Sage Reference Manual: Fixed and Arbitrary Precision Numerical Fields

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The Sage Development Team

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CONTENTS

Sage supports two optimized fixed precision fields for numerical computation, the real double (RealDoubleField) and complex double fields (ComplexDoubleField). Sage also supports arbitrary precision real (RealField) and complex fields (ComplexField), and real and complex interval arithmetic (RealIntervalField and ComplexIntervalField).

Real and complex double elements are optimized implementations that use the GNU Scientific Library for arithmetic and some special functions. Arbitrary precision real and complex numbers are implemented using the MPFR library, which builds on GMP. (Note that Sage doesn't currently use the MPC library.) The interval arithmetic field is implemented using the MPFI library.

In many cases the PARI C-library is used to compute special functions when implementations aren't otherwise available.

CONTENTS

DOUBLE PRECISION REAL NUMBERS

EXAMPLES:

We create the real double vector space of dimension 3:

```
sage: V = RDF^3; V
Vector space of dimension 3 over Real Double Field
```

Notice that this space is unique:

```
sage: V is RDF^3
True
sage: V is FreeModule(RDF, 3)
True
sage: V is VectorSpace(RDF, 3)
True
```

Also, you can instantly create a space of large dimension:

```
sage: V = RDF^10000
```

TESTS:

Test NumPy conversions:

```
sage: RDF(1).__array_interface__
{'typestr': '=f8'}
sage: import numpy
sage: numpy.array([RDF.pi()]).dtype
dtype('float64')
```

class sage.rings.real_double.**RealDoubleElement**
    Bases: sage.structure.element.FieldElement

    An approximation to a real number using double precision floating point numbers. Answers derived from calculations with such approximations may differ from what they would be if those calculations were performed with true real numbers. This is due to the rounding errors inherent to finite precision calculations.

    NaN()
        Return Not-a-Number NaN.

        EXAMPLES:
```
sage: RDF.NaN()
NaN
```

abs()
>   Returns the absolute value of `self`.
>
>   EXAMPLES:
>    ```
>    sage: RDF(1e10).abs()
>    10000000000.0
>    sage: RDF(-1e10).abs()
>    10000000000.0
>    ```

acosh()
>   Return the hyperbolic inverse cosine of `self`.
>
>   EXAMPLES:
>    ```
>    sage: q = RDF.pi()/2
>    sage: i = q.cosh(); i
>    2.5091784786580567
>    sage: abs(i.acosh()-q) < 1e-15
>    True
>    ```

agm(*other*)
>   Return the arithmetic-geometric mean of `self` and `other`. The arithmetic-geometric mean is the common limit of the sequences u_n and v_n, where u_0 is `self`, v_0 is other, u_{n+1} is the arithmetic mean of u_n and v_n, and v_{n+1} is the geometric mean of u_n and v_n. If any operand is negative, the return value is `NaN`.
>
>   EXAMPLES:
>    ```
>    sage: a = RDF(1.5)
>    sage: b = RDF(2.3)
>    sage: a.agm(b)
>    1.8786484558146697
>    ```
>
>   The arithmetic-geometric mean always lies between the geometric and arithmetic mean:
>    ```
>    sage: sqrt(a*b) < a.agm(b) < (a+b)/2
>    True
>    ```

algdep(*n*)
>   Return a polynomial of degree at most n which is approximately satisfied by this number.
>
>   ---
>
>   **Note:** The resulting polynomial need not be irreducible, and indeed usually won't be if this number is a good approximation to an algebraic number of degree less than n.
>
>   ---
>
>   ALGORITHM:
>
>   Uses the PARI C-library `algdep` command.
>
>   EXAMPLE:
>    ```
>    sage: r = sqrt(RDF(2)); r
>    1.4142135623730951
>    sage: r.algebraic_dependency(5)
>    x^2 - 2
>    ```

algebraic_dependency(*n*)
>   Return a polynomial of degree at most n which is approximately satisfied by this number.
>
>   ---
>
>   **Note:** The resulting polynomial need not be irreducible, and indeed usually won't be if this number is a good approximation to an algebraic number of degree less than n.
>
>   ---

ALGORITHM:

Uses the PARI C-library `algdep` command.

EXAMPLE:
```
sage: r = sqrt(RDF(2)); r
1.4142135623730951
sage: r.algebraic_dependency(5)
x^2 - 2
```

arccos()
>   Return the inverse cosine of `self`.

>   EXAMPLES:
```
sage: q = RDF.pi()/3
sage: i = q.cos()
sage: i.arccos() == q
True
```

arcsin()
>   Return the inverse sine of `self`.

>   EXAMPLES:
```
sage: q = RDF.pi()/5
sage: i = q.sin()
sage: i.arcsin() == q
True
```

arcsinh()
>   Return the hyperbolic inverse sine of `self`.

>   EXAMPLES:
```
sage: q = RDF.pi()/2
sage: i = q.sinh(); i
2.3012989023072947
sage: abs(i.arcsinh()-q) < 1e-15
True
```

arctan()
>   Return the inverse tangent of `self`.

>   EXAMPLES:
```
sage: q = RDF.pi()/5
sage: i = q.tan()
sage: i.arctan() == q
True
```

arctanh()
>   Return the hyperbolic inverse tangent of `self`.

>   EXAMPLES:
```
sage: q = RDF.pi()/2
sage: i = q.tanh(); i
0.9171523356672744
sage: i.arctanh() - q  # rel tol 1
4.440892098500626e-16
```

ceil()
> Return the ceiling of `self`.

> EXAMPLES:
> ```
> sage: RDF(2.99).ceil()
> 3
> sage: RDF(2.00).ceil()
> 2
> sage: RDF(-5/2).ceil()
> -2
> ```

ceiling()
> Return the ceiling of `self`.

> EXAMPLES:
> ```
> sage: RDF(2.99).ceil()
> 3
> sage: RDF(2.00).ceil()
> 2
> sage: RDF(-5/2).ceil()
> -2
> ```

conjugate()
> Returns the complex conjugate of this real number, which is the real number itself.

> EXAMPLES:
> ```
> sage: RDF(4).conjugate()
> 4.0
> ```

cos()
> Return the cosine of `self`.

> EXAMPLES:
> ```
> sage: t=RDF.pi()/2
> sage: t.cos()
> 6.123233995736757e-17
> ```

cosh()
> Return the hyperbolic cosine of `self`.

> EXAMPLES:
> ```
> sage: q = RDF.pi()/12
> sage: q.cosh()
> 1.0344656400955106
> ```

coth()
> Return the hyperbolic cotangent of `self`.

> EXAMPLES:
> ```
> sage: RDF(pi).coth()
> 1.003741873197321
> sage: CDF(pi).coth()
> 1.0037418731973213
> ```

csch()
> Return the hyperbolic cosecant of `self`.

EXAMPLES:
```
sage: RDF(pi).csch()
0.08658953753004694
sage: CDF(pi).csch()  # rel tol 1e-15
0.08658953753004696
```

cube_root()
    Return the cubic root (defined over the real numbers) of `self`.

    EXAMPLES:
```
sage: r = RDF(125.0); r.cube_root()
5.000000000000001
sage: r = RDF(-119.0)
sage: r.cube_root()^3 - r  # rel tol 1
-1.4210854715202004e-14
```

erf()
    Return the value of the error function on `self`.

    EXAMPLES:
```
sage: RDF(6).erf()
1.0
```

exp()
    Return e^{self}.

    EXAMPLES:
```
sage: r = RDF(0.0)
sage: r.exp()
1.0

sage: r = RDF('32.3')
sage: a = r.exp(); a
106588847274864.47
sage: a.log()
32.3

sage: r = RDF('-32.3')
sage: r.exp()
9.381844588498685e-15

sage: RDF(1000).exp()
+infinity
```

exp10()
    Return 10^{self}.

    EXAMPLES:
```
sage: r = RDF(0.0)
sage: r.exp10()
1.0

sage: r = RDF(32.0)
sage: r.exp10()
1.0000000000000069e+32
```

```
sage: r = RDF(-32.3)
sage: r.exp10()
5.011872336272702e-33
```

exp2()
    Return 2^{self}.

    EXAMPLES:
```
sage: r = RDF(0.0)
sage: r.exp2()
1.0
```

```
sage: r = RDF(32.0)
sage: r.exp2()
4294967295.9999967
```

```
sage: r = RDF(-32.3)
sage: r.exp2()
1.8911724825302065e-10
```

floor()
    Return the floor of `self`.

    EXAMPLES:
```
sage: RDF(2.99).floor()
2
sage: RDF(2.00).floor()
2
sage: RDF(-5/2).floor()
-3
```

frac()
    Return a real number in $(-1, 1)$. It satisfies the relation: `x = x.trunc() + x.frac()`

    EXAMPLES:
```
sage: RDF(2.99).frac()
0.9900000000000002
sage: RDF(2.50).frac()
0.5
sage: RDF(-2.79).frac()
-0.79
```

gamma()
    Return the value of the Euler gamma function on `self`.

    EXAMPLES:
```
sage: RDF(6).gamma()
120.0
sage: RDF(1.5).gamma()  # rel tol 1e-15
0.8862269254527584
```

hypot(*other*)
    Computes the value $\sqrt{s^2 + o^2}$ where s is `self` and o is `other` in such a way as to avoid overflow.

    EXAMPLES:

```
sage: x = RDF(4e300); y = RDF(3e300);
sage: x.hypot(y)
5e+300
sage: sqrt(x^2+y^2) # overflow
+infinity
```

imag()
    Return the imaginary part of this number, which is zero.

    EXAMPLES:
```
sage: a = RDF(3)
sage: a.imag()
0.0
```

integer_part()
    If in decimal this number is written `n.defg`, returns `n`.

    EXAMPLES:
```
sage: r = RDF('-1.6')
sage: a = r.integer_part(); a
-1
sage: type(a)
<type 'sage.rings.integer.Integer'>
sage: r = RDF(0.0/0.0)
sage: a = r.integer_part()
Traceback (most recent call last):
...
TypeError: Attempt to get integer part of NaN
```

is_NaN()
    Check if `self` is `NaN`.

    EXAMPLES:
```
sage: RDF(1).is_NaN()
False
sage: a = RDF(0)/RDF(0)
sage: a.is_NaN()
True
```

is_infinity()
    Check if `self` is ∞.

    EXAMPLES:
```
sage: a = RDF(2); b = RDF(0)
sage: (a/b).is_infinity()
True
sage: (b/a).is_infinity()
False
```

is_integer()
    Return True if this number is a integer

    EXAMPLES:
```
sage: RDF(3.5).is_integer()
False
sage: RDF(3).is_integer()
True
```

`is_negative_infinity``()`
> Check if `self` is $-\infty$.

> EXAMPLES:
```
sage: a = RDF(2)/RDF(0)
sage: a.is_negative_infinity()
False
sage: a = RDF(-3)/RDF(0)
sage: a.is_negative_infinity()
True
```

`is_positive_infinity``()`
> Check if `self` is $+\infty$.

> EXAMPLES:
```
sage: a = RDF(1)/RDF(0)
sage: a.is_positive_infinity()
True
sage: a = RDF(-1)/RDF(0)
sage: a.is_positive_infinity()
False
```

`is_square``()`
> Return whether or not this number is a square in this field. For the real numbers, this is `True` if and only if `self` is non-negative.

> EXAMPLES:
```
sage: RDF(3.5).is_square()
True
sage: RDF(0).is_square()
True
sage: RDF(-4).is_square()
False
```

`log``(`*base=None*`)`
> Return the logarithm.

> INPUT:

>> •`base` – integer or `None` (default). The base of the logarithm. If `None` is specified, the base is e (the so-called natural logarithm).

> OUTPUT:

> The logarithm of `self`. If `self` is positive, a double floating point number. Infinity if `self` is zero. A imaginary complex floating point number if `self` is negative.

> EXAMPLES:
```
sage: RDF(2).log()
0.6931471805599453
sage: RDF(2).log(2)
1.0
sage: RDF(2).log(pi)
0.6055115613982801
sage: RDF(2).log(10)
0.30102999566398114
sage: RDF(2).log(1.5)
```

```
1.7095112913514547
sage: RDF(0).log()
-infinity
sage: RDF(-1).log()
3.141592653589793*I
sage: RDF(-1).log(2)   # rel tol 1e-15
4.532360141827194*I
```

TESTS:

Make sure that we can take the log of small numbers accurately and the fix doesn't break preexisting values (trac ticket #12557):

```
sage: R = RealField(128)
sage: def check_error(x):
....:    x = RDF(x)
....:    log_RDF = x.log()
....:    log_RR = R(x).log()
....:    diff = R(log_RDF) - log_RR
....:    if abs(diff) < log_RDF.ulp():
....:        return True
....:    print "logarithm check failed for %s (diff = %s ulp)"%            ....:        (x, d
....:    return False
sage: all( check_error(2^x) for x in range(-100,100) )
True
sage: all( check_error(x) for x in sxrange(0.01, 2.00, 0.01) )
True
sage: all( check_error(x) for x in sxrange(0.99, 1.01, 0.001) )
True
sage: RDF(1.000000001).log()
1.000000082240371e-09
sage: RDF(1e-17).log()
-39.14394658089878
sage: RDF(1e-50).log()
-115.12925464970229
```

log10()
    Return log to the base 10 of self.

    EXAMPLES:

```
sage: r = RDF('16.0'); r.log10()
1.2041199826559246
sage: r.log() / RDF(log(10))
1.2041199826559246
sage: r = RDF('39.9'); r.log10()
1.6009728956867482
```

log2()
    Return log to the base 2 of self.

    EXAMPLES:

```
sage: r = RDF(16.0)
sage: r.log2()
4.0

sage: r = RDF(31.9); r.log2()
4.995484518877507
```

logpi()
> Return log to the base π of `self`.
>
> EXAMPLES:
> ```
> sage: r = RDF(16); r.logpi()
> 2.4220462455931204
> sage: r.log() / RDF(log(pi))
> 2.4220462455931204
> sage: r = RDF('39.9'); r.logpi()
> 3.2203023346075152
> ```

multiplicative_order()
> Returns n such that `self^n == 1`.
>
> Only ± 1 have finite multiplicative order.
>
> EXAMPLES:
> ```
> sage: RDF(1).multiplicative_order()
> 1
> sage: RDF(-1).multiplicative_order()
> 2
> sage: RDF(3).multiplicative_order()
> +Infinity
> ```

nan()
> Return Not-a-Number `NaN`.
>
> EXAMPLES:
> ```
> sage: RDF.NaN()
> NaN
> ```

nth_root(n)
> Return the nth root of `self`.
>
> INPUT:
>
> > •n – an integer
>
> OUTPUT:
>
> The output is a complex double if `self` is negative and n is even, otherwise it is a real double.
>
> EXAMPLES:
> ```
> sage: r = RDF(-125.0); r.nth_root(3)
> -5.000000000000001
> sage: r.nth_root(5)
> -2.6265278044037674
> sage: RDF(-2).nth_root(5)^5  # rel tol 1e-15
> -2.000000000000001
> sage: RDF(-1).nth_root(5)^5
> -1.0
> sage: RDF(3).nth_root(10)^10
> 2.9999999999999982
> sage: RDF(-1).nth_root(2)
> 6.123233995736757e-17 + 1.0*I
> sage: RDF(-1).nth_root(4)
> 0.7071067811865476 + 0.7071067811865475*I
> ```

prec()
> Return the precision of this number in bits.
>
> Always returns 53.
>
> EXAMPLES:
> ```
> sage: RDF(0).prec()
> 53
> ```

real()
> Return `self` - we are already real.
>
> EXAMPLES:
> ```
> sage: a = RDF(3)
> sage: a.real()
> 3.0
> ```

restrict_angle()
> Return a number congruent to `self` mod 2π that lies in the interval $(-\pi, \pi]$.
>
> Specifically, it is the unique $x \in (-\pi, \pi]$ such that `self` $= x + 2\pi n$ for some $n \in \mathbf{Z}$.
>
> EXAMPLES:
> ```
> sage: RDF(pi).restrict_angle()
> 3.141592653589793
> sage: RDF(pi + 1e-10).restrict_angle()
> -3.1415926534897936
> sage: RDF(1+10^10*pi).restrict_angle()
> 0.9999977606...
> ```

round()
> Given real number x, rounds up if fractional part is greater than 0.5, rounds down if fractional part is less than 0.5.
>
> EXAMPLES:
> ```
> sage: RDF(0.49).round()
> 0
> sage: a=RDF(0.51).round(); a
> 1
> ```

sech()
> Return the hyperbolic secant of `self`.
>
> EXAMPLES:
> ```
> sage: RDF(pi).sech()
> 0.08626673833405443
> sage: CDF(pi).sech()
> 0.08626673833405443
> ```

sign()
> Returns -1,0, or 1 if `self` is negative, zero, or positive; respectively.
>
> EXAMPLES:
> ```
> sage: RDF(-1.5).sign()
> -1
> sage: RDF(0).sign()
> 0
> ```

```
sage: RDF(2.5).sign()
1
```

sign_mantissa_exponent()
Return the sign, mantissa, and exponent of `self`.

In Sage (as in MPFR), floating-point numbers of precision p are of the form $sm2^{e-p}$, where $s \in \{-1, 1\}$, $2^{p-1} \le m < 2^p$, and $-2^{30}+1 \le e \le 2^{30}-1$; plus the special values $+0$, -0, $+\text{infinity}$, $-\text{infinity}$, and `NaN` (which stands for Not-a-Number).

This function returns s, m, and $e - p$. For the special values:

- $+0$ returns `(1, 0, 0)`

- -0 returns `(-1, 0, 0)`

- the return values for $+\text{infinity}$, $-\text{infinity}$, and `NaN` are not specified.

EXAMPLES:
```
sage: a = RDF(exp(1.0)); a
2.718281828459045
sage: sign,mantissa,exponent = RDF(exp(1.0)).sign_mantissa_exponent()
sage: sign,mantissa,exponent
(1, 6121026514868073, -51)
sage: sign*mantissa*(2**exponent) == a
True
```

The mantissa is always a nonnegative number:
```
sage: RDF(-1).sign_mantissa_exponent()
(-1, 4503599627370496, -52)
```

TESTS:
```
sage: RDF('+0').sign_mantissa_exponent()
(1, 0, 0)
sage: RDF('-0').sign_mantissa_exponent()
(-1, 0, 0)
```

sin()
Return the sine of `self`.

EXAMPLES:
```
sage: RDF(2).sin()
0.9092974268256817
```

sincos()
Return a pair consisting of the sine and cosine of `self`.

EXAMPLES:
```
sage: t = RDF.pi()/6
sage: t.sincos()
(0.49999999999999994, 0.8660254037844387)
```

sinh()
Return the hyperbolic sine of `self`.

EXAMPLES:

```
sage: q = RDF.pi()/12
sage: q.sinh()
0.26480022760227073
```

sqrt (*extend=True*, *all=False*)

The square root function.

INPUT:

- •extend – bool (default: `True`); if `True`, return a square root in a complex field if necessary if `self` is negative; otherwise raise a `ValueError`.

- •all – bool (default: `False`); if `True`, return a list of all square roots.

EXAMPLES:

```
sage: r = RDF(4.0)
sage: r.sqrt()
2.0
sage: r.sqrt()^2 == r
True
```

```
sage: r = RDF(4344)
sage: r.sqrt()
65.90902821313632
sage: r.sqrt()^2 - r
0.0
```

```
sage: r = RDF(-2.0)
sage: r.sqrt()
1.4142135623730951*I
```

```
sage: RDF(2).sqrt(all=True)
[1.4142135623730951, -1.4142135623730951]
sage: RDF(0).sqrt(all=True)
[0.0]
sage: RDF(-2).sqrt(all=True)
[1.4142135623730951*I, -1.4142135623730951*I]
```

str ()

Return the informal string representation of `self`.

EXAMPLES:

```
sage: a = RDF('4.5'); a.str()
'4.5'
sage: a = RDF('49203480923840.2923904823048'); a.str()
'4.92034809238e+13'
sage: a = RDF(1)/RDF(0); a.str()
'+infinity'
sage: a = -RDF(1)/RDF(0); a.str()
'-infinity'
sage: a = RDF(0)/RDF(0); a.str()
'NaN'
```

We verify consistency with RR (mpfr reals):

```
sage: str(RR(RDF(1)/RDF(0))) == str(RDF(1)/RDF(0))
True
sage: str(RR(-RDF(1)/RDF(0))) == str(-RDF(1)/RDF(0))
True
```

```
sage: str(RR(RDF(0)/RDF(0))) == str(RDF(0)/RDF(0))
True
```

tan()

> Return the tangent of `self`.

> EXAMPLES:

```
sage: q = RDF.pi()/3
sage: q.tan()
1.7320508075688767
sage: q = RDF.pi()/6
sage: q.tan()
0.5773502691896256
```

tanh()

> Return the hyperbolic tangent of `self`.

> EXAMPLES:

```
sage: q = RDF.pi()/12
sage: q.tanh()
0.25597778924568454
```

trunc()

> Truncates this number (returns integer part).

> EXAMPLES:

```
sage: RDF(2.99).trunc()
2
sage: RDF(-2.00).trunc()
-2
sage: RDF(0.00).trunc()
0
```

ulp()

> Returns the unit of least precision of `self`, which is the weight of the least significant bit of `self`. This is always a strictly positive number. It is also the gap between this number and the closest number with larger absolute value that can be represented.

> EXAMPLES:

```
sage: a = RDF(pi)
sage: a.ulp()
4.440892098500626e-16
sage: b = a + a.ulp()
```

> Adding or subtracting an ulp always gives a different number:

```
sage: a + a.ulp() == a
False
sage: a - a.ulp() == a
False
sage: b + b.ulp() == b
False
sage: b - b.ulp() == b
False
```

> Since the default rounding mode is round-to-nearest, adding or subtracting something less than half an ulp always gives the same number, unless the result has a smaller ulp. The latter can only happen if the input

---

number is (up to sign) exactly a power of 2:

```
sage: a - a.ulp()/3 == a
True
sage: a + a.ulp()/3 == a
True
sage: b - b.ulp()/3 == b
True
sage: b + b.ulp()/3 == b
True
sage: c = RDF(1)
sage: c - c.ulp()/3 == c
False
sage: c.ulp()
2.220446049250313e-16
sage: (c - c.ulp()).ulp()
1.1102230246251565e-16
```

The ulp is always positive:

```
sage: RDF(-1).ulp()
2.220446049250313e-16
```

The ulp of zero is the smallest positive number in RDF:

```
sage: RDF(0).ulp()
5e-324
sage: RDF(0).ulp()/2
0.0
```

Some special values:

```
sage: a = RDF(1)/RDF(0); a
+infinity
sage: a.ulp()
+infinity
sage: (-a).ulp()
+infinity
sage: a = RDF('nan')
sage: a.ulp() is a
True
```

The ulp method works correctly with small numbers:

```
sage: u = RDF(0).ulp()
sage: u.ulp() == u
True
sage: x = u * (2^52-1)  # largest denormal number
sage: x.ulp() == u
True
sage: x = u * 2^52  # smallest normal number
sage: x.ulp() == u
True
```

zeta()

Return the Riemann zeta function evaluated at this real number.

Note: PARI is vastly more efficient at computing the Riemann zeta function. See the example below for how to use it.

EXAMPLES:
```
sage: RDF(2).zeta()   # rel tol 1e-15
1.6449340668482269
sage: RDF.pi()^2/6
1.6449340668482264
sage: RDF(-2).zeta()
0.0
sage: RDF(1).zeta()
+infinity
```

sage.rings.real_double.**RealDoubleField**()

> Return the unique instance of the real double field.
>
> EXAMPLES:
> ```
> sage: RealDoubleField() is RealDoubleField()
> True
> ```

class sage.rings.real_double.**RealDoubleField_class**

> Bases: sage.rings.ring.Field
>
> An approximation to the field of real numbers using double precision floating point numbers. Answers derived from calculations in this approximation may differ from what they would be if those calculations were performed in the true field of real numbers. This is due to the rounding errors inherent to finite precision calculations.
>
> EXAMPLES:
> ```
> sage: RR == RDF
> False
> sage: RDF == RealDoubleField()   # RDF is the shorthand
> True
>
> sage: RDF(1)
> 1.0
> sage: RDF(2/3)
> 0.6666666666666666
> ```
>
> A `TypeError` is raised if the coercion doesn't make sense:
> ```
> sage: RDF(QQ['x'].0)
> Traceback (most recent call last):
> ...
> TypeError: cannot coerce nonconstant polynomial to float
> sage: RDF(QQ['x'](3))
> 3.0
> ```
>
> One can convert back and forth between double precision real numbers and higher-precision ones, though of course there may be loss of precision:
> ```
> sage: a = RealField(200)(2).sqrt(); a
> 1.4142135623730950488016887242096980785696718753769480731767
> sage: b = RDF(a); b
> 1.4142135623730951
> sage: a.parent()(b)
> 1.4142135623730951454746218587388284504413604736328125000000
> sage: a.parent()(b) == b
> True
> sage: b == RR(a)
> True
> ```
>
> **NaN**()

Return Not-a-Number `NaN`.

EXAMPLES:
```
sage: RDF.NaN()
NaN
```

algebraic_closure()
Return the algebraic closure of `self`, i.e., the complex double field.

EXAMPLES:
```
sage: RDF.algebraic_closure()
Complex Double Field
```

characteristic()
Returns 0, since the field of real numbers has characteristic 0.

EXAMPLES:
```
sage: RDF.characteristic()
0
```

complex_field()
Return the complex field with the same precision as `self`, i.e., the complex double field.

EXAMPLES:
```
sage: RDF.complex_field()
Complex Double Field
```

construction()
Returns the functorial construction of `self`, namely, completion of the rational numbers with respect to the prime at ∞.

Also preserves other information that makes this field unique (i.e. the Real Double Field).

EXAMPLES:
```
sage: c, S = RDF.construction(); S
Rational Field
sage: RDF == c(S)
True
```

euler_constant()
Return Euler's gamma constant to double precision.

EXAMPLES:
```
sage: RDF.euler_constant()
0.5772156649015329
```

factorial(*n*)
Return the factorial of the integer n as a real number.

EXAMPLES:
```
sage: RDF.factorial(100)
9.332621544394415e+157
```

gen(*n=0*)
Return the generator of the real double field.

EXAMPLES:

```
sage: RDF.0
1.0
sage: RDF.gens()
(1.0,)
```

is_exact()
>   Returns `False`, because doubles are not exact.
>
>   EXAMPLE:
> ```
> sage: RDF.is_exact()
> False
> ```

is_finite()
>   Return `False`, since the field of real numbers is not finite.
>
>   Technical note: There exists an upper bound on the double representation.
>
>   EXAMPLES:
> ```
> sage: RDF.is_finite()
> False
> ```

log2()
>   Return $\log(2)$ to the precision of this field.
>
>   EXAMPLES:
> ```
> sage: RDF.log2()
> 0.6931471805599453
> sage: RDF(2).log()
> 0.6931471805599453
> ```

name()
>   The name of `self`.
>
>   EXAMPLES:
> ```
> sage: RDF.name()
> 'RealDoubleField'
> ```

nan()
>   Return Not-a-Number `NaN`.
>
>   EXAMPLES:
> ```
> sage: RDF.NaN()
> NaN
> ```

ngens()
>   Return the number of generators which is always 1.
>
>   EXAMPLES:
> ```
> sage: RDF.ngens()
> 1
> ```

pi()
>   Returns π to double-precision.
>
>   EXAMPLES:

```
sage: RDF.pi()
3.141592653589793
sage: RDF.pi().sqrt()/2
0.8862269254527579
```

prec()
> Return the precision of this real double field in bits.

> Always returns 53.

> EXAMPLES:
```
sage: RDF.precision()
53
```

precision()
> Return the precision of this real double field in bits.

> Always returns 53.

> EXAMPLES:
```
sage: RDF.precision()
53
```

random_element(*min=-1*, *max=1*)
> Return a random element of this real double field in the interval `[min, max]`.

> EXAMPLES:
```
sage: RDF.random_element()
0.7369454235661859
sage: RDF.random_element(min=100, max=110)
102.8159473516245
```

to_prec(*prec*)
> Return the real field to the specified precision. As doubles have fixed precision, this will only return a real
> double field if `prec` is exactly 53.

> EXAMPLES:
```
sage: RDF.to_prec(52)
Real Field with 52 bits of precision
sage: RDF.to_prec(53)
Real Double Field
```

zeta(*n=2*)
> Return an n-th root of unity in the real field, if one exists, or raise a `ValueError` otherwise.

> EXAMPLES:
```
sage: RDF.zeta()
-1.0
sage: RDF.zeta(1)
1.0
sage: RDF.zeta(5)
Traceback (most recent call last):
...
ValueError: No 5th root of unity in self
```

class sage.rings.real_double.**ToRDF**
> Bases: `sage.categories.morphism.Morphism`

Fast morphism from anything with a __float__ method to an RDF element.

EXAMPLES:
```
sage: f = RDF.coerce_map_from(ZZ); f
Native morphism:
  From: Integer Ring
  To:   Real Double Field
sage: f(4)
4.0
sage: f = RDF.coerce_map_from(QQ); f
Native morphism:
  From: Rational Field
  To:   Real Double Field
sage: f(1/2)
0.5
sage: f = RDF.coerce_map_from(int); f
Native morphism:
  From: Set of Python objects of type 'int'
  To:   Real Double Field
sage: f(3r)
3.0
sage: f = RDF.coerce_map_from(float); f
Native morphism:
  From: Set of Python objects of type 'float'
  To:   Real Double Field
sage: f(3.5)
3.5
```

sage.rings.real_double.**is_RealDoubleElement**(*x*)

Check if x is an element of the real double field.

EXAMPLE:
```
sage: from sage.rings.real_double import is_RealDoubleElement
sage: is_RealDoubleElement(RDF(3))
True
sage: is_RealDoubleElement(RIF(3))
False
```

sage.rings.real_double.**is_RealDoubleField**(*x*)

Returns True if x is the field of real double precision numbers.

EXAMPLES:
```
sage: from sage.rings.real_double import is_RealDoubleField
sage: is_RealDoubleField(RDF)
True
sage: is_RealDoubleField(RealField(53))
False
```

sage.rings.real_double.**pool_stats**()

Statistics for the real double pool.

EXAMPLES:

We first pull all elements from the pool (making sure it is empty to illustrate how the pool works):
```
sage: from sage.rings.real_double import time_alloc_list, pool_stats
sage: L = time_alloc_list(50)
sage: pool_stats()
Used pool 0 / 0 times
```

```
Pool contains 0 / 50 items
```

During the operation (in this example, addition), we end up with two temporary elements. After completion of the operation, they are added to the pool:

```
sage: RDF(2.1) + RDF(2.2)
4.300000000000001
sage: pool_stats()
Used pool 0 / 0 times
Pool contains 2 / 50 items
```

Next when we call `time_alloc_list()`, the "created" elements are actually pulled from the pool:

```
sage: time_alloc_list(3)
[2.2, 2.1, 0.0]
```

Note that the number of objects left in the pool depends on the garbage collector:

```
sage: pool_stats()
Used pool 0 / 0 times
Pool contains 1 / 50 items
```

`sage.rings.real_double.`**`time_alloc`**(*n*)

Allocate n `RealDoubleElement` instances.

EXAMPLES:

Since this does not store anything in a python object, the created elements will not be sent to the garbage collector. Therefore they remain in the pool:

```
sage: from sage.rings.real_double import time_alloc, pool_stats
sage: pool_stats()
Used pool 0 / 0 times
Pool contains 7 / 50 items
sage: time_alloc(25)
sage: pool_stats()
Used pool 0 / 0 times
Pool contains 7 / 50 items
```

`sage.rings.real_double.`**`time_alloc_list`**(*n*)

Allocate a list of length n of `RealDoubleElement` instances.

EXAMPLES:

During the operation (in this example, addition), we end up with two temporary elements. After completion of the operation, they are added to the pool:

```
sage: from sage.rings.real_double import time_alloc_list
sage: RDF(2.1) + RDF(2.2)
4.300000000000001
```

Next when we call `time_alloc_list()`, the "created" elements are actually pulled from the pool:

```
sage: time_alloc_list(2)
[2.2, 2.1]
```

TWO

ARBITRARY PRECISION REAL NUMBERS

AUTHORS:

- Kyle Schalm (2005-09)

- William Stein: bug fixes, examples, maintenance

- Didier Deshommes (2006-03-19): examples

- David Harvey (2006-09-20): compatibility with Element._parent

- William Stein (2006-10): default printing truncates to avoid base-2 rounding confusing (fix suggested by Bill Hart)

- Didier Deshommes: special constructor for QD numbers

- Paul Zimmermann (2008-01): added new functions from mpfr-2.3.0, replaced some, e.g., sech = 1/cosh, by their original mpfr version.

- Carl Witty (2008-02): define floating-point rank and associated functions; add some documentation

- Robert Bradshaw (2009-09): decimal literals, optimizations

- Jeroen Demeyer (2012-05-27): set the MPFR exponent range to the maximal possible value (trac ticket #13033)

- Travis Scrimshaw (2012-11-02): Added doctests for full coverage

This is a binding for the MPFR arbitrary-precision floating point library.

We define a class `RealField`, where each instance of `RealField` specifies a field of floating-point numbers with a specified precision and rounding mode. Individual floating-point numbers are of `RealNumber`.

In Sage (as in MPFR), floating-point numbers of precision p are of the form $sm2^{e-p}$, where $s \in \{-1, 1\}$, $2^{p-1} \le m < 2^p$, and $-2^B + 1 \le e \le 2^B - 1$ where $B = 30$ on 32-bit systems and $B = 62$ on 64-bit systems; additionally, there are the special values $+0$, -0, $+\text{infinity}$, $-\text{infinity}$ and NaN (which stands for Not-a-Number).

Operations in this module which are direct wrappers of MPFR functions are "correctly rounded"; we briefly describe what this means. Assume that you could perform the operation exactly, on real numbers, to get a result r. If this result can be represented as a floating-point number, then we return that number.

Otherwise, the result r is between two floating-point numbers. For the directed rounding modes (round to plus infinity, round to minus infinity, round to zero), we return the floating-point number in the indicated direction from r. For round to nearest, we return the floating-point number which is nearest to r.

This leaves one case unspecified: in round to nearest mode, what happens if r is exactly halfway between the two nearest floating-point numbers? In that case, we round to the number with an even mantissa (the mantissa is the number m in the representation above).

Consider the ordered set of floating-point numbers of precision p. (Here we identify $+0$ and -0, and ignore NaN.) We can give a bijection between these floating-point numbers and a segment of the integers, where 0 maps to 0 and

adjacent floating-point numbers map to adjacent integers. We call the integer corresponding to a given floating-point number the "floating-point rank" of the number. (This is not standard terminology; I just made it up.)

EXAMPLES:

A difficult conversion:

```
sage: RR(sys.maxsize)
9.22337203685478e18        # 64-bit
2.14748364700000e9         # 32-bit
```

TESTS:

```
sage: -1e30
-1.00000000000000e30
sage: hex(-1. + 2^-52)
'-0xf.fffffffffffffp-4'
```

Make sure we don't have a new field for every new literal:

```
sage: parent(2.0) is parent(2.0)
True
sage: RealField(100, rnd='RNDZ') is RealField(100, rnd='RNDD')
False
sage: RealField(100, rnd='RNDZ') is RealField(100, rnd='RNDZ')
True
```

class sage.rings.real_mpfr.**QQtoRR**

Bases: sage.categories.map.Map

INPUT:

There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

TESTS:
```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

class sage.rings.real_mpfr.**RRtoRR**

Bases: sage.categories.map.Map

INPUT:

There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

TESTS:

```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

section()
    EXAMPLES:
```
sage: from sage.rings.real_mpfr import RRtoRR
sage: R10 = RealField(10)
sage: R100 = RealField(100)
sage: f = RRtoRR(R100, R10)
sage: f.section()
Generic map:
  From: Real Field with 10 bits of precision
  To:   Real Field with 100 bits of precision
```

sage.rings.real_mpfr.**RealField**(*prec=53*, *sci_not=0*, *rnd='RNDN'*)
    RealField(prec, sci_not, rnd):

    INPUT:

    •prec – (integer) precision; default = 53 prec is the number of bits used to represent the mantissa of a floating-point number. The precision can be any integer between mpfr_prec_min() and mpfr_prec_max(). In the current implementation, mpfr_prec_min() is equal to 2.

