
Sage Reference Manual: Utilities

Release 6.8

The Sage Development Team

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GENERAL INFRASTRUCTURE

1.1 Default Settings

AUTHORS: William Stein and David Kohel

`sage.misc.defaults.latex_variable_names` (*n*, *name=None*)

`sage.misc.defaults.series_precision` ()

Return the Sage-wide precision for series (symbolic, power series, Laurent series).

EXAMPLES:

```
sage: series_precision()
20
```

`sage.misc.defaults.set_default_variable_name` (*name*, *separator=''*)

Change the default variable name and separator.

`sage.misc.defaults.set_series_precision` (*prec*)

Change the Sage-wide precision for series (symbolic, power series, Laurent series).

EXAMPLES:

```
sage: set_series_precision(5)
sage: series_precision()
5
sage: set_series_precision(20)
```

`sage.misc.defaults.variable_names` (*n*, *name=None*)

1.2 Functional notation

These are functions so that you can write `foo(x)` instead of `x.foo()` in certain common cases.

AUTHORS:

- William Stein: Initial version
- David Joyner (2005-12-20): More Examples

`sage.misc.functional.N` (*x*, *prec=None*, *digits=None*, *algorithm=None*)

Returns a numerical approximation of an object *x* with at least *prec* bits (or decimal *digits*) of precision.

Note: Both upper case *N* and lower case *n* are aliases for `numerical_approx()`, and all three may be used as methods.

INPUT:

- `x` - an object that has a `numerical_approx` method, or can be coerced into a real or complex field
- `prec` (optional) - an integer (bits of precision)
- `digits` (optional) - an integer (digits of precision)
- `algorithm` (optional) - a string specifying the algorithm to use for functions that implement more than one

If neither the `prec` or `digits` are specified, the default is 53 bits of precision. If both are specified, then `prec` is used.

EXAMPLES:

```
sage: numerical_approx(pi, 10)
3.1
sage: numerical_approx(pi, digits=10)
3.141592654
sage: numerical_approx(pi^2 + e, digits=20)
12.587886229548403854
sage: n(pi^2 + e)
12.5878862295484
sage: N(pi^2 + e)
12.5878862295484
sage: n(pi^2 + e, digits=50)
12.587886229548403854194778471228813633070946500941
sage: a = CC(-5).n(prec=40)
sage: b = ComplexField(40)(-5)
sage: a == b
True
sage: parent(a) is parent(b)
True
sage: numerical_approx(9)
9.000000000000000
```

You can also usually use method notation.

```
sage: (pi^2 + e).n()
12.5878862295484
sage: (pi^2 + e).N()
12.5878862295484
sage: (pi^2 + e).numerical_approx()
12.5878862295484
```

Vectors and matrices may also have their entries approximated.

```
sage: v = vector(RDF, [1,2,3])
sage: v.n()
(1.000000000000000, 2.000000000000000, 3.000000000000000)

sage: v = vector(CDF, [1,2,3])
sage: v.n()
(1.000000000000000, 2.000000000000000, 3.000000000000000)
sage: _.parent()
Vector space of dimension 3 over Complex Field with 53 bits of precision
sage: v.n(prec=20)
(1.0000, 2.0000, 3.0000)

sage: u = vector(QQ, [1/2, 1/3, 1/4])
sage: n(u, prec=15)
```

```

(0.5000, 0.3333, 0.2500)
sage: n(u, digits=5)
(0.50000, 0.33333, 0.25000)

sage: v = vector(QQ, [1/2, 0, 0, 1/3, 0, 0, 0, 1/4], sparse=True)
sage: u = v.numerical_approx(digits=4)
sage: u.is_sparse()
True
sage: u
(0.5000, 0.0000, 0.0000, 0.3333, 0.0000, 0.0000, 0.0000, 0.2500)

sage: A = matrix(QQ, 2, 3, range(6))
sage: A.n()
[0.0000000000000000  1.0000000000000000  2.0000000000000000]
[ 3.0000000000000000  4.0000000000000000  5.0000000000000000]

sage: B = matrix(Integers(12), 3, 8, srange(24))
sage: N(B, digits=2)
[0.00  1.0  2.0  3.0  4.0  5.0  6.0  7.0]
[ 8.0  9.0 10. 11. 0.00  1.0  2.0  3.0]
[ 4.0  5.0  6.0  7.0  8.0  9.0 10. 11.]

```

Internally, numerical approximations of real numbers are stored in base-2. Therefore, numbers which look the same in their decimal expansion might be different:

```

sage: x=N(pi, digits=3); x
3.14
sage: y=N(3.14, digits=3); y
3.14
sage: x==y
False
sage: x.str(base=2)
'11.001001000100'
sage: y.str(base=2)
'11.001000111101'

```

Increasing the precision of a floating point number is not allowed:

```

sage: CC(-5).n(prec=100)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 100 bits, use at most 53 bits
sage: n(1.3r, digits=20)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 70 bits, use at most 53 bits
sage: RealField(24).pi().n()
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 53 bits, use at most 24 bits

```

As an exceptional case, `digits=1` usually leads to 2 digits (one significant) in the decimal output (see [ticket #11647](#)):

```

sage: N(pi, digits=1)
3.2
sage: N(pi, digits=2)
3.1
sage: N(100*pi, digits=1)

```

```
320.  
sage: N(100*pi, digits=2)  
310.
```

In the following example, `pi` and `3` are both approximated to two bits of precision and then subtracted, which kills two bits of precision:

```
sage: N(pi, prec=2)  
3.0  
sage: N(3, prec=2)  
3.0  
sage: N(pi - 3, prec=2)  
0.00
```

TESTS:

```
sage: numerical_approx(I)  
1.000000000000000*I  
sage: x = QQ['x'].gen()  
sage: F.<k> = NumberField(x^2+2, embedding=sqrt(CC(2))*CC.0)  
sage: numerical_approx(k)  
1.41421356237309*I
```

```
sage: type(numerical_approx(CC(1/2)))  
<type 'sage.rings.complex_number.ComplexNumber'>
```

The following tests [trac ticket #10761](#), in which `n()` would break when called on complex-valued algebraic numbers.

```
sage: E = matrix(3, [3,1,6,5,2,9,7,3,13]).eigenvalues(); E  
[18.16815365088822?, -0.08407682544410650? - 0.2190261484802906?*I, -0.08407682544410650? + 0.2190261484802906?*I]  
sage: E[1].parent()  
Algebraic Field  
sage: [a.n() for a in E]  
[18.1681536508882, -0.0840768254441065 - 0.219026148480291*I, -0.0840768254441065 + 0.219026148480291*I]
```

Make sure we've rounded up $\log(10,2)$ enough to guarantee sufficient precision ([trac ticket #10164](#)):

```
sage: ks = 4*10**5, 10**6  
sage: check_str_length = lambda k: len(str(numerical_approx(1+10**-k, digits=k+1)))-1 >= k+1  
sage: check_precision = lambda k: numerical_approx(1+10**-k, digits=k+1)-1 > 0  
sage: all(check_str_length(k) and check_precision(k) for k in ks)  
True
```

Testing we have sufficient precision for the golden ratio ([trac ticket #12163](#)), note that the decimal point adds 1 to the string length:

```
sage: len(str(n(golden_ratio, digits=5000)))  
5001  
sage: len(str(n(golden_ratio, digits=5000000))) # long time (4s on sage.math, 2012)  
5000001
```

Check that [trac ticket #14778](#) is fixed:

```
sage: n(0, algorithm='foo')  
0.000000000000000
```

`sage.misc.functional.additive_order(x)`

Returns the additive order of x .

EXAMPLES:


```

sage: additive_order(5)
+Infinity
sage: additive_order(Mod(5,11))
11
sage: additive_order(Mod(4,12))
3

```

`sage.misc.functional.base_field(x)`

Returns the base field over which x is defined.

EXAMPLES:

```

sage: R = PolynomialRing(GF(7), 'x')
sage: base_ring(R)
Finite Field of size 7
sage: base_field(R)
Finite Field of size 7

```

This catches base rings which are fields as well, but does not implement a `base_field` method for objects which do not have one:

```

sage: R.base_field()
Traceback (most recent call last):
...
AttributeError: 'PolynomialRing_dense_mod_p_with_category' object has no attribute 'base_field'

```

`sage.misc.functional.base_ring(x)`

Returns the base ring over which x is defined.

EXAMPLES:

```

sage: R = PolynomialRing(GF(7), 'x')
sage: base_ring(R)
Finite Field of size 7

```

`sage.misc.functional.basis(x)`

Returns the fixed basis of x .

EXAMPLES:

```

sage: V = VectorSpace(QQ,3)
sage: S = V.subspace([[1,2,0],[2,2,-1]])
sage: basis(S)
[
(1, 0, -1),
(0, 1, 1/2)
]

```

`sage.misc.functional.category(x)`

Returns the category of x .

EXAMPLES:

```

sage: V = VectorSpace(QQ,3)
sage: category(V)
Category of vector spaces with basis over quotient fields

```

`sage.misc.functional.characteristic_polynomial(x, var='x')`

Returns the characteristic polynomial of x in the given variable.

EXAMPLES:

```
sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
sage: charpoly(A)
x^3 - 15*x^2 - 18*x
sage: charpoly(A, 't')
t^3 - 15*t^2 - 18*t

sage: k.<alpha> = GF(7^10); k
Finite Field in alpha of size 7^10
sage: alpha.charpoly('T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
sage: characteristic_polynomial(alpha, 'T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
```

Ensure the variable name of the polynomial does not conflict with variables used within the matrix, and that non-integral powers of variables don't confuse the computation ([trac ticket #14403](#)):

```
sage: y = var('y')
sage: a = matrix([[x, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])
sage: characteristic_polynomial(a).list()
[x, -3*x - 1, 3*x + 3, -x - 3, 1]
sage: b = matrix([[y^(1/2), 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])
sage: charpoly(b).list()
[sqrt(y), -3*sqrt(y) - 1, 3*sqrt(y) + 3, -sqrt(y) - 3, 1]
```

`sage.misc.functional.charpoly(x, var='x')`

Returns the characteristic polynomial of `x` in the given variable.

EXAMPLES:

```
sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
sage: charpoly(A)
x^3 - 15*x^2 - 18*x
sage: charpoly(A, 't')
t^3 - 15*t^2 - 18*t

sage: k.<alpha> = GF(7^10); k
Finite Field in alpha of size 7^10
sage: alpha.charpoly('T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
sage: characteristic_polynomial(alpha, 'T')
T^10 + T^6 + T^5 + 4*T^4 + T^3 + 2*T^2 + 3*T + 3
```

Ensure the variable name of the polynomial does not conflict with variables used within the matrix, and that non-integral powers of variables don't confuse the computation ([trac ticket #14403](#)):

```
sage: y = var('y')
sage: a = matrix([[x, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])
sage: characteristic_polynomial(a).list()
[x, -3*x - 1, 3*x + 3, -x - 3, 1]
sage: b = matrix([[y^(1/2), 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])
sage: charpoly(b).list()
[sqrt(y), -3*sqrt(y) - 1, 3*sqrt(y) + 3, -sqrt(y) - 3, 1]
```

`sage.misc.functional.coerce(P, x)`

Attempts to coerce `x` to type `P` if possible.

EXAMPLES:

```
sage: type(5)
<type 'sage.rings.integer.Integer'>
sage: type(coerce(QQ, 5))
<type 'sage.rings.rational.Rational'>
```

`sage.misc.functional.cyclotomic_polynomial(n, var='x')`

Returns the n^{th} cyclotomic polynomial.

EXAMPLES:

```
sage: cyclotomic_polynomial(3)
x^2 + x + 1
sage: cyclotomic_polynomial(4)
x^2 + 1
sage: cyclotomic_polynomial(9)
x^6 + x^3 + 1
sage: cyclotomic_polynomial(10)
x^4 - x^3 + x^2 - x + 1
sage: cyclotomic_polynomial(11)
x^10 + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1
```

`sage.misc.functional.decomposition(x)`

Returns the decomposition of x .

EXAMPLES:

```
sage: M = matrix([[2, 3], [3, 4]])
sage: M.decomposition()
[
(Ambient free module of rank 2 over the principal ideal domain Integer Ring, True)
]

sage: G.<a,b> = DirichletGroup(20)
sage: c = a*b
sage: d = c.decomposition(); d
[Dirichlet character modulo 4 of conductor 4 mapping 3 |--> -1,
Dirichlet character modulo 5 of conductor 5 mapping 2 |--> zeta4]
sage: d[0].parent()
Group of Dirichlet characters of modulus 4 over Cyclotomic Field of order 4 and degree 2
```

`sage.misc.functional.denominator(x)`

Returns the denominator of x .

EXAMPLES:

```
sage: denominator(17/11111)
11111
sage: R.<x> = PolynomialRing(QQ)
sage: F = FractionField(R)
sage: r = (x+1)/(x-1)
sage: denominator(r)
x - 1
```

`sage.misc.functional.det(x)`

Returns the determinant of x .

EXAMPLES:

```
sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
sage: det(A)
0
```

`sage.misc.functional.dim(x)`
Returns the dimension of `x`.

EXAMPLES:

```
sage: V = VectorSpace(QQ, 3)
sage: S = V.subspace([[1, 2, 0], [2, 2, -1]])
sage: dimension(S)
2
```

`sage.misc.functional.dimension(x)`
Returns the dimension of `x`.

EXAMPLES:

```
sage: V = VectorSpace(QQ, 3)
sage: S = V.subspace([[1, 2, 0], [2, 2, -1]])
sage: dimension(S)
2
```

`sage.misc.functional.disc(x)`
Returns the discriminant of `x`.

EXAMPLES:

```
sage: R.<x> = PolynomialRing(QQ)
sage: S = R.quotient(x^29 - 17*x - 1, 'alpha')
sage: K = S.number_field()
sage: discriminant(K)
-15975100446626038280218213241591829458737190477345113376757479850566957249523
```

`sage.misc.functional.discriminant(x)`
Returns the discriminant of `x`.

EXAMPLES:

```
sage: R.<x> = PolynomialRing(QQ)
sage: S = R.quotient(x^29 - 17*x - 1, 'alpha')
sage: K = S.number_field()
sage: discriminant(K)
-15975100446626038280218213241591829458737190477345113376757479850566957249523
```

`sage.misc.functional.eta(x)`
Returns the value of the eta function at `x`, which must be in the upper half plane.

The η function is

$$\eta(z) = e^{\pi iz/12} \prod_{n=1}^{\infty} (1 - e^{2\pi inz})$$

EXAMPLES:

```
sage: eta(1+I)
0.7420487758365647 + 0.1988313702299107*I
```

`sage.misc.functional.fcp(x, var='x')`
Returns the factorization of the characteristic polynomial of `x`.

EXAMPLES:

```

sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
sage: fcp(A, 'x')
x * (x^2 - 15*x - 18)

```

`sage.misc.functional.gen(x)`

Returns the generator of x .

EXAMPLES:

```

sage: R.<x> = QQ[]; R
Univariate Polynomial Ring in x over Rational Field
sage: gen(R)
x
sage: gen(GF(7))
1
sage: A = AbelianGroup(1, [23])
sage: gen(A)
f

```

`sage.misc.functional.gens(x)`

Returns the generators of x .

EXAMPLES:

```

sage: R.<x,y> = SR[]
sage: R
Multivariate Polynomial Ring in x, y over Symbolic Ring
sage: gens(R)
(x, y)
sage: A = AbelianGroup(5, [5, 5, 7, 8, 9])
sage: gens(A)
(f0, f1, f2, f3, f4)

```

`sage.misc.functional.hecke_operator(x, n)`

Returns the n -th Hecke operator T_n acting on x .

EXAMPLES:

```

sage: M = ModularSymbols(1, 12)
sage: hecke_operator(M, 5)
Hecke operator T_5 on Modular Symbols space of dimension 3 for Gamma_0(1) of weight 12 with sign

```

`sage.misc.functional.image(x)`

Returns the image of x .

EXAMPLES:

```

sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
sage: image(A)
Vector space of degree 3 and dimension 2 over Rational Field
Basis matrix:
[ 1  0 -1]
[ 0  1  2]

```

`sage.misc.functional.integral(x, *args, **kws)`

Returns an indefinite or definite integral of an object x .

First call `x.integral()` and if that fails make an object and integrate it using Maxima, maple, etc, as specified by `algorithm`.

For symbolic expression calls `sage.calculus.calculus.integral()` - see this function for available options.

EXAMPLES:

```
sage: f = cyclotomic_polynomial(10)
sage: integral(f)
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x
```

```
sage: integral(sin(x), x)
-cos(x)
```

```
sage: y = var('y')
sage: integral(sin(x), y)
y*sin(x)
```

```
sage: integral(sin(x), x, 0, pi/2)
1
sage: sin(x).integral(x, 0, pi/2)
1
sage: integral(exp(-x), (x, 1, oo))
e^(-1)
```

Numerical approximation:

```
sage: h = integral(tan(x)/x, (x, 1, pi/3)); h
integrate(tan(x)/x, x, 1, 1/3*pi)
sage: h.n()
0.07571599101...
```

Specific algorithm can be used for integration:

```
sage: integral(sin(x)^2, x, algorithm='maxima')
1/2*x - 1/4*sin(2*x)
sage: integral(sin(x)^2, x, algorithm='sympy')
-1/2*cos(x)*sin(x) + 1/2*x
```

TESTS:

A symbolic integral from [trac ticket #11445](#) that was incorrect in earlier versions of Maxima:

```
sage: f = abs(x - 1) + abs(x + 1) - 2*abs(x)
sage: integrate(f, (x, -Infinity, Infinity))
2
```

Another symbolic integral, from [trac ticket #11238](#), that used to return zero incorrectly; with Maxima 5.26.0 one gets $1/2\sqrt{\pi}e^{1/4}$, whereas with 5.29.1, and even more so with 5.33.0, the expression is less pleasant, but still has the same value. Unfortunately, the computation takes a very long time with the default settings, so we temporarily use the Maxima setting `domain: real`:

```
sage: sage.calculus.calculus.maxima('domain: real')
real
sage: f = exp(-x) * sinh(sqrt(x))
sage: t = integrate(f, x, 0, Infinity); t
1/4*sqrt(pi)*(erf(1) - 1)*e^(1/4) - 1/4*(sqrt(pi)*(erf(1) - 1) - sqrt(pi) + 2*e^(-1) - 2)*e^(1/4)
sage: t.canonicalize_radical()
1/2*sqrt(pi)*e^(1/4)
sage: sage.calculus.calculus.maxima('domain: complex')
complex
```

An integral which used to return -1 before maxima 5.28. See [trac ticket #12842](#):

```
sage: f = e^(-2*x)/sqrt(1-e^(-2*x))
sage: integrate(f, x, 0, infinity)
1
```

This integral would cause a stack overflow in earlier versions of Maxima, crashing sage. See [trac ticket #12377](#). We don't care about the result here, just that the computation completes successfully:

```
sage: y = (x^2)*exp(x) / (1 + exp(x))^2
sage: _ = integrate(y, x, -1000, 1000)
```

`sage.misc.functional.integral_closure(x)`

Returns the integral closure of x.

EXAMPLES:

```
sage: integral_closure(QQ)
Rational Field
sage: K.<a> = QuadraticField(5)
sage: O2 = K.order(2*a); O2
Order in Number Field in a with defining polynomial x^2 - 5
sage: integral_closure(O2)
Maximal Order in Number Field in a with defining polynomial x^2 - 5
```

`sage.misc.functional.integrate(x, *args, **kws)`

Returns an indefinite or definite integral of an object x.

First call `x.integral()` and if that fails make an object and integrate it using Maxima, maple, etc, as specified by algorithm.

For symbolic expression calls `sage.calculus.calculus.integral()` - see this function for available options.

EXAMPLES:

```
sage: f = cyclotomic_polynomial(10)
sage: integral(f)
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x

sage: integral(sin(x), x)
-cos(x)

sage: y = var('y')
sage: integral(sin(x), y)
y*sin(x)

sage: integral(sin(x), x, 0, pi/2)
1
sage: sin(x).integral(x, 0, pi/2)
1
sage: integral(exp(-x), (x, 1, oo))
e^(-1)
```

Numerical approximation:

```
sage: h = integral(tan(x)/x, (x, 1, pi/3)); h
integrate(tan(x)/x, x, 1, 1/3*pi)
sage: h.n()
0.07571599101...
```

Specific algorithm can be used for integration:

```
sage: integral(sin(x)^2, x, algorithm='maxima')
1/2*x - 1/4*sin(2*x)
sage: integral(sin(x)^2, x, algorithm='sympy')
-1/2*cos(x)*sin(x) + 1/2*x
```

TESTS:

A symbolic integral from [trac ticket #11445](#) that was incorrect in earlier versions of Maxima:

```
sage: f = abs(x - 1) + abs(x + 1) - 2*abs(x)
sage: integrate(f, (x, -Infinity, Infinity))
2
```

Another symbolic integral, from [trac ticket #11238](#), that used to return zero incorrectly; with Maxima 5.26.0 one gets $1/2 \cdot \sqrt{\pi} \cdot e^{1/4}$, whereas with 5.29.1, and even more so with 5.33.0, the expression is less pleasant, but still has the same value. Unfortunately, the computation takes a very long time with the default settings, so we temporarily use the Maxima setting `domain: real`:

```
sage: sage.calculus.calculus.maxima('domain: real')
real
sage: f = exp(-x) * sinh(sqrt(x))
sage: t = integrate(f, x, 0, Infinity); t # long time
1/4*sqrt(pi)*(erf(1) - 1)*e^(1/4) - 1/4*(sqrt(pi)*(erf(1) - 1) - sqrt(pi) + 2*e^(-1) - 2)*e^(1/4)
sage: t.canonicalize_radical() # long time
1/2*sqrt(pi)*e^(1/4)
sage: sage.calculus.calculus.maxima('domain: complex')
complex
```

An integral which used to return -1 before maxima 5.28. See [trac ticket #12842](#):

```
sage: f = e^(-2*x)/sqrt(1-e^(-2*x))
sage: integrate(f, x, 0, infinity)
1
```

This integral would cause a stack overflow in earlier versions of Maxima, crashing sage. See [trac ticket #12377](#). We don't care about the result here, just that the computation completes successfully:

```
sage: y = (x^2)*exp(x) / (1 + exp(x))^2
sage: _ = integrate(y, x, -1000, 1000)
```

`sage.misc.functional.interval(a, b)`

Integers between *a* and *b* *inclusive* (*a* and *b* integers).

EXAMPLES:

```
sage: I = interval(1, 3)
sage: 2 in I
True
sage: 1 in I
True
sage: 4 in I
False
```

`sage.misc.functional.is_commutative(x)`

Returns whether or not *x* is commutative.

EXAMPLES:

```
sage: R = PolynomialRing(QQ, 'x')
sage: is_commutative(R)
True
```


`sage.misc.functional.is_even(x)`

Returns whether or not an integer x is even, e.g., divisible by 2.

EXAMPLES:

```
sage: is_even(-1)
False
sage: is_even(4)
True
sage: is_even(-2)
True
```

`sage.misc.functional.is_field(x)`

Returns whether or not x is a field.

EXAMPLES:

```
sage: R = PolynomialRing(QQ, 'x')
sage: F = FractionField(R)
sage: is_field(F)
True
```

`sage.misc.functional.is_integrally_closed(x)`

Returns whether x is integrally closed.

EXAMPLES:

```
sage: is_integrally_closed(QQ)
True
sage: K.<a> = NumberField(x^2 + 189*x + 394)
sage: R = K.order(2*a)
sage: is_integrally_closed(R)
False
```

`sage.misc.functional.is_odd(x)`

Returns whether or not x is odd. This is by definition the complement of `is_even`.

EXAMPLES:

```
sage: is_odd(-2)
False
sage: is_odd(-3)
True
sage: is_odd(0)
False
sage: is_odd(1)
True
```

`sage.misc.functional.isqrt(x)`

Returns an integer square root, i.e., the floor of a square root.

EXAMPLES:

```
sage: isqrt(10)
3
sage: isqrt(10r)
3
```

`sage.misc.functional.kernel(x)`

Returns the left kernel of x .

EXAMPLES:

```
sage: M = MatrixSpace(QQ,3,2)
sage: A = M([1,2,3,4,5,6])
sage: kernel(A)
Vector space of degree 3 and dimension 1 over Rational Field
Basis matrix:
[ 1 -2  1]
sage: kernel(A.transpose())
Vector space of degree 2 and dimension 0 over Rational Field
Basis matrix:
[]
```

Here are two corner cases: sage: M=MatrixSpace(QQ,0,3) sage: A=M([]) sage: kernel(A) Vector space of degree 0 and dimension 0 over Rational Field Basis matrix: [] sage: kernel(A.transpose()).basis() [(1, 0, 0), (0, 1, 0), (0, 0, 1)]

sage.misc.functional.**krull_dimension**(x)
Returns the Krull dimension of x.

EXAMPLES:

```
sage: krull_dimension(QQ)
0
sage: krull_dimension(ZZ)
1
sage: krull_dimension(ZZ[sqrt(5)])
1
sage: U.<x,y,z> = PolynomialRing(ZZ,3); U
Multivariate Polynomial Ring in x, y, z over Integer Ring
sage: U.krull_dimension()
4
```

sage.misc.functional.**lift**(x)
Lift an object of a quotient ring R/I to R .

EXAMPLES: We lift an integer modulo 3.

```
sage: Mod(2,3).lift()
2
```

We lift an element of a quotient polynomial ring.

```
sage: R.<x> = QQ['x']
sage: S.<xmod> = R.quo(x^2 + 1)
sage: lift(xmod-7)
x - 7
```

sage.misc.functional.**log**(x, b=None)
Returns the log of x to the base b. The default base is e.

INPUT:

- x - number
- b - base (default: None, which means natural log)

OUTPUT: number

Note: In Magma, the order of arguments is reversed from in Sage, i.e., the base is given first. We use the opposite ordering, so the base can be viewed as an optional second argument.

EXAMPLES:

```
sage: log(e^2)
2
sage: log(16,2)
4
sage: log(3.)
1.09861228866811
```

`sage.misc.functional.minimal_polynomial(x, var='x')`

Returns the minimal polynomial of x .

EXAMPLES:

```
sage: a = matrix(ZZ, 2, [1..4])
sage: minpoly(a)
x^2 - 5*x - 2
sage: minpoly(a, 't')
t^2 - 5*t - 2
sage: minimal_polynomial(a)
x^2 - 5*x - 2
sage: minimal_polynomial(a, 'theta')
theta^2 - 5*theta - 2
```

`sage.misc.functional.minpoly(x, var='x')`

Returns the minimal polynomial of x .

EXAMPLES:

```
sage: a = matrix(ZZ, 2, [1..4])
sage: minpoly(a)
x^2 - 5*x - 2
sage: minpoly(a, 't')
t^2 - 5*t - 2
sage: minimal_polynomial(a)
x^2 - 5*x - 2
sage: minimal_polynomial(a, 'theta')
theta^2 - 5*theta - 2
```

`sage.misc.functional.multiplicative_order(x)`

Returns the multiplicative order of self, if self is a unit, or raise `ArithmeticError` otherwise.

EXAMPLES:

```
sage: a = mod(5,11)
sage: multiplicative_order(a)
5
sage: multiplicative_order(mod(2,11))
10
sage: multiplicative_order(mod(2,12))
Traceback (most recent call last):
...
ArithmeticError: multiplicative order of 2 not defined since it is not a unit modulo 12
```

`sage.misc.functional.n(x, prec=None, digits=None, algorithm=None)`

Returns a numerical approximation of an object x with at least `prec` bits (or decimal `digits`) of precision.

Note: Both upper case `N` and lower case `n` are aliases for `numerical_approx()`, and all three may be used as methods.

INPUT:

- `x` - an object that has a `numerical_approx` method, or can be coerced into a real or complex field
- `prec` (optional) - an integer (bits of precision)
- `digits` (optional) - an integer (digits of precision)
- `algorithm` (optional) - a string specifying the algorithm to use for functions that implement more than one

If neither the `prec` or `digits` are specified, the default is 53 bits of precision. If both are specified, then `prec` is used.

EXAMPLES:

```
sage: numerical_approx(pi, 10)
3.1
sage: numerical_approx(pi, digits=10)
3.141592654
sage: numerical_approx(pi^2 + e, digits=20)
12.587886229548403854
sage: n(pi^2 + e)
12.5878862295484
sage: N(pi^2 + e)
12.5878862295484
sage: n(pi^2 + e, digits=50)
12.587886229548403854194778471228813633070946500941
sage: a = CC(-5).n(prec=40)
sage: b = ComplexField(40)(-5)
sage: a == b
True
sage: parent(a) is parent(b)
True
sage: numerical_approx(9)
9.000000000000000
```

You can also usually use method notation.

```
sage: (pi^2 + e).n()
12.5878862295484
sage: (pi^2 + e).N()
12.5878862295484
sage: (pi^2 + e).numerical_approx()
12.5878862295484
```

Vectors and matrices may also have their entries approximated.

```
sage: v = vector(RDF, [1,2,3])
sage: v.n()
(1.000000000000000, 2.000000000000000, 3.000000000000000)

sage: v = vector(CDF, [1,2,3])
sage: v.n()
(1.000000000000000, 2.000000000000000, 3.000000000000000)
sage: _.parent()
Vector space of dimension 3 over Complex Field with 53 bits of precision
sage: v.n(prec=20)
(1.0000, 2.0000, 3.0000)

sage: u = vector(QQ, [1/2, 1/3, 1/4])
sage: n(u, prec=15)
(0.5000, 0.3333, 0.2500)
sage: n(u, digits=5)
```

```

(0.50000, 0.33333, 0.25000)

sage: v = vector(QQ, [1/2, 0, 0, 1/3, 0, 0, 0, 1/4], sparse=True)
sage: u = v.numerical_approx(digits=4)
sage: u.is_sparse()
True
sage: u
(0.5000, 0.0000, 0.0000, 0.3333, 0.0000, 0.0000, 0.0000, 0.2500)

sage: A = matrix(QQ, 2, 3, range(6))
sage: A.n()
[0.0000000000000000  1.0000000000000000  2.0000000000000000]
[ 3.0000000000000000  4.0000000000000000  5.0000000000000000]

sage: B = matrix(Integers(12), 3, 8, srange(24))
sage: N(B, digits=2)
[0.00  1.0  2.0  3.0  4.0  5.0  6.0  7.0]
[ 8.0  9.0 10. 11. 0.00  1.0  2.0  3.0]
[ 4.0  5.0  6.0  7.0  8.0  9.0 10. 11.]

```

Internally, numerical approximations of real numbers are stored in base-2. Therefore, numbers which look the same in their decimal expansion might be different:

```

sage: x=N(pi, digits=3); x
3.14
sage: y=N(3.14, digits=3); y
3.14
sage: x==y
False
sage: x.str(base=2)
'11.001001000100'
sage: y.str(base=2)
'11.001000111101'

```

Increasing the precision of a floating point number is not allowed:

```

sage: CC(-5).n(prec=100)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 100 bits, use at most 53 bits
sage: n(1.3r, digits=20)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 70 bits, use at most 53 bits
sage: RealField(24).pi().n()
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 53 bits, use at most 24 bits

```

As an exceptional case, `digits=1` usually leads to 2 digits (one significant) in the decimal output (see [ticket #11647](#)):

```

sage: N(pi, digits=1)
3.2
sage: N(pi, digits=2)
3.1
sage: N(100*pi, digits=1)
320.
sage: N(100*pi, digits=2)

```

```
310.
```

In the following example, `pi` and `3` are both approximated to two bits of precision and then subtracted, which kills two bits of precision:

```
sage: N(pi, prec=2)
3.0
sage: N(3, prec=2)
3.0
sage: N(pi - 3, prec=2)
0.00
```

TESTS:

```
sage: numerical_approx(I)
1.000000000000000*I
sage: x = QQ['x'].gen()
sage: F.<k> = NumberField(x^2+2, embedding=sqrt(CC(2))*CC(0))
sage: numerical_approx(k)
1.41421356237309*I

sage: type(numerical_approx(CC(1/2)))
<type 'sage.rings.complex_number.ComplexNumber'>
```

The following tests [trac ticket #10761](#), in which `n()` would break when called on complex-valued algebraic numbers.

```
sage: E = matrix(3, [3,1,6,5,2,9,7,3,13]).eigenvalues(); E
[18.16815365088822?, -0.08407682544410650? - 0.2190261484802906?*I, -0.08407682544410650? + 0.2190261484802906*I]
sage: E[1].parent()
Algebraic Field
sage: [a.n() for a in E]
[18.1681536508882, -0.0840768254441065 - 0.219026148480291*I, -0.0840768254441065 + 0.219026148480291*I]
```

Make sure we've rounded up $\log(10,2)$ enough to guarantee sufficient precision ([trac ticket #10164](#)):

```
sage: ks = 4*10**5, 10**6
sage: check_str_length = lambda k: len(str(numerical_approx(1+10**-k,digits=k+1)))-1 >= k+1
sage: check_precision = lambda k: numerical_approx(1+10**-k,digits=k+1)-1 > 0
sage: all(check_str_length(k) and check_precision(k) for k in ks)
True
```

Testing we have sufficient precision for the golden ratio ([trac ticket #12163](#)), note that the decimal point adds 1 to the string length:

```
sage: len(str(n(golden_ratio, digits=5000)))
5001
sage: len(str(n(golden_ratio, digits=5000000))) # long time (4s on sage.math, 2012)
5000001
```

Check that [trac ticket #14778](#) is fixed:

```
sage: n(0, algorithm='foo')
0.000000000000000
```

`sage.misc.functional.ngens(x)`
Returns the number of generators of `x`.

EXAMPLES:

```

sage: R.<x,y> = SR[]; R
Multivariate Polynomial Ring in x, y over Symbolic Ring
sage: ngens(R)
2
sage: A = AbelianGroup(5, [5,5,7,8,9])
sage: ngens(A)
5
sage: ngens(ZZ)
1

```

`sage.misc.functional.norm(x)`

Returns the norm of `x`.

For matrices and vectors, this returns the L2-norm. The L2-norm of a vector $\mathbf{v} = (v_1, v_2, \dots, v_n)$, also called the Euclidean norm, is defined as

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^n |v_i|^2}$$

where $|v_i|$ is the complex modulus of v_i . The Euclidean norm is often used for determining the distance between two points in two- or three-dimensional space.

For complex numbers, the function returns the field norm. If $c = a + bi$ is a complex number, then the norm of c is defined as the product of c and its complex conjugate

$$\text{norm}(c) = \text{norm}(a + bi) = c \cdot \bar{c} = a^2 + b^2.$$

The norm of a complex number is different from its absolute value. The absolute value of a complex number is defined to be the square root of its norm. A typical use of the complex norm is in the integral domain $\mathbf{Z}[i]$ of Gaussian integers, where the norm of each Gaussian integer $c = a + bi$ is defined as its complex norm.

See also:

- `sage.matrix.matrix2.Matrix.norm()`
- `sage.modules.free_module_element.FreeModuleElement.norm()`
- `sage.rings.complex_double.ComplexDoubleElement.norm()`
- `sage.rings.complex_number.ComplexNumber.norm()`
- `sage.symbolic.expression.Expression.norm()`

EXAMPLES:

The norm of vectors:

```

sage: z = 1 + 2*I
sage: norm(vector([z]))
sqrt(5)
sage: v = vector([-1, 2, 3])
sage: norm(v)
sqrt(14)
sage: _ = var("a b c d", domain='real')
sage: v = vector([a, b, c, d])
sage: norm(v)
sqrt(a^2 + b^2 + c^2 + d^2)

```

The norm of matrices:

```
sage: z = 1 + 2*I
sage: norm(matrix([[z]]))
2.23606797749979
sage: M = matrix(ZZ, [[1,2,4,3], [-1,0,3,-10]])
sage: norm(M) # abs tol 1e-14
10.690331129154467
sage: norm(CDF(z))
5.0
sage: norm(CC(z))
5.000000000000000
```

The norm of complex numbers:

```
sage: z = 2 - 3*I
sage: norm(z)
13
sage: a = randint(-10^10, 100^10)
sage: b = randint(-10^10, 100^10)
sage: z = a + b*I
sage: bool(norm(z) == a^2 + b^2)
True
```

The complex norm of symbolic expressions:

```
sage: a, b, c = var("a, b, c")
sage: assume((a, 'real'), (b, 'real'), (c, 'real'))
sage: z = a + b*I
sage: bool(norm(z).simplify() == a^2 + b^2)
True
sage: norm(a + b).simplify()
a^2 + 2*a*b + b^2
sage: v = vector([a, b, c])
sage: bool(norm(v).simplify() == sqrt(a^2 + b^2 + c^2))
True
sage: forget()
```

`sage.misc.functional.numerator(x)`

Returns the numerator of x .

EXAMPLES:

```
sage: R.<x> = PolynomialRing(QQ)
sage: F = FractionField(R)
sage: r = (x+1)/(x-1)
sage: numerator(r)
x + 1
sage: numerator(17/11111)
17
```

`sage.misc.functional.numerical_approx(x, prec=None, digits=None, algorithm=None)`

Returns a numerical approximation of an object x with at least `prec` bits (or decimal `digits`) of precision.

Note: Both upper case N and lower case n are aliases for `numerical_approx()`, and all three may be used as methods.

INPUT:

- x - an object that has a `numerical_approx` method, or can be coerced into a real or complex field
- `prec` (optional) - an integer (bits of precision)

- `digits` (optional) - an integer (digits of precision)
- `algorithm` (optional) - a string specifying the algorithm to use for functions that implement more than one

If neither the `prec` or `digits` are specified, the default is 53 bits of precision. If both are specified, then `prec` is used.

EXAMPLES:

```
sage: numerical_approx(pi, 10)
3.1
sage: numerical_approx(pi, digits=10)
3.141592654
sage: numerical_approx(pi^2 + e, digits=20)
12.587886229548403854
sage: n(pi^2 + e)
12.5878862295484
sage: N(pi^2 + e)
12.5878862295484
sage: n(pi^2 + e, digits=50)
12.587886229548403854194778471228813633070946500941
sage: a = CC(-5).n(prec=40)
sage: b = ComplexField(40)(-5)
sage: a == b
True
sage: parent(a) is parent(b)
True
sage: numerical_approx(9)
9.000000000000000
```

You can also usually use method notation.

```
sage: (pi^2 + e).n()
12.5878862295484
sage: (pi^2 + e).N()
12.5878862295484
sage: (pi^2 + e).numerical_approx()
12.5878862295484
```

Vectors and matrices may also have their entries approximated.

```
sage: v = vector(RDF, [1,2,3])
sage: v.n()
(1.000000000000000, 2.000000000000000, 3.000000000000000)

sage: v = vector(CDF, [1,2,3])
sage: v.n()
(1.000000000000000, 2.000000000000000, 3.000000000000000)
sage: v.parent()
Vector space of dimension 3 over Complex Field with 53 bits of precision
sage: v.n(prec=20)
(1.0000, 2.0000, 3.0000)

sage: u = vector(QQ, [1/2, 1/3, 1/4])
sage: n(u, prec=15)
(0.5000, 0.3333, 0.2500)
sage: n(u, digits=5)
(0.50000, 0.33333, 0.25000)

sage: v = vector(QQ, [1/2, 0, 0, 1/3, 0, 0, 0, 1/4], sparse=True)
```

```
sage: u = v.numerical_approx(digits=4)
sage: u.is_sparse()
True
sage: u
(0.5000, 0.0000, 0.0000, 0.3333, 0.0000, 0.0000, 0.0000, 0.2500)

sage: A = matrix(QQ, 2, 3, range(6))
sage: A.n()
[0.0000000000000000  1.0000000000000000  2.0000000000000000]
[ 3.0000000000000000  4.0000000000000000  5.0000000000000000]

sage: B = matrix(Integers(12), 3, 8, srange(24))
sage: N(B, digits=2)
[0.00  1.0  2.0  3.0  4.0  5.0  6.0  7.0]
[ 8.0  9.0 10. 11. 0.00  1.0  2.0  3.0]
[ 4.0  5.0  6.0  7.0  8.0  9.0 10. 11.]
```

Internally, numerical approximations of real numbers are stored in base-2. Therefore, numbers which look the same in their decimal expansion might be different:

```
sage: x=N(pi, digits=3); x
3.14
sage: y=N(3.14, digits=3); y
3.14
sage: x==y
False
sage: x.str(base=2)
'11.001001000100'
sage: y.str(base=2)
'11.001000111101'
```

Increasing the precision of a floating point number is not allowed:

```
sage: CC(-5).n(prec=100)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 100 bits, use at most 53 bits
sage: n(1.3r, digits=20)
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 70 bits, use at most 53 bits
sage: RealField(24).pi().n()
Traceback (most recent call last):
...
TypeError: cannot approximate to a precision of 53 bits, use at most 24 bits
```

As an exceptional case, `digits=1` usually leads to 2 digits (one significant) in the decimal output (see [trac ticket #11647](#)):

```
sage: N(pi, digits=1)
3.2
sage: N(pi, digits=2)
3.1
sage: N(100*pi, digits=1)
320.
sage: N(100*pi, digits=2)
310.
```

In the following example, `pi` and `3` are both approximated to two bits of precision and then subtracted, which

kills two bits of precision:

```
sage: N(pi, prec=2)
3.0
sage: N(3, prec=2)
3.0
sage: N(pi - 3, prec=2)
0.00
```

TESTS:

```
sage: numerical_approx(I)
1.000000000000000*I
sage: x = QQ['x'].gen()
sage: F.<k> = NumberField(x^2+2, embedding=sqrt(CC(2))*CC.0)
sage: numerical_approx(k)
1.41421356237309*I

sage: type(numerical_approx(CC(1/2)))
<type 'sage.rings.complex_number.ComplexNumber'>
```

The following tests [trac ticket #10761](#), in which `n()` would break when called on complex-valued algebraic numbers.

```
sage: E = matrix(3, [3,1,6,5,2,9,7,3,13]).eigenvalues(); E
[18.16815365088822?, -0.08407682544410650? - 0.2190261484802906?*I, -0.08407682544410650? + 0.2190261484802906?*I]
sage: E[1].parent()
Algebraic Field
sage: [a.n() for a in E]
[18.1681536508882, -0.0840768254441065 - 0.219026148480291*I, -0.0840768254441065 + 0.219026148480291*I]
```

Make sure we've rounded up $\log(10,2)$ enough to guarantee sufficient precision ([trac ticket #10164](#)):

```
sage: ks = 4*10**5, 10**6
sage: check_str_length = lambda k: len(str(numerical_approx(1+10**-k,digits=k+1)))-1 >= k+1
sage: check_precision = lambda k: numerical_approx(1+10**-k,digits=k+1)-1 > 0
sage: all(check_str_length(k) and check_precision(k) for k in ks)
True
```

Testing we have sufficient precision for the golden ratio ([trac ticket #12163](#)), note that the decimal point adds 1 to the string length:

```
sage: len(str(n(golden_ratio, digits=5000)))
5001
sage: len(str(n(golden_ratio, digits=5000000))) # long time (4s on sage.math, 2012)
5000001
```

Check that [trac ticket #14778](#) is fixed:

```
sage: n(0, algorithm='foo')
0.0000000000000000
```

`sage.misc.functional.objgen(x)`

EXAMPLES:

```
sage: R, x = objgen(FractionField(QQ['x']))
sage: R
Fraction Field of Univariate Polynomial Ring in x over Rational Field
sage: x
x
```

`sage.misc.functional.objgens(x)`

EXAMPLES:

```
sage: R, x = objgens(PolynomialRing(QQ, 3, 'x'))
sage: R
Multivariate Polynomial Ring in x0, x1, x2 over Rational Field
sage: x
(x0, x1, x2)
```

`sage.misc.functional.one(R)`

Returns the one element of the ring R.

EXAMPLES:

```
sage: one(RR)
doctest:...: DeprecationWarning: one(R) is deprecated, use R.one() or R(1) instead
See http://trac.sagemath.org/17158 for details.
1.0000000000000000
sage: R.<x> = PolynomialRing(QQ)
sage: one(R)*x == x
True
sage: one(R) in R
True
```

`sage.misc.functional.order(x)`

Returns the order of x. If x is a ring or module element, this is the additive order of x.

EXAMPLES:

```
sage: C = CyclicPermutationGroup(10)
sage: order(C)
10
sage: F = GF(7)
sage: order(F)
7
```

`sage.misc.functional.quo(x, y, *args, **kws)`

Returns the quotient object x/y, e.g., a quotient of numbers or of a polynomial ring x by the ideal generated by y, etc.

EXAMPLES:

```
sage: quotient(5, 6)
5/6
sage: quotient(5., 6.)
0.8333333333333333
sage: R.<x> = ZZ[]; R
Univariate Polynomial Ring in x over Integer Ring
sage: I = Ideal(R, x^2+1)
sage: quotient(R, I)
Univariate Quotient Polynomial Ring in xbar over Integer Ring with modulus x^2 + 1
```

`sage.misc.functional.quotient(x, y, *args, **kws)`

Returns the quotient object x/y, e.g., a quotient of numbers or of a polynomial ring x by the ideal generated by y, etc.

EXAMPLES:

```
sage: quotient(5, 6)
5/6
sage: quotient(5., 6.)
0.8333333333333333
sage: R.<x> = ZZ[]; R
```

```

Univariate Polynomial Ring in x over Integer Ring
sage: I = Ideal(R, x^2+1)
sage: quotient(R, I)
Univariate Quotient Polynomial Ring in xbar over Integer Ring with modulus x^2 + 1

```

`sage.misc.functional.rank(x)`

Returns the rank of x.

EXAMPLES: We compute the rank of a matrix:

```

sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
sage: rank(A)
2

```

We compute the rank of an elliptic curve:

```

sage: E = EllipticCurve([0, 0, 1, -1, 0])
sage: rank(E)
1

```

`sage.misc.functional.regulator(x)`

Returns the regulator of x.

EXAMPLES:

```

sage: regulator(NumberField(x^2-2, 'a'))
0.881373587019543
sage: regulator(EllipticCurve('11a'))
1.000000000000000

```

`sage.misc.functional.round(x, ndigits=0)`

`round(number[, ndigits])` - double-precision real number

Round a number to a given precision in decimal digits (default 0 digits). If no precision is specified this just calls the element's `.round()` method.

EXAMPLES:

```

sage: round(sqrt(2), 2)
1.41
sage: q = round(sqrt(2), 5); q
1.41421
sage: type(q)
<type 'sage.rings.real_double.RealDoubleElement'>
sage: q = round(sqrt(2)); q
1
sage: type(q)
<type 'sage.rings.integer.Integer'>
sage: round(pi)
3
sage: b = 5.499999999999999
sage: round(b)
5

```

Since we use floating-point with a limited range, some roundings can't be performed:

```

sage: round(sqrt(Integer('1'*1000)), 2)
+infinity

```

IMPLEMENTATION: If `ndigits` is specified, it calls Python's builtin `round` function, and converts the result to a

real double field element. Otherwise, it tries the argument's `.round()` method; if that fails, it reverts to the builtin round function, converted to a real double field element.

Note: This is currently slower than the builtin round function, since it does more work - i.e., allocating an RDF element and initializing it. To access the builtin version do `import __builtin__`; `__builtin__.round`.

`sage.misc.functional.squarefree_part(x)`

Returns the square free part of x , i.e., a divisor z such that $x = zy^2$, for a perfect square y^2 .

EXAMPLES:

```
sage: squarefree_part(100)
1
sage: squarefree_part(12)
3
sage: squarefree_part(10)
10
sage: squarefree_part(216r) # see #8976
6

sage: x = QQ['x'].0
sage: S = squarefree_part(-9*x*(x-6)^7*(x-3)^2); S
-9*x^2 + 54*x
sage: S.factor()
(-9) * (x - 6) * x

sage: f = (x^3 + x + 1)^3*(x-1); f
x^10 - x^9 + 3*x^8 + 3*x^5 - 2*x^4 - x^3 - 2*x - 1
sage: g = squarefree_part(f); g
x^4 - x^3 + x^2 - 1
sage: g.factor()
(x - 1) * (x^3 + x + 1)
```

`sage.misc.functional.symbolic_sum(expression, *args, **kws)`

Returns the symbolic sum $\sum_{v=a}^b expression$ with respect to the variable v with endpoints a and b .

