Problem decomposition and approximation for shared mobility applications with endogenous congestion: integrated vehicle assignment and routing in capacitated transportation networks:

Jiangtao Liu (jliu215@asu.edu)
Xuesong Zhou (xzhou74@asu.edu)
School of Sustainable Engineering and the Built Environment

Pitu Mirchandani (pitu@asu.edu)
School of Computing, Informatics, and Decision Systems Engineering

Arizona State University

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

Outline

1. Introduction

2. Key Elements

3. Space-Time-State Network Flow Models

4. Decomposition: Dantzig-Wolfe decomposition

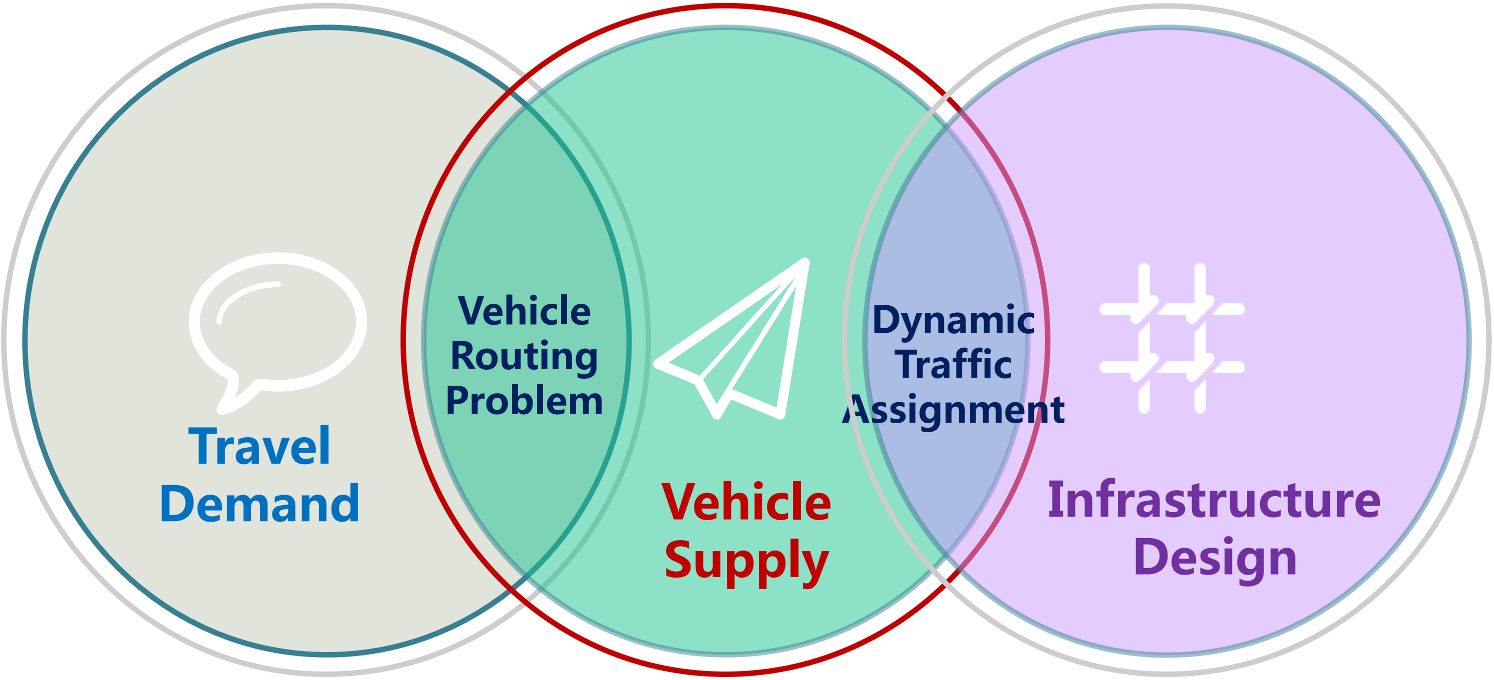
5. Decomposition: Column-pool based approximation

6. Discussion and Preliminary Experiments

7. Summary

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

1. Introduction



How to optimize demand, **supply** and infrastructure?

1. Introduction

Vehicle Routing Problem:

Input:

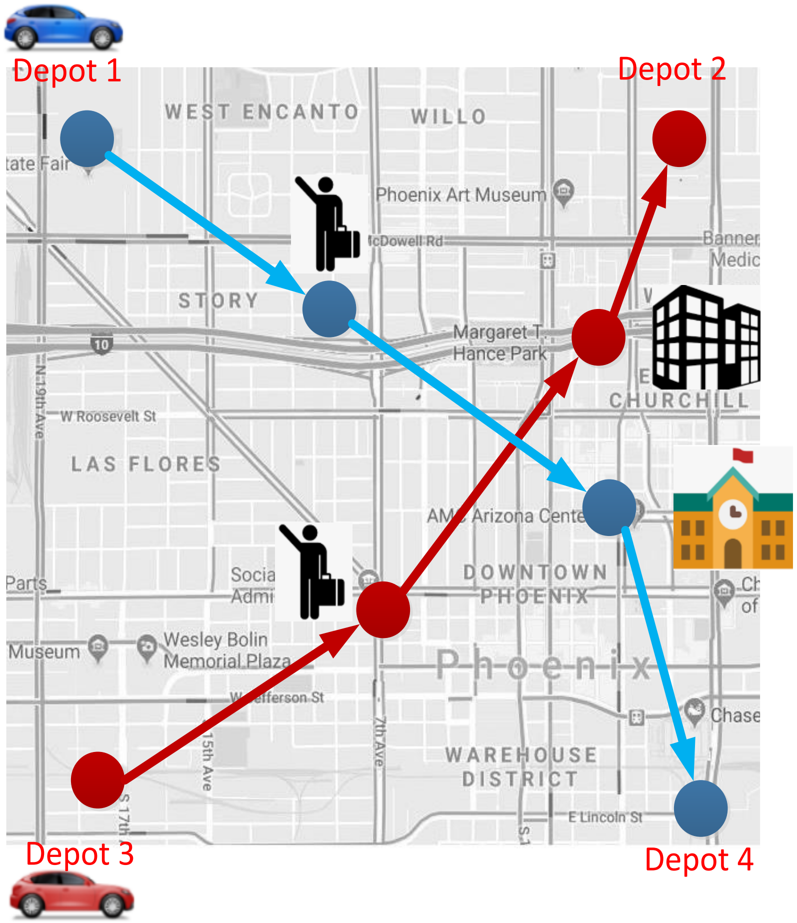
- ❑ **Network**: Virtual point-to-point network
- ❑ **Passenger trip request**: has specific pick-up and drop-off location with time windows
- ❑ **Vehicle carrying capacity:** considered

Goal:

- ❑ System Optimal

Output:

- ❑ **Vehicle-to-passenger assignment**: will be found
- ❑ **Variable**: discrete vehicle routing and scheduling



1. Introduction

Traffic Assignment Problem:

Input:

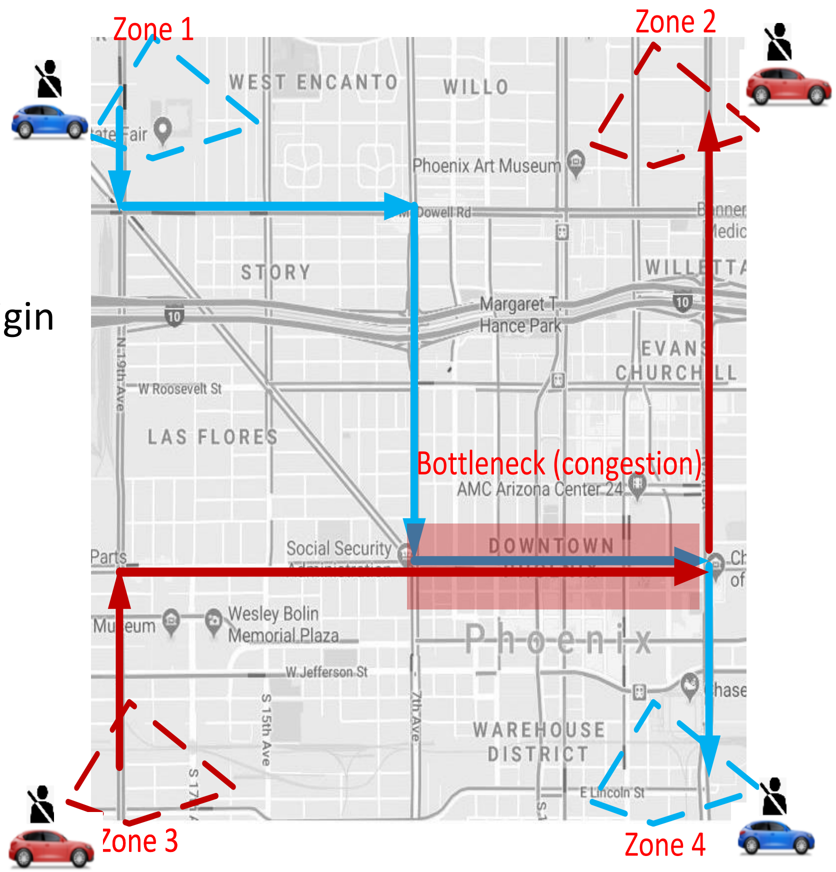
- ❑ **Network**: Physical traffic network
- ❑ **Road capacity**: capture road congestions
- ❑ **Origin and Destination**: vehicle has the same origin and destination with the assigned passengers

Goal:

- ❑ System Optimal or User Equilibrium

Output:

- ❑ continuous vehicle flow on links/paths



1. Introduction

Uncertain elements from the current to the future

Travel Demand	Vehicle Supply	Infrastructure
• **Trip request**: origin and destination with self-owned vehicles or submit pickup and drop-off locations with time windows for mobility providers • **Trip privacy**: alone or rideshare • **Trip mode**: single mode or multiple modes	• **Driving mode**: self-driving or human-driven • **Routing behavior**: selfish or coordinated • **Ownership**: household or mobility providers	• **Link/Lane capacity** change • **Sensor and communication** (V2V, V2I) • **Smart Transportation network** (road, parking, depots, bus line, rail transit line, etc.)

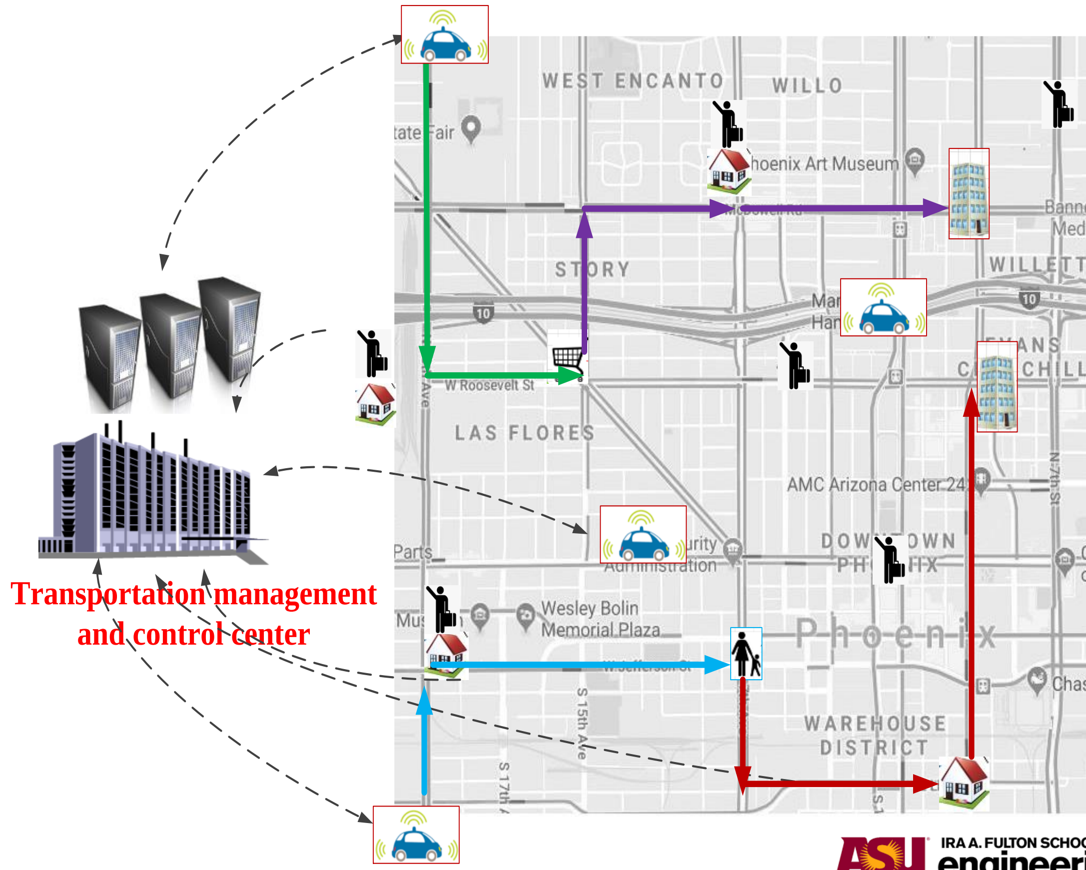
IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

1. Introduction

Special scenario: integration of travel demand, vehicle supply and infrastructure capacity

Keywords:

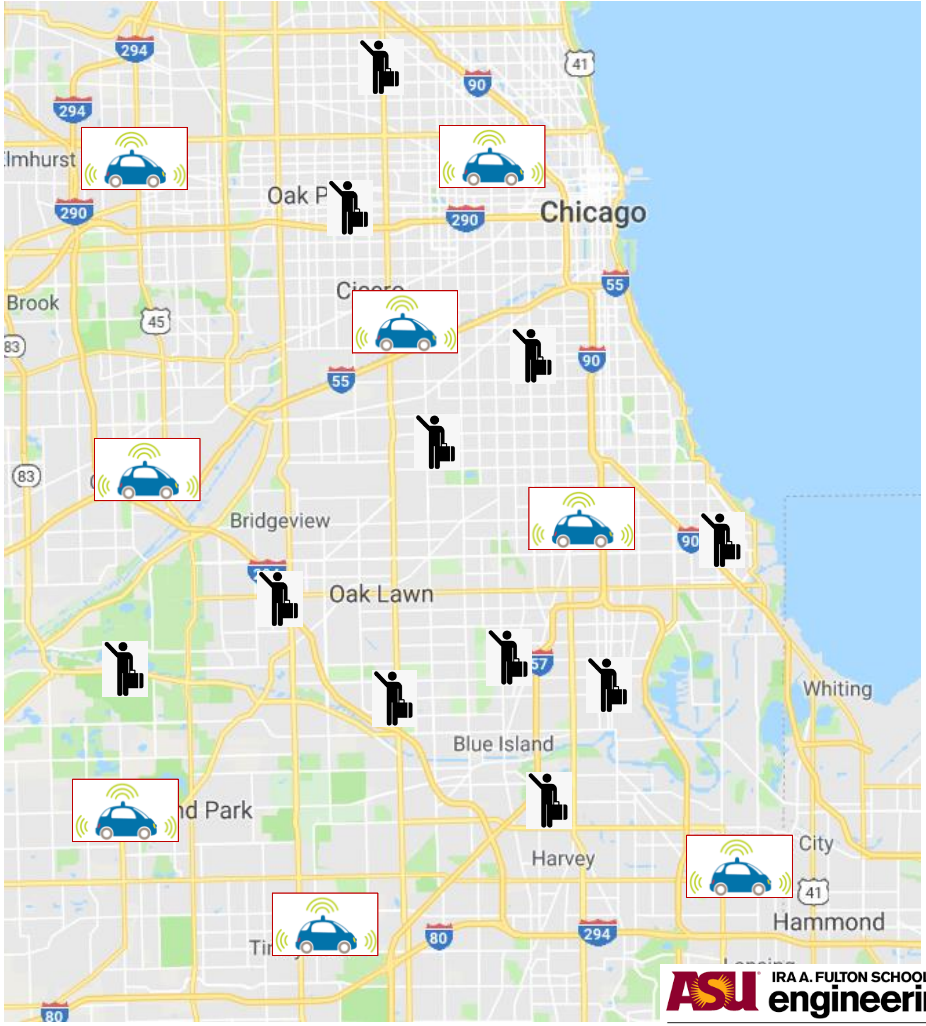
- **Physical traffic network** to consider **traffic congestion**
- **Trip requests** with Pickup and delivery with time windows
- **Autonomous vehicles with carrying capacity** for ride sharing
- **Central control (System optimal)**