    •sci_not – (default: False) if True, always display using scientific notation; if False, display using scientific notation only for very large or very small numbers

    •rnd – (string) the rounding mode:

        –'RNDN' – (default) round to nearest (ties go to the even number): Knuth says this is the best choice to prevent "floating point drift"

        –'RNDD' – round towards minus infinity

        –'RNDZ' – round towards zero

        –'RNDU' – round towards plus infinity

    EXAMPLES:
```
sage: RealField(10)
Real Field with 10 bits of precision
sage: RealField()
Real Field with 53 bits of precision
sage: RealField(100000)
Real Field with 100000 bits of precision
```

Here we show the effect of rounding:

```
sage: R17d = RealField(17,rnd='RNDD')
sage: a = R17d(1)/R17d(3); a.exact_rational()
87381/262144
sage: R17u = RealField(17,rnd='RNDU')
sage: a = R17u(1)/R17u(3); a.exact_rational()
43691/131072
```

Note: The default precision is 53, since according to the MPFR manual: 'mpfr should be able to exactly reproduce all computations with double-precision machine floating-point numbers (double type in C), except the default exponent range is much wider and subnormal numbers are not implemented.'

---

class sage.rings.real_mpfr.**RealField_class**
    Bases: sage.rings.ring.Field

An approximation to the field of real numbers using floating point numbers with any specified precision. Answers derived from calculations in this approximation may differ from what they would be if those calculations were performed in the true field of real numbers. This is due to the rounding errors inherent to finite precision calculations.

See the documentation for the module sage.rings.real_mpfr for more details.

algebraic_closure()
    Return the algebraic closure of self, i.e., the complex field with the same precision.

    EXAMPLES:
```
sage: RR.algebraic_closure()
Complex Field with 53 bits of precision
sage: RR.algebraic_closure() is CC
True
sage: RealField(100,rnd='RNDD').algebraic_closure()
Complex Field with 100 bits of precision
sage: RealField(100).algebraic_closure()
Complex Field with 100 bits of precision
```

catalan_constant()
    Returns Catalan's constant to the precision of this field.

    EXAMPLES:
```
sage: RealField(100).catalan_constant()
0.91596559417721901505460351493
```

characteristic()
    Returns 0, since the field of real numbers has characteristic 0.

    EXAMPLES:
```
sage: RealField(10).characteristic()
0
```

complex_field()
    Return complex field of the same precision.

    EXAMPLES:
```
sage: RR.complex_field()
Complex Field with 53 bits of precision
sage: RR.complex_field() is CC
True
```

```
sage: RealField(100,rnd='RNDD').complex_field()
Complex Field with 100 bits of precision
sage: RealField(100).complex_field()
Complex Field with 100 bits of precision
```

construction()

Return the functorial construction of `self`, namely, completion of the rational numbers with respect to the prime at ∞.

Also preserves other information that makes this field unique (e.g. precision, rounding, print mode).

EXAMPLES:

```
sage: R = RealField(100, rnd='RNDU')
sage: c, S = R.construction(); S
Rational Field
sage: R == c(S)
True
```

euler_constant()

Returns Euler's gamma constant to the precision of this field.

EXAMPLES:

```
sage: RealField(100).euler_constant()
0.57721566490153286060651209008
```

factorial(*n*)

Return the factorial of the integer `n` as a real number.

EXAMPLES:

```
sage: RR.factorial(0)
1.00000000000000
sage: RR.factorial(1000000)
8.26393168833124e5565708
sage: RR.factorial(-1)
Traceback (most recent call last):
...
ArithmeticError: n must be nonnegative
```

gen(*i=0*)

Return the `i`-th generator of `self`.

EXAMPLES:

```
sage: R=RealField(100)
sage: R.gen(0)
1.0000000000000000000000000000
sage: R.gen(1)
Traceback (most recent call last):
...
IndexError: self has only one generator
```

gens()

Return a list of generators.

EXAMPLE:

```
sage: RR.gens()
[1.00000000000000]
```

is_exact()

> Return `False`, since a real field (represented using finite precision) is not exact.
>
> EXAMPLE:
> ```
> sage: RR.is_exact()
> False
> sage: RealField(100).is_exact()
> False
> ```

is_finite()

> Return `False`, since the field of real numbers is not finite.
>
> EXAMPLES:
> ```
> sage: RealField(10).is_finite()
> False
> ```

log2()

> Return $\log(2)$ (i.e., the natural log of 2) to the precision of this field.
>
> EXAMPLES:
> ```
> sage: R=RealField(100)
> sage: R.log2()
> 0.69314718055994530941723212146
> sage: R(2).log()
> 0.69314718055994530941723212146
> ```

name()

> Return the name of `self`, which encodes the precision and rounding convention.
>
> EXAMPLES:
> ```
> sage: RR.name()
> 'RealField53_0'
> sage: RealField(100,rnd='RNDU').name()
> 'RealField100_2'
> ```

ngens()

> Return the number of generators.
>
> EXAMPLES:
> ```
> sage: RR.ngens()
> 1
> ```

pi()

> Return π to the precision of this field.
>
> EXAMPLES:
> ```
> sage: R = RealField(100)
> sage: R.pi()
> 3.1415926535897932384626433833
> sage: R.pi().sqrt()/2
> 0.88622692545275801364908374167
> sage: R = RealField(150)
> sage: R.pi().sqrt()/2
> 0.88622692545275801364908374167057259139877473
> ```

prec()

> Return the precision of `self`.

EXAMPLES:
```
sage: RR.precision()
53
sage: RealField(20).precision()
20
```

precision()
Return the precision of `self`.

EXAMPLES:
```
sage: RR.precision()
53
sage: RealField(20).precision()
20
```

random_element(*min=-1*, *max=1*, *distribution=None*)
Return a uniformly distributed random number between `min` and `max` (default -1 to 1).

> **Warning:** The argument `distribution` is ignored—the random number is from the uniform distribution.

EXAMPLES:
```
sage: RealField(100).random_element(-5, 10)
-1.7093633198207765227646362966
sage: RealField(10).random_element()
-0.11
```

TESTS:
```
sage: RealField(31).random_element()
-0.676162510
sage: RealField(32).random_element()
0.689774422
sage: RealField(33).random_element()
0.396496861
sage: RealField(63).random_element()
-0.339980711116375371
sage: RealField(64).random_element()
-0.0453049884016705260
sage: RealField(65).random_element()
-0.5926714709589708137
sage: RealField(10).random_element()
0.23
sage: RealField(10).random_element()
-0.41
sage: RR.random_element()
-0.0420335212948924
sage: RR.random_element()
-0.616678906367394
```

rounding_mode()
Return the rounding mode.

EXAMPLES:
```
sage: RR.rounding_mode()
'RNDN'
sage: RealField(20,rnd='RNDZ').rounding_mode()
```

```
'RNDZ'
sage: RealField(20,rnd='RNDU').rounding_mode()
'RNDU'
sage: RealField(20,rnd='RNDD').rounding_mode()
'RNDD'
```

scientific_notation(*status=None*)

Set or return the scientific notation printing flag. If this flag is `True` then real numbers with this space as parent print using scientific notation.

INPUT:

   •`status` – boolean optional flag

EXAMPLES:

```
sage: RR.scientific_notation()
False
sage: elt = RR(0.2512); elt
0.251200000000000
sage: RR.scientific_notation(True)
sage: elt
2.51200000000000e-1
sage: RR.scientific_notation()
True
sage: RR.scientific_notation(False)
sage: elt
0.251200000000000
sage: R = RealField(20, sci_not=1)
sage: R.scientific_notation()
True
sage: R(0.2512)
2.5120e-1
```

to_prec(*prec*)

Return the real field that is identical to `self`, except that it has the specified precision.

EXAMPLES:

```
sage: RR.to_prec(212)
Real Field with 212 bits of precision
sage: R = RealField(30, rnd="RNDZ")
sage: R.to_prec(300)
Real Field with 300 bits of precision and rounding RNDZ
```

zeta(*n=2*)

Return an n-th root of unity in the real field, if one exists, or raise a `ValueError` otherwise.

EXAMPLES:

```
sage: R = RealField()
sage: R.zeta()
-1.00000000000000
sage: R.zeta(1)
1.00000000000000
sage: R.zeta(5)
Traceback (most recent call last):
...
ValueError: No 5th root of unity in self
```

class sage.rings.real_mpfr.**RealLiteral**

    Bases: `sage.rings.real_mpfr.RealNumber`

    Real literals are created in preparsing and provide a way to allow casting into higher precision rings.

    base

    literal

class sage.rings.real_mpfr.**RealNumber**

    Bases: `sage.structure.element.RingElement`

    A floating point approximation to a real number using any specified precision. Answers derived from calculations with such approximations may differ from what they would be if those calculations were performed with true real numbers. This is due to the rounding errors inherent to finite precision calculations.

    The approximation is printed to slightly fewer digits than its internal precision, in order to avoid confusing roundoff issues that occur because numbers are stored internally in binary.

    agm(*other*)

        Return the arithmetic-geometric mean of `self` and `other`.

        The arithmetic-geometric mean is the common limit of the sequences u_n and v_n, where u_0 is `self`, v_0 is other, u_{n+1} is the arithmetic mean of u_n and v_n, and v_{n+1} is the geometric mean of u_n and v_n. If any operand is negative, the return value is `NaN`.

        INPUT:

            •`right` – another real number

        OUTPUT:

            •the AGM of `self` and `other`

        EXAMPLES:

```
sage: a = 1.5
sage: b = 2.5
sage: a.agm(b)
1.96811775182478
sage: RealField(200)(a).agm(b)
1.9681177518247777389894630877503739489139488203685819712291
sage: a.agm(100)
28.1189391225320
```

        The AGM always lies between the geometric and arithmetic mean:

```
sage: sqrt(a*b) < a.agm(b) < (a+b)/2
True
```

        It is, of course, symmetric:

```
sage: b.agm(a)
1.96811775182478
```

        and satisfies the relation $AGM(ra, rb) = r\,AGM(a, b)$:

```
sage: (2*a).agm(2*b) / 2
1.96811775182478
sage: (3*a).agm(3*b) / 3
1.96811775182478
```

It is also related to the elliptic integral

$$\int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m\sin^2\theta}}.$$

```
sage: m = (a-b)^2/(a+b)^2
sage: E = numerical_integral(1/sqrt(1-m*sin(x)^2), 0, RR.pi()/2)[0]
sage: RR.pi()/4 * (a+b)/E
1.96811775182478
```

TESTS:
```
sage: 1.5.agm(0)
0.000000000000000
```

algdep(*n*)

Return a polynomial of degree at most n which is approximately satisfied by this number.

Note: The resulting polynomial need not be irreducible, and indeed usually won't be if this number is a good approximation to an algebraic number of degree less than n.

ALGORITHM:

Uses the PARI C-library `algdep` command.

EXAMPLE:
```
sage: r = sqrt(2.0); r
1.41421356237310
sage: r.algebraic_dependency(5)
x^2 - 2
```

algebraic_dependency(*n*)

Return a polynomial of degree at most n which is approximately satisfied by this number.

Note: The resulting polynomial need not be irreducible, and indeed usually won't be if this number is a good approximation to an algebraic number of degree less than n.

ALGORITHM:

Uses the PARI C-library `algdep` command.

EXAMPLE:
```
sage: r = sqrt(2.0); r
1.41421356237310
sage: r.algebraic_dependency(5)
x^2 - 2
```

arccos()

Return the inverse cosine of `self`.

EXAMPLES:
```
sage: q = RR.pi()/3
sage: i = q.cos()
sage: i.arccos() == q
True
```

arccosh()

Return the hyperbolic inverse cosine of `self`.

EXAMPLES:
```
sage: q = RR.pi()/2
sage: i = q.cosh() ; i
2.50917847865806
sage: q == i.arccosh()
True
```

arccoth()
> Return the inverse hyperbolic cotangent of `self`.

> EXAMPLES:
```
sage: q = RR.pi()/5
sage: i = q.coth()
sage: i.arccoth() == q
True
```

arccsch()
> Return the inverse hyperbolic cosecant of `self`.

> EXAMPLES:
```
sage: i = RR.pi()/5
sage: q = i.csch()
sage: q.arccsch() == i
True
```

arcsech()
> Return the inverse hyperbolic secant of `self`.

> EXAMPLES:
```
sage: i = RR.pi()/3
sage: q = i.sech()
sage: q.arcsech() == i
True
```

arcsin()
> Return the inverse sine of `self`.

> EXAMPLES:
```
sage: q = RR.pi()/5
sage: i = q.sin()
sage: i.arcsin() == q
True
sage: i.arcsin() - q
0.000000000000000
```

arcsinh()
> Return the hyperbolic inverse sine of `self`.

> EXAMPLES:
```
sage: q = RR.pi()/7
sage: i = q.sinh() ; i
0.464017630492991
sage: i.arcsinh() - q
0.000000000000000
```

arctan()
> Return the inverse tangent of `self`.

EXAMPLES:
```
sage: q = RR.pi()/5
sage: i = q.tan()
sage: i.arctan() == q
True
```

arctanh()

Return the hyperbolic inverse tangent of `self`.

EXAMPLES:
```
sage: q = RR.pi()/7
sage: i = q.tanh() ; i
0.420911241048535
sage: i.arctanh() - q
0.000000000000000
```

ceil()

Return the ceiling of `self`.

EXAMPLES:
```
sage: (2.99).ceil()
3
sage: (2.00).ceil()
2
sage: (2.01).ceil()
3

sage: ceil(10^16 * 1.0)
10000000000000000
sage: ceil(10^17 * 1.0)
100000000000000000
sage: ceil(RR(+infinity))
Traceback (most recent call last):
...
ValueError: Calling ceil() on infinity or NaN
```

ceiling()

Return the ceiling of `self`.

EXAMPLES:
```
sage: (2.99).ceil()
3
sage: (2.00).ceil()
2
sage: (2.01).ceil()
3

sage: ceil(10^16 * 1.0)
10000000000000000
sage: ceil(10^17 * 1.0)
100000000000000000
sage: ceil(RR(+infinity))
Traceback (most recent call last):
...
ValueError: Calling ceil() on infinity or NaN
```

conjugate()

Return the complex conjugate of this real number, which is the number itself.

EXAMPLES:

```
sage: x = RealField(100)(1.238)
sage: x.conjugate()
1.2380000000000000000000000000
```

cos()

Returnn the cosine of `self`.

EXAMPLES:

```
sage: t=RR.pi()/2
sage: t.cos()
6.12323399573677e-17
```

cosh()

Return the hyperbolic cosine of `self`.

EXAMPLES:

```
sage: q = RR.pi()/12
sage: q.cosh()
1.03446564009551
```

cot()

Return the cotangent of `self`.

EXAMPLES:

```
sage: RealField(100)(2).cot()
-0.45765755436028576375027741043
```

coth()

Return the hyperbolic cotangent of `self`.

EXAMPLES:

```
sage: RealField(100)(2).coth()
1.0373147207275480958778097648
```

csc()

Return the cosecant of `self`.

EXAMPLES:

```
sage: RealField(100)(2).csc()
1.0997501702946164667566973970
```

csch()

Return the hyperbolic cosecant of `self`.

EXAMPLES:

```
sage: RealField(100)(2).csch()
0.27572056477178320775835148216
```

cube_root()

Return the cubic root (defined over the real numbers) of `self`.

EXAMPLES:

```
sage: r = 125.0; r.cube_root()
5.00000000000000
sage: r = -119.0
sage: r.cube_root()^3 - r        # illustrates precision loss
-1.42108547152020e-14
```

eint()
>   Returns the exponential integral of this number.
>
>   EXAMPLES:
> ```
> sage: r = 1.0
> sage: r.eint()
> 1.89511781635594
> ```
>
> ```
> sage: r = -1.0
> sage: r.eint()
> NaN
> ```

epsilon(*field=None*)
>   Returns abs(self) divided by 2^b where b is the precision in bits of self. Equivalently, return abs(self) multiplied by the ulp() of 1.
>
>   This is a scale-invariant version of ulp() and it lies in $[u/2, u)$ where u is self.ulp() (except in the case of zero or underflow).
>
>   INPUT:
>
>   •field – RealField used as parent of the result. If not specified, use parent(self).
>
>   OUTPUT:
>
>   field(self.abs() / 2^self.precision())
>
>   EXAMPLES:
> ```
> sage: RR(2^53).epsilon()
> 1.00000000000000
> sage: RR(0).epsilon()
> 0.000000000000000
> sage: a = RR.pi()
> sage: a.epsilon()
> 3.48786849800863e-16
> sage: a.ulp()/2, a.ulp()
> (2.22044604925031e-16, 4.44089209850063e-16)
> sage: a / 2^a.precision()
> 3.48786849800863e-16
> sage: (-a).epsilon()
> 3.48786849800863e-16
> ```
>
>   We use a different field:
> ```
> sage: a = RealField(256).pi()
> sage: a.epsilon()
> 2.713132368784788677624750042896586252980746500631892201656843478528498954308e-77
> sage: e = a.epsilon(RealField(64))
> sage: e
> 2.71313236878478868e-77
> sage: parent(e)
> Real Field with 64 bits of precision
> sage: e = a.epsilon(QQ)
> Traceback (most recent call last):
> ```

```
...
TypeError: field argument must be a RealField
```

Special values:

```
sage: RR('nan').epsilon()
NaN
sage: parent(RR('nan').epsilon(RealField(42)))
Real Field with 42 bits of precision
sage: RR('+Inf').epsilon()
+infinity
sage: RR('-Inf').epsilon()
+infinity
```

erf()
> Return the value of the error function on `self`.

> EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).erf()
0.995322265018953
sage: R(6).erf()
1.00000000000000
```

erfc()
> Return the value of the complementary error function on `self`, i.e., $1 - \mathtt{erf}(\mathtt{self})$.

> EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).erfc()
0.00467773498104727
sage: R(6).erfc()
2.15197367124989e-17
```

exact_rational()
> Returns the exact rational representation of this floating-point number.

> EXAMPLES:

```
sage: RR(0).exact_rational()
0
sage: RR(1/3).exact_rational()
6004799503160661/18014398509481984
sage: RR(37/16).exact_rational()
37/16
sage: RR(3^60).exact_rational()
42391158275216203520420085760
sage: RR(3^60).exact_rational() - 3^60
6125652559
sage: RealField(5)(-pi).exact_rational()
-25/8
```

> TESTS:

```
sage: RR('nan').exact_rational()
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
sage: RR('-infinity').exact_rational()
```

```
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
```

exp()
> Return $e^{\mathbf{self}}$.

> EXAMPLES:
```
sage: r = 0.0
sage: r.exp()
1.00000000000000

sage: r = 32.3
sage: a = r.exp(); a
1.06588847274864e14
sage: a.log()
32.3000000000000

sage: r = -32.3
sage: r.exp()
9.38184458849869e-15
```

exp10()
> Return $10^{\mathbf{self}}$.

> EXAMPLES:
```
sage: r = 0.0
sage: r.exp10()
1.00000000000000

sage: r = 32.0
sage: r.exp10()
1.00000000000000e32

sage: r = -32.3
sage: r.exp10()
5.01187233627276e-33
```

exp2()
> Return $2^{\mathbf{self}}$.

> EXAMPLES:
```
sage: r = 0.0
sage: r.exp2()
1.00000000000000

sage: r = 32.0
sage: r.exp2()
4.29496729600000e9

sage: r = -32.3
sage: r.exp2()
1.89117248253021e-10
```

expm1()
> Return $e^{\mathbf{self}} - 1$, avoiding cancellation near 0.

> EXAMPLES:

```
sage: r = 1.0
sage: r.expm1()
1.71828182845905

sage: r = 1e-16
sage: exp(r)-1
0.000000000000000
sage: r.expm1()
1.00000000000000e-16
```

floor()

Return the floor of `self`.

EXAMPLES:

```
sage: R = RealField()
sage: (2.99).floor()
2
sage: (2.00).floor()
2
sage: floor(RR(-5/2))
-3
sage: floor(RR(+infinity))
Traceback (most recent call last):
...
ValueError: Calling floor() on infinity or NaN
```

fp_rank()

Returns the floating-point rank of this number. That is, if you list the floating-point numbers of this precision in order, and number them starting with $0.0 \to 0$ and extending the list to positive and negative infinity, returns the number corresponding to this floating-point number.

EXAMPLES:

```
sage: RR(0).fp_rank()
0
sage: RR(0).nextabove().fp_rank()
1
sage: RR(0).nextbelow().nextbelow().fp_rank()
-2
sage: RR(1).fp_rank()
4835703278458516698824705                 # 32-bit
20769187434139310514121985316880385  # 64-bit
sage: RR(-1).fp_rank()
-4835703278458516698824705                 # 32-bit
-20769187434139310514121985316880385  # 64-bit
sage: RR(1).fp_rank() - RR(1).nextbelow().fp_rank()
1
sage: RR(-infinity).fp_rank()
-9671406552413433770278913                 # 32-bit
-41538374868278621023740371006390273  # 64-bit
sage: RR(-infinity).fp_rank() - RR(-infinity).nextabove().fp_rank()
-1
```

fp_rank_delta(*other*)

Return the floating-point rank delta between `self` and `other`. That is, if the return value is positive, this is the number of times you have to call `.nextabove()` to get from `self` to `other`.

EXAMPLES:

```
sage: [x.fp_rank_delta(x.nextabove()) for x in
...       (RR(-infinity), -1.0, 0.0, 1.0, RR(pi), RR(infinity))]
[1, 1, 1, 1, 1, 0]
```

In the 2-bit floating-point field, one subsegment of the floating-point numbers is: 1, 1.5, 2, 3, 4, 6, 8, 12, 16, 24, 32

```
sage: R2 = RealField(2)
sage: R2(1).fp_rank_delta(R2(2))
2
sage: R2(2).fp_rank_delta(R2(1))
-2
sage: R2(1).fp_rank_delta(R2(1048576))
40
sage: R2(24).fp_rank_delta(R2(4))
-5
sage: R2(-4).fp_rank_delta(R2(-24))
-5
```

There are lots of floating-point numbers around 0:

```
sage: R2(-1).fp_rank_delta(R2(1))
4294967298              # 32-bit
18446744073709551618  # 64-bit
```

frac()

Return a real number such that `self = self.trunc() + self.frac()`. The return value will also satisfy $-1 <$ `self.frac()` < 1.

EXAMPLES:

```
sage: (2.99).frac()
0.990000000000000
sage: (2.50).frac()
0.500000000000000
sage: (-2.79).frac()
-0.790000000000000
sage: (-2.79).trunc() + (-2.79).frac()
-2.79000000000000
```

gamma()

Return the value of the Euler gamma function on `self`.

EXAMPLES:

```
sage: R = RealField()
sage: R(6).gamma()
120.000000000000
sage: R(1.5).gamma()
0.886226925452758
```

hex()

Return a hexadecimal floating-point representation of `self`, in the style of C99 hexadecimal floating-point constants.

EXAMPLES:

```
sage: RR(-1/3).hex()
'-0x5.5555555555554p-4'
sage: Reals(100)(123.456e789).hex()
'0xf.721008e90630c8da88f44dd2p+2624'
```

```
sage: (-0.).hex()
'-0x0p+0'
```

```
sage: [(a.hex(), float(a).hex()) for a in [.5, 1., 2., 16.]]
[('0x8p-4', '0x1.0000000000000p-1'),
 ('0x1p+0', '0x1.0000000000000p+0'),
 ('0x2p+0', '0x1.0000000000000p+1'),
 ('0x1p+4', '0x1.0000000000000p+4')]
```

Special values:
```
sage: [RR(s).hex() for s in ['+inf', '-inf', 'nan']]
['inf', '-inf', 'nan']
```

imag()

　　Return the imaginary part of self.

　　(Since self is a real number, this simply returns exactly 0.)

　　EXAMPLES:
```
sage: RR.pi().imag()
0
sage: RealField(100)(2).imag()
0
```

integer_part()

　　If in decimal this number is written n.defg, returns n.

　　OUTPUT: a Sage Integer

　　EXAMPLE:
```
sage: a = 119.41212
sage: a.integer_part()
119
sage: a = -123.4567
sage: a.integer_part()
-123
```

　　A big number with no decimal point:
```
sage: a = RR(10^17); a
1.00000000000000e17
sage: a.integer_part()
100000000000000000
```

is_NaN()

　　Return True if self is Not-a-Number NaN.

　　EXAMPLES:
```
sage: a = RR(0) / RR(0); a
NaN
sage: a.is_NaN()
True
```

is_infinity()

　　Return True if self is ∞ and False otherwise.

　　EXAMPLES:

```
sage: a = RR('1.494') / RR(0); a
+infinity
sage: a.is_infinity()
True
sage: a = -RR('1.494') / RR(0); a
-infinity
sage: a.is_infinity()
True
sage: RR(1.5).is_infinity()
False
sage: RR('nan').is_infinity()
False
```

is_integer()

> Return `True` if this number is a integer.

> EXAMPLES:

```
sage: RR(1).is_integer()
True
sage: RR(0.1).is_integer()
False
```

is_negative_infinity()

> Return `True` if `self` is $-\infty$.

> EXAMPLES:

```
sage: a = RR('1.494') / RR(0); a
+infinity
sage: a.is_negative_infinity()
False
sage: a = -RR('1.494') / RR(0); a
-infinity
sage: RR(1.5).is_negative_infinity()
False
sage: a.is_negative_infinity()
True
```

is_positive_infinity()

> Return `True` if `self` is $+\infty$.

> EXAMPLES:

```
sage: a = RR('1.494') / RR(0); a
+infinity
sage: a.is_positive_infinity()
True
sage: a = -RR('1.494') / RR(0); a
-infinity
sage: RR(1.5).is_positive_infinity()
False
sage: a.is_positive_infinity()
False
```

is_real()

> Return `True` if `self` is real (of course, this always returns `True` for a finite element of a real field).

> EXAMPLES:

```
sage: RR(1).is_real()
True
sage: RR('-100').is_real()
True
```

is_square()

Return whether or not this number is a square in this field. For the real numbers, this is `True` if and only if `self` is non-negative.

EXAMPLES:

```
sage: r = 3.5
sage: r.is_square()
True
sage: r = 0.0
sage: r.is_square()
True
sage: r = -4.0
sage: r.is_square()
False
```

is_unit()

Return `True` if `self` is a unit (has a multiplicative inverse) and `False` otherwise.

EXAMPLES:

```
sage: RR(1).is_unit()
True
sage: RR('0').is_unit()
False
sage: RR('-0').is_unit()
False
sage: RR('nan').is_unit()
False
sage: RR('inf').is_unit()
False
sage: RR('-inf').is_unit()
False
```

j0()

Return the value of the Bessel J function of order 0 at `self`.

EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).j0()
0.223890779141236
```

j1()

Return the value of the Bessel J function of order 1 at `self`.

EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).j1()
0.576724807756873
```

jn(n)

Return the value of the Bessel J function of order n at `self`.

EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).jn(3)
0.128943249474402
sage: R(2).jn(-17)
-2.65930780516787e-15
```

log(*base=None*)

    Return the logarithm of `self` to the `base`.

    EXAMPLES:

```
sage: R = RealField()
sage: R(2).log()
0.693147180559945
sage: log(RR(2))
0.693147180559945
sage: log(RR(2), "e")
0.693147180559945
sage: log(RR(2), e)
0.693147180559945

sage: r = R(-1); r.log()
3.14159265358979*I
sage: log(RR(-1),e)
3.14159265358979*I
sage: r.log(2)
4.53236014182719*I
```

    For the error value NaN (Not A Number), log will return NaN:

```
sage: r = R(NaN); r.log()
NaN
```

log10()

    Return log to the base 10 of `self`.

    EXAMPLES:

```
sage: r = 16.0; r.log10()
1.20411998265592
sage: r.log() / log(10.0)
1.20411998265592

sage: r = 39.9; r.log10()
1.60097289568675

sage: r = 0.0
sage: r.log10()
-infinity

sage: r = -1.0
sage: r.log10()
1.36437635384184*I
```

log1p()

    Return log base e of `1 + self`.

    EXAMPLES:

```
sage: r = 15.0; r.log1p()
2.77258872223978
sage: (r+1).log()
2.77258872223978
```

For small values, this is more accurate than computing `log(1 + self)` directly, as it avoids cancellation issues:

```
sage: r = 3e-10
sage: r.log1p()
2.99999999955000e-10
sage: (1+r).log()
3.00000024777111e-10
sage: r100 = RealField(100)(r)
sage: (1+r100).log()
2.9999999995500000000978021372e-10
```

For small values, this is more accurate than computing `log(1 + self)` directly, as it avoid cancelation issues:

```
sage: r = 3e-10
sage: r.log1p()
2.99999999955000e-10
sage: (1+r).log()
3.00000024777111e-10
sage: r100 = RealField(100)(r)
sage: (1+r100).log()
2.9999999995500000000978021372e-10
```

```
sage: r = 38.9; r.log1p()
3.68637632389582
```

```
sage: r = -1.0
sage: r.log1p()
-infinity
```

```
sage: r = -2.0
sage: r.log1p()
3.14159265358979*I
```

log2()
    Return log to the base 2 of `self`.

    EXAMPLES:
```
sage: r = 16.0
sage: r.log2()
4.00000000000000
```

```
sage: r = 31.9; r.log2()
4.99548451887751
```

```
sage: r = 0.0
sage: r.log2()
-infinity
```

```
sage: r = -3.0; r.log2()
1.58496250072116 + 4.53236014182719*I
```

log_gamma()

Return the logarithm of gamma of `self`.

EXAMPLES:
```
sage: R = RealField(53)
sage: R(6).log_gamma()
4.78749174278205
sage: R(1e10).log_gamma()
2.20258509288811e11
```

multiplicative_order()

Return the multiplicative order of `self`.

EXAMPLES:
```
sage: RR(1).multiplicative_order()
1
sage: RR(-1).multiplicative_order()
2
sage: RR(3).multiplicative_order()
+Infinity
```

nearby_rational(*max_error=None*, *max_denominator=None*)

Find a rational near to `self`. Exactly one of `max_error` or `max_denominator` must be specified.

If `max_error` is specified, then this returns the simplest rational in the range `[self-max_error .. self+max_error]`. If `max_denominator` is specified, then this returns the rational closest to `self` with denominator at most `max_denominator`. (In case of ties, we pick the simpler rational.)

EXAMPLES:
```
sage: (0.333).nearby_rational(max_error=0.001)
1/3
sage: (0.333).nearby_rational(max_error=1)
0
sage: (-0.333).nearby_rational(max_error=0.0001)
-257/772

sage: (0.333).nearby_rational(max_denominator=100)
1/3
sage: RR(1/3 + 1/1000000).nearby_rational(max_denominator=2999999)
777780/2333333
sage: RR(1/3 + 1/1000000).nearby_rational(max_denominator=3000000)
1000003/3000000
sage: (-0.333).nearby_rational(max_denominator=1000)
-333/1000
sage: RR(3/4).nearby_rational(max_denominator=2)
1
sage: RR(pi).nearby_rational(max_denominator=120)
355/113
sage: RR(pi).nearby_rational(max_denominator=10000)
355/113
sage: RR(pi).nearby_rational(max_denominator=100000)
312689/99532
sage: RR(pi).nearby_rational(max_denominator=1)
3
sage: RR(-3.5).nearby_rational(max_denominator=1)
-3
```

TESTS:

```
sage: RR('nan').nearby_rational(max_denominator=1000)
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
sage: RR('nan').nearby_rational(max_error=0.01)
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
sage: RR(oo).nearby_rational(max_denominator=1000)
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
sage: RR(oo).nearby_rational(max_error=0.01)
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
```

nextabove()

> Return the next floating-point number larger than `self`.

> EXAMPLES:

```
sage: RR('-infinity').nextabove()
-2.09857871646739e323228496          # 32-bit
-5.87565378911159e1388255822130839282  # 64-bit
sage: RR(0).nextabove()
2.38256490488795e-323228497          # 32-bit
8.50969131174084e-1388255822130839284  # 64-bit
sage: RR('+infinity').nextabove()
+infinity
sage: RR(-sqrt(2)).str(truncate=False)
'-1.4142135623730951'
sage: RR(-sqrt(2)).nextabove().str(truncate=False)
'-1.4142135623730949'
```

nextbelow()

> Return the next floating-point number smaller than `self`.

> EXAMPLES:

```
sage: RR('-infinity').nextbelow()
-infinity
sage: RR(0).nextbelow()
-2.38256490488795e-323228497          # 32-bit
-8.50969131174084e-1388255822130839284  # 64-bit
sage: RR('+infinity').nextbelow()
2.09857871646739e323228496          # 32-bit
5.87565378911159e1388255822130839282    # 64-bit
sage: RR(-sqrt(2)).str(truncate=False)
'-1.4142135623730951'
sage: RR(-sqrt(2)).nextbelow().str(truncate=False)
'-1.4142135623730954'
```

nexttoward(*other*)

> Return the floating-point number adjacent to `self` which is closer to `other`. If `self` or other is NaN, returns NaN; if `self` equals `other`, returns `self`.

> EXAMPLES:

```
sage: (1.0).nexttoward(2).str(truncate=False)
'1.0000000000000002'
sage: (1.0).nexttoward(RR('-infinity')).str(truncate=False)
'0.99999999999999989'
sage: RR(infinity).nexttoward(0)
2.09857871646739e323228496             # 32-bit
5.87565378911159e1388255822130839282   # 64-bit
sage: RR(pi).str(truncate=False)
'3.1415926535897931'
sage: RR(pi).nexttoward(22/7).str(truncate=False)
'3.1415926535897936'
sage: RR(pi).nexttoward(21/7).str(truncate=False)
'3.1415926535897927'
```

nth_root(*n*, *algorithm=0*)

Return an nth root of `self`.

INPUT:

- `n` – A positive number, rounded down to the nearest integer. Note that n should be less than `sys.maxsize`.

- `algorithm` – Set this to 1 to call mpfr directly, set this to 2 to use interval arithmetic and logarithms, or leave it at the default of 0 to choose the algorithm which is estimated to be faster.

AUTHORS:

- Carl Witty (2007-10)

EXAMPLES:

```
sage: R = RealField()
sage: R(8).nth_root(3)
2.00000000000000
sage: R(8).nth_root(3.7)      # illustrate rounding down
2.00000000000000
sage: R(-8).nth_root(3)
-2.00000000000000
sage: R(0).nth_root(3)
0.000000000000000
sage: R(32).nth_root(-1)
Traceback (most recent call last):
...
ValueError: n must be positive
sage: R(32).nth_root(1.0)
32.0000000000000
sage: R(4).nth_root(4)
1.41421356237310
sage: R(4).nth_root(40)
1.03526492384138
sage: R(4).nth_root(400)
1.00347174850950
sage: R(4).nth_root(4000)
1.00034663365385
sage: R(4).nth_root(4000000)
1.00000034657365
sage: R(-27).nth_root(3)
-3.00000000000000
sage: R(-4).nth_root(3999999)
-1.00000034657374
```

Note that for negative numbers, any even root throws an exception:

```
sage: R(-2).nth_root(6)
Traceback (most recent call last):
...
ValueError: taking an even root of a negative number
```

The nth root of 0 is defined to be 0, for any n:

```
sage: R(0).nth_root(6)
0.000000000000000
sage: R(0).nth_root(7)
0.000000000000000
```

TESTS:

The old and new algorithms should give exactly the same results in all cases:

```
sage: def check(x, n):
....:        answers = []
....:        for sign in (1, -1):
....:            if is_even(n) and sign == -1:
....:                continue
....:            for rounding in ('RNDN', 'RNDD', 'RNDU', 'RNDZ'):
....:                fld = RealField(x.prec(), rnd=rounding)
....:                fx = fld(sign * x)
....:                alg_mpfr = fx.nth_root(n, algorithm=1)
....:                alg_mpfi = fx.nth_root(n, algorithm=2)
....:                assert(alg_mpfr == alg_mpfi)
....:                if sign == 1: answers.append(alg_mpfr)
....:        return answers
```

Check some perfect powers (and nearby numbers):