INPUT:

- `expression` - a symbolic expression
- `v` - a variable or variable name
- `a` - lower endpoint of the sum
- `b` - upper endpoint of the sum
- `algorithm` - (default: 'maxima') one of
 - 'maxima' - use Maxima (the default)
 - 'maple' - (optional) use Maple
 - 'mathematica' - (optional) use Mathematica
 - 'giac' - (optional) use Giac

EXAMPLES:

```
sage: k, n = var('k,n')
sage: sum(k, k, 1, n).factor()
1/2*(n + 1)*n
```

```
sage: sum(1/k^4, k, 1, oo)
1/90*pi^4
```

```
sage: sum(1/k^5, k, 1, oo)
zeta(5)
```

Warning: This function only works with symbolic expressions. To sum any other objects like list elements or function return values, please use python summation, see <http://docs.python.org/library/functions.html#sum>

In particular, this does not work:

```
sage: n = var('n')
sage: list=[1,2,3,4,5]
sage: sum(list[n],n,0,3)
Traceback (most recent call last):
...
TypeError: unable to convert n to an integer
```

Use python `sum()` instead:

```
sage: sum(list[n] for n in range(4))
10
```

Also, only a limited number of functions are recognized in symbolic sums:

```
sage: sum(valuation(n,2),n,1,5)
Traceback (most recent call last):
...
TypeError: unable to convert n to an integer
```

Again, use python `sum()`:

```
sage: sum(valuation(n+1,2) for n in range(5))
3
```

(now back to the Sage `sum` examples)

A well known binomial identity:

```
sage: sum(binomial(n,k), k, 0, n)
2^n
```

The binomial theorem:

```
sage: x, y = var('x, y')
sage: sum(binomial(n,k) * x^k * y^(n-k), k, 0, n)
(x + y)^n
```

```
sage: sum(k * binomial(n, k), k, 1, n)
2^(n - 1)*n
```

```
sage: sum((-1)^k*binomial(n,k), k, 0, n)
0
```

```
sage: sum(2^(-k)/(k*(k+1)), k, 1, oo)
-log(2) + 1
```

Another binomial identity ([trac ticket #7952](#)):

```
sage: t,k,i = var('t,k,i')
sage: sum(binomial(i+t,t),i,0,k)
binomial(k + t + 1, t + 1)
```

Summing a hypergeometric term:

```
sage: sum(binomial(n, k) * factorial(k) / factorial(n+1+k), k, 0, n)
1/2*sqrt(pi)/factorial(n + 1/2)
```

We check a well known identity:

```
sage: bool(sum(k^3, k, 1, n) == sum(k, k, 1, n)^2)
True
```

A geometric sum:

```
sage: a, q = var('a, q')
sage: sum(a*q^k, k, 0, n)
(a*q^(n + 1) - a)/(q - 1)
```

The geometric series:

```
sage: assume(abs(q) < 1)
sage: sum(a*q^k, k, 0, oo)
-a/(q - 1)
```

A divergent geometric series. Don't forget to forget your assumptions:

```
sage: forget()
sage: assume(q > 1)
sage: sum(a*q^k, k, 0, oo)
Traceback (most recent call last):
...
ValueError: Sum is divergent.
```

This summation only Mathematica can perform:

```
sage: sum(1/(1+k^2), k, -oo, oo, algorithm = 'mathematica') # optional - mathematica
pi*coth(pi)
```

Use Maple as a backend for summation:

```
sage: sum(binomial(n,k)*x^k, k, 0, n, algorithm = 'maple') # optional - maple
(x + 1)^n
```

Python ints should work as limits of summation ([trac ticket #9393](#)):

```
sage: sum(x, x, 1r, 5r)
15
```

Note:

1. Sage can currently only understand a subset of the output of Maxima, Maple and Mathematica, so even if the chosen backend can perform the summation the result might not be convertible into a Sage expression.
-

`sage.misc.functional.transpose(x)`

Returns the transpose of `x`.

EXAMPLES:


```

sage: M = MatrixSpace(QQ, 3, 3)
sage: A = M([1, 2, 3, 4, 5, 6, 7, 8, 9])
sage: transpose(A)
[1 4 7]
[2 5 8]
[3 6 9]

```

`sage.misc.functional.xinterval(a, b)`
 Iterator over the integers between a and b, *inclusive*.

EXAMPLES:

```

sage: I = xinterval(2, 5); I
xrange(2, 6)
sage: 5 in I
True
sage: 6 in I
False

```

`sage.misc.functional.zero(R)`
 Returns the zero element of the ring R.

EXAMPLES:

```

sage: zero(RR)
doctest:...: DeprecationWarning: zero(R) is deprecated, use R.zero() or R(0) instead
See http://trac.sagemath.org/17158 for details.
0.0000000000000000
sage: R.<x> = PolynomialRing(QQ)
sage: zero(R) in R
True
sage: zero(R)*x == zero(R)
True

```

1.3 Random Number States

AUTHORS:

- Carl Witty (2008-03): new file

This module manages all the available pseudo-random number generators in Sage. (For the rest of the documentation in this module, we will drop the “pseudo”.)

The goal is to allow algorithms using random numbers to be reproducible from one run of Sage to the next, and (to the extent possible) from one machine to the next (even across different operating systems and architectures).

There are two parts to the API. First we will describe the command line oriented API, for setting random number generator seeds. Then we will describe the library API, for people writing Sage library code that uses random numbers.

1.3.1 Command line oriented API

We’ll start with the simplest usage: setting fixed random number seeds and showing that these lead to reproducible results.

```

sage: K.<x> = QQ[]
sage: G = PermutationGroup([[ (1, 2, 3), (4, 5) ], [ (1, 2) ]])
sage: rgp = Gp()

```

```
sage: def gap_randstring(n):
.....:     current_randstate().set_seed_gap()
.....:     return gap(n).SCRRandomString()
sage: def rtest():
.....:     current_randstate().set_seed_gp(rgp)
.....:     return (ZZ.random_element(1000), RR.random_element(),
.....:           K.random_element(), G.random_element(),
.....:           gap_randstring(5),
.....:           rgp.random(), ntl.ZZ_random(99999),
.....:           random())
```

The above test shows the results of six different random number generators, in three different processes. The random elements from ZZ, RR, and K all derive from a single GMP-based random number generator. The random element from G comes from a GAP subprocess. The random “string” (5-element binary list) is also from a GAP subprocess, using the “classical” GAP random generator. The random number from `rgp` is from a Pari/gp subprocess. NTL’s `ZZ_random` uses a separate NTL random number generator in the main Sage process. And `random()` is from a Python `random.Random` object.

Here we see that setting the random number seed really does make the results of these random number generators reproducible.

```
sage: set_random_seed(0)
sage: rtest()
(303, -0.266166246380421, 1/2*x^2 - 1/95*x - 1/2, (1,3,2), [ 0, 0, 0, 0, 1 ], 265625921, 8045, 0.9663...)
sage: set_random_seed(1)
sage: rtest()
(978, 0.0557699430711638, -3*x^2 - 1/12, (1,3,2), [ 0, 1, 1, 0, 0 ], 807447831, 60359, 0.83350776541...)
sage: set_random_seed(2)
sage: rtest()
(207, -0.0141049486533456, 4*x^2 + 1/2, (1,3,2), [ 0, 0, 1, 0, 1 ], 1642898426, 27695, 0.19982565117...)
sage: set_random_seed(0)
sage: rtest()
(303, -0.266166246380421, 1/2*x^2 - 1/95*x - 1/2, (1,3,2), [ 0, 0, 0, 0, 1 ], 265625921, 8045, 0.9663...)
sage: set_random_seed(1)
sage: rtest()
(978, 0.0557699430711638, -3*x^2 - 1/12, (1,3,2), [ 0, 1, 1, 0, 0 ], 807447831, 60359, 0.83350776541...)
sage: set_random_seed(2)
sage: rtest()
(207, -0.0141049486533456, 4*x^2 + 1/2, (1,3,2), [ 0, 0, 1, 0, 1 ], 1642898426, 27695, 0.19982565117...)
```

Once we’ve set the random number seed, we can check what seed was used. (This is not the current random number state; it does not change when random numbers are generated.)

```
sage: set_random_seed(12345)
sage: initial_seed()
12345L
sage: rtest()
(720, -0.612180244315804, x^2 - x, (2,3), [ 1, 0, 0, 0, 0 ], 1911581957, 14005, 0.9205331599518184)
sage: initial_seed()
12345L
```

If `set_random_seed()` is called with no arguments, then a new seed is automatically selected. On operating systems that support it, the new seed comes from `os.urandom()`; this is intended to be a truly random (not pseudo-random), cryptographically secure number. (Whether it is actually cryptographically secure depends on operating system details that are outside the control of Sage.)

If `os.urandom()` is not supported, then the new seed comes from the current time, which is definitely not cryptographically secure.

```
sage: set_random_seed()
sage: r = rtest()
sage: r          # random
(909, -0.407373370020575, 6/7*x^2 + 1, (1,2,3)(4,5), 985329107, 21461, 0.30047071049504859)
```

After setting a new random number seed with `set_random_seed()`, we can use `initial_seed()` to see what seed was automatically selected, and call `set_random_seed()` to restart the same random number sequence.

```
sage: s = initial_seed()
sage: s          # random
336237747258024892084418842839280045662L
sage: set_random_seed(s)
sage: r2 = rtest()
sage: r == r2
True
```

Whenever Sage starts, `set_random_seed()` is called just before command line interaction starts; so every Sage run starts with a different random number seed. This seed can be recovered with `initial_seed()` (as long as the user has not set a different seed with `set_random_seed()`), so that the results of this run can be reproduced in another run; or this automatically selected seed can be overridden with, for instance, `set_random_seed(0)`.

We can demonstrate this startup behavior by running a new instance of Sage as a subprocess.

```
sage: subsage = Sage()
sage: s = ZZ(subsage('initial_seed()'))
sage: r = ZZ(subsage('ZZ.random_element(2^200)'))
sage: s          # random
161165040149656168853863459174502758403
sage: r          # random
1273828861620427462924151488498075119241254209468761367941442
sage: set_random_seed(s)
sage: r == ZZ.random_element(2^200)
True
```

Note that wrappers of all the random number generation methods from Python's `random` module are available at the Sage command line, and these wrappers are properly affected by `set_random_seed()`.

```
sage: set_random_seed(0)
sage: random(), getrandbits(20), uniform(5.0, 10.0), normalvariate(0, 1)
(0.111439293741037, 539332L, 8.26785106378383, 1.3893337539828183)
sage: set_random_seed(1)
sage: random(), getrandbits(20), uniform(5.0, 10.0), normalvariate(0, 1)
(0.8294022851874259, 624859L, 5.77894484361117, -0.4201366826308758)
sage: set_random_seed(0)
sage: random(), getrandbits(20), uniform(5.0, 10.0), normalvariate(0, 1)
(0.111439293741037, 539332L, 8.26785106378383, 1.3893337539828183)
```

That pretty much covers what you need to know for command-line use of this module. Now let's move to what authors of Sage library code need to know about the module.

1.3.2 Library API

First, we'll cover doctesting. Every docstring now has an implicit `set_random_seed(0)` prepended. Any uses of `# random` that are based on random numbers under the control of this module should be removed, and the reproducible answers inserted instead.

This practice has two potential drawbacks. First, it increases the work of maintaining doctests. For instance, in a long docstring that has many doctests that depend on random numbers, a change near the beginning (for instance, adding a new doctest) may invalidate all later doctests in the docstring. To reduce this downside, you may add calls to `set_random_seed(0)` throughout the docstring (in the extreme case, before every doctest).

Second, the `# random` in the doctest served as a signal to the reader of the docstring that the result was unpredictable and that it would not be surprising to get a different result when trying out the examples in the doctest. If a doctest specifically refers to `ZZ.random_element()` (for instance), this is presumably enough of a signal to render this function of `# random` unnecessary. However, some doctests are not obviously (from the name) random, but do depend on random numbers internally, such as the `composition_series` method of a `PermutationGroup`. In these cases, the convention is to insert the following text at the beginning of the `EXAMPLES` section.

These computations use pseudo-random numbers, so we set the seed for reproducible testing.

```
sage: set_random_seed(0)
```

Note that this call to `set_random_seed(0)` is redundant, since `set_random_seed(0)` is automatically inserted at the beginning of every docstring. However, it makes the example reproducible for somebody who just types the lines from the doctest and doesn't know about the automatic `set_random_seed(0)`.

Next, let's cover setting the random seed from library code. The first rule is that library code should never call `set_random_seed()`. This function is only for command-line use. Instead, if the library code wants to use a different random seed, it should use `with seed(s) :`. This will use the new seed within the scope of the `with` statement, but will revert to the previous seed once the `with` statement is completed. (Or the library can use `seed()` : to get a seed automatically selected using `os.urandom()` or the current time, in the same way as described for `set_random_seed()` above.)

Ideally, using `with seed(s) :` should not affect the outer random number sequence at all; we will call this property "isolation." We achieve isolation for most, but not all, of the random number generators in Sage (we fail for generators, such as NTL, that do not provide an API to retrieve the current random number state).

We'll demonstrate isolation. First, we show the sequence of random numbers that you get without intervening with seed.

```
sage: set_random_seed(0)
sage: r1 = rtest(); r1
(303, -0.266166246380421, 1/2*x^2 - 1/95*x - 1/2, (1,3,2), [ 0, 0, 0, 0, 1 ], 265625921, 8045, 0.9663...)
sage: r2 = rtest(); r2
(105, 0.642309615982449, -x^2 - x - 6, (1,2,3), [ 1, 0, 0, 1, 1 ], 53231108, 1271, 0.001767155077382...)
```

We get slightly different results with an intervening `with seed`.

```
sage: set_random_seed(0)
sage: r1 == rtest()
True
sage: with seed(1): rtest()
(978, 0.0557699430711638, -3*x^2 - 1/12, (1,3,2), [ 0, 1, 1, 0, 0 ], 807447831, 60359, 0.83350776541...)
sage: r2m = rtest(); r2m
(105, 0.642309615982449, -x^2 - x - 6, (1,2,3), [ 1, 0, 0, 1, 1 ], 53231108, 19769, 0.001767155077382...)
sage: r2m == r2
False
```

We can see that `r2` and `r2m` are the same except for the call to `ntl.ZZ_random()`, which produces different results with and without the `with seed`.

However, we do still get a partial form of isolation, even in this case, as we see in this example:

```

sage: set_random_seed(0)
sage: r1 == rtest()
True
sage: with seed(1):
....:     rtest()
....:     rtest()
(978, 0.0557699430711638, -3*x^2 - 1/12, (1,3,2), [ 0, 1, 1, 0, 0 ], 807447831, 60359, 0.8335077654199736)
(138, -0.0404945051288503, 2*x - 24, (2,3), [ 1, 1, 1, 0, 1 ], 1010791326, 10234, 0.00333322308080608)
sage: r2m == rtest()
True

```

The NTL results after the `with seed` don't depend on how many NTL random numbers were generated inside the `with seed`.

```

sage: set_random_seed(0) sage: r1 == rtest() True sage: with seed(1): ....: rtest() (978,
0.0557699430711638, -3*x^2 - 1/12, (1,3,2), [ 0, 1, 1, 0, 0 ], 807447831, 60359, 0.8335077654199736)
sage: r2m == rtest() True

```

(In general, the above code is not exactly equivalent to the `with` statement, because if an exception happens in the body, the real `with` statement will pass the exception information as parameters to the `__exit__` method. However, our `__exit__` method ignores the exception information anyway, so the above is equivalent in our case.)

1.3.3 Generating random numbers in library code

Now we come to the last part of the documentation: actually generating random numbers in library code. First, the easy case. If you generate random numbers only by calling other Sage library code (such as `random_element` methods on parents), you don't need to do anything special; the other code presumably already interacts with this module correctly.

Otherwise, it depends on what random number generator you want to use.

- `gmp_randstate_t` – If you want to use some random number generator that takes a `gmp_randstate_t` (like `mpz_urandomm` or `mpfr_urandomb`), then use code like the following:

```

from sage.misc.randstate cimport randstate, current_randstate
...

cdef randstate rstate = current_randstate()

```

Then a `gmp_randstate_t` is available as `rstate.gmp_state`.

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

- Python – If you want to use the random number generators from the `random` module, you have two choices. The slightly easier choice is to import functions from `sage.misc.prandom`; for instance, you can simply replace `from random import randrange` with `from sage.misc.prandom import randrange`. However, this is slightly less efficient, because the wrappers in `sage.misc.prandom` look up the current `randstate` on each call. If you're generating many random numbers in a row, it's faster to instead do

```

from sage.misc.randstate import current_randstate ...

randrange = current_randstate().python_random().randrange

```

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache the `randstate`, the `Random` object returned by `python_random`, or the bound methods on that `Random` object globally or in a class. (Such caching would break `set_random_seed`).

- GAP – If you are calling code in GAP that uses random numbers, call `set_seed_gap` at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_gap()
```

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

- Pari – If you are calling code in the Pari library that uses random numbers, call `set_seed_pari` at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_pari()
```

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

- Pari/gp – If you are calling code in a Pari/gp subprocess that uses random numbers, call `set_seed_gp` at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_gp()
```

This will set the seed in the gp process in `sage.interfaces.gp.gp`. If you have a different gp process, say in the variable `my_gp`, then call `set_seed_gp(my_gp)` instead.

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

- NTL – If you are calling code in the NTL library that uses random numbers, call `set_seed_ntl` at the beginning of your function, like this:

```
from sage.misc.randstate import current_randstate ...

current_randstate().set_seed_ntl(False)
```

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

- libc – If you are writing code that calls the libc function `random()`: don't! The `random()` function does not give reproducible results across different operating systems, so we can't make portable doctests for the results. Instead, do:

```
from sage.misc.randstate cimport random
```

The `random()` function in `sage.misc.randstate` gives a 31-bit random number, but it uses the `gmp_randstate_t` in the current `randstate`, so it is portable. (This range was chosen for two reasons: it matches the range of `random()` on 32-bit and 64-bit Linux, although not Solaris; and it's the largest range of nonnegative numbers that fits in a 32-bit signed integer.)

However, you may still need to set the libc random number state; for instance, if you are wrapping a library that uses `random()` internally and you don't want to change the library. In that case, call `set_seed_libc` at the beginning of your function, like this:

```

from sage.misc.randstate import current_randstate
...

current_randstate().set_seed_libc(False)

```

Fetch the current `randstate` with `current_randstate()` in every function that wants to use it; don't cache it globally or in a class. (Such caching would break `set_random_seed`).

1.3.4 Classes and methods

`sage.misc.randstate.benchmark_libc()`

This function was used to test whether moving from `libc` to GMP's Mersenne Twister for random numbers would be a significant slowdown.

EXAMPLES:

```

sage: from sage.misc.randstate import benchmark_libc, benchmark_mt
sage: timeit('benchmark_libc()') # random
125 loops, best of 3: 1.95 ms per loop
sage: timeit('benchmark_mt()') # random
125 loops, best of 3: 2.12 ms per loop

```

`sage.misc.randstate.benchmark_mt()`

This function was used to test whether moving from `libc` to GMP's Mersenne Twister for random numbers would be a significant slowdown.

EXAMPLES:

```

sage: from sage.misc.randstate import benchmark_libc, benchmark_mt
sage: timeit('benchmark_libc()') # random
125 loops, best of 3: 1.95 ms per loop
sage: timeit('benchmark_mt()') # random
125 loops, best of 3: 2.11 ms per loop

```

`sage.misc.randstate.current_randstate()`

Return the current random number state.

EXAMPLES:

```

sage: current_randstate()
<sage.misc.randstate.randstate object at 0x...>
sage: current_randstate().python_random().random()
0.111439293741037

```

`sage.misc.randstate.initial_seed()`

Returns the initial seed used to create the current `randstate`.

EXAMPLES:

```

sage: set_random_seed(42)
sage: initial_seed()
42L

```

If you set a random seed (by failing to specify the seed), this is how you retrieve the seed actually chosen by Sage. This can also be used to retrieve the seed chosen for a new Sage run (if the user has not used `set_random_seed()`).

```

sage: set_random_seed()
sage: initial_seed() # random
121030915255244661507561642968348336774L

```

`sage.misc.randstate.random()`

Returns a 31-bit random number. Intended as a drop-in replacement for the libc `random()` function.

EXAMPLES:

```
sage: set_random_seed(31)
sage: from sage.misc.randstate import random
sage: random()
32990711
```

class `sage.misc.randstate.randstate`

Bases: `object`

The `randstate` class. This class keeps track of random number states and seeds. Type `sage.misc.randstate?` for much more information on random numbers in Sage.

ZZ_seed()

When called on the current `randstate`, returns a 128-bit Integer suitable for seeding another random number generator.

EXAMPLES:

```
sage: set_random_seed(1414)
sage: current_randstate().ZZ_seed()
48314508034782595865062786044921182484
```

c_rand_double()

Returns a random floating-point number between 0 and 1.

EXAMPLES:

```
sage: set_random_seed(2718281828)
sage: current_randstate().c_rand_double()
0.22437207488974298
```

c_random()

Returns a 31-bit random number. Intended for internal use only; instead of calling `current_randstate().c_random()`, it is equivalent (but probably faster) to call the `random` method of this `randstate` class.

EXAMPLES:

```
sage: set_random_seed(1207)
sage: current_randstate().c_random()
2008037228
```

We verify the equivalence mentioned above.

```
sage: from sage.misc.randstate import random
sage: set_random_seed(1207)
sage: random()
2008037228
```

long_seed()

When called on the current `randstate`, returns a 128-bit Python long suitable for seeding another random number generator.

EXAMPLES:


```
sage: set_random_seed(1618)
sage: current_randstate().long_seed()
256056279774514099508607350947089272595L
```

`python_random()`

Return a `random.Random` object. The first time it is called on a given `randstate`, a new `random.Random` is created (seeded from the *current* `randstate`); the same object is returned on subsequent calls.

It is expected that `python_random` will only be called on the current `randstate`.

EXAMPLES:

```
sage: set_random_seed(5)
sage: rnd = current_randstate().python_random()
sage: rnd.random()
0.013558022446944151
sage: rnd.randrange(1000)
544
```

`seed()`

Return the initial seed of a `randstate` object. (This is not the current state; it does not change when you get random numbers.)

EXAMPLES:

```
sage: from sage.misc.randstate import randstate
sage: r = randstate(314159)
sage: r.seed()
314159L
sage: r.python_random().random()
0.111439293741037
sage: r.seed()
314159L
```

`set_seed_gap()`

Checks to see if `self` was the most recent `randstate` to seed the GAP random number generator. If not, seeds the generator.

EXAMPLES:

```
sage: set_random_seed(99900000999)
sage: current_randstate().set_seed_gap()
sage: gap.Random(1, 10^50)
1496738263332555434474532297768680634540939580077
sage: gap(35).SCRRandomString()
[ 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0,
  0, 0, 1, 0, 0, 1, 1, 0, 0, 1 ]
```

`set_seed_gp(gp=None)`

Checks to see if `self` was the most recent `randstate` to seed the random number generator in the given instance of `gp`. (If no instance is given, uses the one in `gp`.) If not, seeds the generator.

EXAMPLES:

```
sage: set_random_seed(987654321)
sage: current_randstate().set_seed_gp()
sage: gp.random()
23289294
```

set_seed_libc (*force*)

Checks to see if `self` was the most recent `randstate` to seed the libc random number generator. If not, seeds the libc random number generator. (Do not use the libc random number generator if you have a choice; its randomness is poor, and the random number sequences it produces are not portable across operating systems.)

If the argument `force` is `True`, seeds the generator unconditionally.

EXAMPLES:

```
sage: from sage.misc.randstate import _doctest_libc_random
sage: set_random_seed(0xBAD)
sage: current_randstate().set_seed_libc(False)
sage: _doctest_libc_random()      # random
1070075918
```

set_seed_ntl (*force*)

Checks to see if `self` was the most recent `randstate` to seed the NTL random number generator. If not, seeds the generator. If the argument `force` is `True`, seeds the generator unconditionally.

EXAMPLES:

```
sage: set_random_seed(2008)
```

This call is actually redundant; `ntl.ZZ_random()` will seed the generator itself. However, we put the call in to make the coverage tester happy.

```
sage: current_randstate().set_seed_ntl(False)
sage: ntl.ZZ_random(10^40)
9121810031060285085432621721962973171231
```

set_seed_pari ()

Checks to see if `self` was the most recent `randstate` to seed the Pari random number generator. If not, seeds the generator.

Note: Since pari 2.4.3, pari's random number generator has changed a lot. the seed output by `getrand()` is now a vector of integers.

EXAMPLES:

```
sage: set_random_seed(5551212)
sage: current_randstate().set_seed_pari()
sage: pari.getrand().type()
't_INT'
```

`sage.misc.randstate.seed`
alias of `randstate`

`sage.misc.randstate.set_random_seed` (*seed=None*)

Set the current random number seed from the given `seed` (which must be coercible to a Python long).

If no seed is given, then a seed is automatically selected using `os.urandom()` if it is available, or the current time otherwise.

Type `sage.misc.randstate?` for much more information on random numbers in Sage.

This function is only intended for command line use. Never call this from library code; instead, use `with seed(s) :`

Note that setting the random number seed to 0 is much faster than using any other number.

EXAMPLES:

```
sage: set_random_seed(5)
sage: initial_seed()
5L
```

1.4 Random Numbers with Python API

AUTHORS: – Carl Witty (2008-03): new file

This module has the same functions as the Python standard module `module{random}`, but uses the current sage random number state from `module{sage.misc.randstate}` (so that it can be controlled by the same global random number seeds).

The functions here are less efficient than the functions in `module{random}`, because they look up the current random number state on each call.

If you are going to be creating many random numbers in a row, it is better to use the functions in `module{sage.misc.randstate}` directly.

Here is an example:

(The imports on the next two lines are not necessary, since function `{randrange}` and function `{current_randstate}` are both available by default at the code `{sage:}` prompt; but you would need them to run these examples inside a module.)

```
sage: from sage.misc.prandom import randrange
sage: from sage.misc.randstate import current_randstate
sage: def test1():
...     return sum([randrange(100) for i in range(100)])
sage: def test2():
...     randrange = current_randstate().python_random().randrange
...     return sum([randrange(100) for i in range(100)])
```

Test2 will be slightly faster than test1, but they give the same answer:

```
sage: with seed(0): test1()
5169
sage: with seed(0): test2()
5169
sage: with seed(1): test1()
5097
sage: with seed(1): test2()
5097
sage: timeit('test1()') # random
625 loops, best of 3: 590 us per loop
sage: timeit('test2()') # random
625 loops, best of 3: 460 us per loop
```

The docstrings for the functions in this file are mostly copied from Python’s file `{random.py}`, so those docstrings are “Copyright (c) 2001, 2002, 2003, 2004, 2005, 2006, 2007 Python Software Foundation; All Rights Reserved” and are available under the terms of the Python Software Foundation License Version 2.

`sage.misc.prandom.betavariate(alpha, beta)`
Beta distribution.

Conditions on the parameters are $\alpha > 0$ and $\beta > 0$. Returned values range between 0 and 1.

EXAMPLES:

```
sage: betavariate(0.1, 0.9)
9.75087916621299e-9
```

```
sage: betavariate(0.9, 0.1)
0.941890400939253
```

`sage.misc.prandom.choice(seq)`
Choose a random element from a non-empty sequence.

EXAMPLES:

```
sage: [choice(list(primes(10, 100))) for i in range(5)]
[17, 47, 11, 31, 47]
```

`sage.misc.prandom.expovariate(lambd)`
Exponential distribution.

`lambd` is 1.0 divided by the desired mean. (The parameter would be called “lambda”, but that is a reserved word in Python.) Returned values range from 0 to positive infinity.

EXAMPLES:

```
sage: [expovariate(0.001) for i in range(3)]
[118.152309288166, 722.261959038118, 45.7190543690470]
sage: [expovariate(1.0) for i in range(3)]
[0.404201816061304, 0.735220464997051, 0.201765578600627]
sage: [expovariate(1000) for i in range(3)]
[0.0012068700332283973, 8.340929747302108e-05, 0.00219877067980605]
```

`sage.misc.prandom.gammavariate(alpha, beta)`
Gamma distribution. Not the gamma function!

Conditions on the parameters are $\alpha > 0$ and $\beta > 0$.

EXAMPLES:

```
sage: gammavariate(1.0, 3.0)
6.58282586130638
sage: gammavariate(3.0, 1.0)
3.07801512341612
```

`sage.misc.prandom.gauss(mu, sigma)`
Gaussian distribution.

`mu` is the mean, and `sigma` is the standard deviation. This is slightly faster than the `normalvariate()` function, but is not thread-safe.

EXAMPLES:

```
sage: [gauss(0, 1) for i in range(3)]
[0.9191011757657915, 0.7744526756246484, 0.8638996866800877]
sage: [gauss(0, 100) for i in range(3)]
[24.916051749154448, -62.99272061579273, -8.1993122536718...]
sage: [gauss(1000, 10) for i in range(3)]
[998.7590700045661, 996.1087338511692, 1010.1256817458031]
```

`sage.misc.prandom.getrandbits(k)`
`getrandbits(k) -> x`. Generates a long int with `k` random bits.

EXAMPLES:

```
sage: getrandbits(10)
114L
sage: getrandbits(200)
1251230322675596703523231194384285105081402591058406420468435L
```

```
sage: getrandbits(10)
533L
```

`sage.misc.prandom.lognormvariate(mu, sigma)`
Log normal distribution.

If you take the natural logarithm of this distribution, you'll get a normal distribution with mean `mu` and standard deviation `sigma`. `mu` can have any value, and `sigma` must be greater than zero.

EXAMPLES:

```
sage: [lognormvariate(100, 10) for i in range(3)]
[2.9410355688290246e+37, 2.2257548162070125e+38, 4.142299451717446e+43]
```

`sage.misc.prandom.normalvariate(mu, sigma)`
Normal distribution.

`mu` is the mean, and `sigma` is the standard deviation.

EXAMPLES:

```
sage: [normalvariate(0, 1) for i in range(3)]
[-1.372558980559407, -1.1701670364898928, 0.04324100555110143]
sage: [normalvariate(0, 100) for i in range(3)]
[37.45695875041769, 159.6347743233298, 124.1029321124009]
sage: [normalvariate(1000, 10) for i in range(3)]
[1008.5303090383741, 989.8624892644895, 985.7728921150242]
```

`sage.misc.prandom.paretovariate(alpha)`
Pareto distribution. `alpha` is the shape parameter.

EXAMPLES:

```
sage: [paretovariate(3) for i in range(1, 5)]
[1.0401699394233033, 1.2722080162636495, 1.0153564009379579, 1.1442323078983077]
```

`sage.misc.prandom.randint(a, b)`
Return random integer in range `[a, b]`, including both end points.

EXAMPLES:

```
sage: [randint(0, 2) for i in range(15)]
[0, 1, 0, 0, 1, 0, 2, 0, 2, 1, 2, 2, 0, 2, 2]
sage: randint(-100, 10)
-46
```

`sage.misc.prandom.random()`
Get the next random number in the range `[0.0, 1.0)`.

EXAMPLES:

```
sage: [random() for i in [1 .. 4]]
[0.111439293741037, 0.5143475134191677, 0.04468968524815642, 0.332490606442413]
```

`sage.misc.prandom.randrange(start, stop=None, step=1)`
Choose a random item from range(`start`, `stop`[, `step`]).

This fixes the problem with `randint()` which includes the endpoint; in Python this is usually not what you want.

EXAMPLES:

```
sage: randrange(0, 100, 11)
11
```

```
sage: randrange(5000, 5100)
5051
sage: [randrange(0, 2) for i in range(15)]
[0, 0, 1, 0, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1]
sage: randrange(0, 1000000, 1000)
486000
sage: randrange(-100, 10)
-56
```

`sage.misc.prandom.sample` (*population*, *k*)

Chooses *k* unique random elements from a population sequence.

Returns a new list containing elements from the population while leaving the original population unchanged. The resulting list is in selection order so that all sub-slices will also be valid random samples. This allows raffle winners (the sample) to be partitioned into grand prize and second place winners (the subslices).

Members of the population need not be hashable or unique. If the population contains repeats, then each occurrence is a possible selection in the sample.

To choose a sample in a range of integers, use `xrange` as an argument. This is especially fast and space efficient for sampling from a large population: `sample(xrange(10000000), 60)`

EXAMPLES:

```
sage: sample(["Here", "I", "come", "to", "save", "the", "day"], 3)
['Here', 'to', 'day']
sage: sample(xrange(2^30), 7)
[357009070, 558990255, 196187132, 752551188, 85926697, 954621491, 624802848]
```

`sage.misc.prandom.shuffle` (*x*, *random=None*)

x, *random=random.random* -> shuffle list *x* in place; return None.

Optional arg *random* is a 0-argument function returning a random float in [0.0, 1.0); by default, the `sage.misc.random.random`.

EXAMPLES:

```
sage: shuffle([1 .. 10])
```

`sage.misc.prandom.uniform` (*a*, *b*)

Get a random number in the range [*a*, *b*).

Equivalent to code `{a + (b-a) * random()}`.

EXAMPLES:

```
sage: uniform(0, 1)
0.111439293741037
sage: uniform(e, pi)
0.5143475134191677*pi + 0.48565248658083227*e
sage: RR(_)
2.93601069876846
```

`sage.misc.prandom.vonmisesvariate` (*mu*, *kappa*)

Circular data distribution.

mu is the mean angle, expressed in radians between 0 and 2π , and *kappa* is the concentration parameter, which must be greater than or equal to zero. If *kappa* is equal to zero, this distribution reduces to a uniform random angle over the range 0 to 2π .

EXAMPLES:

```
sage: [vonmisesvariate(1.0r, 3.0r) for i in range(1, 5)] # abs tol 1e-12
[0.898328639355427, 0.6718030007041281, 2.0308777524813393, 1.714325253725145]
```

`sage.misc.prandom.weibullvariate(alpha, beta)`

Weibull distribution.

alpha is the scale parameter and beta is the shape parameter.

EXAMPLES:

```
sage: [weibullvariate(1, 3) for i in range(1, 5)]
[0.490697775546342537, 0.8972185564611213, 0.357573846531942, 0.739377255516847]
```

1.5 The Unknown truth value

AUTHORS:

- Florent Hivert (2010): initial version.

class `sage.misc.unknown.UnknownClass`

Bases: `sage.structure.unique_representation.UniqueRepresentation`,
`sage.structure.sage_object.SageObject`

TESTS:

```
sage: TestSuite(Unknown).run()
```

1.6 Exceptions

This module defines Sage-specific exceptions.

exception `sage.misc.exceptions.OptionalPackageNotFoundError`

Bases: `exceptions.RuntimeError`

This class defines the exception that should be raised when a function, method, or class cannot detect an optional package that it depends on. When an `OptionalPackageNotFoundError` is raised, this means one of the following:

- The required optional package is not installed.
- The required optional package is installed, but the relevant interface to that package is unable to detect the package.

EXAMPLES:

```
sage: from sage.misc.exceptions import OptionalPackageNotFoundError
sage: def find_package(fav_package):
...     try:
...         raise OptionalPackageNotFoundError("Unable to detect optional package: %s" % fav_package)
...     except OptionalPackageNotFoundError:
...         raise
...
sage: find_package("ham and spam")
Traceback (most recent call last):
...
OptionalPackageNotFoundError: Unable to detect optional package: ham and spam
```


PROGRAMMING UTILITIES

2.1 Python 2 and 3 Compatibility

2.1.1 MISSING TITLE

`sage.misc.six.with_metaclass` (*meta*, **bases*)
Create a base class with a metaclass.

TESTS:

```
sage: from sage.misc.six import with_metaclass
sage: class Meta(type): pass
sage: class X(with_metaclass(Meta)): pass
sage: type(X) is Meta
True
sage: issubclass(X, object)
True
sage: class Base(object): pass
sage: class X(with_metaclass(Meta, Base)): pass
sage: type(X) is Meta
True
sage: issubclass(X, Base)
True
sage: class Base2(object): pass
sage: class X(with_metaclass(Meta, Base, Base2)): pass
sage: type(X) is Meta
True
sage: issubclass(X, Base)
True
sage: issubclass(X, Base2)
True
sage: X.__mro__ == (X, Base, Base2, object)
True
```

Check that [trac ticket #18503](#) is fixed, i.e. that `with_metaclass` works with cdef'ed metaclasses:

```
sage: from sage.misc.classcall_metaclass import ClasscallMetaclass
sage: class X(with_metaclass(ClasscallMetaclass)): pass
sage: type(X) is ClasscallMetaclass
True
sage: X.__mro__ == (X, object)
True
```

2.2 Special Base Classes, Decorators, etc.

2.2.1 Abstract methods

class `sage.misc.abstract_method.AbstractMethod` (*f*, *optional=False*)

Bases: `object`

Constructor for abstract methods

EXAMPLES:

```
sage: def f(x):
...     "doc of f"
...     return 1
...
sage: x = abstract_method(f); x
<abstract method f at ...>
sage: x.__doc__
'doc of f'
sage: x.__name__
'f'
sage: x.__module__
'__main__'
```

is_optional ()

Returns whether an abstract method is optional or not.

EXAMPLES:

```
sage: class AbstractClass:
...     @abstract_method
...     def required(): pass
...
...     @abstract_method(optional = True)
...     def optional(): pass
sage: AbstractClass.required.is_optional()
False
sage: AbstractClass.optional.is_optional()
True
```

`sage.misc.abstract_method.abstract_method` (*f=None*, *optional=False*)

Abstract methods

INPUT:

- *f* – a function
- *optional* – a boolean; defaults to `False`

The decorator `abstract_method` can be used to declare methods that should be implemented by all concrete derived classes. This declaration should typically include documentation for the specification for this method.

The purpose is to enforce a consistent and visual syntax for such declarations. It is used by the Sage categories for automated tests (see `Sets.Parent.test_not_implemented`).

EXAMPLES:

We create a class with an abstract method:

```
sage: class A(object):
...
...     @abstract_method
```

```

...     def my_method(self):
...         """
...         The method :meth:`my_method` computes my_method
...
...         EXAMPLES::
...
...         """
...         pass
sage: A.my_method
<abstract method my_method at ...>

```

The current policy is that a `NotImplementedError` is raised when accessing the method through an instance, even before the method is called:

```

sage: x = A()
sage: x.my_method
Traceback (most recent call last):
...
NotImplementedError: <abstract method my_method at ...>

```

It is also possible to mark abstract methods as optional:

```

sage: class A(object):
...     @abstract_method(optional = True)
...     def my_method(self):
...         """
...         The method :meth:`my_method` computes my_method
...
...         EXAMPLES::
...
...         """
...         pass
sage: A.my_method
<optional abstract method my_method at ...>
sage: x = A()
sage: x.my_method
NotImplemented

```

The official mantra for testing whether an optional abstract method is implemented is:

```

# Fixme: sage -t complains about indentation below
# sage: if x.my_method is not NotImplemented:
#     ...     x.my_method()
#     else:
#         print "x.my_method not available. Let's use some other trick."
#     ...
# x.my_method not available. Let's use some other trick.

```

Discussion

The policy details are not yet fixed. The purpose of this first implementation is to let developers experiment with it and give feedback on what's most practical.

The advantage of the current policy is that attempts at using a non implemented methods are caught as early as possible. On the other hand, one cannot use introspection directly to fetch the documentation:

```
sage: x.my_method?          # todo: not implemented
```

Instead one needs to do:

```
sage: A._my_method?        # todo: not implemented
```

This could probably be fixed in `sage.misc.sageinspect`.

TODO: what should be the recommended mantra for existence testing from the class?

TODO: should extra information appear in the output? The name of the class? That of the super class where the abstract method is defined?

TODO: look for similar decorators on the web, and merge

Implementation details

Technically, an `abstract_method` is a non-data descriptor (see Invoking Descriptors in the Python reference manual).

The syntax `@abstract_method` w.r.t. `@abstract_method(optional = True)` is achieved by a little trick which we test here:

```
sage: abstract_method(optional = True)
<function <lambda> at ...>
sage: abstract_method(optional = True)(banner)
<optional abstract method banner at ...>
sage: abstract_method(banner, optional = True)
<optional abstract method banner at ...>
```

```
sage.misc.abstract_method.abstract_methods_of_class(cls)
```

Returns the required and optional abstract methods of the class

EXAMPLES:

```
sage: class AbstractClass:
...     @abstract_method
...     def required1(): pass
...
...     @abstract_method(optional = True)
...     def optional2(): pass
...
...     @abstract_method(optional = True)
...     def optional1(): pass
...
...     @abstract_method
...     def required2(): pass
...
sage: sage.misc.abstract_method.abstract_methods_of_class(AbstractClass)
{'optional': ['optional1', 'optional2'],
 'required': ['required1', 'required2']}
```

2.2.2 Bindable classes

```
class sage.misc.bindable_class.BindableClass
    Bases: object
```

Bindable classes

This class implements a binding behavior for nested classes that derive from it. Namely, if a nested class `Outer.Inner` derives from `BindableClass`, and if `outer` is an instance of `Outer`, then `outer.Inner(...)` is equivalent to `Outer.Inner(outer, ...)`.

EXAMPLES:

Let us consider the following class `Outer` with a nested class `Inner`:

```
sage: from sage.misc.nested_class import NestedClassMetaclass
sage: class Outer:
...     __metaclass__ = NestedClassMetaclass # just a workaround for Python misnaming nested c
...
...     class Inner:
...         def __init__(self, *args):
...             print args
...
...         def f(self, *args):
...             print self, args
...
...         @staticmethod
...         def f_static(*args):
...             print args
...
sage: outer = Outer()
```

By default, when `Inner` is a class nested in `Outer`, accessing `outer.Inner` returns the `Inner` class as is:

```
sage: outer.Inner is Outer.Inner
True
```

In particular, `outer` is completely ignored in the following call:

```
sage: x = outer.Inner(1,2,3)
(1, 2, 3)
```

This is similar to what happens with a static method:

```
sage: outer.f_static(1,2,3)
(1, 2, 3)
```

In some cases, we would want instead `Inner`` to receive `outer` as parameter, like in a usual method call:

```
sage: outer.f(1,2,3)
<__main__.Outer object at ...> (1, 2, 3)
```

To this end, `outer.f` returns a *bound method*:

```
sage: outer.f
<bound method Outer.f of <__main__.Outer object at ...>>
```

so that `outer.f(1,2,3)` is equivalent to:

```
sage: Outer.f(outer, 1,2,3)
<__main__.Outer object at ...> (1, 2, 3)
```

`BindableClass` gives this binding behavior to all its subclasses:

```
sage: from sage.misc.bindable_class import BindableClass
sage: class Outer:
...     __metaclass__ = NestedClassMetaclass # just a workaround for Python misnaming nested c
...
```

```
...     class Inner(BindableClass):
...         " some documentation "
...         def __init__(self, outer, *args):
...             print outer, args
```

Calling `Outer.Inner` returns the (unbound) class as usual:

```
sage: Outer.Inner
<class '__main__.Outer.Inner'>
```

However, `outer.Inner(1, 2, 3)` is equivalent to `Outer.Inner(outer, 1, 2, 3)`:

```
sage: outer = Outer()
sage: x = outer.Inner(1, 2, 3)
<__main__.Outer object at ...> (1, 2, 3)
```

To achieve this, `outer.Inner` returns (some sort of) bound class:

```
sage: outer.Inner
<bound class '__main__.Outer.Inner' of <__main__.Outer object at ...>>
```

Note: This is not actually a class, but an instance of `functools.partial`:

```
sage: type(outer.Inner).mro()
[<class 'sage.misc.bindable_class.BoundClass'>,
 <type 'functools.partial'>,
 <type 'object'>]
```

Still, documentation works as usual:

```
sage: outer.Inner.__doc__
' some documentation '
```

TESTS:

```
sage: from sage.misc.bindable_class import Outer
sage: TestSuite(Outer.Inner).run()
sage: outer = Outer()
sage: TestSuite(outer.Inner).run(skip=["_test_pickling"])
```

```
class sage.misc.bindable_class.BoundClass(*args)
    Bases: functools.partial
```

```
class sage.misc.bindable_class.Inner2
    Bases: sage.misc.bindable_class.BindableClass
    Some documentation for Inner2
```

```
class sage.misc.bindable_class.Outer
    Bases: object
```

A class with a bindable nested class, for testing purposes

```
class Inner
    Bases: sage.misc.bindable_class.BindableClass
    Some documentation for Outer.Inner
```

```
Outer.Inner2
    alias of Inner2
```

2.2.3 Decorators

Python decorators for use in Sage.

AUTHORS:

- Tim Dumol (5 Dec 2009) – initial version.
- Johan S. R. Nielsen (2010) – collect decorators from various modules.
- Johan S. R. Nielsen (8 apr 2011) – improve introspection on decorators.
- Simon King (2011-05-26) – improve introspection of `sage_wraps`. Put this file into the reference manual.
- Julian Rueth (2014-03-19): added `decorator_keywords` decorator

`sage.misc.decorators.decorator_defaults` (*func*)

This function allows a decorator to have default arguments.

Normally, a decorator can be called with or without arguments. However, the two cases call for different types of return values. If a decorator is called with no parentheses, it should be run directly on the function. However, if a decorator is called with parentheses (i.e., arguments), then it should return a function that is then in turn called with the defined function as an argument.

This decorator allows us to have these default arguments without worrying about the return type.

EXAMPLES:

```
sage: from sage.misc.decorators import decorator_defaults
sage: @decorator_defaults
... def my_decorator(f, *args, **kwds):
...     print kwds
...     print args
...     print f.__name__
...
sage: @my_decorator
... def my_fun(a,b):
...     return a,b
...
{}
()
my_fun
sage: @my_decorator(3,4,c=1,d=2)
... def my_fun(a,b):
...     return a,b
...
{'c': 1, 'd': 2}
(3, 4)
my_fun
```

`sage.misc.decorators.decorator_keywords` (*func*)

A decorator for decorators with optional keyword arguments.

EXAMPLES:

```
sage: from sage.misc.decorators import decorator_keywords
sage: @decorator_keywords
....: def preprocess(f=None, processor=None):
....:     def wrapper(*args, **kwargs):
....:         if processor is not None:
....:             args, kwargs = processor(*args, **kwargs)
....:         return f(*args, **kwargs)
....:     return wrapper
```

This decorator can be called with and without arguments:

```
sage: @preprocess
....: def foo(x): return x
sage: foo(None)
sage: foo(1)
1

sage: def normalize(x): return ((0,), {}) if x is None else ((x,), {})
sage: @preprocess(processor=normalize)
....: def foo(x): return x
sage: foo(None)
0
sage: foo(1)
1
```

```
class sage.misc.decorators.infix_operator(precedence)
    Bases: object
```

A decorator for functions which allows for a hack that makes the function behave like an infix operator.

This decorator exists as a convenience for interactive use.

EXAMPLES:

An infix dot product operator:

```
sage: def dot(a,b): return a.dot_product(b)
sage: dot=infix_operator('multiply')(dot)
sage: u=vector([1,2,3])
sage: v=vector([5,4,3])
sage: u *dot* v
22
```

An infix element-wise addition operator:

```
sage: def eadd(a,b):
...     return a.parent([i+j for i,j in zip(a,b)])
sage: eadd=infix_operator('add')(eadd)
sage: u=vector([1,2,3])
sage: v=vector([5,4,3])
sage: u +eadd+ v
(6, 6, 6)
sage: 2*u +eadd+ v
(7, 8, 9)
```

A hack to simulate a postfix operator:

```
sage: def thendo(a,b): return b(a)
sage: thendo=infix_operator('or')(thendo)
sage: x |thendo| cos |thendo| (lambda x: x^2)
cos(x)^2
```

```
class sage.misc.decorators.options(**options)
    Bases: object
```

A decorator for functions which allows for default options to be set and reset by the end user. Additionally, if one needs to, one can get at the original keyword arguments passed into the decorator.

TESTS:


```

sage: from sage.misc.decorators import options
sage: o = options(rgbcolor=(0,0,1))
sage: o.options
{'rgbcolor': (0, 0, 1)}
sage: o = options(rgbcolor=(0,0,1), __original_opts=True)
sage: o.original_opts
True
sage: loads(dumps(o)).options
{'rgbcolor': (0, 0, 1)}

```

Demonstrate that the introspected argument specification of the wrapped function is updated (see [trac ticket #9976](#)):

```

sage: from sage.misc.decorators import options
sage: o = options(rgbcolor=(0,0,1))
sage: def f(*args, **kwargs): print args, list(sorted(kwargs.items()))
sage: f1 = o(f)
sage: from sage.misc.sageinspect import sage_getargspec
sage: sage_getargspec(f1)
ArgSpec(args=['rgbcolor'], varargs='args', keywords='kwargs', defaults=((0, 0, 1),))

```

class `sage.misc.decorators.rename_keyword` (*deprecated=None, deprecation=None, **renames*)
 Bases: `object`

A decorator which renames keyword arguments and optionally deprecates the new keyword.

INPUT:

- `deprecation` – integer. The trac ticket number where the deprecation was introduced.
- the rest of the arguments is a list of keyword arguments in the form `renamed_option='existing_option'`. This will have the effect of renaming `renamed_option` so that the function only sees `existing_option`. If both `renamed_option` and `existing_option` are passed to the function, `existing_option` will override the `renamed_option` value.

EXAMPLES:

```

sage: from sage.misc.decorators import rename_keyword
sage: r = rename_keyword(color='rgbcolor')
sage: r.renames
{'color': 'rgbcolor'}
sage: loads(dumps(r)).renames
{'color': 'rgbcolor'}

```

To deprecate an old keyword:

```

sage: r = rename_keyword(deprecation=13109, color='rgbcolor')

```

`sage.misc.decorators.sage_wraps` (*wrapped, assigned=('__module__', '__name__', '__doc__'), updated=('__dict__',)*)

Decorator factory which should be used in decorators for making sure that meta-information on the decorated callables are retained through the decorator, such that the introspection functions of `sage.misc.sageinspect` retrieves them correctly. This includes documentation string, source, and argument specification. This is an extension of the Python standard library decorator `functools.wraps`.

That the argument specification is retained from the decorated functions implies, that if one uses `sage_wraps` in a decorator which intentionally changes the argument specification, one should add this information to the special attribute `__sage_argspec__` of the wrapping function (for an example, see e.g. `@options` decorator in this module).

EXAMPLES:

Demonstrate that documentation string and source are retained from the decorated function:

```
sage: def square(f):
...     @sage_wraps(f)
...     def new_f(x):
...         return f(x)*f(x)
...     return new_f
sage: @square
... def g(x):
...     "My little function"
...     return x
sage: g(2)
4
sage: g(x)
x^2
sage: g.__doc__
'My little function'
sage: from sage.misc.sageinspect import sage_getsource, sage_getsourcelines, sage_getfile
sage: sage_getsource(g)
'@square...def g(x) ...'
```

Demonstrate that the argument description are retained from the decorated function through the special method (when left unchanged) (see [trac ticket #9976](#)):

```
sage: def diff_arg_dec(f):
...     @sage_wraps(f)
...     def new_f(y, some_def_arg=2):
...         return f(y+some_def_arg)
...     return new_f
sage: @diff_arg_dec
... def g(x):
...     return x
sage: g(1)
3
sage: g(1, some_def_arg=4)
5
sage: from sage.misc.sageinspect import sage_getargspec
sage: sage_getargspec(g)
ArgSpec(args=['x'], varargs=None, keywords=None, defaults=None)
```

Demonstrate that it correctly gets the source lines and the source file, which is essential for interactive code edition; note that we do not test the line numbers, as they may easily change:

```
sage: P.<x,y> = QQ[]
sage: I = P*[x,y]
sage: sage_getfile(I.interreduced_basis)
'.../sage/interfaces/singular.py'
sage: sage_getsourcelines(I.interreduced_basis)
(['    @singular_gb_standard_options\n',
  '    @libsingular_gb_standard_options\n',
  '    def interreduced_basis(self):\n',
  ...
  '        return self.basis.reduced()\n'], ...)
```

Demonstrate that `sage_wraps` works for non-function callables ([trac ticket #9919](#)):

```
sage: def square_for_met(f):
...     @sage_wraps(f)
...     def new_f(self, x):
```

```

...         return f(self, x) * f(self, x)
...     return new_f
sage: class T:
...     @square_for_met
...     def g(self, x):
...         "My little method"
...         return x
sage: t = T()
sage: t.g(2)
4
sage: t.g.__doc__
'My little method'

```

The bug described in [trac ticket #11734](#) is fixed:

```

sage: def square(f):
...     @sage_wraps(f)
...     def new_f(x):
...         return f(x) * f(x)
...     return new_f
sage: f = lambda x: x^2
sage: g = square(f)
sage: g(3) # this line used to fail for some people if these command were manually entered on th
81

```

class `sage.misc.decorators.specialize(*args, **kwargs)`

A decorator generator that returns a decorator that in turn returns a specialized function for function `f`. In other words, it returns a function that acts like `f` with arguments `*args` and `**kwargs` supplied.

INPUT:

•`*args, **kwargs` – arguments to specialize the function for.

OUTPUT:

•a decorator that accepts a function `f` and specializes it with `*args` and `**kwargs`

EXAMPLES:

```

sage: f = specialize(5)(lambda x, y: x+y)
sage: f(10)
15
sage: f(5)
10
sage: @specialize("Bon Voyage")
... def greet(greeting, name):
...     print "{0}, {1}!".format(greeting, name)
sage: greet("Monsieur Jean Valjean")
Bon Voyage, Monsieur Jean Valjean!
sage: greet(name = 'Javert')
Bon Voyage, Javert!

```

class `sage.misc.decorators.suboptions(name, **options)`

Bases: `object`

A decorator for functions which collects all keywords starting with `name+'_'` and collects them into a dictionary which will be passed on to the wrapped function as a dictionary called `name_options`.

The keyword arguments passed into the constructor are taken to be default for the `name_options` dictionary.

EXAMPLES:

```
sage: from sage.misc.decorators import suboptions
sage: s = suboptions('arrow', size=2)
sage: s.name
'arrow_'
sage: s.options
{'size': 2}
```

2.2.4 Constant functions

class sage.misc.constant_function.**ConstantFunction**

Bases: sage.structure.sage_object.SageObject

A class for function objects implementing constant functions.

EXAMPLES:

```
sage: f = ConstantFunction(3)
sage: f
The constant function (...) -> 3
sage: f()
3
sage: f(5)
3
```

Such a function could be implemented as a lambda expression, but this is not (currently) picklable:

```
sage: g = lambda x: 3
sage: g == loads(dumps(g))
Traceback (most recent call last):
...
PicklingError: Can't pickle <type 'function'>: attribute lookup __builtin__.function failed
sage: f == loads(dumps(f))
True
```

Also, in the long run, the information that this function is constant could be used by some algorithms.

TODO:

- Should constant functions have unique representation?
- Should the number of arguments be specified in the input?
- Should this go into `sage.categories.maps`? Then what should be the parent (e.g. for `lambda x: True`)?

TESTS:

These tests do fail if we try to use `UniqueRepresentation`:

```
sage: f = ConstantFunction(True)
sage: g = ConstantFunction(1)
sage: f(), g()
(True, 1)
```

That's because 1 and True cannot be distinguished as keys in a dictionary (arg!):

```
sage: { 1: 'a', True: 'b' }
{1: 'b'}
```

2.2.5 Special Methods for Classes

AUTHORS:

- Nicolas M. Thiery (2009-2011) implementation of `__classcall__`, `__classget__`, `__classcontains__`;
- Florent Hivert (2010-2012): implementation of `__classcall_private__`, documentation, Cythonization and optimization.

