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Autonomous Vehicle Assignment and Routing in Congested Networks

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Outline

1. Introduction

2. Problem Statement

3. Space-Time-State Network Flow Models

4. Dantzig-Wolfe Decomposition Algorithm

5. Preliminary Experiments

6. Summary

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1. Introduction

Ride Sharing Companies

Information-sharing technology

1. Introduction

The “three revolutions” in the future transportation systems

Automation

Shared Use

Electrification

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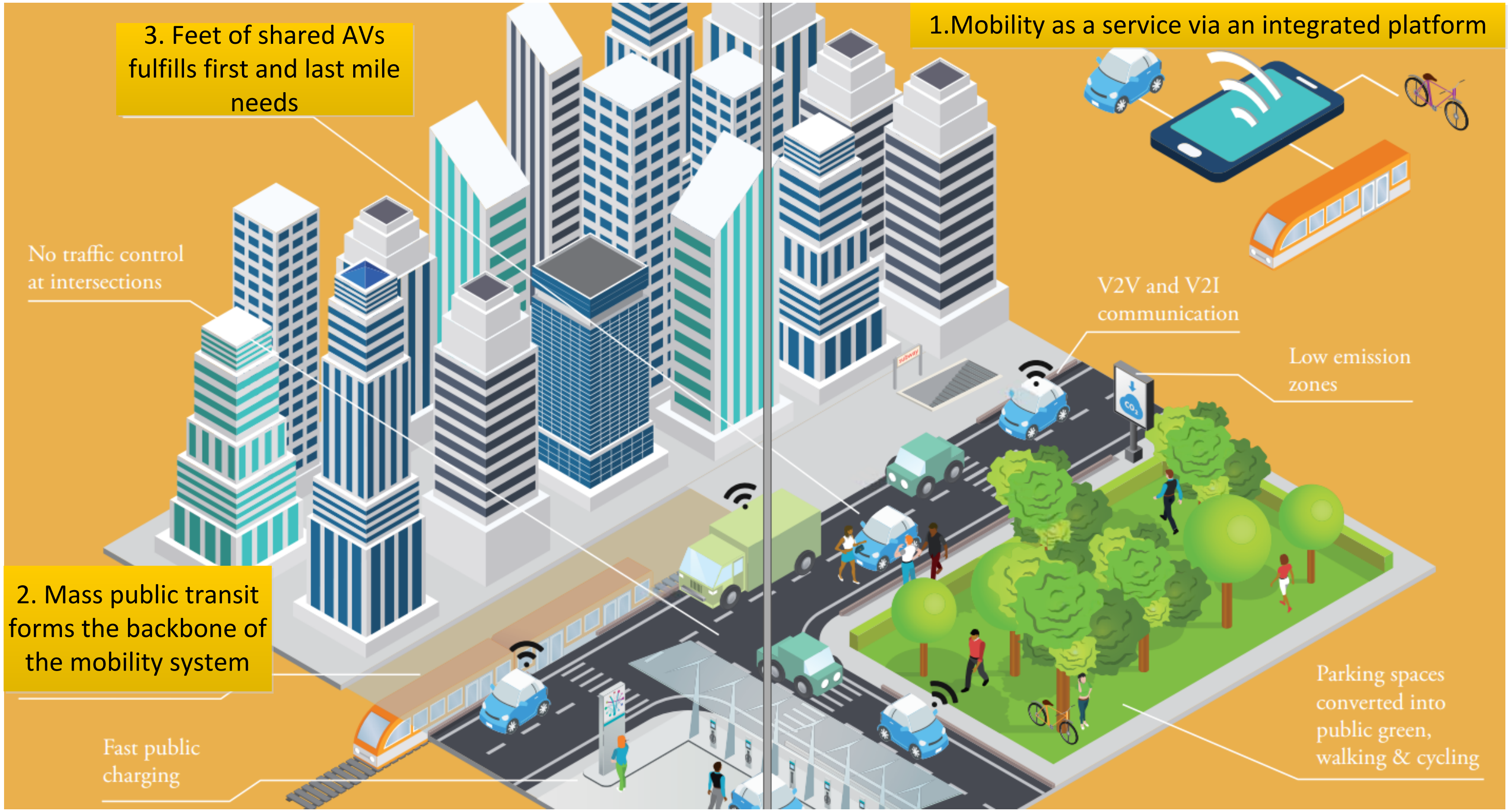
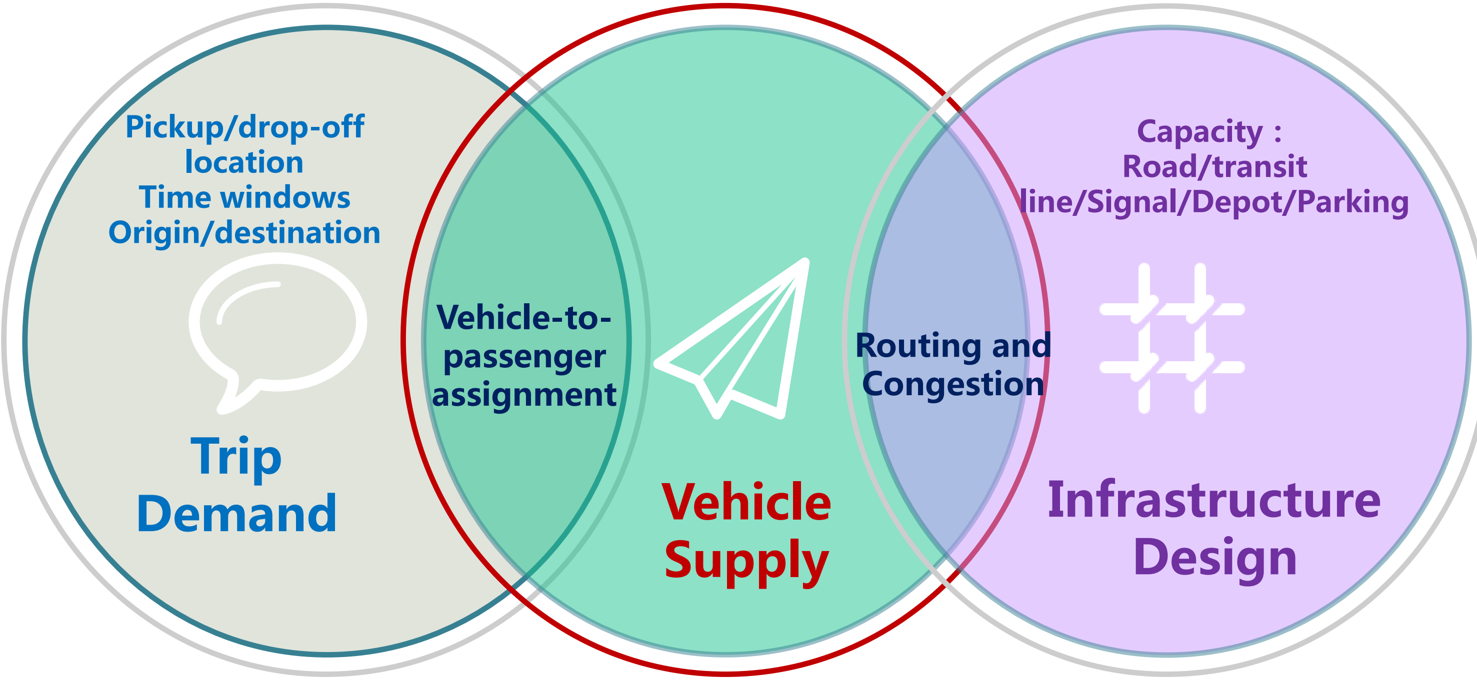
1. Introduction

Future urban transportation systems: **integrated multi-modal scheduled transportation system**



Picture source: McKinsey&Company, 2016. An integrated perspective on the future of mobility.

