; }
    if (a.s == -1) out << "-";
    out << a.back();
    for (int i = a.len()-2; i >= 0; i--) {
        char str[10];
        snprintf(str, 5, "%.4d", a.v[i]);
        out << str;
    }
    return out;
}
int cp3(const BigInt &b) const {
    if (s != b.s) return s > b.s ? 1 : -1;
    if (s == -1) return -(*this).cp3(-b);
    if (len() != b.len()) return len() > b.len() ? 1 : -1;
    for (int i = len()-1; i >= 0; i--)
        if (v[i] != b.v[i]) return v[i] > b.v[i] ? 1 : -1;
    return 0;
}
bool operator < (const BigInt &b) const { return cp3(b) == -1; }
bool operator <= (const BigInt &b) const { return cp3(b) <= 0; }
bool operator >= (const BigInt &b) const { return cp3(b) >= 0; }
bool operator == (const BigInt &b) const { return cp3(b) == 0; }
bool operator != (const BigInt &b) const { return cp3(b) != 0; }
bool operator > (const BigInt &b) const { return cp3(b) == 1; }
BigInt operator - () const {
    BigInt r = (*this);
    r.s = -r.s;
    return r;
}
BigInt operator + (const BigInt &b) const {
    if (s == -1) return -(*this)+(-b);
    if (b.s == -1) return (*this)-(-b);
}

```

```

Bigint r;
int nl = max(len(), b.len());
r.resize(nl + 1);
for (int i=0; i<nl; i++) {
    if (i < len()) r.v[i] += v[i];
    if (i < b.len()) r.v[i] += b.v[i];
    if (r.v[i] >= BIGMOD) {
        r.v[i+1] += r.v[i] / BIGMOD;
        r.v[i] %= BIGMOD;
    }
}
r.n();
return r;
}

Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(*this)-(-b);
    if (b.s == -1) return (*this)+(-b);
    if ((*this) < b) return -(b-(*this));
    Bigint r;
    r.resize(len());
    for (int i=0; i<len(); i++) {
        r.v[i] += v[i];
        if (i < b.len()) r.v[i] -= b.v[i];
        if (r.v[i] < 0) {
            r.v[i] += BIGMOD;
            r.v[i+1]--;
        }
    }
    r.n();
    return r;
}

Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
    r.s = s * b.s;
    for (int i=0; i<len(); i++) {
        for (int j=0; j<b.len(); j++) {
            r.v[i+j] += v[i] * b.v[j];
            if (r.v[i+j] >= BIGMOD) {
                r.v[i+j+1] += r.v[i+j] / BIGMOD;
                r.v[i+j] %= BIGMOD;
            }
        }
    }
    r.n();
    return r;
}

Bigint operator / (const Bigint &b) {
    Bigint r;
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
        int d=0, u=BIGMOD-1;
        while(d<u) {
            int m = (d+u+1)>>1;
            r.v[i] = m;
            if ((r*b2) > (*this)) u = m-1;
            else d = m;
        }
        r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
}

Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
}
};

```

### 5.3 GaussElimination

// by bcw\_codebook

```

const int MAXN = 300;
const double EPS = 1e-8;

```

```

int n;
double A[MAXN][MAXN];

void Gauss() {
    for (int i = 0; i < n; i++) {
        bool ok = 0;
        for (int j = i; j < n; j++) {
            if (fabs(A[j][i]) > EPS) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if (!ok) continue;

        double fs = A[i][i];
        for (int j = i+1; j < n; j++) {
            double r = A[j][i] / fs;
            for (int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}

template<class T>
void Gauss(vector<vector<T>> &A) {
    int n = A.size();
    for (int i = 0; i < n; i++) {
        bool ok = 0;
        for (int j = i; j < n; j++) {
            if (A[j][i] != 0) {
                swap(A[j], A[i]);
                ok = 1;
                break;
            }
        }
        if (!ok) continue;

        T fs = A[i][i];
        for (int j = i+1; j < n; j++) {
            T r = A[j][i] / fs;
            for (int k = i; k < n; k++) {
                A[j][k] -= A[i][k] * r;
            }
        }
    }
}

```

### 5.4 Inverse

```

int inverse[100000];
void invTable(int b, int p) {
    inverse[1] = 1;
    for (int i = 2; i <= b; i++) {
        inverse[i] = (long long)inverse[p%i] * (p-p/i) % p;
    }
}

int inv(int b, int p) {
    return b == 1 ? 1 : ((long long)inv(p % b, p) * (p-p/
        b) % p);
}

```

### 5.5 LinearPrime 歐拉篩

```

const int MAXP = 100; //max prime
vector<int> P; // primes
void build_prime(){
    static bitset<MAXP> ok;
    int np=0;
    for (int i=2; i<MAXP; i++){
        if (ok[i]==0)P.push_back(i), np++;
        for (int j=0; j<np && i*P[j]<MAXP; j++){
            ok[ i*P[j] ] = 1;
            if ( i%P[j]==0 )break;
        }
    }
}

```

```
}
}
```

## 5.6 Miller Rabin

```
typedef long long LL;

inline LL bin_mul(LL a, LL n, const LL& MOD){
    LL re=0;
    while (n>0){
        if (n&1) re += a;
        a += a; if (a>=MOD) a-=MOD;
        n>>=1;
    }
    return re%MOD;
}

inline LL bin_pow(LL a, LL n, const LL& MOD){
    LL re=1;
    while (n>0){
        if (n&1) re = bin_mul(re,a,MOD);
        a = bin_mul(a,a,MOD);
        n>>=1;
    }
    return re;
}

bool is_prime(LL n){
    //static LL sprp[3] = { 2LL, 7LL, 61LL};
    static LL sprp[7] = { 2LL, 325LL, 9375LL,
        28178LL, 450775LL, 9780504LL,
        1795265022LL };
    if (n==1 || (n&1)==0 ) return n==2;
    int u=n-1, t=0;
    while ( (u&1)==0 ) u>>=1, t++;
    for (int i=0; i<3; i++){
        LL x = bin_pow( sprp[i]%n, u, n);
        if (x==0 || x==1 || x==n-1) continue;

        for (int j=1; j<t; j++){
            x=x*x%n;
            if (x==1 || x==n-1) break;
        }
        if (x==n-1) continue;
        return 0;
    }
    return 1;
}
```

## 5.7 Pollard's rho

```
// from PEC
// does not work when n is prime
Int f(Int x, Int mod){
    return add(mul(x, x, mod), 1, mod);
}

Int pollard_rho(Int n) {
    if ( !(n & 1) ) return 2;
    while (true) {
        Int y = 2, x = rand()%(n-1) + 1, res = 1;
        for ( int sz = 2 ; res == 1 ; sz *= 2 ) {
            for ( int i = 0 ; i < sz && res <= 1 ; i++ ) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if ( res != 0 && res != n ) return res;
    }
}
```

## 5.8 數論基本工具

```
Int POW(Int a, Int n, Int mod){
    Int re=1;
    while (n>0){
        if (n&1LL) re = re*a%mod;
```

```
        a = a*a%mod;
        n>>=1;
    }
    return re;
}

Int C(Int n, Int m){
    if (m<0 || m>n) return 0;
    return J[n] * inv(J[m]*J[n-m]%MOD) %MOD;
}
```

## 5.9 Mobius