```
class sage.misc.classcall_metaclass.ClasscallMetaclass
    Bases: sage.misc.nested_class.NestedClassMetaclass
```

A metaclass providing support for special methods for classes.

From the Section [Special method names](#) of the Python Reference Manual:

‘a class `cls` can implement certain operations on its instances that are invoked by special syntax (such as arithmetic operations or subscripting and slicing) by defining methods with special names’.

The purpose of this metaclass is to allow for the class `cls` to implement analogues of those special methods for the operations on the class itself.

Currently, the following special methods are supported:

- `__classcall__` (and `__classcall_private__`) for customizing `cls(...)` (analogue of `__call__`).
- `__classcontains__` for customizing membership testing `x in cls` (analogue of `__contains__`).
- `__classget__` for customizing the binding behavior in `foo.cls` (analogue of `__get__`).

See the documentation of `__call__()` and of `__get__()` and `__contains__()` for the description of the respective protocols.

Warning: For technical reasons, `__classcall__`, `__classcall_private__`, `__classcontains__`, and `__classget__` must be defined as `staticmethod()`’s, even though they receive the class itself as their first argument.

Warning: For efficiency reasons, the resolution for the special methods is done once for all, upon creation of the class. Thus, later dynamic changes to those methods are ignored. But see also `_set_classcall()`.

`ClasscallMetaclass` is an extension of the base type.

TODO: find a good name for this metaclass.

TESTS:

```
sage: PerfectMatchings(2).list()
[[ (1, 2) ]]
```

Note: If a class is put in this metaclass it automatically becomes a new-style class:

```
sage: from sage.misc.classcall_metaclass import ClasscallMetaclass
sage: class Foo:
...     __metaclass__ = ClasscallMetaclass
sage: x = Foo(); x
<__main__.Foo object at 0x...>
sage: isinstance(Foo, object)
True
```

```
sage: isinstance(Foo, type)
True
```

```
sage.misc.classcall_metaclass.typecall(cls, *args, **kws)
Object construction
```

This is a faster equivalent to `type.__call__(cls, <some arguments>)`.

INPUT:

- `cls` – the class used for constructing the instance. It must be a builtin type or a new style class (inheriting from `object`).

EXAMPLES:

```
sage: from sage.misc.classcall_metaclass import typecall
sage: class Foo(object): pass
sage: typecall(Foo)
<__main__.Foo object at 0x...>
sage: typecall(list)
[]
sage: typecall(Integer, 2)
2
```

Warning: `typecall()` doesn't work for old style class (not inheriting from `object`):

```
sage: class Bar: pass
sage: typecall(Bar)
Traceback (most recent call last):
...
TypeError: Argument 'cls' has incorrect type (expected type, got classobj)
```

```
sage.misc.classcall_metaclass.timeCall(T, n, *args)
```

We illustrate some timing when using the classcall mechanism.

EXAMPLES:

```
sage: from sage.misc.classcall_metaclass import (
...     ClasscallMetaclass, CRef, C2, C3, C2C, timeCall)
sage: timeCall(object, 1000)
```

For reference let construct basic objects and a basic Python class:

```
sage: %timeit timeCall(object, 1000) # not tested
625 loops, best of 3: 41.4 µs per loop

sage: i1 = int(1); i3 = int(3) # don't use Sage's Integer
sage: class PRef(object):
...     def __init__(self, i):
...         self.i = i+i1
```

For a Python class, compared to the reference class there is a 10% overhead in using `ClasscallMetaclass` if there is no classcall defined:

```
sage: class P(object):
...     __metaclass__ = ClasscallMetaclass
...     def __init__(self, i):
...         self.i = i+i1
```

```
sage: %timeit timeCall(PRef, 1000, i3)    # not tested
625 loops, best of 3: 420 µs per loop
sage: %timeit timeCall(P, 1000, i3)      # not tested
625 loops, best of 3: 458 µs per loop
```

For a Cython class (not cdef since they doesn't allows metaclasses), the overhead is a little larger:

```
sage: %timeit timeCall(CRef, 1000, i3)    # not tested
625 loops, best of 3: 266 µs per loop
sage: %timeit timeCall(C2, 1000, i3)      # not tested
625 loops, best of 3: 298 µs per loop
```

Let's now compare when there is a classcall defined:

```
sage: class PC(object):
...     __metaclass__ = ClasscallMetaclass
...     @staticmethod
...     def __classcall__(cls, i):
...         return i+i1
sage: %timeit timeCall(C2C, 1000, i3)    # not tested
625 loops, best of 3: 148 µs per loop
sage: %timeit timeCall(PC, 1000, i3)     # not tested
625 loops, best of 3: 289 µs per loop
```

The overhead of the indirection (C(...) -> ClasscallMetaclass.__call__(...) -> C.__classcall__(...)) is unfortunately quite large in this case (two method calls instead of one). In reasonable usecases, the overhead should be mostly hidden by the computations inside the classcall:

```
sage: %timeit timeCall(C2C.__classcall__, 1000, C2C, i3) # not tested
625 loops, best of 3: 33 µs per loop
sage: %timeit timeCall(PC.__classcall__, 1000, PC, i3)   # not tested
625 loops, best of 3: 131 µs per loop
```

Finally, there is no significant difference between Cython's V2 and V3 syntax for metaclass:

```
sage: %timeit timeCall(C2, 1000, i3)    # not tested
625 loops, best of 3: 330 µs per loop
sage: %timeit timeCall(C3, 1000, i3)    # not tested
625 loops, best of 3: 328 µs per loop
```

2.2.6 Metaclasses for Cython extension types

Cython does not support metaclasses, but this module can be used to implement metaclasses for extension types.

Warning: This module has many caveats and you can easily get segfaults if you make a mistake. It relies on undocumented Python and Cython behaviour, so things might break in future versions.

How to use

To enable this metaclass mechanism, you need to put `cimport sage.misc.cython_metaclass` in your module (in the `.pxd` file if you are using one).

In the extension type (a.k.a. `cdef class`) for which you want to define a metaclass, define a method `__getmetaclass__` with a single unused argument. This method should return a type to be used as metaclass:

```
cimport sage.misc.cython_metaclass
cdef class MyCustomType(object):
    def __getmetaclass__(_):
        from foo import MyMetaclass
        return MyMetaclass
```

The above `__getmetaclass__` method is analogous to `__metaclass__ = MyMetaclass` in Python 2.

Warning: `__getmetaclass__` must be defined as an ordinary method taking a single argument, but this argument should not be used in the method (it will be `None`).

When a type `cls` is being constructed with metaclass `meta`, then `meta.__init__(cls, None, None, None)` is called from Cython. In Python, this would be `meta.__init__(cls, name, bases, dict)`.

Warning: The `__getmetaclass__` method is called while the type is being created during the import of the module. Therefore, `__getmetaclass__` should not refer to any global objects, including the type being created or other types defined or imported in the module (unless you are very careful). Note that non-imported `cdef` functions are not Python objects, so those are safe to call.
The same warning applies to the `__init__` method of the metaclass.

Warning: The `__new__` method of the metaclass (including the `__cinit__` method for Cython extension types) is never called if you're using this from Cython. In particular, the metaclass cannot have any attributes or virtual methods.

EXAMPLES:

```
sage: cython("""
....: cimport sage.misc.cython_metaclass
....: cdef class MyCustomType(object):
....:     def __getmetaclass__(_):
....:         class MyMetaclass(type):
....:             def __init__(*args):
....:                 print "Calling MyMetaclass.__init__{}".format(args)
....:                 return MyMetaclass
....:
....: cdef class MyDerivedType(MyCustomType):
....:     pass
....: """)
Calling MyMetaclass.__init__(<type '...MyCustomType'>, None, None, None)
Calling MyMetaclass.__init__(<type '...MyDerivedType'>, None, None, None)
sage: MyCustomType.__class__
<class '...MyMetaclass'>
sage: class MyPythonType(MyDerivedType):
....:     pass
Calling MyMetaclass.__init__(<class '...MyPythonType'>, 'MyPythonType', (<type '...MyDerivedType'>,,)
```

Implementation

All this is implemented by defining

```
#define PyTypeReady(t) Sage_PyType_Ready(t)
```

and then implementing the function `Sage_PyType_Ready(t)` which first calls `PyType_Ready(t)` and then handles the metaclass stuff.

TESTS:

Check that a proper exception is raised if `__getmetaclass__` returns a non-type:

```
sage: cython("""
....: cimport sage.misc.cython_metaclass
....: cdef class MyCustomType(object):
....:     def __getmetaclass__(_):
....:         return 2
....: """)
Traceback (most recent call last):
...
TypeError: __getmetaclass__ did not return a type
```

2.2.7 Metaclass for inheriting comparison functions

This module defines a metaclass `InheritComparisonMetaclass` to inherit comparison functions in Cython extension types. In Python 2, the special methods `__richcmp__`, `__cmp__` and `__hash__` are only inherited as a whole: defining just 1 or 2 of these will prevent the others from being inherited.

To solve this issue, you can use `InheritComparisonMetaclass` as a Cython “metaclass” (see `sage.misc.cython_metaclass` for the general mechanism). If you do this for an extension type which defines neither `__richcmp__` nor `__cmp__`, then both these methods are inherited from the base class (the MRO is not used).

In Sage, this is in particular used for `sage.structure.element.Element` to support comparisons using the coercion framework.

None of this is relevant to Python classes, which inherit comparison methods anyway.

AUTHOR:

- Jeroen Demeyer (2015-05-22): initial version, see [trac ticket #18329](#)

```
class sage.misc.inherit_comparison.InheritComparisonClasscallMetaclass
    Bases:
        sage.misc.inherit_comparison.InheritComparisonMetaclass,
        sage.misc.classcall_metaclass.ClasscallMetaclass
```

```
class sage.misc.inherit_comparison.InheritComparisonMetaclass
    Bases: type
```

If the type does not define `__richcmp__` nor `__cmp__`, inherit both these methods from the base class. The difference with plain extension types is that comparison is inherited even if `__hash__` is defined.

EXAMPLES:

```
sage: cython("""
....: from sage.misc.inherit_comparison cimport InheritComparisonMetaclass
....:
....: cdef class Base(object):
....:     def __richcmp__(left, right, int op):
....:         print("Calling Base.__richcmp__")
....:         return left is right
....:
....: cdef class Derived(Base):
....:     def __hash__(self):
....:         return 1
....:
....: cdef class DerivedWithRichcmp(Base):
....:     def __getmetaclass__(_):
```

```
.....:         from sage.misc.inherit_comparison import InheritComparisonMetaclass
.....:         return InheritComparisonMetaclass
.....:     def __hash__(self):
.....:         return 1
.....:     '''
sage: a = Derived()
sage: a == a
True
sage: b = DerivedWithRichcmp()
sage: b == b
Calling Base.__richcmp__
True
```

2.2.8 Base Class to Support Method Decorators

AUTHOR:

- Martin Albrecht (2009-05): inspired by a conversation with and code by Mike Hansen

class `sage.misc.method_decorator.MethodDecorator` (*f*)
Bases: `sage.structure.sage_object.SageObject`

EXAMPLE:: `sage: from sage.misc.method_decorator import MethodDecorator sage: class Foo: ...`
`@MethodDecorator ... def bar(self, x): ... return x**2 ... sage: J = Foo() sage: J.bar <class`
`'sage.misc.method_decorator.MethodDecorator'>`

2.2.9 Multiplex calls to one object to calls to many objects

AUTHORS:

- Martin Albrecht (2011): initial version

class `sage.misc.object_muxlexer.Multiplex` (**args*)
Object for a list of children such that function calls on this new objects implies that the same function is called on all children.

class `sage.misc.object_muxlexer.MultiplexFunction` (*multiplexer, name*)
A simple wrapper object for functions that are called on a list of objects.

2.2.10 Fast methods via Cython

This module provides extension classes with useful methods of cython speed, that python classes can inherit.

Note: This module provides a cython base class `WithEqualityById` implementing unique instance behaviour, and a cython base class `FastHashable_class`, which has a quite fast hash whose value can be freely chosen at initialisation time.

AUTHOR:

- Simon King (2013-02): Original version
- Simon King (2013-10): Add `Singleton`

class `sage.misc.fast_methods.FastHashable_class`
Bases: `object`

A class that has a fast hash method, returning a pre-assigned value.

NOTE:

This is for internal use only. The class has a cdef attribute `_hash`, that needs to be assigned (for example, by calling the `init` method, or by a direct assignement using cython). This is slower than using `provide_hash_by_id()`, but has the advantage that the hash can be prescribed, by assigning a cdef attribute `_hash`.

TESTS:

```
sage: from sage.misc.fast_methods import FastHashable_class
sage: H = FastHashable_class(123)
sage: hash(H)
123
```

class `sage.misc.fast_methods.Singleton`
 Bases: `sage.misc.fast_methods.WithEqualityById`

A base class for singletons.

A singleton is a class that allows to create not more than a single instance. This instance can also belong to a subclass, but it is not possible to have several subclasses of a singleton all having distinct unique instances.

In order to create a singleton, just add `Singleton` to the list of base classes:

```
sage: from sage.misc.fast_methods import Singleton
sage: class C(Singleton, SageObject):
....:     def __init__(self):
....:         print "creating singleton"
sage: c = C()
creating singleton
sage: c2 = C()
sage: c is c2
True
```

The unique instance of a singleton stays in memory as long as the singleton itself does.

Pickling, copying, hashing, and comparison are provided for by `Singleton` according to the singleton paradigm. Note that pickling fails if the class is replaced by a sub-sub-class after creation of the instance:

```
sage: class D(C):
....:     pass
sage: import __main__          # This is only needed ...
sage: __main__.C = C          # ... in doctests
sage: __main__.D = D          # same here, only in doctests
sage: orig = type(c)
sage: c.__class__ = D
sage: orig == type(c)
False
sage: loads(dumps(c))
Traceback (most recent call last):
...
AssertionError: (('<class '.__main__.D'> is not a direct
subclass of <class 'sage.misc.fast_methods.Singleton'>',),
<class '.__main__.D'>, ())
```

class `sage.misc.fast_methods.WithEqualityById`
 Bases: `object`

Provide hash and equality test based on identity.

Note: This class provides the unique representation behaviour of `UniqueRepresentation`, together with `CachedRepresentation`.

EXAMPLES:

Any instance of `UniqueRepresentation` inherits from `WithEqualityById`.

```
sage: class MyParent (Parent):
...     def __init__(self, x):
...         self.x = x
...     def __cmp__(self, other):
...         return cmp(self.x^2, other.x^2)
...     def __hash__(self):
...         return hash(self.x)
sage: class MyUniqueParent (UniqueRepresentation, MyParent): pass
sage: issubclass(MyUniqueParent, sage.misc.fast_methods.WithEqualityById)
True
```

Inheriting from `WithEqualityById` provides unique representation behaviour. In particular, the comparison inherited from `MyParent` is overloaded:

```
sage: a = MyUniqueParent(1)
sage: b = MyUniqueParent(2)
sage: c = MyUniqueParent(1)
sage: a is c
True
sage: d = MyUniqueParent(-1)
sage: a == d
False
```

Note, however, that Python distinguishes between “comparison by `cmp`” and “comparison by binary relations”:

```
sage: cmp(a, d)
0
```

The comparison inherited from `MyParent` will be used in those cases in which identity does not give sufficient information to find the relation:

```
sage: a < b
True
sage: b > d
True
```

The hash inherited from `MyParent` is replaced by a hash that coincides with `object`’s hash:

```
sage: hash(a) == hash(a.x)
False
sage: hash(a) == object.__hash__(a)
True
```

Warning: It is possible to inherit from `UniqueRepresentation` and then overload equality test in a way that destroys the unique representation property. We strongly recommend against it! You should use `CachedRepresentation` instead.

```
sage: class MyNonUniqueParent (MyUniqueParent):
...     def __eq__(self, other):
...         return self.x^2 == other.x^2
sage: a = MyNonUniqueParent(1)
sage: d = MyNonUniqueParent(-1)
sage: a is MyNonUniqueParent(1)
```

```

True
sage: a == d
True
sage: a is d
False

```

2.3 Lists and Iteration, etc.

2.3.1 Callable dictionaries

```
class sage.misc.callable_dict.CallableDict
```

Bases: dict

Callable dictionary.

This is a trivial subclass of dict with an alternative view as a function.

Typical use cases involve passing a dictionary *d* down to some tool that takes a function as input. The usual idiom in such use cases is to pass the `d.__getitem__` bound method. A pitfall is that this object is not pickleable. When this feature is desired, a `CallableDict` can be used instead. Note however that, with the current implementation, `CallableDict` is slightly slower than `d.__getitem__` (see [trac ticket #6484](#) for benchmarks, and [trac ticket #18330](#) for potential for improvement).

EXAMPLES:

```

sage: from sage.misc.callable_dict import CallableDict
sage: d = CallableDict({'one': 1, 'zwei': 2, 'trois': 3})
sage: d['zwei']
2
sage: d('zwei')
2

```

In case the input is not in the dictionary, a `ValueError` is raised, for consistency with the function call syntax:

```

sage: d[1]
Traceback (most recent call last):
....
KeyError: 1
sage: d(1)
Traceback (most recent call last):
....
ValueError: 1 is not in dict

```

2.3.2 Flatten nested lists

```
sage.misc.flatten.flatten(in_list, ltypes=(<type 'list'>, <type 'tuple'>),
                           max_level=9223372036854775807)
```

Flattens a nested list.

INPUT:

- *in_list* – a list or tuple
- *ltypes* – optional list of particular types to flatten
- *max_level* – the maximum level to flatten

OUTPUT:

a flat list of the entries of `in_list`

EXAMPLES:

```
sage: flatten([[1,1],[1],2])
[1, 1, 1, 2]
sage: flatten([[1,2,3], (4,5), [[[1],[2]]]])
[1, 2, 3, 4, 5, 1, 2]
sage: flatten([[1,2,3], (4,5), [[[1],[2]]]],max_level=1)
[1, 2, 3, 4, 5, [[1], [2]]]
sage: flatten([[[3],[ ]]],max_level=0)
[[[3], [ ]]]
sage: flatten([[[3],[ ]]],max_level=1)
[[3], [ ]]
sage: flatten([[[3],[ ]]],max_level=2)
[3]
```

In the following example, the vector isn't flattened because it is not given in the `ltypes` input.

```
sage: flatten(['Hi',2,vector(QQ,[1,2,3])],(4,5,6))
['Hi', 2, (1, 2, 3), 4, 5, 6]
```

We give the vector type and then even the vector gets flattened:

```
sage: flatten(['Hi',2,vector(QQ,[1,2,3])], (4,5,6), ltypes=(list, tuple,sage.modules.vector_real))
['Hi', 2, 1, 2, 3, 4, 5, 6]
```

We flatten a finite field.

```
sage: flatten(GF(5))
[0, 1, 2, 3, 4]
sage: flatten([GF(5)])
[Finite Field of size 5]
sage: flatten([GF(5)], ltypes = (list, tuple, sage.rings.finite_rings.finite_field_prime_modn.Fi
[0, 1, 2, 3, 4]
```

Degenerate cases:

```
sage: flatten([[],[ ]])
[]
sage: flatten([[[ ]]])
[]
```

2.3.3 Searching a sorted list

This is like the `bisect` library module, but also returns whether or not the element is in the list, which saves having to do an extra comparison. Also, the function names make more sense.

`sage.misc.search.search(v,x)`

Return `(True,i)` where `i` is such that `v[i] == x` if there is such an `i`, or `(False,j)` otherwise, where `j` is the position that `a` should be inserted so that `v` remains sorted.

INPUT: `v` – a list, which is assumed sorted `x` – Python object

OUTPUT: `bool, int`

2.3.4 Iterators

Miscellaneous functions which should eventually be moved upstream into Python's standard itertools module.

TODO: Cython this file as soon as Cython supports `yield`.

`sage.misc.sage_itertools.imap_and_filter_none(function, iterable)`

Returns an iterator over the elements `function(x)`, where `x` iterates through `iterable`, such that `function(x)` is not `None`.

EXAMPLES:

```
sage: from sage.misc.sage_itertools import imap_and_filter_none
sage: p = imap_and_filter_none(lambda x: x if is_prime(x) else None, range(15))
sage: [next(p), next(p), next(p), next(p), next(p), next(p)]
[2, 3, 5, 7, 11, 13]
sage: p = imap_and_filter_none(lambda x: x+x, ['a','b','c','d','e'])
sage: [next(p), next(p), next(p), next(p), next(p)]
['aa', 'bb', 'cc', 'dd', 'ee']
```

`sage.misc.sage_itertools.max_cmp(L, cmp=None)`

Returns the largest item of a list (or iterable) with respect to a comparison function.

INPUT: `L` – an iterable `cmp` – an optional comparison function.

`cmp(x, y)` should return a negative value if $x < y$, 0 if $x == y$, and a positive value if $x > y$.

OUTPUT: the largest item of `L` with respect to `cmp`.

EXAMPLES:

```
sage: from sage.misc.sage_itertools import max_cmp
sage: L = [1, -1, 3, -1, 3, 2]
sage: max_cmp(L)
3
sage: def mycmp(x, y): return y - x
sage: max_cmp(L, mycmp)
-1
```

The input can be any iterable:

```
sage: max_cmp( (x^2 for x in L) )
9
sage: max_cmp( (x^2 for x in L), mycmp)
1
```

Computing the max of an empty iterable raises an error:

```
sage: max_cmp([])
Traceback (most recent call last):
...
ValueError: max() arg is an empty sequence
sage: max_cmp([], mycmp)
Traceback (most recent call last):
...
ValueError: max_cmp() arg is an empty sequence
```

`sage.misc.sage_itertools.min_cmp(L, cmp=None)`

Returns the smallest item of a list (or iterable) with respect to a comparison function.

INPUT: `L` – an iterable `cmp` – an optional comparison function.

`cmp(x, y)` should return a negative value if $x < y$, 0 if $x == y$, and a positive value if $x > y$.

OUTPUT: the smallest item of L with respect to `cmp`.

EXAMPLES:

```
sage: from sage.misc.sage_itertools import min_cmp
sage: L = [1,-1,3,-1,3,2]
sage: min_cmp(L)
-1
sage: def mycmp(x,y): return y - x
sage: min_cmp(L, mycmp)
3
```

The input can be any iterable:

```
sage: min_cmp( (x^2 for x in L) )
1
sage: min_cmp( (x^2 for x in L), mycmp)
9
```

Computing the min of an empty iterable raises an error:

```
sage: min_cmp([])
Traceback (most recent call last):
...
ValueError: min() arg is an empty sequence
sage: min_cmp([], mycmp)
Traceback (most recent call last):
...
ValueError: min_cmp() arg is an empty sequence
```

`sage.misc.sage_itertools.unique_merge(*lists)`

INPUT:

- `lists`: sorted lists (or iterables)

Return an iterator over the elements of each list in `lists`, in sorted order, with duplicates removed.

```
sage: from sage.misc.sage_itertools import unique_merge
sage: list(unique_merge([1,2,2,3,4,7,9],
[0,2,4], [2,5]))
[0, 1, 2, 3, 4, 5, 7, 9]
```

Inspired from: http://rosettacode.org/wiki/Create_a_Sequence_of_unique_elements#Python

2.3.5 Multidimensional enumeration

AUTHORS:

- Joel B. Mohler (2006-10-12)
- William Stein (2006-07-19)
- Jon Hanke

`sage.misc.mrange.cartesian_product_iterator(X)`

Iterate over the Cartesian product.

INPUT:

- `X` - list or tuple of lists

OUTPUT: iterator over the cartesian product of the elements of `X`

EXAMPLES:


```

sage: list(cartesian_product_iterator([[1,2], ['a','b']]))
[(1, 'a'), (1, 'b'), (2, 'a'), (2, 'b')]
sage: list(cartesian_product_iterator([]))
[()]

```

`sage.misc.mrange.mrange(sizes, typ=<type 'list'>)`

Return the multirange list with given sizes and type.

This is the list version of `xmrange`. Use `xmrange` for the iterator.

More precisely, return the iterator over all objects of type `typ` of `n`-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

INPUT:

- `sizes` - a list of nonnegative integers
- `typ` - (default: list) a type or class; more generally, something that can be called with a list as input.

OUTPUT: a list

EXAMPLES:

```

sage: mrange([3,2])
[[0, 0], [0, 1], [1, 0], [1, 1], [2, 0], [2, 1]]
sage: mrange([3,2], tuple)
[(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
sage: mrange([3,2], sum)
[0, 1, 1, 2, 2, 3]

```

Examples that illustrate empty multi-ranges:

```

sage: mrange([5,3,-2])
[]
sage: mrange([5,3,0])
[]

```

This example isn't empty, and shouldn't be. See trac #6561.

```

sage: mrange([])
[[]]

```

AUTHORS:

- Jon Hanke
- William Stein

`sage.misc.mrange.mrange_iter(iter_list, typ=<type 'list'>)`

Return the multirange list derived from the given list of iterators.

This is the list version of `xmrange_iter`. Use `xmrange_iter` for the iterator.

More precisely, return the iterator over all objects of type `typ` of `n`-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

INPUT:

- `iter_list` - a finite iterable of finite iterables
- `typ` - (default: list) a type or class; more generally, something that can be called with a list as input.

OUTPUT: a list

EXAMPLES:

```
sage: mrange_iter([range(3), [0, 2]])
[[0, 0], [0, 2], [1, 0], [1, 2], [2, 0], [2, 2]]
sage: mrange_iter([['Monty', 'Flying'], ['Python', 'Circus']], tuple)
[('Monty', 'Python'), ('Monty', 'Circus'), ('Flying', 'Python'), ('Flying', 'Circus')]
sage: mrange_iter([[2, 3, 5, 7], [1, 2]], sum)
[3, 4, 4, 5, 6, 7, 8, 9]
```

Examples that illustrate empty multi-ranges:

```
sage: mrange_iter([range(5), xrange(3), xrange(-2)])
[]
sage: mrange_iter([range(5), range(3), range(0)])
[]
```

This example isn't empty, and shouldn't be. See trac #6561.

```
sage: mrange_iter([])
[[]]
```

AUTHORS:

•Joel B. Mohler

class sage.misc.mrange.**xmrange**(*sizes*, *typ*=<type 'list'>)

Return the multirange iterate with given sizes and type.

More precisely, return the iterator over all objects of type *typ* of n-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

Use mrange for the non-iterator form.

INPUT:

- sizes* - a list of nonnegative integers
- typ* - (default: list) a type or class; more generally, something that can be called with a list as input.

OUTPUT: a generator

EXAMPLES: We create multi-range iterators, print them and also iterate through a tuple version.

```
sage: z = xmrange([3, 2]); z
xmrange([3, 2])
sage: z = xmrange([3, 2], tuple); z
xmrange([3, 2], <type 'tuple'>)
sage: for a in z:
...     print a
(0, 0)
(0, 1)
(1, 0)
(1, 1)
(2, 0)
(2, 1)
```

We illustrate a few more iterations.

```
sage: list(xmrange([3, 2]))
[[0, 0], [0, 1], [1, 0], [1, 1], [2, 0], [2, 1]]
sage: list(xmrange([3, 2], tuple))
[(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
```

Here we compute the sum of each element of the multi-range iterator:

```
sage: list(xmrange([3,2], sum))
[0, 1, 1, 2, 2, 3]
```

Next we compute the product:

```
sage: list(xmrange([3,2], prod))
[0, 0, 0, 1, 0, 2]
```

Examples that illustrate empty multi-ranges.

```
sage: list(xmrange([5,3,-2]))
[]
sage: list(xmrange([5,3,0]))
[]
```

This example isn't empty, and shouldn't be. See trac #6561.

```
sage: list(xmrange([]))
[[]]
```

We use a multi-range iterator to iterate through the Cartesian product of sets.

```
sage: X = ['red', 'apple', 389]
sage: Y = ['orange', 'horse']
sage: for i,j in xmrange([len(X), len(Y)]):
...     print (X[i], Y[j])
('red', 'orange')
('red', 'horse')
('apple', 'orange')
('apple', 'horse')
(389, 'orange')
(389, 'horse')
```

AUTHORS:

- Jon Hanke
- William Stein

class sage.misc.mrange.**xmrange_iter**(*iter_list*, *typ=<type 'list'>*)
Return the multirange iterate derived from the given iterators and type.

Note: This basically gives you the Cartesian product of sets.

More precisely, return the iterator over all objects of type *typ* of n-tuples of Python ints with entries between 0 and the integers in the sizes list. The iterator is empty if sizes is empty or contains any non-positive integer.

Use `mrange_iter` for the non-iterator form.

INPUT:

- iter_list** - a list of objects usable as iterators (possibly lists)
- typ** - (default: list) a type or class; more generally, something that can be called with a list as input.

OUTPUT: a generator

EXAMPLES: We create multi-range iterators, print them and also iterate through a tuple version.

```
sage: z = xmrange_iter([xrange(3), xrange(2)]); z
xmrange_iter([xrange(3), xrange(2)])
sage: z = xmrange_iter([range(3), range(2)], tuple); z
```

```
xmrange_iter([[0, 1, 2], [0, 1]], <type 'tuple'>)
sage: for a in z:
...     print a
(0, 0)
(0, 1)
(1, 0)
(1, 1)
(2, 0)
(2, 1)
```

We illustrate a few more iterations.

```
sage: list(xmrange_iter([range(3), range(2)]))
[[0, 0], [0, 1], [1, 0], [1, 1], [2, 0], [2, 1]]
sage: list(xmrange_iter([range(3), range(2)], tuple))
[(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
```

Here we compute the sum of each element of the multi-range iterator:

```
sage: list(xmrange_iter([range(3), range(2)], sum))
[0, 1, 1, 2, 2, 3]
```

Next we compute the product:

```
sage: list(xmrange_iter([range(3), range(2)], prod))
[0, 0, 0, 1, 0, 2]
```

Examples that illustrate empty multi-ranges.

```
sage: list(xmrange_iter([xrange(5), xrange(3), xrange(-2)]))
[]
sage: list(xmrange_iter([xrange(5), xrange(3), xrange(0)]))
[]
```

This example isn't empty, and shouldn't be. See trac #6561.

```
sage: list(xmrange_iter([]))
[[]]
```

We use a multi-range iterator to iterate through the Cartesian product of sets.

```
sage: X = ['red', 'apple', 389]
sage: Y = ['orange', 'horse']
sage: for i, j in xmrange_iter([X, Y], tuple):
...     print (i, j)
('red', 'orange')
('red', 'horse')
('apple', 'orange')
('apple', 'horse')
(389, 'orange')
(389, 'horse')
```

AUTHORS:

•Joel B. Mohler

cardinality()

Return the cardinality of this iterator.

EXAMPLES:

```

sage: C = cartesian_product_iterator([xrange(3), xrange(4)])
sage: C.cardinality()
12
sage: C = cartesian_product_iterator([ZZ, QQ])
sage: C.cardinality()
+Infinity
sage: C = cartesian_product_iterator([ZZ, []])
sage: C.cardinality()
0

```

2.3.6 multi_replace

`sage.misc.multireplace.multiple_replace(dict, text)`

Replace in 'text' all occurrences of any key in the given dictionary by its corresponding value. Returns the new string.

2.3.7 Threaded map function

`sage.misc.map_threaded.map_threaded(function, sequence)`

Apply the function to the elements in the sequence by threading recursively through all sub-sequences in the sequence.

EXAMPLES:

```

sage: map_threaded(log, [[1,2], [3,e]])
[[0, log(2)], [log(3), 1]]
sage: map_threaded(log, [(1,2), (3,e)])
[[0, log(2)], [log(3), 1]]
sage: map_threaded(N, [[1,2], [3,e]])
[[1.0000000000000000, 2.0000000000000000], [3.0000000000000000, 2.71828182845905]]
sage: map_threaded((x^2).function(x), [[1,2,3,5], [2,10]])
[[1, 4, 9, 25], [4, 100]]

```

`map_threaded` also works on any object with an `apply_map` method, e.g., on matrices:

```

sage: map_threaded(lambda x: x^2, matrix([[1,2], [3,4]]))
[ 1  4]
[ 9 16]

```

AUTHORS:

- William Stein (2007-12); based on feedback from Peter Doyle.

2.4 Integer Arithmetic

2.4.1 Fast decomposition of small integers into sums of squares

Implement fast version of decomposition of (small) integers into sum of squares by direct method not relying on factorisation.

AUTHORS:

- Vincent Delecroix (2014): first implementation ([trac ticket #16374](#))

`sage.rings.sum_of_squares.four_squares_pyx(n)`

Return a 4-tuple of non-negative integers (i, j, k, l) such that $i^2 + j^2 + k^2 + l^2 = n$ and $i \leq j \leq k \leq l$.

The input must be lesser than $2^{32} = 4294967296$, otherwise an `OverflowError` is raised.

See also:

`four_squares()` is much more suited for large input

EXAMPLES:

```
sage: from sage.rings.sum_of_squares import four_squares_pyx
sage: four_squares_pyx(15447)
(2, 5, 17, 123)
sage: 2^2 + 5^2 + 17^2 + 123^2
15447
```

```
sage: four_squares_pyx(523439)
(3, 5, 26, 723)
sage: 3^2 + 5^2 + 26^2 + 723^2
523439
```

```
sage: four_squares_pyx(2**32)
Traceback (most recent call last):
...
OverflowError: ...
```

TESTS:

```
sage: four_squares_pyx(0)
(0, 0, 0, 0)

sage: s = lambda (x,y,z,t): x**2 + y**2 + z**2 + t**2
sage: all(s(four_squares_pyx(n)) == n for n in xrange(5000,10000))
True
```

`sage.rings.sum_of_squares.is_sum_of_two_squares_pyx(n)`

Return True if n is a sum of two squares and False otherwise.

The input must be smaller than $2^{32} = 4294967296$, otherwise an `OverflowError` is raised.

EXAMPLES:

```
sage: from sage.rings.sum_of_squares import is_sum_of_two_squares_pyx
sage: filter(is_sum_of_two_squares_pyx, range(30))
[0, 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, 20, 25, 26, 29]

sage: is_sum_of_two_squares_pyx(2**32)
Traceback (most recent call last):
...
OverflowError: ...
```

`sage.rings.sum_of_squares.three_squares_pyx(n)`

If n is a sum of three squares return a 3-tuple (i, j, k) of Sage integers such that $i^2 + j^2 + k^2 = n$ and $i \leq j \leq k$. Otherwise raise a `ValueError`.

The input must be lesser than $2^{32} = 4294967296$, otherwise an `OverflowError` is raised.

EXAMPLES:

```
sage: from sage.rings.sum_of_squares import three_squares_pyx
sage: three_squares_pyx(0)
(0, 0, 0)
```

```

sage: three_squares_pyx(1)
(0, 0, 1)
sage: three_squares_pyx(2)
(0, 1, 1)
sage: three_squares_pyx(3)
(1, 1, 1)
sage: three_squares_pyx(4)
(0, 0, 2)
sage: three_squares_pyx(5)
(0, 1, 2)
sage: three_squares_pyx(6)
(1, 1, 2)
sage: three_squares_pyx(7)
Traceback (most recent call last):
...
ValueError: 7 is not a sum of 3 squares
sage: three_squares_pyx(107)
(1, 5, 9)

sage: three_squares_pyx(2**32)
Traceback (most recent call last):
...
OverflowError: ...

```

TESTS:

```

sage: s = lambda (x,y,z) : x**2 + y**2 + z**2
sage: for ijk in Subsets(Subsets(35000,15).random_element(),3):
....:     if s(three_squares_pyx(s(ijk))) != s(ijk):
....:         print "hey"

```

`sage.rings.sum_of_squares.two_squares_pyx(n)`

Return a pair of non-negative integers (i, j) such that $i^2 + j^2 = n$.

If n is not a sum of two squares, a `ValueError` is raised. The input must be lesser than $2^{32} = 4294967296$, otherwise an `OverflowError` is raised.

See also:

`two_squares()` is much more suited for large inputs

EXAMPLES:

```

sage: from sage.rings.sum_of_squares import two_squares_pyx
sage: two_squares_pyx(0)
(0, 0)
sage: two_squares_pyx(1)
(0, 1)
sage: two_squares_pyx(2)
(1, 1)
sage: two_squares_pyx(3)
Traceback (most recent call last):
...
ValueError: 3 is not a sum of 2 squares
sage: two_squares_pyx(106)
(5, 9)

sage: two_squares_pyx(2**32)
Traceback (most recent call last):
...

```

```
OverflowError: ...
```

TESTS:

```
sage: s = lambda (x,y) : x**2 + y**2
sage: for ij in Subsets(Subsets(45000,15).random_element(),2):
....:     if s(two_squares_pyx(s(ij))) != s(ij):
....:         print "hey"

sage: for n in xrange(1,65536):
....:     if two_squares_pyx(n**2) != (0, n):
....:         print "hey"
....:     if two_squares_pyx(n**2+1) != (1, n):
....:         print "ho"
```

2.4.2 Utility classes for multi-modular algorithms

```
class sage.ext.multi_modular.MultiModularBasis
Bases: sage.ext.multi_modular.MultiModularBasis_base
```

Class used for storing a MultiModular bases of a fixed length.