1. Introduction

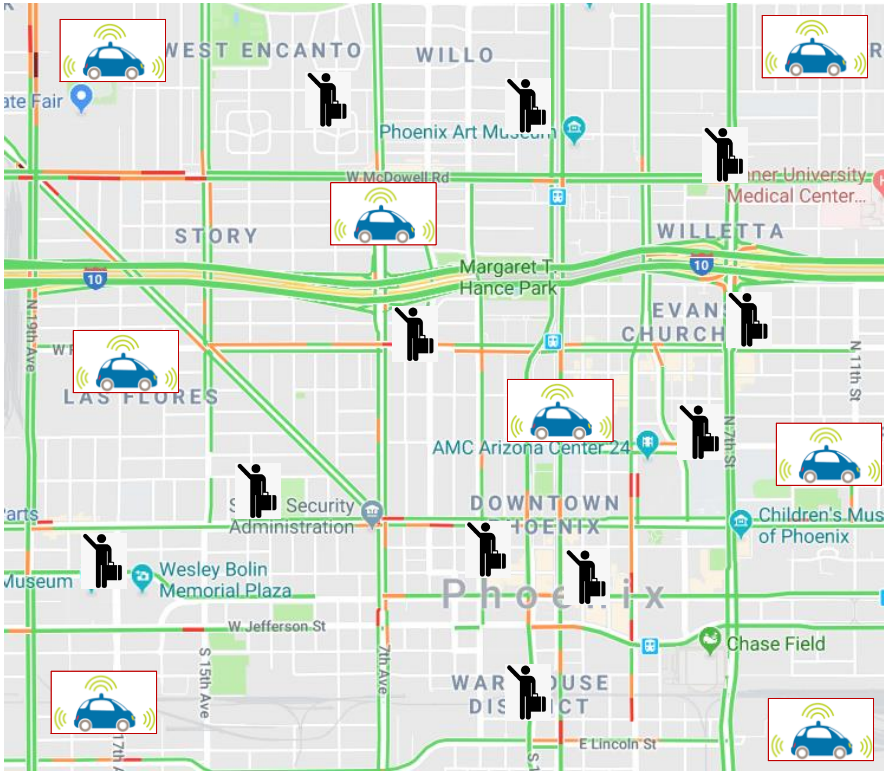


How to optimize demand, **supply** and infrastructure?

1. Introduction

Key questions:

- ❑ How many autonomous vehicles do we need?
- ❑ How many passengers can we serve?
- ❑ How to capture the new traffic congestion?
- ❑ What is the best vehicle routing and vehicle-to-passenger assignment solution?



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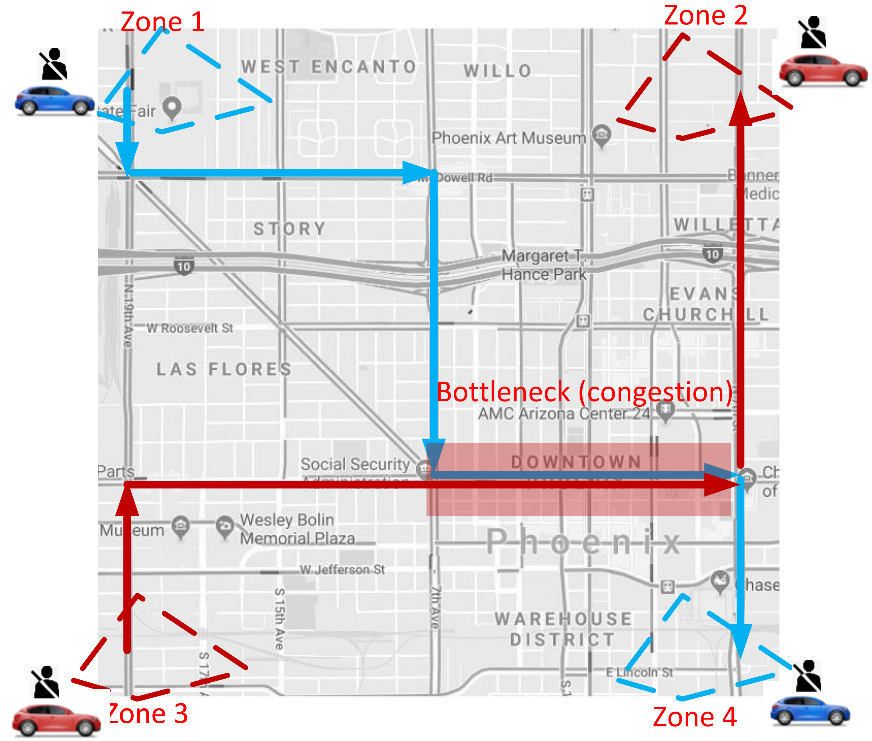
2. Problem Statement

Traffic Assignment Problem:

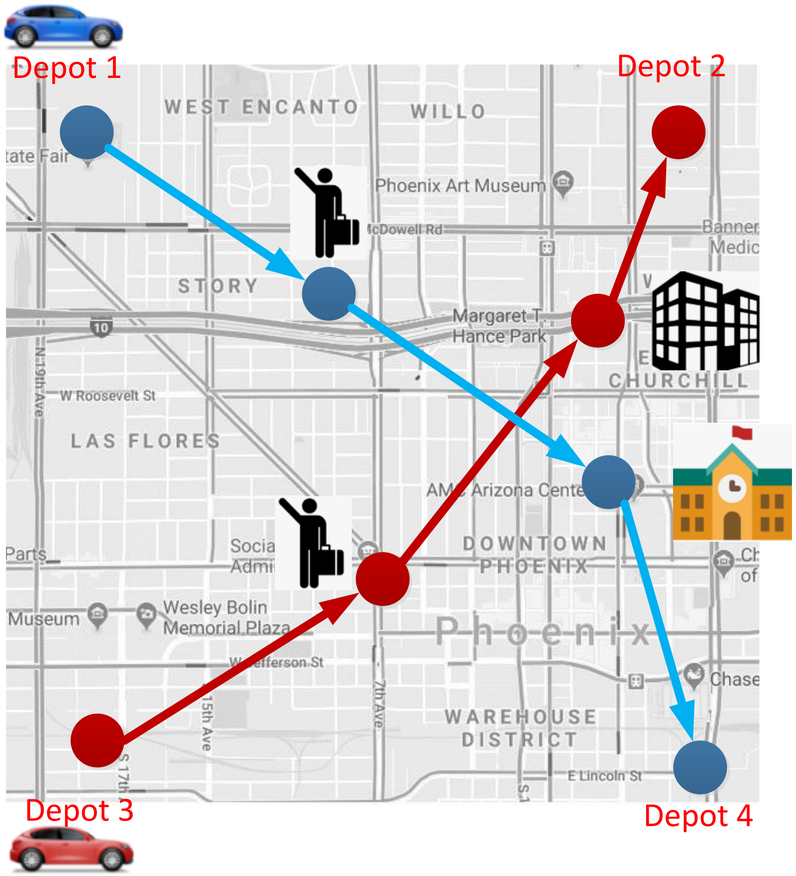
- **Network**: Physical traffic network
- **Objective function**: System Optimal or User Equilibrium
- **Road capacity**: considered to capture road congestions
- **Vehicle-to-passenger assignment**: given in advance
- **Passenger trip request**: has the same origin and destination as that of the assigned vehicles
- **Vehicle carrying capacity**: not explicitly considered
- **Variable**: usually continuous vehicle flow



2. Problem Statement

Vehicle Routing Problem:

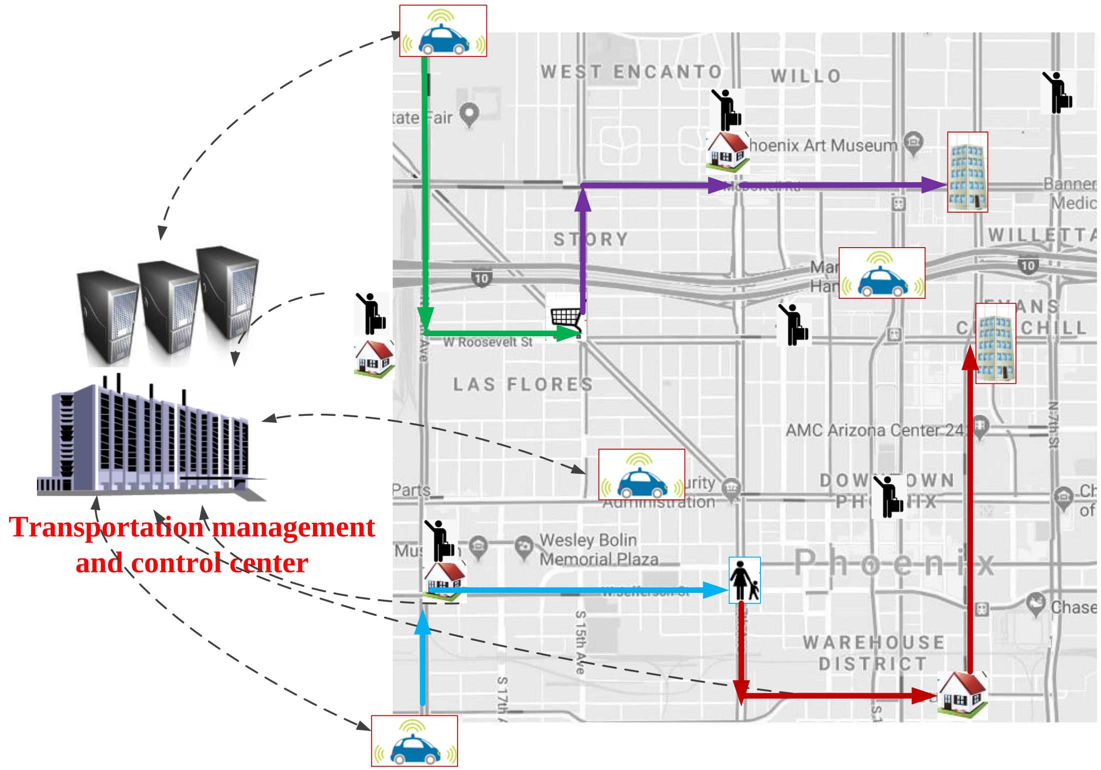
- **Network**: Virtual point-to-point network
- **Objective function**: System Optimal
- **Traffic congestion**: not explicitly considered
- **Vehicle-to-passenger assignment**: will be found
- **Passenger trip request**: has specific pick-up and drop-off location with time windows
- **Vehicle carrying capacity:** considered
- **Variable**: discrete vehicle routing and scheduling