```
void mobius() {
    fill(isPrime, isPrime + MAXN, 1);
    mu[1] = 1, num = 0;
    for (int i = 2; i < MAXN; ++i) {
        if (isPrime[i]) primes[num++] = i, mu[i] = -1;
        static int d;
        for (int j = 0; j < num && (d = i * primes[j])
            < MAXN; ++j) {
            isPrime[d] = false;
            if (i % primes[j] == 0) {
                mu[d] = 0; break;
            } else mu[d] = -mu[i];
        }
    }
}
```

## 5.10 SG

Anti Nim (取走最後一個石子者敗)

先手必勝 **if and only if**

1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。

Anti-SG (決策集合為空的遊戲者贏)

定義 SG 值為 0 時，遊戲結束，

則先手必勝 **if and only if**

1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。

Sprague-Grundy

1. 雙人、回合制
2. 資訊完全公開
3. 無隨機因素
4. 可在有限步內結束
5. 沒有和局
6. 雙方可採取的行動相同

SG(S) 的值為 0：後手(P)必勝

不為 0：先手(N)必勝

```
int mex(set S) {
    // find the min number >= 0 that not in the S
    // e.g. S = {0, 1, 3, 4} mex(S) = 2
}
```

```
state = []
int SG(A) {
    if (A not in state) {
        S = sub_states(A)
        if( len(S) > 1 ) state[A] = reduce(operator.xor, [
            SG(B) for B in S])
        else state[A] = mex(set(SG(B) for B in next_states(
            A)))
    }
    return state[A]
}
```

## 5.11 Theorem

```

/*
Lucas's Theorem
For non-negative integer n,m and prime P,
 $C(m,n) \bmod P = C(m/M,n/M) * C(m\%M,n\%M) \bmod P$ 
= mult_i ( C(m_i,n_i) )
where m_i is the i-th digit of m in base P.
-----
Pick's Theorem
 $A = i + b/2 - 1$ 
-----
Kirchhoff's theorem
 $A_{ii} = \deg(i), A_{ij} = (i,j) \in E ? -1 : 0$ 
Deleting any one row, one column, and cal the det(A)
-----
Nth Catalan recursive function:
 $C_0 = 1, C_{n+1} = C_n * 2(2n+1)/(n+2)$ 
-----
Mobius Formula
 $u(n) = 1$  , if  $n = 1$ 
 $(-1)^m$  , 若  $n$  無平方數因數, 且  $n = p_1 * p_2 * p_3 * \dots * p_k$ 
 $0$  , 若  $n$  有大於 1 的平方數因數
- Property
1. (積性函數)  $u(a)u(b) = u(ab)$ 
2.  $\sum_{d|n} u(d) = [n == 1]$ 
-----
Mobius Inversion Formula
if  $f(n) = \sum_{d|n} g(d)$ 
then  $g(n) = \sum_{d|n} u(n/d)f(d)$ 
 $= \sum_{d|n} u(d)f(n/d)$ 
- Application
the number/power of gcd(i, j) = k
- Trick
分塊,  $O(\sqrt{n})$ 
-----
Chinese Remainder Theorem ( $m_i$  兩兩互質)

 $x = a_1 \pmod{m_1}$ 
 $x = a_2 \pmod{m_2}$ 
....
 $x = a_i \pmod{m_i}$ 

construct a solution:

Let  $M = m_1 * m_2 * m_3 * \dots * m_n$ 
Let  $M_i = M / m_i$ 

 $t_i = 1 / M_i$ 
 $t_i * M_i = 1 \pmod{m_i}$ 

solution  $x = a_1 * t_1 * M_1 + a_2 * t_2 * M_2 + \dots$ 
 $+ a_n * t_n * M_n + k * M$ 
 $= k * M + \sum a_i * t_i * M_i, k \text{ is positive integer.}$ 

under mod M, there is one solution  $x = \sum a_i * t_i * M_i$ 
-----
Burnside's Lemma
 $|G| * |X/G| = \sum (|X^g|) \text{ where } g \text{ in } G$ 
總方法數: 每一種旋轉下不動點的個數總和 除以 旋轉的方法數
*/

```

## 6 Graph

### 6.1 BCC

#### 邊雙連通

任意兩點間至少有兩條不重疊的路徑連接, 找法:

1. 標記出所有的橋
2. 對全圖進行 DFS, 不走橋, 每一次 DFS 就是一個新的邊雙連通

// from BCW

```

struct BccEdge {
    static const int MXN = 100005;
    struct Edge { int v, eid; };
    int n, m, step, par[MXN], dfn[MXN], low[MXN];
    vector<Edge> E[MXN];
    DisjointSet djs;
    void init(int _n) {
        n = _n; m = 0;
        for (int i=0; i<n; i++) E[i].clear();
        djs.init(n);
    }
    void add_edge(int u, int v) {
        E[u].PB({v, m});
        E[v].PB({u, m});
        m++;
    }
    void DFS(int u, int f, int f_eid) {
        par[u] = f;
        dfn[u] = low[u] = step++;
        for (auto it:E[u]) {
            if (it.eid == f_eid) continue;
            int v = it.v;
            if (dfn[v] == -1) {
                DFS(v, u, it.eid);
                low[u] = min(low[u], low[v]);
            } else {
                low[u] = min(low[u], dfn[v]);
            }
        }
    }
    void solve() {
        step = 0;
        memset(dfn, -1, sizeof(int)*n);
        for (int i=0; i<n; i++) {
            if (dfn[i] == -1) DFS(i, i, -1);
        }
        djs.init(n);
        for (int i=0; i<n; i++) {
            if (low[i] < dfn[i]) djs.uni(i, par[i]);
        }
    }
}graph;

```

### 6.2 Prim

```

// edge strucute
struct edge{
    int a, b;
    double data;
    bool operator <(const edge b)const{
        return data > b.data;
    }
};

// main prim algorithm
int n, m, root, aa, bb, cc;
while (cin >> n >> m){
    priority_queue<edge>yee;
    int visit[500] = {}, p[500] = {};
    double a[500][500] = {};
    //undirectional edge aa to bb is weighted cc
    for (int i = 0; i < m; i++){
        cin >> aa >> bb >> cc;
        a[aa][bb] = a[bb][aa] = cc;
    }
    cin >> root;
    yee.push({ 0, root, 0 });
    edge tmp;
    double total = 0;
    while (!yee.empty()){
        tmp = yee.top(); yee.pop();
        if (visit[tmp.b])continue;
        total += tmp.data; p[tmp.b] = tmp.a; visit[tmp.b] = 1;
        for (int i = 1; i <= n; i++){
            if (a[tmp.b][i] != 0 && (!visit[i])){
                yee.push({tmp.b, i, a[tmp.b][i]});
            }
        }
    }
}

```