```
class sage.ext.multi_modular.MultiModularBasis_base
Bases: object
```

This class stores a list of machine-sized prime numbers, and can do reduction and Chinese Remainder Theorem lifting modulo these primes.

Lifting implemented via Garner's algorithm, which has the advantage that all reductions are word-sized. For each i precompute

$$\$prod_j = 1^{i-1} m_j \$ \text{ and } \$prod_j = 1^{i-1} m_j^{-1} \pmod{m_i} \$$$

This class can be initialized in two ways, either with a list of prime moduli or an upper bound for the product of the prime moduli. The prime moduli is generated automatically in the second case:

```
sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([3, 5, 7]); mm
MultiModularBasis with moduli [3, 5, 7]
```

```
sage: height = 52348798724
sage: mm = MultiModularBasis_base(height); mm
MultiModularBasis with moduli [4561, 17351, 28499]
sage: mm = MultiModularBasis_base(height); mm
MultiModularBasis with moduli [32573, 4339, 30859]
sage: mm = MultiModularBasis_base(height); mm
MultiModularBasis with moduli [16451, 14323, 28631]
```

```
sage: mm.prod()//height
128
```

TESTS:

```
sage: mm = MultiModularBasis_base((3,5,7)); mm
MultiModularBasis with moduli [3, 5, 7]
sage: mm = MultiModularBasis_base(primes(10,20)); mm
MultiModularBasis with moduli [11, 13, 17, 19]
```

There is no overflow if the modulus is below MAX_MODULUS


```

sage: from sage.ext.multi_modular import MAX_MODULUS
sage: p0 = previous_prime(MAX_MODULUS)
sage: p1 = previous_prime(p0)
sage: MultiModularBasis_base([p0, p1]).crt([p0-1, p1-1])
-1

```

If we add another bit to the prime length then there is an overflow, as expected:

```

sage: p0 = previous_prime(2*MAX_MODULUS)
sage: p1 = previous_prime(p0)
sage: MultiModularBasis_base([p0, p1]).crt([p0-1, p1-1])
Traceback (most recent call last):
...
ValueError: given modulus cannot be manipulated in a single machine word

```

crt (*b*)

Calculate lift mod $\prod_{i=0}^{\text{len}(b)-1} m_i$.

In the case that offset > 0, $z[j]$ remains unchanged mod $\prod_{i=0}^{\text{offset}-1} m_i$

INPUT:

- *b* - a list of length at most self.n

OUTPUT:

Integer z where $z = b[i] \bmod m_i$ for $0 \leq i < \text{len}(b)$

EXAMPLES:

```

sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([10007, 10009, 10037, 10039, 17351])
sage: res = mm.crt([3,5,7,9]); res
8474803647063985
sage: res % 10007
3
sage: res % 10009
5
sage: res % 10037
7
sage: res % 10039
9

```

extend_with_primes (*plist*, *partial_products=None*, *check=True*)

Extend the stored list of moduli with the given primes in *plist*.

EXAMPLES:

```

sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([1009, 10007]); mm
MultiModularBasis with moduli [1009, 10007]
sage: mm.extend_with_primes([10037, 10039])
4
sage: mm
MultiModularBasis with moduli [1009, 10007, 10037, 10039]

```

list ()

Return a list with the prime moduli.

EXAMPLES:

```
sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([46307, 10007])
sage: mm.list()
[46307, 10007]
```

partial_product (*n*)

Returns a list containing precomputed partial products.

EXAMPLES:

```
sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([46307, 10007]); mm
MultiModularBasis with moduli [46307, 10007]
sage: mm.partial_product(0)
46307
sage: mm.partial_product(1)
463394149
```

TESTS:

```
sage: mm.partial_product(2)
Traceback (most recent call last):
...
IndexError: beyond bound for multi-modular prime list
sage: mm.partial_product(-2)
Traceback (most recent call last):
...
IndexError: negative index not valid
```

precomputation_list ()

Returns a list of the precomputed coefficients

$$\prod_j = 1^{i-1} m_j^{-1} (\text{mod } m_i)$$

where m_i are the prime moduli.

EXAMPLES:

```
sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([46307, 10007]); mm
MultiModularBasis with moduli [46307, 10007]
sage: mm.precomputation_list()
[1, 4013]
```

prod ()

Returns the product of the prime moduli.

EXAMPLES:

```
sage: from sage.ext.multi_modular import MultiModularBasis_base
sage: mm = MultiModularBasis_base([46307]); mm
MultiModularBasis with moduli [46307]
sage: mm.prod()
46307
sage: mm = MultiModularBasis_base([46307, 10007]); mm
MultiModularBasis with moduli [46307, 10007]
sage: mm.prod()
463394149
```

TESTS:

```

sage: mm = MultiModularBasis_base([]); mm
MultiModularBasis with moduli []
sage: len(mm)
0
sage: mm.prod()
1

```

class `sage.ext.multi_modular.MutableMultiModularBasis`

Bases: `sage.ext.multi_modular.MultiModularBasis`

Class used for performing multi-modular methods, with the possibility of removing bad primes.

next_prime()

Picks a new random prime between the bounds given during the initialization of this object, updates the precomputed data, and returns the new prime modulus.

EXAMPLES:

```

sage: from sage.ext.multi_modular import MutableMultiModularBasis
sage: mm = MutableMultiModularBasis([10007])
sage: mm.next_prime()
4561          # 64-bit
4561L         # 32-bit
sage: mm
MultiModularBasis with moduli [10007, 4561]

```

replace_prime(ix)

Replace the prime moduli at the given index with a different one, update the precomputed data accordingly, and return the new prime.

EXAMPLES:

```

sage: from sage.ext.multi_modular import MutableMultiModularBasis
sage: mm = MutableMultiModularBasis([10007, 10009, 10037, 10039])
sage: mm
MultiModularBasis with moduli [10007, 10009, 10037, 10039]
sage: mm.prod()
10092272478850909
sage: mm.precomputation_list()
[1, 5004, 6536, 6060]
sage: mm.partial_product(2)
1005306552331
sage: mm.replace_prime(1)
4561          # 64-bit
4561L         # 32-bit
sage: mm
MultiModularBasis with moduli [10007, 4561, 10037, 10039]
sage: mm.prod()
4598946425820661
sage: mm.precomputation_list()
[1, 2314, 3274, 3013]
sage: mm.partial_product(2)
458108021299

```

2.4.3 Miscellaneous arithmetic functions

`sage.rings.arith.CRT(a, b, m=None, n=None)`

Returns a solution to a Chinese Remainder Theorem problem.

INPUT:

- a, b - two residues (elements of some ring for which extended gcd is available), or two lists, one of residues and one of moduli.
- m, n - (default: None) two moduli, or None.

OUTPUT:

If m, n are not None, returns a solution x to the simultaneous congruences $x \equiv a \pmod{m}$ and $x \equiv b \pmod{n}$, if one exists. By the Chinese Remainder Theorem, a solution to the simultaneous congruences exists if and only if $a \equiv b \pmod{\gcd(m, n)}$. The solution x is only well-defined modulo $\text{lcm}(m, n)$.

If a and b are lists, returns a simultaneous solution to the congruences $x \equiv a_i \pmod{b_i}$, if one exists.

See also:

- `CRT_list()`

EXAMPLES:

Using `crt` by giving it pairs of residues and moduli:

```
sage: crt(2, 1, 3, 5)
11
sage: crt(13, 20, 100, 301)
28013
sage: crt([2, 1], [3, 5])
11
sage: crt([13, 20], [100, 301])
28013
```

You can also use upper case:

```
sage: c = CRT(2, 3, 3, 5); c
8
sage: c % 3 == 2
True
sage: c % 5 == 3
True
```

Note that this also works for polynomial rings:

```
sage: K.<a> = NumberField(x^3 - 7)
sage: R.<y> = K[]
sage: f = y^2 + 3
sage: g = y^3 - 5
sage: CRT(1, 3, f, g)
-3/26*y^4 + 5/26*y^3 + 15/26*y + 53/26
sage: CRT(1, a, f, g)
(-3/52*a + 3/52)*y^4 + (5/52*a - 5/52)*y^3 + (15/52*a - 15/52)*y + 27/52*a + 25/52
```

You can also do this for any number of moduli:

```
sage: K.<a> = NumberField(x^3 - 7)
sage: R.<x> = K[]
sage: CRT([], [])
0
sage: CRT([a], [x])
a
sage: f = x^2 + 3
sage: g = x^3 - 5
```

```

sage: h = x^5 + x^2 - 9
sage: k = CRT([1, a, 3], [f, g, h]); k
(127/26988*a - 5807/386828)*x^9 + (45/8996*a - 33677/1160484)*x^8 + (2/173*a - 6/173)*x^7 + (133/173)*x^6 + (1/173)*x^5 + (1/173)*x^4 + (1/173)*x^3 + (1/173)*x^2 + (1/173)*x + (1/173)
sage: k.mod(f)
1
sage: k.mod(g)
a
sage: k.mod(h)
3

```

If the moduli are not coprime, a solution may not exist:

```

sage: crt(4, 8, 8, 12)
20
sage: crt(4, 6, 8, 12)
Traceback (most recent call last):
...
ValueError: No solution to crt problem since gcd(8,12) does not divide 4-6

sage: x = polygen(QQ)
sage: crt(2, 3, x-1, x+1)
-1/2*x + 5/2
sage: crt(2, x, x^2-1, x^2+1)
-1/2*x^3 + x^2 + 1/2*x + 1
sage: crt(2, x, x^2-1, x^3-1)
Traceback (most recent call last):
...
ValueError: No solution to crt problem since gcd(x^2 - 1, x^3 - 1) does not divide 2-x

sage: crt(int(2), int(3), int(7), int(11))
58

```

sage.rings.arith.**CRT_basis** (*moduli*)

Returns a CRT basis for the given moduli.

INPUT:

- **moduli** - list of pairwise coprime moduli m which admit an extended Euclidean algorithm

OUTPUT:

- a list of elements a_i of the same length as m such that a_i is congruent to 1 modulo m_i and to 0 modulo m_j for $j \neq i$.

Note: The pairwise coprimality of the input is not checked.

EXAMPLES:

```

sage: a1 = ZZ(mod(42, 5))
sage: a2 = ZZ(mod(42, 13))
sage: c1, c2 = CRT_basis([5, 13])
sage: mod(a1*c1+a2*c2, 5*13)
42

```

A polynomial example:

```

sage: x=polygen(QQ)
sage: mods = [x, x^2+1, 2*x-3]
sage: b = CRT_basis(mods)
sage: b

```

```
[-2/3*x^3 + x^2 - 2/3*x + 1, 6/13*x^3 - x^2 + 6/13*x, 8/39*x^3 + 8/39*x]
sage: [[bi % mj for mj in mods] for bi in b]
[[1, 0, 0], [0, 1, 0], [0, 0, 1]]
```

`sage.rings.arith.CRT_list(v, moduli)`

Given a list `v` of elements and a list of corresponding `moduli`, find a single element that reduces to each element of `v` modulo the corresponding `moduli`.

See also:

- `crt()`

EXAMPLES:

```
sage: CRT_list([2,3,2], [3,5,7])
23
sage: x = polygen(QQ)
sage: c = CRT_list([3], [x]); c
3
sage: c.parent()
Univariate Polynomial Ring in x over Rational Field
```

It also works if the `moduli` are not coprime:

```
sage: CRT_list([32,2,2], [60,90,150])
452
```

But with non coprime `moduli` there is not always a solution:

```
sage: CRT_list([32,2,1], [60,90,150])
Traceback (most recent call last):
...
ValueError: No solution to crt problem since gcd(180,150) does not divide 92-1
```

The arguments must be lists:

```
sage: CRT_list([1,2,3], "not a list")
Traceback (most recent call last):
...
ValueError: Arguments to CRT_list should be lists
sage: CRT_list("not a list", [2,3])
Traceback (most recent call last):
...
ValueError: Arguments to CRT_list should be lists
```

The list of `moduli` must have the same length as the list of elements:

```
sage: CRT_list([1,2,3], [2,3,5])
23
sage: CRT_list([1,2,3], [2,3])
Traceback (most recent call last):
...
ValueError: Arguments to CRT_list should be lists of the same length
sage: CRT_list([1,2,3], [2,3,5,7])
Traceback (most recent call last):
...
ValueError: Arguments to CRT_list should be lists of the same length
```

TESTS:

```
sage: CRT([32r, 2r, 2r], [60r, 90r, 150r])
452
```

```
sage.rings.arith.CRT_vectors(X, moduli)
```

Vector form of the Chinese Remainder Theorem: given a list of integer vectors v_i and a list of coprime moduli m_i , find a vector w such that $w = v_i \pmod{m_i}$ for all i . This is more efficient than applying `CRT()` to each entry.

INPUT:

- `X` - list or tuple, consisting of lists/tuples/vectors/etc of integers of the same length
- `moduli` - list of `len(X)` moduli

OUTPUT:

- `list` - application of CRT componentwise.

EXAMPLES:

```
sage: CRT_vectors([[3, 5, 7], [3, 5, 11]], [2, 3])
[3, 5, 5]
```

```
sage: CRT_vectors([vector(ZZ, [2, 3, 1]), Sequence([1, 7, 8], ZZ)], [8, 9])
[10, 43, 17]
```

```
class sage.rings.arith.Euler_Phi
```

Return the value of the Euler phi function on the integer n . We defined this to be the number of positive integers $\leq n$ that are relatively prime to n . Thus if $n \leq 0$ then `euler_phi(n)` is defined and equals 0.

INPUT:

- `n` - an integer

EXAMPLES:

```
sage: euler_phi(1)
1
sage: euler_phi(2)
1
sage: euler_phi(3)
2
sage: euler_phi(12)
4
sage: euler_phi(37)
36
```

Notice that `euler_phi` is defined to be 0 on negative numbers and 0.

```
sage: euler_phi(-1)
0
sage: euler_phi(0)
0
sage: type(euler_phi(0))
<type 'sage.rings.integer.Integer'>
```

We verify directly that the phi function is correct for 21.

```
sage: euler_phi(21)
12
sage: [i for i in range(21) if gcd(21, i) == 1]
[1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20]
```

The length of the list of integers 'i' in range(n) such that the $\gcd(i,n) == 1$ equals $\text{euler_phi}(n)$.

```
sage: len([i for i in range(21) if gcd(21,i) == 1]) == euler_phi(21)
True
```

The phi function also has a special plotting method.

```
sage: P = plot(euler_phi, -3, 71)
```

AUTHORS:

- William Stein
- Alex Clemesha (2006-01-10): some examples

plot (*xmin=1, xmax=50, pointsize=30, rgbcolor=(0, 0, 1), join=True, **kwds*)
Plot the Euler phi function.

INPUT:

- xmin - default: 1
- xmax - default: 50
- pointsize - default: 30
- rgbcolor - default: (0,0,1)
- join - default: True; whether to join the points.
- **kwds - passed on

EXAMPLES:

```
sage: p = Euler_Phi().plot()
sage: p.ymax()
46.0
```

`sage.rings.arith.GCD(a, b=None, **kwargs)`

The greatest common divisor of a and b, or if a is a list and b is omitted the greatest common divisor of all elements of a.

INPUT:

- a, b - two elements of a ring with gcd or
- a - a list or tuple of elements of a ring with gcd

Additional keyword arguments are passed to the respectively called methods.

OUTPUT:

The given elements are first coerced into a common parent. Then, their greatest common divisor *in that common parent* is returned.

EXAMPLES:

```
sage: GCD(97, 100)
1
sage: GCD(97*10^15, 19^20*97^2)
97
sage: GCD(2/3, 4/5)
2/15
sage: GCD([2, 4, 6, 8])
2
sage: GCD(srange(0, 10000, 10)) # fast !!
10
```


Note that to take the gcd of n elements for $n \neq 2$ you must put the elements into a list by enclosing them in `[...]`. Before #4988 the following wrongly returned 3 since the third parameter was just ignored:

```
sage: gcd(3,6,2)
Traceback (most recent call last):
...
TypeError: gcd() takes at most 2 arguments (3 given)
sage: gcd([3,6,2])
1
```

Similarly, giving just one element (which is not a list) gives an error:

```
sage: gcd(3)
Traceback (most recent call last):
...
TypeError: 'sage.rings.integer.Integer' object is not iterable
```

By convention, the gcd of the empty list is (the integer) 0:

```
sage: gcd([])
0
sage: type(gcd([]))
<type 'sage.rings.integer.Integer'>
```

TESTS:

The following shows that indeed coercion takes place before computing the gcd. This behaviour was introduced in [trac ticket #10771](#):

```
sage: R.<x>=QQ[]
sage: S.<x>=ZZ[]
sage: p = S.random_element(degree=(0,10))
sage: q = R.random_element(degree=(0,10))
sage: parent(gcd(1/p,q))
Fraction Field of Univariate Polynomial Ring in x over Rational Field
sage: parent(gcd([1/p,q]))
Fraction Field of Univariate Polynomial Ring in x over Rational Field
```

Make sure we try QQ and not merely ZZ ([trac ticket #13014](#)):

```
sage: bool(gcd(2/5, 3/7) == gcd(SR(2/5), SR(3/7)))
True
```

Make sure that the gcd of Expressions stays symbolic:

```
sage: parent(gcd(2, 4))
Integer Ring
sage: parent(gcd(SR(2), 4))
Symbolic Ring
sage: parent(gcd(2, SR(4)))
Symbolic Ring
sage: parent(gcd(SR(2), SR(4)))
Symbolic Ring
```

Verify that objects without gcd methods but which can't be coerced to ZZ or QQ raise an error:

```
sage: F.<a,b> = FreeMonoid(2)
sage: gcd(a,b)
Traceback (most recent call last):
...
```

```
TypeError: unable to find gcd
```

```
sage.rings.arith.LCM(a, b=None)
```

The least common multiple of a and b , or if a is a list and b is omitted the least common multiple of all elements of a .

Note that LCM is an alias for lcm.

INPUT:

- a, b - two elements of a ring with lcm or
- a - a list or tuple of elements of a ring with lcm

OUTPUT:

First, the given elements are coerced into a common parent. Then, their least common multiple *in that parent* is returned.

EXAMPLES:

```
sage: lcm(97, 100)
9700
sage: LCM(97, 100)
9700
sage: LCM(0, 2)
0
sage: LCM(-3, -5)
15
sage: LCM([1, 2, 3, 4, 5])
60
sage: v = LCM(range(1, 10000))    # *very* fast!
sage: len(str(v))
4349
```

TESTS:

The following tests against a bug that was fixed in [trac ticket #10771](#):

```
sage: lcm(4/1, 2)
4
```

The following shows that indeed coercion takes place before computing the least common multiple:

```
sage: R.<x>=QQ[]
sage: S.<x>=ZZ[]
sage: p = S.random_element(degree=(0, 5))
sage: q = R.random_element(degree=(0, 5))
sage: parent(lcm([1/p, q]))
Fraction Field of Univariate Polynomial Ring in x over Rational Field
```

Make sure we try QQ and not merely ZZ ([trac ticket #13014](#)):

```
sage: bool(lcm(2/5, 3/7) == lcm(SR(2/5), SR(3/7)))
True
```

Make sure that the lcm of Expressions stays symbolic:

```
sage: parent(lcm(2, 4))
Integer Ring
sage: parent(lcm(SR(2), 4))
Symbolic Ring
sage: parent(lcm(2, SR(4)))
```

```
Symbolic Ring
sage: parent(lcm(SR(2), SR(4)))
Symbolic Ring
```

Verify that objects without lcm methods but which can't be coerced to ZZ or QQ raise an error:

```
sage: F.<a,b> = FreeMonoid(2)
sage: lcm(a,b)
Traceback (most recent call last):
...
TypeError: unable to find lcm
```

Check rational and integers (trac ticket #17852):

```
sage: lcm(1/2, 4)
4
sage: lcm(4, 1/2)
4
```

class sage.rings.arith.**Moebius**

Returns the value of the Moebius function of $\text{abs}(n)$, where n is an integer.

DEFINITION: $\mu(n)$ is 0 if n is not square free, and otherwise equals $(-1)^r$, where n has r distinct prime factors.

For simplicity, if $n = 0$ we define $\mu(n) = 0$.

IMPLEMENTATION: Factors or - for integers - uses the PARI C library.

INPUT:

- n - anything that can be factored.

OUTPUT: 0, 1, or -1

EXAMPLES:

```
sage: moebius(-5)
-1
sage: moebius(9)
0
sage: moebius(12)
0
sage: moebius(-35)
1
sage: moebius(-1)
1
sage: moebius(7)
-1

sage: moebius(0) # potentially nonstandard!
0
```

The moebius function even makes sense for non-integer inputs.

```
sage: x = GF(7)['x'].0
sage: moebius(x+2)
-1
```

plot ($xmin=0, xmax=50, pointsize=30, rgbcolor=(0, 0, 1), join=True, **kwds$)

Plot the Moebius function.

INPUT:

- xmin - default: 0
- xmax - default: 50
- pointsize - default: 30
- rgbcolor - default: (0,0,1)
- join - default: True; whether to join the points (very helpful in seeing their order).
- **kwds - passed on

EXAMPLES:

```
sage: p = Moebius().plot()
sage: p.ymax()
1.0
```

range (*start, stop=None, step=None*)

Return the Moebius function evaluated at the given range of values, i.e., the image of the list range(start, stop, step) under the Mobius function.

This is much faster than directly computing all these values with a list comprehension.

EXAMPLES:

```
sage: v = moebius.range(-10,10); v
[1, 0, 0, -1, 1, -1, 0, -1, -1, 1, 0, 1, -1, -1, 0, -1, 1, -1, 0, 0]
sage: v == [moebius(n) for n in range(-10,10)]
True
sage: v = moebius.range(-1000, 2000, 4)
sage: v == [moebius(n) for n in range(-1000,2000, 4)]
True
```

class sage.rings.arith.**Sigma**

Return the sum of the k-th powers of the divisors of n.

INPUT:

- n - integer
- k - integer (default: 1)

OUTPUT: integer

EXAMPLES:

```
sage: sigma(5)
6
sage: sigma(5,2)
26
```

The sigma function also has a special plotting method.

```
sage: P = plot(sigma, 1, 100)
```

This method also works with k-th powers.

```
sage: P = plot(sigma, 1, 100, k=2)
```

AUTHORS:

- William Stein: original implementation
- Craig Citro (2007-06-01): rewrote for huge speedup

TESTS:

```

sage: sigma(100,4)
106811523
sage: sigma(factorial(100),3).mod(144169)
3672
sage: sigma(factorial(150),12).mod(691)
176
sage: RR(sigma(factorial(133),20))
2.80414775675747e4523
sage: sigma(factorial(100),0)
39001250856960000
sage: sigma(factorial(41),1)
229199532273029988767733858700732906511758707916800

```

plot (*xmin=1, xmax=50, k=1, pointsize=30, rgbcolor=(0, 0, 1), join=True, **kwds*)
 Plot the sigma (sum of k-th powers of divisors) function.

INPUT:

- *xmin* - default: 1
- *xmax* - default: 50
- *k* - default: 1
- *pointsize* - default: 30
- *rgbcolor* - default: (0,0,1)
- *join* - default: True; whether to join the points.
- ***kwds* - passed on

EXAMPLES:

```

sage: p = Sigma().plot()
sage: p.ymax()
124.0

```

`sage.rings.arith.XGCD(a, b)`

Return a triple (g, s, t) such that $g = s \cdot a + t \cdot b = \gcd(a, b)$.

Note: One exception is if a and b are not in a principal ideal domain (see [Wikipedia article Principal_ideal_domain](#)), e.g., they are both polynomials over the integers. Then this function can't in general return (g, s, t) as above, since they need not exist. Instead, over the integers, we first multiply g by a divisor of the resultant of a/g and b/g , up to sign.

INPUT:

- a, b - integers or more generally, element of a ring for which the xgcd make sense (e.g. a field or univariate polynomials).

OUTPUT:

- g, s, t - such that $g = s \cdot a + t \cdot b$

Note: There is no guarantee that the returned cofactors (s and t) are minimal.

EXAMPLES:

```

sage: xgcd(56, 44)
(4, 4, -5)

```

```

sage: 4*56 + (-5)*44
4

sage: g, a, b = xgcd(5/1, 7/1); g, a, b
(1, 3, -2)
sage: a*(5/1) + b*(7/1) == g
True

sage: x = polygen(QQ)
sage: xgcd(x^3 - 1, x^2 - 1)
(x - 1, 1, -x)

sage: K.<g> = NumberField(x^2-3)
sage: g.xgcd(g+2)
(1, 1/3*g, 0)

sage: R.<a,b> = K[]
sage: S.<y> = R.fraction_field()[]
sage: xgcd(y^2, a*y+b)
(1, a^2/b^2, ((-a)/b^2)*y + 1/b)
sage: xgcd((b+g)*y^2, (a+g)*y+b)
(1, (a^2 + (2*g)*a + 3)/(b^3 + (g)*b^2), ((-a + (-g))/b^2)*y + 1/b)

```

Here is an example of a `xgcd` for two polynomials over the integers, where the linear combination is not the gcd but the gcd multiplied by the resultant:

```

sage: R.<x> = ZZ[]
sage: gcd(2*x*(x-1), x^2)
x
sage: xgcd(2*x*(x-1), x^2)
(2*x, -1, 2)
sage: (2*(x-1)).resultant(x)
2

```

`sage.rings.arith.algdep(z, degree, known_bits=None, use_bits=None, known_digits=None, use_digits=None, height_bound=None, proof=False)`

Returns a polynomial of degree at most *degree* which is approximately satisfied by the number *z*. Note that the returned polynomial need not be irreducible, and indeed usually won't be if *z* is a good approximation to an algebraic number of degree less than *degree*.

You can specify the number of known bits or digits of *z* with `known_bits=k` or `known_digits=k`. PARI is then told to compute the result using $0.8k$ of these bits/digits. Or, you can specify the precision to use directly with `use_bits=k` or `use_digits=k`. If none of these are specified, then the precision is taken from the input value.

A height bound may be specified to indicate the maximum coefficient size of the returned polynomial; if a sufficiently small polynomial is not found, then `None` will be returned. If `proof=True` then the result is returned only if it can be proved correct (i.e. the only possible minimal polynomial satisfying the height bound, or no such polynomial exists). Otherwise a `ValueError` is raised indicating that higher precision is required.

ALGORITHM: Uses LLL for real/complex inputs, PARI C-library `algdep` command otherwise.

Note that `algebraic_dependency` is a synonym for `algdep`.

INPUT:

- *z* - real, complex, or *p*-adic number
- *degree* - an integer

- height_bound** - an integer (default: None) specifying the maximum coefficient size for the returned polynomial
- proof** - a boolean (default: False), requires height_bound to be set

EXAMPLES:

```
sage: algdep(1.8888888888888888, 1)
9*x - 17
sage: algdep(0.1212121212121212, 1)
33*x - 4
sage: algdep(sqrt(2), 2)
x^2 - 2
```

This example involves a complex number:

```
sage: z = (1/2)*(1 + RDF(sqrt(3)) *CC.0); z
0.5000000000000000 + 0.866025403784439*I
sage: p = algdep(z, 6); p
x^3 + 1
sage: p.factor()
(x + 1) * (x^2 - x + 1)
sage: z^2 - z + 1 # abs tol 2e-16
0.0000000000000000
```

This example involves a p -adic number:

```
sage: K = Qp(3, print_mode = 'series')
sage: a = K(7/19); a
1 + 2*3 + 3^2 + 3^3 + 2*3^4 + 2*3^5 + 3^8 + 2*3^9 + 3^11 + 3^12 + 2*3^15 + 2*3^16 + 3^17 + 2*3^18
sage: algdep(a, 1)
19*x - 7
```

These examples show the importance of proper precision control. We compute a 200-bit approximation to $\sqrt{2}$ which is wrong in the 33'rd bit:

```
sage: z = sqrt(RealField(200)(2)) + (1/2)^33
sage: p = algdep(z, 4); p
227004321085*x^4 - 216947902586*x^3 - 99411220986*x^2 + 82234881648*x - 211871195088
sage: factor(p)
227004321085*x^4 - 216947902586*x^3 - 99411220986*x^2 + 82234881648*x - 211871195088
sage: algdep(z, 4, known_bits=32)
x^2 - 2
sage: algdep(z, 4, known_digits=10)
x^2 - 2
sage: algdep(z, 4, use_bits=25)
x^2 - 2
sage: algdep(z, 4, use_digits=8)
x^2 - 2
```

Using the height_bound and proof parameters, we can see that πi is not the root of an integer polynomial of degree at most 5 and coefficients bounded above by 10:

```
sage: algdep(pi.n(), 5, height_bound=10, proof=True) is None
True
```

For stronger results, we need more precision:

```
sage: algdep(pi.n(), 5, height_bound=100, proof=True) is None
Traceback (most recent call last):
...
ValueError: insufficient precision for non-existence proof
```

```
sage: algdep(pi.n(200), 5, height_bound=100, proof=True) is None
True

sage: algdep(pi.n(), 10, height_bound=10, proof=True) is None
Traceback (most recent call last):
...
ValueError: insufficient precision for non-existence proof
sage: algdep(pi.n(200), 10, height_bound=10, proof=True) is None
True
```

We can also use `proof=True` to get positive results:

```
sage: a = sqrt(2) + sqrt(3) + sqrt(5)
sage: algdep(a.n(), 8, height_bound=1000, proof=True)
Traceback (most recent call last):
...
ValueError: insufficient precision for uniqueness proof
sage: f = algdep(a.n(1000), 8, height_bound=1000, proof=True); f
x^8 - 40*x^6 + 352*x^4 - 960*x^2 + 576
sage: f(a).expand()
0
```

TESTS:

```
sage: algdep(complex("1+2j"), 4)
x^2 - 2*x + 5
```

`sage.rings.arith.algebraic_dependency(z, degree, known_bits=None, use_bits=None, known_digits=None, use_digits=None, height_bound=None, proof=False)`

Returns a polynomial of degree at most *degree* which is approximately satisfied by the number *z*. Note that the returned polynomial need not be irreducible, and indeed usually won't be if *z* is a good approximation to an algebraic number of degree less than *degree*.

You can specify the number of known bits or digits of *z* with `known_bits=k` or `known_digits=k`. PARI is then told to compute the result using $0.8k$ of these bits/digits. Or, you can specify the precision to use directly with `use_bits=k` or `use_digits=k`. If none of these are specified, then the precision is taken from the input value.

A height bound may be specified to indicate the maximum coefficient size of the returned polynomial; if a sufficiently small polynomial is not found, then `None` will be returned. If `proof=True` then the result is returned only if it can be proved correct (i.e. the only possible minimal polynomial satisfying the height bound, or no such polynomial exists). Otherwise a `ValueError` is raised indicating that higher precision is required.

ALGORITHM: Uses LLL for real/complex inputs, PARI C-library `algdep` command otherwise.

Note that `algebraic_dependency` is a synonym for `algdep`.

INPUT:

- *z* - real, complex, or *p*-adic number
- *degree* - an integer
- **height_bound** - an integer (default: `None`) specifying the maximum coefficient size for the returned polynomial
- *proof* - a boolean (default: `False`), requires `height_bound` to be set

EXAMPLES:


```

sage: algdep(1.8888888888888888, 1)
9*x - 17
sage: algdep(0.1212121212121212, 1)
33*x - 4
sage: algdep(sqrt(2), 2)
x^2 - 2

```

This example involves a complex number:

```

sage: z = (1/2)*(1 + RDF(sqrt(3)) *CC.0); z
0.5000000000000000 + 0.866025403784439*I
sage: p = algdep(z, 6); p
x^3 + 1
sage: p.factor()
(x + 1) * (x^2 - x + 1)
sage: z^2 - z + 1 # abs tol 2e-16
0.0000000000000000

```

This example involves a p -adic number:

```

sage: K = Qp(3, print_mode = 'series')
sage: a = K(7/19); a
1 + 2*3 + 3^2 + 3^3 + 2*3^4 + 2*3^5 + 3^8 + 2*3^9 + 3^11 + 3^12 + 2*3^15 + 2*3^16 + 3^17 + 2*3^18
sage: algdep(a, 1)
19*x - 7

```

These examples show the importance of proper precision control. We compute a 200-bit approximation to $\sqrt{2}$ which is wrong in the 33'rd bit:

```

sage: z = sqrt(RealField(200)(2)) + (1/2)^33
sage: p = algdep(z, 4); p
227004321085*x^4 - 216947902586*x^3 - 99411220986*x^2 + 82234881648*x - 211871195088
sage: factor(p)
227004321085*x^4 - 216947902586*x^3 - 99411220986*x^2 + 82234881648*x - 211871195088
sage: algdep(z, 4, known_bits=32)
x^2 - 2
sage: algdep(z, 4, known_digits=10)
x^2 - 2
sage: algdep(z, 4, use_bits=25)
x^2 - 2
sage: algdep(z, 4, use_digits=8)
x^2 - 2

```

Using the `height_bound` and `proof` parameters, we can see that πi is not the root of an integer polynomial of degree at most 5 and coefficients bounded above by 10:

```

sage: algdep(pi.n(), 5, height_bound=10, proof=True) is None
True

```

For stronger results, we need more precision:

```

sage: algdep(pi.n(), 5, height_bound=100, proof=True) is None
Traceback (most recent call last):
...
ValueError: insufficient precision for non-existence proof
sage: algdep(pi.n(200), 5, height_bound=100, proof=True) is None
True

sage: algdep(pi.n(), 10, height_bound=10, proof=True) is None
Traceback (most recent call last):

```

```
...
ValueError: insufficient precision for non-existence proof
sage: algdep(pi.n(200), 10, height_bound=10, proof=True) is None
True
```

We can also use `proof=True` to get positive results:

```
sage: a = sqrt(2) + sqrt(3) + sqrt(5)
sage: algdep(a.n(), 8, height_bound=1000, proof=True)
Traceback (most recent call last):
...
ValueError: insufficient precision for uniqueness proof
sage: f = algdep(a.n(1000), 8, height_bound=1000, proof=True); f
x^8 - 40*x^6 + 352*x^4 - 960*x^2 + 576
sage: f(a).expand()
0
```

TESTS:

```
sage: algdep(complex("1+2j"), 4)
x^2 - 2*x + 5
```

`sage.rings.arith.bernoulli(n, algorithm='default', num_threads=1)`

Return the n -th Bernoulli number, as a rational number.

INPUT:

- `n` - an integer
- `algorithm`:
 - ‘default’ – use ‘flint’ for $n \leq 300000$, and ‘bernmm’ otherwise (this is just a heuristic, and not guaranteed to be optimal on all hardware)
 - ‘flint’ – use the FLINT library
 - ‘pari’ – use the PARI C library
 - ‘gap’ – use GAP
 - ‘gp’ – use PARI/GP interpreter
 - ‘magma’ – use MAGMA (optional)
 - ‘bernmm’ – use bernmm package (a multimodular algorithm)
- `num_threads` - positive integer, number of threads to use (only used for bernmm algorithm)

EXAMPLES:

```
sage: bernoulli(12)
-691/2730
sage: bernoulli(50)
495057205241079648212477525/66
```

We demonstrate each of the alternative algorithms:

```
sage: bernoulli(12, algorithm='flint')
-691/2730
sage: bernoulli(12, algorithm='gap')
-691/2730
sage: bernoulli(12, algorithm='gp')
-691/2730
sage: bernoulli(12, algorithm='magma')           # optional - magma
```

```

-691/2730
sage: bernoulli(12, algorithm='pari')
-691/2730
sage: bernoulli(12, algorithm='bernmm')
-691/2730
sage: bernoulli(12, algorithm='bernmm', num_threads=4)
-691/2730

```

TESTS:

```

sage: algs = ['gap', 'gp', 'pari', 'bernmm', 'flint']
sage: test_list = [ZZ.random_element(2, 2255) for _ in range(500)]
sage: vals = [[bernoulli(i, algorithm = j) for j in algs] for i in test_list] # long time (up to 100s)
sage: union([len(union(x))==1 for x in vals]) # long time (depends on previous line)
[True]
sage: algs = ['gp', 'pari', 'bernmm']
sage: test_list = [ZZ.random_element(2256, 5000) for _ in range(500)]
sage: vals = [[bernoulli(i, algorithm = j) for j in algs] for i in test_list] # long time (up to 100s)
sage: union([len(union(x))==1 for x in vals]) # long time (depends on previous line)
[True]

```

AUTHOR:

•David Joyner and William Stein

`sage.rings.arith.binomial(x, m, **kws)`

Return the binomial coefficient

$$\binom{x}{m} = x(x-1)\cdots(x-m+1)/m!$$

which is defined for $m \in \mathbf{Z}$ and any x . We extend this definition to include cases when $x - m$ is an integer but m is not by

$$\binom{x}{m} = \binom{x}{x-m}$$

If $m < 0$, return 0.

INPUT:

•*x*, *m* - numbers or symbolic expressions. Either *m* or $x-m$ must be an integer.

OUTPUT: number or symbolic expression (if input is symbolic)

EXAMPLES:

```

sage: from sage.rings.arith import binomial
sage: binomial(5, 2)
10
sage: binomial(2, 0)
1
sage: binomial(1/2, 0)
1
sage: binomial(3, -1)
0
sage: binomial(20, 10)
184756
sage: binomial(-2, 5)
-6
sage: binomial(-5, -2)

```

```
0
sage: binomial(RealField() ('2.5'), 2)
1.8750000000000000
sage: n=var('n'); binomial(n,2)
1/2*(n - 1)*n
sage: n=var('n'); binomial(n,n)
1
sage: n=var('n'); binomial(n,n-1)
n
sage: binomial(2^100, 2^100)
1

sage: x = polygen(ZZ)
sage: binomial(x, 3)
1/6*x^3 - 1/2*x^2 + 1/3*x
sage: binomial(x, x-3)
1/6*x^3 - 1/2*x^2 + 1/3*x
```

If $x \in \mathbf{Z}$, there is an optional ‘algorithm’ parameter, which can be ‘mpir’ (faster for small values) or ‘pari’ (faster for large values):

```
sage: a = binomial(100, 45, algorithm='mpir')
sage: b = binomial(100, 45, algorithm='pari')
sage: a == b
True
```

TESTS:

We test that certain binomials are very fast (this should be instant) – see [trac ticket #3309](#):

```
sage: a = binomial(RR(1140000.78), 23310000)
```

We test conversion of arguments to Integers – see [trac ticket #6870](#):

```
sage: binomial(1/2, 1/1)
1/2
sage: binomial(10^20+1/1, 10^20)
10000000000000000000001
sage: binomial(SR(10**7), 10**7)
1
sage: binomial(3/2, SR(1/1))
3/2
```

Some floating point cases – see [trac ticket #7562](#), [trac ticket #9633](#), and [trac ticket #12448](#):

```
sage: binomial(1.,3)
0.0000000000000000
sage: binomial(-2.,3)
-4.0000000000000000
sage: binomial(0.5r, 5)
0.02734375
sage: a = binomial(float(1001), float(1)); a
1001.0
sage: type(a)
<type 'float'>
sage: binomial(float(1000), 1001)
0.0
```

Test more output types:

```

sage: type(binomial(5r, 2))
<type 'int'>
sage: type(binomial(5, 2r))
<type 'sage.rings.integer.Integer'>

sage: type(binomial(5.0r, 2))
<type 'float'>

sage: type(binomial(5/1, 2))
<type 'sage.rings.rational.Rational'>

sage: R = Integers(11)
sage: b = binomial(R(7), R(3))
sage: b
2
sage: b.parent()
Ring of integers modulo 11

```

Test symbolic and uni/multivariate polynomials:

```

sage: x = polygen(ZZ)
sage: binomial(x, 3)
1/6*x^3 - 1/2*x^2 + 1/3*x
sage: binomial(x, 3).parent()
Univariate Polynomial Ring in x over Rational Field

sage: K.<x,y> = Integers(7)[]
sage: binomial(y, 3)
-y^3 + 3*y^2 - 2*y
sage: binomial(y, 3).parent()
Multivariate Polynomial Ring in x, y over Ring of integers modulo 7

sage: n = var('n')
sage: binomial(n, 2)
1/2*(n - 1)*n

```

Invalid inputs:

```

sage: x = polygen(ZZ)
sage: binomial(x, x^2)
Traceback (most recent call last):
...
TypeError: either m or x-m must be an integer

sage: k, i = var('k,i')
sage: binomial(k,i)
Traceback (most recent call last):
...
TypeError: either m or x-m must be an integer

sage: R6 = Zmod(6)
sage: binomial(R6(5), 2)
Traceback (most recent call last):
...
ZeroDivisionError: factorial(2) not invertible in Ring of integers modulo 6

sage: R7 = Zmod(7)
sage: binomial(R7(10), 7)
Traceback (most recent call last):

```

```
...
ZeroDivisionError: factorial(7) not invertible in Ring of integers modulo 7
```

The last two examples failed to execute since $2!$ and $7!$ are respectively not invertible in $\mathbb{Z}/6\mathbb{Z}$ and $\mathbb{Z}/7\mathbb{Z}$. One can check that there is no well defined value for that binomial coefficient in the quotient:

```
sage: R6(binomial(5, 2))
4
sage: R6(binomial(5+6, 2))
1

sage: R7(binomial(3, 7))
0
sage: R7(binomial(10, 7))
1
sage: R7(binomial(17, 7))
2
```

For symbolic manipulation, you should use the function `binomial()` from the module `sage.functions.other`:

```
sage: from sage.functions.other import binomial
sage: binomial(k, i)
binomial(k, i)
```

`sage.rings.arith.binomial_coefficients(n)`

Return a dictionary containing pairs $\{(k_1, k_2) : C_{k,n}\}$ where $C_{k,n}$ are binomial coefficients and $n = k_1 + k_2$.

INPUT:

- n - an integer

OUTPUT: dict

EXAMPLES:

```
sage: sorted(binomial_coefficients(3).items())
[(0, 3), 1), ((1, 2), 3), ((2, 1), 3), ((3, 0), 1)]
```

Notice the coefficients above are the same as below:

```
sage: R.<x,y> = QQ[]
sage: (x+y)^3
x^3 + 3*x^2*y + 3*x*y^2 + y^3
```

AUTHORS:

- Fredrik Johansson

`sage.rings.arith.continuant(v, n=None)`

Function returns the continuant of the sequence v (list or tuple).

Definition: see Graham, Knuth and Patashnik, *Concrete Mathematics*, section 6.7: Continuants. The continuant is defined by

- $K_0() = 1$

- $K_1(x_1) = x_1$

- $K_n(x_1, \dots, x_n) = K_{n-1}(x_n, \dots, x_{n-1})x_n + K_{n-2}(x_1, \dots, x_{n-2})$

If $n = \text{None}$ or $n > \text{len}(v)$ the default $n = \text{len}(v)$ is used.

INPUT:

- v - list or tuple of elements of a ring
- n - optional integer

OUTPUT: element of ring (integer, polynomial, etcetera).

EXAMPLES:

```
sage: continuant([1, 2, 3])
10
sage: p = continuant([2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10])
sage: q = continuant([1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10])
sage: p/q
517656/190435
sage: continued_fraction([2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10]).convergent(14)
517656/190435
sage: x = PolynomialRing(RationalField(), 'x', 5).gens()
sage: continuant(x)
x0*x1*x2*x3*x4 + x0*x1*x2 + x0*x1*x4 + x0*x3*x4 + x2*x3*x4 + x0 + x2 + x4
sage: continuant(x, 3)
x0*x1*x2 + x0 + x2
sage: continuant(x, 2)
x0*x1 + 1
```

We verify the identity

$$K_n(z, z, \dots, z) = \sum_{k=0}^n \binom{n-k}{k} z^{n-2k}$$

for $n = 6$ using polynomial arithmetic:

```
sage: z = QQ['z'].0
sage: continuant((z, z, z, z, z, z, z, z, z, z, z, z, z), 6)
z^6 + 5*z^4 + 6*z^2 + 1

sage: continuant(9)
Traceback (most recent call last):
...
TypeError: object of type 'sage.rings.integer.Integer' has no len()
```

AUTHORS:

- Jaap Spies (2007-02-06)

sage.rings.arith.**crt** ($a, b, m=None, n=None$)

Returns a solution to a Chinese Remainder Theorem problem.

INPUT:

- a, b - two residues (elements of some ring for which extended gcd is available), or two lists, one of residues and one of moduli.
- m, n - (default: None) two moduli, or None.

OUTPUT:

If m, n are not None, returns a solution x to the simultaneous congruences $x \equiv a \pmod{m}$ and $x \equiv b \pmod{n}$, if one exists. By the Chinese Remainder Theorem, a solution to the simultaneous congruences exists if and only if $a \equiv b \pmod{\gcd(m, n)}$. The solution x is only well-defined modulo $\text{lcm}(m, n)$.

If a and b are lists, returns a simultaneous solution to the congruences $x \equiv a_i \pmod{b_i}$, if one exists.

See also:

•`CRT_list()`

EXAMPLES:

Using `crt` by giving it pairs of residues and moduli:

```
sage: crt(2, 1, 3, 5)
11
sage: crt(13, 20, 100, 301)
28013
sage: crt([2, 1], [3, 5])
11
sage: crt([13, 20], [100, 301])
28013
```

You can also use upper case:

```
sage: c = CRT(2, 3, 3, 5); c
8
sage: c % 3 == 2
True
sage: c % 5 == 3
True
```

Note that this also works for polynomial rings:

```
sage: K.<a> = NumberField(x^3 - 7)
sage: R.<y> = K[]
sage: f = y^2 + 3
sage: g = y^3 - 5
sage: CRT(1, 3, f, g)
-3/26*y^4 + 5/26*y^3 + 15/26*y + 53/26
sage: CRT(1, a, f, g)
(-3/52*a + 3/52)*y^4 + (5/52*a - 5/52)*y^3 + (15/52*a - 15/52)*y + 27/52*a + 25/52
```

You can also do this for any number of moduli:

```
sage: K.<a> = NumberField(x^3 - 7)
sage: R.<x> = K[]
sage: CRT([], [])
0
sage: CRT([a], [x])
a
sage: f = x^2 + 3
sage: g = x^3 - 5
sage: h = x^5 + x^2 - 9
sage: k = CRT([1, a, 3], [f, g, h]); k
(127/26988*a - 5807/386828)*x^9 + (45/8996*a - 33677/1160484)*x^8 + (2/173*a - 6/173)*x^7 + (133/173)*x^6 + (133/173)*x^5 + (133/173)*x^4 + (133/173)*x^3 + (133/173)*x^2 + (133/173)*x + 133/173
sage: k.mod(f)
1
sage: k.mod(g)
a
sage: k.mod(h)
3
```

If the moduli are not coprime, a solution may not exist:

```
sage: crt(4, 8, 8, 12)
20
sage: crt(4, 6, 8, 12)
Traceback (most recent call last):
```



```

...
ValueError: No solution to crt problem since gcd(8,12) does not divide 4-6

sage: x = polygen(QQ)
sage: crt(2,3,x-1,x+1)
-1/2*x + 5/2
sage: crt(2,x,x^2-1,x^2+1)
-1/2*x^3 + x^2 + 1/2*x + 1
sage: crt(2,x,x^2-1,x^3-1)
Traceback (most recent call last):
...
ValueError: No solution to crt problem since gcd(x^2 - 1,x^3 - 1) does not divide 2-x

sage: crt(int(2), int(3), int(7), int(11))
58

```

`sage.rings.arith.dedekind_sum(p, q, algorithm='default')`

Return the Dedekind sum $s(p, q)$ defined for integers p, q as

$$s(p, q) = \sum_{i=0}^{q-1} \left(\left(\frac{i}{q} \right) \right) \left(\left(\frac{pi}{q} \right) \right)$$

where

$$\left(\left(x \right) \right) = \begin{cases} x - [x] - \frac{1}{2} & \text{if } x \in \mathbf{Q} \setminus \mathbf{Z} \\ 0 & \text{if } x \in \mathbf{Z}. \end{cases}$$

Warning: Caution is required as the Dedekind sum sometimes depends on the algorithm or is left undefined when p and q are not coprime.

INPUT:

- p, q – integers
- `algorithm` – must be one of the following
 - ‘default’ - (default) use FLINT
 - ‘flint’ - use FLINT
 - ‘pari’ - use PARI (gives different results if p and q are not coprime)

OUTPUT: a rational number

EXAMPLES:

Several small values:

```

sage: for q in range(10): print [dedekind_sum(p,q) for p in range(q+1)]
[0]
[0, 0]
[0, 0, 0]
[0, 1/18, -1/18, 0]
[0, 1/8, 0, -1/8, 0]
[0, 1/5, 0, 0, -1/5, 0]
[0, 5/18, 1/18, 0, -1/18, -5/18, 0]
[0, 5/14, 1/14, -1/14, 1/14, -1/14, -5/14, 0]
[0, 7/16, 1/8, 1/16, 0, -1/16, -1/8, -7/16, 0]
[0, 14/27, 4/27, 1/18, -4/27, 4/27, -1/18, -4/27, -14/27, 0]

```

Check relations for restricted arguments:

```
sage: q = 23; dedekind_sum(1, q); (q-1)*(q-2)/(12*q)
77/46
77/46
sage: p, q = 100, 723      # must be coprime
sage: dedekind_sum(p, q) + dedekind_sum(q, p)
31583/86760
sage: -1/4 + (p/q + q/p + 1/(p*q))/12
31583/86760
```

We check that evaluation works with large input:

```
sage: dedekind_sum(3^54 - 1, 2^93 + 1)
459340694971839990630374299870/29710560942849126597578981379
sage: dedekind_sum(3^54 - 1, 2^93 + 1, algorithm='pari')
459340694971839990630374299870/29710560942849126597578981379
```

We check consistency of the results:

```
sage: dedekind_sum(5, 7, algorithm='default')
-1/14
sage: dedekind_sum(5, 7, algorithm='flint')
-1/14
sage: dedekind_sum(5, 7, algorithm='pari')
-1/14
sage: dedekind_sum(6, 8, algorithm='default')
-1/8
sage: dedekind_sum(6, 8, algorithm='flint')
-1/8
sage: dedekind_sum(6, 8, algorithm='pari')
-1/8
```

REFERENCES:

- [Wikipedia article Dedekind_sum](#)

`sage.rings.arith.differences(lis, n=1)`

Returns the n successive differences of the elements in *lis*.

EXAMPLES:

```
sage: differences(prime_range(50))
[1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4]
sage: differences([i^2 for i in range(1,11)])
[3, 5, 7, 9, 11, 13, 15, 17, 19]
sage: differences([i^3 + 3*i for i in range(1,21)])
[10, 22, 40, 64, 94, 130, 172, 220, 274, 334, 400, 472, 550, 634, 724, 820, 922, 1030, 1144]
sage: differences([i^3 - i^2 for i in range(1,21)], 2)
[10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, 76, 82, 88, 94, 100, 106, 112]
sage: differences([p - i^2 for i, p in enumerate(prime_range(50))], 3)
[-1, 2, -4, 4, -4, 4, 0, -6, 8, -6, 0, 4]
```

AUTHORS:

- Timothy Clemans (2008-03-09)

`sage.rings.arith.divisors(n)`

Returns a list of all positive integer divisors of the nonzero integer n .

INPUT:

- n - the element

EXAMPLES:

```

sage: divisors(-3)
[1, 3]
sage: divisors(6)
[1, 2, 3, 6]
sage: divisors(28)
[1, 2, 4, 7, 14, 28]
sage: divisors(2^5)
[1, 2, 4, 8, 16, 32]
sage: divisors(100)
[1, 2, 4, 5, 10, 20, 25, 50, 100]
sage: divisors(1)
[1]
sage: divisors(0)
Traceback (most recent call last):
...
ValueError: n must be nonzero
sage: divisors(2^3 * 3^2 * 17)
[1, 2, 3, 4, 6, 8, 9, 12, 17, 18, 24, 34, 36, 51, 68, 72, 102, 136, 153, 204, 306, 408, 612, 1224]

```

This function works whenever one has unique factorization:

```

sage: K.<a> = QuadraticField(7)
sage: divisors(K.ideal(7))
[Fractional ideal (1), Fractional ideal (a), Fractional ideal (7)]
sage: divisors(K.ideal(3))
[Fractional ideal (1), Fractional ideal (3), Fractional ideal (-a + 2), Fractional ideal (-a - 2)]
sage: divisors(K.ideal(35))
[Fractional ideal (1), Fractional ideal (5), Fractional ideal (a), Fractional ideal (7), Fractional ideal (a+5)]

```

TESTS:

```

sage: divisors(int(300))
[1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300]

```

`sage.rings.arith.eratosthenes(n)`

Return a list of the primes $\leq n$.

This is extremely slow and is for educational purposes only.

INPUT:

- n - a positive integer

OUTPUT:

- a list of primes less than or equal to n .

EXAMPLES:

```

sage: eratosthenes(3)
[2, 3]
sage: eratosthenes(50)
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47]
sage: len(eratosthenes(100))
25
sage: eratosthenes(213) == prime_range(213)
True

```

`sage.rings.arith.factor(n, proof=None, int_=False, algorithm='pari', verbose=0, **kws)`

Returns the factorization of n . The result depends on the type of n .

If n is an integer, returns the factorization as an object of type `Factorization`.

If n is not an integer, `n.factor(proof=proof, **kwds)` gets called. See `n.factor??` for more documentation in this case.

Warning: This means that applying `factor` to an integer result of a symbolic computation will not factor the integer, because it is considered as an element of a larger symbolic ring.

EXAMPLE:

```
sage: f(n)=n^2
sage: is_prime(f(3))
False
sage: factor(f(3))
9
```

INPUT:

- `n` - an nonzero integer
- `proof` - bool or None (default: None)
- `int_` - bool (default: False) whether to return answers as Python ints
- `algorithm` - string
 - ‘`pari`’ - (default) use the PARI c library
 - ‘`kash`’ - use KASH computer algebra system (requires the optional kash package be installed)
 - ‘`magma`’ - use Magma (requires magma be installed)
- `verbose` - integer (default: 0); PARI’s debug variable is set to this; e.g., set to 4 or 8 to see lots of output during factorization.

OUTPUT:

- factorization of n

The `qsieve` and `ecm` commands give access to highly optimized implementations of algorithms for doing certain integer factorization problems. These implementations are not used by the generic `factor` command, which currently just calls PARI (note that PARI also implements sieve and `ecm` algorithms, but they aren’t as optimized). Thus you might consider using them instead for certain numbers.

The factorization returned is an element of the class `Factorization`; see `Factorization??` for more details, and examples below for usage. A `Factorization` contains both the unit factor (+1 or -1) and a sorted list of (prime, exponent) pairs.

The factorization displays in pretty-print format but it is easy to obtain access to the (prime,exponent) pairs and the unit, to recover the number from its factorization, and even to multiply two factorizations. See examples below.

EXAMPLES:

```
sage: factor(500)
2^2 * 5^3
sage: factor(-20)
-1 * 2^2 * 5
sage: f=factor(-20)
sage: list(f)
[(2, 2), (5, 1)]
sage: f.unit()
-1
sage: f.value()
20
```

```

-20
sage: factor( -next_prime(10^2) * next_prime(10^7) )
-1 * 101 * 10000019

sage: factor(-500, algorithm='kash')      # optional - kash
-1 * 2^2 * 5^3

sage: factor(-500, algorithm='magma')     # optional - magma
-1 * 2^2 * 5^3

sage: factor(0)
Traceback (most recent call last):
...
ArithmeticError: Prime factorization of 0 not defined.
sage: factor(1)
1
sage: factor(-1)
-1
sage: factor(2^(2^7)+1)
59649589127497217 * 5704689200685129054721

```

Sage calls PARI's factor, which has `proof=False` by default. Sage has a global proof flag, set to `True` by default (see `sage.structure.proof.proof`, or `proof.[tab]`). To override the default, call this function with `proof=False`.