2. Problem Statement

Keywords:

- ❑ **Physical traffic network** to consider **traffic congestion**
- ❑ **Trip requests** with Pickup and delivery with time windows
- ❑ **Autonomous vehicles with carrying capacity** for ride sharing
- ❑ **Central control (System optimal)**



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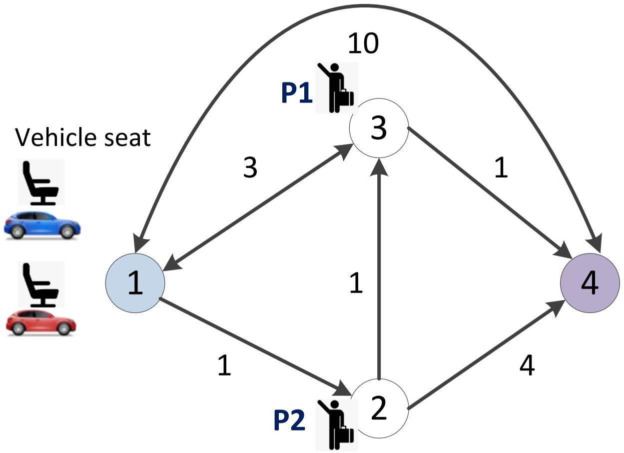
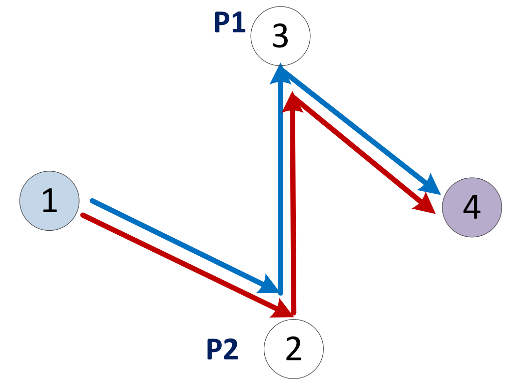
2. Problem Statement

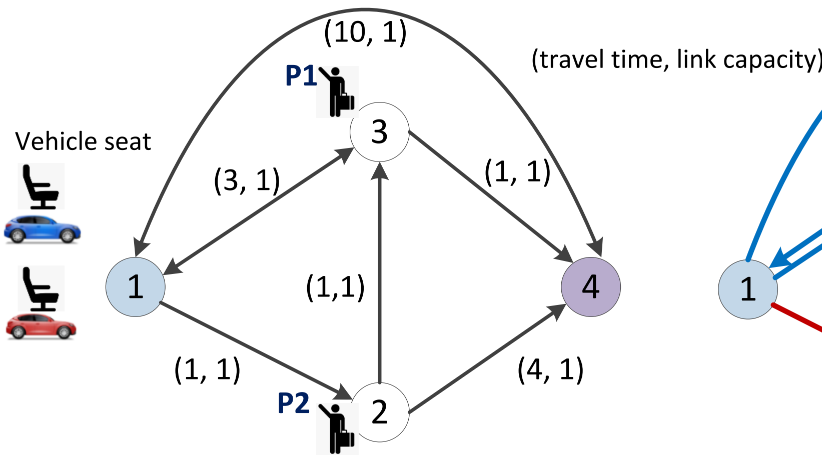
Link Capacity

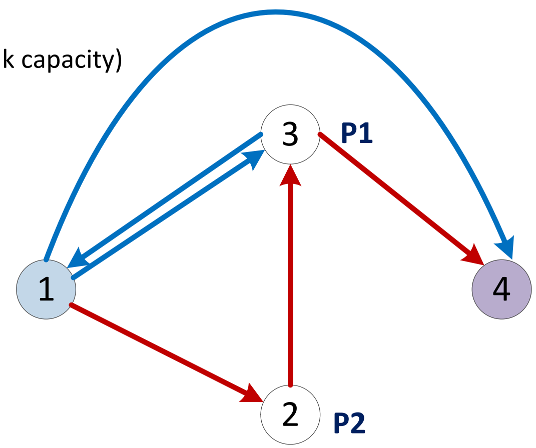
Without link capacity:
Total cost is 6





With link capacity:
Total cost is 15





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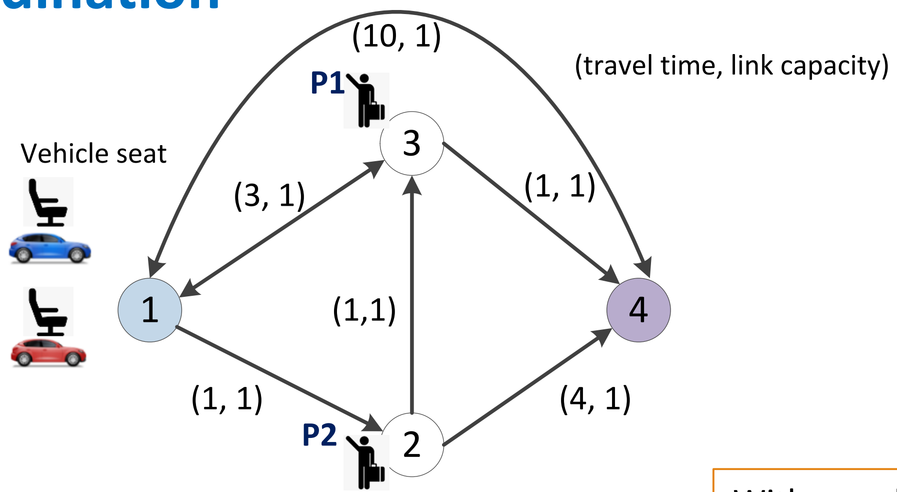
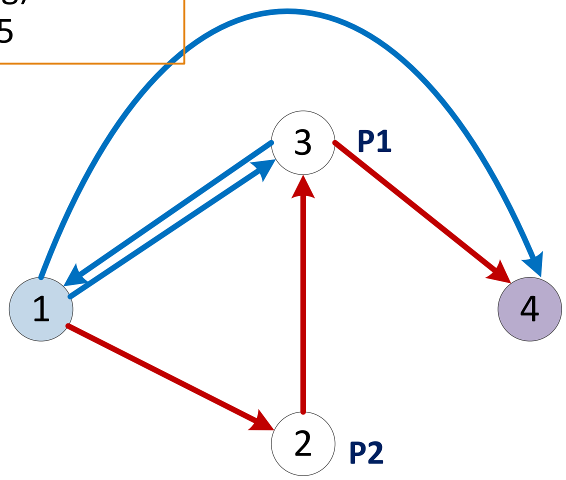
2. Problem Statement

