Uniform charge density plane forms a uniform electric field

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- Notice that we shall,
  - for any point in space above the uniform charge density plane (actually similar for below the plane),
  - put a test charge there,
  - and consider the test charge as the subject
    - (instead of the electric field lines which is a bit more difficult to think about),
  - and we think about the electric field acted on the test charge by the points on the plane with uniform charge.
- Also, below it's not so necessary to consider the (electric) charge of the test charge and the charge of the plane, as only signs would be altered in this way.

1. WLOG
   - (WLOG means *without loss of generality*, saying that all other cases are easy after you understand the case described below),
   - we consider an arbitrary point A above the plane with height h (of course measured by the foot of perpendicular from A to the plane), and a point B vertically above the plane by a distance of h again, so that the height of B is actually twice that of A measured from the plane.
2. Create a *right* circular cone with [a subset of the plane] as base and A as the vertex, and consider the electric force being applied on A by point charge on the circumference of the base of the cone (a circle).
3. The electric force act on A by the plane is the resultant force (vector sum / superposition of electric field) of all such electric force described in the point 2, for any radius $r > 0$.
   - For any point charge on the plane, the electric force acted on the test charge is a vector, but we can take the vertical component because (we can assume, in this case that) other 2 component of forces (3D --- 3 components) are balanced out by other point charges on the plane.
     - May refer to *Active Physics textbook 4 page 37 figure 20.33b*.
   - So we can assume that there is a [corresponded magnitude $a > 0$ of vertical component of the electric force from any point charge on the plane on that test charge] associated with a test charge.
   - Hence we ignore treat the vector as a scalar when defining functions / doing (simple) integration below.
4. For any such cone with base radius r, we double it in length (that is, actually, 7 times more in volume of course) so that now the base radius is $2r$ and with B as vertex.
5. Now the electric force on B by (all) the point charge on the circumference of the base of the cone should be **half** of that considered in point 2, because:
   - the slant height (as distance to point charge) is doubled and the electric force is inversely proportional to distance, and
   - the circumcenter of base circle is doubled and the total electric force experienced by the top of the cone is doubled.
6. Or we may view in by the intuition / concept of *integration*, which is simply *summation* over a continous length / area / whatsoever:
   - We can integrate with respect to r from $r = 0$ to ∞.
   - (Actually in more advanced Mathematics we can define such integral, which usually, is the $\lim_{c\to\infty}\int_0^c \dots d(\dots)$, and notice that even though the integrand (the function) is not defined when $r = 0$ we can also define the integral, because endpoint(s) are not important when integrating.)
   - Let $f_1(r) = f(A, r)$ and $f_2(r) = f(B, r)$,

$$\int_0^\infty f(A,r)dr = \lim_{c\to\infty}\int_0^c f_1(r)dr = \lim_{c\to\infty}\int_0^c 2f_2(2r)dr = \lim_{c\to\infty}\int_0^{2c} f_2(2r)d(2r)$$
$$= \int_0^\infty f(B,r)dr$$