```
sage: check(16.0, 4)
[2.00000000000000, 2.00000000000000, 2.00000000000000, 2.00000000000000]
sage: check((16.0).nextabove(), 4)
[2.00000000000000, 2.00000000000000, 2.00000000000001, 2.00000000000000]
sage: check((16.0).nextbelow(), 4)
[2.00000000000000, 1.99999999999999, 2.00000000000000, 1.99999999999999]
sage: check(((9.0 * 256)^7), 7)
[2304.00000000000, 2304.00000000000, 2304.00000000000, 2304.00000000000]
sage: check(((9.0 * 256)^7).nextabove(), 7)
[2304.00000000000, 2304.00000000000, 2304.00000000001, 2304.00000000000]
sage: check(((9.0 * 256)^7).nextbelow(), 7)
[2304.00000000000, 2303.99999999999, 2304.00000000000, 2303.99999999999]
sage: check(((5.0 / 512)^17), 17)
[0.00976562500000000, 0.00976562500000000, 0.00976562500000000, 0.00976562500000000]
sage: check(((5.0 / 512)^17).nextabove(), 17)
[0.00976562500000000, 0.00976562500000000, 0.00976562500000001, 0.00976562500000000]
sage: check(((5.0 / 512)^17).nextbelow(), 17)
[0.00976562500000000, 0.00976562499999999, 0.00976562500000000, 0.00976562499999999]
```

And check some non-perfect powers:

```
sage: check(2.0, 3)
[1.25992104989487, 1.25992104989487, 1.25992104989488, 1.25992104989487]
sage: check(2.0, 4)
[1.18920711500272, 1.18920711500272, 1.18920711500273, 1.18920711500272]
sage: check(2.0, 5)
[1.14869835499704, 1.14869835499703, 1.14869835499704, 1.14869835499703]
```

And some different precisions:

```
sage: check(RealField(20)(22/7), 19)
[1.0621, 1.0621, 1.0622, 1.0621]
sage: check(RealField(200)(e), 4)
[1.28402541668777414840734205680624364583362808652814630892175, 1.2840254166877741484073420568
```

Check that trac ticket #12105 is fixed:

```
sage: RealField(53)(0.05).nth_root(7 * 10^8)
0.999999995720382
```

prec()

Return the precision of `self`.

EXAMPLES:

```
sage: RR(1.0).precision()
53
sage: RealField(101)(-1).precision()
101
```

precision()

Return the precision of `self`.

EXAMPLES:

```
sage: RR(1.0).precision()
53
sage: RealField(101)(-1).precision()
101
```

real()

Return the real part of `self`.

(Since `self` is a real number, this simply returns `self`.)

EXAMPLES:

```
sage: RR(2).real()
2.00000000000000
sage: RealField(200)(-4.5).real()
-4.5000000000000000000000000000000000000000000000000000000000
```

round()

Rounds `self` to the nearest integer. The rounding mode of the parent field has no effect on this function.

EXAMPLES:

```
sage: RR(0.49).round()
0
sage: RR(0.5).round()
1
sage: RR(-0.49).round()
0
sage: RR(-0.5).round()
-1
```

sec()

Returns the secant of this number

EXAMPLES:

```
sage: RealField(100)(2).sec()
-2.4029979617223809897546004014
```

sech()

Return the hyperbolic secant of `self`.

EXAMPLES:

```
sage: RealField(100)(2).sech()
0.26580222883407969212086273982
```

sign()

Return +1 if `self` is positive, −1 if `self` is negative, and 0 if `self` is zero.

EXAMPLES:

```
sage: R=RealField(100)
sage: R(-2.4).sign()
-1
sage: R(2.1).sign()
1
sage: R(0).sign()
0
```

sign_mantissa_exponent()

Return the sign, mantissa, and exponent of `self`.

In Sage (as in MPFR), floating-point numbers of precision p are of the form $sm2^{e-p}$, where $s \in \{-1, 1\}$, $2^{p-1} \le m < 2^p$, and $-2^{30}+1 \le e \le 2^{30}-1$; plus the special values $+0$, -0, $+\text{infinity}$, $-\text{infinity}$, and NaN (which stands for Not-a-Number).

This function returns s, m, and $e - p$. For the special values:

- +0 returns (1, 0, 0) (analogous to IEEE-754; note that MPFR actually stores the exponent as "smallest exponent possible")

- −0 returns (−1, 0, 0) (analogous to IEEE-754; note that MPFR actually stores the exponent as "smallest exponent possible")

- the return values for +infinity, -infinity, and NaN are not specified.

EXAMPLES:

```
sage: R = RealField(53)
sage: a = R(exp(1.0)); a
2.71828182845905
sage: sign, mantissa, exponent = R(exp(1.0)).sign_mantissa_exponent()
sage: sign, mantissa, exponent
(1, 6121026514868073, -51)
sage: sign*mantissa*(2**exponent) == a
True
```

The mantissa is always a nonnegative number (see [trac ticket #14448](#)):

```
sage: RR(-1).sign_mantissa_exponent()
(-1, 4503599627370496, -52)
```

We can also calculate this also using p-adic valuations:

```
sage: a = R(exp(1.0))
sage: b = a.exact_rational()
sage: valuation, unit = b.val_unit(2)
```

```
sage: (b/abs(b), unit, valuation)
(1, 6121026514868073, -51)
sage: a.sign_mantissa_exponent()
(1, 6121026514868073, -51)
```

TESTS:
```
sage: R('+0').sign_mantissa_exponent()
(1, 0, 0)
sage: R('-0').sign_mantissa_exponent()
(-1, 0, 0)
```

simplest_rational()

> Return the simplest rational which is equal to self (in the Sage sense). Recall that Sage defines the equality operator by coercing both sides to a single type and then comparing; thus, this finds the simplest rational which (when coerced to this RealField) is equal to self.

> Given rationals a/b and c/d (both in lowest terms), the former is simpler if $b < d$ or if $b = d$ and $|a| < |c|$.

> The effect of rounding modes is slightly counter-intuitive. Consider the case of round-toward-minus-infinity. This rounding is performed when coercing a rational to a floating-point number; so the simplest_rational() of a round-to-minus-infinity number will be either exactly equal to or slightly larger than the number.

> EXAMPLES:
```
sage: RRd = RealField(53, rnd='RNDD')
sage: RRz = RealField(53, rnd='RNDZ')
sage: RRu = RealField(53, rnd='RNDU')
sage: def check(x):
...       rx = x.simplest_rational()
...       assert(x == rx)
...       return rx
sage: RRd(1/3) < RRu(1/3)
True
sage: check(RRd(1/3))
1/3
sage: check(RRu(1/3))
1/3
sage: check(RRz(1/3))
1/3
sage: check(RR(1/3))
1/3
sage: check(RRd(-1/3))
-1/3
sage: check(RRu(-1/3))
-1/3
sage: check(RRz(-1/3))
-1/3
sage: check(RR(-1/3))
-1/3
sage: check(RealField(20)(pi))
355/113
sage: check(RR(pi))
245850922/78256779
sage: check(RR(2).sqrt())
131836323/93222358
sage: check(RR(1/2^210))
1/1645504557321205859467264516194506011931735427766374553794641921
```

```
sage: check(RR(2^210))
1645504557321205950811116849375918117252433820865891134852825088
sage: (RR(17).sqrt()).simplest_rational()^2 - 17
-1/348729667233025
sage: (RR(23).cube_root()).simplest_rational()^3 - 23
-1404915133/26474339584203908489584
sage: RRd5 = RealField(5, rnd='RNDD')
sage: RRu5 = RealField(5, rnd='RNDU')
sage: RR5 = RealField(5)
sage: below1 = RR5(1).nextbelow()
sage: check(RRd5(below1))
31/32
sage: check(RRu5(below1))
16/17
sage: check(below1)
21/22
sage: below1.exact_rational()
31/32
sage: above1 = RR5(1).nextabove()
sage: check(RRd5(above1))
10/9
sage: check(RRu5(above1))
17/16
sage: check(above1)
12/11
sage: above1.exact_rational()
17/16
sage: check(RR(1234))
1234
sage: check(RR5(1234))
1185
sage: check(RR5(1184))
1120
sage: RRd2 = RealField(2, rnd='RNDD')
sage: RRu2 = RealField(2, rnd='RNDU')
sage: RR2 = RealField(2)
sage: check(RR2(8))
7
sage: check(RRd2(8))
8
sage: check(RRu2(8))
7
sage: check(RR2(13))
11
sage: check(RRd2(13))
12
sage: check(RRu2(13))
13
sage: check(RR2(16))
14
sage: check(RRd2(16))
16
sage: check(RRu2(16))
13
sage: check(RR2(24))
21
sage: check(RRu2(24))
17
```

```
sage: check(RR2(-24))
-21
sage: check(RRu2(-24))
-24
```

TESTS:

```
sage: RR('nan').simplest_rational()
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
sage: RR('-infinity').simplest_rational()
Traceback (most recent call last):
...
ValueError: Cannot convert NaN or infinity to rational number
```

sin()

Return the sine of `self`.

EXAMPLES:

```
sage: R = RealField(100)
sage: R(2).sin()
0.90929742682568169539601986591
```

sincos()

Return a pair consisting of the sine and cosine of `self`.

EXAMPLES:

```
sage: R = RealField()
sage: t = R.pi()/6
sage: t.sincos()
(0.500000000000000, 0.866025403784439)
```

sinh()

Return the hyperbolic sine of `self`.

EXAMPLES:

```
sage: q = RR.pi()/12
sage: q.sinh()
0.264800227602271
```

sqrt(*extend=True*, *all=False*)

The square root function.

INPUT:

- `extend` – bool (default: `True`); if `True`, return a square root in a complex field if necessary if `self` is negative; otherwise raise a `ValueError`

- `all` – bool (default: `False`); if `True`, return a list of all square roots.

EXAMPLES:

```
sage: r = -2.0
sage: r.sqrt()
1.41421356237310*I
```

```
sage: r = 4.0
sage: r.sqrt()
```

```
2.00000000000000
sage: r.sqrt()^2 == r
True

sage: r = 4344
sage: r.sqrt()
2*sqrt(1086)

sage: r = 4344.0
sage: r.sqrt()^2 == r
True
sage: r.sqrt()^2 - r
0.000000000000000

sage: r = -2.0
sage: r.sqrt()
1.41421356237310*I
```

str (*base=10*, *no_sci=None*, *e=None*, *truncate=1*, *skip_zeroes=0*)
Return a string representation of `self`.

INPUT:

- `base` – base for output

- `no_sci` – if 2, never print using scientific notation; if 1 or `True`, print using scientific notation only for very large or very small numbers; if 0 or `False` always print with scientific notation; if `None` (the default), print how the parent prints.

- `e` – symbol used in scientific notation; defaults to 'e' for base=10, and '@' otherwise

- `truncate` – if `True`, round off the last digits in printing to lessen confusing base-2 roundoff issues.

- `skip_zeroes` – if `True`, skip trailing zeroes in mantissa

EXAMPLES:

```
sage: a = 61/3.0; a
20.3333333333333
sage: a.str(truncate=False)
'20.333333333333332'
sage: a.str(2)
'10100.010101010101010101010101010101010101010101010101'
sage: a.str(no_sci=False)
'2.03333333333333e1'
sage: a.str(16, no_sci=False)
'1.4555555555555@1'
sage: b = 2.0^99
sage: b.str()
'6.33825300114115e29'
sage: b.str(no_sci=False)
'6.33825300114115e29'
sage: b.str(no_sci=True)
'6.33825300114115e29'
sage: c = 2.0^100
sage: c.str()
'1.26765060022823e30'
sage: c.str(no_sci=False)
'1.26765060022823e30'
sage: c.str(no_sci=True)
'1.26765060022823e30'
```

```
sage: c.str(no_sci=2)
'126765060022823000000000000000000.'
sage: 0.5^53
1.11022302462516e-16
sage: 0.5^54
5.55111512312578e-17
sage: (0.01).str()
'0.0100000000000000'
sage: (0.01).str(skip_zeroes=True)
'0.01'
sage: (-10.042).str()
'-10.0420000000000'
sage: (-10.042).str(skip_zeroes=True)
'-10.042'
sage: (389.0).str(skip_zeroes=True)
'389.'
```

Test various bases:

```
sage: print (65536.0).str(base=2)
1.0000000000000000000000000000000000000000000000000000e16
sage: print (65536.0).str(base=36)
1ekg.00000000
sage: print (65536.0).str(base=62)
H32.0000000
sage: print (65536.0).str(base=63)
Traceback (most recent call last):
...
ValueError: base (=63) must be an integer between 2 and 62
```

tan()

Return the tangent of `self`.

EXAMPLES:

```
sage: q = RR.pi()/3
sage: q.tan()
1.73205080756888
sage: q = RR.pi()/6
sage: q.tan()
0.577350269189626
```

tanh()

Return the hyperbolic tangent of `self`.

EXAMPLES:

```
sage: q = RR.pi()/11
sage: q.tanh()
0.278079429295850
```

trunc()

Truncate `self`.

EXAMPLES:

```
sage: (2.99).trunc()
2
sage: (-0.00).trunc()
0
```

```
sage: (0.00).trunc()
0
```

ulp (*field=None*)

>   Returns the unit of least precision of `self`, which is the weight of the least significant bit of `self`. This is always a strictly positive number. It is also the gap between this number and the closest number with larger absolute value that can be represented.

>   INPUT:

>   •`field` – `RealField` used as parent of the result. If not specified, use `parent(self)`.

>   **Note:** The ulp of zero is defined as the smallest representable positive number. For extremely small numbers, underflow occurs and the output is also the smallest representable positive number (the rounding mode is ignored, this computation is done by rounding towards +infinity).

>   **See also:**

>   `epsilon()` for a scale-invariant version of this.

>   EXAMPLES:

```
sage: a = 1.0
sage: a.ulp()
2.22044604925031e-16
sage: (-1.5).ulp()
2.22044604925031e-16
sage: a + a.ulp() == a
False
sage: a + a.ulp()/2 == a
True

sage: a = RealField(500).pi()
sage: b = a + a.ulp()
sage: (a+b)/2 in [a,b]
True
```

>   The ulp of zero is the smallest non-zero number:

```
sage: a = RR(0).ulp()
sage: a
2.38256490488795e-323228497             # 32-bit
8.50969131174084e-1388255822130839284   # 64-bit
sage: a.fp_rank()
1
```

>   The ulp of very small numbers results in underflow, so the smallest non-zero number is returned instead:

```
sage: a.ulp() == a
True
```

>   We use a different field:

```
sage: a = RealField(256).pi()
sage: a.ulp()
3.454467422037777850154540745120159828446400145774512554009481388067436721265e-77
sage: e = a.ulp(RealField(64))
sage: e
3.45446742203777785e-77
sage: parent(e)
```

```
Real Field with 64 bits of precision
sage: e = a.ulp(QQ)
Traceback (most recent call last):
...
TypeError: field argument must be a RealField
```

For infinity and NaN, we get back positive infinity and NaN:

```
sage: a = RR(infinity)
sage: a.ulp()
+infinity
sage: (-a).ulp()
+infinity
sage: a = RR('nan')
sage: a.ulp()
NaN
sage: parent(RR('nan').ulp(RealField(42)))
Real Field with 42 bits of precision
```

y0()

Return the value of the Bessel Y function of order 0 at `self`.

EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).y0()
0.510375672649745
```

y1()

Return the value of the Bessel Y function of order 1 at `self`.

EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).y1()
-0.107032431540938
```

yn(*n*)

Return the value of the Bessel Y function of order n at `self`.

EXAMPLES:

```
sage: R = RealField(53)
sage: R(2).yn(3)
-1.12778377684043
sage: R(2).yn(-17)
7.09038821729481e12
```

zeta()

Return the Riemann zeta function evaluated at this real number.

Note: PARI is vastly more efficient at computing the Riemann zeta function. See the example below for how to use it.

EXAMPLES:

```
sage: R = RealField()
sage: R(2).zeta()
1.64493406684823
sage: R.pi()^2/6
```

```
1.64493406684823
sage: R(-2).zeta()
0.000000000000000
sage: R(1).zeta()
+infinity
```

Computing zeta using PARI is much more efficient in difficult cases. Here's how to compute zeta with at least a given precision:

```
sage: z = pari(2).zeta(precision=53); z
1.64493406684823
sage: pari(2).zeta(precision=128).python().prec()
128
sage: pari(2).zeta(precision=65).python().prec()
128                                                    # 64-bit
96                                                     # 32-bit
```

Note that the number of bits of precision in the constructor only effects the internal precision of the pari number, which is rounded up to the nearest multiple of 32 or 64. To increase the number of digits that gets displayed you must use `pari.set_real_precision`.

```
sage: type(z)
<type 'sage.libs.pari.gen.gen'>
sage: R(z)
1.64493406684823
```

class sage.rings.real_mpfr.**ZZtoRR**

> Bases: sage.categories.map.Map

> INPUT:

> There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

> TESTS:
> ```
> sage: from sage.categories.map import Map
> ```

> Using a hom space:
> ```
> sage: Map(Hom(QQ, ZZ, Rings()))
> Generic map:
>   From: Rational Field
>   To:   Integer Ring
> ```

> Using domain and codomain:
> ```
> sage: Map(QQ['x'], SymmetricGroup(6))
> Generic map:
>   From: Univariate Polynomial Ring in x over Rational Field
>   To:   Symmetric group of order 6! as a permutation group
> ```

sage.rings.real_mpfr.**create_RealField**(*prec=53*, *type='MPFR'*, *rnd='RNDN'*, *sci_not=0*)

> Create a real field with given precision, type, rounding mode and scientific notation.

> Some options are ignored for certain types (RDF for example).

> INPUT:

> - prec – a positive integer

> - type – type of real field:

- –'RDF' – the Sage real field corresponding to native doubles

- –'Interval' – real fields implementing interval arithmetic

- –'RLF' – the real lazy field

- –'MPFR' – floating point real numbers implemented using the MPFR library

- •rnd – rounding mode:

  - –'RNDN' – round to nearest

  - –'RNDZ' – round toward zero

  - –'RNDD' – round down

  - –'RNDU' – round up

- •sci_not – boolean, whether to use scientific notation for printing

OUTPUT:

the appropriate real field

EXAMPLES:

```
sage: from sage.rings.real_mpfr import create_RealField
sage: create_RealField(30)
Real Field with 30 bits of precision
sage: create_RealField(20, 'RDF') # ignores precision
Real Double Field
sage: create_RealField(60, 'Interval')
Real Interval Field with 60 bits of precision
sage: create_RealField(40, 'RLF') # ignores precision
Real Lazy Field
```

sage.rings.real_mpfr.**create_RealNumber**(*s*, *base=10*, *pad=0*, *rnd='RNDN'*, *min_prec=53*)

Return the real number defined by the string s as an element of RealField(prec=n), where n potentially has slightly more (controlled by pad) bits than given by s.

INPUT:

- •s – a string that defines a real number (or something whose string representation defines a number)

- •base – an integer between 2 and 62

- •pad – an integer = 0.

- •rnd – rounding mode:

  - –'RNDN' – round to nearest

  - –'RNDZ' – round toward zero

  - –'RNDD' – round down

  - –'RNDU' – round up

- •min_prec – number will have at least this many bits of precision, no matter what.

EXAMPLES:

```
sage: RealNumber('2.3') # indirect doctest
2.30000000000000
sage: RealNumber(10)
10.0000000000000
sage: RealNumber('1.0000000000000000000000000000000000')
1.00000000000000000000000000000000000
```

```
sage: RealField(200)(1.2)
1.2000000000000000000000000000000000000000000000000000000000
sage: (1.2).parent() is RR
True
```

We can use various bases:

```
sage: RealNumber("10101e2",base=2)
84.0000000000000
sage: RealNumber("deadbeef", base=16)
3.73592855900000e9
sage: RealNumber("deadbeefxxx", base=16)
Traceback (most recent call last):
...
TypeError: Unable to convert x (='deadbeefxxx') to real number.
sage: RealNumber("z", base=36)
35.0000000000000
sage: RealNumber("AAA", base=37)
14070.0000000000
sage: RealNumber("aaa", base=37)
50652.0000000000
sage: RealNumber("3.4", base="foo")
Traceback (most recent call last):
...
TypeError: an integer is required
sage: RealNumber("3.4", base=63)
Traceback (most recent call last):
...
ValueError: base (=63) must be an integer between 2 and 62
```

The rounding mode is respected in all cases:

```
sage: RealNumber("1.5", rnd="RNDU").parent()
Real Field with 53 bits of precision and rounding RNDU
sage: RealNumber("1.500000000000000000000000000000000000", rnd="RNDU").parent()
Real Field with 130 bits of precision and rounding RNDU
```

TESTS:

```
sage: RealNumber('.0000000000000000000000000000000001').prec()
53
sage: RealNumber('-.0000000000000000000000000000000001').prec()
53
```

Make sure we've rounded up `log(10,2)` enough to guarantee sufficient precision (trac ticket #10164):

```
sage: ks = 5*10**5, 10**6
sage: all(RealNumber("1." + "0"*k +"1")-1 > 0 for k in ks)
True
```

class `sage.rings.real_mpfr.`**`double_toRR`**

Bases: `sage.categories.map.Map`

INPUT:

There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

TESTS:

```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

class sage.rings.real_mpfr.**int_toRR**

Bases: sage.categories.map.Map

INPUT:

There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

TESTS:
```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

sage.rings.real_mpfr.**is_RealField**(*x*)

Returns `True` if x is technically of a Python real field type.

EXAMPLES:
```
sage: sage.rings.real_mpfr.is_RealField(RR)
True
sage: sage.rings.real_mpfr.is_RealField(CC)
False
```

sage.rings.real_mpfr.**is_RealNumber**(*x*)

Return `True` if x is of type RealNumber, meaning that it is an element of the MPFR real field with some precision.

EXAMPLES:
```
sage: from sage.rings.real_mpfr import is_RealNumber
sage: is_RealNumber(2.5)
True
sage: is_RealNumber(float(2.3))
```

```
    False
    sage: is_RealNumber(RDF(2))
    False
    sage: is_RealNumber(pi)
    False
```

sage.rings.real_mpfr.**mpfr_get_exp_max**()

Return the current maximal exponent for MPFR numbers.

EXAMPLES:

```
sage: from sage.rings.real_mpfr import mpfr_get_exp_max
sage: mpfr_get_exp_max()
1073741823              # 32-bit
4611686018427387903     # 64-bit
sage: 0.5 << mpfr_get_exp_max()
1.04928935823369e323228496              # 32-bit
2.93782689455579e1388255822130839282   # 64-bit
sage: 0.5 << (mpfr_get_exp_max()+1)
+infinity
```

sage.rings.real_mpfr.**mpfr_get_exp_max_max**()

Get the maximal value allowed for mpfr_set_exp_max().

EXAMPLES:

```
sage: from sage.rings.real_mpfr import mpfr_get_exp_max_max, mpfr_set_exp_max
sage: mpfr_get_exp_max_max()
1073741823              # 32-bit
4611686018427387903     # 64-bit
```

This is really the maximal value allowed:

```
sage: mpfr_set_exp_max(mpfr_get_exp_max_max() + 1)
Traceback (most recent call last):
...
OverflowError: bad value for mpfr_set_exp_max()
```

sage.rings.real_mpfr.**mpfr_get_exp_min**()

Return the current minimal exponent for MPFR numbers.

EXAMPLES:

```
sage: from sage.rings.real_mpfr import mpfr_get_exp_min
sage: mpfr_get_exp_min()
-1073741823             # 32-bit
-4611686018427387903    # 64-bit
sage: 0.5 >> (-mpfr_get_exp_min())
2.38256490488795e-323228497             # 32-bit
8.50969131174084e-1388255822130839284  # 64-bit
sage: 0.5 >> (-mpfr_get_exp_min()+1)
0.000000000000000
```

sage.rings.real_mpfr.**mpfr_get_exp_min_min**()

Get the minimal value allowed for mpfr_set_exp_min().

EXAMPLES:

```
sage: from sage.rings.real_mpfr import mpfr_get_exp_min_min, mpfr_set_exp_min
sage: mpfr_get_exp_min_min()
```

```
    -1073741823           # 32-bit
    -4611686018427387903  # 64-bit
```

This is really the minimal value allowed:
```
sage: mpfr_set_exp_min(mpfr_get_exp_min_min() - 1)
Traceback (most recent call last):
...
OverflowError: bad value for mpfr_set_exp_min()
```

sage.rings.real_mpfr.**mpfr_prec_max**()
    TESTS:
```
sage: from sage.rings.real_mpfr import mpfr_prec_max
sage: mpfr_prec_max()
2147483391
sage: R = RealField(2^31-257)
sage: R
Real Field with 2147483391 bits of precision
sage: R = RealField(2^31-256)
Traceback (most recent call last):
...
ValueError: prec (=2147483392) must be >= 2 and <= 2147483391
```

sage.rings.real_mpfr.**mpfr_prec_min**()
    Return the mpfr variable MPFR_PREC_MIN.

    EXAMPLES:
```
sage: from sage.rings.real_mpfr import mpfr_prec_min
sage: mpfr_prec_min()
2
sage: R = RealField(2)
sage: R(2) + R(1)
3.0
sage: R(4) + R(1)
4.0
sage: R = RealField(1)
Traceback (most recent call last):
...
ValueError: prec (=1) must be >= 2 and <= 2147483391
```

sage.rings.real_mpfr.**mpfr_set_exp_max**(*e*)
    Set the maximal exponent for MPFR numbers.

    EXAMPLES:
```
sage: from sage.rings.real_mpfr import mpfr_get_exp_max, mpfr_set_exp_max
sage: old = mpfr_get_exp_max()
sage: mpfr_set_exp_max(1000)
sage: 0.5 << 1000
5.35754303593134e300
sage: 0.5 << 1001
+infinity
sage: mpfr_set_exp_max(old)
sage: 0.5 << 1001
1.07150860718627e301
```

sage.rings.real_mpfr.**mpfr_set_exp_min**(*e*)
    Set the minimal exponent for MPFR numbers.

EXAMPLES:

```
sage: from sage.rings.real_mpfr import mpfr_get_exp_min, mpfr_set_exp_min
sage: old = mpfr_get_exp_min()
sage: mpfr_set_exp_min(-1000)
sage: 0.5 >> 1000
4.66631809251609e-302
sage: 0.5 >> 1001
0.000000000000000
sage: mpfr_set_exp_min(old)
sage: 0.5 >> 1001
2.33315904625805e-302
```

THREE

ARBITRARY PRECISION REAL INTERVALS

AUTHORS:

- Carl Witty (2007-01-21): based on `real_mpfr.pyx`; changed it to use mpfi rather than mpfr.
- William Stein (2007-01-24): modifications and clean up and docs, etc.
- Niles Johnson (2010-08): trac ticket #3893: `random_element()` should pass on `*args` and `**kwds`.
- Travis Scrimshaw (2012-10-20): Fixing scientific notation output to fix trac ticket #13634.
- Travis Scrimshaw (2012-11-02): Added doctests for full coverage

This is a straightforward binding to the MPFI library; it may be useful to refer to its documentation for more details.

An interval is represented as a pair of floating-point numbers a and b (where $a \leq b$) and is printed as a standard floating-point number with a question mark (for instance, `3.1416?`). The question mark indicates that the preceding digit may have an error of ± 1. These floating-point numbers are implemented using MPFR (the same as the `RealNumber` elements of `RealField_class`).

There is also an alternate method of printing, where the interval prints as `[a ..  b]` (for instance, `[3.1415 .. 3.1416]`).

The interval represents the set $\{x : a \leq x \leq b\}$ (so if $a = b$, then the interval represents that particular floating-point number). The endpoints can include positive and negative infinity, with the obvious meaning. It is also possible to have a `NaN` (Not-a-Number) interval, which is represented by having either endpoint be `NaN`.

PRINTING:

There are two styles for printing intervals: 'brackets' style and 'question' style (the default).

In question style, we print the "known correct" part of the number, followed by a question mark. The question mark indicates that the preceding digit is possibly wrong by ± 1.

```
sage: RIF(sqrt(2))
1.414213562373095?
```

However, if the interval is precise (its lower bound is equal to its upper bound) and equal to a not-too-large integer, then we just print that integer.

```
sage: RIF(0)
0
sage: RIF(654321)
654321
```

```
sage: RIF(123, 125)
124.?
sage: RIF(123, 126)
1.3?e2
```

As we see in the last example, question style can discard almost a whole digit's worth of precision. We can reduce this by allowing "error digits": an error following the question mark, that gives the maximum error of the digit(s) before the question mark. If the error is absent (which it always is in the default printing), then it is taken to be 1.

```
sage: RIF(123, 126).str(error_digits=1)
'125.?2'
sage: RIF(123, 127).str(error_digits=1)
'125.?2'
sage: v = RIF(-e, pi); v
0.?e1
sage: v.str(error_digits=1)
'1.?4'
sage: v.str(error_digits=5)
'0.2117?29300'
```

Error digits also sometimes let us indicate that the interval is actually equal to a single floating-point number:

```
sage: RIF(54321/256)
212.19140625000000?
sage: RIF(54321/256).str(error_digits=1)
'212.19140625000000?0'
```

In brackets style, intervals are printed with the left value rounded down and the right rounded up, which is conservative, but in some ways unsatisfying.

Consider a 3-bit interval containing exactly the floating-point number 1.25. In round-to-nearest or round-down, this prints as 1.2; in round-up, this prints as 1.3. The straightforward options, then, are to print this interval as `[1.2 .. 1.2]` (which does not even contain the true value, 1.25), or to print it as `[1.2 .. 1.3]` (which gives the impression that the upper and lower bounds are not equal, even though they really are). Neither of these is very satisfying, but we have chosen the latter.

```
sage: R = RealIntervalField(3)
sage: a = R(1.25)
sage: a.str(style='brackets')
'[1.2 .. 1.3]'
sage: a == 1.25
True
sage: a == 2
False
```

COMPARISONS:

Comparison operations (==, !=, <, <=, >, >=) return `True` if every value in the first interval has the given relation to every value in the second interval. The `cmp(a, b)` function works differently; it compares two intervals lexicographically. (However, the behavior is not specified if given a non-interval and an interval.)

This convention for comparison operators has good and bad points. The good:

- Expected transitivity properties hold (if `a > b` and `b == c`, then `a > c`; etc.)

- if `a > b`, then `cmp(a, b) == 1`; if `a == b`, then `cmp(a,b) == 0`; if `a < b`, then `cmp(a, b) == -1`

- `a == 0` is true if the interval contains only the floating-point number 0; similarly for `a == 1`

- `a > 0` means something useful (that every value in the interval is greater than 0)

The bad:

- Trichotomy fails to hold: there are values `(a,b)` such that none of `a < b`, `a == b`, or `a > b` are true

- It is not the case that if `cmp(a, b) == 0` then `a == b`, or that if `cmp(a, b) == 1` then `a > b`, or that if `cmp(a, b) == -1` then `a < b`

- There are values `a` and `b` such that `a <= b` but neither `a < b` nor `a == b` hold.

Note: Intervals `a` and `b` overlap iff `not(a != b)`.

EXAMPLES:

```
sage: 0 < RIF(1, 2)
True
sage: 0 == RIF(0)
True
sage: not(0 == RIF(0, 1))
True
sage: not(0 != RIF(0, 1))
True
sage: 0 <= RIF(0, 1)
True
sage: not(0 < RIF(0, 1))
True
sage: cmp(RIF(0), RIF(0, 1))
-1
sage: cmp(RIF(0, 1), RIF(0))
1
sage: cmp(RIF(0, 1), RIF(1))
-1
sage: cmp(RIF(0, 1), RIF(0, 1))
0
```

Comparison with infinity is defined through coercion to the infinity ring where semi-infinite intervals are sent to their central value (plus or minus infinity); This implements the above convention for inequalities:

```
sage: InfinityRing.has_coerce_map_from(RIF)
True
sage: -oo < RIF(-1,1) < oo
True
sage: -oo < RIF(0,oo) <= oo
True
sage: -oo <= RIF(-oo,-1) < oo
True
```

Comparison by equality shows what the semi-infinite intervals actually coerce to:

```
sage: RIF(1,oo) == oo
True
sage: RIF(-oo,-1) == -oo
True
```

For lack of a better value in the infinity ring, the doubly infinite interval coerces to plus infinity:

```
sage: RIF(-oo,oo) == oo
True
```

`sage.rings.real_mpfi.`**`RealInterval`**(*s*, *upper=None*, *base=10*, *pad=0*, *min_prec=53*)
    Return the real number defined by the string s as an element of `RealIntervalField(prec=n)`, where n potentially has slightly more (controlled by pad) bits than given by s.

    INPUT:

- `s` – a string that defines a real number (or something whose string representation defines a number)

- `upper` – (default: `None`) - upper endpoint of interval if given, in which case `s` is the lower endpoint

- `base` – an integer between 2 and 36

- `pad` – (default: 0) an integer

- `min_prec` – number will have at least this many bits of precision, no matter what

EXAMPLES:
```
sage: RealInterval('2.3')
2.300000000000000?
sage: RealInterval(10)
10
sage: RealInterval('1.00000000000000000000000000000000000')
1
sage: RealInterval('1.234567890123456789012345678901234 5')
1.2345678901234567890123456789012345 0?
sage: RealInterval(29308290382930840239842390482, 3^20).str(style='brackets')
'[3.4867844010000000000000000000000e9 .. 2.93082903829308402398423904820e28]'
```

TESTS:

Make sure we've rounded up `log(10,2)` enough to guarantee sufficient precision (trac ticket #10164). This is a little tricky because at the time of writing, we don't support intervals long enough to trip the error. However, at least we can make sure that we either do it correctly or fail noisily:
```
sage: ks = 5*10**5, 10**6
sage: for k in ks:
...       try:
...           z = RealInterval("1." + "1"*k)
...           assert len(str(z))-4 >= k
...       except TypeError:
...           pass
```

sage.rings.real_mpfi.**RealIntervalField**(*prec=53*, *sci_not=False*)
    Construct a `RealIntervalField_class`, with caching.

    INPUT:

- `prec` – (integer) precision; default = 53: The number of bits used to represent the mantissa of a floating-point number. The precision can be any integer between `mpfr_prec_min()` and `mpfr_prec_max()`. In the current implementation, `mpfr_prec_min()` is equal to 2.

- `sci_not` – (default: `False`) whether or not to display using scientific notation

    EXAMPLES:
```
sage: RealIntervalField()
Real Interval Field with 53 bits of precision
sage: RealIntervalField(200, sci_not=True)
Real Interval Field with 200 bits of precision
sage: RealIntervalField(53) is RIF
True
sage: RealIntervalField(200) is RIF
False
sage: RealIntervalField(200) is RealIntervalField(200)
True
```

    See the documentation for `RealIntervalField_class` for many more examples.

class sage.rings.real_mpfi.**RealIntervalFieldElement**

> Bases: sage.structure.element.RingElement

> A real number interval.

> **absolute_diameter**()

>> The diameter of this interval (for $[a..b]$, this is $b - a$), rounded upward, as a RealNumber.

>> EXAMPLES:

>> ```
sage: RIF(1, pi).absolute_diameter()
2.14159265358979
```

> **alea**()

>> Return a floating-point number picked at random from the interval.