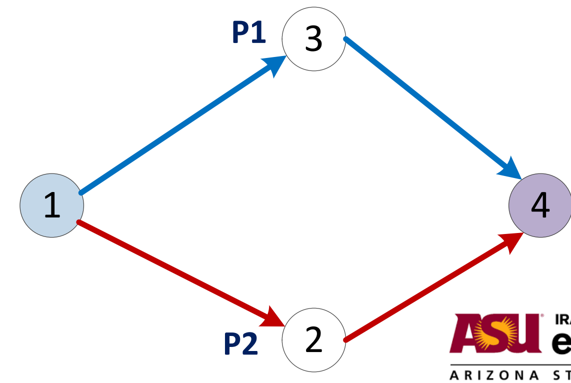
System Optimal Coordination



Without coordination
(selfish routing):
Total cost is 15

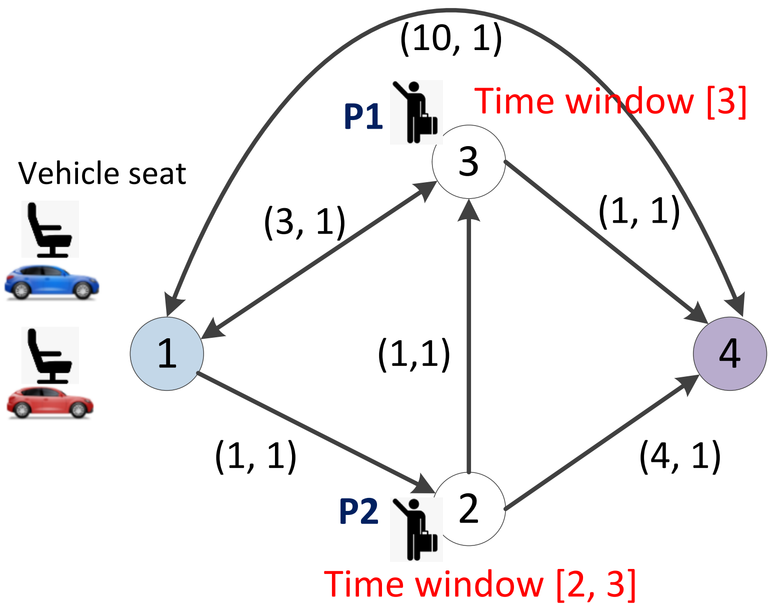


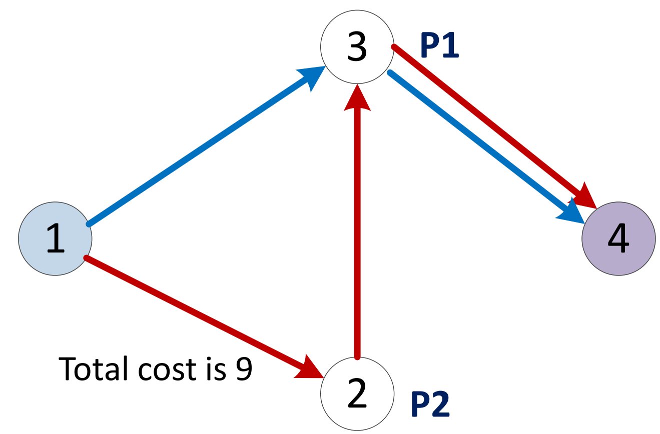
With coordination:
Total cost is 9



2. Problem Statement

Time windows

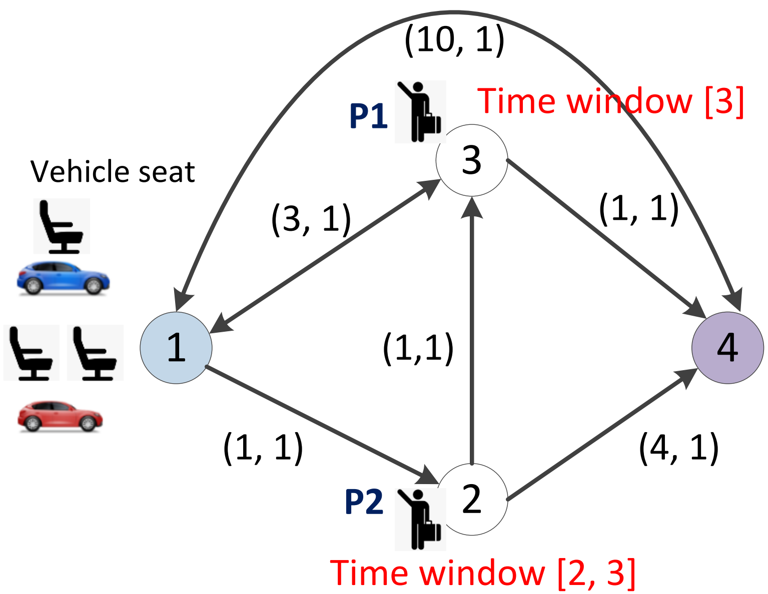


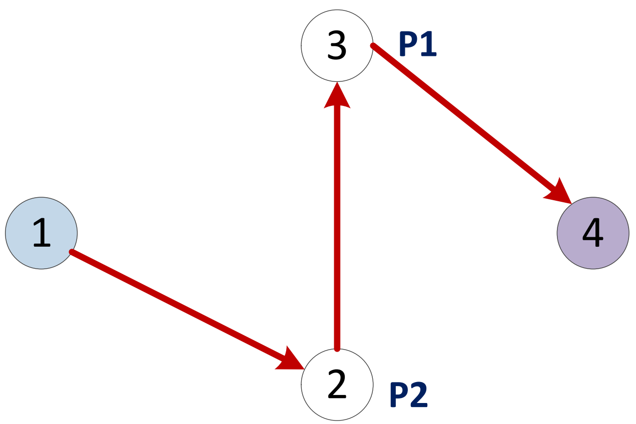


- ❑ The red vehicle can wait until time 3 to pick up passenger 2, so the blue vehicle can pick up passenger 1 at exact time 3.
- ❑ The optimal result doesn't only optimize the vehicle routing, but also the departure time of picked up passengers.

2. Problem Statement

Ridesharing (vehicle carrying capacity)





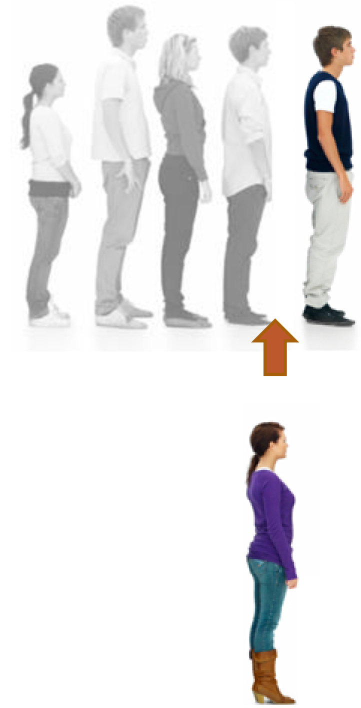
Total cost is 4

- ❑ When the red vehicle's carrying capacity is increased to 2, the total cost is reduced to 4 from 9;
- ❑ Only the red vehicle is required to serve the trip requests.

2. Problem Statement

The Challenge of solving system optimal vehicle routing with pickup and drop-off location and time windows in congested physical traffic networks:

[Time window] [Time window]



System-impact of adjusting one vehicle routing:

- ❑ System marginal vehicle travel cost
- ❑ System marginal passenger service benefit/cost

In this queuing system:

❑ Waiting time for individual: 4 min

After adding one more person in the queue:

❑ Societal travel time: additional 4 min for each person behind: +16 min, and the waiting time of added person is 4 min, so the system marginal waiting time is 20 min.

❑ Societal service benefit: some persons may not be served in their preferred time window and it decreases the service benefit.

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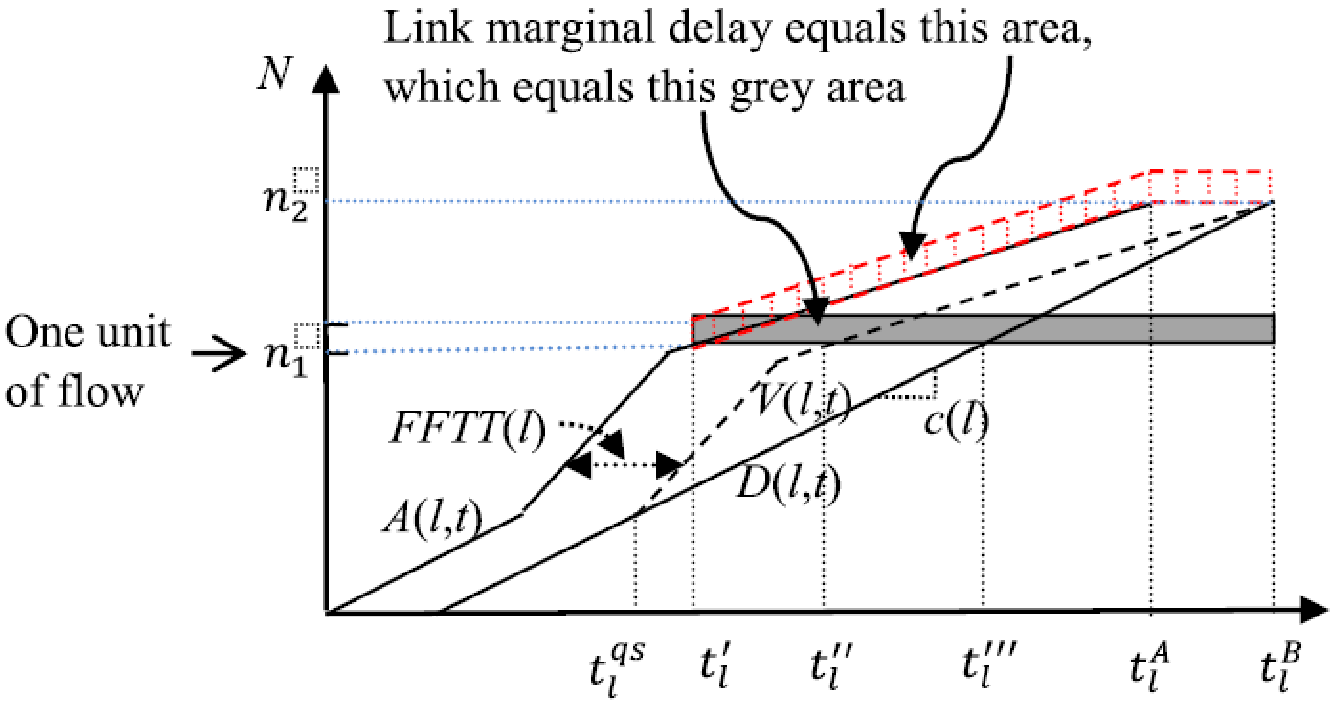
2. Problem Statement