```

    }
    cout << total << endl;
}

```

### 6.3 Bellman Ford

```

int a[100][100], d[100], p[100];

void bellman_ford(int root, int n){
    for (int i = 1; i <= n; i++) d[i] = 1e9;
    d[root] = 0, p[root] = 0;
    for (int i = 0; i < n - 1; i++){
        for (int j = 1; j <= n; j++){
            for (int k = 1; k <= n; k++){
                if (d[j] != 1e9 && a[j][k] != 1e9){
                    if (d[j] + a[j][k] < d[k]){
                        d[k] = d[j] + a[j][k], p[k] = j;
                    }
                }
            }
        }
    }

    bool nega_cyc(int n){
        for (int i = 1; i <= n; i++){
            for (int j = 1; j <= n; j++){
                if (d[i] != 1e9 && a[i][j] != 1e9)
                    if (d[i] + a[i][j] < d[j]){
                        return 0;
                    }
            }
        }
        return 1;
    }

    int main(){
        int n, m, aa, bb, dd;
        while (cin >> n >> m){
            for (int i = 0; i <= n; i++) for (int j = 0; j <= n; j++){
                a[i][j] = E9;
            }
            memset(p, 0, sizeof(p));
            for (int i = 0; i < m; i++){
                cin >> aa >> bb >> dd;
                a[aa][bb] = min(a[aa][bb], dd);
            }
            cin >> aa;
            bellman_ford(aa, n);
            int t = nega_cyc(n);
            if(t){
                for (int i = 1; i <= n; i++) cout << d[i] << " \n"
                    [i==n];
                for (int i = 1; i <= n; i++) cout << p[i] << " \n"
                    [i==n];
            }
            else cout << "There is a negative weight cycle in the graph\n";
        }
    }
}

```

### 6.4 Kruskal

```

struct v {
    int a, b, c;
};

int p[200001]; v a[200001];

bool sor(v a, v b) {
    return a.c < b.c;
}

int find(int x) {
    return (x != p[x] ? (p[x] = find(p[x])) : x);
}

```

```

int main() {
    int n, m, i, j, sum;
    while (cin >> n >> m) {
        sum = 0;
        for (i = 0; i < 200001; i++) p[i] = i;
        for (i = 0; i < m; i++) cin >> a[i].a >> a[i].b >> a[i].c;
        sort(a, a + m, sor);
        for (i = 0, j = 0; j < m; j++) {
            if (find(a[j].a) != find(a[j].b)) {
                i++;
                p[find(a[j].a)] = find(a[j].b);
                sum += a[j].c;
            }
        }
        cout << ((i==n-1)?sum:-1) << endl;
    }
}

```

### 6.5 Dijkstra

```

struct node {
    int num{}, w{};
    bool operator < (const node& other) const {
        return w > other.w;
    }
};

vector<int> dijkstra(int root, const vector<vector<node>>& graph) {
    vector<int> d(graph.size(), INT_MAX >> 1), p(graph.size());
    priority_queue<node> pq;
    d[root] = p[root] = 0;
    pq.push({root, d[root]});
    while (!pq.empty()) {
        node tmp = pq.top(); pq.pop();
        for (const node &i : graph[tmp.num]) {
            if (d[i.num] > d[tmp.num] + i.w) {
                d[i.num] = d[tmp.num] + i.w;
                p[i.num] = tmp.num;
                pq.push({i.num, d[tmp.num]});
            }
        }
    }
    return d;
}

```

### 6.6 Strongly Connected Component(SCC)

```

#define MXN 100005
#define PB push_back
#define FZ(s) memset(s,0,sizeof(s))

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])

```



```

    if (!vst[v]) rDFS(v);
}
void solve(){
    nScc = 0;
    vec.clear();
    FZ(vst);
    for (int i=0; i<n; i++){
        if (!vst[i]) DFS(i);
    }
    reverse(vec.begin(),vec.end());
    FZ(vst);
    for (auto v : vec){
        if (!vst[v]){
            rDFS(v);
            nScc++;
        }
    }
}
};

```

## 6.7 Hungarian

// Maximum Cardinality Bipartite Matching

```

struct Graph {
    static const int MAXN = 5005;
    vector<int> G[MAXN];
    int n;
    int match[MAXN]; // Matching Result
    int vis[MAXN];

    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) G[i].clear();
    }

    bool dfs(int u) {
        for (auto v:G[u]) {
            if (!vis[v]) {
                vis[v] = true;
                if (match[v] == -1 || dfs(match[v])) {
                    match[v] = u;
                    match[u] = v;
                    return true;
                }
            }
        }
        return false;
    }

    int solve() {
        int res = 0;
        memset(match, -1, sizeof(match));
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                memset(vis, 0, sizeof(vis));
                if (dfs(i)) res += 1;
            }
        }
        return res;
    }
} graph;

```

## 6.8 KM

Detect non-perfect-matching:  
 1. set all edge[i][j] as INF  
 2. if solve() >= INF, it is **not** perfectmatching.

// Maximum Weight Perfect Bipartite Matching  
 // allow negative weight!

```

typedef long long Int;
struct KM {
    static const int MAXN = 1050;
    static const int INF = 1LL<<60;
    int n, match[MAXN], vx[MAXN], vy[MAXN];
    Int edge[MAXN][MAXN], lx[MAXN], ly[MAXN], slack[
        MAXN];

```

```

void init(int _n){
    n = _n;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            edge[i][j] = 0;
}
void add_edge(int x, int y, Int w){
    edge[x][y] = w;
}
bool DFS(int x){
    vx[x] = 1;
    for (int y = 0; y < n; y++) {
        if (vy[y]) continue;
        if (lx[x] + ly[y] > edge[x][y]) {
            slack[y] = min(slack[y], lx[x] + ly[y]
                - edge[x][y]);
        } else {
            vy[y] = 1;
            if (match[y] == -1 || DFS(match[y])) {
                match[y] = x;
                return true;
            }
        }
    }
    return false;
}
Int solve() {
    fill(match, match + n, -1);
    fill(lx, lx + n, -INF);
    fill(ly, ly + n, 0);
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            lx[i] = max(lx[i], edge[i][j]);
    for (int i = 0; i < n; i++) {
        fill(slack, slack + n, INF);
        while (true){
            fill(vx, vx + n, 0);
            fill(vy, vy + n, 0);
            if (DFS(i)) break;
            Int d = INF;
            for (int j = 0; j < n; j++)
                if (!vy[j]) d = min(d, slack[j]);
            for (int j = 0; j < n; j++) {
                if (vx[j]) lx[j] -= d;
                if (vy[j]) ly[j] += d;
                else slack[j] -= d;
            }
        }
    }
    Int res = 0;
    for (int i = 0; i < n; i++) {
        res += edge[ match[i] ][i];
    }
    return res;
}
} graph;

```

## 6.9 最小平均環

// from BCW

```

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
    int v,u;
    double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;

```

```

    if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
    }
}
}
double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    for(int i=0; i<n; i++) vst[i] = 0;
    edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i-][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}

```

## 6.10 偵測負環

```

#include <bits/stdc++.h>
using namespace std;

const int INF = 1000000;
const int MAXN = 200;
int n, m, q;
int d[MAXN][MAXN];

int main () {
    while ( cin >> n >> m >> q && n ) {

        for ( int i = 0 ; i <= n ; i++ ) {
            for ( int j = 0 ; j <= n ; j++ ) d[i][j] = (i==j ? 0 : INF);
        }

        for ( int i = 0 ; i < m ; i++ ) {
            int a, b, c;
            cin >> a >> b >> c;
            d[a][b] = min(d[a][b], c);
        }

        for ( int k = 0 ; k < n ; k++ ) {
            for ( int i = 0 ; i < n ; i++ ) {
                for ( int j = 0 ; j < n ; j++ ) {
                    if ( d[i][j] > d[i][k] + d[k][j] &&
                        d[i][k] < INF && d[k][j] < INF ) {
                        //printf("%d > %d + %d\n", d[i][j], d[i][k], d[k][j]);
                        //if ( d[i][k] >= INF || d[k][j] >= INF ) cout << "NO : "
                        << i << " " << j << " " <<
                        k << "--";
                        d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
                    }
                }
            }
        }
    }
}

```