1. Introduction

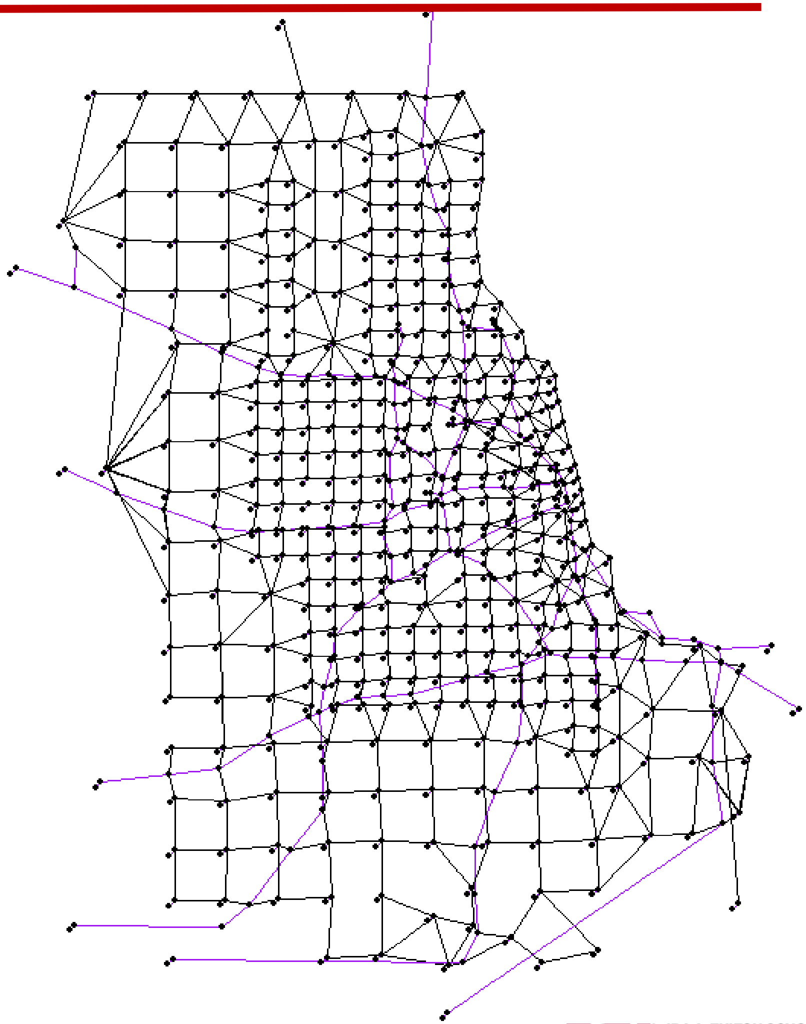
Key questions:

- ❑ How many autonomous vehicles do we need?
- ❑ How many passengers can we serve?
- ❑ How to capture the new traffic congestion?
- ❑ What is the best vehicle routing and vehicle-to-passenger assignment solution?



1. Introduction

# of nodes	1320
# of links	5431
optimization time horizon (min)	60
arc capacity each min (vehicle)	35
# of passenger groups with same OD and departure time	2226
Total trip requests	4452
# of vehicle depots	243

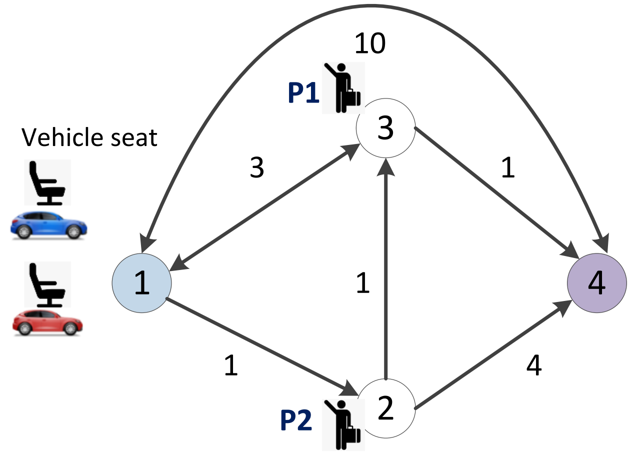


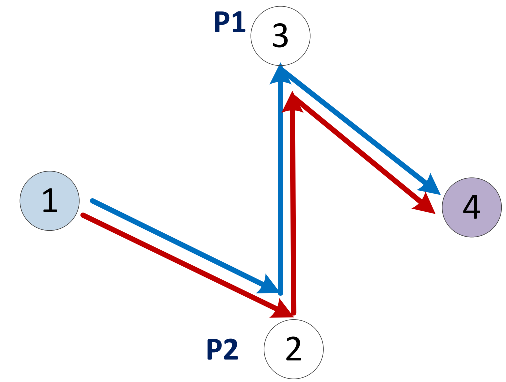
Chicago Sketch Network

2. Key Elements

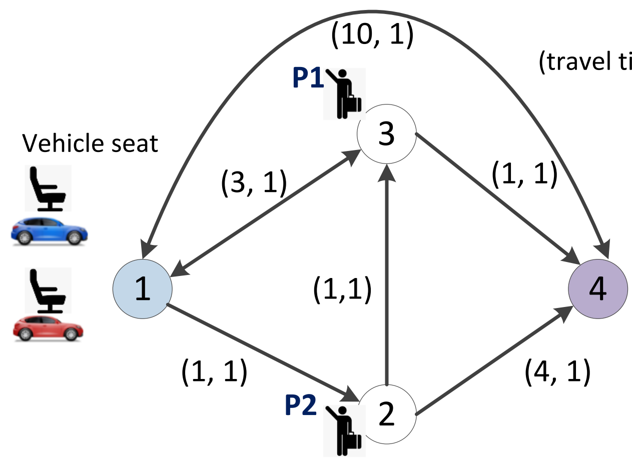
Link Capacity

Without link capacity:
Total cost is 6

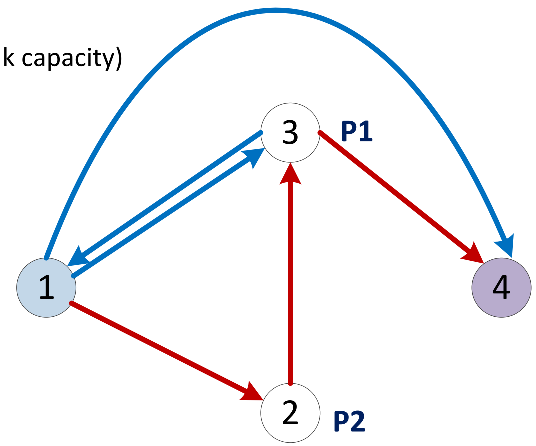




With link capacity:
Total cost is 15

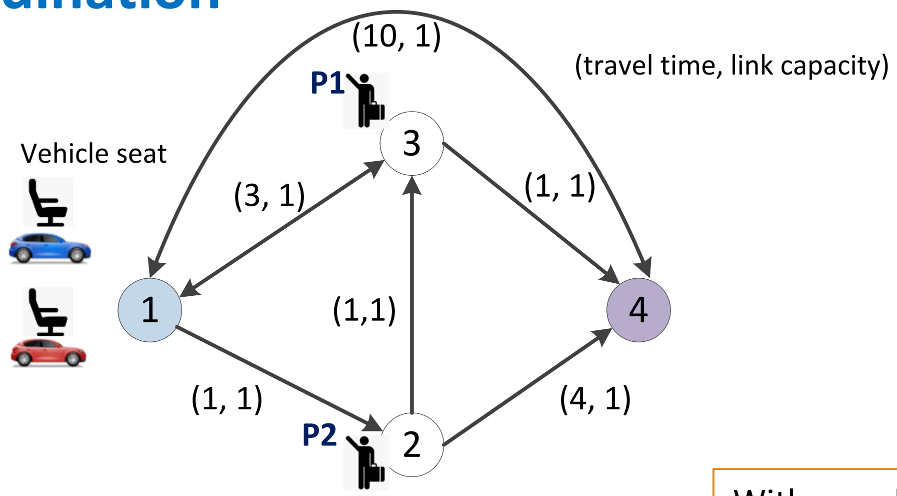


(travel time, link capacity)

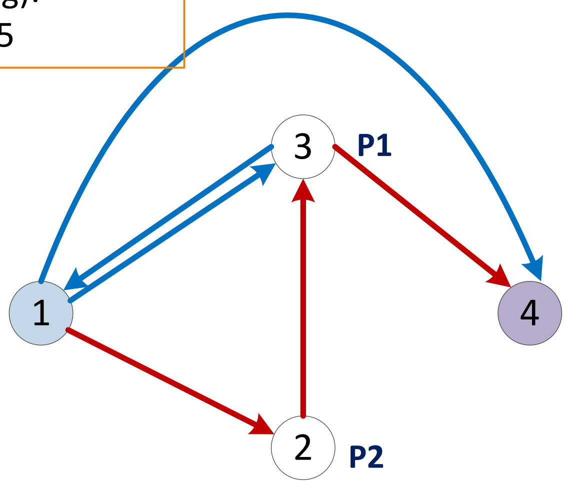


2. Key Elements

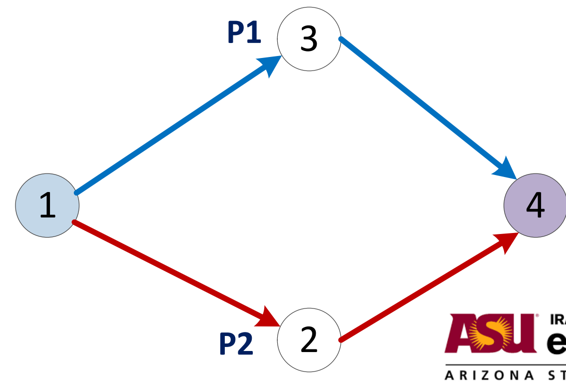
System Optimal Coordination



Without coordination
(selfish routing):
Total cost is 15

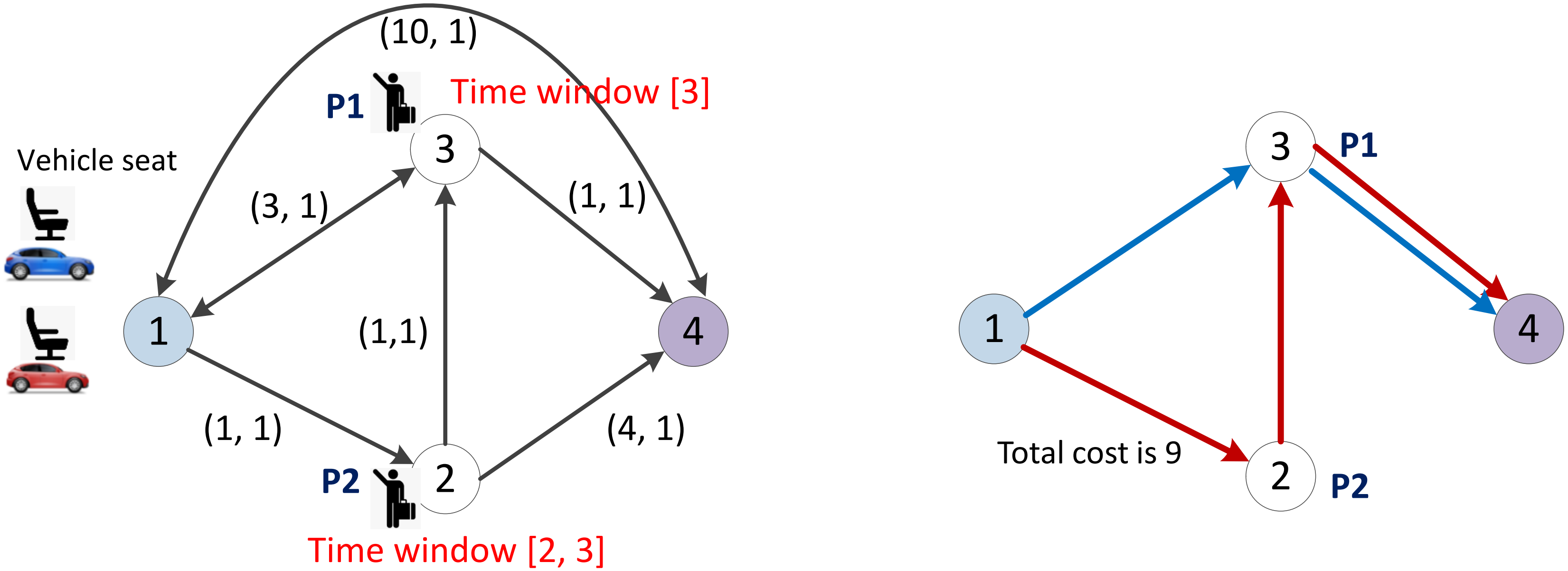


With coordination:
Total cost is 9



2. Key Elements

Time windows

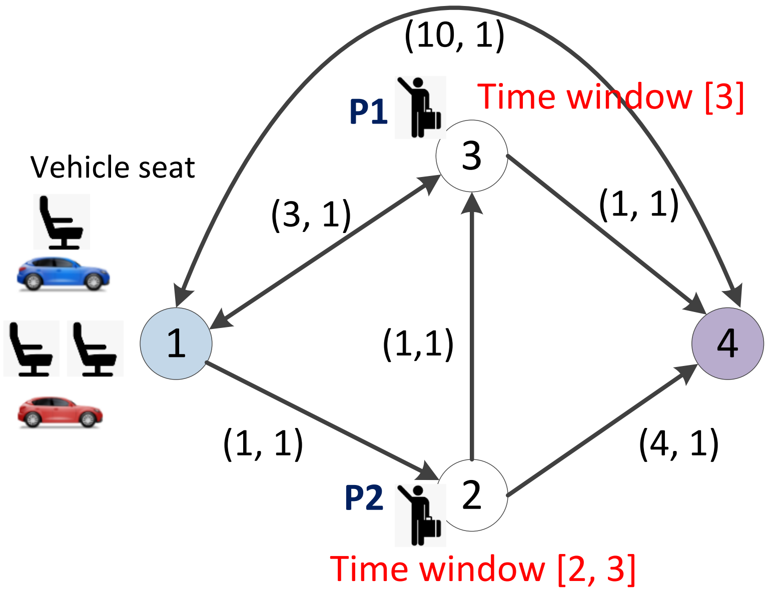


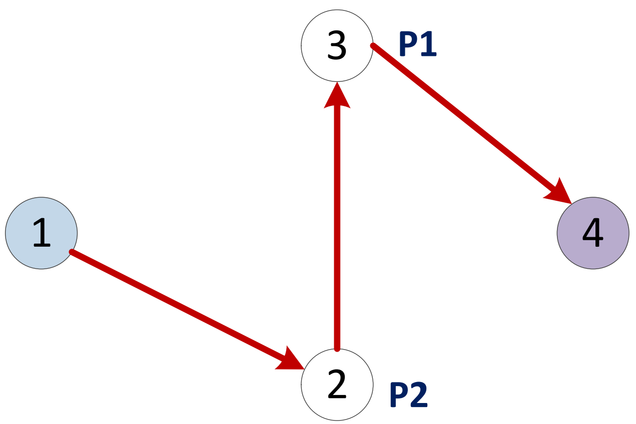
- The red vehicle can wait until time 3 to pick up passenger 2, so the blue vehicle can pick up passenger 1 at exact time 3.
- The optimal result doesn't only optimize the vehicle routing, but also the departure time of picked up passengers.