>> EXAMPLES:

>> ```
sage: RIF(1, 2).alea() # random
1.34696133696137
```

> **algdep**(n)

>> Returns a polynomial of degree at most n which is approximately satisfied by self.

>> ---

>> **Note:** The returned polynomial need not be irreducible, and indeed usually won't be if self is a good approximation to an algebraic number of degree less than n.

>> ---

>> Pari needs to know the number of "known good bits" in the number; we automatically get that from the interval width.

>> ALGORITHM:

>> Uses the PARI C-library algdep command.

>> EXAMPLES:

>> ```
sage: r = sqrt(RIF(2)); r
1.414213562373095?
sage: r.algdep(5)
x^2 - 2
```

>> If we compute a wrong, but precise, interval, we get a wrong answer:

>> ```
sage: r = sqrt(RealIntervalField(200)(2)) + (1/2)^40; r
1.4142135623740045435034616524476131176321718753769480073176680?
sage: r.algdep(5)
7266488*x^5 + 22441629*x^4 - 90470501*x^3 + 23297703*x^2 + 45778664*x + 13681026
```

>> But if we compute an interval that includes the number we mean, we're much more likely to get the right answer, even if the interval is very imprecise:

>> ```
sage: r = r.union(sqrt(2.0))
sage: r.algdep(5)
x^2 - 2
```

>> Even on this extremely imprecise interval we get an answer which is technically correct:

>> ```
sage: RIF(-1, 1).algdep(5)
x
```

> **arccos**()

>> Return the inverse cosine of self.

EXAMPLES:

```
sage: q = RIF.pi()/3; q
1.047197551196598?
sage: i = q.cos(); i
0.500000000000000?
sage: q2 = i.arccos(); q2
1.047197551196598?
sage: q == q2
False
sage: q != q2
False
sage: q2.lower() == q.lower()
False
sage: q - q2
0.?e-15
sage: q in q2
True
```

arccosh()

Return the hyperbolic inverse cosine of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/2
sage: i = q.arccosh() ; i
1.023227478547551?
```

arccoth()

Return the inverse hyperbolic cotangent of `self`.

EXAMPLES:

```
sage: RealIntervalField(100)(2).arccoth()
0.54930614433405484569762261846?
sage: (2.0).arccoth()
0.549306144334055
```

arccsch()

Return the inverse hyperbolic cosecant of `self`.

EXAMPLES:

```
sage: RealIntervalField(100)(2).arccsch()
0.48121182505960344749775891342?
sage: (2.0).arccsch()
0.481211825059603
```

arcsech()

Return the inverse hyperbolic secant of `self`.

EXAMPLES:

```
sage: RealIntervalField(100)(0.5).arcsech()
1.3169578969248167086250463730?
sage: (0.5).arcsech()
1.31695789692482
```

arcsin()

Return the inverse sine of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/5; q
0.6283185307179587?
sage: i = q.sin(); i
0.587785252292474?
sage: q2 = i.arcsin(); q2
0.628318530717959?
sage: q == q2
False
sage: q != q2
False
sage: q2.lower() == q.lower()
False
sage: q - q2
0.?e-15
sage: q in q2
True
```

arcsinh()

Return the hyperbolic inverse sine of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/7
sage: i = q.sinh() ; i
0.464017630492991?
sage: i.arcsinh() - q
0.?e-15
```

arctan()

Return the inverse tangent of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/5; q
0.6283185307179587?
sage: i = q.tan(); i
0.726542528005361?
sage: q2 = i.arctan(); q2
0.628318530717959?
sage: q == q2
False
sage: q != q2
False
sage: q2.lower() == q.lower()
False
sage: q - q2
0.?e-15
sage: q in q2
True
```

arctanh()

Return the hyperbolic inverse tangent of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/7
sage: i = q.tanh() ; i
0.420911241048535?
sage: i.arctanh() - q
0.?e-15
```

bisection()

> Returns the bisection of `self` into two intervals of half the size whose union is `self` and intersection is `center()`.
>
> EXAMPLES:
>
> ```
> sage: a, b = RIF(1,2).bisection()
> sage: a.lower(), a.upper()
> (1.00000000000000, 1.50000000000000)
> sage: b.lower(), b.upper()
> (1.50000000000000, 2.00000000000000)
>
> sage: I = RIF(e, pi)
> sage: a, b = I.bisection()
> sage: a.intersection(b) == I.center()
> True
> sage: a.union(b).endpoints() == I.endpoints()
> True
> ```

ceil()

> Return the ceiling of `self`.
>
> OUTPUT:
>
> integer
>
> EXAMPLES:
>
> ```
> sage: (2.99).ceil()
> 3
> sage: (2.00).ceil()
> 2
> sage: (2.01).ceil()
> 3
> sage: R = RealIntervalField(30)
> sage: a = R(-9.5, -11.3); a.str(style='brackets')
> '[-11.300000012 .. -9.5000000000]'
> sage: a.floor().str(style='brackets')
> '[-12.000000000 .. -10.000000000]'
> sage: a.ceil()
> -10.?
> sage: ceil(a).str(style='brackets')
> '[-11.000000000 .. -9.0000000000]'
> ```

ceiling()

> Return the ceiling of `self`.
>
> OUTPUT:
>
> integer
>
> EXAMPLES:
>
> ```
> sage: (2.99).ceil()
> 3
> sage: (2.00).ceil()
> 2
> sage: (2.01).ceil()
> 3
> sage: R = RealIntervalField(30)
> sage: a = R(-9.5, -11.3); a.str(style='brackets')
> '[-11.300000012 .. -9.5000000000]'
> ```

```
sage: a.floor().str(style='brackets')
'[-12.000000000 .. -10.000000000]'
sage: a.ceil()
-10.?
sage: ceil(a).str(style='brackets')
'[-11.000000000 .. -9.0000000000]'
```

center()

Compute the center of the interval $[a..b]$ which is $(a + b)/2$.

EXAMPLES:

```
sage: RIF(1, 2).center()
1.50000000000000
```

contains_zero()

Return `True` if `self` is an interval containing zero.

EXAMPLES:

```
sage: RIF(0).contains_zero()
True
sage: RIF(1, 2).contains_zero()
False
sage: RIF(-1, 1).contains_zero()
True
sage: RIF(-1, 0).contains_zero()
True
```

cos()

Return the cosine of `self`.

EXAMPLES:

```
sage: t=RIF(pi)/2
sage: t.cos()
0.?e-15
sage: t.cos().str(style='brackets')
'[-1.6081226496766367e-16 .. 6.1232339957367661e-17]'
sage: t.cos().cos()
0.9999999999999999?
```

TESTS:

This looped forever with an earlier version of MPFI, but now it works:

```
sage: RIF(-1, 1).cos().str(style='brackets')
'[0.54030230586813965 .. 1.0000000000000000]'
```

cosh()

Return the hyperbolic cosine of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/12
sage: q.cosh()
1.034465640095511?
```

cot()

Return the cotangent of `self`.

EXAMPLES:

```
sage: RealIntervalField(100)(2).cot()
-0.45765755436028576375027741 0432?
```

coth()
> Return the hyperbolic cotangent of `self`.

> EXAMPLES:
```
sage: RealIntervalField(100)(2).coth()
1.0373147207275480958778097 6477?
```

csc()
> Return the cosecant of `self`.

> EXAMPLES:
```
sage: RealIntervalField(100)(2).csc()
1.0997501702946164667566973 97026?
```

csch()
> Return the hyperbolic cosecant of `self`.

> EXAMPLES:
```
sage: RealIntervalField(100)(2).csch()
0.27572056477178320775835148 2163?
```

diameter()
> If 0 is in `self`, then return `absolute_diameter()`, otherwise return `relative_diameter()`.

> EXAMPLES:
```
sage: RIF(1, 2).diameter()
0.666666666666667
sage: RIF(1, 2).absolute_diameter()
1.00000000000000
sage: RIF(1, 2).relative_diameter()
0.666666666666667
sage: RIF(pi).diameter()
1.41357985842823e-16
sage: RIF(pi).absolute_diameter()
4.44089209850063e-16
sage: RIF(pi).relative_diameter()
1.41357985842823e-16
sage: (RIF(pi) - RIF(3, 22/7)).diameter()
0.142857142857144
sage: (RIF(pi) - RIF(3, 22/7)).absolute_diameter()
0.142857142857144
sage: (RIF(pi) - RIF(3, 22/7)).relative_diameter()
2.03604377705518
```

endpoints(*rnd=None*)
> Return the lower and upper endpoints of `self`.

> EXAMPLES:
```
sage: RIF(1,2).endpoints()
(1.00000000000000, 2.00000000000000)
sage: RIF(pi).endpoints()
(3.14159265358979, 3.14159265358980)
sage: a = CIF(RIF(1,2), RIF(3,4))
```

```
sage: a.real().endpoints()
(1.00000000000000, 2.00000000000000)
```

As with `lower()` and `upper()`, a rounding mode is accepted:

```
sage: RIF(1,2).endpoints('RNDD')[0].parent()
Real Field with 53 bits of precision and rounding RNDD
```

exp()
> Returns $e^{\mathtt{self}}$

> EXAMPLES:

```
sage: r = RIF(0.0)
sage: r.exp()
1

sage: r = RIF(32.3)
sage: a = r.exp(); a
1.065888472748645?e14
sage: a.log()
32.30000000000000?

sage: r = RIF(-32.3)
sage: r.exp()
9.38184458849869?e-15
```

exp2()
> Returns $2^{\mathtt{self}}$

> EXAMPLES:

```
sage: r = RIF(0.0)
sage: r.exp2()
1

sage: r = RIF(32.0)
sage: r.exp2()
4294967296

sage: r = RIF(-32.3)
sage: r.exp2()
1.891172482530207?e-10
```

factorial()
> Return the factorial evaluated on `self`.

> EXAMPLES:

```
sage: RIF(5).factorial()
120
sage: RIF(2.3,5.7).factorial()
1.?e3
sage: RIF(2.3).factorial()
2.683437381955768?
```

> Recover the factorial as integer:

```
sage: f = RealIntervalField(200)(50).factorial()
sage: f
3.0414093201713378043612608166064768844377641568960512000000000?e64
```

```
sage: f.unique_integer()
30414093201713378043612608166064768844377641568960512000000000000
sage: 50.factorial()
30414093201713378043612608166064768844377641568960512000000000000
```

floor()

> Return the floor of self.
>
> EXAMPLES:
> ```
> sage: R = RealIntervalField()
> sage: (2.99).floor()
> 2
> sage: (2.00).floor()
> 2
> sage: floor(RR(-5/2))
> -3
> sage: R = RealIntervalField(100)
> sage: a = R(9.5, 11.3); a.str(style='brackets')
> '[9.5000000000000000000000000000000 .. 11.300000000000000710542735760101]'
> sage: floor(a).str(style='brackets')
> '[9.0000000000000000000000000000000 .. 11.000000000000000000000000000000]'
> sage: a.floor()
> 10.?
> sage: ceil(a)
> 11.?
> sage: a.ceil().str(style='brackets')
> '[10.000000000000000000000000000000 .. 12.000000000000000000000000000000]'
> ```

fp_rank_diameter()

> Computes the diameter of this interval in terms of the "floating-point rank".
>
> The floating-point rank is the number of floating-point numbers (of the current precision) contained in the given interval, minus one. An fp_rank_diameter of 0 means that the interval is exact; an fp_rank_diameter of 1 means that the interval is as tight as possible, unless the number you're trying to represent is actually exactly representable as a floating-point number.
>
> EXAMPLES:
> ```
> sage: RIF(pi).fp_rank_diameter()
> 1
> sage: RIF(12345).fp_rank_diameter()
> 0
> sage: RIF(-sqrt(2)).fp_rank_diameter()
> 1
> sage: RIF(5/8).fp_rank_diameter()
> 0
> sage: RIF(5/7).fp_rank_diameter()
> 1
> sage: a = RIF(pi)^12345; a
> 2.06622879260?e6137
> sage: a.fp_rank_diameter()
> 30524
> sage: (RIF(sqrt(2)) - RIF(sqrt(2))).fp_rank_diameter()
> 9671406088542672151117826              # 32-bit
> 41538374868278620559869609387229186   # 64-bit
> ```
>
> Just because we have the best possible interval, doesn't mean the interval is actually small:

```
sage: a = RIF(pi)^12345678901234567890; a
[2.0985787164673874e323228496 .. +infinity]        # 32-bit
[5.8756537891115869e1388255822130839282 .. +infinity]  # 64-bit
sage: a.fp_rank_diameter()
1
```

gamma()

Return the gamma function evalutated on self.

EXAMPLES:
```
sage: RIF(1).gamma()
1
sage: RIF(5).gamma()
24
sage: a = RIF(3,4).gamma(); a
1.?e1
sage: a.lower(), a.upper()
(2.00000000000000, 6.00000000000000)
sage: RIF(-1/2).gamma()
-3.54490770181104?
sage: gamma(-1/2).n(100) in RIF(-1/2).gamma()
True
sage: 0 in (RealField(2000)(-19/3).gamma() - RealIntervalField(1000)(-19/3).gamma())
True
sage: gamma(RIF(100))
9.33262154439442?e155
sage: gamma(RIF(-10000/3))
1.31280781451?e-10297
```

Verify the result contains the local minima:
```
sage: 0.88560319441088 in RIF(1, 2).gamma()
True
sage: 0.88560319441088 in RIF(0.25, 4).gamma()
True
sage: 0.88560319441088 in RIF(1.4616, 1.46164).gamma()
True
```

```
sage: (-0.99).gamma()
-100.436954665809
sage: (-0.01).gamma()
-100.587197964411
sage: RIF(-0.99, -0.01).gamma().upper()
-1.60118039970055
```

Correctly detects poles:
```
sage: gamma(RIF(-3/2,-1/2))
[-infinity .. +infinity]
```

intersection(*other*)

Return the intersection of two intervals. If the intervals do not overlap, raises a `ValueError`.

EXAMPLES:
```
sage: RIF(1, 2).intersection(RIF(1.5, 3)).str(style='brackets')
'[1.5000000000000000 .. 2.0000000000000000]'
sage: RIF(1, 2).intersection(RIF(4/3, 5/3)).str(style='brackets')
'[1.3333333333333332 .. 1.6666666666666668]'
```

```
sage: RIF(1, 2).intersection(RIF(3, 4))
Traceback (most recent call last):
...
ValueError: intersection of non-overlapping intervals
```

is_NaN()

> Check to see if `self` is Not-a-Number `NaN`.
>
> EXAMPLES:
> ```
> sage: a = RIF(0) / RIF(0.0,0.00); a
> [.. NaN ..]
> sage: a.is_NaN()
> True
> ```

is_exact()

> Return whether this real interval is exact (i.e. contains exactly one real value).
>
> EXAMPLES:
> ```
> sage: RIF(3).is_exact()
> True
> sage: RIF(2*pi).is_exact()
> False
> ```

is_int()

> Checks to see whether this interval includes exactly one integer.
>
> OUTPUT:
>
> If this contains exactly one integer, it returns the tuple `(True, n)`, where n is that integer; otherwise, this returns `(False, None)`.
>
> EXAMPLES:
> ```
> sage: a = RIF(0.8,1.5)
> sage: a.is_int()
> (True, 1)
> sage: a = RIF(1.1,1.5)
> sage: a.is_int()
> (False, None)
> sage: a = RIF(1,2)
> sage: a.is_int()
> (False, None)
> sage: a = RIF(-1.1, -0.9)
> sage: a.is_int()
> (True, -1)
> sage: a = RIF(0.1, 1.9)
> sage: a.is_int()
> (True, 1)
> sage: RIF(+infinity,+infinity).is_int()
> (False, None)
> ```

log(*base='e'*)

> Return the logarithm of `self` to the given `base`.
>
> EXAMPLES:
> ```
> sage: R = RealIntervalField()
> sage: r = R(2); r.log()
> 0.6931471805599453?
> ```

```
sage: r = R(-2); r.log()
0.6931471805599453? + 3.141592653589794?*I
```

log10()

Return log to the base 10 of `self`.

EXAMPLES:

```
sage: r = RIF(16.0); r.log10()
1.204119982655925?
sage: r.log() / log(10.0)
1.204119982655925?

sage: r = RIF(39.9); r.log10()
1.600972895686749?

sage: r = RIF(0.0)
sage: r.log10()
[-infinity .. -infinity]

sage: r = RIF(-1.0)
sage: r.log10()
1.364376353841841?*I
```

log2()

Return `log` to the base 2 of `self`.

EXAMPLES:

```
sage: r = RIF(16.0)
sage: r.log2()
4

sage: r = RIF(31.9); r.log2()
4.995484518877507?

sage: r = RIF(0.0, 2.0)
sage: r.log2()
[-infinity .. 1.0000000000000000]
```

lower(*rnd=None*)

Return the lower bound of this interval

INPUT:

- `rnd` – (string) the rounding mode

    - `'RNDN'` – round to nearest

    - `'RNDD'` – (default) round towards minus infinity

    - `'RNDZ'` – round towards zero

    - `'RNDU'` – round towards plus infinity

The rounding mode does not affect the value returned as a floating-point number, but it does control which variety of `RealField` the returned number is in, which affects printing and subsequent operations.

EXAMPLES:

```
sage: R = RealIntervalField(13)
sage: R.pi().lower().str(truncate=False)
'3.1411'
```

```
sage: x = R(1.2,1.3); x.str(style='brackets')
'[1.1999 .. 1.3001]'
sage: x.lower()
1.19
sage: x.lower('RNDU')
1.20
sage: x.lower('RNDN')
1.20
sage: x.lower('RNDZ')
1.19
sage: x.lower().parent()
Real Field with 13 bits of precision and rounding RNDD
sage: x.lower('RNDU').parent()
Real Field with 13 bits of precision and rounding RNDU
sage: x.lower() == x.lower('RNDU')
True
```

magnitude()

 The largest absolute value of the elements of the interval.

 EXAMPLES:

```
sage: RIF(-2, 1).magnitude()
2.00000000000000
sage: RIF(-1, 2).magnitude()
2.00000000000000
```

max(**_others*)

 Return an interval containing the maximum of `self` and the arguments.

 EXAMPLES:

```
sage: RIF(-1, 1).max(0).endpoints()
(0.000000000000000, 1.00000000000000)
sage: RIF(-1, 1).max(RIF(2, 3)).endpoints()
(2.00000000000000, 3.00000000000000)
sage: RIF(-1, 1).max(RIF(-100, 100)).endpoints()
(-1.00000000000000, 100.000000000000)
sage: RIF(-1, 1).max(RIF(-100, 100), RIF(5, 10)).endpoints()
(5.00000000000000, 100.000000000000)
```

 Note that if the maximum is one of the given elements, that element will be returned.

```
sage: a = RIF(-1, 1)
sage: b = RIF(2, 3)
sage: c = RIF(3, 4)
sage: c.max(a, b) is c
True
sage: b.max(a, c) is c
True
sage: a.max(b, c) is c
True
```

 It might also be convenient to call the method as a function:

```
sage: from sage.rings.real_mpfi import RealIntervalFieldElement
sage: RealIntervalFieldElement.max(a, b, c) is c
True
sage: elements = [a, b, c]
sage: RealIntervalFieldElement.max(*elements) is c
```

```
True
```

The generic max does not always do the right thing:

```
sage: max(0, RIF(-1, 1))
0
sage: max(RIF(-1, 1), RIF(-100, 100)).endpoints()
(-1.00000000000000, 1.00000000000000)
```

See also:

min()

TESTS:

```
sage: a.max('x')
Traceback (most recent call last):
...
TypeError: Unable to convert number to real interval.
```

mignitude()

> The smallest absolute value of the elements of the interval.
>
> EXAMPLES:
>
> ```
> sage: RIF(-2, 1).mignitude()
> 0.000000000000000
> sage: RIF(-2, -1).mignitude()
> 1.00000000000000
> sage: RIF(3, 4).mignitude()
> 3.00000000000000
> ```

min(**_others*)

> Return an interval containing the minimum of self and the arguments.
>
> EXAMPLES:
>
> ```
> sage: a = RIF(-1, 1).min(0).endpoints()
> sage: a[0] == -1.0 and a[1].abs() == 0.0 # in MPFI, the sign of 0.0 is not specified
> True
> sage: RIF(-1, 1).min(pi).endpoints()
> (-1.00000000000000, 1.00000000000000)
> sage: RIF(-1, 1).min(RIF(-100, 100)).endpoints()
> (-100.000000000000, 1.00000000000000)
> sage: RIF(-1, 1).min(RIF(-100, 0)).endpoints()
> (-100.000000000000, 0.000000000000000)
> sage: RIF(-1, 1).min(RIF(-100, 2), RIF(-200, -3)).endpoints()
> (-200.000000000000, -3.00000000000000)
> ```
>
> Note that if the minimum is one of the given elements, that element will be returned.
>
> ```
> sage: a = RIF(-1, 1)
> sage: b = RIF(2, 3)
> sage: c = RIF(3, 4)
> sage: c.min(a, b) is a
> True
> sage: b.min(a, c) is a
> True
> sage: a.min(b, c) is a
> True
> ```

It might also be convenient to call the method as a function:

```
sage: from sage.rings.real_mpfi import RealIntervalFieldElement
sage: RealIntervalFieldElement.min(a, b, c) is a
True
sage: elements = [a, b, c]
sage: RealIntervalFieldElement.min(*elements) is a
True
```

The generic min does not always do the right thing:

```
sage: min(0, RIF(-1, 1))
0
sage: min(RIF(-1, 1), RIF(-100, 100)).endpoints()
(-1.00000000000000, 1.00000000000000)
```

See also:

[max()](#)

TESTS:

```
sage: a.min('x')
Traceback (most recent call last):
...
TypeError: Unable to convert number to real interval.
```

multiplicative_order()

Return n such that `self^n == 1`.

Only ± 1 have finite multiplicative order.

EXAMPLES:

```
sage: RIF(1).multiplicative_order()
1
sage: RIF(-1).multiplicative_order()
2
sage: RIF(3).multiplicative_order()
+Infinity
```

overlaps(*other*)

Return `True` if `self` and other are intervals with at least one value in common. For intervals `a` and `b`, we have `a.overlaps(b)` iff `not(a!=b)`.

EXAMPLES:

```
sage: RIF(0, 1).overlaps(RIF(1, 2))
True
sage: RIF(1, 2).overlaps(RIF(0, 1))
True
sage: RIF(0, 1).overlaps(RIF(2, 3))
False
sage: RIF(2, 3).overlaps(RIF(0, 1))
False
sage: RIF(0, 3).overlaps(RIF(1, 2))
True
sage: RIF(0, 2).overlaps(RIF(1, 3))
True
```

prec()

Returns the precision of `self`.

EXAMPLES:
```
sage: RIF(2.1).precision()
53
sage: RealIntervalField(200)(2.1).precision()
200
```

precision()
>   Returns the precision of `self`.
>
>   EXAMPLES:
>    ```
>    sage: RIF(2.1).precision()
>    53
>    sage: RealIntervalField(200)(2.1).precision()
>    200
>    ```

psi()
>   Return the digamma function evaluated on self.
>
>   INPUT:
>
>   None.
>
>   OUTPUT:
>
>   A `RealIntervalFieldElement`.
>
>   This uses the optional arb library. You may have to install it via `sage -i arb`.
>
>   EXAMPLES:
>    ```
>    sage: psi_1 = RIF(1).psi() # optional - arb
>    sage: psi_1 # optional - arb
>    -0.577215664901533?
>    sage: psi_1.overlaps(-RIF.euler_constant()) # optional - arb
>    True
>    ```

real()
>   Return the real part of `self`.
>
>   (Since `self` is a real number, this simply returns `self`.)
>
>   EXAMPLES:
>    ```
>    sage: RIF(1.2465).real() == RIF(1.2465)
>    True
>    ```

relative_diameter()
>   The relative diameter of this interval (for $[a..b]$, this is $(b - a)/((a + b)/2)$), rounded upward, as a `RealNumber`.
>
>   EXAMPLES:
>    ```
>    sage: RIF(1, pi).relative_diameter()
>    1.03418797197910
>    ```

sec()
>   Return the secant of this number.
>
>   EXAMPLES:
>    ```
>    sage: RealIntervalField(100)(2).sec()
>    -2.4029979617223809897546004142?
>    ```

sech()

Return the hyperbolic secant of `self`.

EXAMPLES:

```
sage: RealIntervalField(100)(2).sech()
0.26580222883407969212086273982 0?
```

simplest_rational(*low_open=False*, *high_open=False*)

Return the simplest rational in this interval. Given rationals a/b and c/d (both in lowest terms), the former is simpler if $b < d$ or if $b = d$ and $|a| < |c|$.

If optional parameters `low_open` or `high_open` are `True`, then treat this as an open interval on that end.

EXAMPLES:

```
sage: RealIntervalField(10)(pi).simplest_rational()
22/7
sage: RealIntervalField(20)(pi).simplest_rational()
355/113
sage: RIF(0.123, 0.567).simplest_rational()
1/2
sage: RIF(RR(1/3).nextabove(), RR(3/7)).simplest_rational()
2/5
sage: RIF(1234/567).simplest_rational()
1234/567
sage: RIF(-8765/432).simplest_rational()
-8765/432
sage: RIF(-1.234, 0.003).simplest_rational()
0
sage: RIF(RR(1/3)).simplest_rational()
6004799503160661/18014398509481984
sage: RIF(RR(1/3)).simplest_rational(high_open=True)
Traceback (most recent call last):
...
ValueError: simplest_rational() on open, empty interval
sage: RIF(1/3, 1/2).simplest_rational()
1/2
sage: RIF(1/3, 1/2).simplest_rational(high_open=True)
1/3
sage: phi = ((RealIntervalField(500)(5).sqrt() + 1)/2)
sage: phi.simplest_rational() == fibonacci(362)/fibonacci(361)
True
```

sin()

Return the sine of `self`.

EXAMPLES:

```
sage: R = RealIntervalField(100)
sage: R(2).sin()
0.90929742682568169539601986591 2?
```

sinh()

Return the hyperbolic sine of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/12
sage: q.sinh()
0.2648002276022707?
```

sqrt()

Return a square root of `self`. Raises an error if `self` is nonpositive.

If you use `square_root()` then an interval will always be returned (though it will be `NaN` if self is nonpositive).

EXAMPLES:
```
sage: r = RIF(4.0)
sage: r.sqrt()
2
sage: r.sqrt()^2 == r
True

sage: r = RIF(4344)
sage: r.sqrt()
65.90902821313633?
sage: r.sqrt()^2 == r
False
sage: r in r.sqrt()^2
True
sage: r.sqrt()^2 - r
0.?e-11
sage: (r.sqrt()^2 - r).str(style='brackets')
'[-9.0949470177292824e-13 .. 1.8189894035458565e-12]'

sage: r = RIF(-2.0)
sage: r.sqrt()
Traceback (most recent call last):
...
ValueError: self (=-2) is not >= 0

sage: r = RIF(-2, 2)
sage: r.sqrt()
Traceback (most recent call last):
...
ValueError: self (=0.?e1) is not >= 0
```

square()

Return the square of `self`.

Note: Squaring an interval is different than multiplying it by itself, because the square can never be negative.

EXAMPLES:
```
sage: RIF(1, 2).square().str(style='brackets')
'[1.0000000000000000 .. 4.0000000000000000]'
sage: RIF(-1, 1).square().str(style='brackets')
'[0.00000000000000000 .. 1.0000000000000000]'
sage: (RIF(-1, 1) * RIF(-1, 1)).str(style='brackets')
'[-1.0000000000000000 .. 1.0000000000000000]'
```

square_root()

Return a square root of `self`. An interval will always be returned (though it will be `NaN` if self is nonpositive).

EXAMPLES:

```
sage: r = RIF(-2.0)
sage: r.square_root()
[.. NaN ..]
sage: r.sqrt()
Traceback (most recent call last):
...
ValueError: self (=-2) is not >= 0
```

str(*base=10*, *style=None*, *no_sci=None*, *e=None*, *error_digits=None*)

Return a string representation of `self`.

INPUT:

- `base` – base for output

- `style` – The printing style; either `'brackets'` or `'question'` (or `None`, to use the current default).

- `no_sci` – if `True` do not print using scientific notation; if `False` print with scientific notation; if `None` (the default), print how the parent prints.

- `e` – symbol used in scientific notation

- `error_digits` – The number of digits of error to print, in `'question'` style.

We support two different styles of printing; `'question'` style and `'brackets'` style. In question style (the default), we print the "known correct" part of the number, followed by a question mark:

```
sage: RIF(pi).str()
'3.141592653589794?'
sage: RIF(pi, 22/7).str()
'3.142?'
sage: RIF(pi, 22/7).str(style='question')
'3.142?'
```

However, if the interval is precisely equal to some integer that's not too large, we just return that integer:

```
sage: RIF(-42).str()
'-42'
sage: RIF(0).str()
'0'
sage: RIF(12^5).str(base=3)
'110122100000'
```

Very large integers, however, revert to the normal question-style printing:

```
sage: RIF(3^7).str()
'2187'
sage: RIF(3^7 * 2^256).str()
'2.5323729916201052?e80'
```

In brackets style, we print the lower and upper bounds of the interval within brackets:

```
sage: RIF(237/16).str(style='brackets')
'[14.812500000000000 .. 14.812500000000000]'
```

Note that the lower bound is rounded down, and the upper bound is rounded up. So even if the lower and upper bounds are equal, they may print differently. (This is done so that the printed representation of the interval contains all the numbers in the internal binary interval.)

For instance, we find the best 10-bit floating point representation of `1/3`:

```
sage: RR10 = RealField(10)
sage: RR(RR10(1/3))
0.333496093750000
```

And we see that the point interval containing only this floating-point number prints as a wider decimal interval, that does contain the number:

```
sage: RIF10 = RealIntervalField(10)
sage: RIF10(RR10(1/3)).str(style='brackets')
'[0.33349 .. 0.33350]'
```

We always use brackets style for `NaN` and infinities:

```
sage: RIF(pi, infinity)
[3.1415926535897931 .. +infinity]
sage: RIF(NaN)
[.. NaN ..]
```

Let's take a closer, formal look at the question style. In its full generality, a number printed in the question style looks like:

MANTISSA ?ERROR eEXPONENT

(without the spaces). The "eEXPONENT" part is optional; if it is missing, then the exponent is 0. (If the base is greater than 10, then the exponent separator is "@" instead of "e".)

The "ERROR" is optional; if it is missing, then the error is 1.

The mantissa is printed in base b, and always contains a decimal point (also known as a radix point, in bases other than 10). (The error and exponent are always printed in base 10.)

We define the "precision" of a floating-point printed representation to be the positional value of the last digit of the mantissa. For instance, in `2.7?e5`, the precision is 10^4; in `8.?`, the precision is 10^0; and in `9.35?` the precision is 10^{-2}. This precision will always be 10^k for some k (or, for an arbitrary base b, b^k).

Then the interval is contained in the interval:

$$\text{mantissa} \cdot b^{\text{exponent}} - \text{error} \cdot b^k .. \text{mantissa} \cdot b^{\text{exponent}} + \text{error} \cdot b^k$$

To control the printing, we can specify a maximum number of error digits. The default is 0, which means that we do not print an error at all (so that the error is always the default, 1).

Now, consider the precisions needed to represent the endpoints (this is the precision that would be produced by `v.lower().str(no_sci=False, truncate=False)`). Our result is no more precise than the less precise endpoint, and is sufficiently imprecise that the error can be represented with the given number of decimal digits. Our result is the most precise possible result, given these restrictions. When there are two possible results of equal precision and with the same error width, then we pick the one which is farther from zero. (For instance, `RIF(0, 123)` with two error digits could print as `61.?62` or `62.?62`. We prefer the latter because it makes it clear that the interval is known not to be negative.)

EXAMPLES:

```
sage: a = RIF(59/27); a
2.185185185185186?
sage: a.str()
'2.185185185185186?'
sage: a.str(style='brackets')
'[2.1851851851851851 .. 2.1851851851851856]'
sage: a.str(16)
'2.2f684bda12f69?'
```

```
sage: a.str(no_sci=False)
'2.185185185185186?e0'
sage: pi_appr = RIF(pi, 22/7)
sage: pi_appr.str(style='brackets')
'[3.1415926535897931 .. 3.1428571428571433]'
sage: pi_appr.str()
'3.142?'
sage: pi_appr.str(error_digits=1)
'3.1422?7'
sage: pi_appr.str(error_digits=2)
'3.14223?64'
sage: pi_appr.str(base=36)
'3.6?'
sage: RIF(NaN)
[.. NaN ..]
sage: RIF(pi, infinity)
[3.1415926535897931 .. +infinity]
sage: RIF(-infinity, pi)
[-infinity .. 3.1415926535897936]
sage: RealIntervalField(210)(3).sqrt()
1.732050807568877293527446341505872366942805253810380628055806980?
sage: RealIntervalField(210)(RIF(3).sqrt())
1.732050807568878?
sage: RIF(3).sqrt()
1.732050807568878?
sage: RIF(0, 3^-150)
1.?e-71
```

TESTS:

Check that [trac ticket #13634](#) is fixed:

```
sage: RIF(0.025)
0.025000000000000002?
sage: RIF.scientific_notation(True)
sage: RIF(0.025)
2.5000000000000002?e-2
sage: RIF.scientific_notation(False)
sage: RIF(0.025)
0.025000000000000002?
```

tan()

Return the tangent of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/3
sage: q.tan()
1.732050807568877?
sage: q = RIF.pi()/6
sage: q.tan()
0.577350269189626?
```

tanh()

Return the hyperbolic tangent of `self`.

EXAMPLES:

```
sage: q = RIF.pi()/11
sage: q.tanh()
```

```
0.2780794292958503?
```

union(*other*)

Return the union of two intervals, or of an interval and a real number (more precisely, the convex hull).

EXAMPLES:

```
sage: RIF(1, 2).union(RIF(pi, 22/7)).str(style='brackets')
'[1.0000000000000000 .. 3.1428571428571433]'
sage: RIF(1, 2).union(pi).str(style='brackets')
'[1.0000000000000000 .. 3.1415926535897936]'
sage: RIF(1).union(RIF(0, 2)).str(style='brackets')
'[0.00000000000000000 .. 2.0000000000000000]'
sage: RIF(1).union(RIF(-1)).str(style='brackets')
'[-1.0000000000000000 .. 1.0000000000000000]'
```

unique_ceil()

Returns the unique ceiling of this interval, if it is well defined, otherwise raises a `ValueError`.

OUTPUT:

•an integer.

EXAMPLES:

```
sage: RIF(pi).unique_ceil()
4
sage: RIF(100*pi).unique_ceil()
315
sage: RIF(100, 200).unique_ceil()
Traceback (most recent call last):
...
ValueError: interval does not have a unique ceil
```

unique_floor()

Returns the unique floor of this interval, if it is well defined, otherwise raises a `ValueError`.

OUTPUT:

•an integer.

EXAMPLES:

```
sage: RIF(pi).unique_floor()
3
sage: RIF(100*pi).unique_floor()
314
sage: RIF(100, 200).unique_floor()
Traceback (most recent call last):
...
ValueError: interval does not have a unique floor
```

unique_integer()

Return the unique integer in this interval, if there is exactly one, otherwise raises a `ValueError`.

EXAMPLES:

```
sage: RIF(pi).unique_integer()
Traceback (most recent call last):
...
ValueError: interval contains no integer
sage: RIF(pi, pi+1).unique_integer()
```

```
4
sage: RIF(pi, pi+2).unique_integer()
Traceback (most recent call last):
...
ValueError: interval contains more than one integer
sage: RIF(100).unique_integer()
100
```

unique_round()

> Returns the unique round (nearest integer) of this interval, if it is well defined, otherwise raises a
> ValueError.
>
> OUTPUT:
>
> > •an integer.
>
> EXAMPLES:

```
sage: RIF(pi).unique_round()
3
sage: RIF(1000*pi).unique_round()
3142
sage: RIF(100, 200).unique_round()
Traceback (most recent call last):
...
ValueError: interval does not have a unique round (nearest integer)
sage: RIF(1.2, 1.7).unique_round()
Traceback (most recent call last):
...
ValueError: interval does not have a unique round (nearest integer)
sage: RIF(0.7, 1.2).unique_round()
1
sage: RIF(-pi).unique_round()
-3
sage: (RIF(4.5).unique_round(), RIF(-4.5).unique_round())
(5, -5)
```

> TESTS:
```
sage: RIF(-1/2, -1/3).unique_round()
Traceback (most recent call last):
...
ValueError: interval does not have a unique round (nearest integer)
sage: RIF(-1/2, 1/3).unique_round()
Traceback (most recent call last):
...
ValueError: interval does not have a unique round (nearest integer)
sage: RIF(-1/3, 1/3).unique_round()
0
sage: RIF(-1/2, 0).unique_round()
Traceback (most recent call last):
...
ValueError: interval does not have a unique round (nearest integer)
sage: RIF(1/2).unique_round()
1
sage: RIF(-1/2).unique_round()
-1
```

```
sage: RIF(0).unique_round()
0
```

upper (*rnd=None*)

Return the upper bound of `self`

INPUT:

- `rnd` – (string) the rounding mode

    - `'RNDN'` – round to nearest

    - `'RNDD'` – (default) round towards minus infinity

    - `'RNDZ'` – round towards zero

    - `'RNDU'` – round towards plus infinity

The rounding mode does not affect the value returned as a floating-point number, but it does control which variety of `RealField` the returned number is in, which affects printing and subsequent operations.

EXAMPLES:

```
sage: R = RealIntervalField(13)
sage: R.pi().upper().str(truncate=False)
'3.1417'

sage: R = RealIntervalField(13)
sage: x = R(1.2,1.3); x.str(style='brackets')
'[1.1999 .. 1.3001]'
sage: x.upper()
1.31
sage: x.upper('RNDU')
1.31
sage: x.upper('RNDN')
1.30
sage: x.upper('RNDD')
1.30
sage: x.upper('RNDZ')
1.30
sage: x.upper().parent()
Real Field with 13 bits of precision and rounding RNDU
sage: x.upper('RNDD').parent()
Real Field with 13 bits of precision and rounding RNDD
sage: x.upper() == x.upper('RNDD')
True
```

class `sage.rings.real_mpfi.`**`RealIntervalField_class`**

Bases: `sage.rings.ring.Field`

Class of the real interval field.

INPUT:

- `prec` – (integer) precision; default = 53 `prec` is the number of bits used to represent the mantissa of a floating-point number. The precision can be any integer between `mpfr_prec_min()` and `mpfr_prec_max()`. In the current implementation, `mpfr_prec_min()` is equal to 2.

- `sci_not` – (default: `False`) whether or not to display using scientific notation

EXAMPLES:

```
sage: RealIntervalField(10)
Real Interval Field with 10 bits of precision
sage: RealIntervalField()
Real Interval Field with 53 bits of precision
sage: RealIntervalField(100000)
Real Interval Field with 100000 bits of precision
```

Note: The default precision is 53, since according to the GMP manual: 'mpfr should be able to exactly reproduce all computations with double-precision machine floating-point numbers (double type in C), except the default exponent range is much wider and subnormal numbers are not implemented.'

EXAMPLES:

Creation of elements.

First with default precision. First we coerce elements of various types, then we coerce intervals:

```
sage: RIF = RealIntervalField(); RIF
Real Interval Field with 53 bits of precision
sage: RIF(3)
3
sage: RIF(RIF(3))
3
sage: RIF(pi)
3.141592653589794?
sage: RIF(RealField(53)('1.5'))
1.5000000000000000?
sage: RIF(-2/19)
-0.1052631578947369?
sage: RIF(-3939)
-3939
sage: RIF(-3939r)
-3939
sage: RIF('1.5')
1.5000000000000000?
sage: R200 = RealField(200)
sage: RIF(R200.pi())
3.141592653589794?
```

The base must be explicitly specified as a named parameter:

```
sage: RIF('101101', base=2)
45
sage: RIF('+infinity')
[+infinity .. +infinity]
sage: RIF('[1..3]').str(style='brackets')
'[1.0000000000000000 .. 3.0000000000000000]'
```

Next we coerce some 2-tuples, which define intervals:

```
sage: RIF((-1.5, -1.3))
-1.4?
sage: RIF((RDF('-1.5'), RDF('-1.3')))
-1.4?
sage: RIF((1/3,2/3)).str(style='brackets')
'[0.33333333333333331 .. 0.66666666666666675]'
```

The extra parentheses aren't needed:

```
sage: RIF(1/3,2/3).str(style='brackets')
'[0.33333333333333331 .. 0.66666666666666675]'
sage: RIF((1,2)).str(style='brackets')
'[1.0000000000000000 .. 2.0000000000000000]'
sage: RIF((1r,2r)).str(style='brackets')
'[1.0000000000000000 .. 2.0000000000000000]'
sage: RIF((pi, e)).str(style='brackets')
'[2.7182818284590455 .. 3.1415926535897932]'
```

Values which can be represented as an exact floating-point number (of the precision of this `RealIntervalField`) result in a precise interval, where the lower bound is equal to the upper bound (even if they print differently). Other values typically result in an interval where the lower and upper bounds are adjacent floating-point numbers.

