Area formula for simple polygons

Tony Ma

2020-10-14

1 Preface

In this paper, we shall try to understand and prove the area formula for general *(simply) polygons*.
However, subtleties in proofs should be **justified** with more rigors, including but limited to the notion of area, and how the area of a curve / straight line is 0.
This paper is being updated on my webpage.

2 Definitions

A *polygon* P is the closed region of the plane bounded by a finite collection of line segments forming a closed curve that does not intersect itself. [1] (Page 1)
Though, I would modify to have my own definition for *polygon*.

Definition 2.1. A *polygon* is the closed region of the infinite plane bounded by its *boundary*, which, with **distinct** points $P_0, \cdots, P_{n-1}$ $(n \in \mathbb{N})$ as vertices (and with $P_n = P_0$ by convention), is a closed curve formed by line segments $P_iP_{i+1}, i = 0, \cdots, n-1$, and the line segments are all disjoint when all the vertices are ignored.

Remark 2.2. Notice that in Definition 2.1 we allow a polygon with line segments '*touching itself*', as well as an '*interior angle of* π', as long as the vertices are all distinct and satisfying the above definition.

3 The formula

Theorem 3.1. *For any simple polygon with* $n \in \mathbb{N}$ *vertices* $(x_n, y_n) = (x_0, y_0), \cdots, (x_{n-1}, y_{n-1})$*, we have: [1]*

$$Area = \frac{\left|\sum_{i=0}^{n-1}(x_iy_{i+1} - x_{i+1}y_i)\right|}{2} = \frac{1}{2}\left|\sum_{i=0}^{n-1}\begin{vmatrix} x_i & x_{i+1} \\ y_i & y_{i+1} \end{vmatrix}\right| \tag{1}$$

References

[1] Satyan L.Devadoss and Joseph O'Rourke. *Discrete and Computational Geometry*. Princeton University Press, 2011. ISBN: 978-0-691-14553-2.