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

2. Key Elements

Ridesharing (vehicle carrying capacity)



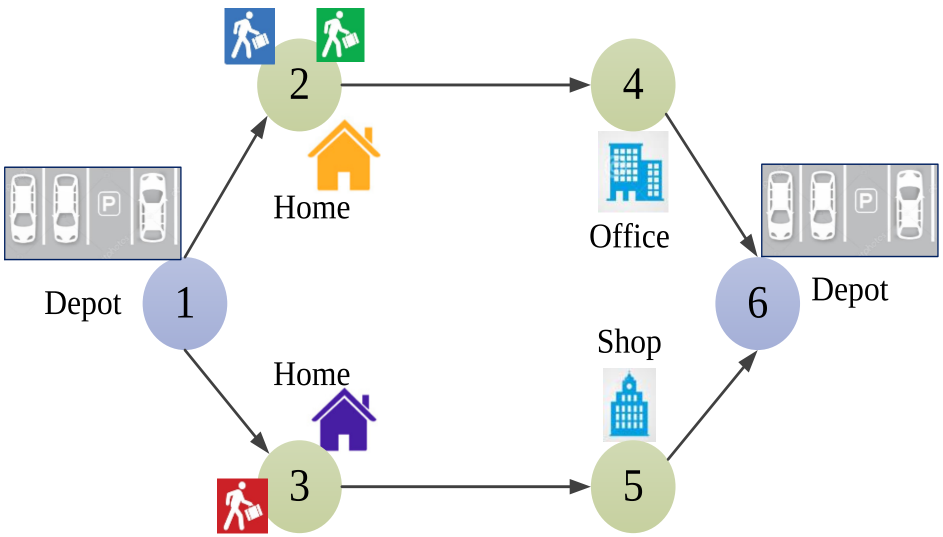


Total cost is 4

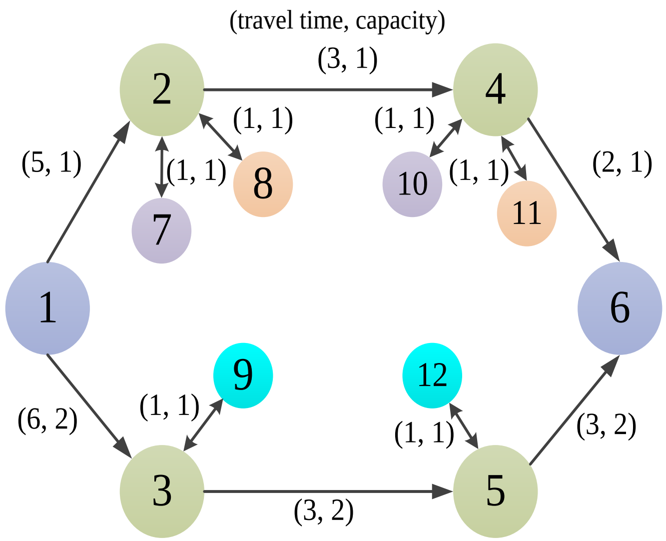
- When the red vehicle's carrying capacity is increased to 2, the total cost is reduced to 4 from 9;
- Only the red vehicle is required to serve the trip requests.

3. Space-time-state network and model formulation

Passenger trip requests: pickup and drop-off locations and time windows



(a) Physical transportation network with vehicles and trip requests

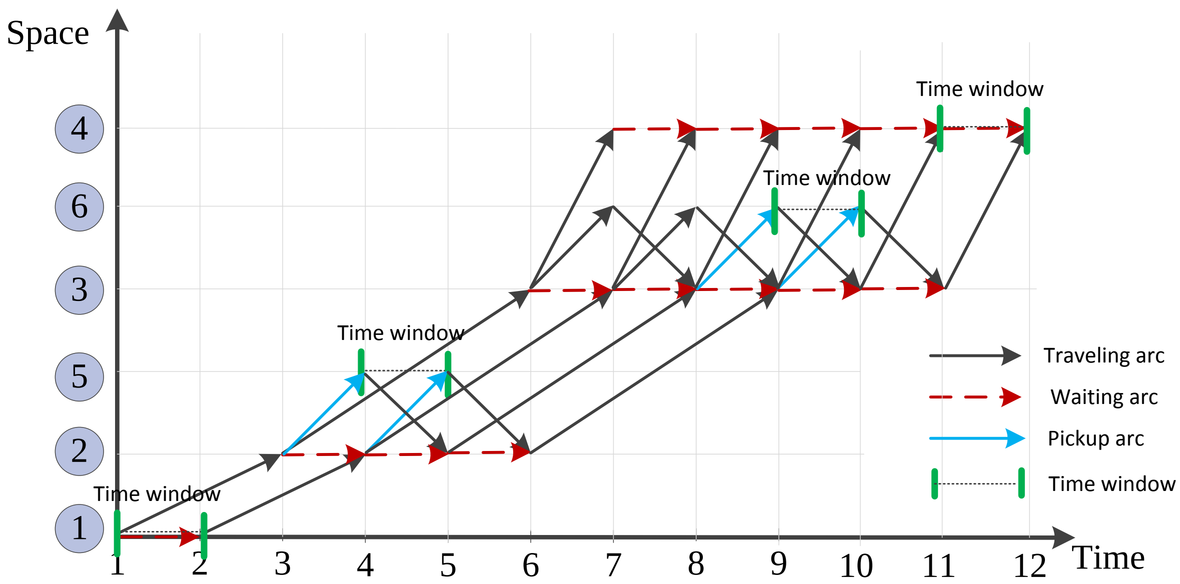


(b) Modified transportation network with virtual pickup and delivery nodes and links

Add virtual pick-up and drop-off nodes and links for each passenger

3. Space-time-state network and model formulation

Time-extended Space-time network construction for physical path $1 \rightarrow 2 \rightarrow 4 \rightarrow 6$



Space-time network

Arc (i,j,t,s) with capacity

Vertex (i,t), (j,s)

Passenger pickup time windows and locations are embedded in this network

3. Space-time-state network and model formulation

Vehicle Carrying States for passenger pickup and drop-off:

which passengers are being carried by this vehicle:

To record the **passenger service status**:

0: the passenger is not served;

1: under served (picked up but not delivered);

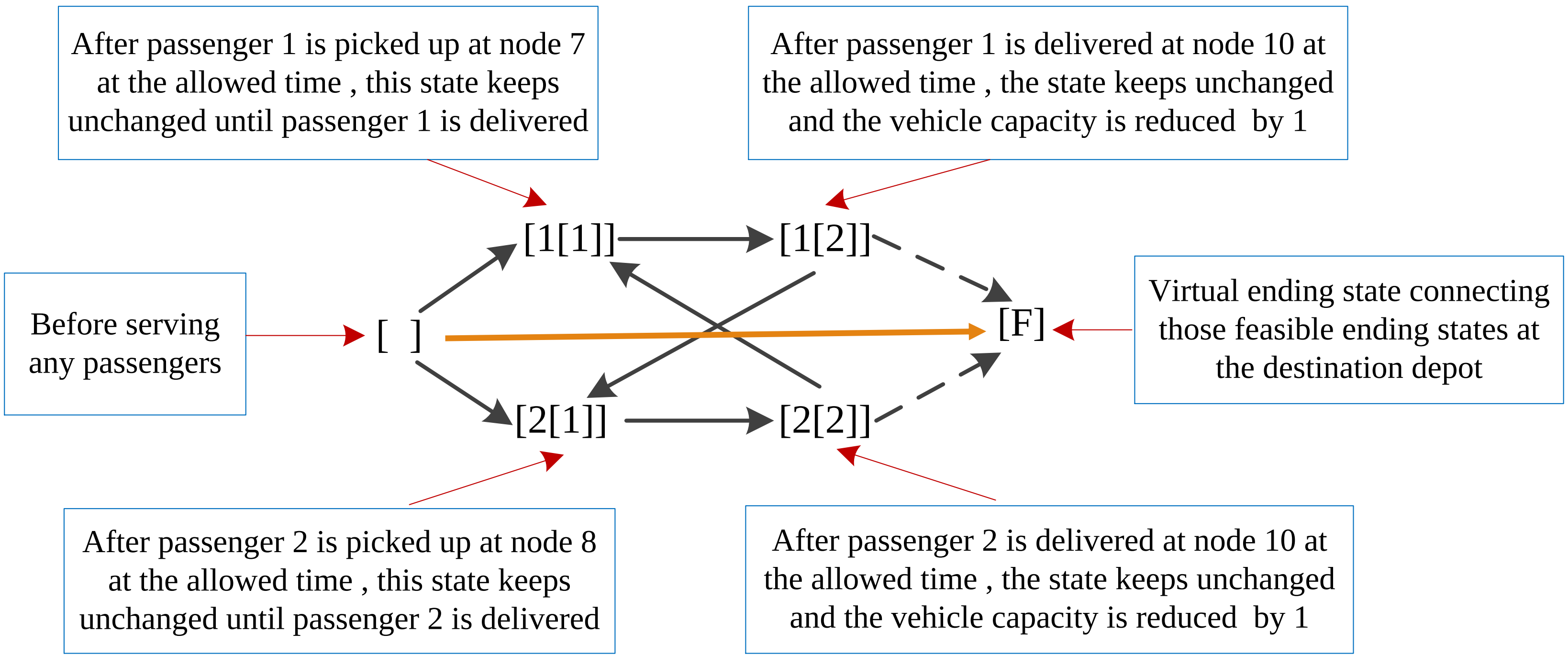
2: finished (delivered)

Example: In the case: if vehicle capacity is 1 and 2 passengers trip requests,

All possible states: [ ], [1[1]], [1[2]], [2[1]] or [2[2]]

3. Space-time-state network and model formulation

Vehicle State Transition with specific pickup and drop-off locations within time windows



Vehicle carrying state transition graph

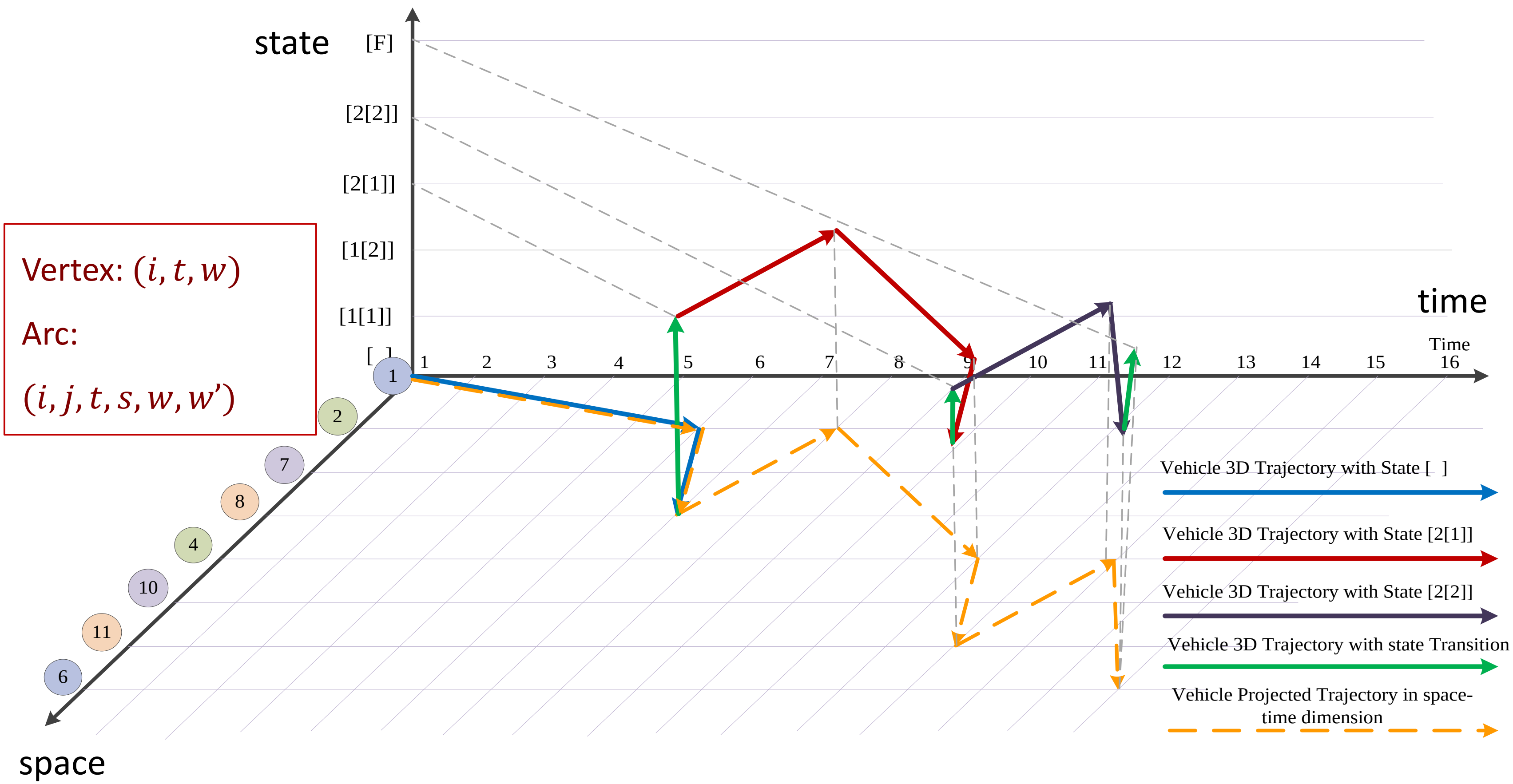
3. Space-time-state network and model formulation

One possible vehicle trajectory in a space-time-state network

Vertex: (i, t, w)

Arc:

(i, j, t, s, w, w')



3. Space-time-state network and model formulation

Arc-based agent-based formulation:

Objective function:

$$Min\ Z = \sum_a \sum_{(i,j,t,s,w,w')} (c^a_{i,j,t,s,w,w'} \times x^a_{i,j,t,s,w,w'}) \quad (1)$$

Subject to,

(i) **Vehicle supply**: Arc-based flow balance constraint for each vehicle

$$\sum_{i,t,w:(i,j,t,s,w,w')} x^a_{i,j,t,s,w,w'} - \sum_{i,t,w:(j,i,s,t,w',w)} x^a_{j,i,s,t,w',w} = \begin{cases} -1 & j = O(a), s = DT(a), w = [0,0,\dots,0] \\ 1 & j = D(a), s = T, w = [0,0,\dots,0] \\ 0 & otherwise \end{cases}, \forall a \quad (2)$$

(ii) **Travel demand**: Passenger p's pick-up request constraint

$$\sum_a \sum_{i,t,s:(i,j,t,s,w,w') \in A(p)} x^a_{i,j,t,s,w,w'} = 1, \forall p \quad (3)$$

(iii) **Infrastructure supply**: Tight road capacity constraint (endogenous congestion)

$$\sum_a \sum_w x^a_{i,j,t,s,w,w'} \leq cap_{i,j,t,s}, \forall (i,j,t,s) \quad (4)$$

(iv) Binary definitional constraint

$$x^a_{i,j,t,s,w,w'} \in \{0,1\} \quad (5)$$

Remark: state definition *w* differs for passenger pickup only and passenger pickup and drop-off

3. Space-time-state network and model formulation

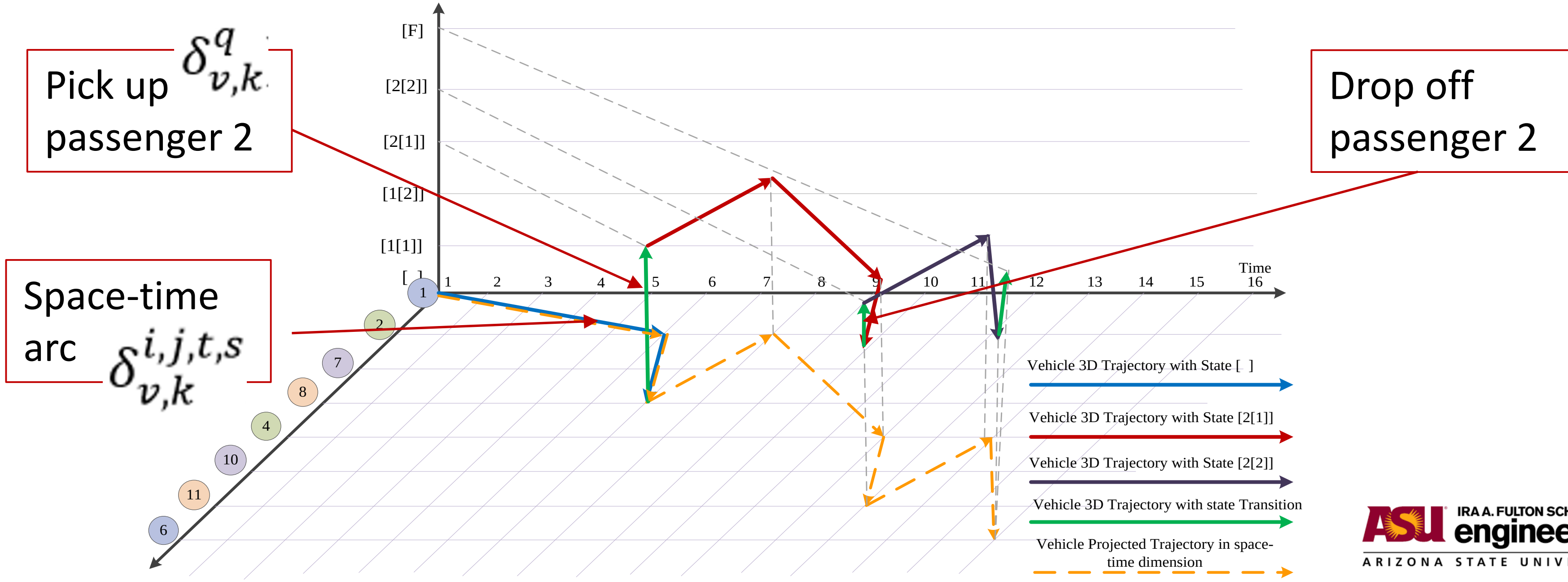
Path-based flow-based formulation:

Assumptions:

(1) **Vehicles** and **Passengers** can be grouped by its origin, destination and required service time period

(2) **All possible paths of vehicle groups** can be **enumerated** in advance.