Marginal cost calculation in system optimal dynamic traffic assignment (SODTA)



Ghali and Smith (1995)

Ghali, M.O. and Smith, M.J., 1995. A model for the dynamic system optimum traffic assignment problem. Transportation Research Part B: Methodological, 29(3), pp.155-170.

2. Problem Statement

Our Approach 1: Marginal Cost Calculation

Step 1: Build virtual pickup and drop-off links in physical traffic networks, and its service pricing is converted to generalized link travel cost

Step 2: find one initial solution as the input

Step 3: Perform network loading within a space-time-state network

3.1 use cumulative arrival and departure counts to derive the link marginal travel cost.

3.2 update the marginal service link benefit of passengers (not served or served by multiple vehicles)

Step 4: find the new least-marginal-cost route for each vehicles, and go to step 3; otherwise, stop.

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2. Problem Statement

Our Approach 2: Dantzig-Wolfe Decomposition

Restricted master problem

- ❑ Road capacity constraint
- ❑ Passenger service constraint

Solved by standard solvers

Marginal costs from the solver →

← Vehicle routing solution

Subproblems (Time-dependent State dependent shortest path problem for each vehicle)

Solved by the beam-search algorithm in space-time-state networks

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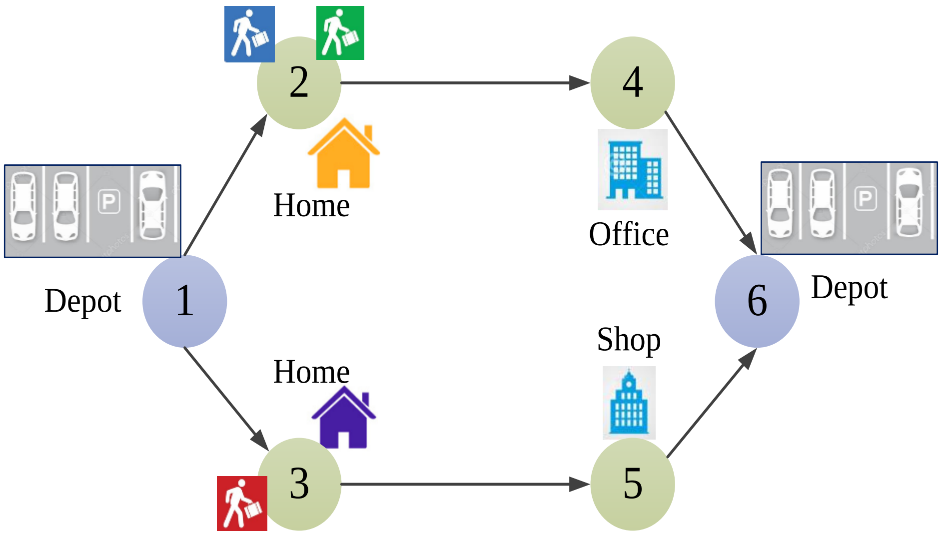
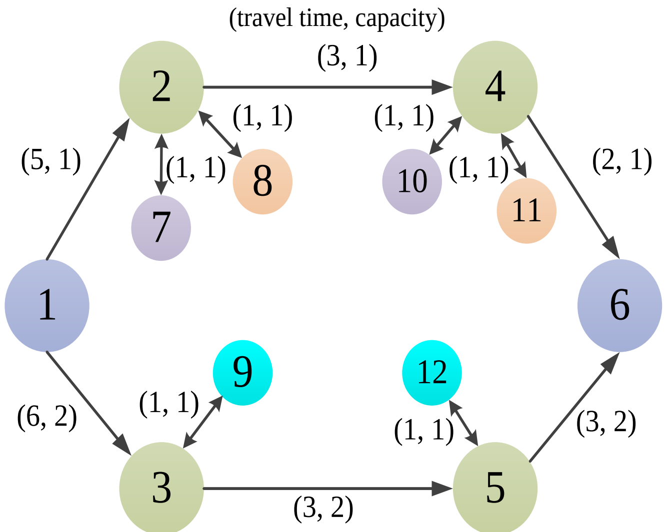
3. Space-time-state network and model formulation



(a) Physical transportation network with vehicles and trip requests

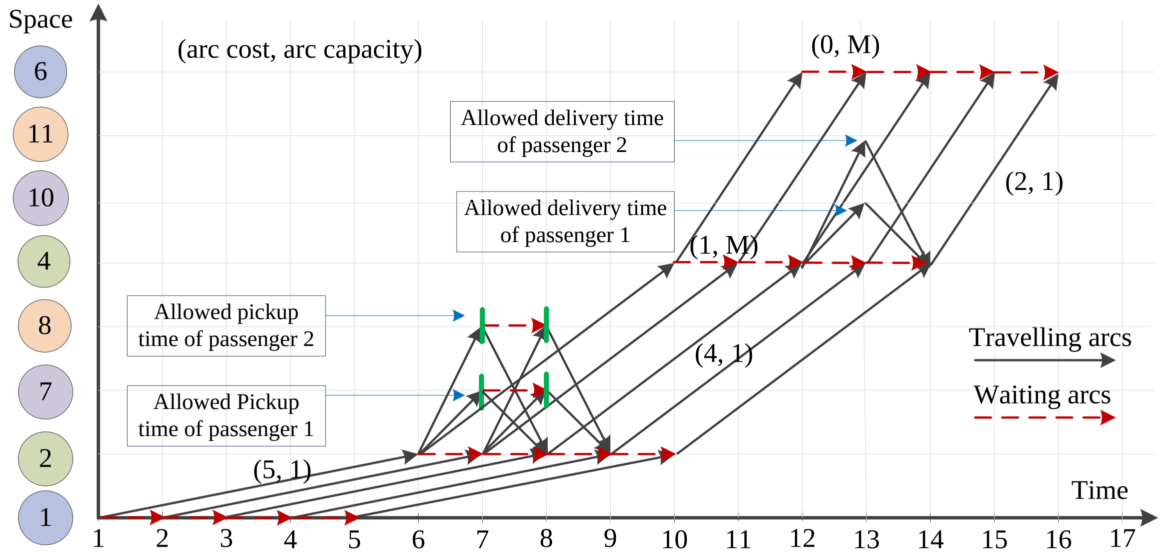


(b) Modified transportation network with virtual pickup and delivery nodes and links

Add virtual pick-up and drop-off nodes and links for each passenger

3. Space-time-state network and model formulation

Time-extended Space-time network construction for physical path $1 \rightarrow 2 \rightarrow 4 \rightarrow 6$



Space-time network

Arc (i,j,t,s) with capacity

Vertex (i,t), (j,s)

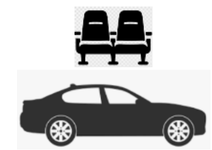
Passenger pickup and drop-off time windows and locations are embedded in this network

3. Space-time-state network and model formulation

Challenge: how to model the logit of passenger pickup and delivery with vehicle carrying capacity

One more dimension-> **Vehicle Carrying States**:

which passengers are being carried by this vehicle:



To record the **passenger service status**:

0: the passenger is not served;

1: under served (picked up but not delivered);

2: finished (delivered)

Example: In the case: if vehicle capacity is 1 and 2 passengers trip requests,

All possible states: [ ], [1[1]], [1[2]], [2[1]] or [2[2]]

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3. Space-time-state network and model formulation

Vehicle carrying state transition logit: focusing on specific passenger 1

- 0->0: vehicle departs at the origin depot
- 0->1[1]: passenger 1 is picked up at his location within time window.
- 1[1]->1[2]: passenger 1 is dropped off at his location within time window
- 1[2]->0: vehicle arrives at the destination depot.