```
sage: def check(x):
...         return (x, x.lower() == x.upper())
sage: check(RIF(pi))
(3.141592653589794?, False)
sage: check(RIF(RR(pi)))
(3.1415926535897932?, True)
sage: check(RIF(1.5))
(1.5000000000000000?, True)
sage: check(RIF('1.5'))
(1.5000000000000000?, True)
sage: check(RIF(0.1))
(0.10000000000000001?, True)
sage: check(RIF(1/10))
(0.10000000000000000?, False)
sage: check(RIF('0.1'))
(0.10000000000000000?, False)
```

Similarly, when specifying both ends of an interval, the lower end is rounded down and the upper end is rounded up:

```
sage: outward = RIF(1/10, 7/10); outward.str(style='brackets')
'[0.099999999999999991 .. 0.70000000000000007]'
sage: nearest = RIF(RR(1/10), RR(7/10)); nearest.str(style='brackets')
'[0.10000000000000000 .. 0.69999999999999996]'
sage: nearest.lower() - outward.lower()
1.38777878078144e-17
sage: outward.upper() - nearest.upper()
1.11022302462516e-16
```

Some examples with a real interval field of higher precision:

```
sage: R = RealIntervalField(100)
sage: R(3)
3
sage: R(R(3))
3
sage: R(pi)
3.14159265358979323846264338328?
sage: R(-2/19)
-0.105263157894736842105263157948?
sage: R(e,pi).str(style='brackets')
'[2.71828182845904523536028747135 .. 3.14159265358979323846264338325]'
```

TESTS:

```
sage: RIF._lower_field() is RealField(53, rnd='RNDD')
True
sage: RIF._upper_field() is RealField(53, rnd='RNDU')
True
sage: RIF._middle_field() is RR
True
sage: TestSuite(RIF).run()
```

characteristic()

Returns 0, since the field of real numbers has characteristic 0.

EXAMPLES:
```
sage: RealIntervalField(10).characteristic()
0
```

complex_field()

Return complex field of the same precision.

EXAMPLES:
```
sage: RIF.complex_field()
Complex Interval Field with 53 bits of precision
```

construction()

Returns the functorial construction of `self`, namely, completion of the rational numbers with respect to the prime at ∞, and the note that this is an interval field.

Also preserves other information that makes this field unique (e.g. precision, print mode).

EXAMPLES:
```
sage: R = RealIntervalField(123)
sage: c, S = R.construction(); S
Rational Field
sage: R == c(S)
True
```

euler_constant()

Returns Euler's gamma constant to the precision of this field.

EXAMPLES:
```
sage: RealIntervalField(100).euler_constant()
0.57721566490153286060651209008?
```

gen(*i=0*)

Return the `i`-th generator of `self`.

EXAMPLES:
```
sage: RIF.gen(0)
1
sage: RIF.gen(1)
Traceback (most recent call last):
...
IndexError: self has only one generator
```

gens()

Return a list of generators.

EXAMPLE:

```
sage: RIF.gens()
[1]
```

is_exact()

Returns whether or not this field is exact, which is always `False`.

EXAMPLES:

```
sage: RIF.is_exact()
False
```

is_finite()

Return `False`, since the field of real numbers is not finite.

EXAMPLES:

```
sage: RealIntervalField(10).is_finite()
False
```

log2()

Returns $\log(2)$ to the precision of this field.

EXAMPLES:

```
sage: R=RealIntervalField(100)
sage: R.log2()
0.6931471805599453094172321214582
sage: R(2).log()
0.6931471805599453094172321214582
```

name()

Return the name of `self`.

EXAMPLES:

```
sage: RIF.name()
'IntervalRealIntervalField53'
sage: RealIntervalField(200).name()
'IntervalRealIntervalField200'
```

ngens()

Return the number of generators of `self`, which is 1.

EXAMPLES:

```
sage: RIF.ngens()
1
```

pi()

Returns π to the precision of this field.

EXAMPLES:

```
sage: R = RealIntervalField(100)
sage: R.pi()
3.141592653589793238462643383282
sage: R.pi().sqrt()/2
0.8862269254527580136490837416?
sage: R = RealIntervalField(150)
sage: R.pi().sqrt()/2
0.8862269254527580136490837416705725913987747282
```

prec()

> Return the precision of this field (in bits).
>
> EXAMPLES:
> ```
> sage: RIF.precision()
> 53
> sage: RealIntervalField(200).precision()
> 200
> ```

precision()

> Return the precision of this field (in bits).
>
> EXAMPLES:
> ```
> sage: RIF.precision()
> 53
> sage: RealIntervalField(200).precision()
> 200
> ```

random_element(**args*, ***kwds*)

> Return a random element of `self`. Any arguments or keywords are passed onto the random element function in real field.
>
> By default, this is uniformly distributed in $[-1, 1]$.
>
> EXAMPLES:
> ```
> sage: RIF.random_element()
> 0.15363619378561300?
> sage: RIF.random_element()
> -0.50298737524751780?
> sage: RIF.random_element(-100, 100)
> 60.958996432224126?
> ```
>
> Passes extra positional or keyword arguments through:
> ```
> sage: RIF.random_element(min=0, max=100)
> 2.5572702830891970?
> sage: RIF.random_element(min=-100, max=0)
> -1.5803457307118123?
> ```

scientific_notation(*status=None*)

> Set or return the scientific notation printing flag.
>
> If this flag is `True` then real numbers with this space as parent print using scientific notation.
>
> INPUT:
>
> > •`status` – boolean optional flag
>
> EXAMPLES:
> ```
> sage: RIF(0.025)
> 0.025000000000000002?
> sage: RIF.scientific_notation(True)
> sage: RIF(0.025)
> 2.5000000000000002?e-2
> sage: RIF.scientific_notation(False)
> sage: RIF(0.025)
> 0.025000000000000002?
> ```

to_prec(*prec*)

    Returns a real interval field to the given precision.

    EXAMPLES:

```
sage: RIF.to_prec(200)
Real Interval Field with 200 bits of precision
sage: RIF.to_prec(20)
Real Interval Field with 20 bits of precision
sage: RIF.to_prec(53) is RIF
True
```

zeta(*n=2*)

    Return an n-th root of unity in the real field, if one exists, or raise a `ValueError` otherwise.

    EXAMPLES:

```
sage: R = RealIntervalField()
sage: R.zeta()
-1
sage: R.zeta(1)
1
sage: R.zeta(5)
Traceback (most recent call last):
...
ValueError: No 5th root of unity in self
```

sage.rings.real_mpfi.**is_RealIntervalField**(*x*)

    Check if x is a `RealIntervalField_class`.

    EXAMPLES:

```
sage: sage.rings.real_mpfi.is_RealIntervalField(RIF)
True
sage: sage.rings.real_mpfi.is_RealIntervalField(RealIntervalField(200))
True
```

sage.rings.real_mpfi.**is_RealIntervalFieldElement**(*x*)

    Check if x is a `RealIntervalFieldElement`.

    EXAMPLES:

```
sage: sage.rings.real_mpfi.is_RealIntervalFieldElement(RIF(2.2))
True
sage: sage.rings.real_mpfi.is_RealIntervalFieldElement(RealIntervalField(200)(2.2))
True
```

FIELD OF ARBITRARY PRECISION REAL NUMBER INTERVALS

sage.rings.real_interval_field.**is_RealIntervalField**(*x*)

    Check if x is a `RealIntervalField_class`.

    EXAMPLES:

```
sage: from sage.rings.real_interval_field import is_RealIntervalField as is_RIF
sage: is_RIF(RIF)
True
```

sage.rings.real_interval_field.**is_RealIntervalFieldElement**(*x*)

    Check if x is a `RealIntervalFieldElement`.

    EXAMPLES:

```
sage: from sage.rings.real_interval_field import is_RealIntervalFieldElement as is_RIFE
sage: is_RIFE(RIF(2.5))
True
```

FILE: SAGE/RINGS/REAL_INTERVAL_ABSOLUTE.PYX (STARTING AT LINE 1)

class sage.rings.real_interval_absolute.**Factory**

Bases: sage.structure.factory.UniqueFactory

INPUT:

> •name – string. A name in the global namespace referring to self or a fully qualified path name to self, which is used to locate the factory on unpickling.

EXAMPLES:

```
sage: from sage.structure.factory import UniqueFactory
sage: fake_factory = UniqueFactory('ZZ')
sage: loads(dumps(fake_factory))
Integer Ring
sage: fake_factory = UniqueFactory('sage.rings.all.QQ')
sage: loads(dumps(fake_factory))
Rational Field
```

create_key(*prec*)

The only piece of data is the precision.

TESTS:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: RealIntervalAbsoluteField.create_key(1000)
1000
```

create_object(*version*, *prec*)

Ensures uniqueness.

TESTS:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: RealIntervalAbsoluteField(23) is RealIntervalAbsoluteField(23) # indirect doctest
True
```

class sage.rings.real_interval_absolute.**MpfrOp**

Bases: object

This class is used to endow absolute real interval field elements with all the methods of (relative) real interval field elements.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(100)
```

```
sage: R(1).sin()
0.841470984807896506652502321631?
```

class sage.rings.real_interval_absolute.**RealIntervalAbsoluteElement**

Bases: sage.structure.element.FieldElement

Create a RealIntervalAbsoluteElement.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(50)
sage: R(1)
1
sage: R(1/3)
0.333333333333334?
sage: R(1.3)
1.300000000000000?
sage: R(pi)
3.141592653589794?
sage: R((11, 12))
12.?
sage: R((11, 11.00001))
11.00001?

sage: R100 = RealIntervalAbsoluteField(100)
sage: R(R100((5,6)))
6.?
sage: R100(R((5,6)))
6.?
```

abs()

Return the absolute value of self.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(100)
sage: R(1/3).abs()
0.3333333333333333333333333333334?
sage: R(-1/3).abs()
0.3333333333333333333333333333334?
sage: R((-1/3, 1/2)).abs()
1.?
sage: R((-1/3, 1/2)).abs().endpoints()
(0, 1/2)
sage: R((-3/2, 1/2)).abs().endpoints()
(0, 3/2)
```

absolute_diameter()

Return the diameter self.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10)
sage: R(1/4).absolute_diameter()
0
sage: a = R(pi)
sage: a.absolute_diameter()
1/1024
```

```
sage: a.upper() - a.lower()
1/1024
```

contains_zero()

Return whether `self` contains zero.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10)
sage: R(10).contains_zero()
False
sage: R((10,11)).contains_zero()
False
sage: R((0,11)).contains_zero()
True
sage: R((-10,11)).contains_zero()
True
sage: R((-10,-1)).contains_zero()
False
sage: R((-10,0)).contains_zero()
True
sage: R(pi).contains_zero()
False
```

diameter()

Return the diameter `self`.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10)
sage: R(1/4).absolute_diameter()
0
sage: a = R(pi)
sage: a.absolute_diameter()
1/1024
sage: a.upper() - a.lower()
1/1024
```

endpoints()

Return the left and right endpoints of `self`, as a tuple.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10)
sage: R(1/4).endpoints()
(1/4, 1/4)
sage: R((1,2)).endpoints()
(1, 2)
```

is_negative()

Return whether `self` is definitely negative.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(100)
sage: R(10).is_negative()
False
```

```
sage: R((10,11)).is_negative()
False
sage: R((0,11)).is_negative()
False
sage: R((-10,11)).is_negative()
False
sage: R((-10,-1)).is_negative()
True
sage: R(pi).is_negative()
False
```

is_positive()

Return whether `self` is definitely positive.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10)
sage: R(10).is_positive()
True
sage: R((10,11)).is_positive()
True
sage: R((0,11)).is_positive()
False
sage: R((-10,11)).is_positive()
False
sage: R((-10,-1)).is_positive()
False
sage: R(pi).is_positive()
True
```

lower()

Return the lower bound of `self`.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(50)
sage: R(1/4).lower()
1/4
```

midpoint()

Return the midpoint of `self`.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(100)
sage: R(1/4).midpoint()
1/4
sage: R(pi).midpoint()
7964883625991394727376702227905/2535301200456458802993406410752
sage: R(pi).midpoint().n()
3.14159265358979
```

mpfi_prec()

Return the precision needed to represent this value as an mpfi interval.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10)
sage: R(10).mpfi_prec()
14
sage: R(1000).mpfi_prec()
20
```

sqrt()

Return the square root of `self`.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(100)
sage: R(2).sqrt()
1.41421356237309504880168872421?
sage: R((4,9)).sqrt().endpoints()
(2, 3)
```

upper()

Return the upper bound of `self`.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(50)
sage: R(1/4).upper()
1/4
```

sage.rings.real_interval_absolute.**RealIntervalAbsoluteField**(*args*, **kwds*)

This field is similar to the `RealIntervalField` except instead of truncating everything to a fixed relative precision, it maintains a fixed absolute precision.

Note that unlike the standard real interval field, elements in this field can have different size and experience coefficient blowup. On the other hand, it avoids precision loss on addition and subtraction. This is useful for, e.g., series computations for special functions.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10); R
Real Interval Field with absolute precision 2^-10
sage: R(3/10)
0.300?
sage: R(1000003/10)
100000.300?
sage: R(1e100) + R(1) - R(1e100)
1
```

class sage.rings.real_interval_absolute.**RealIntervalAbsoluteField_class**

Bases: sage.rings.ring.Field

This field is similar to the `RealIntervalField` except instead of truncating everything to a fixed relative precision, it maintains a fixed absolute precision.

Note that unlike the standard real interval field, elements in this field can have different size and experience coefficient blowup. On the other hand, it avoids precision loss on addition and subtraction. This is useful for, e.g., series computations for special functions.

EXAMPLES:

```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(10); R
Real Interval Field with absolute precision 2^-10
sage: R(3/10)
0.300?
sage: R(1000003/10)
100000.300?
sage: R(1e100) + R(1) - R(1e100)
1
```

absprec()

Returns the absolute precision of self.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import RealIntervalAbsoluteField
sage: R = RealIntervalAbsoluteField(100)
sage: R.absprec()
100
sage: RealIntervalAbsoluteField(5).absprec()
5
```

sage.rings.real_interval_absolute.**shift_ceil**(*x*, *shift*)

Return $x/2^s$ where s is the value of shift, rounded towards $+\infty$. For internal use.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import shift_ceil
sage: shift_ceil(15, 2)
4
sage: shift_ceil(-15, 2)
-3
sage: shift_ceil(32, 2)
8
sage: shift_ceil(-32, 2)
-8
```

sage.rings.real_interval_absolute.**shift_floor**(*x*, *shift*)

Return $x/2^s$ where s is the value of shift, rounded towards $-\infty$. For internal use.

EXAMPLES:
```
sage: from sage.rings.real_interval_absolute import shift_floor
sage: shift_floor(15, 2)
3
sage: shift_floor(-15, 2)
-4
```

THESE CLASSES ARE VERY LAZY, IN THE SENSE THAT IT DOESN'T REALLY DO ANYTHING

but simply sits between exact rings of characteristic 0 and the real numbers. The values are actually computed when they are cast into a field of fixed precision.

The main purpose of these classes is to provide a place for exact rings (e.g. number fields) to embed for the coercion model (as only one embedding can be specified in the forward direction).

sage.rings.real_lazy.**ComplexLazyField**()
>   Returns the lazy complex field.
>
>   EXAMPLES:
>
>   There is only one lazy complex field:
>   ```
>   sage: ComplexLazyField() is ComplexLazyField()
>   True
>   ```

class sage.rings.real_lazy.**ComplexLazyField_class**
>   Bases: sage.rings.real_lazy.LazyField
>
>   This class represents the set of complex numbers to unspecified precision. For the most part it simply wraps exact elements and defers evaluation until a specified precision is requested.
>
>   For more information, see the documentation of the RLF.
>
>   EXAMPLES:
>   ```
>   sage: a = CLF(-1).sqrt()
>   sage: a
>   1*I
>   sage: CDF(a)
>   1.0*I
>   sage: ComplexField(200)(a)
>   1.0000000000000000000000000000000000000000000000000000000000*I
>   ```
>
>   TESTS:
>   ```
>   sage: TestSuite(CLF).run(skip=["_test_prod"])
>   ```
>
>   ---
>
>   **Note:** The following TestSuite failure:
>   ```
>   sage: CLF._test_prod()
>   Traceback (most recent call last):
>   ...
>   AssertionError: False is not true
>   ```
>
>   is due to (acceptable?) numerical noise:

```
sage: x = CLF.I
sage: x*x == x^2
False
sage: x*x
-1
sage: x^2
-0.9999999999999999? + 0.?e-15*I
```

construction()

> Returns the functorial construction of `self`, namely, algebraic closure of the real lazy field.
>
> EXAMPLES:
> ```
> sage: c, S = CLF.construction(); S
> Real Lazy Field
> sage: CLF == c(S)
> True
> ```

gen(*i=0*)

> Return the i-th generator of `self`.
>
> EXAMPLES:
> ```
> sage: CLF.gen()
> 1*I
> sage: ComplexField(100)(CLF.gen())
> 1.0000000000000000000000000000*I
> ```

interval_field(*prec=None*)

> Returns the interval field that represents the same mathematical field as `self`.
>
> EXAMPLES:
> ```
> sage: CLF.interval_field()
> Complex Interval Field with 53 bits of precision
> sage: CLF.interval_field(333)
> Complex Interval Field with 333 bits of precision
> sage: CLF.interval_field() is CIF
> True
> ```

class sage.rings.real_lazy.**LazyAlgebraic**

> Bases: `sage.rings.real_lazy.LazyFieldElement`
>
> This represents an algebraic number, specified by a polynomial over **Q** and a real or complex approximation.
>
> EXAMPLES:
> ```
> sage: x = polygen(QQ)
> sage: from sage.rings.real_lazy import LazyAlgebraic
> sage: a = LazyAlgebraic(RLF, x^2-2, 1.5)
> sage: a
> 1.414213562373095?
> ```

> **eval**(*R*)
>
> > Convert `self` into an element of R.
> >
> > EXAMPLES:
> > ```
> > sage: from sage.rings.real_lazy import LazyAlgebraic
> > sage: a = LazyAlgebraic(CLF, QQ['x'].cyclotomic_polynomial(7), 0.6+0.8*CC.0)
> > sage: a
> > ```

```
0.6234898018587335? + 0.7818314824680299?*I
sage: ComplexField(150)(a) # indirect doctest
0.62348980185873353052500488400423981063227473 + 0.78183148246802980870844452667405775023233
```

```
sage: a = LazyAlgebraic(CLF, QQ['x'].0^2-7, -2.0)
sage: RR(a)
-2.64575131106459
sage: RR(a)^2
7.00000000000000
```

class sage.rings.real_lazy.**LazyBinop**

Bases: `sage.rings.real_lazy.LazyFieldElement`

A lazy element representing a binary (usually arithmetic) operation between two other lazy elements.

EXAMPLES:
```
sage: from sage.rings.real_lazy import LazyBinop
sage: a = LazyBinop(RLF, 2, 1/3, operator.add)
sage: a
2.333333333333334?
sage: Reals(200)(a)
2.3333333333333333333333333333333333333333333333333333333333333
```

> **depth**()
>
> Return the depth of `self` as an arithmetic expression.
>
> This is the maximum number of dependent intermediate expressions when evaluating `self`, and is used to determine the precision needed to get the final result to the desired number of bits.
>
> It is equal to the maximum of the right and left depths, plus one.
>
> EXAMPLES:
> ```
> sage: from sage.rings.real_lazy import LazyBinop
> sage: a = LazyBinop(RLF, 6, 8, operator.mul)
> sage: a.depth()
> 1
> sage: b = LazyBinop(RLF, 2, a, operator.sub)
> sage: b.depth()
> 2
> ```

> **eval**(*R*)
>
> Convert the operands to elements of `R`, then perform the operation on them.
>
> EXAMPLES:
> ```
> sage: from sage.rings.real_lazy import LazyBinop
> sage: a = LazyBinop(RLF, 6, 8, operator.add)
> sage: a.eval(RR)
> 14.0000000000000
> ```
>
> A bit absurd:
> ```
> sage: a.eval(str)
> '68'
> ```

class sage.rings.real_lazy.**LazyConstant**

Bases: `sage.rings.real_lazy.LazyFieldElement`

This class represents a real or complex constant (such as `pi` or `I`).

TESTS:
```
sage: a = RLF.pi(); a
3.141592653589794?
sage: RealField(300)(a)
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482
```
```
sage: from sage.rings.real_lazy import LazyConstant
sage: a = LazyConstant(RLF, 'euler_constant')
sage: RealField(200)(a)
0.57721566490153286060651209008240243104215933593992359880577
```

eval(*R*)

> Convert `self` into an element of `R`.
>
> EXAMPLES:
> ```
> sage: from sage.rings.real_lazy import LazyConstant
> sage: a = LazyConstant(RLF, 'e')
> sage: RDF(a) # indirect doctest
> 2.718281828459045
> sage: a = LazyConstant(CLF, 'I')
> sage: CC(a)
> 1.00000000000000*I
> ```

class `sage.rings.real_lazy.`**`LazyField`**

> Bases: `sage.rings.ring.Field`
>
> The base class for lazy real fields.

> ---
> **Warning:** LazyField uses `__getattr__`(), to implement:
> ```
> sage: CLF.pi
> 3.141592653589794?
> ```
>
> I (NT, 20/04/2012) did not manage to have `__getattr__` call `Parent.__getattr__`() in case of failure; hence we can't use this `__getattr__` trick for extension types to recover the methods from categories. Therefore, at this point, no concrete subclass of this class should be an extension type (which is probably just fine):
> ```
> sage: RLF.__class__
> <class 'sage.rings.real_lazy.RealLazyField_class_with_category'>
> sage: CLF.__class__
> <class 'sage.rings.real_lazy.ComplexLazyField_class_with_category'>
> ```
> ---

algebraic_closure()

> Returns the algebraic closure of `self`, i.e., the complex lazy field.
>
> EXAMPLES:
> ```
> sage: RLF.algebraic_closure()
> Complex Lazy Field
> ```
> ```
> sage: CLF.algebraic_closure()
> Complex Lazy Field
> ```

interval_field(*prec=None*)

> Abstract method to create the corresponding interval field.

TESTS:
```
sage: RLF.interval_field() # indirect doctest
Real Interval Field with 53 bits of precision
```

class sage.rings.real_lazy.**LazyFieldElement**
> Bases: sage.structure.element.FieldElement

> INPUT:

>> •parent - a SageObject

> **approx**()
>> Returns self as an element of an interval field.

>> EXAMPLES:
>> ```
>> sage: CLF(1/6).approx()
>> 0.1666666666666667?
>> sage: CLF(1/6).approx().parent()
>> Complex Interval Field with 53 bits of precision
>> ```

>> When the absolute value is involved, the result might be real:
>> ```
>> sage: z = exp(CLF(1 + I/2)); z
>> 2.38551673095914? + 1.303213729686996?*I
>> sage: r = z.abs(); r
>> 2.71828182845905?
>> sage: parent(z.approx())
>> Complex Interval Field with 53 bits of precision
>> sage: parent(r.approx())
>> Real Interval Field with 53 bits of precision
>> ```

> **depth**()
>> Abstract method for returning the depth of self as an arithmetic expression.

>> This is the maximum number of dependent intermediate expressions when evaluating self, and is used to determine the precision needed to get the final result to the desired number of bits.

>> It is equal to the maximum of the right and left depths, plus one.

>> EXAMPLES:
>> ```
>> sage: from sage.rings.real_lazy import LazyBinop
>> sage: a = LazyBinop(RLF, 6, 8, operator.mul)
>> sage: a.depth()
>> 1
>> ```

> **eval**(*R*)
>> Abstract method for converting self into an element of R.

>> EXAMPLES:
>> ```
>> sage: a = RLF(12)
>> sage: a.eval(ZZ)
>> 12
>> ```

class sage.rings.real_lazy.**LazyNamedUnop**
> Bases: sage.rings.real_lazy.LazyUnop

> This class is used to represent the many named methods attached to real numbers, and is instantiated by the __getattr__ method of LazyElements.

> EXAMPLES:

```
sage: from sage.rings.real_lazy import LazyNamedUnop
sage: a = LazyNamedUnop(RLF, 1, 'arcsin')
sage: RR(a)
1.57079632679490
sage: a = LazyNamedUnop(RLF, 9, 'log', extra_args=(3,))
sage: RR(a)
2.00000000000000
```

approx()
> Does something reasonable with functions that are not defined on the interval fields.
>
> TESTS:
> ```
> sage: from sage.rings.real_lazy import LazyNamedUnop
> sage: LazyNamedUnop(RLF, 8, 'sqrt') # indirect doctest
> 2.828427124746190?
> ```

eval(*R*)
> Convert self into an element of R.
>
> TESTS:
> ```
> sage: from sage.rings.real_lazy import LazyNamedUnop
> sage: a = LazyNamedUnop(RLF, 4, 'sqrt')
> sage: RR(a) # indirect doctest
> 2.00000000000000
> sage: a.sqrt()
> 1.414213562373095?
> sage: RealField(212)(a)
> 2.00000000000000000000000000000000000000000000000000000000000000
> sage: float(a)
> 2.0
> ```
>
> Now for some extra arguments:
> ```
> sage: a = RLF(100)
> sage: a.log(10)
> 2
> sage: float(a.log(10))
> 2.0
> ```

class sage.rings.real_lazy.**LazyUnop**
> Bases: `sage.rings.real_lazy.LazyFieldElement`
>
> Represents a unevaluated single function of one variable.
>
> EXAMPLES:
> ```
> sage: from sage.rings.real_lazy import LazyUnop
> sage: a = LazyUnop(RLF, 3, sqrt); a
> 1.732050807568878?
> sage: a._arg
> 3
> sage: a._op
> <function sqrt at ...>
> sage: Reals(100)(a)
> 1.7320508075688772935274463415
> sage: Reals(100)(a)^2
> 3.0000000000000000000000000000
> ```

> **depth()**

Return the depth of `self` as an arithmetic expression.

This is the maximum number of dependent intermediate expressions when evaluating `self`, and is used to determine the precision needed to get the final result to the desired number of bits.

It is equal to one more than the depth of its operand.

EXAMPLES:

```
sage: from sage.rings.real_lazy import LazyUnop
sage: a = LazyUnop(RLF, 3, sqrt)
sage: a.depth()
1
sage: b = LazyUnop(RLF, a, sin)
sage: b.depth()
2
```

eval(*R*)

Convert `self` into an element of R.

EXAMPLES:

```
sage: from sage.rings.real_lazy import LazyUnop
sage: a = LazyUnop(RLF, 3, sqrt)
sage: a.eval(ZZ)
sqrt(3)
```

class `sage.rings.real_lazy.`**LazyWrapper**

Bases: `sage.rings.real_lazy.LazyFieldElement`

A lazy element that simply wraps an element of another ring.

EXAMPLES:

```
sage: from sage.rings.real_lazy import LazyWrapper
sage: a = LazyWrapper(RLF, 3)
sage: a._value
3
```

depth()

Returns the depth of `self` as an expression, which is always 0.

EXAMPLES:

```
sage: RLF(4).depth()
0
```

eval(*R*)

Convert `self` into an element of R.

EXAMPLES:

```
sage: a = RLF(12)
sage: a.eval(ZZ)
12
sage: a.eval(ZZ).parent()
Integer Ring
```

class `sage.rings.real_lazy.`**LazyWrapperMorphism**

Bases: `sage.categories.morphism.Morphism`

This morphism coerces elements from anywhere into lazy rings by creating a wrapper element (as fast as possible).

EXAMPLES:
```
sage: from sage.rings.real_lazy import LazyWrapperMorphism
sage: f = LazyWrapperMorphism(QQ, RLF)
sage: a = f(3); a
3
sage: type(a)
<type 'sage.rings.real_lazy.LazyWrapper'>
sage: a._value
3
sage: a._value.parent()
Rational Field
```

sage.rings.real_lazy.**RealLazyField**()
>   Return the lazy real field.
>
>   EXAMPLES:
>
>   There is only one lazy real field:
>    ```
>    sage: RealLazyField() is RealLazyField()
>    True
>    ```

class sage.rings.real_lazy.**RealLazyField_class**
>   Bases: sage.rings.real_lazy.LazyField
>
>   This class represents the set of real numbers to unspecified precision. For the most part it simply wraps exact elements and defers evaluation until a specified precision is requested.
>
>   Its primary use is to connect the exact rings (such as number fields) to fixed precision real numbers. For example, to specify an embedding of a number field K into $\mathbf{R}$ one can map into this field and the coercion will then be able to carry the mapping to real fields of any precision.
>
>   EXAMPLES:
>    ```
>    sage: a = RLF(1/3)
>    sage: a
>    0.3333333333333334?
>    sage: a + 1/5
>    0.5333333333333334?
>    sage: a = RLF(1/3)
>    sage: a
>    0.3333333333333334?
>    sage: a + 5
>    5.333333333333334?
>    sage: RealField(100)(a+5)
>    5.3333333333333333333333333333
>    ```
>
>   TESTS:
>    ```
>    sage: TestSuite(RLF).run()
>    ```
>
>   **construction**()
>   >   Returns the functorial construction of self, namely, the completion of the rationals at infinity to infinite precision.
>   >
>   >   EXAMPLES:
>    >    ```
>    >    sage: c, S = RLF.construction(); S
>    >    Rational Field
>    >    sage: RLF == c(S)
>    >    True
>    >    ```

gen(*i=0*)

Return the i-th generator of `self`.

EXAMPLES:

```
sage: RLF.gen()
1
```

interval_field(*prec=None*)

Returns the interval field that represents the same mathematical field as `self`.

EXAMPLES:

```
sage: RLF.interval_field()
Real Interval Field with 53 bits of precision
sage: RLF.interval_field(200)
Real Interval Field with 200 bits of precision
```

`sage.rings.real_lazy.`**make_element**(*parent*, **args*)

Create an element of `parent`.

EXAMPLES:

```
sage: a = RLF(pi) + RLF(sqrt(1/2)) # indirect doctest
sage: loads(dumps(a)) == a
True
```

DOUBLE PRECISION COMPLEX NUMBERS

Sage supports arithmetic using double-precision complex numbers. A double-precision complex number is a complex number x + I*y with x, y 64-bit (8 byte) floating point numbers (double precision).

The field `ComplexDoubleField` implements the field of all double-precision complex numbers. You can refer to this field by the shorthand CDF. Elements of this field are of type `ComplexDoubleElement`. If x and y are coercible to doubles, you can create a complex double element using ComplexDoubleElement(x,y). You can coerce more general objects z to complex doubles by typing either ComplexDoubleField(x) or CDF(x).

EXAMPLES:

```
sage: ComplexDoubleField()
Complex Double Field
sage: CDF
Complex Double Field
sage: type(CDF.0)
<type 'sage.rings.complex_double.ComplexDoubleElement'>
sage: ComplexDoubleElement(sqrt(2),3)
1.4142135623730951 + 3.0*I
sage: parent(CDF(-2))
Complex Double Field


sage: CC == CDF
False
sage: CDF is ComplexDoubleField()    # CDF is the shorthand
True
sage: CDF == ComplexDoubleField()
True
```

The underlying arithmetic of complex numbers is implemented using functions and macros in GSL (the GNU Scientific Library), and should be very fast. Also, all standard complex trig functions, log, exponents, etc., are implemented using GSL, and are also robust and fast. Several other special functions, e.g. eta, gamma, incomplete gamma, etc., are implemented using the PARI C library.

AUTHORS:

- William Stein (2006-09): first version

- Travis Scrimshaw (2012-10-18): Added doctests to get full coverage

- Jeroen Demeyer (2013-02-27): fixed all PARI calls (trac ticket #14082)

class sage.rings.complex_double.**ComplexDoubleElement**
    Bases: sage.structure.element.FieldElement

    An approximation to a complex number using double precision floating point numbers. Answers derived from calculations with such approximations may differ from what they would be if those calculations were performed

with true complex numbers. This is due to the rounding errors inherent to finite precision calculations.

abs()

This function returns the magnitude $|z|$ of the complex number z.