(3) The **space-time-state** path of each vehicle group has the **mapping** information about **vehicle-to-passenger** and **vehicle-to-arc** relation.



3. Space-time-state network and model formulation

Path-based flow-based formulation:

$$\min \textstyle\sum_{(v,k)}(c_v^k \times y_v^k) \tag{6}$$

Subject to

(i) **Vehicle supply:** Path-based vehicle group flow balance constraint:

$$\textstyle\sum_k y_v^k = d(v), \forall v \tag{7}$$

(ii) **Travel demand**: Pickup requests on passenger group q:

$$\textstyle\sum_{(v,k)}(y_v^k \times \delta_{v,k}^q) = g(q), \forall q \tag{8}$$

(iii) **Infrastructure supply**: Road capacity constraints (endogenous congestion):

$$\textstyle\sum_{(v,k)}(y_v^k \times \delta_{v,k}^{i,j,t,s}) \le cap_{i,j,t,s}, \forall(i,j,t,s) \tag{9}$$

(iv) Positive continuous variable:

$$y_v^k \ge 0 \tag{10}$$

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

3. Space-time-state network and model formulation

Simplified format of our models

1. Primal problem

$$\min cx$$
$$Ax = b$$
$$Bx \leq d$$

Flow-balance constraint

Coupling constraint (trip request, link capacity)

2. Lagrangian Relaxation

$$\min cx + \mu(d - Bx)$$
$$Ax = b$$

3. Cutting plan method

$$max_{\mu} \min_{x}[cx + \mu(d - Bx)]$$
$$Ax = b$$

4. Column generation/DW

$$\min c(\sum_i \lambda_i \times x_i)$$
$$B\sum_i \lambda_i \times x_i \leq d$$
$$\sum_i \lambda_i = 1$$
$$\lambda_i \geq 0$$

Restricted master problem

5. Augmented Lagrangian

$$\min cx - \mu Bx - \pi$$
$$Ax = b$$

Subproblems

4. Dantzig-Wolfe Decomposition

$$\min c_1x_1 + c_2x_2 + c_3x_3$$

Objective function

$$a_1x_1 = b_1$$

$$a_2x_2 = b_2$$

Flow-balance constraint for each vehicle

$$a_3x_3 = b_3$$

$$a_4x_1 + a_5x_2 + a_6x_6 \leq b_4$$

Coupling constraints (passenger pickup, road capacity)

4. Dantzig-Wolfe Decomposition

Restricted Master Problem

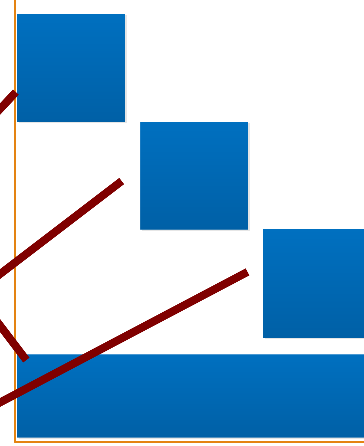
$$\min c_1 \sum_k (\lambda_1^k \times x_1^k) + c_2 \sum_k (\lambda_2^k \times x_2^k) + c_3 \sum_k (\lambda_3^k \times x_3^k)$$

$$m_1 \sum_k (\lambda_1^k \times x_1^k) + m_2 \sum_k (\lambda_2^k \times x_2^k) + m_3 \sum_k (\lambda_3^k \times x_3^k) \leq b_4$$

$$\sum_k \lambda_i^k = 1, \forall i$$

$$\lambda_i^k \geq 0$$

$$\min c_1x_1 + c_2x_2 + c_3x_3$$



$$a_1x_1 = b_1$$

$$a_2x_2 = b_2$$

$$a_3x_3 = b_3$$

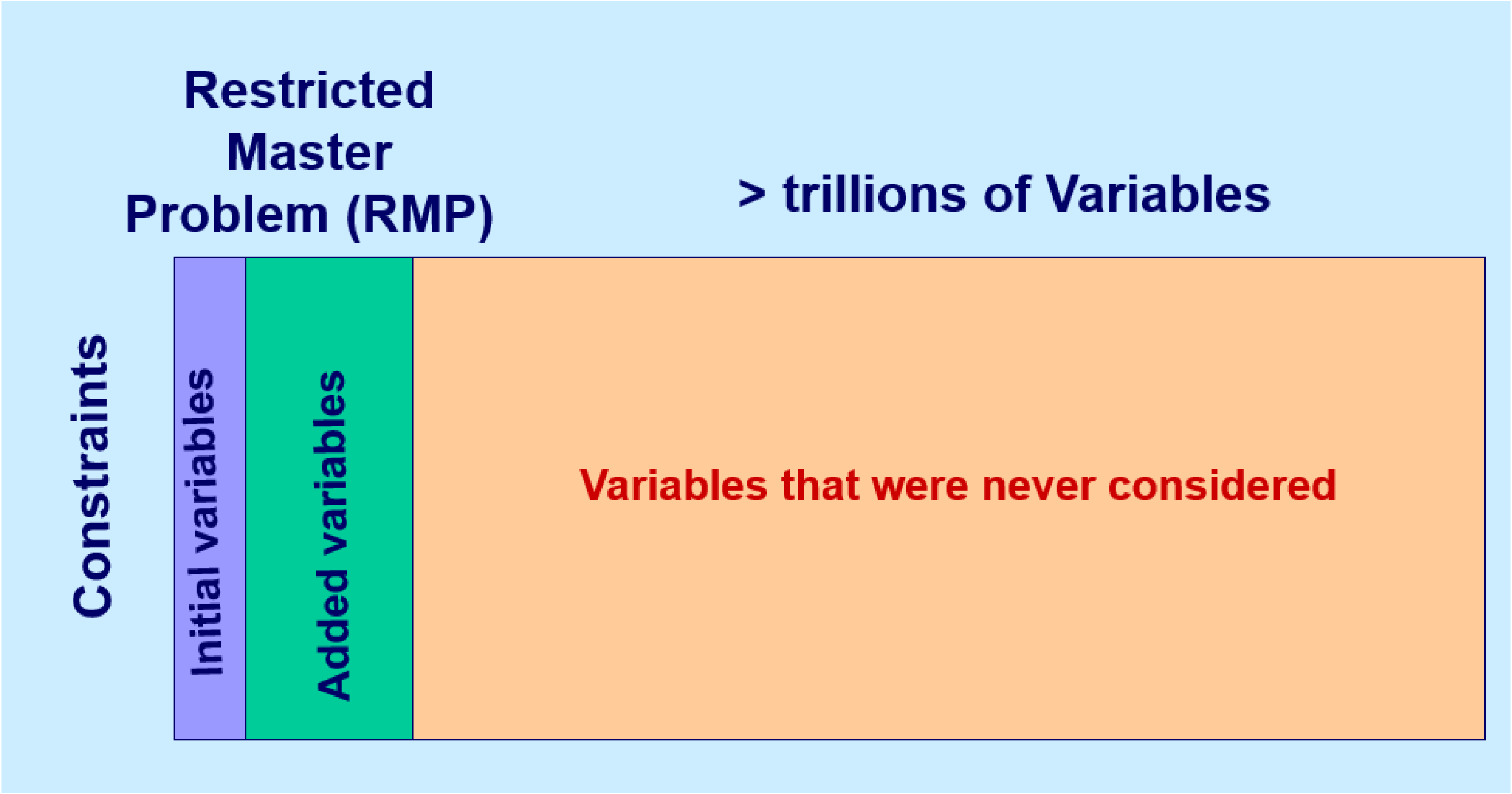
$$m_1x_1 + m_2x_2 + m_3x_3 \leq b_4$$

Sub-Problems

$$\min c_ix_i - \pi \times m_ix_i - \mu_i$$

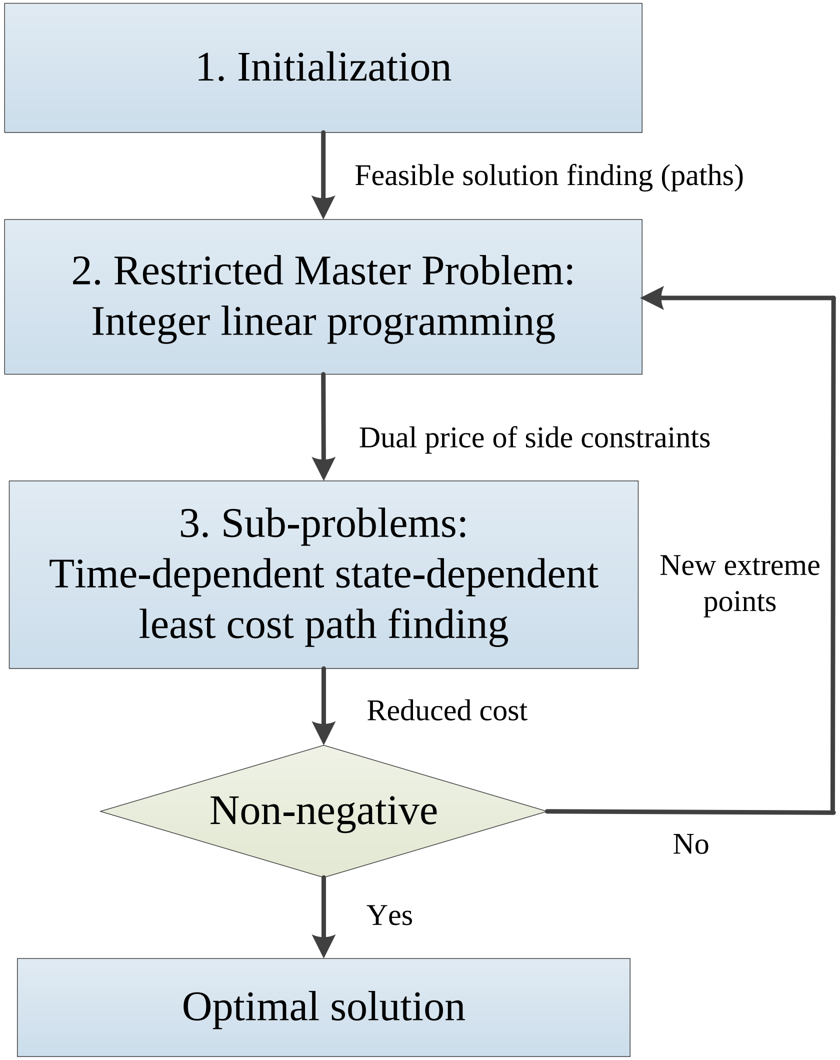
$$a_ix_i = b_i$$

4. Dantzig-Wolfe Decomposition



https://ocw.mit.edu/courses/sloan-school-of-management/15-082j-network-optimization-fall-2010/lecture-notes/

4. Dantzig-Wolfe Decomposition



4. Dantzig-Wolfe Decomposition

Restricted master problem

Objective function

$$Min \sum_v \sum_k (c^k_{a,o,d} \times \lambda^k_{a,o,d})$$

Passenger pickup constraint

$$\sum_v \sum_k (\lambda^k_{a,o,d} \times \delta^{a,o,d}_{k,p}) = 1, \forall\, p$$

Space-time arc capacity constraint

$$\sum_{a,o,d} \sum_k (\lambda^k_{a,o,d} \times \beta^k_{a,o,d,(i,j,t,s)}) \leq cap_{i,j,t,s}, \ \forall (i,j,t,s)$$

Path weight constraint

$$\sum_k \lambda^{a,o,d}_k = 1, \forall (a,o,d)$$

$$\lambda^{a,o,d}_k = \{0,1\}$$

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

4. Dantzig-Wolfe Decomposition

Subproblems (TDSDSP)

The sub-problem for each vehicle a:

$$Min\ Z' = \sum_{(i,j,t,s,w,w')}(c^a_{i,j,t,s,w,w'} \times x^a_{i,j,t,s,w,w'}) - \sum_p \sum_{(i,j,t,s,w,w')\in A(p)} (\boxed{\pi_p} \times x^a_{i,j,t,s,w,w'}) - \sum_{(i,j,t,s)}(\boxed{\mu_{i,j,t,s}} \times \sum_w x^a_{i,j,t,s,w,w'}) - \boxed{\omega_a}$$

Flow balance constraint for each vehicle

$$\sum_{i,t,w:(i,j,t,s,w,w')} x^a_{i,j,t,s,w,w'} - \sum_{i,t,w:(j,i,s,t,w',w)} x^a_{j,i,s,t,w',w} = \begin{cases} -1 & j = O(a), s = DT(a), w = [0,0,\dots,0] \\ 1 & j = D(a), s = T, w = [0,0,\dots,0] \\ 0 & otherwise \end{cases} \quad ,\forall a$$

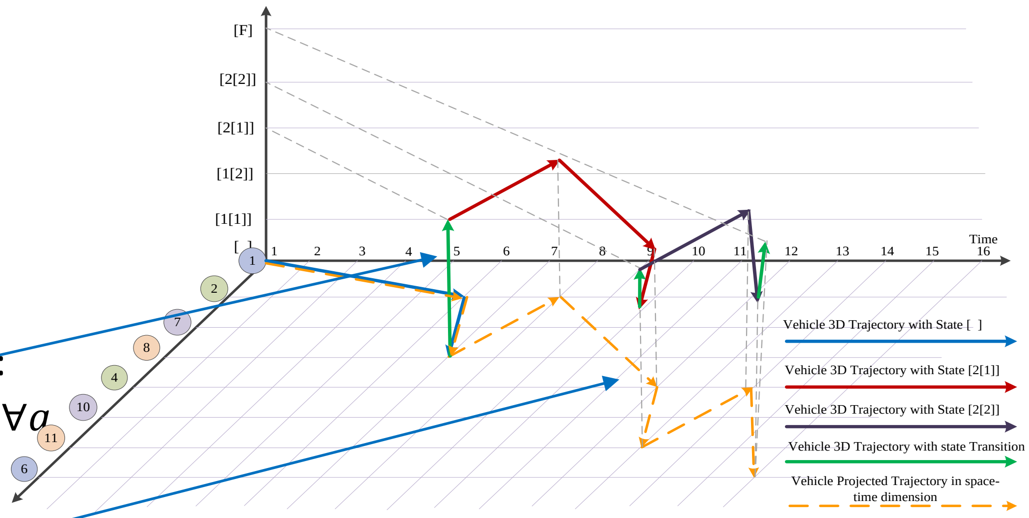
Dual price of of passenger pickup constraints

Dual price of congestion constraints

Dual price of path weight constraints

5. Column-pool based Approximation

Each column is a path with information and connection of passenger trip requests, and vehicle passed space-time arcs



$$\min \sum_{(o,d,k)} (c_{o,d}^{k} \times y_{o,d}^{k})$$

(1) Pickup requests on passenger group q:

$$\sum_{(o,d,k)} (y_{o,d}^{k} \times \boxed{\delta_{o,d,k}^{q}}) = g(q), \forall q$$

(2) Road capacity constraints:

$$\sum_{(o,d,k)} (y_{o,d}^{k} \times \boxed{\delta_{o,d,k}^{i,j,t,s}}) \leq cap_{i,j,t,s}, \forall (i,j,t,s)$$

(3) Positive continuous variable (path vehicle flow):

$$y_{o,d}^{k} \geq 0$$

Compared with arc-based formulation, column-based model greatly reduces the number of variables.