```

    }
}

for ( int i = 0 ; i < n ; i++ ) {
    for ( int j = 0 ; j < n ; j++ ) {
        for ( int k = 0 ; k < n && d[i][j] != -
            INF ; k++ ) {
            if ( d[k][k] < 0 && d[i][k] != INF
                && d[k][j] != INF )
                d[i][j] = -INF;
        }
    }
}

int u, v;
for (int i=0; i<q; i++){
    scanf("%d%d",&u,&v);

    if (d[u][v] == INF) printf("Impossible\n");
    else if (d[u][v] == -INF) printf("-Infinity\n");
    else printf("%d\n",d[u][v]);
}

puts("");
}

return 0;
}

```

## 6.11 Tarjan

割點

點  $u$  為割點 **if and only if** 滿足 1. **or** 2.

1.  $u$  為樹根，且  $u$  有多於一個子樹。
2.  $u$  不為樹根，且滿足存在  $(u, v)$  為樹枝邊（或稱父子邊，即  $u$  為  $v$  在搜索樹中的父親），使得  $DFN(u) \leq Low(v)$ 。

橋

一條無向邊  $(u, v)$  是橋 **if and only if**  $(u, v)$  為樹枝邊，且滿足  $DFN(u) < Low(v)$ 。

```

// 0 base
struct TarjanSCC{
    static const int MAXN = 1000006;
    int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
    vector<int> G[MAXN];
    stack<int> stk;
    bool ins[MAXN];

    void tarjan(int u){
        dfn[u] = low[u] = ++count;
        stk.push(u);
        ins[u] = true;

        for(auto v:G[u]){
            if(!dfn[v]){
                tarjan(v);
                low[u] = min(low[u], low[v]);
            }else if(ins[v]){
                low[u] = min(low[u], dfn[v]);
            }
        }

        if(dfn[u] == low[u]){
            int v;
            do {
                v = stk.top();
                stk.pop();
                scc[v] = scn;
                ins[v] = false;
            } while(v != u);
            scn++;
        }
    }

    void getSCC(){
        memset(dfn,0,sizeof(dfn));
        memset(low,0,sizeof(low));
        memset(ins,0,sizeof(ins));
    }
}

```

```

memset(scc,0,sizeof(scc));
count = scn = 0;
for(int i = 0 ; i < n ; i++){
    if(!dfn[i]) tarjan(i);
}
}SCC;

```

## 6.12 Topological Sort

```

#define N 87

bool adj[N][N];    // adjacency matrix
int visit[N];      // record visited coordinations in DFS
int order[N], n;   // save the order

bool cycle;        // detect the cycle

void DFS(int s)
{
    // back edge occured, detected the cycle
    if (visit[s] == 1) {cycle = true; return;}
    // forward edge and cross edge;C
    if (visit[s] == 2) return;

    visit[s] = 1;
    for (int t=0; t<N; ++t){
        if (adj[s][t]) DFS(t);
    }
    visit[s] = 2;
    order[n--] = s;    // record the order
}

void topological_ordering()
{
    memset(visit, 0, sizeof(visit));
    cycle = false;
    n = N - 1;

    for (int s=0; s<9; ++s)
        if (!v[s])
            DFS(s);

    if (cycle) cout << "The graph has the cycle!";
    else{
        for (int i=0; i<N; ++i)
            cout << order[i];
    }
}

```

## 7 Data Structure

### 7.1 2D Range Tree

```

// remember sort x !!!!!
typedef int T;
const int LGN = 20;
const int MAXN = 100005;

struct Point{
    T x, y;
    friend bool operator < (Point a, Point b){
        return tie(a.x,a.y) < tie(b.x,b.y);
    }
};

struct TREE{
    Point pt;
    int toleft;
}tree[LGN][MAXN];
struct SEG{
    T mx, Mx;
    int sz;
    TREE *st;
}seg[MAXN*4];

```

```

vector<Point> P;

void build(int l, int r, int o, int deep){
    seg[o].mx = P[l].x;
    seg[o].Mx = P[r].x;
    seg[o].sz = r-l+1;

    if(l == r){
        tree[deep][r].pt = P[r];
        tree[deep][r].toleft = 0;
        seg[o].st = &tree[deep][r];
        return;
    }
    int mid = (l+r)>>1;
    build(l,mid,o+o,deep+1);
    build(mid+1,r,o+o+1,deep+1);

    TREE *ptr = &tree[deep][l];
    TREE *pl = &tree[deep+1][l], *nl = &tree[deep+1][mid+1];
    TREE *pr = &tree[deep+1][mid+1], *nr = &tree[deep+1][r+1];

    int cnt = 0;
    while(pl != nl && pr != nr) {
        *(ptr) = pl->pt.y <= pr->pt.y ? cnt++, *(pl++);
        *(pr++)
        ptr -> toleft = cnt; ptr++;
    }
    while(pl != nl) *(ptr) = *(pl++), ptr -> toleft = ++cnt, ptr++;
    while(pr != nr) *(ptr) = *(pr++), ptr -> toleft = cnt, ptr++;
}

int main(){
    int n; cin >> n;
    for(int i = 0 ; i < n ; i++){
        T x,y; cin >> x >> y;
        P.push_back((Point){x,y});
    }
    sort(P.begin(),P.end());
    build(0,n-1,1,0);
}

```

### 7.2 Segment Tree

```

struct Node{
    int mx; // 區間最大值
    int tag; // 子樹裡所有人的'值'都要加上 tag
};

vector<Node> seg;

// 節點 id 的整個區間要加上 tag
void addtag(int tag, int id){
    seg[id].mx += tag; // 最大值會加上 tag
    seg[id].tag += tag; // 注意可能本來就有標記了，所以是 +=
}

// 更新子節點資訊並把標記移到子節點身上
void push(int id){
    addtag(seg[id].tag, lc);
    addtag(seg[id].tag, rc);
    seg[id].tag = 0; // 標記被移到子節點上所以要改成 0
}

// 區間 [L,r] 加上 v
void modify(int l, int r, int v, int L, int R, int id){
    if(l <= L && R <= r){
        addtag(v, id);
        return;
    }
    push(id);
    if(r <= M) modify(l, r, v, L, M, lc);
    else if(l > M) modify(l, r, v, M + 1, R, rc);
}

```

```

    else{
        modify(l, r, v, L, M, lc);
        modify(l, r, v, M + 1, R, rc);
    }
    seg[id].mx = max(seg[lc].mx, seg[rc].mx);
}

int query(int l, int r, int L, int R, int id){
    if(l <= L && R <= r) return seg[id].mx;
    push(id);
    int M = (L + R) / 2;
    if(r <= M) return query(l, r, L, M, lc);
    else if(l > M) return query(l, r, M + 1, R, rc);
    else return max(query(l, r, L, M, lc),
                    query(l, r, M + 1, R, rc));
}

```

### 7.3 ZKW 線段樹