See also:

• norm()

EXAMPLES:
```
sage: CDF(2,3).abs()
3.605551275463989
```

abs2()

This function returns the squared magnitude $|z|^2$ of the complex number z, otherwise known as the complex norm.

See also:

• norm()

EXAMPLES:
```
sage: CDF(2,3).abs2()
13.0
```

agm(*right*, *algorithm='optimal'*)

Return the Arithmetic-Geometric Mean (AGM) of `self` and `right`.

INPUT:

• `right` (complex) – another complex number

• `algorithm` (string, default `"optimal"`) – the algorithm to use (see below).

OUTPUT:

(complex) A value of the AGM of self and right. Note that this is a multi-valued function, and the algorithm used affects the value returned, as follows:

• `'pari'`: Call the agm function from the pari library.

• `'optimal'`: Use the AGM sequence such that at each stage (a, b) is replaced by $(a_1, b_1) = ((a + b)/2, \pm\sqrt{ab})$ where the sign is chosen so that $|a_1 - b_1| \leq |a_1 + b_1|$, or equivalently $\Re(b_1/a_1) \geq 0$. The resulting limit is maximal among all possible values.

• `'principal'`: Use the AGM sequence such that at each stage (a, b) is replaced by $(a_1, b_1) = ((a + b)/2, \pm\sqrt{ab})$ where the sign is chosen so that $\Re(b_1/a_1) \geq 0$ (the so-called principal branch of the square root).

EXAMPLES:
```
sage: i = CDF(I)
sage: (1+i).agm(2-i)  # rel tol 1e-15
1.6278054848727064 + 0.1368275483973686*I
```

An example to show that the returned value depends on the algorithm parameter:
```
sage: a = CDF(-0.95,-0.65)
sage: b = CDF(0.683,0.747)
sage: a.agm(b, algorithm='optimal')
-0.3715916523517613 + 0.31989466020683*I
```

```
sage: a.agm(b, algorithm='principal')  # rel tol 1e-15
0.33817546298618006 - 0.013532696956540503*I
sage: a.agm(b, algorithm='pari')
-0.37159165235176134 + 0.31989466020683005*I
```

Some degenerate cases:
```
sage: CDF(0).agm(a)
0.0
sage: a.agm(0)
0.0
sage: a.agm(-a)
0.0
```

algdep(n)

Returns a polynomial of degree at most n which is approximately satisfied by this complex number. Note that the returned polynomial need not be irreducible, and indeed usually won't be if z is a good approximation to an algebraic number of degree less than n.

ALGORITHM: Uses the PARI C-library algdep command.

EXAMPLES:
```
sage: z = (1/2)*(1 + RDF(sqrt(3)) *CDF.0); z
0.5 + 0.8660254037844387*I
sage: p = z.algdep(5); p
x^3 + 1
sage: p.factor()
(x + 1) * (x^2 - x + 1)
sage: abs(z^2 - z + 1) < 1e-14
True

sage: CDF(0,2).algdep(10)
x^2 + 4
sage: CDF(1,5).algdep(2)
x^2 - 2*x + 26
```

arccos()

This function returns the complex arccosine of the complex number z, $\arccos(z)$. The branch cuts are on the real axis, less than -1 and greater than 1.

EXAMPLES:
```
sage: CDF(1,1).arccos()
0.9045568943023814 - 1.0612750619050357*I
```

arccosh()

This function returns the complex hyperbolic arccosine of the complex number z, $\operatorname{arccosh}(z)$. The branch cut is on the real axis, less than 1.

EXAMPLES:
```
sage: CDF(1,1).arccosh()
1.0612750619050357 + 0.9045568943023814*I
```

arccot()

This function returns the complex arccotangent of the complex number z, $\operatorname{arccot}(z) = \arctan(1/z)$.

EXAMPLES:

```
sage: CDF(1,1).arccot()   # rel tol 1e-15
0.5535743588970452 - 0.4023594781085251*I
```

arccoth()

This function returns the complex hyperbolic arccotangent of the complex number z, $\mathrm{arccoth}(z) = \mathrm{arctanh}(1/z)$.

EXAMPLES:

```
sage: CDF(1,1).arccoth()   # rel tol 1e-15
0.4023594781085251 - 0.5535743588970452*I
```

arccsc()

This function returns the complex arccosecant of the complex number z, $\mathrm{arccsc}(z) = \mathrm{arcsin}(1/z)$.

EXAMPLES:

```
sage: CDF(1,1).arccsc()   # rel tol 1e-15
0.45227844715119064 - 0.5306375309525178*I
```

arccsch()

This function returns the complex hyperbolic arccosecant of the complex number z, $\mathrm{arccsch}(z) = \mathrm{arcsin}(1/z)$.

EXAMPLES:

```
sage: CDF(1,1).arccsch()   # rel tol 1e-15
0.5306375309525178 - 0.45227844715119064*I
```

arcsec()

This function returns the complex arcsecant of the complex number z, $\mathrm{arcsec}(z) = \mathrm{arccos}(1/z)$.

EXAMPLES:

```
sage: CDF(1,1).arcsec()   # rel tol 1e-15
1.118517879643706 + 0.5306375309525178*I
```

arcsech()

This function returns the complex hyperbolic arcsecant of the complex number z, $\mathrm{arcsech}(z) = \mathrm{arccosh}(1/z)$.

EXAMPLES:

```
sage: CDF(1,1).arcsech()   # rel tol 1e-15
0.5306375309525176 - 1.118517879643706*I
```

arcsin()

This function returns the complex arcsine of the complex number z, $\mathrm{arcsin}(z)$. The branch cuts are on the real axis, less than -1 and greater than 1.

EXAMPLES:

```
sage: CDF(1,1).arcsin()
0.6662394324925152 + 1.0612750619050357*I
```

arcsinh()

This function returns the complex hyperbolic arcsine of the complex number z, $\mathrm{arcsinh}(z)$. The branch cuts are on the imaginary axis, below $-i$ and above i.

EXAMPLES:

```
sage: CDF(1,1).arcsinh()
1.0612750619050357 + 0.6662394324925152*I
```

arctan()

This function returns the complex arctangent of the complex number z, $\arctan(z)$. The branch cuts are on the imaginary axis, below $-i$ and above i.

EXAMPLES:

```
sage: CDF(1,1).arctan()
1.0172219678978514 + 0.4023594781085251*I
```

arctanh()

This function returns the complex hyperbolic arctangent of the complex number z, $\operatorname{arctanh}(z)$. The branch cuts are on the real axis, less than -1 and greater than 1.

EXAMPLES:

```
sage: CDF(1,1).arctanh()
0.4023594781085251 + 1.0172219678978514*I
```

arg()

This function returns the argument of `self`, the complex number z, denoted by $\arg(z)$, where $-\pi < \arg(z) <= \pi$.

EXAMPLES:

```
sage: CDF(1,0).arg()
0.0
sage: CDF(0,1).arg()
1.5707963267948966
sage: CDF(0,-1).arg()
-1.5707963267948966
sage: CDF(-1,0).arg()
3.141592653589793
```

argument()

This function returns the argument of the `self`, the complex number z, in the interval $-\pi < arg(z) \leq \pi$.

EXAMPLES:

```
sage: CDF(6).argument()
0.0
sage: CDF(i).argument()
1.5707963267948966
sage: CDF(-1).argument()
3.141592653589793
sage: CDF(-1 - 0.000001*i).argument()
-3.1415916535897934
```

conj()

This function returns the complex conjugate of the complex number z:

$$\overline{z} = x - iy.$$

EXAMPLES:

```
sage: z = CDF(2,3); z.conj()
2.0 - 3.0*I
```

conjugate()
> This function returns the complex conjugate of the complex number z:
>
> $$\overline{z} = x - iy.$$
>
> EXAMPLES:
> ```
> sage: z = CDF(2,3); z.conjugate()
> 2.0 - 3.0*I
> ```

cos()
> This function returns the complex cosine of the complex number z:
>
> $$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$$
>
> EXAMPLES:
> ```
> sage: CDF(1,1).cos()
> 0.8337300251311491 - 0.9888977057628651*I
> ```

cosh()
> This function returns the complex hyperbolic cosine of the complex number z:
>
> $$\cosh(z) = \frac{e^{z} + e^{-z}}{2}.$$
>
> EXAMPLES:
> ```
> sage: CDF(1,1).cosh()
> 0.8337300251311491 + 0.9888977057628651*I
> ```

cot()
> This function returns the complex cotangent of the complex number z:
>
> $$\cot(z) = \frac{1}{\tan(z)}.$$
>
> EXAMPLES:
> ```
> sage: CDF(1,1).cot()   # rel tol 1e-15
> 0.21762156185440268 - 0.8680141428959249*I
> ```

coth()
> This function returns the complex hyperbolic cotangent of the complex number z:
>
> $$\coth(z) = \frac{1}{\tanh(z)}.$$
>
> EXAMPLES:
> ```
> sage: CDF(1,1).coth()   # rel tol 1e-15
> 0.8680141428959249 - 0.21762156185440268*I
> ```

csc()
> This function returns the complex cosecant of the complex number z:
>
> $$\csc(z) = \frac{1}{\sin(z)}.$$
>
> EXAMPLES:

```
sage: CDF(1,1).csc()    # rel tol 1e-15
0.6215180171704284 - 0.30393100162842646*I
```

csch()

This function returns the complex hyperbolic cosecant of the complex number z:

$$\text{csch}(z) = \frac{1}{\sinh(z)}.$$

EXAMPLES:

```
sage: CDF(1,1).csch()   # rel tol 1e-15
0.30393100162842646 - 0.6215180171704284*I
```

dilog()

Returns the principal branch of the dilogarithm of x, i.e., analytic continuation of the power series

$$\log_2(x) = \sum_{n \geq 1} x^n/n^2.$$

EXAMPLES:

```
sage: CDF(1,2).dilog()
-0.059474798673809476 + 2.0726479717747566*I
sage: CDF(10000000,10000000).dilog()
-134.411774490731 + 38.79396299904504*I
```

eta(*omit_frac=0*)

Return the value of the Dedekind η function on self.

INPUT:

 • `self` - element of the upper half plane (if not, raises a ValueError).

 • `omit_frac` - (bool, default: `False`), if `True`, omit the $e^{\pi i z/12}$ factor.

OUTPUT: a complex double number

ALGORITHM: Uses the PARI C library.

The η function is

$$\eta(z) = e^{\pi i z/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i n z})$$

EXAMPLES:

We compute a few values of `eta()`:

```
sage: CDF(0,1).eta()
0.7682254223260566
sage: CDF(1,1).eta()
0.7420487758365647 + 0.1988313702299107*I
sage: CDF(25,1).eta()
0.7420487758365647 + 0.1988313702299107*I
```

`eta()` works even if the inputs are large:

```
sage: CDF(0, 10^15).eta()
0.0
sage: CDF(10^15, 0.1).eta()   # abs tol 1e-10
-0.115342592727 - 0.19977923088*I
```

We compute a few values of `eta()`, but with the fractional power of e omitted:

```
sage: CDF(0,1).eta(True)
0.9981290699259585
```

We compute `eta()` to low precision directly from the definition:

```
sage: z = CDF(1,1); z.eta()
0.7420487758365647 + 0.1988313702299107*I
sage: i = CDF(0,1); pi = CDF(pi)
sage: exp(pi * i * z / 12) * prod([1-exp(2*pi*i*n*z) for n in range(1,10)])
0.7420487758365647 + 0.19883137022991068*I
```

The optional argument allows us to omit the fractional part:

```
sage: z.eta(omit_frac=True)
0.9981290699259585
sage: pi = CDF(pi)
sage: prod([1-exp(2*pi*i*n*z) for n in range(1,10)])   # abs tol 1e-12
0.998129069926 + 4.59084695545e-19*I
```

We illustrate what happens when z is not in the upper half plane:

```
sage: z = CDF(1)
sage: z.eta()
Traceback (most recent call last):
...
ValueError: value must be in the upper half plane
```

You can also use functional notation:

```
sage: z = CDF(1,1)
sage: eta(z)
0.7420487758365647 + 0.1988313702299107*I
```

exp()

This function returns the complex exponential of the complex number z, $\exp(z)$.

EXAMPLES:

```
sage: CDF(1,1).exp()
1.4686939399158851 + 2.2873552871788423*I
```

We numerically verify a famous identity to the precision of a double:

```
sage: z = CDF(0, 2*pi); z
6.283185307179586*I
sage: exp(z)   # rel tol 1e-4
1.0 - 2.4492935982947064e-16*I
```

gamma()

Return the gamma function $\Gamma(z)$ evaluated at `self`, the complex number z.

EXAMPLES:

```
sage: CDF(5,0).gamma()
24.0
sage: CDF(1,1).gamma()
0.49801566811835607 - 0.15494982830181067*I
sage: CDF(0).gamma()
Infinity
sage: CDF(-1,0).gamma()
Infinity
```

gamma_inc(*t*)

 Return the incomplete gamma function evaluated at this complex number.

 EXAMPLES:
```
sage: CDF(1,1).gamma_inc(CDF(2,3))
0.0020969148636468277 - 0.059981913655449706*I
sage: CDF(1,1).gamma_inc(5)
-0.001378130936215849 + 0.006519820023119819*I
sage: CDF(2,0).gamma_inc(CDF(1,1))
0.7070920963459381 - 0.4203536409598115*I
```

imag()

 Return the imaginary part of this complex double.

 EXAMPLES:
```
sage: a = CDF(3,-2)
sage: a.imag()
-2.0
sage: a.imag_part()
-2.0
```

imag_part()

 Return the imaginary part of this complex double.

 EXAMPLES:
```
sage: a = CDF(3,-2)
sage: a.imag()
-2.0
sage: a.imag_part()
-2.0
```

is_infinity()

 Check if self is ∞.

 EXAMPLES:
```
sage: CDF(1, 2).is_infinity()
False
sage: CDF(0, oo).is_infinity()
True
```

is_integer()

 Returns True if this number is a integer

 EXAMPLES:
```
sage: CDF(0.5).is_integer()
False
sage: CDF(I).is_integer()
False
sage: CDF(2).is_integer()
True
```

is_negative_infinity()

 Check if self is $-\infty$.

 EXAMPLES:

```
sage: CDF(1, 2).is_negative_infinity()
False
sage: CDF(-oo, 0).is_negative_infinity()
True
sage: CDF(0, -oo).is_negative_infinity()
False
```

is_positive_infinity()
>   Check if self is $+\infty$.

>   EXAMPLES:
```
sage: CDF(1, 2).is_positive_infinity()
False
sage: CDF(oo, 0).is_positive_infinity()
True
sage: CDF(0, oo).is_positive_infinity()
False
```

is_square()
>   This function always returns True as $\mathbf{C}$ is algebraically closed.

>   EXAMPLES:
```
sage: CDF(-1).is_square()
True
```

log(*base=None*)
>   This function returns the complex natural logarithm to the given base of the complex number z, $\log(z)$.
>   The branch cut is the negative real axis.

>   INPUT:

>   •``base`` - default: e, the base of the natural logarithm

>   EXAMPLES:
```
sage: CDF(1,1).log()
0.34657359027997264 + 0.7853981633974483*I
```

>   This is the only example different from the GSL:
```
sage: CDF(0,0).log()
-infinity
```

log10()
>   This function returns the complex base-10 logarithm of the complex number z, $\log_{10}(z)$.

>   The branch cut is the negative real axis.

>   EXAMPLES:
```
sage: CDF(1,1).log10()
0.15051499783199057 + 0.3410940884604603*I
```

log_b(*b*)
>   This function returns the complex base-b logarithm of the complex number z, $\log_b(z)$. This quantity is
>   computed as the ratio $\log(z)/\log(b)$.

>   The branch cut is the negative real axis.

>   EXAMPLES:

```
sage: CDF(1,1).log_b(10)   # rel tol 1e-15
0.15051499783199057 + 0.3410940884604603*I
```

logabs()

This function returns the natural logarithm of the magnitude of the complex number z, $\log|z|$.

This allows for an accurate evaluation of $\log|z|$ when $|z|$ is close to 1. The direct evaluation of `log(abs(z))` would lead to a loss of precision in this case.

EXAMPLES:

```
sage: CDF(1.1,0.1).logabs()
0.09942542937258267
sage: log(abs(CDF(1.1,0.1)))
0.09942542937258259
```

```
sage: log(abs(ComplexField(200)(1.1,0.1)))
0.099425429372582595066319157757531449594489450091985182495705
```

norm()

This function returns the squared magnitude $|z|^2$ of the complex number z, otherwise known as the complex norm. If $c = a + bi$ is a complex number, then the norm of c is defined as the product of c and its complex conjugate:

$$\text{norm}(c) = \text{norm}(a + bi) = c \cdot \bar{c} = a^2 + b^2.$$

The norm of a complex number is different from its absolute value. The absolute value of a complex number is defined to be the square root of its norm. A typical use of the complex norm is in the integral domain $\mathbf{Z}[i]$ of Gaussian integers, where the norm of each Gaussian integer $c = a + bi$ is defined as its complex norm.

See also:

- `abs()`

- `abs2()`

- `sage.misc.functional.norm()`

- `sage.rings.complex_number.ComplexNumber.norm()`

EXAMPLES:

```
sage: CDF(2,3).norm()
13.0
```

nth_root(*n*, *all=False*)

The `n`-th root function.

INPUT:

- `all` – bool (default: `False`); if `True`, return a list of all `n`-th roots.

EXAMPLES:

```
sage: a = CDF(125)
sage: a.nth_root(3)
5.000000000000001
sage: a = CDF(10, 2)
sage: [r^5 for r in a.nth_root(5, all=True)]   # rel tol 1e-14
[9.999999999999998 + 2.0*I, 9.999999999999993 + 2.000000000000002*I, 9.999999999999996 + 1.9
```

```
sage: abs(sum(a.nth_root(111, all=True)))  # rel tol 0.1
1.1057313523818259e-13
```

prec()
> Returns the precision of this number (to be more similar to `ComplexNumber`). Always returns 53.
>
> EXAMPLES:
> ```
> sage: CDF(0).prec()
> 53
> ```

real()
> Return the real part of this complex double.
>
> EXAMPLES:
> ```
> sage: a = CDF(3,-2)
> sage: a.real()
> 3.0
> sage: a.real_part()
> 3.0
> ```

real_part()
> Return the real part of this complex double.
>
> EXAMPLES:
> ```
> sage: a = CDF(3,-2)
> sage: a.real()
> 3.0
> sage: a.real_part()
> 3.0
> ```

sec()
> This function returns the complex secant of the complex number z:
>
> $$\sec(z) = \frac{1}{\cos(z)}.$$
>
> EXAMPLES:
> ```
> sage: CDF(1,1).sec()  # rel tol 1e-15
> 0.4983370305551868 + 0.591083841721045*I
> ```

sech()
> This function returns the complex hyperbolic secant of the complex number z:
>
> $$\operatorname{sech}(z) = \frac{1}{\cosh(z)}.$$
>
> EXAMPLES:
> ```
> sage: CDF(1,1).sech()  # rel tol 1e-15
> 0.4983370305551868 - 0.591083841721045*I
> ```

sin()
> This function returns the complex sine of the complex number z:
>
> $$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}.$$
>
> EXAMPLES:

```
sage: CDF(1,1).sin()
1.2984575814159773 + 0.6349639147847361*I
```

sinh()

This function returns the complex hyperbolic sine of the complex number z:

$$\sinh(z) = \frac{e^z - e^{-z}}{2}.$$

EXAMPLES:
```
sage: CDF(1,1).sinh()
0.6349639147847361 + 1.2984575814159773*I
```

sqrt (*all=False*, ***kwds*)

The square root function.

INPUT:

•`all` - bool (default: `False`); if `True`, return a list of all square roots.

If all is `False`, the branch cut is the negative real axis. The result always lies in the right half of the complex plane.

EXAMPLES:

We compute several square roots:
```
sage: a = CDF(2,3)
sage: b = a.sqrt(); b   # rel tol 1e-15
1.6741492280355401 + 0.8959774761298381*I
sage: b^2   # rel tol 1e-15
2.0 + 3.0*I
sage: a^(1/2)
1.6741492280355401 + 0.895977476129838*I
```

We compute the square root of -1:
```
sage: a = CDF(-1)
sage: a.sqrt()
1.0*I
```

We compute all square roots:
```
sage: CDF(-2).sqrt(all=True)
[1.4142135623730951*I, -1.4142135623730951*I]
sage: CDF(0).sqrt(all=True)
[0.0]
```

tan()

This function returns the complex tangent of the complex number z:

$$\tan(z) = \frac{\sin(z)}{\cos(z)}.$$

EXAMPLES:
```
sage: CDF(1,1).tan()
0.27175258531951174 + 1.0839233273386946*I
```

tanh()

This function returns the complex hyperbolic tangent of the complex number z:

$$\tanh(z) = \frac{\sinh(z)}{\cosh(z)}.$$

EXAMPLES:
```
sage: CDF(1,1).tanh()
1.0839233273386946 + 0.27175258531951174*I
```

zeta()

Return the Riemann zeta function evaluated at this complex number.

EXAMPLES:
```
sage: z = CDF(1, 1)
sage: z.zeta()
0.5821580597520036 - 0.9268485643308071*I
sage: zeta(z)
0.5821580597520036 - 0.9268485643308071*I
sage: zeta(CDF(1))
Infinity
```

sage.rings.complex_double.**ComplexDoubleField**()

Returns the field of double precision complex numbers.

EXAMPLE:
```
sage: ComplexDoubleField()
Complex Double Field
sage: ComplexDoubleField() is CDF
True
```

class sage.rings.complex_double.**ComplexDoubleField_class**

Bases: sage.rings.ring.Field

An approximation to the field of complex numbers using double precision floating point numbers. Answers derived from calculations in this approximation may differ from what they would be if those calculations were performed in the true field of complex numbers. This is due to the rounding errors inherent to finite precision calculations.

ALGORITHM:

Arithmetic is done using GSL (the GNU Scientific Library).

algebraic_closure()

Returns the algebraic closure of self, i.e., the complex double field.

EXAMPLES:
```
sage: CDF.algebraic_closure()
Complex Double Field
```

characteristic()

Return the characteristic of the complex double field, which is 0.

EXAMPLES:
```
sage: CDF.characteristic()
0
```

construction()

> Returns the functorial construction of `self`, namely, algebraic closure of the real double field.
>
> EXAMPLES:
>
> ```
> sage: c, S = CDF.construction(); S
> Real Double Field
> sage: CDF == c(S)
> True
> ```

gen(*n=0*)

> Return the generator of the complex double field.
>
> EXAMPLES:
>
> ```
> sage: CDF.0
> 1.0*I
> sage: CDF.gen(0)
> 1.0*I
> ```

is_exact()

> Returns whether or not this field is exact, which is always `False`.
>
> EXAMPLES:
>
> ```
> sage: CDF.is_exact()
> False
> ```

ngens()

> The number of generators of this complex field as an **R**-algebra.
>
> There is one generator, namely `sqrt(-1)`.
>
> EXAMPLES:
>
> ```
> sage: CDF.ngens()
> 1
> ```

pi()

> Returns π as a double precision complex number.
>
> EXAMPLES:
>
> ```
> sage: CDF.pi()
> 3.141592653589793
> ```

prec()

> Return the precision of this complex double field (to be more similar to `ComplexField`). Always returns 53.
>
> EXAMPLES:
>
> ```
> sage: CDF.prec()
> 53
> ```

precision()

> Return the precision of this complex double field (to be more similar to `ComplexField`). Always returns 53.
>
> EXAMPLES:
>
> ```
> sage: CDF.prec()
> 53
> ```

random_element(*xmin=-1*, *xmax=1*, *ymin=-1*, *ymax=1*)
Return a random element of this complex double field with real and imaginary part bounded by `xmin`, `xmax`, `ymin`, `ymax`.

EXAMPLES:
```
sage: CDF.random_element()
-0.43681052967509904 + 0.7369454235661859*I
sage: CDF.random_element(-10,10,-10,10)
-7.088740263015161 - 9.54135400334003*I
sage: CDF.random_element(-10^20,10^20,-2,2)
-7.587654737635711e+19 + 0.925549022838656*I
```

real_double_field()
The real double field, which you may view as a subfield of this complex double field.

EXAMPLES:
```
sage: CDF.real_double_field()
Real Double Field
```

to_prec(*prec*)
Returns the complex field to the specified precision. As doubles have fixed precision, this will only return a complex double field if prec is exactly 53.

EXAMPLES:
```
sage: CDF.to_prec(53)
Complex Double Field
sage: CDF.to_prec(250)
Complex Field with 250 bits of precision
```

zeta(*n=2*)
Return a primitive n-th root of unity in this CDF, for $n \geq 1$.

INPUT:

   •n – a positive integer (default: 2)

OUTPUT: a complex n-th root of unity.

EXAMPLES:
```
sage: CDF.zeta(7)   # rel tol 1e-15
0.6234898018587336 + 0.7818314824680298*I
sage: CDF.zeta(1)
1.0
sage: CDF.zeta()
-1.0
sage: CDF.zeta() == CDF.zeta(2)
True

sage: CDF.zeta(0.5)
Traceback (most recent call last):
...
ValueError: n must be a positive integer
sage: CDF.zeta(0)
Traceback (most recent call last):
...
ValueError: n must be a positive integer
sage: CDF.zeta(-1)
Traceback (most recent call last):
```

```
        ...
        ValueError: n must be a positive integer
```

class `sage.rings.complex_double.`**`FloatToCDF`**

    Bases: `sage.categories.morphism.Morphism`

    Fast morphism from anything with a `__float__` method to an RDF element.

    EXAMPLES:

```
sage: f = CDF.coerce_map_from(ZZ); f
Native morphism:
  From: Integer Ring
  To:   Complex Double Field
sage: f(4)
4.0
sage: f = CDF.coerce_map_from(QQ); f
Native morphism:
  From: Rational Field
  To:   Complex Double Field
sage: f(1/2)
0.5
sage: f = CDF.coerce_map_from(int); f
Native morphism:
  From: Set of Python objects of type 'int'
  To:   Complex Double Field
sage: f(3r)
3.0
sage: f = CDF.coerce_map_from(float); f
Native morphism:
  From: Set of Python objects of type 'float'
  To:   Complex Double Field
sage: f(3.5)
3.5
```

`sage.rings.complex_double.`**`is_ComplexDoubleElement`**(*x*)

    Return `True` if x is a [ComplexDoubleElement](#).

    EXAMPLES:

```
sage: from sage.rings.complex_double import is_ComplexDoubleElement
sage: is_ComplexDoubleElement(0)
False
sage: is_ComplexDoubleElement(CDF(0))
True
```

`sage.rings.complex_double.`**`is_ComplexDoubleField`**(*x*)

    Return `True` if x is the complex double field.

    EXAMPLE:

```
sage: from sage.rings.complex_double import is_ComplexDoubleField
sage: is_ComplexDoubleField(CDF)
True
sage: is_ComplexDoubleField(ComplexField(53))
False
```

FIELD OF ARBITRARY PRECISION COMPLEX NUMBERS

AUTHORS:

- William Stein (2006-01-26): complete rewrite

- Niles Johnson (2010-08): trac ticket #3893: `random_element()` should pass on `*args` and `**kwds`.

- Travis Scrimshaw (2012-10-18): Added documentation for full coverage.

sage.rings.complex_field.**ComplexField**(*prec=53*, *names=None*)

Return the complex field with real and imaginary parts having prec *bits* of precision.

EXAMPLES:

```
sage: ComplexField()
Complex Field with 53 bits of precision
sage: ComplexField(100)
Complex Field with 100 bits of precision
sage: ComplexField(100).base_ring()
Real Field with 100 bits of precision
sage: i = ComplexField(200).gen()
sage: i^2
-1.0000000000000000000000000000000000000000000000000000000000
```

class sage.rings.complex_field.**ComplexField_class**(*prec=53*)

Bases: sage.rings.ring.Field

An approximation to the field of complex numbers using floating point numbers with any specified precision. Answers derived from calculations in this approximation may differ from what they would be if those calculations were performed in the true field of complex numbers. This is due to the rounding errors inherent to finite precision calculations.

EXAMPLES:

```
sage: C = ComplexField(); C
Complex Field with 53 bits of precision
sage: Q = RationalField()
sage: C(1/3)
0.333333333333333
sage: C(1/3, 2)
0.333333333333333 + 2.00000000000000*I
sage: C(RR.pi())
3.14159265358979
sage: C(RR.log2(), RR.pi())
0.693147180559945 + 3.14159265358979*I
```

We can also coerce rational numbers and integers into C, but coercing a polynomial will raise an exception:

```
sage: Q = RationalField()
sage: C(1/3)
0.333333333333333
sage: S = PolynomialRing(Q, 'x')
sage: C(S.gen())
Traceback (most recent call last):
...
TypeError: unable to coerce to a ComplexNumber: <type 'sage.rings.polynomial.polynomial_rational
```

This illustrates precision:

```
sage: CC = ComplexField(10); CC(1/3, 2/3)
0.33 + 0.67*I
sage: CC
Complex Field with 10 bits of precision
sage: CC = ComplexField(100); CC
Complex Field with 100 bits of precision
sage: z = CC(1/3, 2/3); z
0.33333333333333333333333333333 + 0.66666666666666666666666666667*I
```

We can load and save complex numbers and the complex field:

```
sage: loads(z.dumps()) == z
True
sage: loads(CC.dumps()) == CC
True
sage: k = ComplexField(100)
sage: loads(dumps(k)) == k
True
```

This illustrates basic properties of a complex field:

```
sage: CC = ComplexField(200)
sage: CC.is_field()
True
sage: CC.characteristic()
0
sage: CC.precision()
200
sage: CC.variable_name()
'I'
sage: CC == ComplexField(200)
True
sage: CC == ComplexField(53)
False
sage: CC == 1.1
False
```

algebraic_closure()
    Return the algebraic closure of self (which is itself).

    EXAMPLES:
```
sage: CC
Complex Field with 53 bits of precision
sage: CC.algebraic_closure()
Complex Field with 53 bits of precision
sage: CC = ComplexField(1000)
sage: CC.algebraic_closure() is CC
True
```

characteristic()
>   Return the characteristic of **C**, which is 0.
>
>   EXAMPLES:
>    ```
>    sage: ComplexField().characteristic()
>    0
>    ```

construction()
>   Returns the functorial construction of `self`, namely the algebraic closure of the real field with the same
>   precision.
>
>   EXAMPLES:
>    ```
>    sage: c, S = CC.construction(); S
>    Real Field with 53 bits of precision
>    sage: CC == c(S)
>    True
>    ```

gen(*n=0*)
>   Return the generator of the complex field.
>
>   EXAMPLES:
>    ```
>    sage: ComplexField().gen(0)
>    1.00000000000000*I
>    ```

is_exact()
>   Return whether or not this field is exact, which is always `False`.
>
>   EXAMPLES:
>    ```
>    sage: ComplexField().is_exact()
>    False
>    ```

is_field(*proof=True*)
>   Return `True` since the complex numbers are a field.
>
>   EXAMPLES:
>    ```
>    sage: CC.is_field()
>    True
>    ```

is_finite()
>   Return `False` since there are infinite number of complex numbers.
>
>   EXAMPLES:
>    ```
>    sage: CC.is_finite()
>    False
>    ```

ngens()
>   The number of generators of this complex field as an **R**-algebra.
>
>   There is one generator, namely `sqrt(-1)`.
>
>   EXAMPLES:
>    ```
>    sage: ComplexField().ngens()
>    1
>    ```

pi()
>   Returns π as a complex number.

EXAMPLES:
```
sage: ComplexField().pi()
3.14159265358979
sage: ComplexField(100).pi()
3.14159265358979323846264338333
```

prec()

Return the precision of this complex field.

EXAMPLES:
```
sage: ComplexField().prec()
53
sage: ComplexField(15).prec()
15
```

precision()

Return the precision of this complex field.

EXAMPLES:
```
sage: ComplexField().prec()
53
sage: ComplexField(15).prec()
15
```

random_element(*component_max=1, *args, **kwds*)

Returns a uniformly distributed random number inside a square centered on the origin (by default, the square $[-1, 1] \times [-1, 1]$).

Passes additional arguments and keywords to underlying real field.

EXAMPLES:
```
sage: [CC.random_element() for _ in range(5)]
[0.153636193785613 - 0.502987375247518*I,
 0.609589964322241 - 0.948854594338216*I,
 0.968393085385764 - 0.148483595843485*I,
 -0.908976099636549 + 0.126219184235123*I,
 0.461226845462901 - 0.0420335212948924*I]
sage: CC6 = ComplexField(6)
sage: [CC6.random_element(2^-20) for _ in range(5)]
[-5.4e-7 - 3.3e-7*I, 2.1e-7 + 8.0e-7*I, -4.8e-7 - 8.6e-7*I, -6.0e-8 + 2.7e-7*I, 6.0e-8 + 1.8
sage: [CC6.random_element(pi^20) for _ in range(5)]
[6.7e8 - 5.4e8*I, -9.4e8 + 5.0e9*I, 1.2e9 - 2.7e8*I, -2.3e9 - 4.0e9*I, 7.7e9 + 1.2e9*I]
```

Passes extra positional or keyword arguments through:
```
sage: [CC.random_element(distribution='1/n') for _ in range(5)]
[-0.900931453455899 - 0.932172283929307*I,
 0.327862582226912 + 0.828104487111727*I,
 0.246299162813240 + 0.588214960163442*I,
 0.892970599589521 - 0.266744694790704*I,
 0.878458776600692 - 0.905641181799996*I]
```

scientific_notation(*status=None*)

Set or return the scientific notation printing flag.

If this flag is `True` then complex numbers with this space as parent print using scientific notation.

EXAMPLES:

```
sage: C = ComplexField()
sage: C((0.025, 2))
0.0250000000000000 + 2.00000000000000*I
sage: C.scientific_notation(True)
sage: C((0.025, 2))
2.50000000000000e-2 + 2.00000000000000e0*I
sage: C.scientific_notation(False)
sage: C((0.025, 2))
0.0250000000000000 + 2.00000000000000*I
```

to_prec(*prec*)

Returns the complex field to the specified precision.

EXAMPLES:

```
sage: CC.to_prec(10)
Complex Field with 10 bits of precision
sage: CC.to_prec(100)
Complex Field with 100 bits of precision
```

zeta(*n=2*)

Return a primitive n-th root of unity.

INPUT:

- n - an integer (default: 2)

OUTPUT: a complex n-th root of unity.

EXAMPLES:

```
sage: C = ComplexField()
sage: C.zeta(2)
-1.00000000000000
sage: C.zeta(5)
0.309016994374947 + 0.951056516295154*I
```

sage.rings.complex_field.**is_ComplexField**(*x*)

Check if x is a complex field.

EXAMPLES:

```
sage: from sage.rings.complex_field import is_ComplexField as is_CF
sage: is_CF(ComplexField())
True
sage: is_CF(ComplexField(12))
True
sage: is_CF(CC)
True
```

sage.rings.complex_field.**late_import**()

Import the objects/modules after build (when needed).