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

5. Column-pool based Approximation

$\delta_{v,k}^{q}$ Incidence matrix of column with passenger trip requests

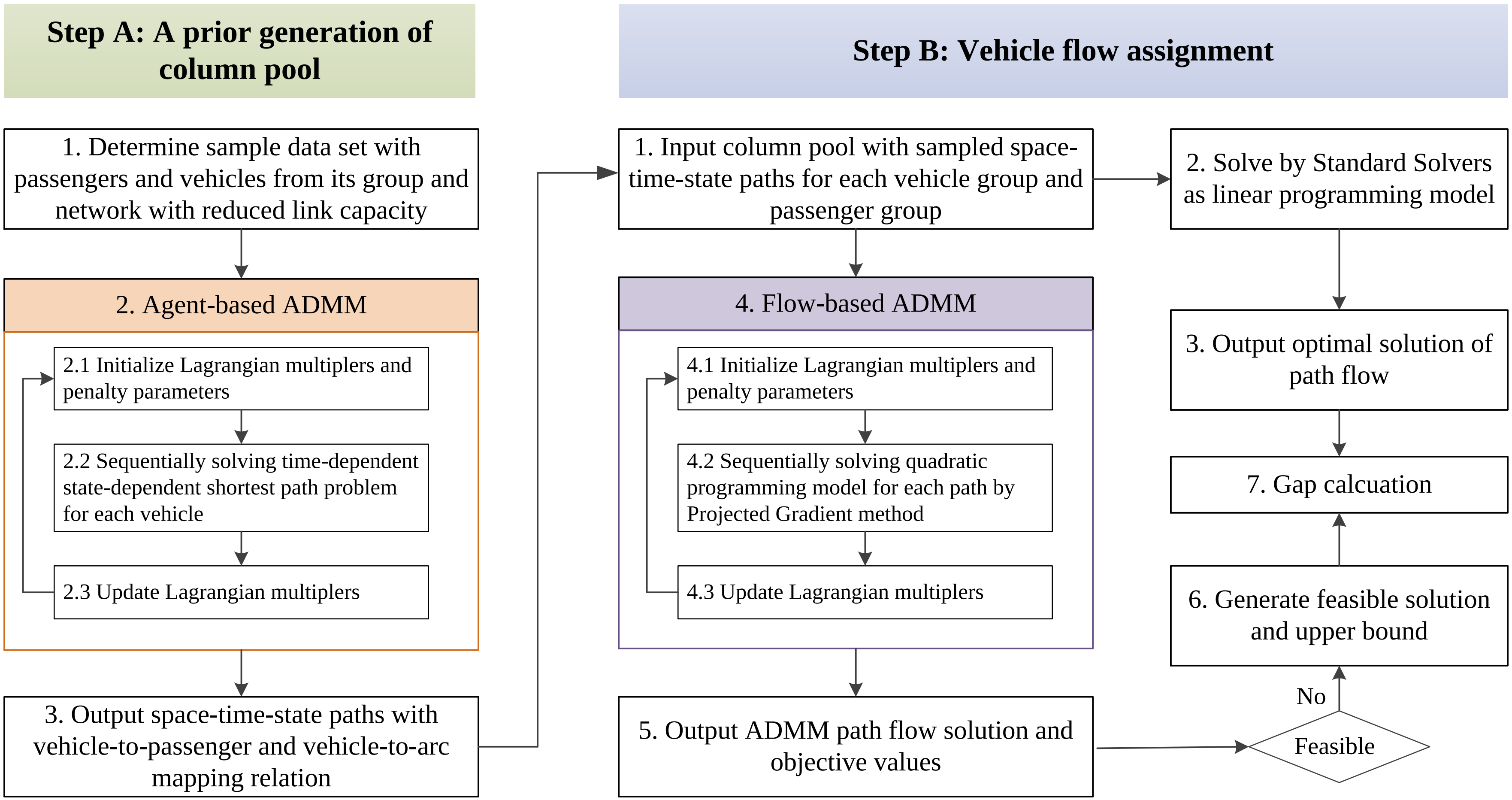
Column\ trip requests	p1	p2	p3	p4
y1	1	1	0	0
y2	0	1	0	1
y3	0	0	1	1

$\delta_{v,k}^{i,j,t,s}$ Incidence matrix of column with arc

Column\space-time arc	arc 1(i,j,t,s)	arc 2(i,j,t,s)	arc 3(i,j,t,s)	arc 4(i,j,t,s)
y1	1	0	0	1
y2	0	1	1	0
y3	1	0	0	1

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

5. Column-pool based Approximation

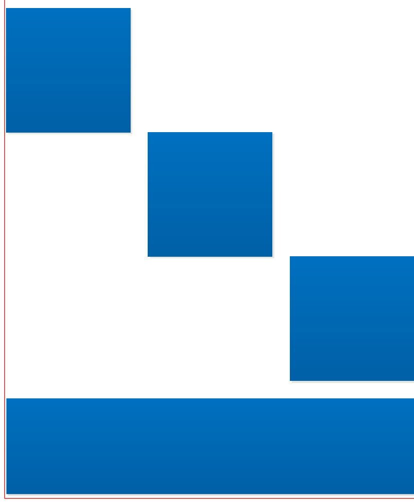


5. Column-pool based Approximation

Alternating Direction Method of Multipliers (ADMM)

Primal Block Angular

$$\min c_1x_1 + c_2x_2 + c_3x_3$$



$$a_1x_1 = b_1$$

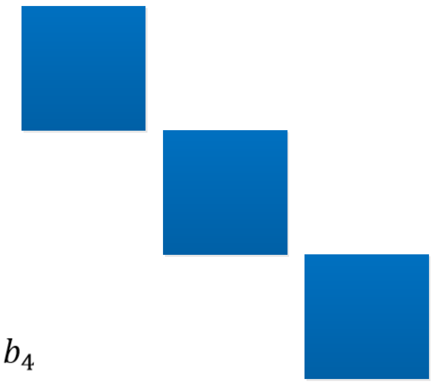
$$a_2x_2 = b_2$$

$$a_3x_3 = b_3$$

$$m_1x_1 + m_2x_2 + m_3x_3 \le b_4$$

Dualization and Augmentation

$$min \quad c_1x_1 + c_2x_2 + c_3x_3 + \lambda\boxed{(m_1x_1 + m_2x_2 + m_3x_3 - b_4)} + \frac{\rho}{2}\boxed{(m_1x_1 + m_2x_2 + m_3x_3 - b_4)^2}$$



$$a_1x_1 = b_1$$

$$a_2x_2 = b_2$$

$$a_3x_3 = b_3$$

Objective function:

$$L(\boldsymbol{x}, \lambda, \rho) = c_1x_1 + c_2x_2 + c_3x_3 + \lambda(m_1x_1 + m_2x_2 + m_3x_3 - b_4) + \frac{\rho}{2}(m_1x_1 + m_2x_2 + m_3x_3 - b_4)^2$$

Solutions: $x_3^{n+1} = argminL(x_1^{n+1}, x_2^{n+1}, x_3, \lambda^n, \rho)$

Multiplier update: $\lambda^{n+1} = \lambda^n + \rho(m_1x_1^{n+1} + m_2x_2^{n+1} + m_3x_3^{n+1} - b_4)$

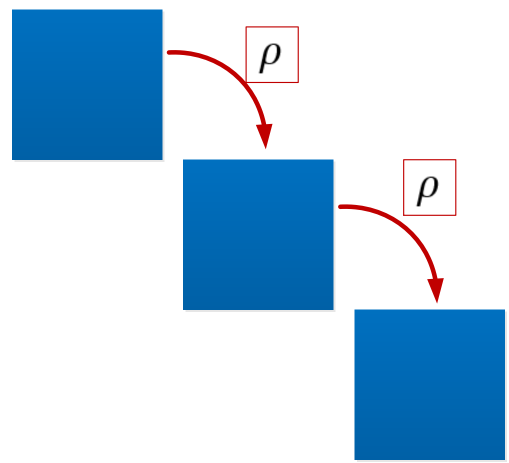
IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

5. Column-pool based Approximation

Sequentially solving each subproblem

Update λ

$$
\min \quad c_1x_1 + c_2x_2 + c_3x_3 + \lambda(m_1x_1 + m_2x_2 + m_3x_3 - b_4) + \frac{\rho}{2}(m_1x_1 + m_2x_2 + m_3x_3 - b_4)^2
$$



$$a_1x_1 = b_1$$

$$a_2x_2 = b_2$$

$$a_3x_3 = b_3$$

λ can be updated by the cutting plane method

5. Column-pool based Approximation

Arc-based Agent-based formulation

Objective function:

$$Min\ Z = L(\boldsymbol{x^a}, \boldsymbol{\pi_p}, \boldsymbol{\pi_{(i,j,t,s)}}) = \sum_a \sum_{(i,j,t,s,w,w')} \left(c_{i,j,t,s,w,w'} \times x^a_{i,j,t,s,w,w'}\right) + \sum_p [\pi_p \times (\sum_a \sum_{(i,j,t,s,w,w') \in A(p)} (x^a_{i,j,t,s,w,w'} \times \delta^a_{i,j,t,s}) - 1)] + \frac{\rho_1}{2} \sum_p \left[\sum_a \sum_{(i,j,t,s,w,w') \in A(p)} (x^a_{i,j,t,s,w,w'} \times \delta^a_{i,j,t,s}) - 1\right]^2 + \sum_{(i,j,t,s)} [\pi_{(i,j,t,s)} \times (\sum_a \sum_w x^a_{i,j,t,s,w,w'} - cap_{i,j,t,s})] + \frac{\rho_2}{2} \sum_{(i,j,t,s)} \left[\sum_a \sum_w x^a_{i,j,t,s,w,w'} - cap_{i,j,t,s}\right]^2$$

At each iteration:

- Solve each sub-problem sequentially for each vehicle
- Each sub-problem is a time-dependent state-dependent shortest path problem for each vehicle
- Update Lagrangian multipliers for passenger pickup constraints and arc capacity constraints

5. Column-pool based Approximation

ADMM for agent-based model

// initialization
Set up initial values for all Lagrangian multiplers and penalty parameters
for $n = 1$ to n_{max} // total number of iterations
  for $a = 1$ to a_{max} //total number of vehicles
    Find the time-dependent state-dependent shortest path for vehicle a with the fixed solution of other vehicles
    Update the network-arc costs based on the new solution of vehicle a for vehicle $a + 1$
  end // vehicle
  Update Lagrangian multipliers of passenger pickup constraints and arc capacity constraints
end // iterations

At iteration $n + 1$ of ADMM:

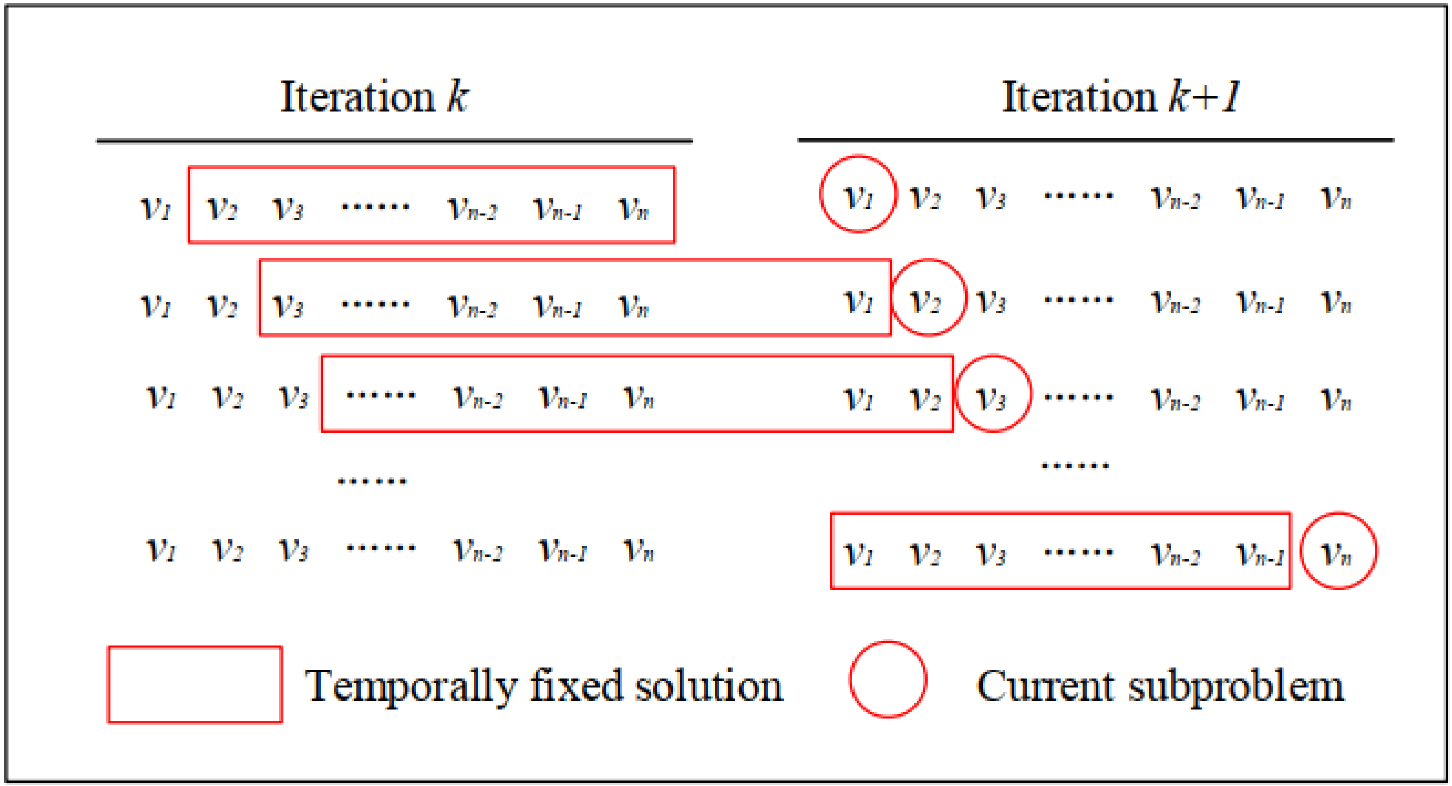
$$x_a^{n+1} = \arg\min \{L(x_1^{n+1}, x_2^{n+1}, \dots, x_a, x_{a+1}^n, \dots, x_{a_{\max}}^n, \boldsymbol{\pi}_{\boldsymbol{p}}^n, \boldsymbol{\pi}_{\boldsymbol{i,j,t,s}}^n)\}$$

$$\pi_p^{n+1} = \pi_p^n - \rho_1[\textstyle\sum_a \sum_{(i,j,t,s,w,w')\in A(p)} (x_{i,j,t,s,w,w'}^{a,n+1} \times \delta_{i,j,t,s}^a) - 1]$$

$$\pi_{i,j,t,s}^{n+1} = \max\{0, \pi_{i,j,t,s}^n - \rho_2[\textstyle\sum_a \sum_w x_{i,j,t,s,w,w'}^{a,n+1} - cap_{i,j,t,s}]$$

5. Column-pool based Approximation

At each iteration for each vehicle:



5. Column-pool based Approximation

Flow-based path-based ADMM:

Quadratic objective functions:

$$\min \sum_k (c^k \times y^k) + \sum_q (\lambda_q \times [(\sum_k (y^k \times \delta_q^k) - g(q)]) + \frac{\rho_1}{2} \sum_q \left(\left(\sum_k (y^k \times \delta_q^k\right) - g(q) \right)^2 +$$

$$\sum_{i,j,t,s} (\mu_{i,j,t,s} \times [\sum_k (y^k \times \delta_{i,j,t,s}^k) - cap_{i,j,t,s}]) + \frac{\rho_2}{2} \sum_{i,j,t,s} (\sum_k (y^k \times \delta_{i,j,t,s}^k) - cap_{i,j,t,s})^2$$

Its Hessian Matrix can be derived as,

$$H = \begin{vmatrix} \sigma_1 \sum_p \delta_p^1 + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^1 & \sigma_1 \sum_p \delta_p^1 \delta_p^2 + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^1 \delta_{i,j,t,s}^2 & \cdots & \sigma_1 \sum_p \delta_p^1 \delta_p^{kmax} + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^1 \delta_{i,j,t,s}^{kmax} \\ \sigma_1 \sum_p \delta_p^1 \delta_p^2 + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^1 \delta_{i,j,t,s}^2 & \sigma_1 \sum_p \delta_p^2 + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^2 & \cdots & \sigma_1 \sum_p \delta_p^2 \delta_p^{kmax} + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^2 \delta_{i,j,t,s}^{kmax} \\ \vdots & \vdots & \cdots & \vdots \\ \sigma_1 \sum_p \delta_p^1 \delta_p^{kmax} + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^1 \delta_{i,j,t,s}^{kmax} & \sigma_1 \sum_p \delta_p^2 \delta_p^{kmax} + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^2 \delta_{i,j,t,s}^{kmax} & \cdots & \sigma_1 \sum_p \delta_p^{kmax} + \sigma_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^{kmax} \end{vmatrix}$$

Since it is difficult to directly obtain its inverse matrix H^-, especially in large-scale networks, we apply ADMM to decompose the primal problem to sequentially solve the subproblem for each column.