```

const int M=1e5+111;
int n,m,q;
int sum[M<<2],mn[M<<2],mx[M<<2],add[M<<2];

int read() {
    int x;
    cin >> x;
    return x;
}

void build(){
    for(m=1;m<=n;m<=1){
        for(int i=m+1;i<=m+n;++i)
            sum[i]=mn[i]=mx[i]=read();
        for(int i=m-1;i--){
            sum[i]=sum[i<<1]+sum[i<<1|1];
            mn[i]=min(mn[i<<1],mn[i<<1|1]);
            mx[i]=max(mx[i<<1],mx[i<<1|1]);
            mx[i<<1]-=mx[i],mx[i<<1|1]-=mx[i];
        }
    }

void update_node(int x,int v,int A=0){
    x+=m,mx[x]+=v,mn[x]+=v,sum[x]+=v;
    for(;x>1;x>>=1){
        sum[x]+=v;
        A=min(mn[x],mn[x^1]);
        mn[x]-=A,mn[x^1]-=A,mn[x>>1]+=A;
        A=max(mx[x],mx[x^1]);
        mx[x]-=A,mx[x^1]-=A,mx[x>>1]+=A;
    }

void update_part(int s,int t,int v){
    int A=0,lc=0,rc=0,len=1;
    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<=1){
        if(s&1^1) add[s^1]+=v,lc+=len, mn[s^1]+=v,mx[s^1]+=v;
        if(t&1) add[t^1]+=v,rc+=len, mn[t^1]+=v,mx[t^1]+=v;
        sum[s>>1]+=v*lc, sum[t>>1]+=v*rc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s>>1]+=A;
        A=min(mn[t],mn[t^1]),mn[t]-=A,mn[t^1]-=A,mn[t>>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s>>1]+=A;
        A=max(mx[t],mx[t^1]),mx[t]-=A,mx[t^1]-=A,mx[t>>1]+=A;
    }
    for(lc+=rc;s>>=1;s>>=1){
        sum[s>>1]+=v*lc;
        A=min(mn[s],mn[s^1]),mn[s]-=A,mn[s^1]-=A,mn[s>>1]+=A;
        A=max(mx[s],mx[s^1]),mx[s]-=A,mx[s^1]-=A,mx[s>>1]+=A;
    }

int query_node(int x,int ans=0){
    for(x+=m;x>>=1) ans+=mn[x]; return ans;
}

int query_sum(int s,int t){
    int lc=0,rc=0,len=1,ans=0;

```

```

    for(s+=m-1,t+=m+1;s^t^1;s>>=1,t>>=1,len<=1){
        if(s&1^1) ans+=sum[s^1]+len*add[s^1],lc+=len;
        if(t&1) ans+=sum[t^1]+len*add[t^1],rc+=len;
        if(add[s>>1]) ans+=add[s>>1]*lc;
        if(add[t>>1]) ans+=add[t>>1]*rc;
    }
    for(lc+=rc,s>>=1;s>>=1) if(add[s]) ans+=add[s]*lc;
    return ans;
}

int query_min(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mn[s],R+=mn[t];
        if(s&1^1) L=min(L,mn[s^1]);
        if(t&1) R=min(R,mn[t^1]);
    }
    for(ans=min(L,R),s>>=1;s>>=1) ans+=mn[s];
    return ans;
}

int query_max(int s,int t,int L=0,int R=0,int ans=0){
    if(s==t) return query_node(s);
    for(s+=m,t+=m;s^t^1;s>>=1,t>>=1){
        L+=mx[s],R+=mx[t];
        if(s&1^1) L=max(L,mx[s^1]);
        if(t&1) R=max(R,mx[t^1]);
    }
    for(ans=max(L,R),s>>=1;s>>=1) ans+=mx[s];
    return ans;
}

```

### 7.4 Sparse Table

```

const int MAXN = 200005;
const int lgN = 20;
/* Sp[i][j] 為區間 [i, i + 2^j - 1] 的值 */
/* 從 i 開始 長度為 2^j */
/* 解決可重複貢獻問題 */
struct SP{ //sparse table
    int Sp[MAXN][lgN];
    function<int(int,int)> opt;
    void build(vector<int> &nums){ // 0 base
        for (int i = 0; i < nums.size(); i++) Sp[i][0]=nums[i];

        for (int h = 1; h < lgN; h++) {
            int len = 1 << (h - 1), i=0;
            for (; i + len < nums.size(); i++)
                Sp[i][h] = opt(Sp[i][h-1], Sp[i+len][h-1]);
            for (; i < nums.size(); i++)
                Sp[i][h] = Sp[i][h-1];
        }
    }

    int query(int l, int r){
        int h = __lg(r-l+1);
        int len = 1<<h;
        return opt(Sp[l][h], Sp[r-len+1][h] );
    }
};

```

### 7.5 Lazy Tag

```

void modify(type value, int l, int r, int L, int R,
            vertex v){
    if(l == L && r == R){
        //打懶標在v上;
        return;
    }
    int M = (L + R) / 2;
    if(r <= M) modify(value, l, r, L, M, //v的左子節點);
    else if(l > M) modify(value, l, r, M + 1, R, //v的右子節點);
    else{
        modify(value, l, M, L, M, v的左子節點);
        modify(value, M + 1, r, M + 1, R, //v的右子節點);
    }
}

```

```

    }
    //用兩個子節點的答案更新v的答案;
}

```

## 7.6 BIT 樹狀樹組

```

class Bitree {
public:
    /* bit 一定是 1 indexed */
    vector<int> data;
    Bitree(const vector<int> &nums) {
        data.resize(nums.size() + 1, 0);
        for(int i = 0; i < nums.size(); i++) {
            update(i, nums[i]);
        }
    }
    void update(int x, int val) {
        x++; /*變成 1 indexed*/
        for(; x < data.size(); x += lowbit(x)) {
            data[x] += val;
        }
    }
    int query(int x) {
        x++; /*變成 1 indexed*/
        int result = 0;
        for(; x > 0; x -= lowbit(x)) {
            result += data[x];
        }
        return result;
    }
    static int lowbit(int x) {
        return x & (-x);
    }
};

```

## 7.7 並查集 union find

```

struct DisjointSet {
    vector<int> parent, sz; // parent[i] = 父節點, sz[
    i] = 集合大小
    void init(int n) {
        parent.resize(n + 1);
        sz.assign(n + 1, 1);
        for (int i = 0; i <= n; i++) {
            parent[i] = i;
        }
    }
    int find(int x) {
        if (parent[x] != x) {
            parent[x] = find(parent[x]); // 路徑壓縮
        }
        return parent[x];
    }
    bool unite(int x, int y) {
        x = find(x);
        y = find(y);
        if (x == y) return false; // 已在同一集合
        // 啟發式合併：小的掛到大的下面
        if (sz[x] < sz[y]) swap(x, y);
        parent[y] = x;
        sz[x] += sz[y];
        return true;
    }
    bool same(int x, int y) {
        return find(x) == find(y);
    }
};

```

## 8 String

### 8.1 KMP