```

sage: factor(3^89-1, proof=False)
2 * 179 * 1611479891519807 * 5042939439565996049162197

sage: factor(2^197 + 1) # long time (2s)
3 * 197002597249 * 1348959352853811313 * 251951573867253012259144010843

```

Any object which has a `factor` method can be factored like this:

```

sage: K.<i> = QuadraticField(-1)
sage: factor(122 - 454*i)
(-3*i - 2) * (-i - 2)^3 * (i + 1)^3 * (i + 4)

```

To access the data in a factorization:

```

sage: f = factor(420); f
2^2 * 3 * 5 * 7
sage: [x for x in f]
[(2, 2), (3, 1), (5, 1), (7, 1)]
sage: [p for p,e in f]
[2, 3, 5, 7]
sage: [e for p,e in f]
[2, 1, 1, 1]
sage: [p^e for p,e in f]
[4, 3, 5, 7]

```

`sage.rings.arith.factorial(n, algorithm='gmp')`
 Compute the factorial of n , which is the product $1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$.

INPUT:

- n - an integer
- `algorithm` - string (default: 'gmp'):
 - 'gmp' - use the GMP C-library factorial function

–'pari' - use PARI's factorial function

OUTPUT: an integer

EXAMPLES:

```
sage: from sage.rings.arith import factorial
sage: factorial(0)
1
sage: factorial(4)
24
sage: factorial(10)
3628800
sage: factorial(1) == factorial(0)
True
sage: factorial(6) == 6*5*4*3*2
True
sage: factorial(1) == factorial(0)
True
sage: factorial(71) == 71* factorial(70)
True
sage: factorial(-32)
Traceback (most recent call last):
...
ValueError: factorial -- must be nonnegative
```

PERFORMANCE: This discussion is valid as of April 2006. All timings below are on a Pentium Core Duo 2Ghz MacBook Pro running Linux with a 2.6.16.1 kernel.

- It takes less than a minute to compute the factorial of 10^7 using the GMP algorithm, and the factorial of 10^6 takes less than 4 seconds.
- The GMP algorithm is faster and more memory efficient than the PARI algorithm. E.g., PARI computes 10^7 factorial in 100 seconds on the core duo 2Ghz.
- For comparison, computation in Magma ≤ 2.12 -10 of $n!$ is best done using `*[1..n]`. It takes 113 seconds to compute the factorial of 10^7 and 6 seconds to compute the factorial of 10^6 . Mathematica V5.2 compute the factorial of 10^7 in 136 seconds and the factorial of 10^6 in 7 seconds. (Mathematica is notably very efficient at memory usage when doing factorial calculations.)

`sage.rings.arith.falling_factorial` (x, a)

Returns the falling factorial $(x)_a$.

The notation in the literature is a mess: often $(x)_a$, but there are many other notations: GKP: Concrete Mathematics uses $x^{\underline{a}}$.

Definition: for integer $a \geq 0$ we have $x(x-1) \cdots (x-a+1)$. In all other cases we use the GAMMA-function: $\frac{\Gamma(x+1)}{\Gamma(x-a+1)}$.

INPUT:

- x - element of a ring
- a - a non-negative integer or

OR

- x and a - any numbers

OUTPUT: the falling factorial

EXAMPLES:

```

sage: falling_factorial(10, 3)
720
sage: falling_factorial(10, RR('3.0'))
720.0000000000000
sage: falling_factorial(10, RR('3.3'))
1310.11633396601
sage: falling_factorial(10, 10)
3628800
sage: factorial(10)
3628800
sage: a = falling_factorial(1+I, I); a
gamma(I + 2)
sage: CC(a)
0.652965496420167 + 0.343065839816545*I
sage: falling_factorial(1+I, 4)
4*I + 2
sage: falling_factorial(I, 4)
-10

sage: M = MatrixSpace(ZZ, 4, 4)
sage: A = M([1,0,1,0,1,0,1,0,1,0,10,10,1,0,1,1])
sage: falling_factorial(A, 2) # A(A - I)
[ 1  0 10 10]
[ 1  0 10 10]
[ 20  0 101 100]
[ 2  0 11 10]

sage: x = ZZ['x'].0
sage: falling_factorial(x, 4)
x^4 - 6*x^3 + 11*x^2 - 6*x

```

TESTS:

Check that [trac ticket #14858](#) is fixed:

```

sage: falling_factorial(-4, SR(2))
20

```

Check that [trac ticket #16770](#) is fixed:

```

sage: d = var('d')
sage: type(falling_factorial(d, 0))
<type 'sage.symbolic.expression.Expression'>

```

AUTHORS:

•Jaap Spies (2006-03-05)

`sage.rings.arith.four_squares(n)`

Write the integer n as a sum of four integer squares.

INPUT:

• n – an integer

OUTPUT: a tuple (a, b, c, d) of non-negative integers such that $n = a^2 + b^2 + c^2 + d^2$ with $a \leq b \leq c \leq d$.

EXAMPLES:

```

sage: four_squares(3)
(0, 1, 1, 1)
sage: four_squares(13)

```

[illegible]

TESTS:

```
sage: for _ in xrange(100):
....:     n = ZZ.random_element(2**32, 2**34)
....:     aa, bb, cc, dd = four_squares(n)
....:     assert aa**2 + bb**2 + cc**2 + dd**2 == n
```

```
sage.rings.arith.fundamental_discriminant(D)
```

Return the discriminant of the quadratic extension $K = Q(\sqrt{D})$, i.e. an integer d congruent to either 0 or 1, mod 4, and such that, at most, the only square dividing it is 4.

INPUT:

- D - an integer

OUTPUT:

- an integer, the fundamental discriminant

EXAMPLES:

```
sage: fundamental_discriminant(102)
408
sage: fundamental_discriminant(720)
5
sage: fundamental_discriminant(2)
8
```

```
sage.rings.arith.gcd(a, b=None, **kwargs)
```

The greatest common divisor of a and b, or if a is a list and b is omitted the greatest common divisor of all elements of a.

INPUT:

- a, b - two elements of a ring with gcd or
- a - a list or tuple of elements of a ring with gcd

Additional keyword arguments are passed to the respectively called methods.

OUTPUT:

The given elements are first coerced into a common parent. Then, their greatest common divisor *in that common parent* is returned.

EXAMPLES:

```
sage: GCD(97, 100)
1
sage: GCD(97*10^15, 19^20*97^2)
97
sage: GCD(2/3, 4/5)
```



```

2/15
sage: GCD([2,4,6,8])
2
sage: GCD(srange(0,10000,10)) # fast !!
10

```

Note that to take the gcd of n elements for $n \neq 2$ you must put the elements into a list by enclosing them in `[...]`. Before #4988 the following wrongly returned 3 since the third parameter was just ignored:

```

sage: gcd(3,6,2)
Traceback (most recent call last):
...
TypeError: gcd() takes at most 2 arguments (3 given)
sage: gcd([3,6,2])
1

```

Similarly, giving just one element (which is not a list) gives an error:

```

sage: gcd(3)
Traceback (most recent call last):
...
TypeError: 'sage.rings.integer.Integer' object is not iterable

```

By convention, the gcd of the empty list is (the integer) 0:

```

sage: gcd([])
0
sage: type(gcd([]))
<type 'sage.rings.integer.Integer'>

```

TESTS:

The following shows that indeed coercion takes place before computing the gcd. This behaviour was introduced in [trac ticket #10771](#):

```

sage: R.<x>=QQ[]
sage: S.<x>=ZZ[]
sage: p = S.random_element(degree=(0,10))
sage: q = R.random_element(degree=(0,10))
sage: parent(gcd(1/p,q))
Fraction Field of Univariate Polynomial Ring in x over Rational Field
sage: parent(gcd([1/p,q]))
Fraction Field of Univariate Polynomial Ring in x over Rational Field

```

Make sure we try QQ and not merely ZZ ([trac ticket #13014](#)):

```

sage: bool(gcd(2/5, 3/7) == gcd(SR(2/5), SR(3/7)))
True

```

Make sure that the gcd of Expressions stays symbolic:

```

sage: parent(gcd(2, 4))
Integer Ring
sage: parent(gcd(SR(2), 4))
Symbolic Ring
sage: parent(gcd(2, SR(4)))
Symbolic Ring
sage: parent(gcd(SR(2), SR(4)))
Symbolic Ring

```

Verify that objects without gcd methods but which can't be coerced to ZZ or QQ raise an error:

```
sage: F.<a,b> = FreeMonoid(2)
sage: gcd(a,b)
Traceback (most recent call last):
...
TypeError: unable to find gcd
```

`sage.rings.arith.get_gcd(order)`

Return the fastest gcd function for integers of size no larger than *order*.

EXAMPLES:

```
sage: sage.rings.arith.get_gcd(4000)
<built-in method gcd_int of sage.rings.fast_arith.arith_int object at ...>
sage: sage.rings.arith.get_gcd(400000)
<built-in method gcd_longlong of sage.rings.fast_arith.arith_llong object at ...>
sage: sage.rings.arith.get_gcd(4000000000)
<function gcd at ...>
```

`sage.rings.arith.get_inverse_mod(order)`

Return the fastest inverse_mod function for integers of size no larger than *order*.

EXAMPLES:

```
sage: sage.rings.arith.get_inverse_mod(6000)
<built-in method inverse_mod_int of sage.rings.fast_arith.arith_int object at ...>
sage: sage.rings.arith.get_inverse_mod(600000)
<built-in method inverse_mod_longlong of sage.rings.fast_arith.arith_llong object at ...>
sage: sage.rings.arith.get_inverse_mod(6000000000)
<function inverse_mod at ...>
```

`sage.rings.arith.hilbert_conductor(a, b)`

This is the product of all (finite) primes where the Hilbert symbol is -1. What is the same, this is the (reduced) discriminant of the quaternion algebra (a, b) over \mathbb{Q} .

INPUT:

- a, b – integers

OUTPUT:

- squarefree positive integer

EXAMPLES:

```
sage: hilbert_conductor(-1, -1)
2
sage: hilbert_conductor(-1, -11)
11
sage: hilbert_conductor(-2, -5)
5
sage: hilbert_conductor(-3, -17)
17
```

AUTHOR:

- Gonzalo Tornaria (2009-03-02)

`sage.rings.arith.hilbert_conductor_inverse(d)`

Finds a pair of integers (a, b) such that `hilbert_conductor(a, b) == d`. The quaternion algebra (a, b) over \mathbb{Q} will then have (reduced) discriminant *d*.

INPUT:

- d – square-free positive integer

OUTPUT: pair of integers

EXAMPLES:

```
sage: hilbert_conductor_inverse(2)
(-1, -1)
sage: hilbert_conductor_inverse(3)
(-1, -3)
sage: hilbert_conductor_inverse(6)
(-1, 3)
sage: hilbert_conductor_inverse(30)
(-3, -10)
sage: hilbert_conductor_inverse(4)
Traceback (most recent call last):
...
ValueError: d needs to be squarefree
sage: hilbert_conductor_inverse(-1)
Traceback (most recent call last):
...
ValueError: d needs to be positive
```

AUTHOR:

- Gonzalo Tornaria (2009-03-02)

TESTS:

```
sage: for i in xrange(100):
...     d = ZZ.random_element(2**32).squarefree_part()
...     if hilbert_conductor(*hilbert_conductor_inverse(d)) != d:
...         print "hilbert_conductor_inverse failed for d =", d
```

`sage.rings.arith.hilbert_symbol(a, b, p, algorithm='pari')`

Returns 1 if $ax^2 + by^2$ p -adically represents a nonzero square, otherwise returns -1 . If either a or b is 0, returns 0.

INPUT:

- a, b - integers
- p - integer; either prime or -1 (which represents the archimedean place)
- algorithm - string
 - 'pari' - (default) use the PARI C library
 - 'direct' - use a Python implementation
 - 'all' - use both PARI and direct and check that the results agree, then return the common answer

OUTPUT: integer (0, -1, or 1)

EXAMPLES:

```
sage: hilbert_symbol(-1, -1, -1, algorithm='all')
-1
sage: hilbert_symbol(2, 3, 5, algorithm='all')
1
sage: hilbert_symbol(4, 3, 5, algorithm='all')
1
sage: hilbert_symbol(0, 3, 5, algorithm='all')
0
```

```
sage: hilbert_symbol (-1, -1, 2, algorithm='all')
-1
sage: hilbert_symbol (1, -1, 2, algorithm='all')
1
sage: hilbert_symbol (3, -1, 2, algorithm='all')
-1

sage: hilbert_symbol(QQ(-1)/QQ(4), -1, 2) == -1
True
sage: hilbert_symbol(QQ(-1)/QQ(4), -1, 3) == 1
True
```

AUTHORS:

- William Stein and David Kohel (2006-01-05)

`sage.rings.arith.integer_ceil(x)`
Return the ceiling of x .

EXAMPLES:

```
sage: integer_ceil(5.4)
6
sage: integer_ceil(x)
Traceback (most recent call last):
...
NotImplementedError: computation of ceil of x not implemented
```

`sage.rings.arith.integer_floor(x)`
Return the largest integer $\leq x$.

INPUT:

- x - an object that has a floor method or is coercible to int

OUTPUT: an Integer

EXAMPLES:

```
sage: integer_floor(5.4)
5
sage: integer_floor(float(5.4))
5
sage: integer_floor(-5/2)
-3
sage: integer_floor(RDF(-5/2))
-3

sage: integer_floor(x)
Traceback (most recent call last):
...
NotImplementedError: computation of floor of x not implemented
```

`sage.rings.arith.inverse_mod(a, m)`
The inverse of the ring element a modulo m .

If no special `inverse_mod` is defined for the elements, it tries to coerce them into integers and perform the inversion there

```
sage: inverse_mod(7, 1)
0
sage: inverse_mod(5, 14)
```

```

3
sage: inverse_mod(3,-5)
2

```

`sage.rings.arith.is_power_of_two(n)`

This function returns True if and only if n is a power of 2

INPUT:

- n - integer

OUTPUT:

- True - if n is a power of 2
- False - if not

EXAMPLES:

```

sage: is_power_of_two(1024)
True
sage: is_power_of_two(1)
True
sage: is_power_of_two(24)
False
sage: is_power_of_two(0)
False
sage: is_power_of_two(-4)
False

```

`sage.rings.arith.is_prime(n)`

Return True if n is a prime number, and False otherwise.

Use a provable primality test or a strong pseudo-primality test depending on the global arithmetic proof flag.

INPUT:

- n - the object for which to determine primality

See also:

- `is_pseudoprime()`
- `sage.rings.integer.Integer.is_prime()`

AUTHORS:

- Kevin Stueve kstueve@uw.edu (2010-01-17): delegated calculation to `n.is_prime()`

EXAMPLES:

```

sage: is_prime(389)
True
sage: is_prime(2000)
False
sage: is_prime(2)
True
sage: is_prime(-1)
False
sage: is_prime(1)
False
sage: is_prime(-2)

```

```
False
```

```
sage: a = 2**2048 + 981
sage: is_prime(a)      # not tested - takes ~ 1min
sage: proof.arithmetic(False)
sage: is_prime(a)      # instantaneous!
True
sage: proof.arithmetic(True)
```

`sage.rings.arith.is_prime_power(n, flag=None, get_data=False)`

Test whether n is a positive power of a prime number

This function simply calls the method `Integer.is_prime_power()` of `Integers`.

INPUT:

- n – an integer
- `get_data` – if set to `True`, return a pair (p, k) such that this integer equals p^k instead of `True` or $(self, 0)$ instead of `False`

EXAMPLES:

```
sage: is_prime_power(389)
True
sage: is_prime_power(2000)
False
sage: is_prime_power(2)
True
sage: is_prime_power(1024)
True
sage: is_prime_power(1024, get_data=True)
(2, 10)
```

The same results can be obtained with:

```
sage: 389.is_prime_power()
True
sage: 2000.is_prime_power()
False
sage: 2.is_prime_power()
True
sage: 1024.is_prime_power()
True
sage: 1024.is_prime_power(get_data=True)
(2, 10)
```

TESTS:

```
sage: is_prime_power(-1)
False
sage: is_prime_power(1)
False
sage: is_prime_power(QQ(997^100))
True
sage: is_prime_power(1/2197)
Traceback (most recent call last):
...
TypeError: no conversion of this rational to integer
sage: is_prime_power("foo")
Traceback (most recent call last):
```

```
...
TypeError: unable to convert 'foo' to an integer
```

`sage.rings.arith.is_pseudoprime(n, flag=None)`

Test whether n is a pseudo-prime

The result is *NOT* proven correct - *this is a pseudo-primality test!*.

INPUT:

- n – an integer

Note: We do not consider negatives of prime numbers as prime.

EXAMPLES:

```
sage: is_pseudoprime(389)
True
sage: is_pseudoprime(2000)
False
sage: is_pseudoprime(2)
True
sage: is_pseudoprime(-1)
False
sage: factor(-6)
-1 * 2 * 3
sage: is_pseudoprime(1)
False
sage: is_pseudoprime(-2)
False
```

TESTS:

Deprecation warning from [trac ticket #16878](#):

```
sage: is_pseudoprime(127, flag=0)
doctest:...: DeprecationWarning: the keyword 'flag' is deprecated and no longer used
See http://trac.sagemath.org/16878 for details.
True
```

`sage.rings.arith.is_pseudoprime_power(n, get_data=False)`

Test if n is a power of a pseudoprime.

The result is *NOT* proven correct - *this IS a pseudo-primality test!*. Note that a prime power is a positive power of a prime number so that 1 is not a prime power.

INPUT:

- n - an integer
- `get_data` - (boolean) instead of a boolean return a pair (p, k) so that n equals p^k and p is a pseudoprime or $(n, 0)$ otherwise.

EXAMPLES:

```
sage: is_pseudoprime_power(389)
True
sage: is_pseudoprime_power(2000)
False
sage: is_pseudoprime_power(2)
True
sage: is_pseudoprime_power(1024)
```

```
True
sage: is_pseudoprime_power(-1)
False
sage: is_pseudoprime_power(1)
False
sage: is_pseudoprime_power(997^100)
True
```

Use of the `get_data` keyword:

```
sage: is_pseudoprime_power(3^1024, get_data=True)
(3, 1024)
sage: is_pseudoprime_power(2^256, get_data=True)
(2, 256)
sage: is_pseudoprime_power(31, get_data=True)
(31, 1)
sage: is_pseudoprime_power(15, get_data=True)
(15, 0)
```

`sage.rings.arith.is_pseudoprime_small_power(n, bound=None, get_data=False)`

Deprecated version of `is_pseudoprime_power`.

EXAMPLES:

```
sage: is_pseudoprime_small_power(1234)
doctest:...: DeprecationWarning: the function is_pseudoprime_small_power() is deprecated, use is
See http://trac.sagemath.org/16878 for details.
False
sage: is_pseudoprime_small_power(3^1024, get_data=True)
[(3, 1024)]
```

`sage.rings.arith.is_square(n, root=False)`

Returns whether or not `n` is square, and if `n` is a square also returns the square root. If `n` is not square, also returns `None`.

INPUT:

- `n` - an integer
- `root` - whether or not to also return a square root (default: `False`)

OUTPUT:

- `bool` - whether or not a square
- `object` - (optional) an actual square if found, and `None` otherwise.

EXAMPLES:

```
sage: is_square(2)
False
sage: is_square(4)
True
sage: is_square(2.2)
True
sage: is_square(-2.2)
False
sage: is_square(CDF(-2.2))
True
sage: is_square((x-1)^2)
True
```



```
sage: is_square(4, True)
(True, 2)
```

`sage.rings.arith.is_squarefree(n)`
Test whether n is square free.

EXAMPLES:

```
sage: is_squarefree(100)
False
sage: is_squarefree(101)
True

sage: R = ZZ['x']
sage: x = R.gen()
sage: is_squarefree((x^2+x+1) * (x-2))
True
sage: is_squarefree((x-1)**2 * (x-3))
False

sage: O = ZZ[sqrt(-1)]
sage: I = O.gen(1)
sage: is_squarefree(I+1)
True
sage: is_squarefree(O(2))
False
sage: O(2).factor()
(-I) * (I + 1)^2
```

This method fails on domains which are not Unique Factorization Domains:

```
sage: O = ZZ[sqrt(-5)]
sage: a = O.gen(1)
sage: is_squarefree(a - 3)
Traceback (most recent call last):
...
ArithmeticError: non-principal ideal in factorization
```

`sage.rings.arith.jacobi_symbol(a, b)`
The Jacobi symbol of integers a and b , where b is odd.

Note: The `kronecker_symbol()` command extends the Jacobi symbol to all integers b .

If

$$b = p_1^{e_1} * \dots * p_r^{e_r}$$

then

$$(a|b) = (a|p_1)^{e_1} \dots (a|p_r)^{e_r}$$

where $(a|p_j)$ are Legendre Symbols.

INPUT:

- a - an integer
- b - an odd integer

EXAMPLES:

```
sage: jacobi_symbol(10,777)
-1
sage: jacobi_symbol(10,5)
0
sage: jacobi_symbol(10,2)
Traceback (most recent call last):
...
ValueError: second input must be odd, 2 is not odd
```

`sage.rings.arith.kronecker(x,y)`
Synonym for `kronecker_symbol()`.

The Kronecker symbol $(x|y)$.

INPUT:

- `x` - integer
- `y` - integer

OUTPUT:

- an integer

EXAMPLES:

```
sage: kronecker(3,5)
-1
sage: kronecker(3,15)
0
sage: kronecker(2,15)
1
sage: kronecker(-2,15)
-1
sage: kronecker(2/3,5)
1
```

`sage.rings.arith.kronecker_symbol(x,y)`
The Kronecker symbol $(x|y)$.

INPUT:

- `x` - integer
- `y` - integer

EXAMPLES:

```
sage: kronecker_symbol(13,21)
-1
sage: kronecker_symbol(101,4)
1
```

IMPLEMENTATION: Using GMP.

`sage.rings.arith.lcm(a,b=None)`

The least common multiple of `a` and `b`, or if `a` is a list and `b` is omitted the least common multiple of all elements of `a`.

Note that LCM is an alias for `lcm`.

INPUT:

- `a, b` - two elements of a ring with `lcm` or

- `a` - a list or tuple of elements of a ring with `lcm`

OUTPUT:

First, the given elements are coerced into a common parent. Then, their least common multiple *in that parent* is returned.

EXAMPLES:

```
sage: lcm(97, 100)
9700
sage: LCM(97, 100)
9700
sage: LCM(0, 2)
0
sage: LCM(-3, -5)
15
sage: LCM([1, 2, 3, 4, 5])
60
sage: v = LCM(range(1, 10000))    # *very* fast!
sage: len(str(v))
4349
```

TESTS:

The following tests against a bug that was fixed in [trac ticket #10771](#):

```
sage: lcm(4/1, 2)
4
```

The following shows that indeed coercion takes place before computing the least common multiple:

```
sage: R.<x>=QQ[]
sage: S.<x>=ZZ[]
sage: p = S.random_element(degree=(0, 5))
sage: q = R.random_element(degree=(0, 5))
sage: parent(lcm([1/p, q]))
Fraction Field of Univariate Polynomial Ring in x over Rational Field
```

Make sure we try QQ and not merely ZZ ([trac ticket #13014](#)):

```
sage: bool(lcm(2/5, 3/7) == lcm(SR(2/5), SR(3/7)))
True
```

Make sure that the `lcm` of Expressions stays symbolic:

```
sage: parent(lcm(2, 4))
Integer Ring
sage: parent(lcm(SR(2), 4))
Symbolic Ring
sage: parent(lcm(2, SR(4)))
Symbolic Ring
sage: parent(lcm(SR(2), SR(4)))
Symbolic Ring
```

Verify that objects without `lcm` methods but which can't be coerced to ZZ or QQ raise an error:

```
sage: F.<a,b> = FreeMonoid(2)
sage: lcm(a, b)
Traceback (most recent call last):
...
TypeError: unable to find lcm
```

Check rational and integers ([trac ticket #17852](#)):

```
sage: lcm(1/2, 4)
4
sage: lcm(4, 1/2)
4
```

`sage.rings.arith.legendre_symbol(x, p)`

The Legendre symbol $(x|p)$, for p prime.

Note: The `kronecker_symbol()` command extends the Legendre symbol to composite moduli and $p = 2$.

INPUT:

- x - integer
- p - an odd prime number

EXAMPLES:

```
sage: legendre_symbol(2, 3)
-1
sage: legendre_symbol(1, 3)
1
sage: legendre_symbol(1, 2)
Traceback (most recent call last):
...
ValueError: p must be odd
sage: legendre_symbol(2, 15)
Traceback (most recent call last):
...
ValueError: p must be a prime
sage: kronecker_symbol(2, 15)
1
sage: legendre_symbol(2/3, 7)
-1
```

`sage.rings.arith.mqrr_rational_reconstruction(u, m, T)`

Maximal Quotient Rational Reconstruction.

For research purposes only - this is pure Python, so slow.

INPUT:

- u, m, T - integers such that $m > u \geq 0, T > 0$.

OUTPUT:

Either integers n, d such that $d > 0$, $\gcd(n, d) = 1$, $n/d = u \bmod m$, and $T \cdot d \cdot |n| < m$, or None.

Reference: Monagan, Maximal Quotient Rational Reconstruction: An Almost Optimal Algorithm for Rational Reconstruction (page 11)

This algorithm is probabilistic.

EXAMPLES:

```
sage: mqrr_rational_reconstruction(21, 3100, 13)
(21, 1)
```

`sage.rings.arith.multinomial(*ks)`

Return the multinomial coefficient

INPUT:

- An arbitrary number of integer arguments k_1, \dots, k_n
- A list of integers $[k_1, \dots, k_n]$

OUTPUT:

Returns the integer:

$$\binom{k_1 + \dots + k_n}{k_1, \dots, k_n} = \frac{(\sum_{i=1}^n k_i)!}{\prod_{i=1}^n k_i!} = \prod_{i=1}^n \binom{\sum_{j=1}^i k_j}{k_i}$$

EXAMPLES:

```
sage: multinomial(0, 0, 2, 1, 0, 0)
3
sage: multinomial([0, 0, 2, 1, 0, 0])
3
sage: multinomial(3, 2)
10
sage: multinomial(2^30, 2, 1)
618970023101454657175683075
sage: multinomial([2^30, 2, 1])
618970023101454657175683075
```

AUTHORS:

- Gabriel Ebner

`sage.rings.arith.multinomial_coefficients(m, n)`

Return a dictionary containing pairs $\{(k_1, k_2, \dots, k_m) : C_{k,n}\}$ where $C_{k,n}$ are multinomial coefficients such that $n = k_1 + k_2 + \dots + k_m$.

INPUT:

- m - integer
- n - integer

OUTPUT: dict

EXAMPLES:

```
sage: sorted(multinomial_coefficients(2, 5).items())
[((0, 5), 1), ((1, 4), 5), ((2, 3), 10), ((3, 2), 10), ((4, 1), 5), ((5, 0), 1)]
```

Notice that these are the coefficients of $(x + y)^5$:

```
sage: R.<x,y> = QQ[]
sage: (x+y)^5
x^5 + 5*x^4*y + 10*x^3*y^2 + 10*x^2*y^3 + 5*x*y^4 + y^5
```

```
sage: sorted(multinomial_coefficients(3, 2).items())
[((0, 0, 2), 1), ((0, 1, 1), 2), ((0, 2, 0), 1), ((1, 0, 1), 2), ((1, 1, 0), 2), ((2, 0, 0), 1)]
```

ALGORITHM: The algorithm we implement for computing the multinomial coefficients is based on the following result:

..math:

$$\binom{n}{k_1, \dots, k_m} = \frac{\binom{n}{k_1+1} \sum_{i=2}^m \binom{n}{k_1+1, \dots, k_{i-1}, \dots}}$$

e.g.:

```
sage: k = (2, 4, 1, 0, 2, 6, 0, 0, 3, 5, 7, 1) # random value
sage: n = sum(k)
sage: s = 0
sage: for i in range(1, len(k)):
...     ki = list(k)
...     ki[0] += 1
...     ki[i] -= 1
...     s += multinomial(n, *ki)
sage: multinomial(n, *k) == (k[0] + 1) / (n - k[0]) * s
True
```

TESTS:

```
sage: multinomial_coefficients(0, 0)
{(): 1}
sage: multinomial_coefficients(0, 3)
{}
```

`sage.rings.arith.next_prime(n, proof=None)`

The next prime greater than the integer `n`. If `n` is prime, then this function does not return `n`, but the next prime after `n`. If the optional argument `proof` is `False`, this function only returns a pseudo-prime, as defined by the PARI `nextprime` function. If it is `None`, uses the global default (see `sage.structure.proof.proof`)

INPUT:

- `n` - integer
- `proof` - bool or `None` (default: `None`)

EXAMPLES:

```
sage: next_prime(-100)
2
sage: next_prime(1)
2
sage: next_prime(2)
3
sage: next_prime(3)
5
sage: next_prime(4)
5
```

Notice that the `next_prime(5)` is not 5 but 7.

```
sage: next_prime(5)
7
sage: next_prime(2004)
2011
```

`sage.rings.arith.next_prime_power(n)`

Return the smallest prime power greater than `n`.

Note that if `n` is a prime power, then this function does not return `n`, but the next prime power after `n`.

This function just calls the method `Integer.next_prime_power()` of `Integers`.

See also:

- `is_prime_power()` (and `Integer.is_prime_power()`)
- `previous_prime_power()` (and `Integer.previous_prime_power()`)

EXAMPLES:

```
sage: next_prime_power(1)
2
sage: next_prime_power(2)
3
sage: next_prime_power(10)
11
sage: next_prime_power(7)
8
sage: next_prime_power(99)
101
```

The same results can be obtained with:

```
sage: 1.next_prime_power()
2
sage: 2.next_prime_power()
3
sage: 10.next_prime_power()
11
```

Note that 2 is the smallest prime power:

```
sage: next_prime_power(-10)
2
sage: next_prime_power(0)
2
```

`sage.rings.arith.next_probable_prime(n)`

Returns the next probable prime after self, as determined by PARI.

INPUT:

- n - an integer

EXAMPLES:

```
sage: next_probable_prime(-100)
2
sage: next_probable_prime(19)
23
sage: next_probable_prime(int(99999999))
1000000007
sage: next_probable_prime(2^768)
155251809230070893514897948846250255525688601711669661113905203802605095268637688633087840882864
```

`sage.rings.arith.nth_prime(n)`

Return the n -th prime number (1-indexed, so that 2 is the 1st prime.)

INPUT:

- n - a positive integer

OUTPUT:

- the n -th prime number

EXAMPLES:

```
sage: nth_prime(3)
5
sage: nth_prime(10)
29
```

```
sage: nth_prime(0)
Traceback (most recent call last):
...
ValueError: nth prime meaningless for non-positive n (=0)
```

TESTS:

```
sage: all(prime_pi(nth_prime(j)) == j for j in range(1, 1000, 10))
True
```

`sage.rings.arith.number_of_divisors(n)`

Return the number of divisors of the integer n .

INPUT:

- n - a nonzero integer

OUTPUT:

- an integer, the number of divisors of n

EXAMPLES:

```
sage: number_of_divisors(100)
9
sage: number_of_divisors(-720)
30
```

`sage.rings.arith.odd_part(n)`

The odd part of the integer n . This is $n/2^v$, where $v = \text{valuation}(n, 2)$.

EXAMPLES:

```
sage: odd_part(5)
5
sage: odd_part(4)
1
sage: odd_part(factorial(31))
122529844256906551386796875
```

`sage.rings.arith.power_mod(a, n, m)`

The n -th power of a modulo the integer m .

EXAMPLES:

```
sage: power_mod(0, 0, 5)
Traceback (most recent call last):
...
ArithmeticError: 0^0 is undefined.
sage: power_mod(2, 390, 391)
285
sage: power_mod(2, -1, 7)
4
sage: power_mod(11, 1, 7)
4
sage: R.<x> = ZZ[]
sage: power_mod(3*x, 10, 7)
4*x^10

sage: power_mod(11, 1, 0)
Traceback (most recent call last):
```



```
...
ZeroDivisionError: modulus must be nonzero.
```

`sage.rings.arith.previous_prime(n)`

The largest prime $< n$. The result is provably correct. If $n \leq 1$, this function raises a `ValueError`.

EXAMPLES:

```
sage: previous_prime(10)
7
sage: previous_prime(7)
5
sage: previous_prime(8)
7
sage: previous_prime(7)
5
sage: previous_prime(5)
3
sage: previous_prime(3)
2
sage: previous_prime(2)
Traceback (most recent call last):
...
ValueError: no previous prime
sage: previous_prime(1)
Traceback (most recent call last):
...
ValueError: no previous prime
sage: previous_prime(-20)
Traceback (most recent call last):
...
ValueError: no previous prime
```

`sage.rings.arith.previous_prime_power(n)`

Return the largest prime power smaller than n .

The result is provably correct. If n is smaller or equal than 2 this function raises an error.

This function simply call the method `Integer.previous_prime_power()` of `Integers`.

See also:

- `is_prime_power()` (and `Integer.is_prime_power()`)
- `next_prime_power()` (and `Integer.next_prime_power()`)

EXAMPLES:

```
sage: previous_prime_power(3)
2
sage: previous_prime_power(10)
9
sage: previous_prime_power(7)
5
sage: previous_prime_power(127)
125
```

The same results can be obtained with:

```
sage: 3.previous_prime_power()
2
```

```
sage: 10.previous_prime_power()
9
sage: 7.previous_prime_power()
5
sage: 127.previous_prime_power()
125
```

Input less than or equal to 2 raises errors:

```
sage: previous_prime_power(2)
Traceback (most recent call last):
...
ValueError: no prime power less than 2
sage: previous_prime_power(-10)
Traceback (most recent call last):
...
ValueError: no prime power less than 2

sage: n = previous_prime_power(2^16 - 1)
sage: while is_prime(n):
....:     n = previous_prime_power(n)
sage: factor(n)
251^2
```

`sage.rings.arith.prime_divisors(n)`

The prime divisors of n .

INPUT:

• n – any object which can be factored

OUTPUT:

A list of prime factors of n . For integers, this list is sorted in increasing order.

EXAMPLES:

```
sage: prime_divisors(1)
[]
sage: prime_divisors(100)
[2, 5]
sage: prime_divisors(2004)
[2, 3, 167]
```

If n is negative, we do *not* include -1 among the prime divisors, since -1 is not a prime number:

```
sage: prime_divisors(-100)
[2, 5]
```

For polynomials we get all irreducible factors:

```
sage: R.<x> = PolynomialRing(QQ)
sage: prime_divisors(x^12 - 1)
[x - 1, x + 1, x^2 - x + 1, x^2 + 1, x^2 + x + 1, x^4 - x^2 + 1]
```

`sage.rings.arith.prime_factors(n)`

The prime divisors of n .

INPUT:

• n – any object which can be factored

OUTPUT:

A list of prime factors of n . For integers, this list is sorted in increasing order.

EXAMPLES:

```
sage: prime_divisors(1)
[]
sage: prime_divisors(100)
[2, 5]
sage: prime_divisors(2004)
[2, 3, 167]
```

If n is negative, we do *not* include -1 among the prime divisors, since -1 is not a prime number:

```
sage: prime_divisors(-100)
[2, 5]
```

For polynomials we get all irreducible factors:

```
sage: R.<x> = PolynomialRing(QQ)
sage: prime_divisors(x^12 - 1)
[x - 1, x + 1, x^2 - x + 1, x^2 + 1, x^2 + x + 1, x^4 - x^2 + 1]
```

`sage.rings.arith.prime_powers(start, stop=None)`

List of all positive primes powers between `start` and `stop-1`, inclusive. If the second argument is omitted, returns the prime powers up to the first argument.

INPUT:

- `start` - an integer. If two inputs are given, a lower bound for the returned set of prime powers. If this is the only input, then it is an upper bound.
- `stop` - an integer (default: None). An upper bound for the returned set of prime powers.

OUTPUT:

The set of all prime powers between `start` and `stop` or, if only one argument is passed, the set of all prime powers between 1 and `start`. The number n is a prime power if $n = p^k$, where p is a prime number and k is a positive integer. Thus, 1 is not a prime power.

EXAMPLES:

```
sage: prime_powers(20)
[2, 3, 4, 5, 7, 8, 9, 11, 13, 16, 17, 19]
sage: len(prime_powers(1000))
193
sage: len(prime_range(1000))
168

sage: a = [z for z in range(95,1234) if is_prime_power(z)]
sage: b = prime_powers(95,1234)
sage: len(b)
194
sage: len(a)
194
sage: a[:10]
[97, 101, 103, 107, 109, 113, 121, 125, 127, 128]
sage: b[:10]
[97, 101, 103, 107, 109, 113, 121, 125, 127, 128]
sage: a == b
True
```

```
sage: prime_powers(100) == [i for i in range(100) if is_prime_power(i)]
True

sage: prime_powers(10, 7)
[]
sage: prime_powers(-5)
[]
sage: prime_powers(-1, 3)
[2]
```

TESTS:

Check that output are always Sage integers (trac ticket #922):

```
sage: v = prime_powers(10)
sage: type(v[0])
<type 'sage.rings.integer.Integer'>

sage: prime_powers(0, 1)
[]
sage: prime_powers(2)
[]
sage: prime_powers(3)
[2]

sage: prime_powers("foo")
Traceback (most recent call last):
...
TypeError: unable to convert 'foo' to an integer

sage: prime_powers(6, "bar")
Traceback (most recent call last):
...
TypeError: unable to convert 'bar' to an integer
```

Check that long input are accepted (trac ticket #17852):

```
sage: prime_powers(61)
[2, 3, 4, 5]
sage: prime_powers(61, 101)
[7, 8, 9]
```

`sage.rings.arith.prime_to_m_part(n, m)`

Returns the prime-to-m part of n, i.e., the largest divisor of n that is coprime to m.

INPUT:

- n - Integer (nonzero)
- m - Integer

OUTPUT: Integer

EXAMPLES:

```
sage: 240.prime_to_m_part(2)
15
sage: 240.prime_to_m_part(3)
80
sage: 240.prime_to_m_part(5)
48
```

```
sage.rings.arith.primes (start, stop=None, proof=None)
```

This command is like the `xrange` command, except it only iterates over primes. In some cases it is better to use `primes` than `prime_range`, because `primes` does not build a list of all primes in the range in memory all at once. However, it is potentially much slower since it simply calls the `next_prime()` function repeatedly, and `next_prime()` is slow.

- `start` - an integer - lower bound for the primes
- `stop` - an integer (or infinity) optional argument - giving upper (open) bound for the primes
- `proof` - bool or None (default: None) If True, the function yields only proven primes. If False, the function uses a pseudo-primality test, which is much faster for really big numbers but does not provide a proof of primality. If None, uses the global default (see `sage.structure.proof.proof`)

- an iterator over primes from start to stop-1, inclusive

[illegible]

```
sage: for a in range(-10, 50):
...     for b in range(-10, 50):
...         assert list(primes(a,b)) == list(filter(is_prime, xrange(a,b)))
...
sage: sum(primes(-10, 9973, proof=False)) == sum(filter(is_prime, range(-10, 9973)))
True
sage: for p in primes(10, infinity):
...     if p > 20: break
...     print p
...
11
13
17
19
sage: next(p for p in primes(10,oo)) # checks alternate infinity notation
11
```

`sage.rings.arith.primes_first_n(n, leave_pari=False)`

Return the first n primes.

INPUT:

- n - a nonnegative integer

OUTPUT:

- a list of the first n prime numbers.

EXAMPLES:

```
sage: primes_first_n(10)
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
sage: len(primes_first_n(1000))
1000
sage: primes_first_n(0)
[]
```

`sage.rings.arith.primitive_root(n, check=True)`

Return a positive integer that generates the multiplicative group of integers modulo n , if one exists; otherwise, raise a `ValueError`.

A primitive root exists if $n = 4$ or $n = p^k$ or $n = 2p^k$, where p is an odd prime and k is a nonnegative number.

INPUT:

- n - a non-zero integer
- `check` - bool (default: `True`); if `False`, then n is assumed to be a positive integer possessing a primitive root, and behavior is undefined otherwise.

OUTPUT:

A primitive root of n . If n is prime, this is the smallest primitive root.

EXAMPLES:

```
sage: primitive_root(23)
5
sage: primitive_root(-46)
5
sage: primitive_root(25)
2
sage: print [primitive_root(p) for p in primes(100)]
[1, 2, 2, 3, 2, 2, 3, 2, 5, 2, 3, 2, 6, 3, 5, 2, 2, 2, 2, 7, 5, 3, 2, 3, 5]
sage: primitive_root(8)
Traceback (most recent call last):
...
ValueError: no primitive root
```

Note: It takes extra work to check if n has a primitive root; to avoid this, use `check=False`, which may slightly speed things up (but could also result in undefined behavior). For example, the second call below is an order of magnitude faster than the first:

```
sage: n = 10^50 + 151    # a prime
sage: primitive_root(n)
11
sage: primitive_root(n, check=False)
11
```

TESTS:

Various special cases:

```
sage: primitive_root(-1)
0
sage: primitive_root(0)
Traceback (most recent call last):
...
ValueError: no primitive root
sage: primitive_root(1)
0
sage: primitive_root(2)
1
sage: primitive_root(3)
2
sage: primitive_root(4)
3
```

We test that various numbers without primitive roots give an error - see [trac ticket #10836](#):

```
sage: primitive_root(15)
Traceback (most recent call last):
...
ValueError: no primitive root
sage: primitive_root(16)
Traceback (most recent call last):
...
ValueError: no primitive root
sage: primitive_root(1729)
Traceback (most recent call last):
...
ValueError: no primitive root
sage: primitive_root(4*7^8)
Traceback (most recent call last):
...
ValueError: no primitive root
```

`sage.rings.arith.quadratic_residues(n)`

Return a sorted list of all squares modulo the integer n in the range $0 \leq x < |n|$.

EXAMPLES:

```
sage: quadratic_residues(11)
[0, 1, 3, 4, 5, 9]
sage: quadratic_residues(1)
[0]
sage: quadratic_residues(2)
[0, 1]
sage: quadratic_residues(8)
[0, 1, 4]
sage: quadratic_residues(-10)
[0, 1, 4, 5, 6, 9]
sage: v = quadratic_residues(1000); len(v);
159
```

`sage.rings.arith.radical(n, *args, **kws)`

Return the product of the prime divisors of n .

This calls `n.radical(*args, **kwargs)`. If that doesn't work, it does `n.factor(*args, **kwargs)` and returns the product of the prime factors in the resulting factorization.

EXAMPLES:

```
sage: radical(2 * 3^2 * 5^5)
30
sage: radical(0)
Traceback (most recent call last):
...
ArithmeticError: Radical of 0 not defined.
sage: K.<i> = QuadraticField(-1)
sage: radical(K(2))
i + 1
```

The next example shows how to compute the radical of a number, assuming no prime > 100000 has exponent > 1 in the factorization:

```
sage: n = 2^1000-1; n / radical(n, limit=100000)
125
```

`sage.rings.arith.random_prime(n, proof=None, lbound=2)`

Returns a random prime p between $lbound$ and n (i.e. $lbound \leq p \leq n$). The returned prime is chosen uniformly at random from the set of prime numbers less than or equal to n .

INPUT:

- n - an integer ≥ 2 .
- `proof` - bool or None (default: None) If False, the function uses a pseudo-primality test, which is much faster for really big numbers but does not provide a proof of primality. If None, uses the global default (see `sage.structure.proof.proof`)
- `lbound` - an integer ≥ 2 lower bound for the chosen primes

EXAMPLES:

```
sage: random_prime(100000)
88237
sage: random_prime(2)
2
```

Here we generate a random prime between 100 and 200:

```
sage: random_prime(200, lbound=100)
149
```

If all we care about is finding a pseudo prime, then we can pass in `proof=False`

```
sage: random_prime(200, proof=False, lbound=100)
149
```

TESTS:

```
sage: type(random_prime(2))
<type 'sage.rings.integer.Integer'>
sage: type(random_prime(100))
<type 'sage.rings.integer.Integer'>
sage: random_prime(1, lbound=-2) #caused Sage hang #10112
Traceback (most recent call last):
...
ValueError: n must be greater than or equal to 2
sage: random_prime(126, lbound=114)
```



```
Traceback (most recent call last):
...
ValueError: There are no primes between 114 and 126 (inclusive)
```

AUTHORS:

- Jon Hanke (2006-08-08): with standard Stein cleanup
- Jonathan Bober (2007-03-17)

```
sage.rings.arith.rational_reconstruction(a, m, algorithm='fast')
```

This function tries to compute x/y , where x/y is a rational number in lowest terms such that the reduction of x/y modulo m is equal to a and the absolute values of x and y are both $\leq \sqrt{m/2}$. If such x/y exists, that pair is unique and this function returns it. If no such pair exists, this function raises `ZeroDivisionError`.

An efficient algorithm for computing rational reconstruction is very similar to the extended Euclidean algorithm. For more details, see Knuth, Vol 2, 3rd ed, pages 656-657.

INPUT:

- `a` – an integer
- `m` – a modulus
- `algorithm` – (default: 'fast')
- 'fast' – a fast implementation using direct MPIR calls in Cython.

OUTPUT:

Numerator and denominator n, d of the unique rational number $r = n/d$, if it exists, with n and $|d| \leq \sqrt{N/2}$. Return $(0, 0)$ if no such number exists.

The algorithm for rational reconstruction is described (with a complete nontrivial proof) on pages 656-657 of Knuth, Vol 2, 3rd ed. as the solution to exercise 51 on page 379. See in particular the conclusion paragraph right in the middle of page 657, which describes the algorithm thus:

This discussion proves that the problem can be solved efficiently by applying Algorithm 4.5.2X with $u = m$ and $v = a$, but with the following replacement for step X2: If $v_3 \leq \sqrt{m/2}$, the algorithm terminates. The pair $(x, y) = (|v_2|, v_3 * \text{sign}(v_2))$ is then the unique solution, provided that x and y are coprime and $x \leq \sqrt{m/2}$; otherwise there is no solution. (Alg 4.5.2X is the extended Euclidean algorithm.)

Knuth remarks that this algorithm is due to Wang, Kornerup, and Gregory from around 1983.

EXAMPLES:

```
sage: m = 100000
sage: (119*inverse_mod(53,m)) %m
11323
sage: rational_reconstruction(11323,m)
119/53

sage: rational_reconstruction(400,1000)
Traceback (most recent call last):
...
ArithmeticError: rational reconstruction of 400 (mod 1000) does not exist

sage: rational_reconstruction(3, 292393)
3
sage: a = Integers(292393)(45/97); a
204977
sage: rational_reconstruction(a, 292393, algorithm='fast')
```

```
45/97
sage: rational_reconstruction(293048, 292393)
Traceback (most recent call last):
...
ArithmeticError: rational reconstruction of 655 (mod 292393) does not exist
sage: rational_reconstruction(0, 0)
Traceback (most recent call last):
...
ZeroDivisionError: rational reconstruction with zero modulus
sage: rational_reconstruction(0, 1, algorithm="foobar")
Traceback (most recent call last):
...
ValueError: unknown algorithm 'foobar'
```

sage.rings.arith.**rising_factorial**(x, a)

Returns the rising factorial $(x)^a$.

The notation in the literature is a mess: often $(x)^a$, but there are many other notations: GKP: Concrete Mathematics uses $x^{\overline{a}}$.

The rising factorial is also known as the Pochhammer symbol, see Maple and Mathematica.

Definition: for integer $a \geq 0$ we have $x(x+1) \cdots (x+a-1)$. In all other cases we use the GAMMA-function: $\frac{\Gamma(x+a)}{\Gamma(x)}$.

INPUT:

- x - element of a ring
- a - a non-negative integer or
- x and a - any numbers

OUTPUT: the rising factorial

EXAMPLES:

```
sage: rising_factorial(10,3)
1320

sage: rising_factorial(10,RR('3.0'))
1320.000000000000

sage: rising_factorial(10,RR('3.3'))
2826.38895824964

sage: a = rising_factorial(1+I, I); a
gamma(2*I + 1)/gamma(I + 1)
sage: CC(a)
0.266816390637832 + 0.122783354006372*I

sage: a = rising_factorial(I, 4); a
-10
```

See `falling_factorial(I, 4)`.