5. Column-pool based Approximation

Flow-based path-based ADMM:

- Solve each variable $y_{o,d}^{k}$ sequentially by ADMM

$$y_k^{n+1} = \arg\min\{L(y_1^{n+1}, y_2^{n+1}, \dots, y_k, y_{k+1}^{n}, \dots, y_{k_{\max}}^{n}, \boldsymbol{\lambda}_q^{n}, \boldsymbol{\mu}_{i,j,t,s}^{n})\}$$

At each iteration of ADMM, Lagrangian multipliers are updated as follows,

Passenger group trip requests:

$$\lambda_q^{n+1} = \lambda_q^{n} + \rho_1((\textstyle\sum_k (y_k^n \times \delta_q^k) - g(q))$$

Arc capacity constraints:

$$\mu_{i,j,t,s}^{n+1} = \max\{0, \mu_{i,j,t,s}^{n} + \rho_2(\textstyle\sum_k (y_k^n \times \delta_{i,j,t,s}^{k}) - cap_{i,j,t,s})\}$$

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

5. Column-pool based Approximation

Projected Gradient Method (Rosen, 1960) to solve each subproblem

$$y_k^{n+1} = \arg\min\{L(y_1^{n+1}, y_2^{n+1}, \dots, y_k, y_{k+1}^n, \dots, y_{k_{\max}}^n, \boldsymbol{\lambda}_q^n, \boldsymbol{\mu}_{i,j,t,s}^n)\}$$

$$y_k^{n+1} = \max\{0, y_k^n - \frac{1}{s} \times L(y_k^n)'\}$$

Where $L(y_k^n)' = c^k + \sum_q \lambda_q \times \delta_q^k + \rho_1 \left(\sum_q \delta_q^k \left(\left(\sum_k (y_k^n \times \delta_q^k\right) - g(q)\right)\right) + \sum_{i,j,t,s} \mu_{i,j,t,s} \times \delta_{i,j,t,s}^k + \rho_2 (\sum_{i,j,t,s} \delta_{i,j,t,s}^k (\sum_k (y_k^n \times \delta_{i,j,t,s}^k) - cap_{i,j,t,s}))$, and $s = \frac{\partial^2 L(x)}{\partial x^2} = \rho_1 \sum_q \delta_q^k + \rho_2 \sum_{i,j,t,s} \delta_{i,j,t,s}^k$

Projected Gradient Method has been used in solving the path-based nonlinear programming models in equilibrium traffic assignment (Larsson and Patriksson, 1992; Jayakrishnan et al, 1994; Florian et al., 2009), and it is more efficient, compared with arc-based nonlinear programming models, but needs more memory use.

5. Column-pool based Approximation

Beam-search algorithm for finding the time-dependent state-dependent shortest paths:

```
//definition: vehicle: v, node: n, time: t, state: w, vehicle location-dependent time-dependent states: td_state[v][n][t][w]
    for t = departure time to ending time T
        for n = 0 to total_number_of_nodes N
            //beam-search: find the best k vehicle states with least travel costs from depot to current node and time
            state_size = min{k, state size of vehicle v at node n and time t}
            for w = 0 to state_size
                Current_node = n
                for to_node =1 to the outbound_node_size of current_node
                    if (to_node is passenger pickup or drop-off node)
                        Update the vehicle state td_state[v][n][t][w] with passenger pickup or drop-off,
                        current_node, current_time, travel cost from the depot to current node and time with benefits
                        of serving passengers, based on previous node n, previous time t and link travel time,
                        previous state w, and the whole state transition logic.
                    if (to_node is physical network node)
                        Update the vehicle state td_state[v][n][t][w] with current_node, current_time and
                        current travel cost, and state w doesn't change.
                    if (to_node is destination node of vehicle v)
                        Update the vehicle state td_state[v][n][t][w] and update the corresponding
                        Vector vehicle_ending_state [v], which will be used to find the least cost route for vehicle
                        v after all loops.
                end // downstream node of one link
            end // states
        end // nodes
    end// times
```

Discussion: path marginal cost calculation

System-impact of adjusting one vehicle routing:

- ❑ System marginal vehicle travel cost
- ❑ System marginal passenger service benefit/cost

In this queuing system:

- ❑ Waiting time for individual: 4 min

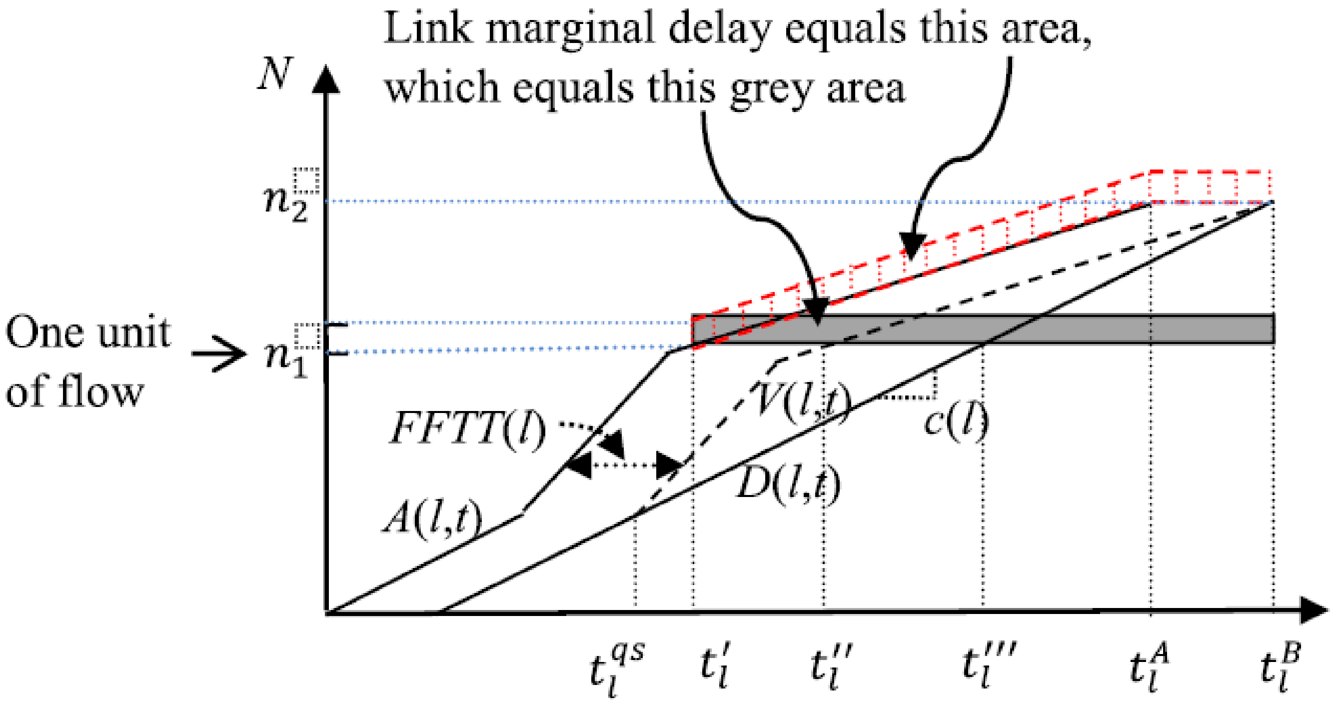
After adding one more person in the queue:

- ❑ Societal travel time: additional 4 min for each person behind: +16 min, and the waiting time of added person is 4 min, so the system marginal waiting time is 20 min.
- ❑ Societal service benefit: some persons may not be served in their preferred time window and it decreases the service benefit.

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

6. Discussion and Preliminary Experiments

Marginal cost calculation in system optimal dynamic traffic assignment (SODTA)



Ghali and Smith (1995)

Ghali, M.O. and Smith, M.J., 1995. A model for the dynamic system optimum traffic assignment problem. Transportation Research Part B: Methodological, 29(3), pp.155-170.

6. Discussion and Preliminary Experiments

Step 1: Build virtual pickup and drop-off links in physical traffic networks, and its service pricing is converted to generalized link travel cost

Step 2: find one initial solution as the input

Step 3: Perform network loading within a space-time-state network

3.1 use cumulative arrival and departure counts to derive the link marginal travel cost.

3.2 update the marginal service link benefit of passengers (not served or served by multiple vehicles)

Step 4: find the new least-marginal-cost route for each vehicles, and go to step 3; otherwise, stop.

Path marginal cost is probably related to the **Lagrangian multipliers** in ADMM and the **dual prices** in Dantzig-Wolfe decomposition.

6. Discussion and Preliminary Experiments

Capture queue spillback:

Inflow arc capacity constraint:

$$\sum_w x_{i,j',t-FFTT_{i,j}+1,t,w,w'} \leq Cap_{i,j',t-FFTT_{i,j}+1,t}, \forall (i,j') \in L_{inflow}, \forall t \quad (11)$$

Outflow arc capacity constraint:

$$\sum_w x_{j',j,t,t+1,w,w'} \leq y_{j',j,t,t+1}, \forall (j',j) \in L_{outflow}, \forall t \quad (12)$$

Outflow arc capacity balance constraint at points without merger and diverge:

$$y_{j',j,t,t+1} \leq Cap_{j,i,t+1,s} \quad (13)$$

Outflow arc capacity balance constraint at merger points:

$$\sum_{(j',t)} y_{j',j,t,t+1} \leq Cap_{j,i,t+1,s}, \forall (j, t+1) \in A_m \quad (14)$$

Outflow arc capacity balance constraint at diverge points:

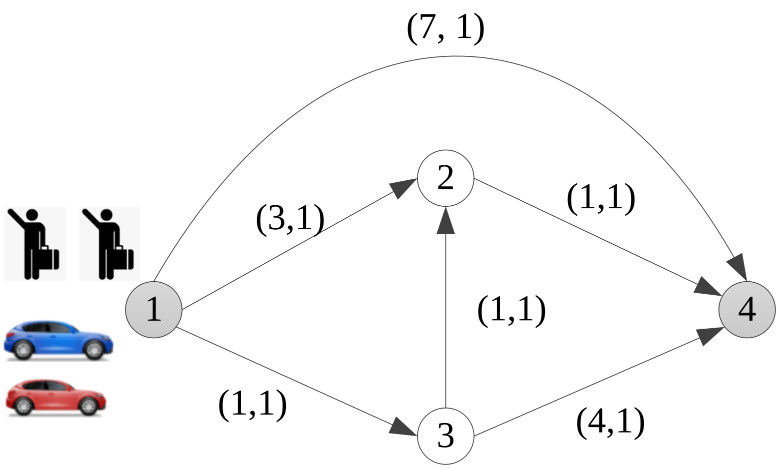
$$y_{j',j,t,t+1} \leq \sum_{(i,s)} Cap_{j,i,t+1,s} \forall (j, t+1) \in A_d \quad (15)$$

Link storage capacity constraint:

$$\sum_w x_{j',j',t-1,t,w,w'} + \sum_w \sum_{s=t-FFTT_{i,j'}}^{t-1} x_{i,j',s,t,w,w'} \leq Len_{i,j'} \times n_{i,j'} \times Jam_{i,j'}, \forall (i,j') \in L_{inflow}, \forall t \quad (16)$$

6. Discussion and Preliminary Experiments

- There are 2 vehicles, and each vehicle picks up one passenger from origin node 1 to destination node 4.
- Our goal is to minimize the total vehicle travel cost by using Dantzig-Wolfe decomposition approach.



Path ID	Node Sequence	Path Cost	Path Trajectory
Path 1	1→2→4	4	
Path 2	1→3→4	5	
Path 3	1→4	7	
Path 4	1→3→2→4	3	

(link cost, link capacity)

6. Discussion and Preliminary Experiments

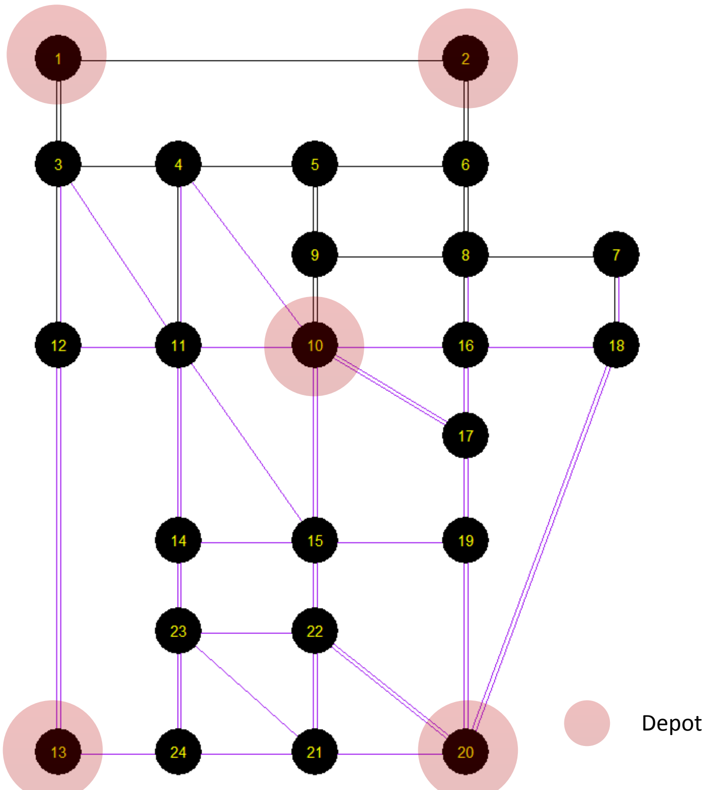
Variable λ_i is the weight of path i of total demand; $\mu_{i,j}$ is the dual price of capacity constraint of link (i, j); π is the dual price of path flow weight constraint.

	Sub problem	**New column: path 4**
Iteration 1	Master problem	$\lambda_4 = 1, \mu_{1,3} = -1, \mu_{1,2} = 0, \mu_{3,2} = 0, \mu_{2,4} = 0, \mu_{3,4} = 0, \mu_{1,4} = 0, \pi = 2$
Iteration 2	Sub problem	New column: path 1
	Master problem	$\lambda_1 = 1, \mu_{2,4} = -1, \mu_{1,2} = 0, \mu_{3,2} = 0, \mu_{1,3} = 0, \mu_{3,4} = 0, \mu_{1,4} = 0, \pi = 2$
Iteration 3	Sub problem	New column: path 3
	Master problem	$\lambda_4 = 0.5, \lambda_3 = 0.5, \mu_{1,3} = -1, \mu_{2,4} = -3, \mu_{1,2} = 0, \mu_{3,2} = 0, \mu_{3,4} = 0, \mu_{1,4} = 0, \pi = 14$
Iteration 4	Sub problem	New column: path 2
	Master problem	$\lambda_1 = 0.5, \lambda_2 = 0.5, \mu_{1,3} = -2, \mu_{2,4} = -2, \mu_{1,2} = -1, \mu_{3,2} = 0, \mu_{3,4} = 0, \mu_{1,4} = 0, \pi = 14$

The reduced cost is $4 + 5 - (-2) - (-2) - (-1) - 14 = 0$ and reach the optimal solution.