TESTS:

```
sage: sage.rings.complex_field.late_import()
```

ARBITRARY PRECISION COMPLEX NUMBERS

AUTHORS:

- William Stein (2006-01-26): complete rewrite

- Joel B. Mohler (2006-12-16): naive rewrite into pyrex

- William Stein(2007-01): rewrite of Mohler's rewrite

- Vincent Delecroix (2010-01): plot function

- Travis Scrimshaw (2012-10-18): Added documentation for full coverage

class sage.rings.complex_number.**CCtoCDF**

Bases: sage.categories.map.Map

INPUT:

There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

TESTS:
```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

class sage.rings.complex_number.**ComplexNumber**

Bases: sage.structure.element.FieldElement

A floating point approximation to a complex number using any specified precision. Answers derived from calculations with such approximations may differ from what they would be if those calculations were performed with true complex numbers. This is due to the rounding errors inherent to finite precision calculations.

EXAMPLES:
```
sage: I = CC.0
sage: b = 1.5 + 2.5*I
```

```
sage: loads(b.dumps()) == b
True
```

additive_order()

> Return the additive order of `self`.
>
> EXAMPLES:
> ```
> sage: CC(0).additive_order()
> 1
> sage: CC.gen().additive_order()
> +Infinity
> ```

agm(*right*, *algorithm='optimal'*)

> Return the Arithmetic-Geometric Mean (AGM) of `self` and `right`.
>
> INPUT:
>
> > •`right` (complex) – another complex number
> >
> > •`algorithm` (string, default "optimal") – the algorithm to use (see below).
>
> OUTPUT:
>
> (complex) A value of the AGM of `self` and `right`. Note that this is a multi-valued function, and the algorithm used affects the value returned, as follows:
>
> > •"pari": Call the sgm function from the pari library.
> >
> > •**"optimal": Use the AGM sequence such that at each stage** (a, b) is replaced by $(a_1, b_1) = ((a + b)/2, \pm\sqrt{ab})$ where the sign is chosen so that $|a_1 - b_1| \leq |a_1 + b_1|$, or equivalently $\Re(b_1/a_1) \geq 0$. The resulting limit is maximal among all possible values.
> >
> > •**"principal": Use the AGM sequence such that at each stage** (a, b) is replaced by $(a_1, b_1) = ((a + b)/2, \pm\sqrt{ab})$ where the sign is chosen so that $\Re(b_1) \geq 0$ (the so-called principal branch of the square root).
>
> The values $AGM(a, 0)$, $AGM(0, a)$, and $AGM(a, -a)$ are all taken to be 0.
>
> EXAMPLES:
> ```
> sage: a = CC(1,1)
> sage: b = CC(2,-1)
> sage: a.agm(b)
> 1.62780548487271 + 0.136827548397369*I
> sage: a.agm(b, algorithm="optimal")
> 1.62780548487271 + 0.136827548397369*I
> sage: a.agm(b, algorithm="principal")
> 1.62780548487271 + 0.136827548397369*I
> sage: a.agm(b, algorithm="pari")
> 1.62780548487271 + 0.136827548397369*I
> ```
>
> An example to show that the returned value depends on the algorithm parameter:
> ```
> sage: a = CC(-0.95,-0.65)
> sage: b = CC(0.683,0.747)
> sage: a.agm(b, algorithm="optimal")
> -0.371591652351761 + 0.319894660206830*I
> sage: a.agm(b, algorithm="principal")
> 0.338175462986180 - 0.0135326969565405*I
> sage: a.agm(b, algorithm="pari")
> -0.371591652351761 + 0.319894660206830*I
> sage: a.agm(b, algorithm="optimal").abs()
> ```

```
0.490319232466314
sage: a.agm(b, algorithm="principal").abs()
0.338446122230459
sage: a.agm(b, algorithm="pari").abs()
0.490319232466314
```

TESTS:

An example which came up in testing:
```
sage: I = CC(I)
sage: a =  0.501648970493109 + 1.11877240294744*I
sage: b =  1.05946309435930 + 1.05946309435930*I
sage: a.agm(b)
0.774901870587681 + 1.10254945079875*I

sage: a = CC(-0.32599972608379413, 0.60395514542928641)
sage: b = CC( 0.6062314525690593,  0.1425693337776659)
sage: a.agm(b)
0.199246281325876 + 0.478401702759654*I
sage: a.agm(-a)
0.000000000000000
sage: a.agm(0)
0.000000000000000
sage: CC(0).agm(a)
0.000000000000000
```

Consistency:
```
sage: a = 1 + 0.5*I
sage: b = 2 - 0.25*I
sage: a.agm(b) - ComplexField(100)(a).agm(b)
0.000000000000000
sage: ComplexField(200)(a).agm(b) - ComplexField(500)(a).agm(b)
0.0000000000000000000000000000000000000000000000000000000000000
sage: ComplexField(500)(a).agm(b) - ComplexField(1000)(a).agm(b)
0.00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000
```

algdep(*n*, ***kwds*)

    Returns a polynomial of degree at most n which is approximately satisfied by this complex number. Note that the returned polynomial need not be irreducible, and indeed usually won't be if z is a good approximation to an algebraic number of degree less than n.

    ALGORITHM: Uses the PARI C-library algdep command.

    INPUT: Type algdep? at the top level prompt. All additional parameters are passed onto the top-level algdep command.

    EXAMPLE:
```
sage: C = ComplexField()
sage: z = (1/2)*(1 + sqrt(3.0) *C.0); z
0.500000000000000 + 0.866025403784439*I
sage: p = z.algdep(5); p
x^3 + 1
sage: p.factor()
(x + 1) * (x^2 - x + 1)
sage: z^2 - z + 1
1.11022302462516e-16
```

algebraic_dependancy(*n*)

Returns a polynomial of degree at most n which is approximately satisfied by this complex number. Note that the returned polynomial need not be irreducible, and indeed usually won't be if z is a good approximation to an algebraic number of degree less than n.

ALGORITHM: Uses the PARI C-library algdep command.

INPUT: Type algdep? at the top level prompt. All additional parameters are passed onto the top-level algdep command.

EXAMPLE:
```
sage: C = ComplexField()
sage: z = (1/2)*(1 + sqrt(3.0) *C.0); z
0.500000000000000 + 0.866025403784439*I
sage: p = z.algebraic_dependancy(5); p
x^3 + 1
sage: p.factor()
(x + 1) * (x^2 - x + 1)
sage: z^2 - z + 1
1.11022302462516e-16
```

arccos()

Return the arccosine of `self`.

EXAMPLES:
```
sage: (1+CC(I)).arccos()
0.904556894302381 - 1.06127506190504*I
```

arccosh()

Return the hyperbolic arccosine of `self`.

EXAMPLES:
```
sage: (1+CC(I)).arccosh()
1.06127506190504 + 0.904556894302381*I
```

arccoth()

Return the hyperbolic arccotangent of `self`.

EXAMPLES:
```
sage: ComplexField(100)(1,1).arccoth()
0.40235947810852509365018983331 - 0.55357435889704525150853273009*I
```

arccsch()

Return the hyperbolic arccosecant of `self`.

EXAMPLES:
```
sage: ComplexField(100)(1,1).arccsch()
0.53063753095251782601650945811 - 0.45227844715119068206365839783*I
```

arcsech()

Return the hyperbolic arcsecant of `self`.

EXAMPLES:
```
sage: ComplexField(100)(1,1).arcsech()
0.53063753095251782601650945811 - 1.1185178796437059371676632938*I
```

arcsin()

Return the arcsine of `self`.

EXAMPLES:
```
sage: (1+CC(I)).arcsin()
0.666239432492515 + 1.06127506190504*I
```

arcsinh()
Return the hyperbolic arcsine of `self`.

EXAMPLES:
```
sage: (1+CC(I)).arcsinh()
1.06127506190504 + 0.666239432492515*I
```

arctan()
Return the arctangent of `self`.

EXAMPLES:
```
sage: (1+CC(I)).arctan()
1.01722196789785 + 0.402359478108525*I
```

arctanh()
Return the hyperbolic arctangent of `self`.

EXAMPLES:
```
sage: (1+CC(I)).arctanh()
0.402359478108525 + 1.01722196789785*I
```

arg()
See `argument()`.

EXAMPLES:
```
sage: i = CC.0
sage: (i^2).arg()
3.14159265358979
```

argument()
The argument (angle) of the complex number, normalized so that $-\pi < \theta \leq \pi$.

EXAMPLES:
```
sage: i = CC.0
sage: (i^2).argument()
3.14159265358979
sage: (1+i).argument()
0.785398163397448
sage: i.argument()
1.57079632679490
sage: (-i).argument()
-1.57079632679490
sage: (RR('-0.001') - i).argument()
-1.57179632646156
```

conjugate()
Return the complex conjugate of this complex number.

EXAMPLES:
```
sage: i = CC.0
sage: (1+i).conjugate()
1.00000000000000 - 1.00000000000000*I
```

cos()
Return the cosine of `self`.

EXAMPLES:
```
sage: (1+CC(I)).cos()
0.833730025131149 - 0.988897705762865*I
```

cosh()
Return the hyperbolic cosine of `self`.

EXAMPLES:
```
sage: (1+CC(I)).cosh()
0.833730025131149 + 0.988897705762865*I
```

cotan()
Return the cotangent of `self`.

EXAMPLES:
```
sage: (1+CC(I)).cotan()
0.217621561854403 - 0.868014142895925*I
sage: i = ComplexField(200).0
sage: (1+i).cotan()
0.21762156185440268136513424360523807352075436916785404091068 - 0.868014142895924948635849200
sage: i = ComplexField(220).0
sage: (1+i).cotan()
0.21762156185440268136513424360523807352075436916785404091068124239 - 0.868014142895924948636
```

coth()
Return the hyperbolic cotangent of `self`.

EXAMPLES:
```
sage: ComplexField(100)(1,1).coth()
0.86801414289592494863584920892 - 0.21762156185440268136513424361*I
```

csc()
Return the cosecant of `self`.

EXAMPLES:
```
sage: ComplexField(100)(1,1).csc()
0.62151801717042842123490780586 - 0.30393100162842645033448560451*I
```

csch()
Return the hyperbolic cosecant of `self`.

EXAMPLES:
```
sage: ComplexField(100)(1,1).csch()
0.30393100162842645033448560451 - 0.62151801717042842123490780586*I
```

dilog()
Returns the complex dilogarithm of `self`.

The complex dilogarithm, or Spence's function, is defined by

$$Li_2(z) = -\int_0^z \frac{\log|1-\zeta|}{\zeta}d(\zeta) = \sum_{k=1}^{\infty}\frac{z^k}{k}$$

Note that the series definition can only be used for $|z| < 1$.

EXAMPLES:
```
sage: a = ComplexNumber(1,0)
sage: a.dilog()
1.64493406684823
sage: float(pi^2/6)
1.6449340668482262

sage: b = ComplexNumber(0,1)
sage: b.dilog()
-0.205616758356028 + 0.915965594177219*I

sage: c = ComplexNumber(0,0)
sage: c.dilog()
0.000000000000000
```

eta(*omit_frac=False*)

Return the value of the Dedekind η function on `self`, intelligently computed using $\mathbb{SL}(2, \mathbf{Z})$ transformations.

The η function is

$$\eta(z) = e^{\pi i z/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i n z})$$

INPUT:

- `self` – element of the upper half plane (if not, raises a `ValueError`).

- `omit_frac` – (bool, default: `False`), if `True`, omit the $e^{\pi i z/12}$ factor.

OUTPUT: a complex number

ALGORITHM: Uses the PARI C library.

EXAMPLES:

First we compute $\eta(1 + i)$:
```
sage: i = CC.0
sage: z = 1+i; z.eta()
0.742048775836565 + 0.198831370229911*I
```

We compute eta to low precision directly from the definition:
```
sage: z = 1 + i; z.eta()
0.742048775836565 + 0.198831370229911*I
sage: pi = CC(pi)          # otherwise we will get a symbolic result.
sage: exp(pi * i * z / 12) * prod([1-exp(2*pi*i*n*z) for n in range(1,10)])
0.742048775836565 + 0.198831370229911*I
```

The optional argument allows us to omit the fractional part:
```
sage: z = 1 + i
sage: z.eta(omit_frac=True)
0.998129069925959
sage: prod([1-exp(2*pi*i*n*z) for n in range(1,10)])
0.998129069925958 + 4.59099857829247e-19*I
```

We illustrate what happens when z is not in the upper half plane:
```
sage: z = CC(1)
sage: z.eta()
```

```
Traceback (most recent call last):
...
ValueError: value must be in the upper half plane
```

You can also use functional notation:

```
sage: eta(1+CC(I))
0.742048775836565 + 0.198831370229911*I
```

exp()

Compute e^z or $\exp(z)$.

EXAMPLES:

```
sage: i = ComplexField(300).0
sage: z = 1 + i
sage: z.exp()
1.46869393991588515713896759732660426132695673662900872279767567631093696585951213872272450
```

gamma()

Return the Gamma function evaluated at this complex number.

EXAMPLES:

```
sage: i = ComplexField(30).0
sage: (1+i).gamma()
0.49801567 - 0.15494983*I
```

TESTS:

```
sage: CC(0).gamma()
Infinity
```

```
sage: CC(-1).gamma()
Infinity
```

gamma_inc(*t*)

Return the incomplete Gamma function evaluated at this complex number.

EXAMPLES:

```
sage: C, i = ComplexField(30).objgen()
sage: (1+i).gamma_inc(2 + 3*i)   # abs tol 2e-10
0.0020969149 - 0.059981914*I
sage: (1+i).gamma_inc(5)
-0.0013781309 + 0.0065198200*I
sage: C(2).gamma_inc(1 + i)
0.70709210 - 0.42035364*I
sage: CC(2).gamma_inc(5)
0.0404276819945128
```

TESTS:

Check that trac ticket #7099 is fixed:

```
sage: C = ComplexField(400)
sage: C(2 + I).gamma_inc(C(3 + I))   # abs tol 1e-120
0.121515644664508695525971545977439666159749344176962379708992904126499444842886620664991650
```

imag()

Return imaginary part of `self`.

EXAMPLES:
```
sage: i = ComplexField(100).0
sage: z = 2 + 3*i
sage: x = z.imag(); x
3.0000000000000000000000000000
sage: x.parent()
Real Field with 100 bits of precision
sage: z.imag_part()
3.0000000000000000000000000000
```

imag_part()

Return imaginary part of `self`.

EXAMPLES:
```
sage: i = ComplexField(100).0
sage: z = 2 + 3*i
sage: x = z.imag(); x
3.0000000000000000000000000000
sage: x.parent()
Real Field with 100 bits of precision
sage: z.imag_part()
3.0000000000000000000000000000
```

is_imaginary()

Return `True` if `self` is imaginary, i.e. has real part zero.

EXAMPLES:
```
sage: CC(1.23*i).is_imaginary()
True
sage: CC(1+i).is_imaginary()
False
```

is_infinity()

Check if `self` is ∞.

EXAMPLES:
```
sage: CC(1, 2).is_infinity()
False
sage: CC(0, oo).is_infinity()
True
```

is_integer()

Return `True` if `self` is a integer

EXAMPLES:
```
sage: CC(3).is_integer()
True
sage: CC(1,2).is_integer()
False
```

is_negative_infinity()

Check if `self` is $-\infty$.

EXAMPLES:
```
sage: CC(1, 2).is_negative_infinity()
False
sage: CC(-oo, 0).is_negative_infinity()
```

```
      True
      sage: CC(0, -oo).is_negative_infinity()
      False
```

is_positive_infinity()
Check if self is $+\infty$.

EXAMPLES:
```
sage: CC(1, 2).is_positive_infinity()
False
sage: CC(oo, 0).is_positive_infinity()
True
sage: CC(0, oo).is_positive_infinity()
False
```

is_real()
Return True if self is real, i.e. has imaginary part zero.

EXAMPLES:
```
sage: CC(1.23).is_real()
True
sage: CC(1+i).is_real()
False
```

is_square()
This function always returns true as **C** is algebraically closed.

EXAMPLES:
```
sage: a = ComplexNumber(2,1)
sage: a.is_square()
True
```

C is algebraically closed, hence every element is a square:
```
sage: b = ComplexNumber(5)
sage: b.is_square()
True
```

log(*base=None*)
Complex logarithm of z with branch chosen as follows: Write $z = \rho e^{i\theta}$ with $-\pi < \theta <= pi$. Then $\log(z) = \log(\rho) + i\theta$.

> **Warning:** Currently the real log is computed using floats, so there is potential precision loss.

EXAMPLES:
```
sage: a = ComplexNumber(2,1)
sage: a.log()
0.804718956217050 + 0.463647609000806*I
sage: log(a.abs())
0.804718956217050
sage: a.argument()
0.463647609000806

sage: b = ComplexNumber(float(exp(42)),0)
sage: b.log()
41.9999999999971
```

```
sage: c = ComplexNumber(-1,0)
sage: c.log()
3.14159265358979*I
```

The option of a base is included for compatibility with other logs:
```
sage: c = ComplexNumber(-1,0)
sage: c.log(2)
4.53236014182719*I
```

If either component (real or imaginary) of the complex number is NaN (not a number), log will return the complex NaN:
```
sage: c = ComplexNumber(NaN,2)
sage: c.log()
NaN - NaN*I
```

multiplicative_order()
    Return the multiplicative order of this complex number, if known, or raise a `NotImplementedError`.

    EXAMPLES:
```
sage: C.<i> = ComplexField()
sage: i.multiplicative_order()
4
sage: C(1).multiplicative_order()
1
sage: C(-1).multiplicative_order()
2
sage: C(i^2).multiplicative_order()
2
sage: C(-i).multiplicative_order()
4
sage: C(2).multiplicative_order()
+Infinity
sage: w = (1+sqrt(-3.0))/2; w
0.500000000000000 + 0.866025403784439*I
sage: abs(w)
1.00000000000000
sage: w.multiplicative_order()
Traceback (most recent call last):
...
NotImplementedError: order of element not known
```

norm()
    Returns the norm of this complex number.

    If $c = a + bi$ is a complex number, then the norm of c is defined as the product of c and its complex conjugate:

$$\text{norm}(c) = \text{norm}(a + bi) = c \cdot \overline{c} = a^2 + b^2.$$

    The norm of a complex number is different from its absolute value. The absolute value of a complex number is defined to be the square root of its norm. A typical use of the complex norm is in the integral domain $\mathbf{Z}[i]$ of Gaussian integers, where the norm of each Gaussian integer $c = a + bi$ is defined as its complex norm.

    See also:

        •`sage.misc.functional.norm()`

> •`sage.rings.complex_double.ComplexDoubleElement.norm()`

EXAMPLES:

This indeed acts as the square function when the imaginary component of `self` is equal to zero:

```
sage: a = ComplexNumber(2,1)
sage: a.norm()
5.00000000000000
sage: b = ComplexNumber(4.2,0)
sage: b.norm()
17.6400000000000
sage: b^2
17.6400000000000
```

nth_root(*n*, *all=False*)

The *n*-th root function.

INPUT:

> •`all` - bool (default: `False`); if `True`, return a list of all *n*-th roots.

EXAMPLES:

```
sage: a = CC(27)
sage: a.nth_root(3)
3.00000000000000
sage: a.nth_root(3, all=True)
[3.00000000000000, -1.50000000000000 + 2.59807621135332*I, -1.50000000000000 - 2.59807621135
sage: a = ComplexField(20)(2,1)
sage: [r^7 for r in a.nth_root(7, all=True)]
[2.0000 + 1.0000*I, 2.0000 + 1.0000*I, 2.0000 + 1.0000*I, 2.0000 + 1.0000*I, 2.0000 + 1.0000
```

plot(***kargs*)

Plots this complex number as a point in the plane

The accepted options are the ones of `point2d()`. Type `point2d.options` to see all options.

Note: Just wraps the sage.plot.point.point2d method

EXAMPLES:

You can either use the indirect:

```
sage: z = CC(0,1)
sage: plot(z)
Graphics object consisting of 1 graphics primitive
```

or the more direct:

```
sage: z = CC(0,1)
sage: z.plot()
Graphics object consisting of 1 graphics primitive
```

prec()

Return precision of this complex number.

EXAMPLES:

```
sage: i = ComplexField(2000).0
sage: i.prec()
2000
```

real()
> Return real part of `self`.

> EXAMPLES:
```
sage: i = ComplexField(100).0
sage: z = 2 + 3*i
sage: x = z.real(); x
2.0000000000000000000000000000
sage: x.parent()
Real Field with 100 bits of precision
sage: z.real_part()
2.0000000000000000000000000000
```

real_part()
> Return real part of `self`.

> EXAMPLES:
```
sage: i = ComplexField(100).0
sage: z = 2 + 3*i
sage: x = z.real(); x
2.0000000000000000000000000000
sage: x.parent()
Real Field with 100 bits of precision
sage: z.real_part()
2.0000000000000000000000000000
```

sec()
> Return the secant of `self`.

> EXAMPLES:
```
sage: ComplexField(100)(1,1).sec()
0.49833703055518678521380589177 + 0.59108384172104504805039169297*I
```

sech()
> Return the hyperbolic secant of `self`.

> EXAMPLES:
```
sage: ComplexField(100)(1,1).sech()
0.49833703055518678521380589177 - 0.59108384172104504805039169297*I
```

sin()
> Return the sine of `self`.

> EXAMPLES:
```
sage: (1+CC(I)).sin()
1.29845758141598 + 0.634963914784736*I
```

sinh()
> Return the hyperbolic sine of `self`.

> EXAMPLES:
```
sage: (1+CC(I)).sinh()
0.634963914784736 + 1.29845758141598*I
```

sqrt(*all=False*)
> The square root function, taking the branch cut to be the negative real axis.

INPUT:

- •`all` - bool (default: `False`); if `True`, return a list of all square roots.

EXAMPLES:

```
sage: C.<i> = ComplexField(30)
sage: i.sqrt()
0.70710678 + 0.70710678*I
sage: (1+i).sqrt()
1.0986841 + 0.45508986*I
sage: (C(-1)).sqrt()
1.0000000*I
sage: (1 + 1e-100*i).sqrt()^2
1.0000000 + 1.0000000e-100*I
sage: i = ComplexField(200).0
sage: i.sqrt()
0.70710678118654752440084436210484903928483593768847403658834 + 0.70710678118654752440084436
```

str(*base=10*, *truncate=True*, *istr='I'*)

Return a string representation of `self`.

INPUTS:

- •`base` – (Default: 10) The base to use for printing

- •`truncate` – (Default: `True`) Whether to print fewer digits than are available, to mask errors in the last bits.

- •`istr` – (Default: `I`) String representation of the complex unit

EXAMPLES:

```
sage: a = CC(pi + I*e)
sage: a.str()
'3.14159265358979 + 2.71828182845905*I'
sage: a.str(truncate=False)
'3.1415926535897931 + 2.7182818284590451*I'
sage: a.str(base=2)
'11.001001000011111101101010100010001000010110100011000 + 10.1011011111100001010100010110001
sage: CC(0.5 + 0.625*I).str(base=2)
'0.10000000000000000000000000000000000000000000000000000 + 0.1010000000000000000000000000000000
sage: a.str(base=16)
'3.243f6a8885a30 + 2.b7e151628aed2*I'
sage: a.str(base=36)
'3.53i5ab8p5fc + 2.puw5nggjf8f*I'
sage: CC(0)
0.000000000000000
sage: CC.0.str(istr='%i')
'1.00000000000000*%i'
```

tan()

Return the tangent of `self`.

EXAMPLES:

```
sage: (1+CC(I)).tan()
0.271752585319512 + 1.08392332733869*I
```

tanh()

Return the hyperbolic tangent of `self`.

EXAMPLES:

```
sage: (1+CC(I)).tanh()
1.08392332733869 + 0.271752585319512*I
```

zeta()

    Return the Riemann zeta function evaluated at this complex number.

    EXAMPLES:

```
sage: i = ComplexField(30).gen()
sage: z = 1 + i
sage: z.zeta()
0.58215806 - 0.92684856*I
sage: zeta(z)
0.58215806 - 0.92684856*I

sage: CC(1).zeta()
Infinity
```

class sage.rings.complex_number.**RRtoCC**

    Bases: sage.categories.map.Map

    EXAMPLES:

```
sage: from sage.rings.complex_number import RRtoCC
sage: RRtoCC(RR, CC)
Natural map:
  From: Real Field with 53 bits of precision
  To:   Complex Field with 53 bits of precision
```

sage.rings.complex_number.**cmp_abs**(a, b)

    Returns -1, 0, or 1 according to whether $|a|$ is less than, equal to, or greater than $|b|$.

    Optimized for non-close numbers, where the ordering can be determined by examining exponents.

    EXAMPLES:

```
sage: from sage.rings.complex_number import cmp_abs
sage: cmp_abs(CC(5), CC(1))
1
sage: cmp_abs(CC(5), CC(4))
1
sage: cmp_abs(CC(5), CC(5))
0
sage: cmp_abs(CC(5), CC(6))
-1
sage: cmp_abs(CC(5), CC(100))
-1
sage: cmp_abs(CC(-100), CC(1))
1
sage: cmp_abs(CC(-100), CC(100))
0
sage: cmp_abs(CC(-100), CC(1000))
-1
sage: cmp_abs(CC(1,1), CC(1))
1
sage: cmp_abs(CC(1,1), CC(2))
-1
sage: cmp_abs(CC(1,1), CC(1,0.99999))
1
sage: cmp_abs(CC(1,1), CC(1,-1))
```

```
0
sage: cmp_abs(CC(0), CC(1))
-1
sage: cmp_abs(CC(1), CC(0))
1
sage: cmp_abs(CC(0), CC(0))
0
sage: cmp_abs(CC(2,1), CC(1,2))
0
```

sage.rings.complex_number.**create_ComplexNumber**(*s_real*, *s_imag=None*, *pad=0*, *min_prec=53*)

Return the complex number defined by the strings s_real and s_imag as an element of ComplexField(prec=n), where n potentially has slightly more (controlled by pad) bits than given by s.

INPUT:

- s_real – a string that defines a real number (or something whose string representation defines a number)

- s_imag – a string that defines a real number (or something whose string representation defines a number)

- pad – an integer at least 0.

- min_prec – number will have at least this many bits of precision, no matter what.

EXAMPLES:

```
sage: ComplexNumber('2.3')
2.30000000000000
sage: ComplexNumber('2.3','1.1')
2.30000000000000 + 1.10000000000000*I
sage: ComplexNumber(10)
10.0000000000000
sage: ComplexNumber(10,10)
10.0000000000000 + 10.0000000000000*I
sage: ComplexNumber(1.000000000000000000000000000,2)
1.000000000000000000000000000 + 2.000000000000000000000000000*I
sage: ComplexNumber(1,2.000000000000000000000)
1.00000000000000000000 + 2.00000000000000000000*I

sage: sage.rings.complex_number.create_ComplexNumber(s_real=2,s_imag=1)
2.00000000000000 + 1.00000000000000*I
```

TESTS:

Make sure we've rounded up log(10,2) enough to guarantee sufficient precision (trac ticket #10164):

```
sage: s = "1." + "0"*10**6 + "1"
sage: sage.rings.complex_number.create_ComplexNumber(s,0).real()-1 == 0
False
sage: sage.rings.complex_number.create_ComplexNumber(0,s).imag()-1 == 0
False
```

sage.rings.complex_number.**is_ComplexNumber**(*x*)

Returns True if x is a complex number. In particular, if x is of the ComplexNumber type.

EXAMPLES:

```
sage: from sage.rings.complex_number import is_ComplexNumber
sage: a = ComplexNumber(1,2); a
1.00000000000000 + 2.00000000000000*I
```

```
sage: is_ComplexNumber(a)
True
sage: b = ComplexNumber(1); b
1.00000000000000
sage: is_ComplexNumber(b)
True
```

Note that the global element `I` is of type `SymbolicConstant`. However, elements of the class `ComplexField_class` are of type `ComplexNumber`:

```
sage: c = 1 + 2*I
sage: is_ComplexNumber(c)
False
sage: d = CC(1 + 2*I)
sage: is_ComplexNumber(d)
True
```

sage.rings.complex_number.**make_ComplexNumber0**(*fld*, *mult_order*, *re*, *im*)

Create a complex number for pickling.

EXAMPLES:

```
sage: a = CC(1 + I)
sage: loads(dumps(a)) == a # indirect doctest
True
```

sage.rings.complex_number.**set_global_complex_round_mode**(*n*)

Set the global complex rounding mode.

> **Warning:** Do not call this function explicitly. The default rounding mode is `n = 0`.

EXAMPLES:

```
sage: sage.rings.complex_number.set_global_complex_round_mode(0)
```

ARBITRARY PRECISION COMPLEX NUMBERS USING GNU MPC

This is a binding for the MPC arbitrary-precision floating point library. It is adapted from `real_mpfr.pyx` and `complex_number.pyx`.

We define a class `MPComplexField`, where each instance of `MPComplexField` specifies a field of floating-point complex numbers with a specified precision shared by the real and imaginary part and a rounding mode stating the rounding mode directions specific to real and imaginary parts.

Individual floating-point numbers are of class `MPComplexNumber`.

For floating-point representation and rounding mode description see the documentation for the `sage.rings.real_mpfr`.

AUTHORS:

- Philippe Theveny (2008-10-13): initial version.

- Alex Ghitza (2008-11): cache, generators, random element, and many doctests.

- Yann Laigle-Chapuy (2010-01): improves compatibility with CC, updates.

- Jeroen Demeyer (2012-02): reformat documentation, make MPC a standard package.

- Travis Scrimshaw (2012-10-18): Added doctests for full coverage.

EXAMPLES:

```
sage: MPC = MPComplexField(42)
sage: a = MPC(12, '15.64E+32'); a
12.0000000000 + 1.56400000000e33*I
sage: a *a *a *a
5.98338564121e132 − 1.83633318912e101*I
sage: a + 1
13.0000000000 + 1.56400000000e33*I
sage: a / 3
4.00000000000 + 5.21333333333e32*I
sage: MPC("infinity + NaN *I")
+infinity + NaN*I
```

class `sage.rings.complex_mpc.`**CCtoMPC**

    Bases: `sage.categories.map.Map`

    INPUT:

    There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

    TESTS:

```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

class sage.rings.complex_mpc.**INTEGERtoMPC**

Bases: sage.categories.map.Map

INPUT:

There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

TESTS:
```
sage: from sage.categories.map import Map
```

Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

sage.rings.complex_mpc.**MPComplexField**(*prec=53*, *rnd='RNDNN'*, *names=None*)

Return the complex field with real and imaginary parts having prec *bits* of precision.

EXAMPLES:
```
sage: MPComplexField()
Complex Field with 53 bits of precision
sage: MPComplexField(100)
Complex Field with 100 bits of precision
sage: MPComplexField(100).base_ring()
Real Field with 100 bits of precision
sage: i = MPComplexField(200).gen()
sage: i^2
-1.0000000000000000000000000000000000000000000000000000000000
```

class sage.rings.complex_mpc.**MPComplexField_class**

Bases: sage.rings.ring.Field

Initialize self.

INPUT:

- `prec` – (integer) precision; default = 53

  prec is the number of bits used to represent the matissa of both the real and imaginary part of complex floating-point number.

- `rnd` – (string) the rounding mode; default = `'RNDNN'`

  Rounding mode is of the form `'RNDxy'` where `x` and `y` are the rounding mode for respectively the real and imaginary parts and are one of:

  - `'N'` for rounding to nearest

  - `'Z'` for rounding towards zero

  - `'U'` for rounding towards plus infinity

  - `'D'` for rounding towards minus infinity

  For example, `'RNDZU'` indicates to round the real part towards zero, and the imaginary part towards plus infinity.

EXAMPLES:
```
sage: MPComplexField(17)
Complex Field with 17 bits of precision
sage: MPComplexField()
Complex Field with 53 bits of precision
sage: MPComplexField(1042,'RNDDZ')
Complex Field with 1042 bits of precision and rounding RNDDZ
```

ALGORITHMS: Computations are done using the MPC library.

characteristic()
> Return 0, since the field of complex numbers has characteristic 0.
>
> EXAMPLES:
> ```
> sage: MPComplexField(42).characteristic()
> 0
> ```

gen(*n=0*)
> Return the generator of this complex field over its real subfield.
>
> EXAMPLES:
> ```
> sage: MPComplexField(34).gen()
> 1.00000000*I
> ```

is_exact()
> Returns whether or not this field is exact, which is always `False`.
>
> EXAMPLES:
> ```
> sage: MPComplexField(42).is_exact()
> False
> ```

is_finite()
> Return `False`, since the field of complex numbers is not finite.
>
> EXAMPLES:
> ```
> sage: MPComplexField(17).is_finite()
> False
> ```

name()

> Return the name of the complex field.

> EXAMPLES:
> ```
> sage: C = MPComplexField(10, 'RNDNZ'); C.name()
> 'MPComplexField10_RNDNZ'
> ```

ngens()

> Return 1, the number of generators of this complex field over its real subfield.

> EXAMPLES:
> ```
> sage: MPComplexField(34).ngens()
> 1
> ```

prec()

> Return the precision of this field of complex numbers.

> EXAMPLES:
> ```
> sage: MPComplexField().prec()
> 53
> sage: MPComplexField(22).prec()
> 22
> ```

random_element(*min=0, max=1*)

> Return a random complex number, uniformly distributed with real and imaginary parts between min and max (default 0 to 1).

> EXAMPLES:
> ```
> sage: MPComplexField(100).random_element(-5, 10)   # random
> 1.9305310520925994224072377281 + 0.94745292506956219710477444855*I
> sage: MPComplexField(10).random_element()   # random
> 0.12 + 0.23*I
> ```

rounding_mode()

> Return rounding modes used for each part of a complex number.

> EXAMPLES:
> ```
> sage: MPComplexField().rounding_mode()
> 'RNDNN'
> sage: MPComplexField(rnd='RNDZU').rounding_mode()
> 'RNDZU'
> ```

rounding_mode_imag()

> Return rounding mode used for the imaginary part of complex number.

> EXAMPLES:
> ```
> sage: MPComplexField(rnd='RNDZU').rounding_mode_imag()
> 'RNDU'
> ```

rounding_mode_real()

> Return rounding mode used for the real part of complex number.

> EXAMPLES:
> ```
> sage: MPComplexField(rnd='RNDZU').rounding_mode_real()
> 'RNDZ'
> ```

class sage.rings.complex_mpc.**MPComplexNumber**

    Bases: sage.structure.element.FieldElement

    A floating point approximation to a complex number using any specified precision common to both real and imaginary part.

    agm(*right*, *algorithm='optimal'*)

        Returns the algebraic geometrc mean of self and right.

        EXAMPLES:

```
sage: MPC = MPComplexField()
sage: u = MPC(1, 4)
sage: v = MPC(-2,5)
sage: u.agm(v, algorithm="pari")
-0.410522769709397 + 4.60061063922097*I
sage: u.agm(v, algorithm="principal")
1.24010691168158 - 0.472193567796433*I
sage: u.agm(v, algorithm="optimal")
-0.410522769709397 + 4.60061063922097*I
```

    algebraic_dependancy(*n*, ***kwds*)

        Returns a polynomial of degree at most n which is approximately satisfied by this complex number. Note that the returned polynomial need not be irreducible, and indeed usually won't be if z is a good approximation to an algebraic number of degree less than n.

        ALGORITHM: Uses the PARI C-library algdep command.

        INPUT: Type algdep? at the top level prompt. All additional parameters are passed onto the top-level algdep command.

        EXAMPLES:

```
sage: MPC = MPComplexField()
sage: z = (1/2)*(1 + sqrt(3.0) * MPC.0); z
0.500000000000000 + 0.866025403784439*I
sage: p = z.algebraic_dependency(5)
sage: p.factor()
(x + 1) * (x^2 - x + 1)^2
sage: z^2 - z + 1
1.11022302462516e-16
```

    algebraic_dependency(*n*, ***kwds*)

        Returns a polynomial of degree at most n which is approximately satisfied by this complex number. Note that the returned polynomial need not be irreducible, and indeed usually won't be if z is a good approximation to an algebraic number of degree less than n.

        ALGORITHM: Uses the PARI C-library algdep command.

        INPUT: Type algdep? at the top level prompt. All additional parameters are passed onto the top-level algdep command.

        EXAMPLES:

```
sage: MPC = MPComplexField()
sage: z = (1/2)*(1 + sqrt(3.0) * MPC.0); z
0.500000000000000 + 0.866025403784439*I
sage: p = z.algebraic_dependency(5)
sage: p.factor()
(x + 1) * (x^2 - x + 1)^2
sage: z^2 - z + 1
1.11022302462516e-16
```

arccos()

Return the arccosine of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: arccos(u)
1.11692611683177 - 2.19857302792094*I
```

arccosh()

Return the hyperbolic arccos of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: arccosh(u)
2.19857302792094 + 1.11692611683177*I
```

arccoth()

Return the hyperbolic arccotangent of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).arccoth()
0.40235947810852509365018983331 - 0.55357435889704525150853273009*I
```

arccsch()

Return the hyperbolic arcsine of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).arccsch()
0.53063753095251782601650945811 - 0.45227844715119068206365839783*I
```

arcsech()

Return the hyperbolic arcsecant of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).arcsech()
0.53063753095251782601650945811 - 1.1185178796437059371676632938*I
```

arcsin()

Return the arcsine of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: arcsin(u)
0.453870209963122 + 2.19857302792094*I
```

arcsinh()

Return the hyperbolic arcsine of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
```

```
sage: arcsinh(u)
2.18358521656456 + 1.09692154883014*I
```

arctan()

Return the arctangent of this complex number.

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: u = MPC(-2, 4)
sage: arctan(u)
-1.46704821357730 + 0.200586618131234*I
```

arctanh()

Return the hyperbolic arctangent of this complex number.