- Once one passenger is picked up, he/she will always be dropped off, because 1[1]->0 is not a feasible state transition.
- No passenger will be served if the vehicle carrying capacity is fully used.

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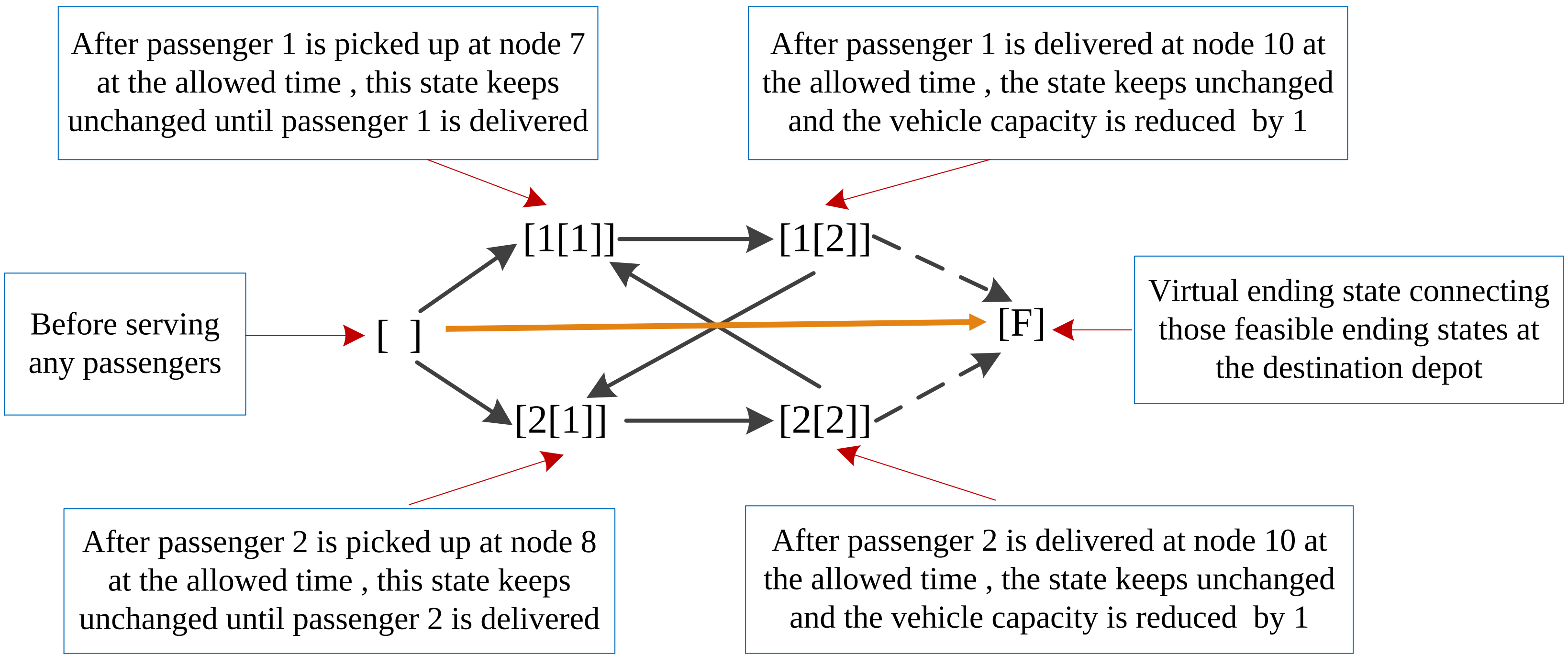
3. Space-time-state network and model formulation

Vehicle State Transition with specific locations and time intervals



Vehicle carrying state transition graph

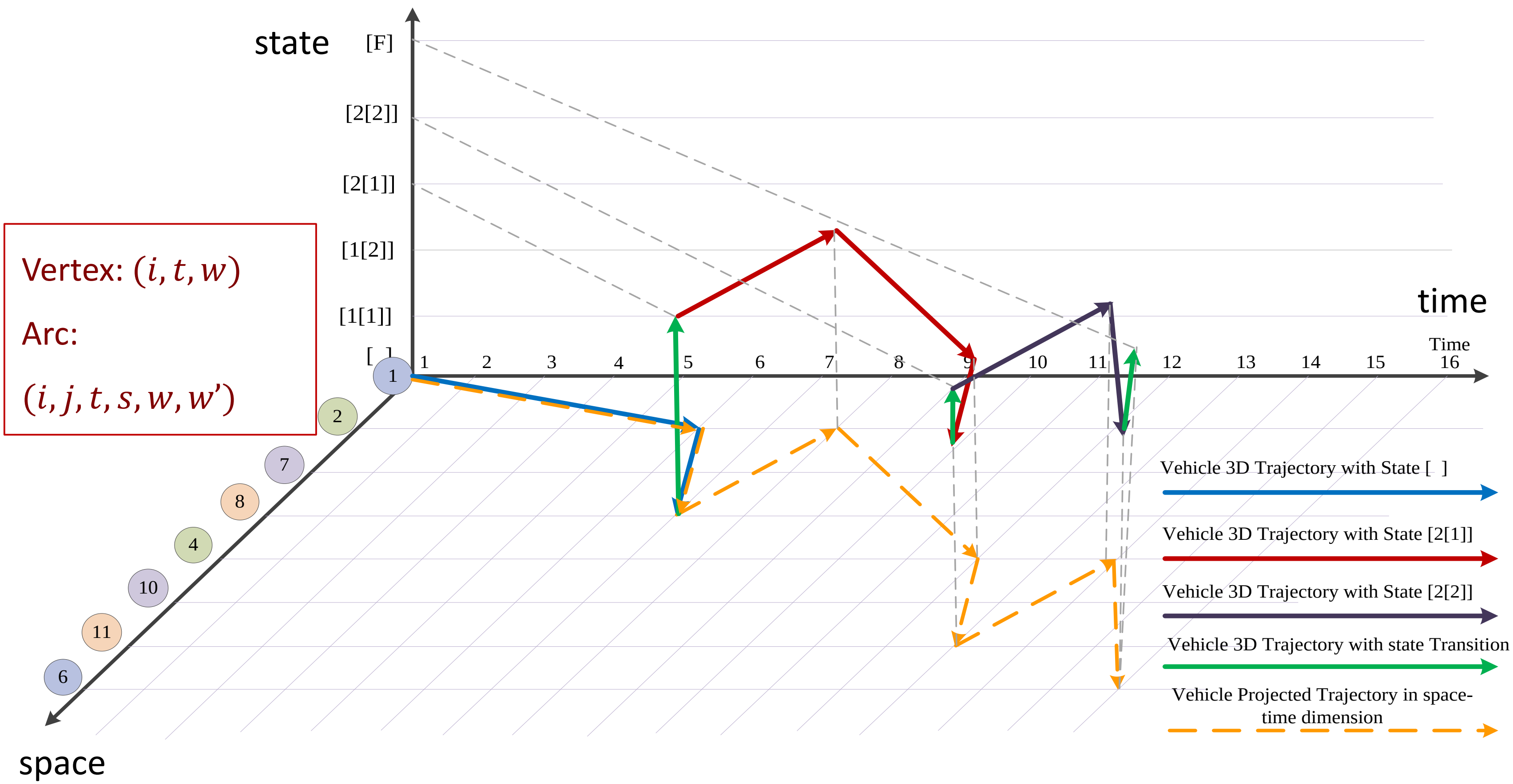
3. Space-time-state network and model formulation

One possible vehicle trajectory in a space-time-state network

Vertex: (i, t, w)

Arc:

(i, j, t, s, w, w')



3. Space-time-state network and model formulation

Mathematical formulation:

$$Min\ Z = \sum_v \sum_{(i,j,t,s,w,w')} (c_{i,j,t,s} \times x^v_{i,j,t,s,w,w'}) (1)$$

(1) Flow balance constraint for each vehicle:

$$\sum_{(i,j,t,s)} x^v_{i,j,t,s,w,w'} - \sum_{(j,i,s,t)} x^v_{i,j,t,s,w,w'} = \begin{cases} -1\ j = O(v), s = DT(v), w = [0,0,\ldots] \\ 1 \quad\ j = D(v), s = T, w = [0,0,\ldots] \\ 0 \qquad otherwise \end{cases}, \forall\ v\ (2)$$