6. Discussion and Preliminary Experiments

Requests with pickup and drop-off and time windows under capacitated networks



Sioux Falls Network

# of nodes	24
# of links	84
# of trip requests (pickup and drop-off with time windows)	30
# of available autonomous vehicles	30
# of depots	5
optimization time horizon (time unit)	110
Vehicle capacity (person)	1

Source code: https://github.com/TonyLiu2015/VRPLite-DW

6. Discussion and Preliminary Experiments

Initial feasible solution

Vehicle_No	Passenger_No	Vehicle_No	Passenger_No	Vehicle_No	Passenger_No
1	[15]	11	[20]	21	[23]
2	[8]	12	[26]	22	[25]
3	[1]	13	[16]	23	[22]
4	[7]	14	[18]	24	[19]
5	[9]	15	[2]	25	[4]
6	[11]	16	[10]	26	[5]
7	[29]	17	[3]	27	[24]
8	[28]	18	[12]	28	[14]
9	[17]	19	[27]	29	[13]
10	[21]	20	[30]	30	[6]

6. Discussion and Preliminary Experiments

Dantzig-Wolfe decomposition algorithm solution:

- ❑ Each passenger has specific pickup and drop-off location and time windows
- ❑ The vehicle benefit of serving one passenger is 20
- ❑ The vehicle waiting cost is the half of the waiting time

	Number of required vehicles	Total travel cost
Initial solution	30	1096
vehicle carrying capacity is 1	27	967.5
vehicle carrying capacity is 2	25	869.5

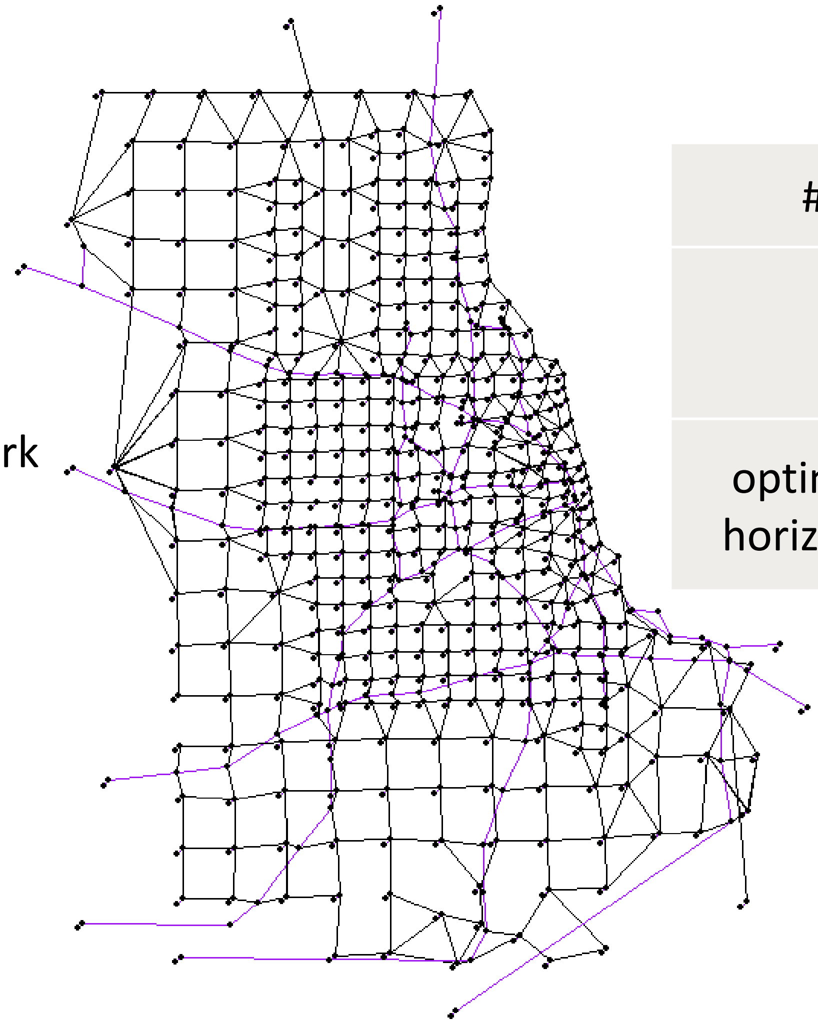
Take **vehicle 9** as an example:

- ❑ In initial solution: picks up passenger 17 -> drops off passenger 17;
- ❑ Vehicle carrying capacity is 1: picks up passenger 17 -> drops off passenger 17-> picks up passenger 29 -> drops off passenger 29
- ❑ Vehicle carrying capacity is 2: picks up passenger 17 -> drops off passenger 17-> picks up passenger 30-> picks up passenger 29-> drops off passenger 29-> drops off passenger 30

6. Discussion and Preliminary Experiments

Requests with pickup only and time windows under capacitated networks

Chicago network



# of nodes	1320
# of links	5431
optimization time horizon (time unit)	60

6. Discussion and Preliminary Experiments

Step A: Prior generation of column pool

Scenario 1: Sample data set

- 10 pairs of vehicle groups and passenger groups.
- Each pair has 243 vehicles and 387 passengers trip requests
- The space-time arc capacity in each minute is 5.
- Vehicle carrying capacity is 1
- 2430 binary variables and 332,160 constraints

Scenario 2: Sample data set

- 20 pair of vehicle groups and passengers groups.
- Each pair has 243 vehicles and 387 passengers trip requests
- The space-time arc capacity in each minute is 5.
- Vehicle carrying capacity is 1
- 4860 binary variables and 338,460 constraints

6. Discussion and Preliminary Experiments

Solution from Agent-based ADMM

Scenario 1:

- 1789 vehicles find their paths/columns to serve 1084 passengers
- 23,357 space-time arcs are generated based on vehicles' space-time paths
- Computation time: about 70 seconds each iteration

Scenario 2:

- 3686 vehicles find their paths/columns to serve 2226 passengers
- 36,454 space-time arcs are generated based on vehicles' space-time paths
- Computation time: about 140 seconds each iteration

Remark: each passenger has a specific pickup location, time window and destination, and vehicle can only pick up passengers within a same pair of groups.

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

6. Discussion and Preliminary Experiments

Step B: Flow-based ADMM implemented by C++

Experiment 1:

- 1084 passenger groups and each passenger group has 4 passenger trip requests
- The space-time arc capacity in each minute is 35
- 1789 positive continuous variables/columns and 24,441 constraints
- Computation time: 700 CPU seconds for running 250 iterations

Experiment 2:

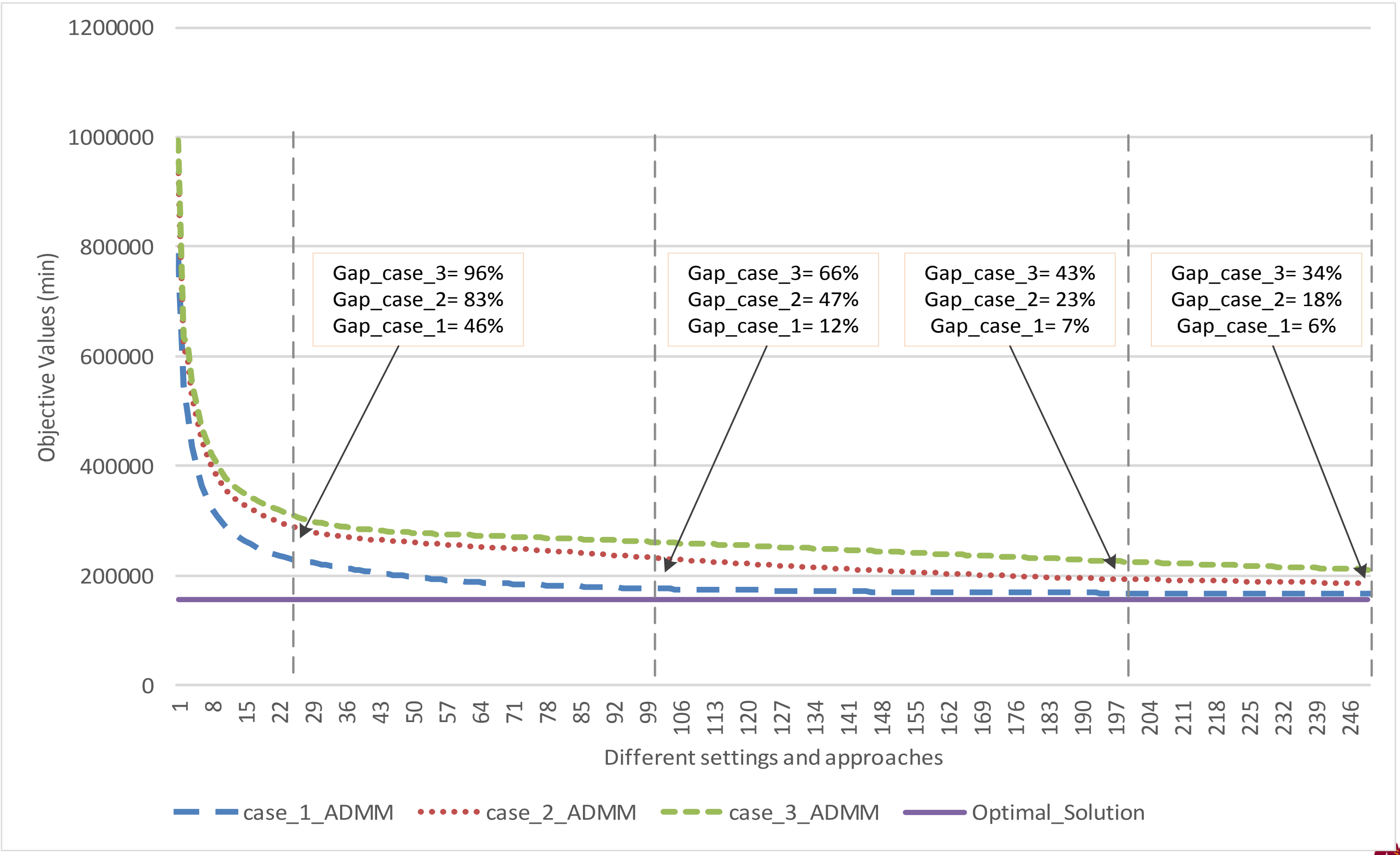
- 2226 passenger groups and each passenger group has 2 passenger trip requests
- The space-time arc capacity in each minute is 35
- 3686 positive continuous variables/columns and 38,680 constraints
- Computation time: 2735 CPU seconds to finish 250 iterations

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

6. Discussion and Preliminary Experiments

Experiment 1

Case 1: $\rho_1 = 3$ and $\rho_2 = 1$; Case 2: $\rho_1 = 3$ and $\rho_2 = 3$; Case 3: $\rho_1 = 3$ and $\rho_2 = 5$.

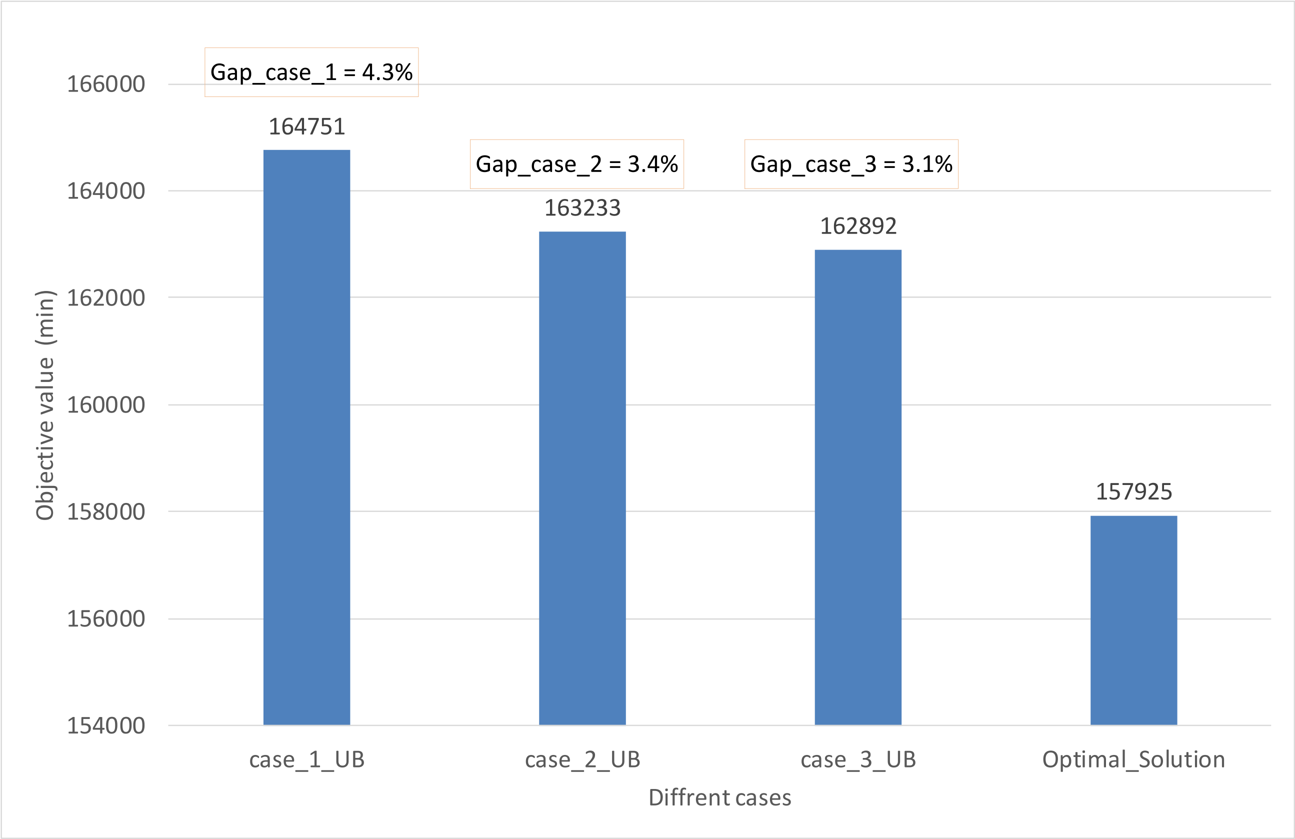


Solution of each iteration of ADMM in three cases and CPLEX

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

6. Discussion and Preliminary Experiments

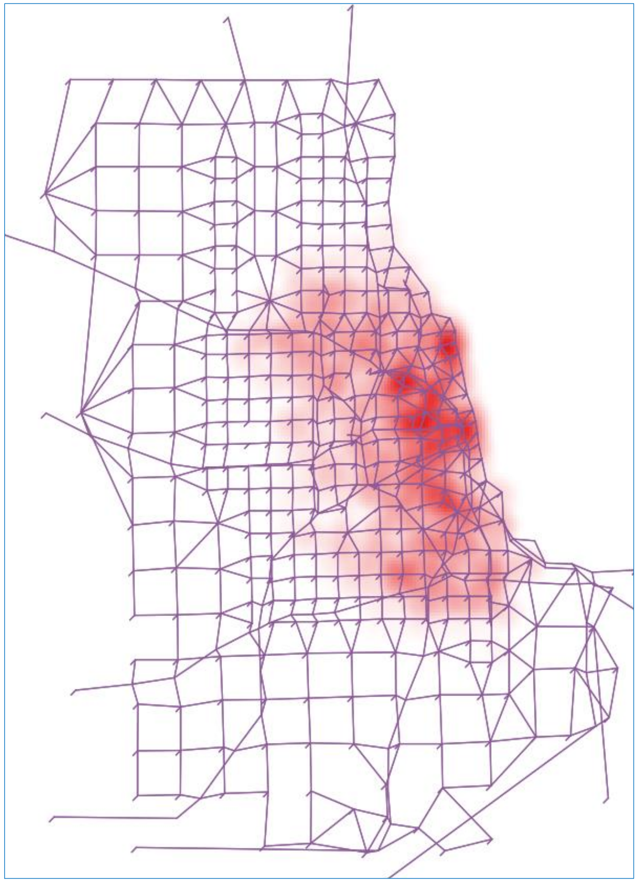
Experiment 1



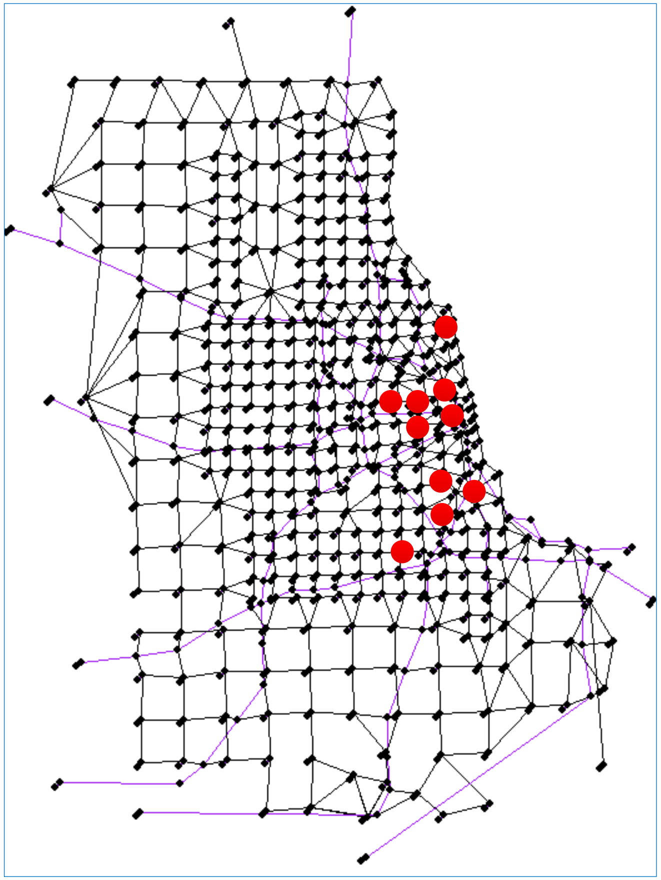
Comparison of objective values of upper bound and CPLEX