```
sage: x = polygen(ZZ)
sage: rising_factorial(x, 4)
x^4 + 6*x^3 + 11*x^2 + 6*x
```

TESTS:

Check that trac ticket #14858 is fixed:

```
sage: bool(rising_factorial(-4, 2) ==
....:      rising_factorial(-4, SR(2)) ==
....:      rising_factorial(SR(-4), SR(2)))
True
```

Check that trac ticket #16770 is fixed:

```
sage: d = var('d')
sage: type(rising_factorial(d, 0))
<type 'sage.symbolic.expression.Expression'>
```

AUTHORS:

- Jaap Spies (2006-03-05)

`sage.rings.arith.sort_complex_numbers_for_display(nums)`

Given a list of complex numbers (or a list of tuples, where the first element of each tuple is a complex number), we sort the list in a “pretty” order. First come the real numbers (with zero imaginary part), then the complex numbers sorted according to their real part. If two complex numbers have a real part which is sufficiently close, then they are sorted according to their imaginary part.

This is not a useful function mathematically (not least because there’s no principled way to determine whether the real components should be treated as equal or not). It is called by various polynomial root-finders; its purpose is to make doctest printing more reproducible.

We deliberately choose a cumbersome name for this function to discourage use, since it is mathematically meaningless.

EXAMPLES:

```
sage: import sage.rings.arith
sage: sort_c = sort_complex_numbers_for_display
sage: nums = [CDF(i) for i in range(3)]
sage: for i in range(3):
....:     nums.append(CDF(i + RDF.random_element(-3e-11, 3e-11),
....:                    RDF.random_element()))
....:     nums.append(CDF(i + RDF.random_element(-3e-11, 3e-11),
....:                    RDF.random_element()))
sage: shuffle(nums)
sage: sort_c(nums)
[0.0, 1.0, 2.0, -2.862406201002009e-11 - 0.7088740263015161*I, 2.2108362706985576e-11 - 0.436810
```

`sage.rings.arith.squarefree_divisors(x)`

Iterator over the squarefree divisors (up to units) of the element x.

Depends on the output of the `prime_divisors` function.

INPUT:

- x – an element of any ring for which the `prime_divisors` function works.

EXAMPLES:

```
sage: list(squarefree_divisors(7))
[1, 7]
sage: list(squarefree_divisors(6))
[1, 2, 3, 6]
sage: list(squarefree_divisors(12))
[1, 2, 3, 6]
```

TESTS:

Check that the first divisor (i.e. 1) is a Sage integer (see [trac ticket #17852](#)):

```
sage: a = next(squarefree_divisors(14))
sage: a
1
sage: type(a)
<type 'sage.rings.integer.Integer'>
```

`sage.rings.arith.subfactorial(n)`

Subfactorial or rencontres numbers, or derangements: number of permutations of n elements with no fixed points.

INPUT:

- n - non negative integer

OUTPUT:

- integer - function value

EXAMPLES:

```
sage: subfactorial(0)
1
sage: subfactorial(1)
0
sage: subfactorial(8)
14833
```

AUTHORS:

- Jaap Spies (2007-01-23)

`sage.rings.arith.sum_of_k_squares(k, n)`

Write the integer n as a sum of k integer squares if possible; otherwise raise a `ValueError`.

INPUT:

- k – a non-negative integer
- n – an integer

OUTPUT: a tuple (x_1, \dots, x_k) of non-negative integers such that their squares sum to n .

EXAMPLES:

```
sage: sum_of_k_squares(2, 9634)
(15, 97)
sage: sum_of_k_squares(3, 9634)
(0, 15, 97)
sage: sum_of_k_squares(4, 9634)
(1, 2, 5, 98)
sage: sum_of_k_squares(5, 9634)
(0, 1, 2, 5, 98)
sage: sum_of_k_squares(6, 11^1111-1)
(19215400822645944253860920437586326284, 37204645194585992174252915693267578306, 347365481947739
sage: sum_of_k_squares(7, 0)
(0, 0, 0, 0, 0, 0, 0)
sage: sum_of_k_squares(30, 999999)
(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 3, 7, 44, 999)
sage: sum_of_k_squares(1, 9)
(3,)
sage: sum_of_k_squares(1, 10)
Traceback (most recent call last):
```

```

...
ValueError: 10 is not a sum of 1 square
sage: sum_of_k_squares(1, -10)
Traceback (most recent call last):
...
ValueError: -10 is not a sum of 1 square
sage: sum_of_k_squares(0, 9)
Traceback (most recent call last):
...
ValueError: 9 is not a sum of 0 squares
sage: sum_of_k_squares(0, 0)
()
sage: sum_of_k_squares(7, -1)
Traceback (most recent call last):
...
ValueError: -1 is not a sum of 7 squares
sage: sum_of_k_squares(-1, 0)
Traceback (most recent call last):
...
ValueError: k = -1 must be non-negative

```

sage.rings.arith.**three_squares**(*n*)

Write the integer *n* as a sum of three integer squares if possible; otherwise raise a `ValueError`.

INPUT:

- *n* – an integer

OUTPUT: a tuple (a, b, c) of non-negative integers such that $n = a^2 + b^2 + c^2$ with $a \leq b \leq c$.

EXAMPLES:

```

sage: three_squares(389)
(1, 8, 18)
sage: three_squares(946)
(9, 9, 28)
sage: three_squares(2986)
(3, 24, 49)
sage: three_squares(7^100)
(0, 0, 1798465042647412146620280340569649349251249)
sage: three_squares(11^111-1)
(616274160655975340150706442680, 901582938385735143295060746161, 6270382387635744140394001363065)
sage: three_squares(7 * 2^41)
(1048576, 2097152, 3145728)
sage: three_squares(7 * 2^42)
Traceback (most recent call last):
...
ValueError: 30786325577728 is not a sum of 3 squares
sage: three_squares(0)
(0, 0, 0)
sage: three_squares(-1)
Traceback (most recent call last):
...
ValueError: -1 is not a sum of 3 squares

```

TESTS:

```

sage: for _ in xrange(100):
....:     a = ZZ.random_element(2**16, 2**20)
....:     b = ZZ.random_element(2**16, 2**20)
....:     c = ZZ.random_element(2**16, 2**20)

```

```
....: n = a**2 + b**2 + c**2
....: aa,bb,cc = three_squares(n)
....: assert aa**2 + bb**2 + cc**2 == n
```

ALGORITHM:

See <http://www.schorn.ch/howto.html>

```
sage.rings.arith.trial_division(n, bound=None)
```

Return the smallest prime divisor \leq bound of the positive integer n, or n if there is no such prime. If the optional argument bound is omitted, then bound \leq n.

INPUT:

- `n` - a positive integer
- `bound` - (optional) a positive integer

OUTPUT:

- `int` - a prime `p=bound` that divides `n`, or `n` if there is no such prime.

EXAMPLES:

```
sage: trial_division(15)
3
sage: trial_division(91)
7
sage: trial_division(11)
11
sage: trial_division(387833, 300)
387833
sage: # 300 is not big enough to split off a
sage: # factor, but 400 is.
sage: trial_division(387833, 400)
389
```

```
sage.rings.arith.two_squares(n)
```

Write the integer n as a sum of two integer squares if possible; otherwise raise a `ValueError`.

INPUT:

- n – an integer

OUTPUT: a tuple (a, b) of non-negative integers such that $n = a^2 + b^2$ with $a \leq b$.

EXAMPLES:

[illegible]

```
(0, 0)
sage: two_squares(-1)
Traceback (most recent call last):
...
ValueError: -1 is not a sum of 2 squares
```

TESTS:

```
sage: for _ in xrange(100):
....:     a = ZZ.random_element(2**16, 2**20)
....:     b = ZZ.random_element(2**16, 2**20)
....:     n = a**2 + b**2
....:     aa,bb = two_squares(n)
....:     assert aa**2 + bb**2 == n
```

ALGORITHM:

See <http://www.schorn.ch/howto.html>

`sage.rings.arith.valuation(m, *args, **kws)`

Return the valuation of `m`.

This function simply calls the `m.valuation()` method. See the documentation of `m.valuation()` for a more precise description.

Note that the use of this functions is discouraged as it is better to use `m.valuation()` directly.

Note: This is not always a valuation in the mathematical sense. For more information see: `sage.rings.finite_rings.integer_mod.IntegerMod_int.valuation`

EXAMPLES:

```
sage: valuation(512, 2)
9
sage: valuation(1, 2)
0
sage: valuation(5/9, 3)
-2
```

Valuation of 0 is defined, but valuation with respect to 0 is not:

```
sage: valuation(0, 7)
+Infinity
sage: valuation(3, 0)
Traceback (most recent call last):
...
ValueError: You can only compute the valuation with respect to a integer larger than 1.
```

Here are some other examples:

```
sage: valuation(100, 10)
2
sage: valuation(200, 10)
2
sage: valuation(243, 3)
5
sage: valuation(243*10007, 3)
5
sage: valuation(243*10007, 10007)
1
```

```
sage: y = QQ['y'].gen()
sage: valuation(y^3, y)
3
sage: x = QQ[['x']].gen()
sage: valuation((x^3-x^2)/(x-4))
2
sage: valuation(4r, 2r)
2
sage: valuation(1r, 1r)
Traceback (most recent call last):
...
ValueError: You can only compute the valuation with respect to a integer larger than 1.
```

`sage.rings.arith.xgcd(a, b)`
Return a triple (g, s, t) such that $g = s \cdot a + t \cdot b = \gcd(a, b)$.

Note: One exception is if a and b are not in a principal ideal domain (see [Wikipedia article Principal_ideal_domain](#)), e.g., they are both polynomials over the integers. Then this function can't in general return (g, s, t) as above, since they need not exist. Instead, over the integers, we first multiply g by a divisor of the resultant of a/g and b/g , up to sign.

INPUT:

- a, b - integers or more generally, element of a ring for which the `xcgd` make sense (e.g. a field or univariate polynomials).

OUTPUT:

- g, s, t - such that $g = s \cdot a + t \cdot b$

Note: There is no guarantee that the returned cofactors (s and t) are minimal.

EXAMPLES:

```
sage: xgcd(56, 44)
(4, 4, -5)
sage: 4*56 + (-5)*44
4

sage: g, a, b = xgcd(5/1, 7/1); g, a, b
(1, 3, -2)
sage: a*(5/1) + b*(7/1) == g
True

sage: x = polygen(QQ)
sage: xgcd(x^3 - 1, x^2 - 1)
(x - 1, 1, -x)

sage: K.<g> = NumberField(x^2-3)
sage: g.xgcd(g+2)
(1, 1/3*g, 0)

sage: R.<a,b> = K[]
sage: S.<y> = R.fraction_field()[]
sage: xgcd(y^2, a*y+b)
(1, a^2/b^2, ((-a)/b^2)*y + 1/b)
sage: xgcd((b+g)*y^2, (a+g)*y+b)
(1, (a^2 + (2*g)*a + 3)/(b^3 + (g)*b^2), ((-a + (-g))/b^2)*y + 1/b)
```


Here is an example of a `xgcd` for two polynomials over the integers, where the linear combination is not the gcd but the gcd multiplied by the resultant:

```
sage: R.<x> = ZZ[]
sage: gcd(2*x*(x-1), x^2)
x
sage: xgcd(2*x*(x-1), x^2)
(2*x, -1, 2)
sage: (2*(x-1)).resultant(x)
2
```

```
sage.rings.arith.xkcd(n='')
```

This function is similar to the `xgcd` function, but behaves in a completely different way.

INPUT:

- `n` - an integer (optional)

OUTPUT:

This function outputs nothing it just prints something. Note that this function does not feel itself at ease in a html deprived environment.

EXAMPLES:

```
sage: xkcd(353) # optional - internet
<html><font color='black'><h1>Python</h1>>> ")
sage: with atomic_write(target_file, append=True) as f, .....:         atomic_write(target_file, append=True) as g:
.....:     f.write("AAA"); f.close()
.....:     g.write("BBB"); g.close()
sage: open(target_file, "r").read()
'>>> AAA'

```

`sage.misc.temporary_file.delete_tmpfiles()`

Remove the directory SAGE_TMP.

TESTS:

This is automatically run when Sage exits, test this by running a separate session of Sage:

```

sage: from sage.tests.cmdline import test_executable
sage: child_SAGE_TMP, err, ret = test_executable(["sage", "-c", "print SAGE_TMP"])
sage: err, ret
('', 0)
sage: os.path.exists(child_SAGE_TMP) # indirect doctest
False

```

The parent directory should exist:

```

sage: parent_SAGE_TMP = os.path.normpath(child_SAGE_TMP + '/../')
sage: os.path.isdir(parent_SAGE_TMP)
True

```

`sage.misc.temporary_file.graphics_filename(ext='.png')`

Deprecated SageNB graphics filename

You should just use `tmp_filename()`.

When run from the Sage notebook, return the next available canonical filename for a plot/graphics file in the current working directory. Otherwise, return a temporary file inside `SAGE_TMP`.

INPUT:

- `ext` – (default: `".png"`) A file extension (including the dot) for the filename.

OUTPUT:

The path of the temporary file created. In the notebook, this is a filename without path in the current directory. Otherwise, this an absolute path.

EXAMPLES:

```
sage: from sage.misc.tmpdir import tmpdir
sage: print tmpdir() # random, typical filename for sagenb
sage0.png
```

TESTS:

When doctesting, this returns instead a random temporary file. We check that it's a file inside `SAGE_TMP` and that the extension is correct:

```
sage: fn = tmpdir(ext=".jpeg")
sage: fn.startswith(str(SAGE_TMP))
True
sage: fn.endswith('.jpeg')
True
```

Historically, it was also possible to omit the dot. This has been changed in [trac ticket #16640](#) but it will still work for now:

```
sage: fn = tmpdir("jpeg")
doctest:...: DeprecationWarning: extension must now include the dot
See http://trac.sagemath.org/16640 for details.
sage: fn.endswith('.jpeg')
True
```

```
sage.misc.tmpdir.tmpdir(name='dir_', ext='')
Create and return a temporary directory in $HOME/.sage/temp/hostname/pid/
```

The temporary directory is deleted automatically when Sage exits.

INPUT:

- `name` – (default: `"dir_"`) A prefix for the directory name.
- `ext` – (default: `" "`) A suffix for the directory name.

OUTPUT:

The absolute path of the temporary directory created, with a trailing slash (or whatever the path separator is on your OS).

EXAMPLES:

```
sage: d = tmpdir('dir_testing_', '.extension')
sage: d # random output
'/home/username/.sage/temp/hostname/7961/dir_testing_XgRu4p.extension/'
sage: os.chdir(d)
sage: _ = open('file_inside_d', 'w')
```

Temporary directories are inaccessible by other users:

```
sage: os.stat(d).st_mode & 0o077
0
```

`sage.misc.tmp_file.tmp_filename(name='tmp_', ext='')`
 Create and return a temporary file in `$HOME/.sage/temp/hostname/pid/`

The temporary file is deleted automatically when Sage exits.

Warning: If you need a particular file extension always use `tmp_filename(ext=".foo")`, this will ensure that the file does not yet exist. If you were to use `tmp_filename()+".foo"`, then you might overwrite an existing file!

INPUT:

- `name` – (default: `"tmp_"`) A prefix for the file name.
- `ext` – (default: `" "`) A suffix for the file name. If you want a filename extension in the usual sense, this should start with a dot.

OUTPUT:

The absolute path of the temporary file created.

EXAMPLES:

```
sage: fn = tmp_filename('just_for_testing_', '.extension')
sage: fn # random
'/home/username/.sage/temp/hostname/8044/just_for_testing_tVVHsn.extension'
sage: _ = open(fn, 'w')
```

Temporary files are unaccessible by other users:

```
sage: os.stat(fn).st_mode & 0o077
0
```

2.5.2 get_remote_file

`sage.misc.remote_file.get_remote_file(filename, verbose=True)`

INPUT:

- `filename` – the URL of a file on the web, e.g., `"http://modular.math.washington.edu/myfile.txt"`
- `verbose` – whether to display download status

OUTPUT:

creates a file in the temp directory and returns the absolute path to that file.

EXAMPLES:

```
sage: g = get_remote_file("http://sagemath.org/ack.html", verbose=False) # optional - internet
sage: len(open(g).read()) # optional - internet; random
10198
```

`sage.misc.remote_file.report_hook(block, size, total)`

2.5.3 Message delivery

Various interfaces to messaging services. Currently:

- `pushover` - a platform for sending and receiving push notifications

is supported.

AUTHORS:

- Martin Albrecht (2012) - initial implementation

`sage.misc.messaging.pushover` (*message*, ***kws*)

Send a push notification with *message* to user using <https://pushover.net/>.

Pushover is a platform for sending and receiving push notifications. On the server side, it provides an HTTP API for queueing messages to deliver to devices. On the device side, iOS and Android clients receive those push notifications, show them to the user, and store them for offline viewing.

An account on <https://pushover.net> is required and the Pushover app must be installed on your phone for this function to be able to deliver messages to you.

INPUT:

- *message* - your message
- *user* - the user key (not e-mail address) of your user (or you), viewable when logged into the Pushover dashboard. (default: `None`)
- *device* - your user's device identifier to send the message directly to that device, rather than all of the user's devices (default: `None`)
- *title* - your message's title, otherwise uses your app's name (default: `None`)
- *url* - a supplementary URL to show with your message (default: `None`)
- *url_title* - a title for your supplementary URL (default: `None`)
- *priority* - set to 1 to display as high-priority and bypass quiet hours, or -1 to always send as a quiet notification (default: 0)
- *timestamp* - set to a unix timestamp to have your message show with a particular time, rather than now (default: `None`)
- *sound* - set to the name of one of the sounds supported by device clients to override the user's default sound choice (default: `None`)
- *token* - your application's API token (default: Sage's default App token)

EXAMPLE:

```
sage: import sage.misc.messaging
sage: sage.misc.messaging.pushover("Hi, how are you?", user="XXX") # not tested
```

To set default values populate `pushover_defaults`:

```
sage: sage.misc.messaging.pushover_defaults["user"] = "USER_TOKEN"
sage: sage.misc.messaging.pushover("Hi, how are you?") # not tested
```

Note: You may want to populate `sage.misc.messaging.pushover_defaults` with default values such as the default user in `$HOME/.sage/init.sage`.

2.5.4 Miscellaneous operating system functions

`sage.misc.sage_ostools.have_program` (*program*, *path=None*)

Return `True` if a program executable is found in the path given by *path*.

INPUT:

- `program` - a string, the name of the program to check.
- `path` - string or None. Paths to search for program, separated by `os.pathsep`. If None, use the `PATH` environment variable.

OUTPUT: bool

EXAMPLES:

```
sage: from sage.misc.sage_ostools import have_program
sage: have_program('ls')
True
sage: have_program('there_is_not_a_program_with_this_name')
False
sage: have_program('sage', path=SAGE_ROOT)
True
sage: have_program('ls', path=SAGE_ROOT)
False
```

2.6 Database Access

2.6.1 Relational (sqlite) Databases Module

INFO:

This module implements classes (`SQLDatabase` and `SQLQuery` (pythonic implementation for the user with little or no knowledge of sqlite)) that wrap the basic functionality of sqlite.

Databases are constructed via a triple indexed dictionary called a skeleton. A skeleton should be constructed to fit the following format:

```
| - skeleton -- a triple-indexed dictionary
|   - outer key -- table name
|     - inner key -- column name
|       - inner inner key -- one of the following:
|         - ``primary_key`` -- boolean, whether column has been set as
|           primary key
|         - ``index`` -- boolean, whether column has been set as index
|         - ``unique`` -- boolean, whether column has been set as unique
|         - ``sql`` -- one of ``TEXT``, ``BOOLEAN``, ``INTEGER``,
|           ``REAL``, or other user defined type
```

An example skeleton of a database with one table, that table with one column:

```
{'table1': {'col1': {'primary_key': False, 'index': True, 'sql': 'REAL'}}
```

`SQLDatabases` can also be constructed via the `add`, `drop`, and `commit` functions. The `vacuum` function is also useful for restoring hard disk space after a database has shrunk in size.

A `SQLQuery` can be constructed by providing a `query_dict`, which is a dictionary with the following sample format:

```
{'table_name': 'tblname', 'display_cols': ['col1', 'col2', 'col3'], 'expression': [col, operator,
```

Finally a `SQLQuery` also allows the user to directly input the query string for a database, and also supports the `'?` syntax by allowing an argument for a tuple of parameters to query.

For full details, please see the tutorial. `sage.graphs.graph_database.py` is an example of implementing a database class in Sage using this interface.

AUTHORS:

- R. Andrew Ohana (2011-07-16): refactored and rewrote most of the code; merged the Generic classes into the non-Generic versions; changed the skeleton format to include a boolean indicating whether the column stores unique keys; changed the index names so as to avoid potential ambiguity
- Emily A. Kirkman (2008-09-20): added functionality to generate plots and reformat output in `show`
- Emily A. Kirkman and Robert L. Miller (2007-06-17): initial version

class `sage.databases.sql_db.SQLDatabase` (*filename=None, read_only=None, skeleton=None*)
Bases: `sage.structure.sage_object.SageObject`

A SQL Database object corresponding to a database file.

INPUT:

- `filename` – a string
- `skeleton` – a triple-indexed dictionary:

```
| - outer key -- table name
|   - inner key -- column name
|     - inner inner key -- one of the following:
|       - ``primary_key`` -- boolean, whether column has been set
|         as primary key
|       - ``index`` -- boolean, whether column has been set as
|         index
|       - ``unique`` -- boolean, whether column has been set as
|         unique
|       - ``sql`` -- one of ``TEXT``, ``BOOLEAN``,
|         ``INTEGER``, ``REAL``, or other user defined type
```

TUTORIAL:

The `SQLDatabase` class is for interactively building databases intended for queries. This may sound redundant, but it is important. If you want a database intended for quick lookup of entries in very large tables, much like a hash table (such as a Python dictionary), a `SQLDatabase` may not be what you are looking for. The strength of `SQLDatabases` is in queries, searches through the database with complicated criteria.

For example, we create a new database for storing isomorphism classes of simple graphs:

```
sage: D = SQLDatabase()
```

In order to generate representatives for the classes, we will import a function which generates all labeled graphs (noting that this is not the optimal way):

```
sage: from sage.groups.perm_gps.partn_ref.refinement_graphs import all_labeled_graphs
```

We will need a table in the database in which to store the graphs, and we specify its structure with a Python dictionary, each of whose keys is the name of a column:

```
sage: table_skeleton = {
...   'graph6': {'sql': 'TEXT', 'index': True, 'primary_key': True},
...   'vertices': {'sql': 'INTEGER'},
...   'edges': {'sql': 'INTEGER'}
... }
```


Then we create the table:

```
sage: D.create_table('simon', table_skeleton)
```

```
sage: D.show('simon')
```

edges	graph6	vertices

Now that we have the table, we will begin to populate the table with rows. First, add the graph on zero vertices.:

```
sage: G = Graph()
```

```
sage: D.add_row('simon', (0, G.graph6_string(), 0))
```

```
sage: D.show('simon')
```

edges	graph6	vertices

0	?	0

Next, add the graph on one vertex.:

```
sage: G.add_vertex()
```

```
0
```

```
sage: D.add_row('simon', (0, G.graph6_string(), 1))
```

```
sage: D.show('simon')
```

edges	graph6	vertices

0	?	0
0	@	1

Say we want a database of graphs on four or less vertices:

```
sage: labels = {}
```

```
sage: for i in range(2, 5):
```

```
...     labels[i] = []
```

```
...     for g in all_labeled_graphs(i):
```

```
...         g = g.canonical_label()
```

```
...         if g not in labels[i]:
```

```
...             labels[i].append(g)
```

```
...             D.add_row('simon', (g.size(), g.graph6_string(), g.order()))
```

```
sage: D.show('simon') # random
```

edges	graph6	vertices

0	?	0
0	@	1
0	A?	2
1	A_	2
0	B?	3
1	BG	3
2	BW	3
3	Bw	3
0	C?	4
1	C@	4
2	CB	4
3	CF	4
3	CJ	4
2	CK	4
3	CL	4
4	CN	4
4	C]	4
5	C^	4
6	C~	4

We can then query the database – let's ask for all the graphs on four vertices with three edges. We do so by

creating two queries and asking for rows that satisfy them both:

```
sage: Q = SQLQuery(D, {'table_name':'simon', 'display_cols':['graph6'], 'expression':['vertices']}
sage: Q2 = SQLQuery(D, {'table_name':'simon', 'display_cols':['graph6'], 'expression':['edges'],
sage: Q = Q.intersect(Q2)
sage: len(Q.query_results())
3
sage: Q.query_results() # random
[(u'CF', u'CF'), (u'CJ', u'CJ'), (u'CL', u'CL')]
```

NOTE: The values of `display_cols` are always concatenated in intersections and unions.

Of course, we can save the database to file:

```
sage: replace_with_your_own_filepath = tmp_dir()
sage: D.save(replace_with_your_own_filepath + 'simon.db')
```

Now the database's hard link is to this file, and not the temporary db file. For example, let's say we open the same file with another class instance. We can load the file as an immutable database:

```
sage: E = SQLDatabase(replace_with_your_own_filepath + 'simon.db')
sage: E.show('simon') # random
```

edges	graph6	vertices
0	?	0
0	@	1
0	A?	2
1	A_	2
0	B?	3
1	BG	3
2	BW	3
3	Bw	3
0	C?	4
1	C@	4
2	CB	4
3	CF	4
3	CJ	4
2	CK	4
3	CL	4
4	CN	4
4	C]	4
5	C^	4
6	C~	4

```
sage: E.drop_table('simon')
Traceback (most recent call last):
...
RuntimeError: Cannot drop tables from a read only database.
```

add_column (*table_name*, *col_name*, *col_dict*, *default*='NULL')

Add a column named *col_name* to table *table_name*, whose data types are described by *col_dict*.

The format for this is:

```
{'coll':{'primary_key':False, 'unique': True, 'index':True, 'sql':'REAL'}}
```

INPUT:

- *col_dict* – a dictionary:
 - key – column name
 - *inner key – one of the following:

- primary_key – boolean, whether column has been set as primary key
- index – boolean, whether column has been set as index
- unique – boolean, whether column has been set as unique
- sql – one of 'TEXT', 'BOOLEAN', 'INTEGER', 'REAL', or other user defined type

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}})
sage: for n in range(20): MonicPolys.add_row('simon', (n,))
sage: MonicPolys.add_column('simon', 'n_squared', {'sql':'INTEGER', 'index':False}, 0)
sage: MonicPolys.show('simon')
```

n_squared	n
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9
0	10
0	11
0	12
0	13
0	14
0	15
0	16
0	17
0	18
0	19

add_data (table_name, rows, entry_order=None)

INPUT:

- rows – a list of tuples that represent one row of data to add (types should match col types in order)
- entry_order – an ordered list or tuple overrides normal order with user defined order

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon', {'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_rows('simon', [(0,0), (1,1), (1,2)])
sage: DB.add_rows('simon', [(0,0), (4,0), (5,1)], ['b2', 'a1'])
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 0), (1, 1), (1, 2), (0, 0), (0, 4), (1, 5)]
```

add_row (table_name, values, entry_order=None)

Add the row described by values to the table table_name. Values should be a tuple, of same length and order as columns in given table.

NOTE:

If values is of length one, be sure to specify that it is a tuple of length one, by using a comma, e.g.:

```
sage: values = (6,)
```

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon', {'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon', (0,1))
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 1)]
```

add_rows (*table_name*, *rows*, *entry_order=None*)**INPUT:**

- *rows* – a list of tuples that represent one row of data to add (types should match col types in order)
- *entry_order* – an ordered list or tuple overrides normal order with user defined order

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon', {'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_rows('simon', [(0,0), (1,1), (1,2)])
sage: DB.add_rows('simon', [(0,0), (4,0), (5,1)], ['b2', 'a1'])
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 0), (1, 1), (1, 2), (0, 0), (0, 4), (1, 5)]
```

commit ()

Commits changes to file.

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon', {'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon', (0,1))
sage: DB.add_data('simon', [(0,0), (1,1), (1,2)])
sage: DB.add_data('simon', [(0,0), (4,0), (5,1)], ['b2', 'a1'])
sage: DB.drop_column('simon', 'b2')
sage: DB.commit()
sage: DB.vacuum()
```

create_table (*table_name*, *table_skeleton*)

Creates a new table in the database.

To create a table, a column structure must be specified. The form for this is a Python dict, for example:

```
{ 'coll': { 'sql': 'INTEGER', 'index': False, 'unique': True, 'primary_key': False }, ... }
```

INPUT:

- *table_name* – a string
- *table_skeleton* – a double-indexed dictionary
 - outer key – column name
 - inner key – one of the following:
 - *primary_key* – boolean, whether column has been set as primary key
 - *index* – boolean, whether column has been set as index

- unique – boolean, whether column has been set as unique
- sql – one of 'TEXT', 'BOOLEAN', 'INTEGER', 'REAL', or other user defined type

NOTE:

Some SQL features, such as automatically incrementing primary key, require the full word 'INTEGER', not just 'INT'.

EXAMPLES:

```
sage: D = SQLDatabase()
sage: table_skeleton = {
...   'graph6':{'sql':'TEXT', 'index':True, 'primary_key':True},
...   'vertices':{'sql':'INTEGER'},
...   'edges':{'sql':'INTEGER'}
... }
sage: D.create_table('simon', table_skeleton)
sage: D.show('simon')
```

edges	graph6	vertices

delete_rows (query)

Uses a SQLQuery instance to modify (delete rows from) the database.

SQLQuery must have no join statements. (As of now, you can only delete from one table at a time – ideas and patches are welcome).

To remove all data that satisfies a SQLQuery, send the query as an argument to delete_rows. Be careful, test your query first.

Recommended use: have some kind of row identification primary key column that you use as a parameter in the query. (See example below).

INPUT:

- query – a SQLQuery (Delete the rows returned when query is run).

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('lucy',{'id':{'sql':'INTEGER', 'primary_key':True, 'index':True}, 'a1':
sage: DB.add_rows('lucy', [(0,1,1), (1,1,4), (2,0,7), (3,1,384), (4,1,978932)], ['id','a1','b2'])
sage: DB.show('lucy')
```

a1	id	b2

1	0	1
1	1	4
0	2	7
1	3	384
1	4	978932

```
sage: Q = SQLQuery(DB, {'table_name':'lucy', 'display_cols':['id','a1','b2'], 'expression':
sage: DB.delete_rows(Q)
sage: DB.show('lucy')
```

a1	id	b2

1	0	1
1	1	4
0	2	7

drop_column (table_name, col_name)

Drop the column col_name from table table_name.

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}})
sage: for n in range(20): MonicPolys.add_row('simon', (n,))
sage: MonicPolys.add_column('simon', 'n_squared', {'sql':'INTEGER'}, 0)
sage: MonicPolys.drop_column('simon', 'n_squared')
sage: MonicPolys.show('simon')
```

```
n
-----
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
```

drop_data_from_table (*table_name*)

Removes all rows from *table_name*.

EXAMPLES:

```
sage: D = SQLDatabase()
sage: D.create_table('simon', {'coll':{'sql':'INTEGER'}})
sage: D.add_row('simon', (9,))
sage: D.show('simon')
```

```
coll
-----
9
```

```
sage: D.drop_data_from_table('simon')
sage: D.show('simon')
```

```
coll
-----
```

drop_index (*table_name*, *index_name*)

Set the column *index_name* in table *table_name* to not be an index. See `make_index()`

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2':{'sql':'INTEGE
sage: MonicPolys.drop_index('simon', 'n')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': False,
                'primary_key': False,
                'sql': 'INTEGER',
                'unique': False},
```

```

'n2': {'index': False,
      'primary_key': False,
      'sql': 'INTEGER',
      'unique': False}}}

```

drop_primary_key (*table_name*, *col_name*)

Set the column *col_name* in table *table_name* not to be a primary key.

A primary key is something like an index, but its main purpose is to link different tables together. This allows searches to be executed on multiple tables that represent maybe different data about the same objects.

NOTE:

This function only changes the column to be non-primary, it does not delete it.

EXAMPLES:

```

sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2':{'sql':'INTE
sage: MonicPolys.make_primary_key('simon', 'n2')
sage: MonicPolys.drop_primary_key('simon', 'n2')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
                'primary_key': False,
                'sql': 'INTEGER',
                'unique': False},
          'n2': {'index': False,
                'primary_key': False,
                'sql': 'INTEGER',
                'unique': True}}}}

```

drop_table (*table_name*)

Delete table *table_name* from database.

INPUT:

- *table_name* – a string

EXAMPLES:

```

sage: D = SQLDatabase()
sage: D.create_table('simon', {'coll':{'sql':'INTEGER'}})
sage: D.show('simon')
coll
-----
sage: D.drop_table('simon')
sage: D.get_skeleton()
{}

```

drop_unique (*table_name*, *col_name*)

Set the column *col_name* in table *table_name* not store unique values.

NOTE:

This function only removes the requirement for entries in *col_name* to be unique, it does not delete it.

EXAMPLES:

```

sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2':{'sql':'INTE
sage: MonicPolys.make_unique('simon', 'n2')

```

```
sage: MonicPolys.drop_unique('simon', 'n2')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
  'primary_key': False,
  'sql': 'INTEGER',
  'unique': False},
  'n2': {'index': False,
  'primary_key': False,
  'sql': 'INTEGER',
  'unique': False}}}
```

get_connection (*ignore_warning=None*)

Returns a pysqlite connection to the database.

You most likely want `get_cursor()` instead, which is used for executing sqlite commands on the database.

Recommended for more advanced users only.

EXAMPLES:

```
sage: D = SQLiteDatabase(read_only=True)
sage: con = D.get_connection()
doctest:...: RuntimeWarning: Database is read only, using the connection can alter the store
sage: con = D.get_connection(True)
sage: D.ignore_warnings = True
sage: con = D.get_connection()
sage: t = con.execute('CREATE TABLE simon(n INTEGER, n2 INTEGER)')
sage: for n in range(10):
...     t = con.execute('INSERT INTO simon VALUES(%d,%d)'%(n,n^2))
sage: D.show('simon')
n                n2
-----
0                0
1                1
2                4
3                9
4               16
5               25
6               36
7               49
8               64
9               81
```

get_cursor (*ignore_warning=None*)

Returns a pysqlite cursor for the database connection.

A cursor is an input from which you can execute sqlite commands on the database.

Recommended for more advanced users only.

EXAMPLES:

```
sage: DB = SQLiteDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon', (0,1))
sage: DB.add_rows('simon', [(0,0), (1,1), (1,2)])
sage: DB.add_rows('simon', [(0,0), (4,0), (5,1)], ['b2','a1'])
sage: cur = DB.get_cursor()
sage: (cur.execute('select * from simon')).fetchall()
[(0, 1), (0, 0), (1, 1), (1, 2), (0, 0), (0, 4), (1, 5)]
```


get_skeleton (*check=False*)

Returns a dictionary representing the hierarchical structure of the database, in the following format:

```
| - skeleton -- a triple-indexed dictionary
|   - outer key -- table name
|     - inner key -- column name
|       - inner inner key -- one of the following:
|         - ``primary_key`` -- boolean, whether column has been set as
|           primary key
|         - ``index`` -- boolean, whether column has been set as index
|         - ``unique`` -- boolean, whether column has been set as unique
|         - ``sql`` -- one of ``TEXT``, ``BOOLEAN``, ``INTEGER``,
|           ``REAL``, or other user defined type
```

For example:

```
{'table1':{'coll':{'primary_key':False, 'index':True, 'unique': False, 'sql':'REAL'}}}
```

INPUT:

- check – if True, checks to make sure the database’s actual structure matches the skeleton on record.

EXAMPLES:

```
sage: GDB = GraphDatabase()
sage: GDB.get_skeleton()           # slightly random output
{u'aut_grp': {u'aut_grp_size': {'index': True,
                                'unique': False,
                                'primary_key': False,
                                'sql': u'INTEGER'},
                                ...
                                u'num_vertices': {'index': True,
                                                  'unique': False,
                                                  'primary_key': False,
                                                  'sql': u'INTEGER'}}}}
```

make_index (*col_name, table_name, unique=False*)

Set the column *col_name* in table *table_name* to be an index, that is, a column set up to do quick searches on.

INPUT:

- col_name – a string
- table_name – a string
- unique – requires that there are no multiple entries in the column, makes searching faster

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2':{'sql':'IN
sage: MonicPolys.make_index('n2','simon')
sage: MonicPolys.get_skeleton()
{'simon': {'n': {'index': True,
                'primary_key': False,
                'sql': 'INTEGER',
                'unique': False},
            'n2': {'index': True,
                  'primary_key': False,
```

```
'sql': 'INTEGER',  
'unique': False}}}
```

make_primary_key (*table_name*, *col_name*)

Set the column *col_name* in table *table_name* to be a primary key.

A primary key is something like an index, but its main purpose is to link different tables together. This allows searches to be executed on multiple tables that represent maybe different data about the same objects.

NOTE:

Some SQL features, such as automatically incrementing primary key, require the full word 'INTEGER', not just 'INT'.

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()  
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2':{'sql':'IN  
sage: MonicPolys.make_primary_key('simon', 'n2')  
sage: MonicPolys.get_skeleton()  
{'simon': {'n': {'index': True,  
    'primary_key': False,  
    'sql': 'INTEGER',  
    'unique': False},  
  'n2': {'index': False,  
    'primary_key': True,  
    'sql': 'INTEGER',  
    'unique': True}}}}
```

make_unique (*table_name*, *col_name*)

Set the column *col_name* in table *table_name* to store unique values.

NOTE:

This function only adds the requirement for entries in *col_name* to be unique, it does not change the values.

EXAMPLES:

```
sage: MonicPolys = SQLDatabase()  
sage: MonicPolys.create_table('simon', {'n':{'sql':'INTEGER', 'index':True}, 'n2':{'sql':'IN  
sage: MonicPolys.make_unique('simon', 'n2')  
sage: MonicPolys.get_skeleton()  
{'simon': {'n': {'index': True,  
    'primary_key': False,  
    'sql': 'INTEGER',  
    'unique': False},  
  'n2': {'index': False,  
    'primary_key': False,  
    'sql': 'INTEGER',  
    'unique': True}}}}
```

query (*args, **kws)

Creates a SQLQuery on this database. For full class details, type SQLQuery? and press shift+enter.

EXAMPLES:

```
sage: D = SQLDatabase()  
sage: D.create_table('simon', {'wolf':{'sql':'BOOLEAN'}, 'tag':{'sql':'INTEGER'}})  
sage: q = D.query({'table_name':'simon', 'display_cols':['tag'], 'expression':['wolf','=',1]}
```

```

sage: q.get_query_string()
'SELECT simon.tag FROM simon WHERE simon.wolf = ?'
sage: q.__param_tuple__
('1',)
sage: q = D.query(query_string='SELECT tag FROM simon WHERE wolf=?',param_tuple=(1,))
sage: q.get_query_string()
'SELECT tag FROM simon WHERE wolf=?'
sage: q.__param_tuple__
('1',)

```

rename_table (*table_name*, *new_name*)

Renames the table *table_name* to *new_name*.

EXAMPLES:

```

sage: D = SQLiteDatabase()
sage: D.create_table('simon', {'coll': {'sql': 'INTEGER'}})
sage: D.show('simon')
coll
-----
sage: D.rename_table('simon', 'lucy')
sage: D.show('simon')
Traceback (most recent call last):
...
RuntimeError: Failure to fetch data.
sage: D.show('lucy')
coll
-----

```

save (*filename*)

Save the database to the specified location.

EXAMPLES:

```

sage: MonicPolys = SQLiteDatabase()
sage: MonicPolys.create_table('simon', {'n': {'sql': 'INTEGER', 'index': True}})
sage: for n in range(20): MonicPolys.add_row('simon', (n,))
sage: tmp = tmp_dir() # replace with your own file path
sage: MonicPolys.save(tmp+'sage.db')
sage: N = SQLiteDatabase(tmp+'sage.db')
sage: N.show('simon')
n
-----
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

```

```
17
18
19
```

show (*table_name*, ****kws**)

Show an entire table from the database.

EXAMPLES:

```
sage: DB = SQLiteDatabase()
sage: DB.create_table('simon', {'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_data('simon', [(0,0), (1,1), (1,2)])
sage: DB.show('simon')
a1                      b2
-----
0                          0
1                          1
1                          2
```

vacuum ()

Cleans the extra hard disk space used up by a database that has recently shrunk.

EXAMPLES:

```
sage: DB = SQLiteDatabase()
sage: DB.create_table('simon', {'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_row('simon', (0,1))
sage: DB.add_data('simon', [(0,0), (1,1), (1,2)])
sage: DB.add_data('simon', [(0,0), (4,0), (5,1)], ['b2', 'a1'])
sage: DB.drop_column('simon', 'b2')
sage: DB.commit()
sage: DB.vacuum()
```

class sage.databases.sql_db.**SQLQuery** (*database*, **args*, ****kws**)

Bases: sage.structure.sage_object.SageObject

A query for a SQLite database.

INPUT:

- *database* – a SQLiteDatabase object
- *query_dict* – a dictionary specifying the query itself. The format is:

```
{ 'table_name': 'tblname', 'display_cols': ['col1', 'col2', 'col3'], 'expression': [col, operator
```

NOTE: Every SQL type we are using is ultimately represented as a string, so if you wish to save actual strings to a database, you actually need to do something like: ““value””.

See the documentation of SQLiteDatabase for an introduction to using SQLite in Sage.

EXAMPLES:

```
sage: D = SQLiteDatabase()
sage: D.create_table('simon', {'a1':{'sql':'bool', 'primary_key':False}, 'b2':{'sql':'int'}})
sage: D.add_data('simon', [(0,0), (1,2), (2,4)])
sage: r = SQLQuery(D, { 'table_name': 'simon', 'display_cols': ['a1'], 'expression': ['b2', '<=', 3] })
sage: r.show()
a1
-----
```

0
1

get_query_string()

Returns a copy of the query string.

EXAMPLES:

```
sage: G = GraphDatabase()
sage: q = 'SELECT graph_id,graph6,num_vertices,num_edges FROM graph_data WHERE graph_id<=(?)'
sage: param = (22,5)
sage: SQLQuery(G,q,param).get_query_string()
'SELECT graph_id,graph6,num_vertices,num_edges FROM graph_data
WHERE graph_id<=(?) AND num_vertices=(?)'
sage: r = 'SELECT graph6 FROM graph_data WHERE num_vertices<=3'
sage: SQLQuery(G,r).get_query_string()
'SELECT graph6 FROM graph_data WHERE num_vertices<=3'
```

intersect (*other*, *join_table=None*, *join_dict=None*, *in_place=False*)

Returns a new SQLQuery that is the intersection of self and other. *join_table* and *join_dict* can be None iff the two queries only search one table in the database. All display columns will be concatenated in order: self display cols + other display cols.

INPUT:

- *other* – the SQLQuery to intersect with
- *join_table* – base table to join on (This table should have at least one column in each table to join on).
- *join_dict* – a dictionary that represents the join structure for the new query. (Must include a mapping for all tables, including those previously joined in either query). Structure is given by:

```
{'join_table1':('corr_base_coll1', 'coll1'), 'join_table2':('corr_base_coll2', 'coll2')}
```

where *join_table1* is to be joined with *join_table* on *join_table.corr_base_coll1* = *join_table1.coll1*

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.create_table('lucy',{'a1':{'sql':'bool'}, 'b2':{'sql':'int'}})
sage: DB.add_data('simon', [(0,5),(1,4)])
sage: DB.add_data('lucy', [(1,1),(1,4)])
sage: q = SQLQuery(DB, {'table_name':'lucy', 'display_cols':['b2'], 'expression':['a1','=',1]})
sage: r = SQLQuery(DB, {'table_name':'simon', 'display_cols':['a1'], 'expression':['b2','<=']})
sage: s = q.intersect(r, 'simon', {'lucy':('a1','a1')})
sage: s.get_query_string()
'SELECT lucy.b2,simon.a1 FROM simon INNER JOIN lucy ON
simon.a1=lucy.a1 WHERE ( lucy.a1 = ? ) AND ( simon.b2 <= ? )'
sage: s.query_results()
[(1, 1), (4, 1)]
sage: s = q.intersect(r)
Traceback (most recent call last):
...
ValueError: Input queries query different tables but join
parameters are NoneType
sage: s.__query_string__ == q.__query_string__
```

```
False
sage: q.intersect(r, 'simon', {'lucy':('a1','a1')}, True)
sage: q.__query_string__ == s.__query_string__
True
```

query_results()

Runs the query by executing the `__query_string__`. Returns the results of the query in a list.

EXAMPLES:

```
sage: G = GraphDatabase()
sage: q = 'SELECT graph_id,graph6,num_vertices,num_edges FROM graph_data WHERE graph_id<=(?)'
sage: param = (22,5)
sage: Q = SQLQuery(G,q,param)
sage: Q.query_results()
[(18, u'D??', 5, 0), (19, u'D?C', 5, 1), (20, u'D?K', 5, 2),
 (21, u'D@O', 5, 2), (22, u'D?[' , 5, 3)]
sage: R = SQLQuery(G,{'table_name':'graph_data', 'display_cols':['graph6'], 'expression':['r
sage: R.query_results()
[(u'C?',), (u'C@',), (u'CB',), (u'CK',), (u'CF',), (u'CJ',),
 (u'CL',), (u'CN',), (u'C]',), (u'C^',), (u'C~',)]
```

show (kws)**

Displays the result of the query in table format.