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: arctanh(u)
0.0964156202029962 + 1.37153510396169*I
```

argument()

The argument (angle) of the complex number, normalized so that $-\pi < \theta \leq \pi$.

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: i = MPC.0
sage: (i^2).argument()
3.14159265358979
sage: (1+i).argument()
0.785398163397448
sage: i.argument()
1.57079632679490
sage: (-i).argument()
-1.57079632679490
sage: (RR('-0.001') - i).argument()
-1.57179632646156
```

conjugate()

Return the complex conjugate of this complex number:

$$\text{conjugate}(a + ib) = a - ib.$$

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: i = MPC(0, 1)
sage: (1+i).conjugate()
1.00000000000000 - 1.00000000000000*I
```

cos()

Return the cosine of this complex number:

$$\cos(a + ib) = \cos a \cosh b - i \sin a \sinh b.$$

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: cos(u)
-11.3642347064011 - 24.8146514856342*I
```

cosh()

Return the hyperbolic cosine of this complex number:

$$\cosh(a + ib) = \cosh a \cos b + i \sinh a \sin b.$$

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: cosh(u)
-2.45913521391738 - 2.74481700679215*I
```

cotan()

Return the cotangent of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(53)
sage: (1+MPC(I)).cotan()
0.217621561854403 - 0.868014142895925*I
sage: i = MPComplexField(200).0
sage: (1+i).cotan()
0.21762156185440268136513424360523807352075436916785404091068 - 0.8680141428959249486358492C
sage: i = MPComplexField(220).0
sage: (1+i).cotan()
0.21762156185440268136513424360523807352075436916785404091068124239 - 0.8680141428959249486
```

coth()

Return the hyperbolic cotangent of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).coth()
0.86801414289592494863584920892 - 0.21762156185440268136513424361*I
```

csc()

Return the cosecent of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).csc()
0.62151801717042842123490780586 - 0.30393100162842645033448560451*I
```

csch()

Return the hyperbolic cosecent of this complex number.

EXAMPLES:
```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).csch()
0.30393100162842645033448560451 - 0.62151801717042842123490780586*I
```

dilog()

Return the complex dilogarithm of `self`.

The complex dilogarithm, or Spence's function, is defined by

$$Li_2(z) = -\int_0^z \frac{\log|1-\zeta|}{\zeta} d(\zeta) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}.$$

Note that the series definition can only be used for $|z| < 1$.

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: a = MPC(1,0)
sage: a.dilog()
1.64493406684823
sage: float(pi^2/6)
1.6449340668482262

sage: b = MPC(0,1)
sage: b.dilog()
-0.205616758356028 + 0.915965594177219*I

sage: c = MPC(0,0)
sage: c.dilog()
0
```

eta (*omit_frac=False*)

Return the value of the Dedekind η function on `self`, intelligently computed using $\mathbb{SL}(2, \mathbf{Z})$ transformations.

The η function is

$$\eta(z) = e^{\pi i z/12} \prod_{n=1}^{\infty} (1 - e^{2\pi i n z})$$

INPUT:

- `self` - element of the upper half plane (if not, raises a `ValueError`).

- `omit_frac` - (bool, default: `False`), if `True`, omit the $e^{\pi i z/12}$ factor.

OUTPUT: a complex number

ALGORITHM: Uses the PARI C library.

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: i = MPC.0
sage: z = 1+i; z.eta()
0.742048775836565 + 0.198831370229911*I
```

exp ()

Return the exponential of this complex number:

$$\exp(a + ib) = \exp(a)(\cos b + i \sin b).$$

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: exp(u)
-4.82980938326939 - 5.59205609364098*I
```

gamma()
>   Return the Gamma function evaluated at this complex number.
>
>   EXAMPLES:
>    ```
>    sage: MPC = MPComplexField(30)
>    sage: i = MPC.0
>    sage: (1+i).gamma()
>    0.49801567 - 0.15494983*I
>    ```
>
>   TESTS:
>    ```
>    sage: MPC(0).gamma()
>    Infinity
>    ```
>    ```
>    sage: MPC(-1).gamma()
>    Infinity
>    ```

gamma_inc(*t*)
>   Return the incomplete Gamma function evaluated at this complex number.
>
>   EXAMPLES:
>    ```
>    sage: C, i = MPComplexField(30).objgen()
>    sage: (1+i).gamma_inc(2 + 3*i)  # abs tol 2e-10
>    0.0020969149 - 0.059981914*I
>    sage: (1+i).gamma_inc(5)
>    -0.0013781309 + 0.0065198200*I
>    sage: C(2).gamma_inc(1 + i)
>    0.70709210 - 0.42035364*I
>    ```

imag()
>   Return imaginary part of self.
>
>   EXAMPLES:
>    ```
>    sage: C = MPComplexField(100)
>    sage: z = C(2, 3)
>    sage: x = z.imag(); x
>    3.0000000000000000000000000000
>    sage: x.parent()
>    Real Field with 100 bits of precision
>    ```

is_imaginary()
>   Return True if self is imaginary, i.e. has real part zero.
>
>   EXAMPLES:
>    ```
>    sage: C200 = MPComplexField(200)
>    sage: C200(1.23*i).is_imaginary()
>    True
>    sage: C200(1+i).is_imaginary()
>    False
>    ```

is_real()
>   Return True if self is real, i.e. has imaginary part zero.
>
>   EXAMPLES:
>    ```
>    sage: C200 = MPComplexField(200)
>    sage: C200(1.23).is_real()
>    True
>    ```

```
sage: C200(1+i).is_real()
False
```

is_square()

    This function always returns true as $\mathbf{C}$ is algebraically closed.

    EXAMPLES:
```
sage: C200 = MPComplexField(200)
sage: a = C200(2,1)
sage: a.is_square()
True
```

    $\mathbf{C}$ is algebraically closed, hence every element is a square:
```
sage: b = C200(5)
sage: b.is_square()
True
```

log()

    Return the logarithm of this complex number with the branch cut on the negative real axis:

$$\log(z) = \log|z| + i\arg(z).$$

    EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: log(u)
1.49786613677700 + 1.10714871779409*I
```

norm()

    Return the norm of a complex number, rounded with the rounding mode of the real part. The norm is the square of the absolute value:

$$\mathrm{norm}(a+ib) = a^2 + b^2.$$

    OUTPUT:

    A floating-point number in the real field of the real part (same precision, same rounding mode).

    EXAMPLES:

    This indeed acts as the square function when the imaginary component of self is equal to zero:
```
sage: MPC = MPComplexField()
sage: a = MPC(2,1)
sage: a.norm()
5.00000000000000
sage: b = MPC(4.2,0)
sage: b.norm()
17.6400000000000
sage: b^2
17.6400000000000
```

nth_root(*n*, *all=False*)

    The n-th root function.

    INPUT:

        •`all` - bool (default: `False`); if `True`, return a list of all n-th roots.

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: a = MPC(27)
sage: a.nth_root(3)
3.00000000000000
sage: a.nth_root(3, all=True)
[3.00000000000000, -1.50000000000000 + 2.59807621135332*I, -1.50000000000000 - 2.59807621135
```

prec()

Return precision of this complex number.

EXAMPLES:

```
sage: i = MPComplexField(2000).0
sage: i.prec()
2000
```

real()

Return the real part of `self`.

EXAMPLES:

```
sage: C = MPComplexField(100)
sage: z = C(2, 3)
sage: x = z.real(); x
2.0000000000000000000000000000
sage: x.parent()
Real Field with 100 bits of precision
```

sec()

Return the secant of this complex number.

EXAMPLES:

```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).sec()
0.49833703055518678521380589177 + 0.59108384172104504805039169297*I
```

sech()

Return the hyperbolic secant of this complex number.

EXAMPLES:

```
sage: MPC = MPComplexField(100)
sage: MPC(1,1).sech()
0.49833703055518678521380589177 - 0.59108384172104504805039169297*I
```

sin()

Return the sine of this complex number:

$$\sin(a + ib) = \sin a \cosh b + i \cos x \sinh b.$$

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: sin(u)
24.8313058489464 - 11.3566127112182*I
```

sinh()

Return the hyperbolic sine of this complex number:

$$\sinh(a + ib) = \sinh a \cos b + i \cosh a \sin b.$$

EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: sinh(u)
-2.37067416935200 - 2.84723908684883*I
```

sqr()
Return the square of a complex number:

$$(a + ib)^2 = (a^2 - b^2) + 2iab.$$

EXAMPLES:
```
sage: C = MPComplexField()
sage: a = C(5, 1)
sage: a.sqr()
24.0000000000000 + 10.0000000000000*I
```

sqrt()
Return the square root, taking the branch cut to be the negative real axis:

$$\sqrt{z} = \sqrt{|z|}(\cos(\arg(z)/2) + i\sin(\arg(z)/2)).$$

EXAMPLES:
```
sage: C = MPComplexField()
sage: a = C(24, 10)
sage: a.sqrt()
5.00000000000000 + 1.00000000000000*I
```

str(*base=10*, *truncate=True*)
Return a string of self.

INPUT:

  •`base` – base for output

  •`truncate` – if `True`, round off the last digits in printing to lessen confusing base-2 roundoff issues.

EXAMPLES:
```
sage: MPC = MPComplexField(64)
sage: z = MPC(-4, 3)/7
sage: z.str()
'-0.571428571428571429 + 0.428571428571428571*I'
sage: z.str(16)
'-0.92492492492492490 + 0.6db6db6db6db6db70*I'
sage: z.str(truncate=True)
'-0.571428571428571429 + 0.428571428571428571*I'
sage: z.str(2, True)
'-0.1001001001001001001001001001001001001001001001001001001001001001 + 0.0110110110110110110
```

tan()
Return the tangent of this complex number:

$$\tan(a + ib) = (\sin 2a + i\sinh 2b)/(\cos 2a + \cosh 2b).$$

EXAMPLES:

```
sage: MPC = MPComplexField()
sage: u = MPC(-2, 4)
sage: tan(u)
0.000507980623470039 + 1.00043851320205*I
```

tanh()
> Return the hyperbolic tangent of this complex number:

$$\tanh(a + ib) = (\sinh 2a + i \sin 2b)/(\cosh 2a + \cos 2b).$$

> EXAMPLES:
```
sage: MPC = MPComplexField()
sage: u = MPC(2, 4)
sage: tanh(u)
1.00468231219024 + 0.0364233692474037*I
```

zeta()
> Return the Riemann zeta function evaluated at this complex number.

> EXAMPLES:
```
sage: i = MPComplexField(30).gen()
sage: z = 1 + i
sage: z.zeta()
0.58215806 - 0.92684856*I
```

class sage.rings.complex_mpc.**MPCtoMPC**
> Bases: sage.categories.map.Map

> INPUT:

> There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

> TESTS:
```
sage: from sage.categories.map import Map
```

> Using a hom space:
```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

> Using domain and codomain:
```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

section()
> EXAMPLES:
```
sage: from sage.rings.complex_mpc import *
sage: C10 = MPComplexField(10)
sage: C100 = MPComplexField(100)
sage: f = MPCtoMPC(C100, C10)
sage: f.section()
Generic map:
```

```
             From: Complex Field with 10 bits of precision
             To:   Complex Field with 100 bits of precision
```

class sage.rings.complex_mpc.**MPFRtoMPC**

    Bases: sage.categories.map.Map

    INPUT:

    There can be one or two arguments of this init method. If it is one argument, it must be a hom space. If it is two arguments, it must be two parent structures that will be domain and codomain of the map-to-be-created.

    TESTS:

```
sage: from sage.categories.map import Map
```

    Using a hom space:

```
sage: Map(Hom(QQ, ZZ, Rings()))
Generic map:
  From: Rational Field
  To:   Integer Ring
```

    Using domain and codomain:

```
sage: Map(QQ['x'], SymmetricGroup(6))
Generic map:
  From: Univariate Polynomial Ring in x over Rational Field
  To:   Symmetric group of order 6! as a permutation group
```

sage.rings.complex_mpc.**late_import**()

    Import the objects/modules after build (when needed).

    TESTS:

```
sage: sage.rings.complex_mpc.late_import()
```

sage.rings.complex_mpc.**split_complex_string**(*string*, *base=10*)

    Split and return in that order the real and imaginary parts of a complex in a string.

    This is an internal function.

    EXAMPLES:

```
sage: sage.rings.complex_mpc.split_complex_string('123.456e789')
('123.456e789', None)
sage: sage.rings.complex_mpc.split_complex_string('123.456e789*I')
(None, '123.456e789')
sage: sage.rings.complex_mpc.split_complex_string('123.+456e789*I')
('123.', '+456e789')
sage: sage.rings.complex_mpc.split_complex_string('123.456e789', base=2)
(None, None)
```

FIELD OF ARBITRARY PRECISION COMPLEX INTERVALS

AUTHORS:

- William Stein wrote complex_field.py.

- William Stein (2006-01-26): complete rewrite

Then `complex_field.py` was copied to `complex_interval_field.py` and heavily modified:

- Carl Witty (2007-10-24): rewrite for intervals

- Niles Johnson (2010-08): trac ticket #3893: `random_element()` should pass on `*args` and `**kwds`.

- Travis Scrimshaw (2012-10-18): Added documentation to get full coverage.

Note: The `ComplexIntervalField` differs from `ComplexField` in that `ComplexIntervalField` only gives the digits with exact precision, then a `?` signifying that that digit can have an error of $+/-1$.

---

sage.rings.complex_interval_field.**ComplexIntervalField**(*prec=53*, *names=None*)
    Return the complex interval field with real and imaginary parts having `prec` *bits* of precision.

    EXAMPLES:
```
sage: ComplexIntervalField()
Complex Interval Field with 53 bits of precision
sage: ComplexIntervalField(100)
Complex Interval Field with 100 bits of precision
sage: ComplexIntervalField(100).base_ring()
Real Interval Field with 100 bits of precision
sage: i = ComplexIntervalField(200).gen()
sage: i^2
-1
sage: i^i
0.207879576350761908546955619834978770033877841631769608075136?
```

class sage.rings.complex_interval_field.**ComplexIntervalField_class**(*prec=53*)
    Bases: sage.rings.ring.Field

    The field of complex (interval) numbers.

    EXAMPLES:
```
sage: C = ComplexIntervalField(); C
Complex Interval Field with 53 bits of precision
sage: Q = RationalField()
sage: C(1/3)
0.3333333333333334?
sage: C(1/3, 2)
0.3333333333333334? + 2*I
```

We can also coerce rational numbers and integers into C, but coercing a polynomial will raise an exception:

```
sage: Q = RationalField()
sage: C(1/3)
0.3333333333333334?
sage: S = PolynomialRing(Q, 'x')
sage: C(S.gen())
Traceback (most recent call last):
...
TypeError: unable to coerce to a ComplexIntervalFieldElement
```

This illustrates precision:

```
sage: CIF = ComplexIntervalField(10); CIF(1/3, 2/3)
0.334? + 0.667?*I
sage: CIF
Complex Interval Field with 10 bits of precision
sage: CIF = ComplexIntervalField(100); CIF
Complex Interval Field with 100 bits of precision
sage: z = CIF(1/3, 2/3); z
0.333333333333333333333333333334? + 0.666666666666666666666666666667?*I
```

We can load and save complex numbers and the complex interval field:

```
sage: cmp(loads(z.dumps()), z)
0
sage: loads(CIF.dumps()) == CIF
True
sage: k = ComplexIntervalField(100)
sage: loads(dumps(k)) == k
True
```

This illustrates basic properties of a complex (interval) field:

```
sage: CIF = ComplexIntervalField(200)
sage: CIF.is_field()
True
sage: CIF.characteristic()
0
sage: CIF.precision()
200
sage: CIF.variable_name()
'I'
sage: CIF == ComplexIntervalField(200)
True
sage: CIF == ComplexIntervalField(53)
False
sage: CIF == 1.1
False
sage: CIF = ComplexIntervalField(53)

sage: CIF.category()
Category of fields
sage: TestSuite(CIF).run()
```

characteristic()

> Return the characteristic of the complex (interval) field, which is 0.

> EXAMPLES:

```
sage: CIF.characteristic()
0
```

gen (*n=0*)

Return the generator of the complex (interval) field.

EXAMPLES:
```
sage: CIF.0
1*I
sage: CIF.gen(0)
1*I
```

is_exact ()

The complex interval field is not exact.

EXAMPLES:
```
sage: CIF.is_exact()
False
```

is_field (*proof=True*)

Return `True`, since the complex numbers are a field.

EXAMPLES:
```
sage: CIF.is_field()
True
```

is_finite ()

Return `False`, since the complex numbers are infinite.

EXAMPLES:
```
sage: CIF.is_finite()
False
```

ngens ()

The number of generators of this complex (interval) field as an $\mathbf{R}$-algebra.

There is one generator, namely `sqrt(-1)`.

EXAMPLES:
```
sage: CIF.ngens()
1
```

pi ()

Returns π as an element in the complex (interval) field.

EXAMPLES:
```
sage: ComplexIntervalField(100).pi()
3.14159265358979323846264338328?
```

prec ()

Returns the precision of `self` (in bits).

EXAMPLES:
```
sage: CIF.prec()
53
sage: ComplexIntervalField(200).prec()
200
```

precision()
>   Returns the precision of `self` (in bits).
>
>   EXAMPLES:
>   ```
>   sage: CIF.prec()
>   53
>   sage: ComplexIntervalField(200).prec()
>   200
>   ```

random_element(**args*, ***kwds*)
>   Create a random element of `self`.
>
>   This simply chooses the real and imaginary part randomly, passing arguments and keywords to the underlying real interval field.
>
>   EXAMPLES:
>   ```
>   sage: CIF.random_element()
>   0.15363619378561300? - 0.50298737524751780?*I
>   sage: CIF.random_element(10, 20)
>   18.047949821611205? + 10.255727028308920?*I
>   ```
>
>   Passes extra positional or keyword arguments through:
>   ```
>   sage: CIF.random_element(max=0, min=-5)
>   -0.079017286535590259? - 2.8712089896087117?*I
>   ```

scientific_notation(*status=None*)
>   Set or return the scientific notation printing flag.
>
>   If this flag is `True` then complex numbers with this space as parent print using scientific notation.
>
>   EXAMPLES:
>   ```
>   sage: CIF((0.025, 2))
>   0.025000000000000002? + 2*I
>   sage: CIF.scientific_notation(True)
>   sage: CIF((0.025, 2))
>   2.5000000000000002?e-2 + 2*I
>   sage: CIF.scientific_notation(False)
>   sage: CIF((0.025, 2))
>   0.025000000000000002? + 2*I
>   ```

to_prec(*prec*)
>   Returns a complex interval field with the given precision.
>
>   EXAMPLES:
>   ```
>   sage: CIF.to_prec(150)
>   Complex Interval Field with 150 bits of precision
>   sage: CIF.to_prec(15)
>   Complex Interval Field with 15 bits of precision
>   sage: CIF.to_prec(53) is CIF
>   True
>   ```

zeta(*n=2*)
>   Return a primitive n-th root of unity.

>   **Todo**

Implement `ComplexIntervalFieldElement` multiplicative order and set this output to have multiplicative order n.

INPUT:

> •n – an integer (default: 2)

OUTPUT:

A complex n-th root of unity.

EXAMPLES:

```
sage: CIF.zeta(2)
-1
sage: CIF.zeta(5)
0.309016994374948? + 0.9510565162951536?*I
```

sage.rings.complex_interval_field.**is_ComplexIntervalField**(*x*)

> Check if x is a `ComplexIntervalField`.

EXAMPLES:

```
sage: from sage.rings.complex_interval_field import is_ComplexIntervalField as is_CIF
sage: is_CIF(CIF)
True
sage: is_CIF(CC)
False
```

sage.rings.complex_interval_field.**late_import**()

> Import the objects/modules after build (when needed).

TESTS:

```
sage: sage.rings.complex_interval_field.late_import()
```

ARBITRARY PRECISION COMPLEX INTERVALS

This is a simple complex interval package, using intervals which are axis-aligned rectangles in the complex plane. It has very few special functions, and it does not use any special tricks to keep the size of the intervals down.

AUTHORS:

These authors wrote `complex_number.pyx`:

- William Stein (2006-01-26): complete rewrite

- Joel B. Mohler (2006-12-16): naive rewrite into pyrex

- William Stein(2007-01): rewrite of Mohler's rewrite

Then `complex_number.pyx` was copied to `complex_interval.pyx` and heavily modified:

- Carl Witty (2007-10-24): rewrite to become a complex interval package

- Travis Scrimshaw (2012-10-18): Added documentation to get full coverage.

Todo

Implement `ComplexIntervalFieldElement` multiplicative order similar to `ComplexNumber` multiplicative order with `_set_multiplicative_order(n)` and `ComplexNumber.multiplicative_order()` methods.

---

class `sage.rings.complex_interval.`**`ComplexIntervalFieldElement`**
>   Bases: `sage.structure.element.FieldElement`

>   A complex interval.

>   EXAMPLES:
>   ```
>   sage: I = CIF.gen()
>   sage: b = 1.5 + 2.5*I
>   sage: TestSuite(b).run()
>   ```

>   **`arg`**`()`
>>   Same as `argument()`.

>>   EXAMPLES:
>>       ```
>>       sage: i = CIF.0
>>       sage: (i^2).arg()
>>       3.141592653589794?
>>       ```

>   **`argument`**`()`
>>   The argument (angle) of the complex number, normalized so that $-\pi < \theta.lower() \leq \pi$.

>>   We raise a `ValueError` if the interval strictly contains 0, or if the interval contains only 0.

> **Warning:** We do not always use the standard branch cut for argument! If the interval crosses the negative real axis, then the argument will be an interval whose lower bound is less than π and whose upper bound is more than π; in effect, we move the branch cut away from the interval.

EXAMPLES:
```
sage: i = CIF.0
sage: (i^2).argument()
3.141592653589794?
sage: (1+i).argument()
0.785398163397449?
sage: i.argument()
1.570796326794897?
sage: (-i).argument()
-1.570796326794897?
sage: (RR('-0.001') - i).argument()
-1.571796326461564?
sage: CIF(2).argument()
0
sage: CIF(-2).argument()
3.141592653589794?
```

Here we see that if the interval crosses the negative real axis, then the argument can exceed π, and we we violate the standard interval guarantees in the process:
```
sage: CIF(-2, RIF(-0.1, 0.1)).argument().str(style='brackets')
'[3.0916342578678501 .. 3.1915510493117365]'
sage: CIF(-2, -0.1).argument()
-3.091634257867851?
```

bisection()

Returns the bisection of self into four intervals whose union is self and intersection is center().

EXAMPLES:
```
sage: z = CIF(RIF(2, 3), RIF(-5, -4))
sage: z.bisection()
(3.? - 5.?*I, 3.? - 5.?*I, 3.? - 5.?*I, 3.? - 5.?*I)
sage: for z in z.bisection():
...       print z.real().endpoints(), z.imag().endpoints()
(2.00000000000000, 2.50000000000000) (-5.00000000000000, -4.50000000000000)
(2.50000000000000, 3.00000000000000) (-5.00000000000000, -4.50000000000000)
(2.00000000000000, 2.50000000000000) (-4.50000000000000, -4.00000000000000)
(2.50000000000000, 3.00000000000000) (-4.50000000000000, -4.00000000000000)

sage: z = CIF(RIF(sqrt(2), sqrt(3)), RIF(e, pi))
sage: a, b, c, d = z.bisection()
sage: a.intersection(b).intersection(c).intersection(d) == CIF(z.center())
True

sage: zz = a.union(b).union(c).union(c)
sage: zz.real().endpoints() == z.real().endpoints()
True
sage: zz.imag().endpoints() == z.imag().endpoints()
True
```

center()

Returns the closest floating-point approximation to the center of the interval.

EXAMPLES:

```
sage: CIF(RIF(1, 2), RIF(3, 4)).center()
1.50000000000000 + 3.50000000000000*I
```

conjugate()

Return the complex conjugate of this complex number.

EXAMPLES:

```
sage: i = CIF.0
sage: (1+i).conjugate()
1 - 1*I
```

contains_zero()

Returns `True` if `self` is an interval containing zero.

EXAMPLES:

```
sage: CIF(0).contains_zero()
True
sage: CIF(RIF(-1, 1), 1).contains_zero()
False
```

crosses_log_branch_cut()

Returns `True` if this interval crosses the standard branch cut for `log()` (and hence for exponentiation) and for argument. (Recall that this branch cut is infinitesimally below the negative portion of the real axis.)

EXAMPLES:

```
sage: z = CIF(1.5, 2.5) - CIF(0, 2.50000000000000001); z
1.5000000000000000? + -1.?e-15*I
sage: z.crosses_log_branch_cut()
False
sage: CIF(-2, RIF(-0.1, 0.1)).crosses_log_branch_cut()
True
```

diameter()

Returns a somewhat-arbitrarily defined "diameter" for this interval.

The diameter of an interval is the maximum of the diameter of the real and imaginary components, where diameter on a real interval is defined as absolute diameter if the interval contains zero, and relative diameter otherwise.

EXAMPLES:

```
sage: CIF(RIF(-1, 1), RIF(13, 17)).diameter()
2.00000000000000
sage: CIF(RIF(-0.1, 0.1), RIF(13, 17)).diameter()
0.266666666666667
sage: CIF(RIF(-1, 1), 15).diameter()
2.00000000000000
```

exp()

Compute e^z or $\exp(z)$ where z is the complex number `self`.

EXAMPLES:

```
sage: i = ComplexIntervalField(300).0
sage: z = 1 + i
sage: z.exp()
1.4686939399158851571389675973266042613269567366290087227976756763109369658595121387227245038
```

imag()
    Return imaginary part of `self`.

    EXAMPLES:
```
sage: i = ComplexIntervalField(100).0
sage: z = 2 + 3*i
sage: x = z.imag(); x
3
sage: x.parent()
Real Interval Field with 100 bits of precision
```

intersection(*other*)
    Returns the intersection of the two complex intervals `self` and `other`.

    EXAMPLES:
```
sage: CIF(RIF(1, 3), RIF(1, 3)).intersection(CIF(RIF(2, 4), RIF(2, 4))).str(style='brackets'
'[2.0000000000000000 .. 3.0000000000000000] + [2.0000000000000000 .. 3.0000000000000000]*I'
sage: CIF(RIF(1, 2), RIF(1, 3)).intersection(CIF(RIF(3, 4), RIF(2, 4)))
Traceback (most recent call last):
...
ValueError: intersection of non-overlapping intervals
```

is_exact()
    Returns whether this complex interval is exact (i.e. contains exactly one complex value).

    EXAMPLES:
```
sage: CIF(3).is_exact()
True
sage: CIF(0, 2).is_exact()
True
sage: CIF(-4, 0).sqrt().is_exact()
True
sage: CIF(-5, 0).sqrt().is_exact()
False
sage: CIF(0, 2*pi).is_exact()
False
sage: CIF(e).is_exact()
False
sage: CIF(1e100).is_exact()
True
sage: (CIF(1e100) + 1).is_exact()
False
```

is_square()
    This function always returns `True` as $\mathbf{C}$ is algebraically closed.

    EXAMPLES:
```
sage: CIF(2, 1).is_square()
True
```

log(*base=None*)
    Complex logarithm of z.

    > **Warning:** This does always not use the standard branch cut for complex log! See the docstring for `argument()` to see what we do instead.

    EXAMPLES:

```
sage: a = CIF(RIF(3, 4), RIF(13, 14))
sage: a.log().str(style='brackets')
'[2.5908917751460420 .. 2.6782931373360067] + [1.2722973952087170 .. 1.3597029935721503]*I'
sage: a.log().exp().str(style='brackets')
'[2.7954667135098274 .. 4.2819545928390213] + [12.751682453911920 .. 14.237018048974635]*I'
sage: a in a.log().exp()
True
```

If the interval crosses the negative real axis, then we don't use the standard branch cut (and we violate the interval guarantees):

```
sage: CIF(-3, RIF(-1/4, 1/4)).log().str(style='brackets')
'[1.0986122886681095 .. 1.1020725100903968] + [3.0584514217013518 .. 3.2247338854782349]*I'
sage: CIF(-3, -1/4).log()
1.102072510090397? - 3.058451421701352?*I
```

Usually if an interval contains zero, we raise an exception:

```
sage: CIF(RIF(-1,1),RIF(-1,1)).log()
Traceback (most recent call last):
...
ValueError: Can't take the argument of interval strictly containing zero
```

But we allow the exact input zero:

```
sage: CIF(0).log()
[-infinity .. -infinity]
```

If a base is passed from another function, we can accommodate this:

```
sage: CIF(-1,1).log(2)
0.500000000000000? + 3.399270106370396?*I
```

norm()

Returns the norm of this complex number.

If $c = a + bi$ is a complex number, then the norm of c is defined as the product of c and its complex conjugate:

$$extnorm(c) = extnorm(a + bi) = c \cdot \overline{c} = a^2 + b^2.$$

The norm of a complex number is different from its absolute value. The absolute value of a complex number is defined to be the square root of its norm. A typical use of the complex norm is in the integral domain $\mathbf{Z}[i]$ of Gaussian integers, where the norm of each Gaussian integer $c = a + bi$ is defined as its complex norm.

See also:

- sage.rings.complex_double.ComplexDoubleElement.norm()

EXAMPLES:

```
sage: CIF(2, 1).norm()
5
sage: CIF(1, -2).norm()
5
```

overlaps(*other*)

Returns True if self and other are intervals with at least one value in common.

EXAMPLES:

```
sage: CIF(0).overlaps(CIF(RIF(0, 1), RIF(-1, 0)))
True
sage: CIF(1).overlaps(CIF(1, 1))
False
```

plot (*pointsize=10*, ***kwds*)

Plot a complex interval as a rectangle.

EXAMPLES:
```
sage: sum(plot(CIF(RIF(1/k, 1/k), RIF(-k, k))) for k in [1..10])
Graphics object consisting of 20 graphics primitives
```

Exact and nearly exact points are still visible:
```
sage: plot(CIF(pi, 1), color='red') + plot(CIF(1, e), color='purple') + plot(CIF(-1, -1))
Graphics object consisting of 6 graphics primitives
```

A demonstration that $z \mapsto z^2$ acts chaotically on $|z| = 1$:
```
sage: z = CIF(0, 2*pi/1000).exp()
sage: g = Graphics()
sage: for i in range(40):
...        z = z^2
...        g += z.plot(color=(1./(40-i), 0, 1))
...
sage: g
Graphics object consisting of 80 graphics primitives
```

prec ()

Return precision of this complex number.

EXAMPLES:
```
sage: i = ComplexIntervalField(2000).0
sage: i.prec()
2000
```

real ()

Return real part of self.

EXAMPLES:
```
sage: i = ComplexIntervalField(100).0
sage: z = 2 + 3*i
sage: x = z.real(); x
2
sage: x.parent()
Real Interval Field with 100 bits of precision
```

sqrt (*all=False*, ***kwds*)

The square root function.

> **Warning:** We approximate the standard branch cut along the negative real axis, with sqrt(-r^2)
> = i*r for positive real r; but if the interval crosses the negative real axis, we pick the root with
> positive imaginary component for the entire interval.

INPUT:

- all – bool (default: False); if True, return a list of all square roots.

EXAMPLES:
```
sage: CIF(-1).sqrt()^2
-1
sage: sqrt(CIF(2))
1.414213562373095?
sage: sqrt(CIF(-1))
1*I
sage: sqrt(CIF(2-I))^2
2.000000000000000? - 1.000000000000000?*I
sage: CC(-2-I).sqrt()^2
-2.00000000000000 - 1.00000000000000*I
```

Here, we select a non-principal root for part of the interval, and violate the standard interval guarantees:
```
sage: CIF(-5, RIF(-1, 1)).sqrt().str(style='brackets')
'[-0.22250788030178321 .. 0.22250788030178296] + [2.2251857651053086 .. 2.2581008643532262]*
sage: CIF(-5, -1).sqrt()
0.222507880301783? - 2.247111425095870?*I
```

str(*base=10, style=None*)
 Returns a string representation of `self`.

 EXAMPLES:
```
sage: CIF(1.5).str()
'1.5000000000000000?'
sage: CIF(1.5, 2.5).str()
'1.5000000000000000? + 2.5000000000000000?*I'
sage: CIF(1.5, -2.5).str()
'1.5000000000000000? - 2.5000000000000000?*I'
sage: CIF(0, -2.5).str()
'-2.5000000000000000?*I'
sage: CIF(1.5).str(base=3)
'1.1111111111111111111111111111111111112?'
sage: CIF(1, pi).str(style='brackets')
'[1.0000000000000000 .. 1.0000000000000000] + [3.1415926535897931 .. 3.1415926535897936]*I'
```

 See also:

> •`RealIntervalFieldElement.str()`

union(*other*)
 Returns the smallest complex interval including the two complex intervals `self` and `other`.

 EXAMPLES:
```
sage: CIF(0).union(CIF(5, 5)).str(style='brackets')
'[0.0000000000000000 .. 5.0000000000000000] + [0.00000000000000000 .. 5.0000000000000000]*I
```

sage.rings.complex_interval.**create_ComplexIntervalFieldElement**(*s_real,*
                                                                    s_imag=None,
                                                                    pad=0,
                                                                    min_prec=53)
 Return the complex number defined by the strings `s_real` and `s_imag` as an element of
 `ComplexIntervalField(prec=n)`, where n potentially has slightly more (controlled by pad) bits than
 given by s.

 INPUT:

> •`s_real` – a string that defines a real number (or something whose string representation defines a number)

- `s_imag` – a string that defines a real number (or something whose string representation defines a number)

- `pad` – an integer at least 0.

- `min_prec` – number will have at least this many bits of precision, no matter what.

EXAMPLES:

```
sage: ComplexIntervalFieldElement('2.3')
2.300000000000000?
sage: ComplexIntervalFieldElement('2.3','1.1')
2.300000000000000? + 1.100000000000000?*I
sage: ComplexIntervalFieldElement(10)
10
sage: ComplexIntervalFieldElement(10,10)
10 + 10*I
sage: ComplexIntervalFieldElement(1.000000000000000000000000000,2)
1 + 2*I
sage: ComplexIntervalFieldElement(1,2.00000000000000000000)
1 + 2*I
sage: ComplexIntervalFieldElement(1.23456789012345678901234.5, 5.4321098654321987654321)
1.234567890123456789012350? + 5.432109865432198765432000?*I
```

TESTS:

Make sure we've rounded up `log(10,2)` enough to guarantee sufficient precision (trac ticket #10164). This is a little tricky because at the time of writing, we don't support intervals long enough to trip the error. However, at least we can make sure that we either do it correctly or fail noisily:

```
sage: c_CIFE = sage.rings.complex_interval.create_ComplexIntervalFieldElement
sage: for kp in range(2,6):
...       s = '1.' + '0'*10**kp + '1'
...       try:
...           assert c_CIFE(s,0).real()-1 != 0
...           assert c_CIFE(0,s).imag()-1 != 0
...       except TypeError:
...           pass
```

sage.rings.complex_interval.**is_ComplexIntervalFieldElement**(*x*)

Check if x is a `ComplexIntervalFieldElement`.

EXAMPLES:

```
sage: from sage.rings.complex_interval import is_ComplexIntervalFieldElement as is_CIFE
sage: is_CIFE(CIF(2))
True
sage: is_CIFE(CC(2))
False
```

sage.rings.complex_interval.**make_ComplexIntervalFieldElement0**(*fld*, *re*, *im*)

Construct a `ComplexIntervalFieldElement` for pickling.

TESTS:

```
sage: a = CIF(1 + I)
sage: loads(dumps(a)) == a # indirect doctest
True
```

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