6. Discussion and Preliminary Experiments

Experiment 1



(a) The heat map on waiting flows in experiment 1



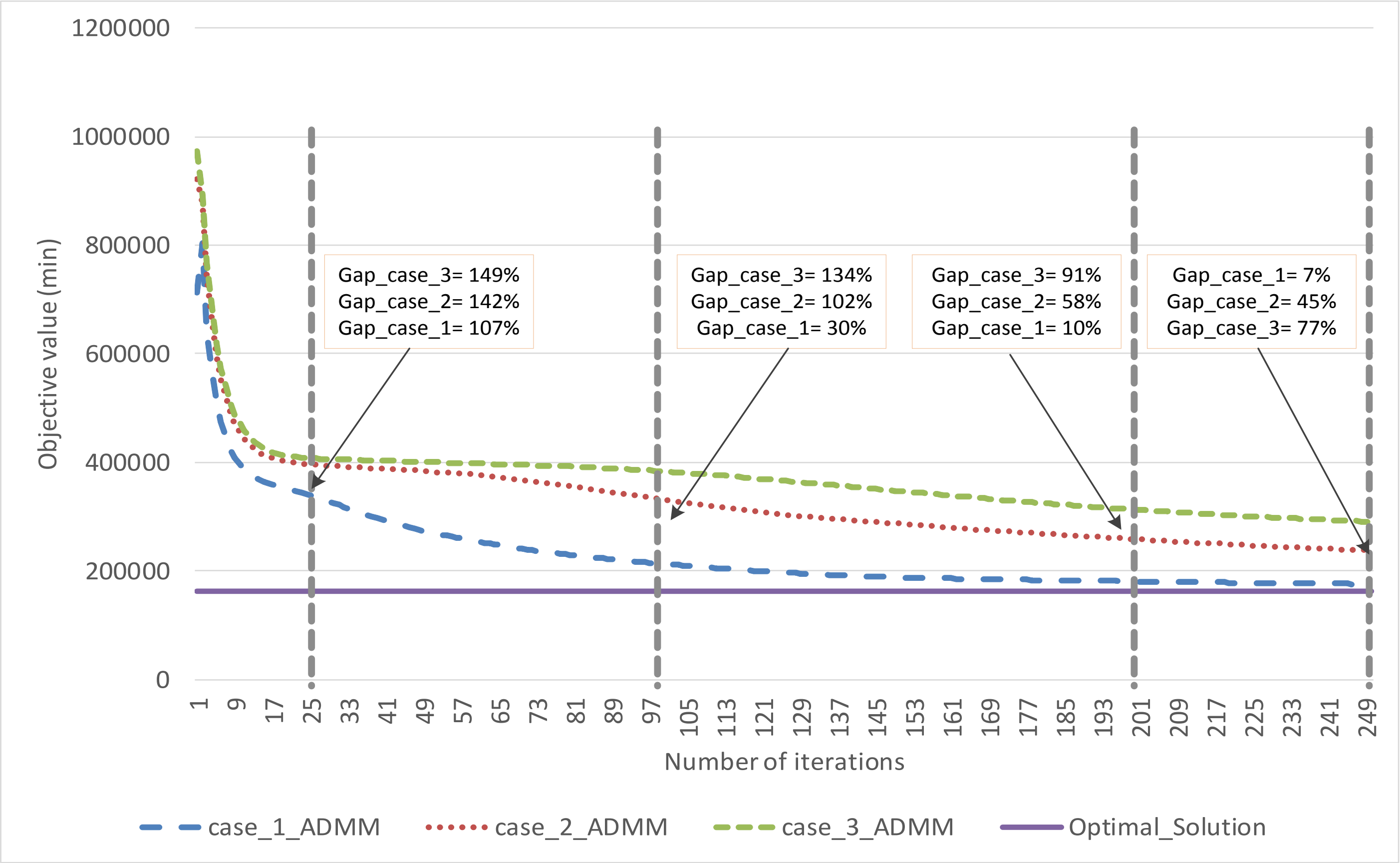
(b) Top 10 of congested nodes in experiment 1

Visualization of congested nodes in experiment 1

6. Discussion and Preliminary Experiments

Experiment 2

Case 1: $\rho_1 = 3$ and $\rho_2 = 1$; Case 2: $\rho_1 = 3$ and $\rho_2 = 3$; Case 3: $\rho_1 = 3$ and $\rho_2 = 5$.



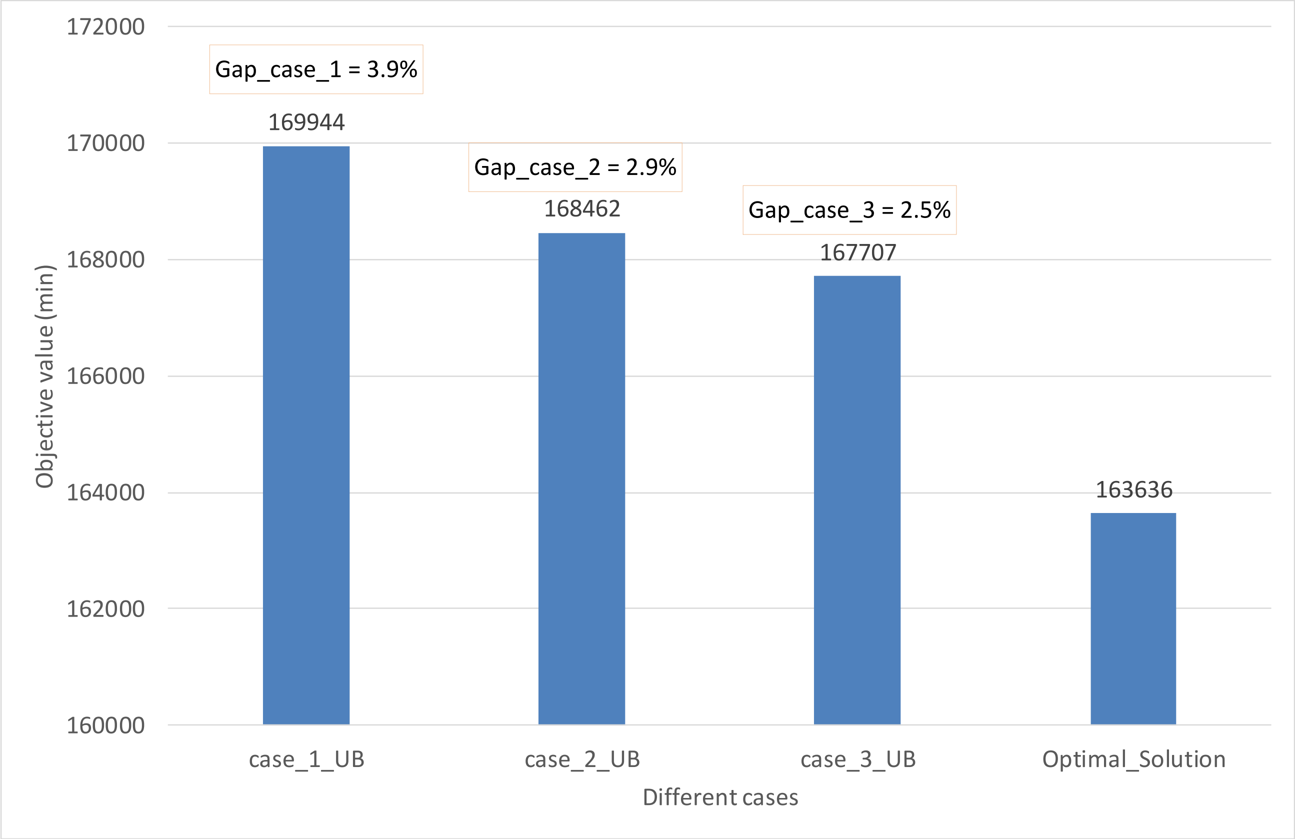
Solution of each iteration of ADMM in three cases and CPLEX

IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

6. Discussion and Preliminary Experiments

Experiment 2

Case 1: $\rho_1 = 3$ and $\rho_2 = 1$; Case 2: $\rho_1 = 3$ and $\rho_2 = 3$; Case 3: $\rho_1 = 3$ and $\rho_2 = 5$.

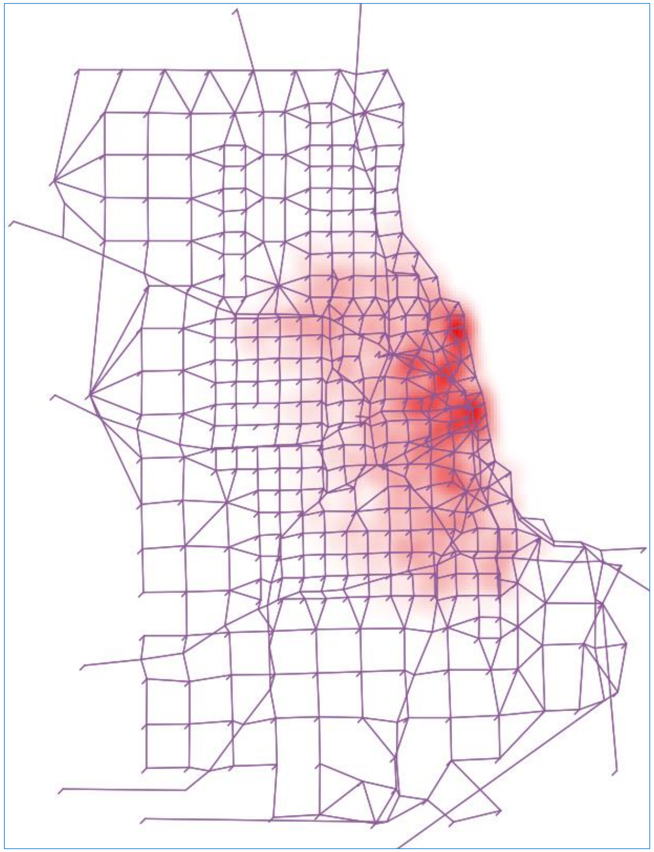


Comparison of objective values of upper bound and CPLEX

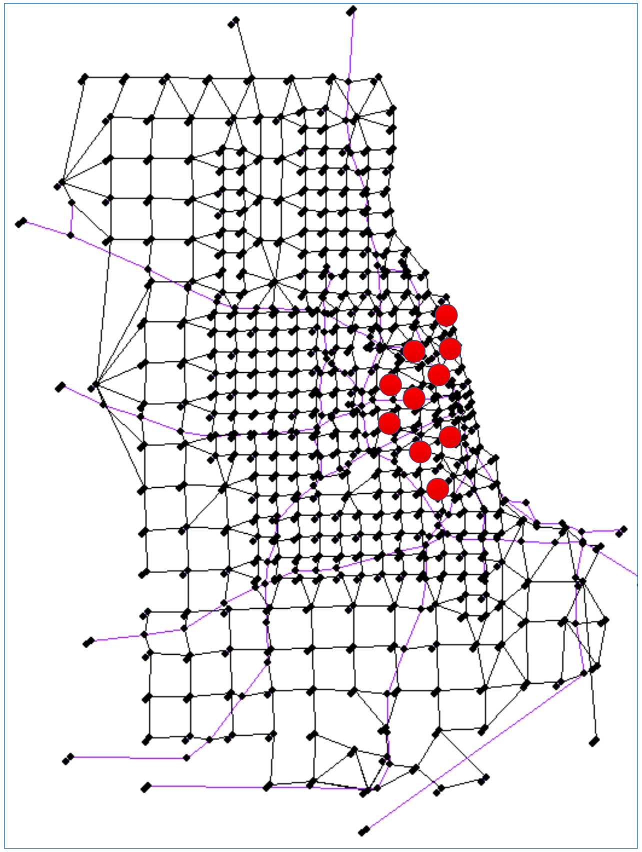
IRA A. FULTON SCHOOLS OF engineering ARIZONA STATE UNIVERSITY

6. Discussion and Preliminary Experiments

Experiment 2



(a) The heat map on waiting flows in experiment 2



(b) Top 10 of congested nodes in experiment 2

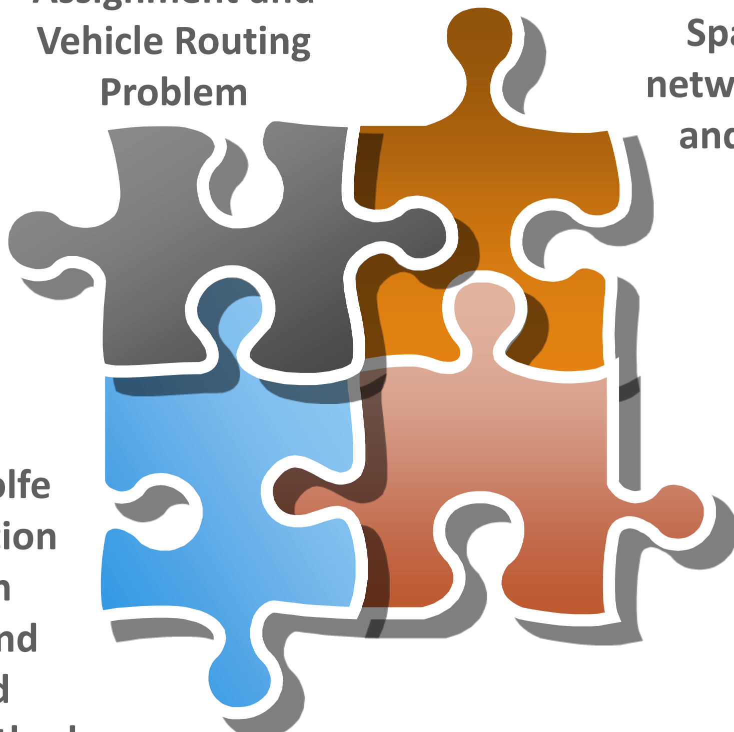
Visualization of congested nodes in experiment 2

7. Summary

Required knowledge:

Dynamic Traffic Assignment and Vehicle Routing Problem

Space-time-state network construction and network flow model



Dantzig-Wolfe decomposition algorithm , ADMM and Projected Gradient method

Time-dependent State-dependent shortest path problem

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

Selected References

Space-time-state network flow models and vehicle routing problem:

1. Mahmoudi, M. and Zhou, X., 2016. Finding optimal solutions for vehicle routing problem with pickup and delivery services with time windows: A dynamic programming approach based on state–space–time network representations. Transportation Research Part B: Methodological, 89, 19-42.
2. Liu, J., Kang, J., Zhou, X., Pendyala, R., 2018. Network-oriented household activity pattern problem for system optimization. Transportation Research Part C 94, 250-269
3. Zhou, X., Tong, L., Mahmoudi, M., Zhuge, L., Yao, Y., Zhang, Y., Shang, P., Liu, J. and Shi, T., 2018. Open-source VRPLite Package for Vehicle Routing with Pickup and Delivery: A Path Finding Engine for Scheduled Transportation Systems. Urban Rail Transit, 4(2), 68-85.

Dynamic Traffic Assignment and Traffic flow model:

4. Lu, C.C., Liu, J., Qu, Y., Peeta, S., Rouphail, N.M. and Zhou, X., 2016. Eco-system optimal time-dependent flow assignment in a congested network. *Transportation Research Part B: Methodological*, *94*, pp.217-239.
5. Zhou, X. and Taylor, J., 2014. DTALite: A queue-based mesoscopic traffic simulator for fast model evaluation and calibration. *Cogent Engineering*, *1*(1), p.961345.

Dantzig-Wolfe Decomposition algorithm

6. https://en.wikipedia.org/wiki/Dantzig%E2%80%93Wolfe_decomposition

Alternating Direction Method of Multipliers (ADMM)

7. https://web.stanford.edu/~boyd/papers/pdf/admm_slides.pdf

IRA A. FULTON SCHOOLS OF engineering
ARIZONA STATE UNIVERSITY

THANK YOU