```

template<typename T>
void build_KMP(int n, T *s, int *f){ // 1 base
    f[0]=-1, f[1]=0;
    for (int i=2; i<=n; i++){
        int w = f[i-1];
        while (w>=0 && s[w+1]!=s[i])w = f[w];
        f[i]=w+1;
    }
}

template<typename T>
int KMP(int n, T *a, int m, T *b){
    build_KMP(m,b,f);
    int ans=0;

    for (int i=1, w=0; i<=n; i++){
        while ( w>=0 && b[w+1]!=a[i] )w = f[w];
        w++;
        if (w==m){
            ans++;
            w=f[w];
        }
    }
    return ans;
}

```

### 8.2 smallest rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}
/*
Booth 演算法
用於尋找一個字串的字典序最小的循環旋轉
*/
Contact GitHub API Training Shop Blog About

```

### 8.3 Suffix Array

```

/*he[i]保存了在了後綴數組中相鄰兩個後綴的最長公共前綴長度
*sa[i]表示的是字典序排名為i的後綴是誰（字典序越小的排名越靠前）
*rk[i]表示的是後綴我所對應的排名是多少 */

const int MAX = 1020304;
int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
    int len = strlen(ip);
    int alp = 256;
    memset(ct, 0, sizeof(ct));
    for(int i=0;i<len;i++) ct[ip[i]+1]++;
    for(int i=1;i<alp;i++) ct[i]+=ct[i-1];
    for(int i=0;i<len;i++) rk[i]=ct[ip[i]];
    for(int i=1;i<len;i*=2){
        for(int j=0;j<len;j++){
            if(j+i>len) tp[j][1]=0;
            else tp[j][1]=rk[j+i]+1;
            tp[j][0]=rk[j];
        }
        memset(ct, 0, sizeof(ct));
        for(int j=0;j<len;j++) ct[tp[j][1]+1]++;
        for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
        for(int j=0;j<len;j++) tsa[ct[tp[j][1]]+1]=j;
        memset(ct, 0, sizeof(ct));
    }
}

```

```

for(int j=0;j<len;j++) ct[tp[j][0]+1]++;
for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
for(int j=0;j<len;j++)
    sa[ct[tp[tsa[j]][0]]++] = tsa[j];
rk[sa[0]] = 0;
for(int j=1;j<len;j++){
    if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
        tp[sa[j]][1] == tp[sa[j-1]][1] )
        rk[sa[j]] = rk[sa[j-1]];
    else
        rk[sa[j]] = j;
}
}
for(int i=0,h=0;i<len;i++){
    if(rk[i]==0) h=0;
    else{
        int j=sa[rk[i]-1];
        h=max(0,h-1);
        for(;ip[i+h]==ip[j+h];h++);
    }
    he[rk[i]] = h;
}
}

```

## 8.4 Z-value

```

z[0] = 0;
for ( int bst = 0, i = 1; i < len ; i++ ) {
    if ( z[bst] + bst <= i ) z[i] = 0;
    else z[i] = min(z[i - bst], z[bst] + bst - i);
    while ( str[i + z[i]] == str[z[i]] ) z[i]++;
    if ( i + z[i] > bst + z[bst] ) bst = i;
}

// 回文版

void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
    // centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
            s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}

```

## 8.5 旋轉哈希

```

typedef unsigned __int128 ull1;

ull1 power(ull1 a, ull1 n, ull1 m) {
    ull1 re = 1;
    while (n > 0) {
        if (n & 1) re = re * a % m;
        a = a * a % m;
        n >>= 1;
    }
    return re;
}

ull1 inv(ull1 a, ull1 m) {
    return power(a, m - 2, m);
}

struct Rh {
    const ull1 p, mod;
    vector<ull1> ps{1};
    Rh(ull1 p, ull1 mod) : p(p), mod(mod) {}
    vector<ull1> build(const string &s) {
        vector<ull1> h(s.size() + 1);
        h[0] = 0;

```

```

        ps.resize(s.size() + 1);
        for (int i = 0; i < s.size(); i++) {
            ps[i + 1] = ps[i] * p % mod;
            h[i + 1] = (h[i] + s[i] * ps[i + 1] % mod) % mod;
        }
        return h;
    }
    ull1 subhash(const vector<ull1> &h, int l, int r) {
        // [l, r] 指原字串
        return ((h[r + 1] - h[l]) * inv(ps[l], mod)) % mod;
    }
};

constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >> 29) + (m << 35 >> 3) + 1;
    ret = (ret & mod) + (ret>>61);
    ret = (ret & mod) + (ret>>61);
    return ret-1;
}

```

## 8.6 後綴自動機

```

struct state {
    int len{}, link{};
    array<int, 26> next{};
};

struct SAM {
    int sz{}, last{};
    vector<state> st;
    SAM(int maxlen) : st(maxlen * 2) {
        st[0].len = 0;
        st[0].link = -1;
        sz++;
        last = 0;
    }

    void insert(char c) {
        insert_impl(c - 'a');
    }

    void insert_impl(char c) {
        int cur = sz++;
        st[cur].len = st[last].len + 1;
        int p = last;
        while(p != -1 && !st[p].next[c]) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if(p == -1) {
            st[cur].link = 0;
        }
        else {
            int q = st[p].next[c];
            if(st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            }
            else {
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                st[clone].next = st[q].next;
                st[clone].link = st[q].link;
                while(p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
    }
};

```

## 9 Others

### 9.1 矩陣樹定理

新的方法介紹

下面我們介紹一個新的方法—Matrix-Tree定理(Kirchhoff矩陣-樹定理)。

Matrix-Tree定理是解決生成樹數問題最有力的武器之一。它首先於1847年被Kirchhoff證明。在介紹定理之前，我們先先明確幾個概念：

1.  $G$ 的度數矩陣 $D[G]$ 是一個 $n \times n$ 的矩陣，並且滿足：當 $i \neq j$ 時， $d_{ij} = 0$ ；當 $i = j$ 時， $d_{ij}$ 等於 $v_i$ 的度數。

2.  $G$ 的鄰接矩陣 $A[G]$ 也是一個 $n \times n$ 的矩陣，且滿足：若 $v_i$ 、 $v_j$ 之間有邊直接相連，則 $a_{ij} = 1$ ，否則為 $0$ 。

我們定義 $G$ 的Kirchhoff矩陣(也稱為拉普拉斯算子) $C[G]$ 為 $C[G] = D[G] - A[G]$ ，

則Matrix-Tree定理可以描述為： $G$ 的所有不同的生成樹的個數等於其Kirchhoff矩陣 $C[G]$ 任何一個 $n-1$ 階主子式的行列式的絕對值。

所謂 $n-1$ 階主子式，就是對於 $r (1 \leq r \leq n)$ ，將 $C[G]$ 的第 $r$ 行、第 $r$ 列同時去掉後所得到的新矩陣，以 $Cr[G]$ 表示。

生成樹計數

演算法步驟：

1、建構拉普拉斯矩陣

```
Matrix[i][j] =
degree(i) , i==j
-1 , i-j有邊
0 , 其他情況
```

2、去掉第 $r$ 行，第 $r$ 列 ( $r$ 任意)