KEYWORDS:

- `max_field_size` – how wide each field can be
- `format_cols` – a dictionary that allows the user to specify the format of a column's output by supplying a function. The format of the dictionary is:

```
{'column_name':(lambda x: format_function(x))}
```

- `plot_cols` – a dictionary that allows the user to specify that a plot should be drawn by the object generated by a data slice. Note that plot kws are permitted. The dictionary format is:

```
{'column_name':((lambda x: plot_function(x)),**kws)}
```

- `relabel_cols` – a dictionary to specify a relabeling of column headers. The dictionary format is:

```
{'table_name':{'old_col_name':'new_col_name'}}
```

- `id_col` – reference to a column that can be used as an object identifier for each row

EXAMPLES:

```
sage: DB = SQLDatabase()
sage: DB.create_table('simon',{'a1':{'sql':'bool', 'primary_key':False}, 'b2':{'sql':'int'}})
sage: DB.add_data('simon',[(0,0),(1,1),(1,2)])
sage: r = SQLQuery(DB, {'table_name':'simon', 'display_cols':['a1'], 'expression':['b2','<=']}
sage: r.show()
a1
-----
0
1
```

```

1
sage: D = GraphDatabase()
sage: from sage.graphs.graph_database import valid_kwds, data_to_degseq
sage: relabel = {}
sage: for col in valid_kwds:
...     relabel[col] = ' '.join([word.capitalize() for word in col.split('_')])
sage: q = GraphQuery(display_cols=['graph6', 'degree_sequence'], num_vertices=4)
sage: SQLQuery.show(q, format_cols={'degree_sequence': (lambda x,y: data_to_degseq(x,y))}, re
Graph6          Degree Sequence
-----
C?              [0, 0, 0, 0]
C@              [0, 0, 1, 1]
CB              [0, 1, 1, 2]
CF              [1, 1, 1, 3]
CJ              [0, 2, 2, 2]
CK              [1, 1, 1, 1]
CL              [1, 1, 2, 2]
CN              [1, 2, 2, 3]
C]              [2, 2, 2, 2]
C^              [2, 2, 3, 3]
C~              [3, 3, 3, 3]

```

union (*other*, *join_table=None*, *join_dict=None*, *in_place=False*)

Returns a new `SQLQuery` that is the union of self and other. `join_table` and `join_dict` can be None iff the two queries only search one table in the database. All display columns will be concatenated in order: self display cols + other display cols.

INPUT:

- `other` – the `SQLQuery` to union with
- `join_table` – base table to join on (This table should have at least one column in each table to join on).
- `join_dict` – a dictionary that represents the join structure for the new query. (Must include a mapping for all tables, including those previously joined in either query). Structure is given by:

```
{'join_table1': ('corr_base_coll1', 'coll1'), 'join_table2': ('corr_base_coll2', 'coll2')}
```

where `join_table1`` is to be joined with ```join_table` on
`join_table.corr_base_coll1=join_table1.coll1`

EXAMPLES:

```

sage: DB = SQLDatabase()
sage: DB.create_table('simon', {'a1': {'sql': 'bool'}, 'b2': {'sql': 'int'}})
sage: DB.create_table('lucy', {'a1': {'sql': 'bool'}, 'b2': {'sql': 'int'}})
sage: DB.add_data('simon', [(0,5), (1,4)])
sage: DB.add_data('lucy', [(1,1), (1,4)])
sage: q = SQLQuery(DB, {'table_name': 'lucy', 'display_cols': ['b2'], 'expression': ['a1', '=', 1]})
sage: r = SQLQuery(DB, {'table_name': 'simon', 'display_cols': ['a1'], 'expression': ['b2', '<=', 1]})
sage: s = q.union(r, 'simon', {'lucy': ('a1', 'a1')})
sage: s.get_query_string()
'SELECT lucy.b2,simon.a1 FROM simon INNER JOIN lucy ON
simon.a1=lucy.a1 WHERE ( lucy.a1 = ? ) OR ( simon.b2 <= ? )'
sage: s.query_results()
[(1, 1), (4, 1)]

```

`sage.databases.sql_db.construct_skeleton(database)`

Constructs a database skeleton from the sql data. The skeleton data structure is a triple indexed dictionary of the following format:

```
| - skeleton -- a triple-indexed dictionary
|   - outer key -- table name
|     - inner key -- column name
|       - inner inner key -- one of the following:
|         - ``primary_key`` -- boolean, whether column has been set as
|           primary key
|         - ``index`` -- boolean, whether column has been set as index
|         - ``unique`` -- boolean, whether column has been set as unique
|         - ``sql`` -- one of ``TEXT``, ``BOOLEAN``, ``INTEGER``,
|           ``REAL``, or other user defined type
```

An example skeleton of a database with one table, that table with one column:

```
{'table1':{'coll':{'primary_key':False, 'unique': True, 'index':True, 'sql':'REAL'}}
```

EXAMPLES:

```
sage: G = SQLDatabase(GraphDatabase().__dblocation__, False)
sage: from sage.databases.sql_db import construct_skeleton
sage: construct_skeleton(G).keys()
[u'aut_grp', u'degrees', u'spectrum', u'misc', u'graph_data']
```

`sage.databases.sql_db.regexp(expr, item)`

Function to define regular expressions in pysqlite. Returns True if parameter item matches the regular expression parameter expr. Returns False otherwise (i.e.: no match).

REFERENCES:

EXAMPLES:

```
sage: from sage.databases.sql_db import regexp
sage: regexp('.s.*', 'cs')
True
sage: regexp('.s.*', 'ccs')
False
sage: regexp('.s.*', 'cscccc')
True
```

`sage.databases.sql_db.verify_column(col_dict)`

Verify that col_dict is in proper format, and return a dict with default values filled in. Proper format:

```
{'primary_key':False, 'index':False, 'unique': False, 'sql':'REAL'}
```

EXAMPLES:

```
sage: from sage.databases.sql_db import verify_column
sage: col = {'sql':'BOOLEAN'}
sage: verify_column(col)
{'index': False, 'primary_key': False, 'sql': 'BOOLEAN', 'unique': False}
sage: col = {'primary_key':True, 'sql':'INTEGER'}
sage: verify_column(col)
{'index': True, 'primary_key': True, 'sql': 'INTEGER', 'unique': True}
sage: verify_column({})
Traceback (most recent call last):
...
ValueError: SQL type must be declared, e.g. {'sql':'REAL'}.
```


`sage.databases.sql_db.verify_operator(operator)`

Checks that `operator` is one of the allowed strings. Legal operators include the following strings:

- '='
- '<='
- '>='
- '<'
- '>'
- '<>'
- 'like'
- 'regexp'
- 'is null'
- 'is not null'

EXAMPLES:

```
sage: from sage.databases.sql_db import verify_operator
sage: verify_operator('<=')
True
sage: verify_operator('regexp')
True
sage: verify_operator('is null')
True
sage: verify_operator('not_an_operator')
Traceback (most recent call last):
...
TypeError: not_an_operator is not a legal operator.
```

`sage.databases.sql_db.verify_type(type)`

Verify that the specified `type` is one of the allowed strings.

EXAMPLES:

```
sage: from sage.databases.sql_db import verify_type
sage: verify_type('INT')
True
sage: verify_type('int')
True
sage: verify_type('float')
Traceback (most recent call last):
...
TypeError: float is not a legal type.
```

2.7 Media

2.7.1 Wrapper for Graphics Files

class `sage.structure.graphics_file.GraphicsFile(filename, mime_type=None)`

Bases: `sage.structure.sage_object.SageObject`

Wrapper around a graphics file.

data()

Return a byte string containing the image file.

filename()

launch_viewer()

Launch external viewer for the graphics file.

Note: Does not actually launch a new process when doctesting.

EXAMPLES:

```
sage: from sage.structure.graphics_file import GraphicsFile
sage: g = GraphicsFile('/tmp/test.png', 'image/png')
sage: g.launch_viewer()
```

mime()

sagenb_embedding()

Embed in SageNB

This amounts to just placing the file in the cell directory. The notebook will then try to guess what we want with it.

save_as(filename)

Make the file available under a new filename.

INPUT:

- filename – string. The new filename.

The newly-created filename will be a hardlink if possible. If not, an independent copy is created.

class sage.structure.graphics_file.**Mime**

Bases: object

classmethod extension(mime_type)

Return file extension.

INPUT:

- mime_type – mime type as string.

OUTPUT:

String containing the usual file extension for that type of file. Excludes `os.extsep`.

EXAMPLES:

```
sage: from sage.structure.graphics_file import Mime
sage: Mime.extension('image/png')
'png'
```

classmethod validate(value)

Check that input is known mime type

INPUT:

- value – string.

OUTPUT:

Unicode string of that mime type. A `ValueError` is raised if input is incorrect / unknown.

EXAMPLES:

```

sage: from sage.structure.graphics_file import Mime
sage: Mime.validate('image/png')
u' image/png'
sage: Mime.validate('foo/bar')
Traceback (most recent call last):
...
ValueError: unknown mime type

```

```

sage.structure.graphics_file.graphics_from_save(save_function, preferred_mime_types,
                                                allowed_mime_types=None, fig-
                                                size=None, dpi=None)

```

Helper function to construct a graphics file.

INPUT:

- `save_function` – callable that can save graphics to a file and accepts options like `sage.plot.graphics.Graphics.save()`.
- `preferred_mime_types` – list of mime types. The graphics output mime types in order of preference (i.e. best quality to worst).
- `allowed_mime_types` – set of mime types (as strings). The graphics types that we can display. Output, if any, will be one of those.
- `figsize` – pair of integers (optional). The desired graphics size in pixels. Suggested, but need not be respected by the output.
- `dpi` – integer (optional). The desired resolution in dots per inch. Suggested, but need not be respected by the output.

OUTPUT:

Return an instance of `sage.structure.graphics_file.GraphicsFile` encapsulating a suitable image file. Image is one of the `preferred_mime_types`. If `allowed_mime_types` is specified, the resulting file format matches one of these.

Alternatively, this function can return `None` to indicate that textual representation is preferable and/or no graphics with the desired mime type can be generated.

2.7.2 Work with WAV files

A WAV file is a header specifying format information, followed by a sequence of bytes, representing the state of some audio signal over a length of time.

A WAV file may have any number of channels. Typically, they have 1 (mono) or 2 (for stereo). The data of a WAV file is given as a sequence of frames. A frame consists of samples. There is one sample per channel, per frame. Every wav file has a sample width, or, the number of bytes per sample. Typically this is either 1 or 2 bytes.

The `wav` module supplies more convenient access to this data. In particular, see the docstring for `Wave.channel_data()`.

The header contains information necessary for playing the WAV file, including the number of frames per second, the number of bytes per sample, and the number of channels in the file.

AUTHORS:

- Bobby Moretti and Gonzolo Tornaria (2007-07-01): First version
- William Stein (2007-07-03): add more
- Bobby Moretti (2007-07-03): add doctests

class `sage.media.wav.Wave` (*data=None, **kwds*)
Bases: `sage.structure.sage_object.SageObject`

A class wrapping a wave audio file.

INPUT:

You must call `Wave()` with either `data = filename`, where `filename` is the name of a wave file, or with each of the following options:

- `channels` – the number of channels in the wave file (1 for mono, 2 for stereo, etc...
- `width` – the number of bytes per sample
- `framerate` – the number of frames per second
- `nframes` – the number of frames in the data stream
- `bytes` – a string object containing the bytes of the data stream

Slicing:

Slicing a `Wave` object returns a new wave object that has been trimmed to the bytes that you have given it.

Indexing:

Getting the `nth` item in a `Wave` object will give you the value of the `nth` frame.

channel_data (*n*)
Get the data from a given channel.

INPUT: *n* – the channel number to get

OUTPUT: A list of signed ints, each containing the value of a frame.

convolve (*right, channel=0*)
NOT DONE!

Convolution of self and other, i.e., add their fft's, then inverse fft back.

domain (*npoints=None*)
Used internally for plotting. Get the x-values for the various points to plot.

getframerate ()
Returns the number of frames per second in this wave object.

OUTPUT: The frame rate of this sound file.

getlength ()
Returns the length of this file (in seconds).

OUTPUT: The running time of the entire WAV object.

getnchannels ()
Returns the number of channels in this wave object.

OUTPUT: The number of channels in this wave file.

getnframes ()
The total number of frames in this wave object.

OUTPUT: The number of frames in this WAV.

getsampwidth ()
Returns the number of bytes per sample in this wave object.

OUTPUT: The number of bytes in each sample.

listen()

Listen to (or download) this wave file.

Creates a link to this wave file in the notebook.

plot (*npoints=None, channel=0, plotjoined=True, **kwds*)

Plots the audio data.

INPUT:

- **npoints** – number of sample points to take; if not given, draws all known points.
- **channel** – 0 or 1 (if stereo). default: 0
- **plotjoined** – whether to just draw dots or draw lines between sample points

OUTPUT:

a plot object that can be shown.

plot_fft (*npoints=None, channel=0, half=True, **kwds*)

plot_raw (*npoints=None, channel=0, plotjoined=True, **kwds*)

readframes (*n*)

Reads out the raw data for the first *n* frames of this wave object.

INPUT: *n* – the number of frames to return

OUTPUT: A list of bytes (in string form) representing the raw wav data.

save (*filename='sage.wav'*)

Save this wave file to disk, either as a Sage sobj or as a .wav file.

INPUT:

filename – the path of the file to save. If filename ends with 'wav', then save as a wave file, otherwise, save a Sage object.

If no input is given, save the file as 'sage.wav'.

set_values (*values, channel=0*)

Used internally for plotting. Get the y-values for the various points to plot.

slice_seconds (*start, stop*)

Slices the wave from start to stop.

INPUT: *start* – the time index from which to begin the slice (in seconds) *stop* – the time index from which to end the slice (in seconds)

OUTPUT: A Wave object whose data is this object's data, sliced between the given time indices

values (*npoints=None, channel=0*)

Used internally for plotting. Get the y-values for the various points to plot.

vector (*npoints=None, channel=0*)

2.8 Warnings

2.8.1 Stopgaps

exception `sage.misc.stopgap.StopgapWarning`

Bases: `exceptions.Warning`

This class is used to warn users of a known issue that may produce mathematically incorrect results.

`sage.misc.stopgap.set_state(mode)`

Enable or disable the stopgap warnings.

INPUT:

- `mode` – (bool); if True, enable stopgaps; otherwise, disable.

EXAMPLES:

```
sage: import sage.misc.stopgap
sage: sage.misc.stopgap.set_state(False)
sage: sage.misc.stopgap.stopgap("Displays nothing.", 12345)
sage: sage.misc.stopgap.set_state(True)
sage: sage.misc.stopgap.stopgap("Displays something.", 123456)
doctest:...:
*****
Displays something.
This issue is being tracked at http://trac.sagemath.org/sage_trac/ticket/123456.
*****
sage: sage.misc.stopgap.set_state(False)
```

`sage.misc.stopgap.stopgap(message, ticket_no)`

Issue a stopgap warning.

INPUT:

- `message` - an explanation of how an incorrect answer might be produced.
- `ticket_no` - an integer, giving the number of the Trac ticket tracking the underlying issue.

EXAMPLES:

```
sage: import sage.misc.stopgap
sage: sage.misc.stopgap.set_state(True)
sage: sage.misc.stopgap.stopgap("Computation of heights on elliptic curves over number fields ca
doctest:...:
*****
Computation of heights on elliptic curves over number fields can return very imprecise results.
This issue is being tracked at http://trac.sagemath.org/sage_trac/ticket/12509.
*****
sage: sage.misc.stopgap.stopgap("This is not printed", 12509)
sage: sage.misc.stopgap.set_state(False) # so rest of doctesting fine
```

2.8.2 Handling Superseded Functionality

The main mechanism in Sage to deal with superseded functionality is to add a deprecation warning. This will be shown once, the first time that the deprecated function is called.

Note that all doctests in the following use the trac ticket number `trac ticket #13109`, which is where this mandatory argument to `deprecation()` was introduced.

Functions and classes

class `sage.misc.superseded.DeprecatedFunctionAlias(trac_number, func, module)`

Bases: object

A wrapper around methods or functions which automatically print the correct deprecation message. See `deprecated_function_alias()`.

AUTHORS:

- Florent Hivert (2009-11-23), with the help of Mike Hansen.
- Luca De Feo (2011-07-11), printing the full module path when different from old path

`sage.misc.superseded.deprecated_callable_import` (*trac_number*, *module_name*, *globs*,
locs, *fromlist*, *message=None*)

Imports a list of callables into the namespace from which it is called. These callables however give a deprecation warning whenever they are called. This is primarily used from deprecating things from Sage's `all.py` files.

INPUT:

- trac_number* – integer. The trac ticket number where the deprecation is introduced.
- param module_name* – string or None. The name of the module from which to import the callables or None.
- globs* – dictionary. The `globals()` from where this is being called.
- locs* – dictionary. The `locals()` from where this is being called.
- fromlist* – list of strings. The list the names of the callables to deprecate
- message* – string. Message to display when the deprecated functions are called.

Note: If *module_name* is None, then no importing will be done, and it will be assumed that the functions have already been imported and are present in *globs*

Warning: This should really only be used for functions.

EXAMPLES:

```
sage: from sage.misc.superseded import deprecated_callable_import
sage: is_prime(3)
True
sage: message = "Using %(name)s from here is deprecated."
sage: deprecated_callable_import(13109, None, globals(), locals(), ['is_prime'], message)
sage: is_prime(3)
doctest:...: DeprecationWarning:
Using is_prime from here is deprecated.
See http://trac.sagemath.org/13109 for details.
True
sage: del is_prime
sage: deprecated_callable_import(13109, 'sage.rings.arith', globals(), locals(), ['is_prime'])
sage: is_prime(3)
doctest:...: DeprecationWarning:
Using is_prime from here is deprecated. If you need to use it, please import it directly from s
See http://trac.sagemath.org/13109 for details.
True
```

`sage.misc.superseded.deprecated_function_alias` (*trac_number*, *func*)

Create an aliased version of a function or a method which raise a deprecation warning message.

If *f* is a function or a method, write `g = deprecated_function_alias(trac_number, f)` to make a deprecated aliased version of *f*.

INPUT:

- trac_number* – integer. The trac ticket number where the deprecation is introduced.
- func* – the function or method to be aliased

EXAMPLES:

```
sage: from sage.misc.superseded import deprecated_function_alias
sage: g = deprecated_function_alias(13109, number_of_partitions)
sage: g(5)
doctest:...: DeprecationWarning: g is deprecated. Please use sage.combinat.partition.number_of_p
See http://trac.sagemath.org/13109 for details.
7
```

This also works for methods:

```
sage: class cls(object):
....:     def new_meth(self): return 42
....:     old_meth = deprecated_function_alias(13109, new_meth)
sage: cls().old_meth()
doctest:...: DeprecationWarning: old_meth is deprecated. Please use new_meth instead.
See http://trac.sagemath.org/13109 for details.
42
```

trac ticket #11585:

```
sage: def a(): pass
sage: b = deprecated_function_alias(13109, a)
sage: b()
doctest:...: DeprecationWarning: b is deprecated. Please use a instead.
See http://trac.sagemath.org/13109 for details.
```

AUTHORS:

- Florent Hivert (2009-11-23), with the help of Mike Hansen.
- Luca De Feo (2011-07-11), printing the full module path when different from old path

`sage.misc.superseded.deprecation(trac_number, message, stacklevel=4)`
Issue a deprecation warning.

INPUT:

- `trac_number` – integer. The trac ticket number where the deprecation is introduced.
- `message` – string. An explanation why things are deprecated and by what it should be replaced.
- `stack_level` – (default: 4) an integer. This is passed on to `warnings.warn()`.

EXAMPLES:

```
sage: def foo():
....:     sage.misc.superseded.deprecation(13109, 'the function foo is replaced by bar')
sage: foo()
doctest:...: DeprecationWarning: the function foo is replaced by bar
See http://trac.sagemath.org/13109 for details.
```

See also:

`experimental()`, `warning()`.

class `sage.misc.superseded.experimental(trac_number, stacklevel=4)`
Bases: `object`

A decorator which warns about the experimental/unstable status of the decorated class/method/function.

INPUT:

- `trac_number` – an integer. The trac ticket number where this code was introduced.

- `stack_level` – (default: 4) an integer. This is passed on to `warnings.warn()`.

EXAMPLES:

```
sage: @sage.misc.superseded.experimental(trac_number=79997)
.....: def foo(*args, **kwargs):
.....:     print args, kwargs
sage: foo(7, what='Hello')
doctest:...: FutureWarning: This class/method/function is
marked as experimental. It, its functionality or its
interface might change without a formal deprecation.
See http://trac.sagemath.org/79997 for details.
(7,) {'what': 'Hello'}
```

```
sage: class bird(SageObject):
.....:     @sage.misc.superseded.experimental(trac_number=99999)
.....:     def __init__(self, *args, **kwargs):
.....:         print "piep", args, kwargs
sage: _ = bird(99)
doctest:...: FutureWarning: This class/method/function is
marked as experimental. It, its functionality or its
interface might change without a formal deprecation.
See http://trac.sagemath.org/99999 for details.
piep (99,) {}
```

See also:

`experimental()`, `warning()`, `deprecation()`.

`sage.misc.superseded.experimental_warning(trac_number, message, stacklevel=4)`

Issue a warning that the functionality or class is experimental and might change in future.

INPUT:

- `trac_number` – an integer. The trac ticket number where the experimental functionality was introduced.
- `message` – a string. An explanation what is going on.
- `stack_level` – (default: 4) an integer. This is passed on to `warnings.warn()`.

EXAMPLES:

```
sage: def foo():
.....:     sage.misc.superseded.experimental_warning(
.....:         66666, 'This function is experimental and '
.....:         'might change in future.')
sage: foo()
doctest:...: FutureWarning: This function is experimental and
might change in future.
See http://trac.sagemath.org/66666 for details.
```

See also:

`mark_as_experimental()`, `warning()`, `deprecation()`.

`sage.misc.superseded.warning(trac_number, message, warning_class=<type 'exceptions.Warning'>, stacklevel=3)`

Issue a warning.

INPUT:

- `trac_number` – integer. The trac ticket number where the deprecation is introduced.
- `message` – string. An explanation what is going on.

- `warning_class` – (default: `Warning`) a class inherited from a Python `Warning`.
- `stack_level` – (default: 3) an integer. This is passed on to `warnings.warn()`.

EXAMPLES:

```
sage: def foo():
....:     sage.misc.superseded.warning(
....:         99999,
....:         'The syntax will change in future.',
....:         FutureWarning)
sage: foo()
doctest:...: FutureWarning: The syntax will change in future.
See http://trac.sagemath.org/99999 for details.
```

See also:

`deprecation()`, `experimental()`, `exceptions.Warning`.

2.9 Miscellaneous Useful Functions

2.9.1 Miscellaneous functions

AUTHORS:

- William Stein
- William Stein (2006-04-26): added workaround for Windows where most users' home directory has a space in it.
- Robert Bradshaw (2007-09-20): Ellipsis range/iterator.

TESTS:

The following test, verifying that [trac ticket #16181](#) has been resolved, needs to stay at the beginning of this file so that its context is not poisoned by other tests:

```
sage: sage.misc.misc.inject_variable('a', 0)
sage: a
0
```

Check the fix from [trac ticket #8323](#):

```
sage: 'name' in globals()
False
sage: 'func' in globals()
False
```

Test deprecation:

```
sage: sage.misc.misc.mul([3,4])
doctest:...: DeprecationWarning:
Importing prod from here is deprecated. If you need to use it, please import it directly from sage.m
See http://trac.sagemath.org/17460 for details.
12
```

```
class sage.misc.misc.AttrCallObject(name, args, kwds)
    Bases: object
```

TESTS:

```
sage: f = attrcall('core', 3); f
*.core(3)
sage: TestSuite(f).run()
```

class `sage.misc.misc.BackslashOperator`

Implements Matlab-style backslash operator for solving systems:

$A \setminus b$

The parser converts this to multiplications using `BackslashOperator()`.

EXAMPLES:

```
sage: prepare("A \ matrix(QQ,2,1,[1/3,'2/3'])")
"A * BackslashOperator() * matrix(QQ,Integer(2),Integer(1),[Integer(1)/Integer(3),'2/3'])"
sage: prepare("A \ matrix(QQ,2,1,[1/3,2*3])")
'A * BackslashOperator() * matrix(QQ,Integer(2),Integer(1),[Integer(1)/Integer(3),Integer(2)*Integer(3)])'
sage: prepare("A \ B + C")
'A * BackslashOperator() * B + C'
sage: prepare("A \ eval('C+D')")
"A * BackslashOperator() * eval('C+D') "
sage: prepare("A \ x / 5")
'A * BackslashOperator() * x / Integer(5)'
sage: prepare("A^3 \ b")
'A**Integer(3) * BackslashOperator() * b'
```

class `sage.misc.misc.GlobalCputime(t)`

Container for CPU times of subprocesses.

AUTHOR:

- Martin Albrecht - (2008-12): initial version

EXAMPLE:

Objects of this type are returned if `subprocesses=True` is passed to `cputime()`:

```
sage: cputime(subprocesses=True) # indirect doctest, output random
0.2347431
```

We can use it to keep track of the CPU time spent in Singular for example:

```
sage: t = cputime(subprocesses=True)
sage: P = PolynomialRing(QQ,7,'x')
sage: I = sage.rings.ideal.Katsura(P)
sage: gb = I.groebner_basis() # calls Singular
sage: cputime(subprocesses=True) - t # output random
0.462987
```

For further processing we can then convert this container to a float:

```
sage: t = cputime(subprocesses=True)
sage: float(t) #output somewhat random
2.1088339999999999
```

See also:

`cputime()`

sage.misc.misc.alarm(seconds)

Raise an `AlarmInterrupt` exception in a given number of seconds. This is useful for automatically interrupting long computations and can be trapped using exception handling.

Use `cancel_alarm()` to cancel a previously scheduled alarm.

INPUT:

- seconds – positive number, may be floating point

EXAMPLES:

```
sage: alarm(0.5); factor(2^1031-1)
Traceback (most recent call last):
...
AlarmInterrupt
sage: alarm(0)
Traceback (most recent call last):
...
ValueError: alarm() time must be positive
```

`sage.misc.misc.assert_attribute(x, attr, init=None)`

If the object `x` has the attribute `attr`, do nothing. If not, set `x.attr` to `init`.

`sage.misc.misc.attrcall(name, *args, **kwargs)`

Returns a callable which takes in an object, gets the method named `name` from that object, and calls it with the specified arguments and keywords.

INPUT:

- name - a string of the name of the method you want to call
- args, kwargs - arguments and keywords to be passed to the method

EXAMPLES:

```
sage: f = attrcall('core', 3); f
*.core(3)
sage: [f(p) for p in Partitions(5)]
[[2], [1, 1], [1, 1], [3, 1, 1], [2], [2], [1, 1]]
```

class `sage.misc.misc.cached_attribute(method, name=None)`

Bases: `object`

Computes attribute value and caches it in the instance.

`sage.misc.misc.call_method(obj, name, *args, **kwargs)`

Call the method `name` on `obj`.

This has to exist somewhere in Python!!!

See also:

`operator.methodcaller()` `attrcall()`

EXAMPLES:

```
sage: from sage.misc.misc import call_method
sage: call_method(1, "__add__", 2)
3
```

`sage.misc.misc.cancel_alarm()`

Cancel a previously scheduled alarm (if any) set by `alarm()`.

EXAMPLES:

```
sage: alarm(0.5)
sage: cancel_alarm()
```

```
sage: cancel_alarm() # Calling more than once doesn't matter
sage: sleep(0.6)    # sleep succeeds
```

`sage.misc.misc.cmp_props` (*left, right, props*)

`sage.misc.misc.coeff_repr` (*c, is_latex=False*)

`sage.misc.misc.compose` (*f, g*)

Return the composition of one-variable functions: $f \circ g$

See also `self_compose()` and `nest()`

INPUT:

- f – a function of one variable
- g – another function of one variable

OUTPUT: A function, such that `compose(f,g)(x) = f(g(x))`

EXAMPLES:

```
sage: def g(x): return 3*x
sage: def f(x): return x + 1
sage: h1 = compose(f, g)
sage: h2 = compose(g, f)
sage: _ = var('x')
sage: h1(x)
3*x + 1
sage: h2(x)
3*x + 3
```

```
:: sage: _ = function('f g') sage: _ = var('x') sage: compose(f,g)(x) f(g(x))
```

`sage.misc.misc.cputime` (*t=0, subprocesses=False*)

Return the time in CPU seconds since Sage started, or with optional argument *t*, return the time since *t*. This is how much time Sage has spent using the CPU. If `subprocesses=False` this does not count time spent in subprocesses spawned by Sage (e.g., Gap, Singular, etc.). If `subprocesses=True` this function tries to take all subprocesses with a working `cputime()` implementation into account.

The measurement for the main Sage process is done via a call to `resource.getrusage()`, so it avoids the wraparound problems in `time.clock()` on Cygwin.

INPUT:

- *t* - (optional) time in CPU seconds, if *t* is a result from an earlier call with `subprocesses=True`, then `subprocesses=True` is assumed.
- `subprocesses` – (optional), include subprocesses (default: `False`)

OUTPUT:

- float - time in CPU seconds if `subprocesses=False`
- `GlobalCputime` - object which holds CPU times of subprocesses otherwise

EXAMPLES:

```
sage: t = cputime()
sage: F = gp.factor(2^199-1)
sage: cputime(t) # somewhat random
0.0109990000000000092
```

```
sage: t = cputime(subprocesses=True)
sage: F = gp.factor(2^199-1)
sage: cputime(t) # somewhat random
0.091999

sage: w = walltime()
sage: F = gp.factor(2^199-1)
sage: walltime(w) # somewhat random
0.58425593376159668
```

Note: Even with `subprocesses=True` there is no guarantee that the CPU time is reported correctly because subprocesses can be started and terminated at any given time.

`sage.misc.misc.ellipsis_iter(*args, **kws)`
Same as `ellipsis_range`, but as an iterator (and may end with an `Ellipsis`).

See also `ellipsis_range`.

Use (1,2,...) notation.

EXAMPLES:

```
sage: A = ellipsis_iter(1,2,Ellipsis)
sage: [next(A) for _ in range(10)]
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
sage: next(A)
11
sage: A = ellipsis_iter(1,3,5,Ellipsis)
sage: [next(A) for _ in range(10)]
[1, 3, 5, 7, 9, 11, 13, 15, 17, 19]
sage: A = ellipsis_iter(1,2,Ellipsis,5,10,Ellipsis)
sage: [next(A) for _ in range(10)]
[1, 2, 3, 4, 5, 10, 11, 12, 13, 14]
```

TESTS:

These were carefully chosen tests, only to be changed if the semantics of ellipsis ranges change. In other words, if they don't pass it's probably a bug in the implementation, not in the doctest.

```
sage: list(1,...,10)
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
sage: list(1,3,...,10)
[1, 3, 5, 7, 9]
sage: list(1,...,10,...,20)
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
sage: list(1,3,...,10,...,20)
[1, 3, 5, 7, 9, 10, 12, 14, 16, 18, 20]
sage: list(1,3,...,10,10,...,20)
[1, 3, 5, 7, 9, 10, 12, 14, 16, 18, 20]
sage: list(0,2,...,10,10,...,20,20,...,25)
[0, 2, 4, 6, 8, 10, 10, 12, 14, 16, 18, 20, 20, 22, 24]
sage: list(10,...,1)
[]
sage: list(10,11,...,1)
[]
sage: list(10,9,...,1)
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
sage: list(100,...,10,...,20)
[10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
```

```

sage: list(0,...,10,...,-20)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
sage: list(100,...,10,...,-20)
[]
sage: list(100,102,...,10,...,20)
[10, 12, 14, 16, 18, 20]

```

`sage.misc.misc.ellipsis_range(*args,**kws)`

Return arithmetic sequence determined by the numeric arguments and ellipsis. Best illustrated by examples.

Use `[1,2,...,n]` notation.

EXAMPLES:

```

sage: ellipsis_range(1, Ellipsis, 11, 100)
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 100]
sage: ellipsis_range(0, 2, Ellipsis, 10, Ellipsis, 20)
[0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]
sage: ellipsis_range(0, 2, Ellipsis, 11, Ellipsis, 20)
[0, 2, 4, 6, 8, 10, 11, 13, 15, 17, 19]
sage: ellipsis_range(0, 2, Ellipsis, 11, Ellipsis, 20, step=3)
[0, 2, 5, 8, 11, 14, 17, 20]
sage: ellipsis_range(10, Ellipsis, 0)
[]

```

TESTS: These were carefully chosen tests, only to be changed if the semantics of ellipsis ranges change. In other words, if they don't pass it's probably a bug in the implementation, not in the doctest.

Note 10 only appears once (though it is in both ranges).

```

sage: ellipsis_range(0, Ellipsis, 10, Ellipsis, 20, step=2)
[0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

```

Sometimes one or more ranges is empty.

```

sage: ellipsis_range(100, Ellipsis, 10, Ellipsis, 20, step=2)
[10, 12, 14, 16, 18, 20]
sage: ellipsis_range(0, Ellipsis, 10, Ellipsis, -20, step=2)
[0, 2, 4, 6, 8, 10]
sage: ellipsis_range(100, Ellipsis, 10, Ellipsis, -20, step=2)
[]

```

We always start on the leftmost point of the range.

```

sage: ellipsis_range(0, Ellipsis, 10, Ellipsis, 20, step=3)
[0, 3, 6, 9, 10, 13, 16, 19]
sage: ellipsis_range(100, Ellipsis, 10, Ellipsis, 20, step=3)
[10, 13, 16, 19]
sage: ellipsis_range(0, Ellipsis, 10, Ellipsis, -20, step=3)
[0, 3, 6, 9]
sage: ellipsis_range(100, Ellipsis, 10, Ellipsis, -20, step=3)
[]
sage: ellipsis_range(0, 1, Ellipsis, -10)
[]
sage: ellipsis_range(0, 1, Ellipsis, -10, step=1)
[0]
sage: ellipsis_range(100, 0, 1, Ellipsis, -10)
[100]

```

Note the duplicate 5 in the output.

```
sage: ellipsis_range(0, Ellipsis, 5, 5, Ellipsis, 10)
[0, 1, 2, 3, 4, 5, 5, 6, 7, 8, 9, 10]
```

Examples in which the step determines the parent of the elements:

```
sage: [1..3, step=0.5]
[1.0000000000000000, 1.5000000000000000, 2.0000000000000000, 2.5000000000000000, 3.0000000000000000]
sage: v = [1..5, step=1/1]; v
[1, 2, 3, 4, 5]
sage: parent(v[2])
Rational Field
```

`sage.misc.misc.embedded()`

Return True if this copy of Sage is running embedded in the Sage notebook.

EXAMPLES:

```
sage: sage.misc.misc.embedded()      # output True if in the notebook
False
```

`sage.misc.misc.exists(S, P)`

If S contains an element x such that $P(x)$ is True, this function returns True and the element x . Otherwise it returns False and None.

Note that this function is NOT suitable to be used in an if-statement or in any place where a boolean expression is expected. For those situations, use the Python built-in

`any(P(x) for x in S)`

INPUT:

- S - object (that supports enumeration)
- P - function that returns True or False

OUTPUT:

- bool - whether or not P is True for some element x of S
- object - x

EXAMPLES: lambda functions are very useful when using the exists function:

```
sage: exists([1,2,5], lambda x : x > 7)
(False, None)
sage: exists([1,2,5], lambda x : x > 3)
(True, 5)
```

The following example is similar to one in the MAGMA handbook. We check whether certain integers are a sum of two (small) cubes:

```
sage: cubes = [t**3 for t in range(-10,11)]
sage: exists([(x,y) for x in cubes for y in cubes], lambda v : v[0]+v[1] == 218)
(True, (-125, 343))
sage: exists([(x,y) for x in cubes for y in cubes], lambda v : v[0]+v[1] == 219)
(False, None)
```

`sage.misc.misc.forall(S, P)`

If $P(x)$ is true every x in S , return True and None. If there is some element x in S such that P is not True, return False and x .

Note that this function is NOT suitable to be used in an if-statement or in any place where a boolean expression is expected. For those situations, use the Python built-in

$\text{all}(P(x) \text{ for } x \text{ in } S)$

INPUT:

- S - object (that supports enumeration)
- P - function that returns True or False

OUTPUT:

- `bool` - whether or not P is True for all elements of S
- `object` - x

EXAMPLES: lambda functions are very useful when using the `forall` function. As a toy example we test whether certain integers are greater than 3.

```
sage: forall([1,2,5], lambda x : x > 3)
(False, 1)
sage: forall([1,2,5], lambda x : x > 0)
(True, None)
```

Next we ask whether every positive integer less than 100 is a product of at most 2 prime factors:

```
sage: forall(range(1,100), lambda n : len(factor(n)) <= 2)
(False, 30)
```

The answer is no, and 30 is a counterexample. However, every positive integer 100 is a product of at most 3 primes.

```
sage: forall(range(1,100), lambda n : len(factor(n)) <= 3)
(True, None)
```

`sage.misc.misc.generic_cmp(x, y)`
Compare x and y and return -1, 0, or 1.

This is similar to `x.__cmp__(y)`, but works even in some cases when a `.__cmp__` method isn't defined.

`sage.misc.misc.get_main_globals()`
Return the main global namespace.

EXAMPLES:

```
sage: from sage.misc.misc import get_main_globals
sage: G = get_main_globals()
sage: bla = 1
sage: G['bla']
1
sage: bla = 2
sage: G['bla']
2
sage: G['ble'] = 5
sage: ble
5
```

This is analogous to `globals()`, except that it can be called from any function, even if it is in a Python module:

```
sage: def f():
....:     G = get_main_globals()
....:     assert G['bli'] == 14
....:     G['blo'] = 42
sage: bli = 14
sage: f()
```

```
sage: blo
42
```

ALGORITHM:

The main global namespace is discovered by going up the frame stack until the frame for the `__main__` module is found. Should this frame not be found (this should not occur in normal operation), an exception “ValueError: call stack is not deep enough” will be raised by `_getframe`.

See `inject_variable_test()` for a real test that this works within deeply nested calls in a function defined in a Python module.

```
sage.misc.misc.get_verbose()
Return the global Sage verbosity level.
```

INPUT: int level: an integer between 0 and 2, inclusive.

OUTPUT: changes the state of the verbosity flag.

EXAMPLES:

```
sage: get_verbose()
0
sage: set_verbose(2)
sage: get_verbose()
2
sage: set_verbose(0)
```

```
sage.misc.misc.get_verbose_files()
```

```
sage.misc.misc.getitem(v, n)
Variant of getitem that coerces to an int if a TypeError is raised.
```

(This is not needed anymore - classes should define an `__index__` method.)

Thus, e.g., `getitem(v, n)` will work even if v is a Python list and n is a Sage integer.

EXAMPLES:

```
sage: v = [1, 2, 3]
```

The following used to fail in Sage $\leq 1.3.7$. Now it works fine:

```
sage: v[ZZ(1)]
2
```

This always worked.

```
sage: getitem(v, ZZ(1))
2
```

```
sage.misc.misc.inject_variable(name, value)
Inject a variable into the main global namespace.
```

INPUT:

- name – a string
- value – anything

EXAMPLES:

```
sage: from sage.misc.misc import inject_variable
sage: inject_variable("a", 314)
```

```
sage: a
314
```

A warning is issued the first time an existing value is overwritten:

```
sage: inject_variable("a", 271)
doctest:...: RuntimeWarning: redefining global value 'a'
sage: a
271
sage: inject_variable("a", 272)
sage: a
272
```

That's because warn seem to not reissue twice the same warning:

```
sage: from warnings import warn sage: warn("blah") doctest:...: UserWarning: blah sage:
warn("blah")
```

Use with care!

`sage.misc.misc.inject_variable_test` (*name, value, depth*)

A function for testing deep calls to `inject_variable`

TESTS:

```
sage: from sage.misc.misc import inject_variable_test
sage: inject_variable_test("a0", 314, 0)
sage: a0
314
sage: inject_variable_test("a1", 314, 1)
sage: a1
314
sage: inject_variable_test("a2", 314, 2)
sage: a2
314
sage: inject_variable_test("a2", 271, 2)
doctest:...: RuntimeWarning: redefining global value 'a2'
sage: a2
271
```

`sage.misc.misc.is_in_string` (*line, pos*)

Returns True if the character at position *pos* in *line* occurs within a string.

EXAMPLES:

```
sage: from sage.misc.misc import is_in_string
sage: line = 'test(\'#\#\' )'
sage: is_in_string(line, line.rfind('#'))
True
sage: is_in_string(line, line.rfind(' '))
False
```

`sage.misc.misc.is_iterator` (*it*)

Tests if it is an iterator.

The mantra `if hasattr(it, 'next')` was used to tests if *it* is an iterator. This is not quite correct since *it* could have a `next` methods with a different semantic.

EXAMPLES:

```
sage: it = iter([1,2,3])
sage: is_iterator(it)
```

```
True

sage: class wrong():
...     def __init__(self): self.n = 5
...     def next(self):
...         self.n -= 1
...         if self.n == 0: raise StopIteration
...         return self.n
sage: x = wrong()
sage: is_iterator(x)
False
sage: list(x)
Traceback (most recent call last):
...
TypeError: iteration over non-sequence

sage: class good(wrong):
...     def __iter__(self): return self
sage: x = good()
sage: is_iterator(x)
True
sage: list(x)
[4, 3, 2, 1]

sage: P = Partitions(3)
sage: is_iterator(P)
False
sage: is_iterator(iter(P))
True
```

class `sage.misc.misc.lazy_prop` (*calculate_function*)
Bases: `object`

`sage.misc.misc.nest` (*f*, *n*, *x*)
Return $f(f(\dots f(x)\dots))$, where the composition occurs *n* times.

See also `compose()` and `self_compose()`

INPUT:

- *f* – a function of one variable
- *n* – a nonnegative integer
- *x* – any input for *f*

OUTPUT: $f(f(\dots f(x)\dots))$, where the composition occurs *n* times

EXAMPLES:

```
sage: def f(x): return x^2 + 1
sage: x = var('x')
sage: nest(f, 3, x)
((x^2 + 1)^2 + 1)^2 + 1
```

```
sage: _ = function('f')
sage: _ = var('x')
sage: nest(f, 10, x)
f(f(f(f(f(f(f(f(f(x))))))))))
```

```

sage: _ = function('f')
sage: _ = var('x')
sage: nest(f, 0, x)
x

```

`sage.misc.misc.newton_method_sizes(N)`

Returns a sequence of integers $1 = a_1 \leq a_2 \leq \dots \leq a_n = N$ such that $a_j = \lceil a_{j+1}/2 \rceil$ for all j .

This is useful for Newton-style algorithms that double the precision at each stage. For example if you start at precision 1 and want an answer to precision 17, then it's better to use the intermediate stages 1, 2, 3, 5, 9, 17 than to use 1, 2, 4, 8, 16, 17.

INPUT:

- N - positive integer

EXAMPLES:

```

sage: newton_method_sizes(17)
[1, 2, 3, 5, 9, 17]
sage: newton_method_sizes(16)
[1, 2, 4, 8, 16]
sage: newton_method_sizes(1)
[1]

```

AUTHORS:

- David Harvey (2006-09-09)

`sage.misc.misc.pad_zeros(s, size=3)`

EXAMPLES:

```

sage: pad_zeros(100)
'100'
sage: pad_zeros(10)
'010'
sage: pad_zeros(10, 5)
'00010'
sage: pad_zeros(389, 5)
'00389'
sage: pad_zeros(389, 10)
'0000000389'

```

`sage.misc.misc.powerset(X)`

Iterator over the *list* of all subsets of the iterable X , in no particular order. Each list appears exactly once, up to order.

INPUT:

- X - an iterable

OUTPUT: iterator of lists

EXAMPLES:

```

sage: list(powerset([1,2,3]))
[[], [1], [2], [1, 2], [3], [1, 3], [2, 3], [1, 2, 3]]
sage: [z for z in powerset([0,[1,2]])]
[[], [0], [[1, 2]], [0, [1, 2]]]

```

Iterating over the power set of an infinite set is also allowed:

```
sage: i = 0
sage: for x in powerset(ZZ):
...     if i > 10:
...         break
...     else:
...         i += 1
...     print x,
[] [0] [1] [0, 1] [-1] [0, -1] [1, -1] [0, 1, -1] [2] [0, 2] [1, 2]
```

You may also use subsets as an alias for powerset:

```
sage: subsets([1,2,3])
<generator object powerset at 0x...>
sage: list(subsets([1,2,3]))
[[], [1], [2], [1, 2], [3], [1, 3], [2, 3], [1, 2, 3]]
```

The reason we return lists instead of sets is that the elements of sets must be hashable and many structures on which one wants the powerset consist of non-hashable objects.

AUTHORS:

- William Stein
- Nils Bruin (2006-12-19): rewrite to work for not-necessarily finite objects X.

sage.misc.misc.**prop**(f)

sage.misc.misc.**random_sublist**(X, s)

Return a pseudo-random sublist of the list X where the probability of including a particular element is s.

INPUT:

- X - list
- s - floating point number between 0 and 1

OUTPUT: list

EXAMPLES:

```
sage: S = [1,7,3,4,18]
sage: random_sublist(S, 0.5)
[1, 3, 4]
sage: random_sublist(S, 0.5)
[1, 3]
```

sage.misc.misc.**repr_lincomb**(terms, coeffs=None, is_latex=False, scalar_mult='*',
strip_one=False, repr_monomial=None, latex_scalar_mult=None)

Compute a string representation of a linear combination of some formal symbols.

INPUT:

- terms – list of terms, as pairs (support, coefficient)
- is_latex – whether to produce latex (default: False)
- scalar_mult – string representing the multiplication (default: '*')
- latex_scalar_mult – latex string representing the multiplication (default: '' if scalar_mult is '*'; otherwise scalar_mult)
- coeffs – for backward compatibility

OUTPUT:

•str - a string

EXAMPLES:

```
sage: repr_lincomb([( 'a',1), ( 'b',-2), ( 'c',3)])
'a - 2*b + 3*c'
sage: repr_lincomb([( 'a',0), ( 'b',-2), ( 'c',3)])
'-2*b + 3*c'
sage: repr_lincomb([( 'a',0), ( 'b',2), ( 'c',3)])
'2*b + 3*c'
sage: repr_lincomb([( 'a',1), ( 'b',0), ( 'c',3)])
'a + 3*c'
sage: repr_lincomb([( 'a',-1), ( 'b','2+3*x'), ( 'c',3)])
'-a + (2+3*x)*b + 3*c'
sage: repr_lincomb([( 'a', '1+x^2'), ( 'b', '2+3*x'), ( 'c', 3)])
'(1+x^2)*a + (2+3*x)*b + 3*c'
sage: repr_lincomb([( 'a', '1+x^2'), ( 'b', '-2+3*x'), ( 'c', 3)])
'(1+x^2)*a + (-2+3*x)*b + 3*c'
sage: repr_lincomb([( 'a', 1), ( 'b', -2), ( 'c', -3)])
'a - 2*b - 3*c'
sage: t = PolynomialRing(RationalField(), 't').gen()
sage: repr_lincomb([( 'a', -t), ( 's', t - 2), ( 't', t^2 + 2)])
'-t*a + (t-2)*s + (t^2+2)*t'
```

Examples for scalar_mult:

```
sage: repr_lincomb([( 'a',1), ( 'b',2), ( 'c',3)], scalar_mult='*')
'a + 2*b + 3*c'
sage: repr_lincomb([( 'a',2), ( 'b',0), ( 'c',-3)], scalar_mult='**')
'2**a - 3**c'
sage: repr_lincomb([( 'a',-1), ( 'b',2), ( 'c',3)], scalar_mult='**')
'-a + 2**b + 3**c'
```

Examples for scalar_mult and is_latex:

```
sage: repr_lincomb([( 'a',-1), ( 'b',2), ( 'c',3)], is_latex=True)
'-a + 2b + 3c'
sage: repr_lincomb([( 'a',-1), ( 'b',-1), ( 'c',3)], is_latex=True, scalar_mult='*')
'-a - b + 3c'
sage: repr_lincomb([( 'a',-1), ( 'b',2), ( 'c',-3)], is_latex=True, scalar_mult='**')
'-a + 2**b - 3**c'
sage: repr_lincomb([( 'a',-2), ( 'b',-1), ( 'c',-3)], is_latex=True, latex_scalar_mult='*')
'-2*a - b - 3*c'
```

Examples for strip_one:

```
sage: repr_lincomb([ ( 'a',1), (1,-2), ( '3',3) ])
'a - 2*1 + 3*3'
sage: repr_lincomb([ ( 'a',-1), (1,1), ( '3',3) ])
'-a + 1 + 3*3'
sage: repr_lincomb([ ( 'a',1), (1,-2), ( '3',3) ], strip_one = True)
'a - 2 + 3*3'
sage: repr_lincomb([ ( 'a',-1), (1,1), ( '3',3) ], strip_one = True)
'-a + 1 + 3*3'
sage: repr_lincomb([ ( 'a',1), (1,-1), ( '3',3) ], strip_one = True)
'a - 1 + 3*3'
```

Examples for repr_monomial:

```
sage: repr_lincomb([('a',1), ('b',2), ('c',3)], repr_monomial = lambda s: s+"1")
'a1 + 2*b1 + 3*c1'
```

TESTS:

For backward compatibility (will be deprecated):

```
sage: repr_lincomb(['a','b','c'], [1,2,3])
doctest:...: DeprecationWarning: calling 'repr_lincomb(monoms, coeffs)' is deprecated; please sp
See http://trac.sagemath.org/12484 for details.
'a + 2*b + 3*c'
```

`sage.misc.misc.sage_makedirs(dir)`

Python version of `mkdir -p`: try to create a directory, and also create all intermediate directories as necessary. Succeed silently if the directory already exists (unlike `os.makedirs()`). Raise other errors (like permission errors) normally.