(2) Passenger $\boldsymbol{p}$ pick-up request at $(\boldsymbol{o}, \boldsymbol{d}, \boldsymbol{\tau})$:

$$\sum_v \sum_{(i,j,t,s,w,w') \in A(p,o,d,\tau)} x^v_{i,j,t,s,w,w'} = 1, \forall\ p (3)$$

(3) Tight road capacity constraint (point queue model)

$$\sum_v \sum_w x^v_{i,j,t,s,w,w'} \le cap_{i,j,t,s}, \forall\ (i,j,t,s) \quad (4)$$

(4) Binary variables

$$x^v_{i,j,t,s,w,w'} = \{0,1\} \qquad (5)$$

3. Space-time-state network and model formulation

Capture queue spillback:

Inflow arc capacity constraint:

$\sum_w x_{i,j',t-FFTT_{i,j}+1,t,w,w'} \le Cap_{i,j',t-FFTT_{i,j}+1,t}, \forall(i,j') \in L_{inflow}, \forall t$ (6)

Outflow arc capacity constraint:

$\sum_w x_{j',j,t,t+1,w,w'} \le Cap_{j',j,t,t+1}, \forall(j',j) \in L_{outflow}, \forall t$ (7)

Link storage capacity constraint:

$\sum_w x_{j',j',t-1,t,w,w'} + \sum_w \sum_{s=t-FFTT_{i,j'}}^{t-1} x_{i,j',s,t,w,w'} \le Len_{i,j'} \times n_{i,j'} \times Jam_{i,j'}, \forall(i,j') \in L_{inflow}, \forall t$ (8)

Newell's simplified kinematic wave model (Newell, 1993)

$\sum_w \sum_{s=t-BWTT(i,j')}^{t} x_{j',j',s-1,s,w,w'} + \sum_w \sum_{s=t-FFTT_{i,j'}-BWTT_{i,j'}}^{t-1} x_{i,j',s,s+FFTT_{i,j'},w,w'} \le Len_{i,j'} \times n_{i,j'} \times Jam_{i,j'}, \forall(i,j') \in L_{inflow}, \forall t$ (9)

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4. Dantzig-Wolfe Decomposition Algorithm

Objective function

$$Min\ Z = \sum_{v} \sum_{(i,j,t,s,w,w')} (c^v_{i,j,t,s,w,w'} \times x^v_{i,j,t,s,w,w'})$$

Subject to,

(1) Flow balance constraint for each vehicle:

$$\sum_{i,t,w:(i,j,t,s,w,w')} x^v_{i,j,t,s,w,w'} - \sum_{i,t,w:(j,i,s,t,w',w)} x^v_{j,i,s,t,w',w} = \begin{cases} -1 & j = O(v), s = DT(v), w = [0,0,\dots,0] \\ 1 & j = D(v), s = T, w = [F] \\ 0 & otherwise \end{cases}, \forall v$$

(2) Passenger p's pick-up request constraint

$$\sum_{v} \sum_{i,t,s:(i,j,t,s,w,w')\in A(p)} x^v_{i,j,t,s,w,w'} = 1, \forall p$$

(3) Tight road capacity constraint (point queue model)

$$\sum_{v} \sum_{w} x^v_{i,j,t,s,w,w'} \leq cap_{i,j,t,s}, \forall (i,j,t,s)$$

(4) Binary definitional constraint

$$x^v_{i,j,t,s,w,w'} \in \{0,1\}$$

Special block: time-dependent state-dependent shortest path problem

- A special block can be solved by our VRP solution engine
- can be the subproblem in **Dantzig-Wolfe decomposition**

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4. Dantzig-Wolfe Decomposition Algorithm

Restricted master problem

$$Min \sum_v \sum_{(i,j,t,s,w,w')} [c^v_{i,j,t,s,w,w'} \times \sum_k (\lambda^v_k \times x^v_k \times \delta_{i,j,t,s,w,w'})]$$

Pick-up constraint:

$$\sum_v \sum_{(i,j,t,s,w,w') \in A(p)} \sum_k (\lambda^v_k \times x^v_k \times \delta^{v,k}_{i,j,t,s,w,w'}) = 1, \forall\, p$$

Capacity constraint:

$$\sum_v \sum_w \sum_k (\lambda^v_k \times x^v_k \times \beta^{v,k}_{i,j,t,s,w,w'}) \leq cap_{i,j,t,s}, \forall\, (i,j,t,s)$$

Column selection:

$$\sum_k \lambda^v_k = 1, \forall v$$

$$\lambda^v_k = \{0,1\}$$

4. Dantzig-Wolfe Decomposition Algorithm

Subproblems (TDSDSP)

$$Min\ Z' = \sum_{(i,j,t,s,w,w')}(c^{v}_{i,j,t,s,w,w'} \times x^{v}_{i,j,t,s,w,w'}) - \sum_{p}\sum_{(i,j,t,s,w,w')\in A(p)}(\boxed{\pi_p} \times x^{v}_{i,j,t,s,w,w'}) - \sum_{(i,j,t,s)}(\boxed{\mu_{i,j,t,s}} \times \sum_{w} x^{v}_{i,j,t,s,w,w'}) - \boxed{\omega_v} \quad (17)$$

Subject to

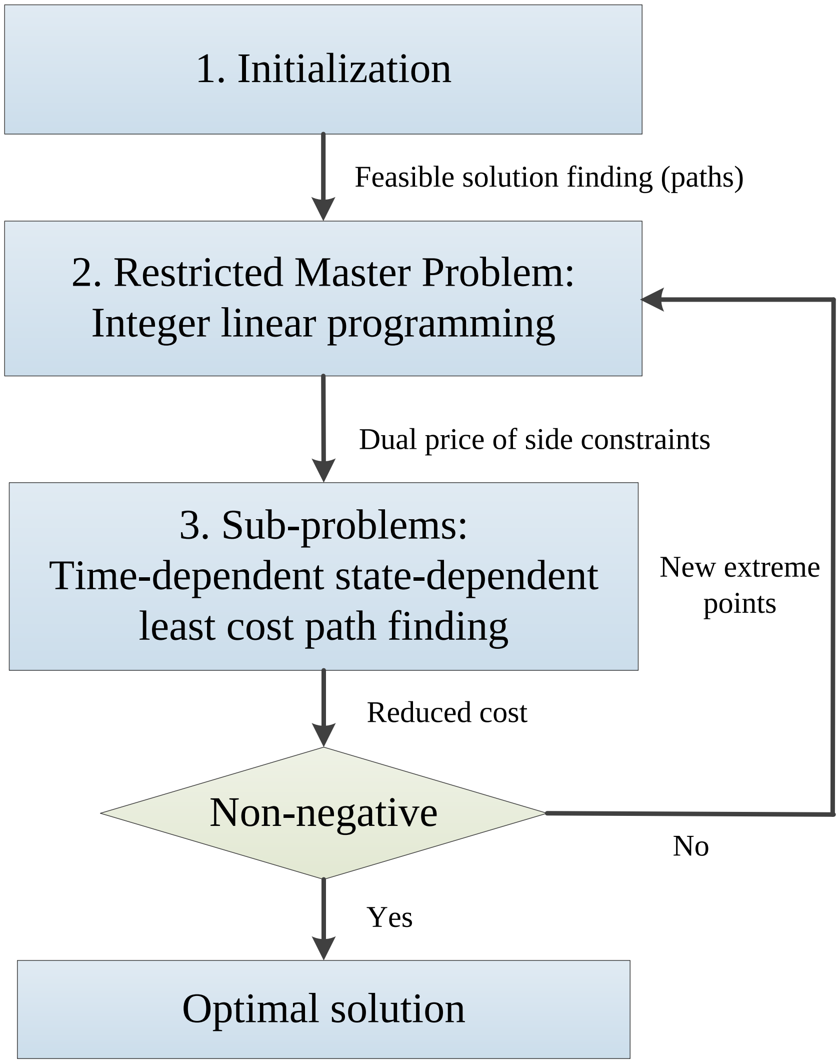
$$\sum_{(i,j,t,s,w,w')} x^{v}_{i,j,t,s,w,w'} - \sum_{(j,i,s,t,w',w)} x^{v}_{j,i,s,t,w',w} = \begin{cases} -1 & j = O(v), s = DT(v), w = [0,0,\ldots,0] \\ 1 & j = D(v), s = T, w = [F] \\ 0 & otherwise \end{cases} \quad (18)$$