3、計算矩陣的行列式

```
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn(double x)
{
    if(fabs(x) < eps)return 0;
    if(x < 0)return -1;
    else return 1;
}
double b[MAXN][MAXN];
double det(double a[][MAXN],int n)
{
    int i, j, k, sign = 0;
    double ret = 1;
    for(i = 0; i < n; i++)
    for(j = 0; j < n; j++) b[i][j] = a[i][j];
    for(i = 0; i < n; i++)
    {
        if(sgn(b[i][i]) == 0)
        {
            for(j = i + 1; j < n; j++)
            if(sgn(b[j][i]) != 0) break;
            if(j == n)return 0;
            for(k = i; k < n; k++) swap(b[i][k],b[j][k]);
            sign++;
        }
        ret *= b[i][i];
        for(k = i + 1; k < n; k++) b[i][k] /= b[i][i];
        for(j = i + 1; j < n; j++)
        for(k = i + 1; k < n; k++) b[j][k] -= b[j][i] * b[i][k];
    }
    if(sign & 1)ret = -ret;
    return ret;
}
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
{
```

```
int T;
int n,m;
int u,v;
scanf("%d",&T);
while(T-->0)
{
    scanf("%d%d",&n,&m);
    memset(g,0,sizeof(g));
    while(m-->0)
    {
        scanf("%d%d",&u,&v);
        u--;v--;
        g[u][v] = g[v][u] = 1;
    }
    memset(a,0,sizeof(a));
    for(int i = 0; i < n; i++)
    for(int j = 0; j < n; j++)
    if(i != j && g[i][j])
    {
        a[i][i]++;
        a[i][j] = -1;
    }
    double ans = det(a,n-1);
    printf("%.0Lf\n",ans);
}
return 0;
```

### 9.2 1D/1D dp 優化

```
#include<bits/stdc++.h>

int t, n, L;
int p;
char s[MAXN][35];
ll sum[MAXN] = {0};
long double dp[MAXN] = {0};
int prevd[MAXN] = {0};

long double pw(long double a, int n) {
    if ( n == 1 ) return a;
    long double b = pw(a, n/2);
    if ( n & 1 ) return b*b*a;
    else return b*b;
}

long double f(int i, int j) {
    // cout << (sum[i] - sum[j]+i-j-1-L) << endl;
    return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
}

struct INV {
    int L, R, pos;
};
INV stk[MAXN*10];
int top = 1, bot = 1;
void update(int i) {
    while ( top > bot && i < stk[top].L && f(stk[top].L, i) < f(stk[top].L, stk[top].pos) ) {
        stk[top - 1].R = stk[top].R;
        top--;
    }
    int lo = stk[top].L, hi = stk[top].R, mid, pos = stk[top].pos;
    //if ( i >= lo ) lo = i + 1;
    while ( lo != hi ) {
        mid = lo + (hi - lo) / 2;
        if ( f(mid, i) < f(mid, pos) ) hi = mid;
        else lo = mid + 1;
    }
    if ( hi < stk[top].R ) {
        stk[top + 1] = (INV) { hi, stk[top].R, i };
        stk[top++].R = hi;
    }
}

int main() {
    cin >> t;
    while ( t-- ) {
        cin >> n >> L >> p;
        dp[0] = sum[0] = 0;
        for ( int i = 1 ; i <= n ; i++ ) {
```



```

    cin >> s[i];
    sum[i] = sum[i-1] + strlen(s[i]);
    dp[i] = numeric_limits<long double>::max();
}
stk[top] = (INV) {1, n + 1, 0};
for ( int i = 1 ; i <= n ; i++ ) {
    if ( i >= stk[bot].R ) bot++;
    dp[i] = f(i, stk[bot].pos);
    update(i);
    cout << (ll) f(i, stk[bot].pos) << endl;
}
if ( dp[n] > 1e18 ) {
    cout << "Too hard to arrange" << endl;
} else {
    vector<PI> as;
    cout << (ll)dp[n] << endl;
}
}
return 0;
}

```

### 9.3 Theorm - DP optimization

Monotonicity & 1D/1D DP & 2D/1D DP

Definition xD/yD

1D/1D DP[j] = min(0≤i<j) { DP[i] + w(i, j) }; DP[0] = k  
 2D/1D DP[i][j] = min(i<k≤j) { DP[i][k - 1] + DP[k][j] }  
 + w(i, j); DP[i][i] = 0

Monotonicity

	c	d
a	w(a, c)	w(a, d)
b	w(b, c)	w(b, d)

Monge Condition

Concave(凹四邊形不等式):  $w(a, c) + w(b, d) \geq w(a, d) + w(b, c)$

Convex (凸四邊形不等式):  $w(a, c) + w(b, d) \leq w(a, d) + w(b, c)$

Totally Monotone

Concave(凹單調):  $w(a, c) \leq w(b, d) \rightarrow w(a, d) \leq w(b, c)$

Convex (凸單調):  $w(a, c) \geq w(b, d) \rightarrow w(a, d) \geq w(b, c)$

1D/1D DP  $O(n^2) \rightarrow O(n \lg n)$

\*\*CONSIDER THE TRANSITION POINT\*\*

Solve 1D/1D Concave by Stack

Solve 1D/1D Convex by Deque

2D/1D Convex DP (Totally Monotone)  $O(n^3) \rightarrow O(n^2)$

$h(i, j - 1) \leq h(i, j) \leq h(i + 1, j)$

### 9.4 Stable Marriage

// normal stable marriage problem

// input:

//3

//Albert Laura Nancy Marcy

//Brad Marcy Nancy Laura

//Chuck Laura Marcy Nancy

//Laura Chuck Albert Brad

//Marcy Albert Chuck Brad

//Nancy Brad Albert Chuck

#include<bits/stdc++.h>

using namespace std;

const int MAXN = 505;

int n;

int favor[MAXN][MAXN]; // favor[boy\_id][rank] = girl\_id

int order[MAXN][MAXN]; // order[girl\_id][boy\_id] = rank

int current[MAXN]; // current[boy\_id] = rank; boy\_id will pursue current[boy\_id] girl.

int girl\_current[MAXN]; // girl[girl\_id] = boy\_id;

void initialize() {

for ( int i = 0 ; i < n ; i++ ) {

current[i] = 0;

girl\_current[i] = n;

order[i][n] = n;

}

}

map<string, int> male, female;

string bname[MAXN], gname[MAXN];

int fit = 0;

void stable\_marriage() {

queue<int> que;

for ( int i = 0 ; i < n ; i++ ) que.push(i);

while ( !que.empty() ) {

int boy\_id = que.front();

que.pop();

int girl\_id = favor[boy\_id][current[boy\_id]];

current[boy\_id] ++;

if ( order[girl\_id][boy\_id] < order[girl\_id][

girl\_current[girl\_id]] ) {

if ( girl\_current[girl\_id] < n ) que.push( girl\_current[girl\_id]); // if not the first time

girl\_current[girl\_id] = boy\_id;

} else {

que.push(boy\_id);

}

}

}

int main() {

cin >> n;

for ( int i = 0 ; i < n ; i++ ) {

string p, t;

cin >> p;

male[p] = i;

bname[i] = p;

for ( int j = 0 ; j < n ; j++ ) {

cin >> t;

if ( !female.count(t) ) {

gname[fit] = t;

female[t] = fit++;