Dual variable/ marginal cost of passenger pickup constraints

Dual variable/ marginal cost of congestion constraints

Dual variable/ marginal cost of path weight constraints

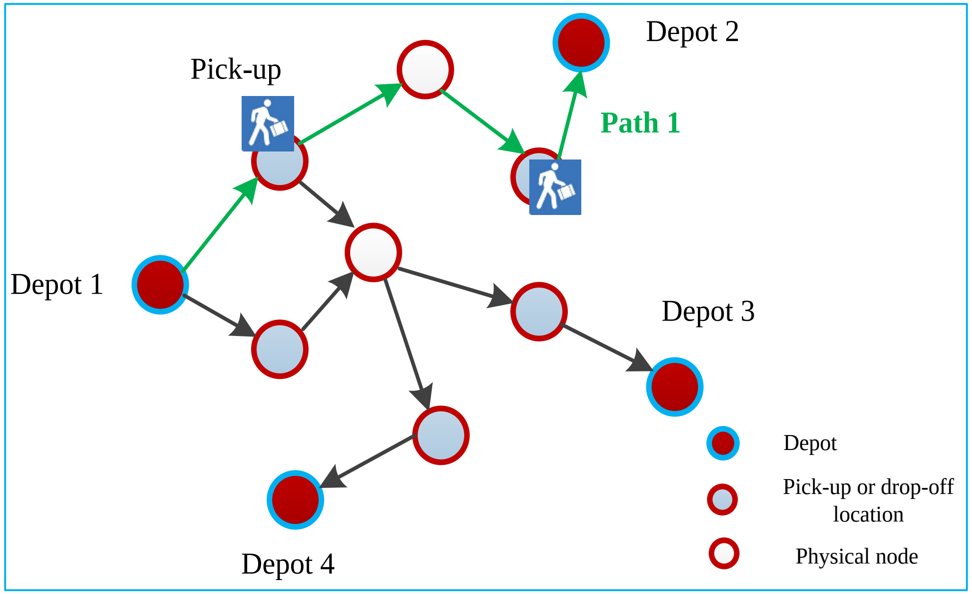
4. Dantzig-Wolfe Decomposition Algorithm



4. Dantzig-Wolfe Decomposition Algorithm

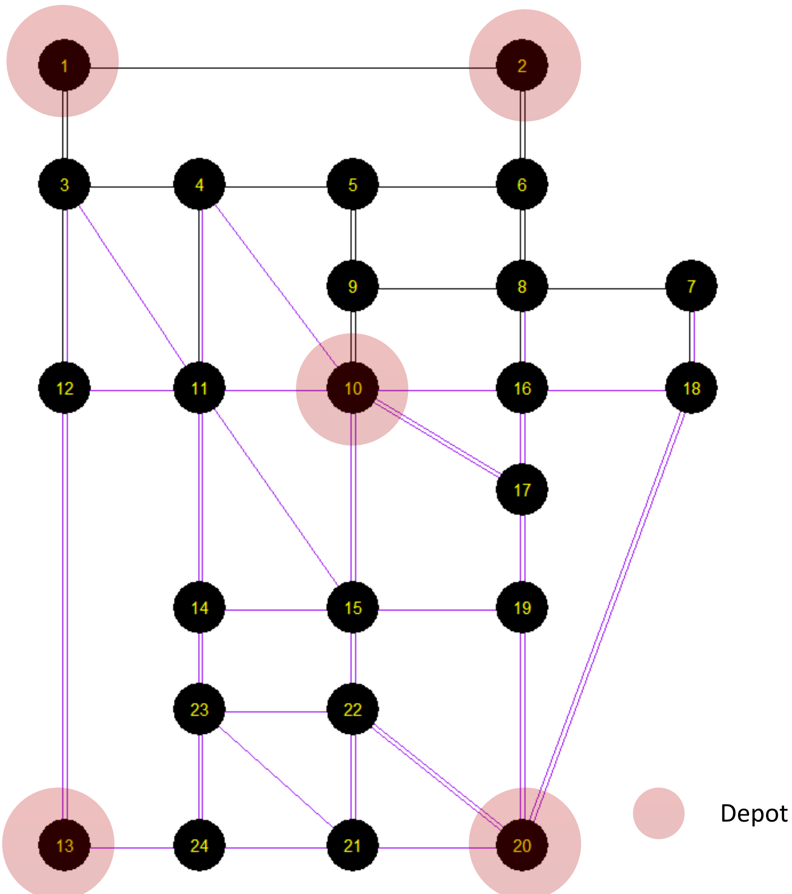
A tree-based path representation

- One-to-all shortest path tree is generated for one vehicle
- Directly obtain the shortest path for vehicles with a same depot and departure time but different destination depot
- Does not need to run the shortest path algorithm again to improve the computation efficiency.



A tree-based path representation

5. Preliminary Experiments



Sioux Falls Network

# of nodes	24
# of links	84
# of trip requests (pickup and drop-off with time windows)	30
# of available autonomous vehicles	30
# of depots	5
optimization time horizon (time unit)	110
Vehicle capacity (person)	1

5. Preliminary Experiments

Initial feasible solution

Vehicle_No	Passenger_No	Vehicle_No	Passenger_No	Vehicle_No	Passenger_No
1	[15]	11	[20]	21	[23]
2	[8]	12	[26]	22	[25]
3	[1]	13	[16]	23	[22]
4	[7]	14	[18]	24	[19]
5	[9]	15	[2]	25	[4]
6	[11]	16	[10]	26	[5]
7	[29]	17	[3]	27	[24]
8	[28]	18	[12]	28	[14]
9	[17]	19	[27]	29	[13]
10	[21]	20	[30]	30	[6]

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5. Preliminary Experiments

Dantzig-Wolfe decomposition algorithm solution:

- ❑ Each passenger has specific pickup and drop-off location and time windows
- ❑ The vehicle benefit of serving one passenger is 20
- ❑ The vehicle waiting cost is the half of the waiting time

	Number of required vehicles	Total travel cost
Initial solution	30	1096
vehicle carrying capacity is 1	27	967.5
vehicle carrying capacity is 2	25	869.5

Take **vehicle 9** as an example:

- ❑ In initial solution: picks up passenger 17 -> drops off passenger 17;
- ❑ Vehicle carrying capacity is 1: picks up passenger 17 -> drops off passenger 17-> picks up passenger 29 -> drops off passenger 29
- ❑ Vehicle carrying capacity is 2: picks up passenger 17 -> drops off passenger 17-> picks up passenger 30-> picks up passenger 29-> drops off passenger 29-> drops off passenger 30

6. Summary

Our goals:

- ❑ Minimize the system-level travel cost, including vehicle travel time and service benefits
- ❑ Satisfy passengers' trip requests with pickup and drop-off location and time windows
- ❑ Consider the road congestion incurred by assigned vehicles

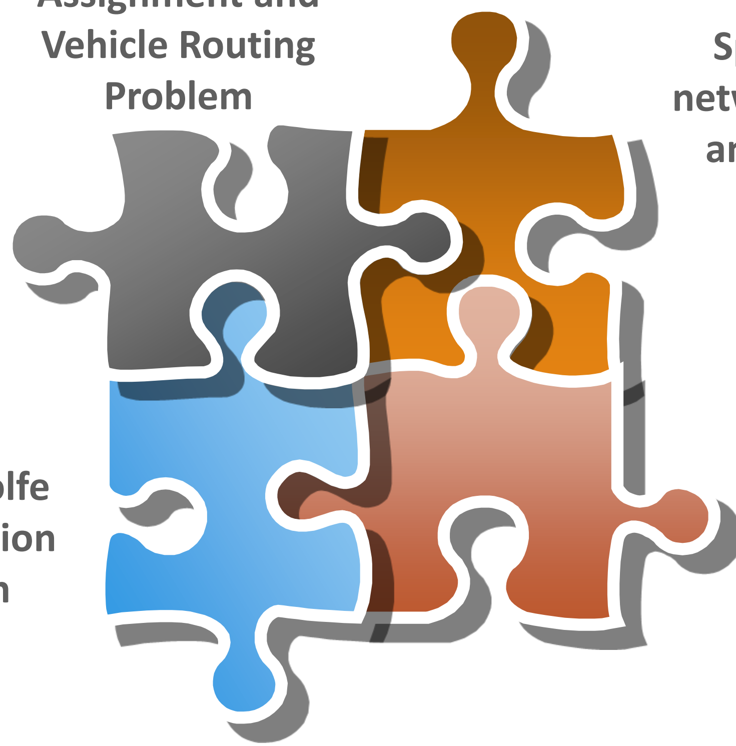
Future Extension: multi-modal scheduled transportation system: human-driven vehicles, autonomous vehicles, and public transit systems.

6. Summary

Required knowledge:

Dynamic Traffic Assignment and Vehicle Routing Problem

Space-time-state network construction and network flow model



Dantzig-Wolfe decomposition algorithm

Time-dependent State-dependent shortest path problem

References

Space-time-state network flow models and vehicle routing problem:

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