}

favor[i][j] = female[t];

}

}

for ( int i = 0 ; i < n ; i++ ) {

string p, t;

cin >> p;

for ( int j = 0 ; j < n ; j++ ) {

cin >> t;

order[female[p]][male[t]] = j;

}

}

initialize();

stable\_marriage();

for ( int i = 0 ; i < n ; i++ ) {

cout << bname[i] << " " << gname[favor[i][current[i] - 1]] << endl;

}

}

### 9.5 莫隊

/\* nums 長度 N ;; query 長度為 M \*/

```

/* O(N * sqrt(M)) */
struct Query {
    int l, r, id;
};

void add(int pos) {
    /*更新狀態*/
    /*將pos所在的移入集合*/
}

void del(int pos) {
    /*更新狀態*/
    /*將pos所在的移出集合*/
}

int bsz = n / sqrt(m); /*分塊大小 block size*/
sort(query.begin(), query.end(), [bsz](const Query &a,
    const Query &b){
    if(a.l / bsz != b.l / bsz) {
        return a.l < b.l;
    }
    return (a.l / bsz & 1 ? a.r < b.r : a.r > b.r);
});

int l = 1;
int r = 0;

vector<pair<int, int>> res(m);

for(int i = 0; i < query.size(); i++) {
    auto &q = query[i];
    /*順序不能換*/
    while (l > q.l) add(--l);
    while (r < q.r) add(++r);
    while (l < q.l) del(l++);
    while (r > q.r) del(r--);
    res[q.id] = /* 根據當前狀態求解 */
}

```

## 9.6 矩陣乘法

```

#define MOD INT_MAX
vector<vector<int>> operator *(const vector<vector<int>> &a,
    const vector<vector<int>> &b) {
    vector<vector<int>> re(a.size(), vector<int>(b[0].
        size()));
    for (int i = 0; i < a.size(); i++) {
        for (int j = 0; j < b[0].size(); j++) {
            for (int k = 0; k < b.size(); k++) {
                re[i][j] += (a[i][k] * b[k][j]) % MOD;
            }
        }
    }
    return re;
}

```

## 9.7 c++ 小抄

```

//pbds tree
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> tr;

tr.find_by_order(k) // O(LogN) 取得第k大的元素
tr.order_of_key(ele) // O(LogN) 得到ele是tree中第幾大(
    有幾個元素小於ele)

//pbds pair priority_queue
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;

```

```

priority_queue<int, less<int>, pairing_heap_tag> pq;
auto it = pq.push(x);
// type of it = priority_queue<int, less<int>,
    pairing_heap_tag>::point_iterator
pq.pop();
pq.top();
pq.join(b);
pq.empty();
pq.size();
pq.modify(it, 6); // O(LogN)
pq.erase(it);

//builtin functions
__builtin_popcount(x); // 1的個數
__builtin_popcountll(x); // for Long Long
__builtin_clz(x); // 前導0的個數
__builtin_ctz(x); // 後導0的個數
__builtin_parity(x); // 奇偶性

//溢位檢查
ret = __builtin_add_overflow(a, b, &res) // if ret = 1
    a+b 溢位
ret = __builtin_sub_overflow(a, b, &res) // if ret = 1
    a-b 溢位
ret = __builtin_mul_overflow(a, b, &res) // if ret = 1
    a*b 溢位
ret = __builtin_add_overflow_p(a, b, 0LL) // if ret = 1
    溢位 第三個參數是判斷的類型

//vector SIMD
typedef int v4si __attribute__((vector_size(4 * sizeof
    (int))));

//大質數表
{1000000007, 1000000009, 1000000021, 1000000033,
    1000000087, 1000000093, 1000000097, 1000000123,
    1000000321};

//mt19937
#include <random>
#include <chrono>

int getRandom(int l, int r) {
    static auto seed = std::chrono::system_clock::now()
        .time_since_epoch().count();
    static std::mt19937 gen(seed);
    std::uniform_int_distribution<int> dis(l, r);
    return dis(gen);
}

//sorted vector 去重
vec.erase(unique(vec.begin(), vec.end()), vec.end());

//std::valarray
valarray<int> a(初始值, 數量);
valarray<int> a(10);
valarray<int> b(10);
valarray<int> c = a + b;
valarray<int> d = a * b;
valarray<int> e = a + 10;
valarray<int> f = a * 10;
valarray<int> g = a.cshift(1); //循環左移
valarray<bool> equal = a == b;
int sum = a.sum();
int max = a.max();
int min = a.min();
std::valarray<int> g = a.apply([](int x) { return x * x
    ; });

//regex ***very slow***
#include <regex>
using namespace std;
bool res = regex_match("abc", regex("a.c"));
bool res = regex_match("abc", regex("A.c", regex::icase
    )); //忽略大小寫

```

## 9.8 python 小抄

```
#!/usr/bin/env python3
# 帕斯卡三角形
n = 10
dp = [ [1 for j in range(n)] for i in range(n) ]
for i in range(1,n):
    for j in range(1,n):
        dp[i][j] = dp[i][j-1] + dp[i-1][j]

for i in range(n):
    print( ' '.join( '{:5d}'.format(x) for x in dp[i] )
          )

# EOF1
while True:
    try:
        n, m = map(int, input().split())
    except:
        break

# EOF2
import sys
for s in sys.stdin:
    print(eval(s.replace("//", "\\/")))

# input a sequence of number
a = [ int(x) for x in input().split() ]
a.sort()
print( ' '.join( str(x)+' ' for x in a ) )

# LCS
ncase = int( input() )
for _ in range(ncase):
    n, m = [int(x) for x in input().split()]
    a, b = "$"+input(), "$"+input()
    dp = [ [int(0) for j in range(m+1)] for i in range(
        n+1) ]
    for i in range(1,n+1):
        for j in range(1,m+1):
            dp[i][j] = max(dp[i-1][j], dp[i][j-1])
            if a[i]==b[j]:
                dp[i][j] = max(dp[i][j], dp[i-1][j-1]+1)

    for i in range(1,n+1):
        print(dp[i][1:])
    print('a={:s}, b={:s}, |LCS(a,b)|={:d}'.format(a
        [1:], b[1:], dp[n][m]))

# list, dict, string
a = [1, 3, 4, 65, 65]
b = list.copy() # b = [1, 3, 4, 65], list a 跟 list b
互相獨立
cnt = list.count(65) # cnt == 2
loc = list.index(65) # loc == 3, find the leftmost
element, if not found then return ERROR
list.sort(reverse = True|False, key = None|lambda x:x
[1]) # list.sort has side effect but no return
value

# stack # C++
stack = [3,4,5]
stack.append(6) # push()
stack.pop() # pop()
stack[-1] # top()
len(stack) # size() O(1)

# queue # C++
from collections import deque
queue = deque([3,4,5])
queue.append(6) # push()
queue.popleft() # pop()
queue[0] # front()
len(queue) # size() O(1)
```

## 10 Persistence

### 9.9 萬年曆

$$h = \left( q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor + 5J \right) \bmod 7$$

h : 星期 (0 = 星期六, 1 = 星期日, 2 = 星期一, ...)

q : 日期 (日)

m : 月份 (3= 三月, 4= 四月, ...; 1、2 月視為前一年的 13、14 月)