EXAMPLES:

```
sage: from sage.misc.misc import sage_makedirs
sage: sage_makedirs(DOT_SAGE) # no output
```

The following fails because we are trying to create a directory in place of an ordinary file (the main Sage executable):

```
sage: sage_executable = os.path.join(SAGE_ROOT, 'sage')
sage: sage_makedirs(sage_executable)
Traceback (most recent call last):
...
OSError: ...
```

`sage.misc.misc.self_compose(f,n)`

Return the function f composed with itself n times.

See `nest()` if you want $f(f(\dots(f(x))\dots))$ for known x .

INPUT:

- f – a function of one variable
- n – a nonnegative integer

OUTPUT: A function, the result of composing f with itself n times

EXAMPLES:

```
sage: def f(x): return x^2 + 1
sage: g = self_compose(f, 3)
sage: x = var('x')
sage: g(x)
((x^2 + 1)^2 + 1)^2 + 1

sage: def f(x): return x + 1
sage: g = self_compose(f, 10000)
sage: g(0)
10000

sage: x = var('x')
sage: self_compose(sin, 0)(x)
x
```


`sage.misc.misc.set_verbose(level, files='all')`

Set the global Sage verbosity level.

INPUT:

- `level` - an integer between 0 and 2, inclusive.
- **files** (default: 'all'): list of files to make verbose, or 'all' to make ALL files verbose (the default).

OUTPUT: changes the state of the verbosity flag and possibly appends to the list of files that are verbose.

EXAMPLES:

```
sage: set_verbose(2)
sage: verbose("This is Sage.", level=1) # not tested
VERBOSE1 (?): This is Sage.
sage: verbose("This is Sage.", level=2) # not tested
VERBOSE2 (?): This is Sage.
sage: verbose("This is Sage.", level=3) # not tested
[no output]
sage: set_verbose(0)
```

`sage.misc.misc.set_verbose_files(file_name)`

`sage.misc.misc.sourcefile(object)`

Work out which source or compiled file an object was defined in.

`sage.misc.misc.srange(start, end=None, step=1, universe=None, check=True, include_endpoint=False, endpoint_tolerance=1e-05)`

Return list of numbers $a, a+step, \dots, a+k*step$, where $a+k*step < b$ and $a+(k+1)*step \geq b$ over exact rings, and makes a best attempt for inexact rings (see note below).

This provides one way to iterate over Sage integers as opposed to Python int's. It also allows you to specify step sizes for such an iteration. Note, however, that what is returned is a full list of Integers and not an iterator. It is potentially much slower than the Python range function, depending on the application. The function `xrange()` provides an iterator with similar functionality which would usually be more efficient than using `srange()`.

INPUT:

- `a` - number
- `b` - number (default: None)
- `step` - number (default: 1)
- `universe` - Parent or type where all the elements should live (default: deduce from inputs)
- `check` - make sure `a`, `b`, and `step` all lie in the same universe
- `include_endpoint` - whether or not to include the endpoint (default: False)
- `endpoint_tolerance` - used to determine whether or not the endpoint is hit for inexact rings (default 1e-5)

OUTPUT:

- list

If `b` is None, then `b` is set equal to `a` and `a` is set equal to the 0 in the parent of `b`.

Unlike `range`, `a` and `b` can be any type of numbers, and the resulting list involves numbers of that type.

Note: The list elements are computed via repeated addition rather than multiplication, which may produce slightly different results with inexact rings. For example:

```
sage: sum([1.1] * 10) == 1.1 * 10
False
```

Also, the question of whether the endpoint is hit exactly for a given $a + k \cdot \text{step}$ is fuzzy for an inexact ring. If $a + k \cdot \text{step} = b$ for some k within `endpoint_tolerance` of being integral, it is considered an exact hit, thus avoiding spurious values falling just below the endpoint.

Note: This function is called `srange` to distinguish it from the built-in Python `range` command. The `s` at the beginning of the name stands for “Sage”.

EXAMPLES:

```
sage: v = srange(5); v
[0, 1, 2, 3, 4]
sage: type(v[2])
<type 'sage.rings.integer.Integer'>
sage: srange(1, 10)
[1, 2, 3, 4, 5, 6, 7, 8, 9]
sage: srange(10, 1, -1)
[10, 9, 8, 7, 6, 5, 4, 3, 2]
sage: srange(10, 1, -1, include_endpoint=True)
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
sage: srange(1, 10, universe=RDF)
[1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0]

sage: srange(1, 10, 1/2)
[1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 19/2]
sage: srange(1, 5, 0.5)
[1.0000000000000000, 1.5000000000000000, 2.0000000000000000, 2.5000000000000000, 3.0000000000000000, 3.5000000000000000]
sage: srange(0, 1, 0.4)
[0.0000000000000000, 0.4000000000000000, 0.8000000000000000]
sage: srange(1.0, 5.0, include_endpoint=True)
[1.0000000000000000, 2.0000000000000000, 3.0000000000000000, 4.0000000000000000, 5.0000000000000000]
sage: srange(1.0, 1.1)
[1.0000000000000000]
sage: srange(1.0, 1.0)
[]
sage: V = VectorSpace(QQ, 2)
sage: srange(V([0,0]), V([5,5]), step=V([2,2]))
[(0, 0), (2, 2), (4, 4)]
```

Including the endpoint:

```
sage: srange(0, 10, step=2, include_endpoint=True)
[0, 2, 4, 6, 8, 10]
sage: srange(0, 10, step=3, include_endpoint=True)
[0, 3, 6, 9]
```

Try some inexact rings:

```
sage: srange(0.5, 1.1, 0.1, universe=RDF, include_endpoint=False)
[0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999, 0.9999999999999999]
sage: srange(0.5, 1, 0.1, universe=RDF, include_endpoint=False)
[0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999]
sage: srange(0.5, 0.9, 0.1, universe=RDF, include_endpoint=False)
[0.5, 0.6, 0.7, 0.7999999999999999]
sage: srange(0, 1.1, 0.1, universe=RDF, include_endpoint=True)
[0.0, 0.1, 0.2, 0.30000000000000004, 0.4, 0.5, 0.6, 0.7, 0.7999999999999999, 0.8999999999999999]
```

```
sage: xrange(0, 0.2, 0.1, universe=RDF, include_endpoint=True)
[0.0, 0.1, 0.2]
sage: xrange(0, 0.3, 0.1, universe=RDF, include_endpoint=True)
[0.0, 0.1, 0.2, 0.3]
```

TESTS:

These are doctests from [trac ticket #6409](#):

```
sage: xrange(1,0,include_endpoint=True)
[]
sage: xrange(1,QQ(0),include_endpoint=True)
[]
sage: xrange(3,0,-1,include_endpoint=True)
[3, 2, 1, 0]
sage: xrange(1,1,0) # trac ticket #11753
Traceback (most recent call last):
...
ValueError: xrange() step argument must not be zero
```

`sage.misc.misc.strunc(s, n=60)`

Truncate at first space after position n, adding '...' if nontrivial truncation.

`sage.misc.misc.subsets(X)`

Iterator over the *list* of all subsets of the iterable X, in no particular order. Each list appears exactly once, up to order.

INPUT:

- X - an iterable

OUTPUT: iterator of lists

EXAMPLES:

```
sage: list(powerset([1,2,3]))
[[], [1], [2], [1, 2], [3], [1, 3], [2, 3], [1, 2, 3]]
sage: [z for z in powerset([0,[1,2]])]
[[], [0], [[1, 2]], [0, [1, 2]]]
```

Iterating over the power set of an infinite set is also allowed:

```
sage: i = 0
sage: for x in powerset(ZZ):
...     if i > 10:
...         break
...     else:
...         i += 1
...     print x,
[] [0] [1] [0, 1] [-1] [0, -1] [1, -1] [0, 1, -1] [2] [0, 2] [1, 2]
```

You may also use `subsets` as an alias for `powerset`:

```
sage: subsets([1,2,3])
<generator object powerset at 0x...>
sage: list(subsets([1,2,3]))
[[], [1], [2], [1, 2], [3], [1, 3], [2, 3], [1, 2, 3]]
```

The reason we return lists instead of sets is that the elements of sets must be hashable and many structures on which one wants the powerset consist of non-hashable objects.

AUTHORS:

- William Stein
- Nils Bruin (2006-12-19): rewrite to work for not-necessarily finite objects X.

```
sage.misc.misc.sxrange(start, end=None, step=1, universe=None, check=True, include_endpoint=False, endpoint_tolerance=1e-05)
```

Return an iterator over numbers a , $a+step$, ..., $a+k*step$, where $a+k*step < b$ and $a+(k+1)*step > b$.

INPUT: universe – Parent or type where all the elements should live (default: deduce from inputs) check – make sure a , b , and $step$ all lie in the same universe include_endpoint – whether or not to include the endpoint (default: False) endpoint_tolerance – used to determine whether or not the endpoint is hit for inexact rings (default 1e-5)

- a - number
- b - number
- step - number (default: 1)

OUTPUT: iterator

Unlike range, a and b can be any type of numbers, and the resulting iterator involves numbers of that type.

See also:

`srange()`

Note: This function is called `sxrange` to distinguish it from the builtin Python `xrange` command.

EXAMPLES:

```
sage: list(xsrange(1,10))
[1, 2, 3, 4, 5, 6, 7, 8, 9]
sage: Q = RationalField()
sage: list(xsrange(1, 10, Q('1/2'))))
[1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 19/2]
sage: list(xsrange(1, 5, 0.5))
[1.0000000000000000, 1.5000000000000000, 2.0000000000000000, 2.5000000000000000, 3.0000000000000000, 3.5000000000000000]
sage: list(xsrange(0, 1, 0.4))
[0.0000000000000000, 0.4000000000000000, 0.8000000000000000]
```

Negative ranges are also allowed:

```
sage: list(xsrange(4,1,-1))
[4, 3, 2]
sage: list(sxrange(4,1,-1))
[4, 3, 2]
sage: list(sxrange(4,1,-1/2))
[4, 7/2, 3, 5/2, 2, 3/2]
```

TESTS:

These are doctests from trac ticket #6409:

```
sage: list(xsrange(1,QQ(0),include_endpoint=True))
[]
sage: list(xsrange(1,QQ(0),-1,include_endpoint=True))
[1, 0]
sage: xsrange(1,1,0) # trac ticket #11753
```

```
Traceback (most recent call last):
...
ValueError: xrange() step argument must not be zero
```

`sage.misc.misc.to_gmp_hex(n)`

`sage.misc.misc.todo(msg='')`

`sage.misc.misc.typecheck(x, C, var='x')`

Check that `x` is of instance `C`. If not raise a `TypeError` with an error message.

`sage.misc.misc.union(x, y=None)`

Return the union of `x` and `y`, as a list. The resulting list need not be sorted and can change from call to call.

INPUT:

- `x` - iterable
- `y` - iterable (may optionally omitted)

OUTPUT: list

EXAMPLES:

```
sage: answer = union([1,2,3,4], [5,6]); answer
[1, 2, 3, 4, 5, 6]
sage: union([1,2,3,4,5,6], [5,6]) == answer
True
sage: union((1,2,3,4,5,6), [5,6]) == answer
True
sage: union((1,2,3,4,5,6), set([5,6])) == answer
True
```

`sage.misc.misc.uniq(x)`

Return the sublist of all elements in the list `x` that is sorted and is such that the entries in the sublist are unique.

EXAMPLES:

```
sage: v = uniq([1,1,8,-5,3,-5,'a','x','a'])
sage: v
# potentially random ordering of output
['a', 'x', -5, 1, 3, 8]
sage: set(v) == set(['a', 'x', -5, 1, 3, 8])
True
```

`sage.misc.misc.unset_verbose_files(file_name)`

`sage.misc.misc.verbose(msg='', t=0, level=1, caller_name=None)`

Print a message if the current verbosity is at least level.

INPUT:

- `msg` - str, a message to print
- `t` - int, optional, if included, will also print `cputime(t)`, - which is the time since time `t`. Thus `t` should have been obtained with `t=cputime()`
- `level` - int, (default: 1) the verbosity level of what we are printing
- `caller_name` - string (default: None), the name of the calling function; in most cases Python can deduce this, so it need not be provided.

OUTPUT: possibly prints a message to stdout; also returns `cputime()`

EXAMPLE:

```
sage: set_verbose(1)
sage: t = cputime()
sage: t = verbose("This is Sage.", t, level=1, caller_name="william")      # not tested
VERBOSE1 (william): This is Sage. (time = 0.0)
sage: set_verbose(0)
```

`sage.misc.misc.walltime(t=0)`

Return the wall time in second, or with optional argument `t`, return the wall time since time `t`. “Wall time” means the time on a wall clock, i.e., the actual time.

INPUT:

- `t` - (optional) float, time in CPU seconds

OUTPUT:

- float - time in seconds

EXAMPLES:

```
sage: w = walltime()
sage: F = factor(2^199-1)
sage: walltime(w)      # somewhat random
0.8823847770690918
```

`sage.misc.misc.word_wrap(s, ncols=85)`

`sage.misc.misc.xsrange(start, end=None, step=1, universe=None, check=True, include_endpoint=False, endpoint_tolerance=1e-05)`

Return an iterator over numbers `a`, `a+step`, ..., `a+k*step`, where `a+k*step < b` and `a+(k+1)*step > b`.

INPUT: `universe` – Parent or type where all the elements should live (default: deduce from inputs) `check` – make sure `a`, `b`, and `step` all lie in the same universe `include_endpoint` – whether or not to include the endpoint (default: False) `endpoint_tolerance` – used to determine whether or not the endpoint is hit for inexact rings (default 1e-5)

- `a` - number
- `b` - number
- `step` - number (default: 1)

OUTPUT: iterator

Unlike `range`, `a` and `b` can be any type of numbers, and the resulting iterator involves numbers of that type.

See also:

`srange()`

Note: This function is called `xsrange` to distinguish it from the builtin Python `xrange` command.

EXAMPLES:

```
sage: list(xsrange(1,10))
[1, 2, 3, 4, 5, 6, 7, 8, 9]
sage: Q = RationalField()
sage: list(xsrange(1, 10, Q('1/2'))))
[1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2, 5, 11/2, 6, 13/2, 7, 15/2, 8, 17/2, 9, 19/2]
sage: list(xsrange(1, 5, 0.5))
[1.0000000000000000, 1.5000000000000000, 2.0000000000000000, 2.5000000000000000, 3.0000000000000000, 3.5000000000000000]
```

```
sage: list(xsrange(0, 1, 0.4))
[0.0000000000000000, 0.4000000000000000, 0.8000000000000000]
```

Negative ranges are also allowed:

```
sage: list(xsrange(4, 1, -1))
[4, 3, 2]
sage: list(xsrange(4, 1, -1))
[4, 3, 2]
sage: list(xsrange(4, 1, -1/2))
[4, 7/2, 3, 5/2, 2, 3/2]
```

TESTS:

These are doctests from trac ticket #6409:

```
sage: list(xsrange(1, QQ(0), include_endpoint=True))
[]
sage: list(xsrange(1, QQ(0), -1, include_endpoint=True))
[1, 0]
sage: xsrange(1, 1, 0) # trac ticket #11753
Traceback (most recent call last):
...
ValueError: xsrange() step argument must not be zero
```

2.9.2 Miscellaneous functions (Cython)

This file contains support for products, running totals, balanced sums, and bitset tests.

AUTHORS:

- William Stein (2005)
- Joel B. Mohler (2007-10-03): Reimplemented in Cython and optimized
- Robert Bradshaw (2007-10-26): Balanced product tree, other optimizations, (lazy) generator support
- Robert Bradshaw (2008-03-26): Balanced product tree for generators and iterators
- Stefan van Zwam (2013-06-06): Added bitset tests, some docstring cleanup

```
class sage.misc.misc_c.NonAssociative(left, right=None)
```

This class is to test the balance nature of prod.

EXAMPLES:

```
sage: from sage.misc.misc_c import NonAssociative
sage: L = [NonAssociative(label) for label in 'abcdef']
sage: prod(L)
(((a*b)*c)*((d*e)*f))
sage: L = [NonAssociative(label) for label in range(20)]
sage: prod(L, recursion_cutoff=5)
((((((0*1)*2)*3)*4)*(((5*6)*7)*8)*9))*((((10*11)*12)*13)*14)*(((15*16)*17)*18)*19)))
sage: prod(L, recursion_cutoff=1)
((((((0*1)*2)*(3*4))*((5*6)*7)*(8*9)))*(((10*11)*12)*(13*14)))*(((15*16)*17)*(18*19))))
sage: L = [NonAssociative(label) for label in range(14)]
sage: prod(L, recursion_cutoff=1)
((((0*1)*(2*3))*((4*5)*6))*((7*8)*(9*10))*((11*12)*13)))
```

```
sage.misc.misc_c.balanced_sum(x, z=None, recursion_cutoff=5)
```

Return the sum of the elements in the list `x`. If optional argument `z` is not given, start the sum with the first element of the list, otherwise use `z`. The empty product is the int 0 if `z` is not specified, and is `z` if given. The sum is computed recursively, where the sum is split up if the list is greater than `recursion_cutoff`. `recursion_cutoff` must be at least 3.

This assumes that your addition is associative; we don't promise which end of the list we start at.

EXAMPLES:

```
sage: balanced_sum([1, 2, 34])
37
sage: balanced_sum([2, 3], 5)
10
sage: balanced_sum((1, 2, 3), 5)
11
```

Order should be preserved:

```
sage: balanced_sum([[i] for i in range(10)], [], recursion_cutoff=3)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

We make copies when appropriate so that we don't accidentally modify the arguments:

```
sage: range(10e4) == balanced_sum([[i] for i in range(10e4)], [])
True
sage: range(10e4) == balanced_sum([[i] for i in range(10e4)], [])
True
```

TESTS:

```
sage: balanced_sum((1..3)) # nonempty, z=None
6
sage: balanced_sum((1..-1)) # empty, z=None
0
sage: balanced_sum((1..3), 5) # nonempty, z is not None
11
sage: balanced_sum((1..-1), 5) # empty, z is not None
5
```

AUTHORS:

- Joel B. Mohler (2007-10-03): Reimplemented in Cython and optimized
- Robert Bradshaw (2007-10-26): Balanced product tree, other optimizations, (lazy) generator support

```
sage.misc.misc_c.iterator_prod(L, z=None)
```

Attempt to do a balanced product of an arbitrary and unknown length sequence (such as a generator). Intermediate multiplications are always done with subproducts of the same size (measured by the number of original factors) up until the iterator terminates. This is optimal when and only when there are exactly a power of two number of terms.

A `StopIteration` is raised if the iterator is empty and `z` is not given.

EXAMPLES:

```
sage: from sage.misc.misc_c import iterator_prod
sage: iterator_prod(1..5)
120
sage: iterator_prod([], z='anything')
'anything'

sage: from sage.misc.misc_c import NonAssociative
```



```

sage: L = [NonAssociative(label) for label in 'abcdef']
sage: iterator_prod(L)
((a*b)*(c*d))*(e*f)

```

`sage.misc.misc_c.normalize_index(key, size)`

Normalize an index key and return a valid index or list of indices within the range(0, size).

INPUT:

- key – the index key, which can be either an integer, a tuple/list of integers, or a slice.
- size – the size of the collection

OUTPUT:

A tuple (SINGLE, VALUE), where SINGLE is True (i.e., 1) if VALUE is an integer and False (i.e., 0) if VALUE is a list.

EXAMPLES:

```

sage: from sage.misc.misc_c import normalize_index
sage: normalize_index(-6,5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index(-5,5)
[0]
sage: normalize_index(-4,5)
[1]
sage: normalize_index(-3,5)
[2]
sage: normalize_index(-2,5)
[3]
sage: normalize_index(-1,5)
[4]
sage: normalize_index(0,5)
[0]
sage: normalize_index(1,5)
[1]
sage: normalize_index(2,5)
[2]
sage: normalize_index(3,5)
[3]
sage: normalize_index(4,5)
[4]
sage: normalize_index(5,5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index(6,5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index((4,-6),5)
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index((-2,3),5)
[3, 3]
sage: normalize_index((5,0),5)

```

```
Traceback (most recent call last):
...
IndexError: index out of range
sage: normalize_index((-5,2),5)
[0, 2]
sage: normalize_index((0,-2),5)
[0, 3]
sage: normalize_index((2,-3),5)
[2, 2]
sage: normalize_index((3,3),5)
[3, 3]
sage: normalize_index((-2,-5),5)
[3, 0]
sage: normalize_index((-2,-4),5)
[3, 1]
sage: normalize_index([-2,-1,3],5)
[3, 4, 3]
sage: normalize_index([4,2,1],5)
[4, 2, 1]
sage: normalize_index([-2,-3,-4],5)
[3, 2, 1]
sage: normalize_index([3,-2,-3],5)
[3, 3, 2]
sage: normalize_index([-5,2,-3],5)
[0, 2, 2]
sage: normalize_index([4,4,-5],5)
[4, 4, 0]
sage: s=slice(None,None,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,None,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,None,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,-2,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,-2,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,-2,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,4,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,4,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(None,4,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,None,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,None,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,None,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,-2,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,-2,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,-2,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,4,None); normalize_index(s,5)==range(5)[s]
```

```

True
sage: s=slice(-2,4,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(-2,4,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,None,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,None,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,None,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,-2,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,-2,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,-2,4); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,4,None); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,4,-2); normalize_index(s,5)==range(5)[s]
True
sage: s=slice(4,4,4); normalize_index(s,5)==range(5)[s]
True

```

`sage.misc.misc_c.prod(x, z=None, recursion_cutoff=5)`

Return the product of the elements in the list x.

If optional argument z is not given, start the product with the first element of the list, otherwise use z. The empty product is the int 1 if z is not specified, and is z if given.

This assumes that your multiplication is associative; we don't promise which end of the list we start at.

EXAMPLES:

```

sage: prod([1,2,34])
68
sage: prod([2,3], 5)
30
sage: prod((1,2,3), 5)
30
sage: F = factor(-2006); F
-1 * 2 * 17 * 59
sage: prod(F)
-2006

```

AUTHORS:

- Joel B. Mohler (2007-10-03): Reimplemented in Cython and optimized
- Robert Bradshaw (2007-10-26): Balanced product tree, other optimizations, (lazy) generator support
- Robert Bradshaw (2008-03-26): Balanced product tree for generators and iterators

`sage.misc.misc_c.running_total(L, start=None)`

Return a list where the i-th entry is the sum of all entries up to (and including) i.

INPUT:

- L – the list
- start – (optional) a default start value

EXAMPLES:

```
sage: running_total(range(5))
[0, 1, 3, 6, 10]
sage: running_total("abcdef")
['a', 'ab', 'abc', 'abcd', 'abcde', 'abcdef']
sage: running_total("abcdef", start="%")
['%a', '%ab', '%abc', '%abcd', '%abcde', '%abcdef']
sage: running_total([1..10], start=100)
[101, 103, 106, 110, 115, 121, 128, 136, 145, 155]
sage: running_total([])
[]
```

2.10 Lazyness

2.10.1 Lazy attributes

AUTHORS:

- Nicolas Thiery (2008): Initial version
- Nils Bruin (2013-05): Cython version

```
class sage.misc.lazy_attribute.lazy_attribute(f)
    Bases: sage.misc.lazy_attribute._lazy_attribute
```

A lazy attribute for an object is like a usual attribute, except that, instead of being computed when the object is constructed (i.e. in `__init__`), it is computed on the fly the first time it is accessed.

For constant values attached to an object, lazy attributes provide a shorter syntax and automatic caching (unlike methods), while playing well with inheritance (like methods): a subclass can easily override a given attribute; you don't need to call the super class constructor, etc.

Technically, a `lazy_attribute` is a non-data descriptor (see Invoking Descriptors in the Python reference manual).

EXAMPLES:

We create a class whose instances have a lazy attribute `x`:

```
sage: class A(object):
.....:     def __init__(self):
.....:         self.a=2 # just to have some data to calculate from
.....:
.....:     @lazy_attribute
.....:     def x(self):
.....:         print "calculating x in A"
.....:         return self.a + 1
.....:
```

For an instance `a` of `A`, `a.x` is calculated the first time it is accessed, and then stored as a usual attribute:

```
sage: a = A()
sage: a.x
calculating x in A
3
sage: a.x
3
```

Implementation details

We redo the same example, but opening the hood to see what happens to the internal dictionary of the object:

```
sage: a = A()
sage: a.__dict__
{'a': 2}
sage: a.x
calculating x in A
3
sage: a.__dict__
{'a': 2, 'x': 3}
sage: a.x
3
sage: timeit('a.x') # random
625 loops, best of 3: 89.6 ns per loop
```

This shows that, after the first calculation, the attribute `x` becomes a usual attribute; in particular, there is no time penalty to access it.

A lazy attribute may be set as usual, even before its first access, in which case the lazy calculation is completely ignored:

```
sage: a = A()
sage: a.x = 4
sage: a.x
4
sage: a.__dict__
{'a': 2, 'x': 4}
```

Class binding results in the lazy attribute itself:

```
sage: A.x
<sage.misc.lazy_attribute.lazy_attribute object at ...>
```

Conditional definitions

The function calculating the attribute may return `NotImplemented` to declare that, after all, it is not able to do it. In that case, the attribute lookup proceeds in the super class hierarchy:

```
sage: class B(A):
....:     @lazy_attribute
....:     def x(self):
....:         if hasattr(self, "y"):
....:             print "calculating x from y in B"
....:             return self.y
....:         else:
....:             print "y not there; B does not define x"
....:             return NotImplemented
....:
sage: b = B()
sage: b.x
y not there; B does not define x
calculating x in A
3
sage: b = B()
sage: b.y = 1
sage: b.x
```

```
calculating x from y in B
1
```

Attribute existence testing

Testing for the existence of an attribute with `hasattr` currently always triggers its full calculation, which may not be desirable when the calculation is expensive:

```
sage: a = A()
sage: hasattr(a, "x")
calculating x in A
True
```

It would be great if we could take over the control somehow, if at all possible without a special implementation of `hasattr`, so as to allow for something like:

```
sage: class A(object):
.....:     @lazy_attribute
.....:     def x(self, existence_only=False):
.....:         if existence_only:
.....:             print "testing for x existence"
.....:             return True
.....:         else:
.....:             print "calculating x in A"
.....:             return 3
.....:
sage: a = A()
sage: hasattr(a, "x") # todo: not implemented
testing for x existence
sage: a.x
calculating x in A
3
sage: a.x
3
```

Here is a full featured example, with both conditional definition and existence testing:

```
sage: class B(A):
.....:     @lazy_attribute
.....:     def x(self, existence_only=False):
.....:         if hasattr(self, "y"):
.....:             if existence_only:
.....:                 print "testing for x existence in B"
.....:                 return True
.....:             else:
.....:                 print "calculating x from y in B"
.....:                 return self.y
.....:         else:
.....:             print "y not there; B does not define x"
.....:             return NotImplemented
.....:
sage: b = B()
sage: hasattr(b, "x") # todo: not implemented
y not there; B does not define x
testing for x existence
True
sage: b.x
y not there; B does not define x
```

```

calculating x in A
3
sage: b = B()
sage: b.y = 1
sage: hasattr(b, "x") # todo: not implemented
testing for x existence in B
True
sage: b.x
calculating x from y in B
1

```

lazy attributes and introspection

Todo

Make the following work nicely:

```

sage: b.x?                # todo: not implemented
sage: b.x??               # todo: not implemented

```

Right now, the first one includes the doc of this class, and the second one brings up the code of this class, both being not very useful.

TESTS:

Partial support for old style classes

Old style and new style classes play a bit differently with @property and attribute setting:

```

sage: class A:
....:     @property
....:     def x(self):
....:         print "calculating x"
....:         return 3
....:
sage: a = A()
sage: a.x = 4
sage: a.__dict__
{'x': 4}
sage: a.x
4
sage: a.__dict__['x']=5
sage: a.x
5

sage: class A (object):
....:     @property
....:     def x(self):
....:         print "calculating x"
....:         return 3
....:
sage: a = A()
sage: a.x = 4
Traceback (most recent call last):
...

```

```
AttributeError: can't set attribute
sage: a.__dict__
{}
sage: a.x
calculating x
3
sage: a.__dict__['x']=5
sage: a.x
calculating x
3
```

In particular, lazy_attributes need to be implemented as non-data descriptors for new style classes, so as to leave access to setattr. We now check that this implementation also works for old style classes (conditional definition does not work yet):

```
sage: class A:
.....:     def __init__(self):
.....:         self.a=2 # just to have some data to calculate from
.....:
.....:     @lazy_attribute
.....:     def x(self):
.....:         print "calculating x"
.....:         return self.a + 1
.....:
sage: a = A()
sage: a.__dict__
{'a': 2}
sage: a.x
calculating x
3
sage: a.__dict__
{'a': 2, 'x': 3}
sage: a.x
3
sage: timeit('a.x') # random
625 loops, best of 3: 115 ns per loop

sage: a = A()
sage: a.x = 4
sage: a.x
4
sage: a.__dict__
{'a': 2, 'x': 4}

sage: class B(A):
.....:     @lazy_attribute
.....:     def x(self):
.....:         if hasattr(self, "y"):
.....:             print "calculating x from y in B"
.....:             return self.y
.....:         else:
.....:             print "y not there; B does not define x"
.....:             return NotImplemented
.....:
sage: b = B()
sage: b.x                                     # todo: not implemented
y not there; B does not define x
calculating x in A
```



```

3
sage: b = B()
sage: b.y = 1
sage: b.x
calculating x from y in B
1

```

Lazy attributes and Cython

This attempts to check that lazy attributes work with built-in functions like `len` methods:

```

sage: class A:
.....:     def __len__(x):
.....:         return int(5)
.....:     len = lazy_attribute(len)
.....:
sage: A().len
5

```

Since [trac ticket #11115](#), extension classes derived from `Parent` can inherit a lazy attribute, such as `element_class`:

```

sage: cython_code = ["from sage.structure.parent cimport Parent",
.....: "from sage.structure.element cimport Element",
.....: "cdef class MyElement(Element): pass",
.....: "cdef class MyParent(Parent):",
.....: "    Element = MyElement"]
sage: cython('\n'.join(cython_code))
sage: P = MyParent(category=Rings())
sage: P.element_class      # indirect doctest
<type '...MyElement'>

```

About descriptor specifications

The specifications of descriptors (see 3.4.2.3 Invoking Descriptors in the Python reference manual) are incomplete w.r.t. inheritance, and maybe even ill-implemented. We illustrate this on a simple class hierarchy, with an instrumented descriptor:

```

sage: class descriptor(object):
.....:     def __get__(self, obj, cls):
.....:         print cls
.....:         return 1
sage: class A(object):
.....:     x = descriptor()
sage: class B(A):
.....:     pass
.....:

```

This is fine:

```

sage: A.x
<class '__main__.A'>
1

```

The behaviour for the following case is not specified (see Instance Binding) when `x` is not in the dictionary of `B` but in that of some super category:

```
sage: B().x
<class '__main__.B'>
1
```

It would seem more natural (and practical!) to get A rather than B.

From the specifications for Super Binding, it would be expected to get A and not B as cls parameter:

```
sage: super(B, B()).x
<class '__main__.B'>
1
```

Due to this, the natural implementation runs into an infinite loop in the following example:

```
sage: class A(object):
.....:     @lazy_attribute
.....:     def unimplemented_A(self):
.....:         return NotImplemented
.....:     @lazy_attribute
.....:     def unimplemented_AB(self):
.....:         return NotImplemented
.....:     @lazy_attribute
.....:     def unimplemented_B_implemented_A(self):
.....:         return 1
.....:
sage: class B(A):
.....:     @lazy_attribute
.....:     def unimplemented_B(self):
.....:         return NotImplemented
.....:     @lazy_attribute
.....:     def unimplemented_AB(self):
.....:         return NotImplemented
.....:     @lazy_attribute
.....:     def unimplemented_B_implemented_A(self):
.....:         return NotImplemented
.....:
sage: class C(B):
.....:     pass
.....:
```

This is the simplest case where, without workaround, we get an infinite loop:

```
sage: hasattr(B(), "unimplemented_A") # todo: not implemented
False
```

Todo

Improve the error message:

```
sage: B().unimplemented_A # todo: not implemented
Traceback (most recent call last):
...
AttributeError: 'super' object has no attribute 'unimplemented_A'
```

We now make some systematic checks:

```
sage: B().unimplemented_A
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_A'
```

```

sage: B().unimplemented_B
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_B'
sage: B().unimplemented_AB
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_AB'
sage: B().unimplemented_B_implemented_A
1

sage: C().unimplemented_A()
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_A'
sage: C().unimplemented_B()
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_B'
sage: C().unimplemented_AB()
Traceback (most recent call last):
...
AttributeError: '...' object has no attribute 'unimplemented_AB'
sage: C().unimplemented_B_implemented_A # todo: not implemented
1

```

class sage.misc.lazy_attribute.**lazy_class_attribute**(f)

Bases: sage.misc.lazy_attribute.lazy_attribute

A lazy class attribute for an class is like a usual class attribute, except that, instead of being computed when the class is constructed, it is computed on the fly the first time it is accessed, either through the class itself or through one of its objects.

This is very similar to `lazy_attribute` except that the attribute is a class attribute. More precisely, once computed, the lazy class attribute is stored in the class rather than in the object. The lazy class attribute is only computed once for all the objects:

```

sage: class C1(object):
....:     @lazy_class_attribute
....:     def x(cls):
....:         print "computing x"
....:         return 1
sage: C1.x
computing x
1
sage: C1.x
1

```

As for a any usual class attribute it is also possible to access it from an object:

```

sage: b = C1()
sage: b.x
1

```

First access from an object also properly triggers the computation:

```

sage: class C11(object):
....:     @lazy_class_attribute
....:     def x(cls):
....:         print "computing x"

```

```
.....:         return 1
sage: C11().x
computing x
1
sage: C11().x
1
```

..WARNING:

The behavior of lazy class attributes with respect to inheritance is not specified. It currently depends on the evaluation order::

```
sage: class A(object):
.....:     @lazy_class_attribute
.....:     def x(cls):
.....:         print "computing x"
.....:         return str(cls)
.....:     @lazy_class_attribute
.....:     def y(cls):
.....:         print "computing y"
.....:         return str(cls)
sage: class B(A):
.....:     pass

sage: A.x
computing x
"<class '__main__.A'>"
sage: B.x
"<class '__main__.A'>"

sage: B.y
computing y
"<class '__main__.B'>"
sage: A.y
computing y
"<class '__main__.A'>"
sage: B.y
"<class '__main__.B'>"
```

TESTS:

```
sage: "x" in b.__dict__
False
```

2.10.2 Lazy format strings

class `sage.misc.lazy_format.LazyFormat`

Bases: `str`

Lazy format strings

An instance of `LazyFormat` behaves like a usual format string, except that the evaluation of the `__repr__` method of the formatted arguments is postponed until actual printing.

EXAMPLES:

Under normal circumstances, `Lazyformat` strings behave as usual:

```
sage: from sage.misc.lazy_format import LazyFormat
sage: LazyFormat("Got '%s'; expected a list")%3
Got '3'; expected a list
sage: LazyFormat("Got '%s'; expected %s")%(3, 2/3)
Got '3'; expected 2/3
```

To demonstrate the laziness, let us build an object with a broken `__repr__` method:

```
sage: class IDontLikeBeingPrinted(object):
...     def __repr__(self):
...         raise ValueError("Don't ever try to print me !")
```

There is no error when binding a lazy format with the broken object:

```
sage: lf = LazyFormat("<%s>")%IDontLikeBeingPrinted()
```

The error only occurs upon printing:

```
sage: lf
<repr(<sage.misc.lazy_format.LazyFormat at 0x...>) failed: ValueError: Don't ever try to print m
```

Common use case:

Most of the time, `__repr__` methods are only called during user interaction, and therefore need not be fast; and indeed there are objects `x` in Sage such `x.__repr__()` is time consuming.

There are however some uses cases where many format strings are constructed but not actually printed. This includes error handling messages in `unittest` or `TestSuite` executions:

```
sage: QQ._tester().assertTrue(0 in QQ,
...                          "%s doesn't contain 0"%QQ)
```

In the above `QQ.__repr__()` has been called, and the result immediately discarded. To demonstrate this we replace `QQ` in the format string argument with our broken object:

```
sage: QQ._tester().assertTrue(True,
...                          "%s doesn't contain 0"%IDontLikeBeingPrinted())
Traceback (most recent call last):
...
ValueError: Don't ever try to print me !
```

This behavior can induce major performance penalties when testing. Note that this issue does not impact the usual assert:

```
sage: assert True, "%s is wrong"%IDontLikeBeingPrinted()
```

We now check that `LazyFormat` indeed solves the assertion problem:

```
sage: QQ._tester().assertTrue(True,
...                          LazyFormat("%s is wrong")%IDontLikeBeingPrinted())
sage: QQ._tester().assertTrue(False,
...                          LazyFormat("%s is wrong")%IDontLikeBeingPrinted())
Traceback (most recent call last):
...
AssertionError: <unprintable AssertionError object>
```

2.10.3 Lazy imports

This module allows one to lazily import objects into a namespace, where the actual import is delayed until the object is actually called or inspected. This is useful for modules that are expensive to import or may cause circular references, though there is some overhead in its use.

EXAMPLES:

```
sage: from sage.misc.lazy_import import lazy_import
sage: lazy_import('sage.rings.all', 'ZZ')
sage: type(ZZ)
<type 'sage.misc.lazy_import.LazyImport'>
sage: ZZ(4.0)
4
```

By default, a warning is issued if a lazy import module is resolved during Sage's startup. In case a lazy import's sole purpose is to break a circular reference and it is known to be resolved at startup time, one can use the `at_startup` option:

```
sage: lazy_import('sage.rings.all', 'ZZ', at_startup=True)
```

This option can also be used as an intermediate step toward not importing by default a module that is used in several places, some of which can already afford to lazy import the module but not all.

A lazy import that is marked as “`at_startup`” will print a message if it is actually resolved after the startup, so that the developer knows that (s)he can remove the flag:

```
sage: ZZ
Option ``at_startup=True`` for lazy import ZZ not needed anymore
Integer Ring
```

See also:

```
lazy_import(), LazyImport
```

AUTHOR:

- Robert Bradshaw

```
class sage.misc.lazy_import.LazyImport
    Bases: object
```

EXAMPLES:

```
sage: from sage.misc.lazy_import import LazyImport
sage: my_integer = LazyImport('sage.rings.all', 'Integer')
sage: my_integer(4)
4
sage: my_integer('101', base=2)
5
sage: my_integer(3/2)
Traceback (most recent call last):
...
TypeError: no conversion of this rational to integer
```

```
sage.misc.lazy_import.finish_startup()
```

This function must be called exactly once at the end of the Sage import process

TESTS:

```
sage: from sage.misc.lazy_import import finish_startup
sage: finish_startup()
Traceback (most recent call last):
...
AssertionError: finish_startup() must be called exactly once
```

`sage.misc.lazy_import.get_star_imports(module_name)`

Lookup the list of names in a module that would be imported with “import *” either via a cache or actually importing.

EXAMPLES:

```
sage: from sage.misc.lazy_import import get_star_imports
sage: 'get_star_imports' in get_star_imports('sage.misc.lazy_import')
True
sage: 'EllipticCurve' in get_star_imports('sage.schemes.all')
True
```

TESTS:

```
sage: import os, tempfile
sage: fd, cache_file = tempfile.mkstemp()
sage: os.write(fd, 'invalid')
7
sage: os.close(fd)
sage: import sage.misc.lazy_import as lazy
sage: lazy.get_cache_file = (lambda: cache_file)
sage: lazy.star_imports = None
sage: lazy.get_star_imports('sage.schemes.all')
doctest:...: UserWarning: star_imports cache is corrupted
[...]
sage: os.remove(cache_file)
```

`sage.misc.lazy_import.is_during_startup()`

Return whether Sage is currently starting up

OUTPUT:

Boolean

TESTS:

```
sage: from sage.misc.lazy_import import is_during_startup
sage: is_during_startup()
False
```

`sage.misc.lazy_import.lazy_import(module, names, _as=None, namespace=None, overwrite=True, at_startup=False, deprecation=None)`

Create a lazy import object and inject it into the caller’s global namespace. For the purposes of introspection and calling, this is like performing a lazy “from module import name” where the import is delayed until the object actually is used or inspected.

INPUT:

- `module` – a string representing the module to import
- `names` – a string or list of strings representing the names to import from module
- `_as` – (optional) a string or list of strings representing the aliases of the names imported
- `namespace` – the namespace where importing the names; by default, import the names to current namespace

- `overwrite` – (default: `True`) if set to `True` and a name is already in the namespace, overwrite it with the lazy_import-ed name
- `at_startup` – a boolean (default: `False`); whether the lazy import is supposed to be resolved at startup time
- `deprecation` – (optional) if not `None`, a deprecation warning will be issued when the object is actually imported; `deprecation` should be either a trac number (integer) or a pair (`trac_number`, `message`)

See also:

`sage.misc.lazy_import`, `LazyImport`

EXAMPLES:

```
sage: from sage.misc.lazy_import import lazy_import
sage: lazy_import('sage.rings.all', 'ZZ')
sage: type(ZZ)
<type 'sage.misc.lazy_import.LazyImport'>
sage: ZZ(4.0)
4
sage: lazy_import('sage.rings.all', 'RDF', 'my_RDF')
sage: my_RDF._get_object() is RDF
True
sage: my_RDF(1/2)
0.5

sage: lazy_import('sage.all', ['QQ', 'RR'], ['my_QQ', 'my_RR'])
sage: my_QQ._get_object() is QQ
True
sage: my_RR._get_object() is RR
True
```

Upon the first use, the object is injected directly into the calling namespace:

```
sage: lazy_import('sage.all', 'ZZ', 'my_ZZ')
sage: my_ZZ is ZZ
False
sage: my_ZZ(37)
37
sage: my_ZZ is ZZ
True
```

We check that `lazy_import()` also works for methods:

```
sage: class Foo(object):
...     lazy_import('sage.all', 'plot')
sage: class Bar(Foo):
...     pass
sage: type(Foo.__dict__['plot'])
<type 'sage.misc.lazy_import.LazyImport'>
sage: 'EXAMPLES' in Bar.plot.__doc__
True
sage: type(Foo.__dict__['plot'])
<type 'function'>
```

If deprecated then a deprecation warning is issued:

```
sage: lazy_import('sage.all', 'Qp', 'my_Qp', deprecation=14275)
sage: my_Qp(5)
doctest:...: DeprecationWarning:
```


Importing `my_Qp` from here is deprecated. If you need to use it, please import it directly from `sage.rings.finite_fields.finite_field`.
 See <http://trac.sagemath.org/14275> for details.
 5-adic Field with capped relative precision 20

An example of deprecation with a message:

```
sage: lazy_import('sage.all', 'Qp', 'my_Qp_msg', deprecation=(14275, "This is an example."))
sage: my_Qp_msg(5)
doctest:...: DeprecationWarning: This is an example.
See http://trac.sagemath.org/14275 for details.
5-adic Field with capped relative precision 20
```

`sage.misc.lazy_import.save_cache_file()`
 Used to save the cached * import names.

TESTS:

```
sage: import sage.misc.lazy_import
sage: sage.misc.lazy_import.save_cache_file()
```

`sage.misc.lazy_import.test_fake_startup()`
 For testing purposes only.

Switch the startup lazy import guard back on.

EXAMPLES:

```
sage: sage.misc.lazy_import.test_fake_startup()
sage: from sage.misc.lazy_import import lazy_import
sage: lazy_import('sage.rings.all', 'ZZ', 'my_ZZ')
sage: my_ZZ(123)
```

```
-----
Resolving lazy import ZZ during startup
Calling stack:
...
-----
```

```
123
sage: sage.misc.lazy_import.finish_startup()
```

2.10.4 Lazy lists

A lazy list is an iterator that behaves like a list and possesses a cache mechanism. A lazy list is potentially infinite and speed performances of the cache is comparable with Python lists. One major difference with original Python list is that lazy list are immutable. The advantage is that slices share memory.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: P = lazy_list(Primes())
sage: P[100]
547
sage: P[10:34]
lazy list [31, 37, 41, ...]
sage: P[12:23].list()
[41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83]

sage: f = lazy_list((i**2-3*i for i in xrange(10)))
sage: for i in f: print i,
```

```
0 -2 -2 0 4 10 18 28 40 54
sage: i1 = iter(f)
sage: i2 = iter(f)
sage: print next(i1), next(i1)
0 -2
sage: print next(i2), next(i2)
0 -2
sage: print next(i1), next(i2)
-2 -2
```

It is possible to prepend a list to a lazy list:

```
sage: from itertools import count
sage: l = [3,7] + lazy_list(i**2 for i in count())
sage: l
lazy list [3, 7, 0, ...]
```

But, naturally, not the other way around:

```
sage: lazy_list(i-1 for i in count()) + [3,2,5]
Traceback (most recent call last):
...
TypeError: can only add list to lazy_list
```

class sage.misc.lazy_list.**lazy_list**

Bases: object

Lazy list.

INPUT:

- iterator – an iterable or an iterator
- cache – None (default) or a list - used to initialize the cache.
- start, stop, step – None (default) or a non-negative integer - parameters for slices

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: from itertools import count
sage: m = lazy_list(count()); m
lazy list [0, 1, 2, ...]

sage: m2 = lazy_list(count(), start=8, stop=20551, step=2)
sage: m2
lazy list [8, 10, 12, ...]

sage: x = iter(m)
sage: print next(x), next(x), next(x)
0 1 2
sage: y = iter(m)
sage: print next(y), next(y), next(y)
0 1 2
sage: print next(x), next(y)
3 3
sage: loads(dumps(m))
lazy list [0, 1, 2, ...]
```

Note:

- `lazy_list` interprets the constant `(size_t)-1` as infinity
- all entry indices are strictly less than `stop` so that `lazy_list` agrees with `range(start, stop)`

get(*i*)

Return the element at position *i*.

If the index is not an integer, then raise a `TypeError`. If the argument is negative then raise a `ValueError`. Finally, if the argument is beyond the size of that lazy list it raises a `IndexError`.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: from itertools import chain, repeat
sage: f = lazy_list(chain(iter([1,2,3]), repeat('a')))
sage: f.get(0)
1
sage: f.get(3)
'a'
sage: f.get(0)
1
sage: f.get(4)
'a'

sage: g = f[:10]
sage: g.get(5)
'a'
sage: g.get(10)
Traceback (most recent call last):
...
IndexError: lazy list index out of range
sage: g.get(1/2)
Traceback (most recent call last):
...
TypeError: rational is not an integer
```

info()

Print information about `self` on standard output.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: P = lazy_list(iter(Primes()))[10:21474838:4]
sage: P.info()
cache length 0
start          10
stop          21474838
step           4
sage: P[0]
31
sage: P.info()
cache length 11
start          10
stop          21474838
step           4
```

list()

Return the list made of the elements of `self`.

Note: If the iterator is sufficiently large, this will build a list of length `(size_t)-1` which should be

beyond the capacity of your RAM!

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: P = lazy_list(iter(Primes()))
sage: P[2:143:5].list()
[5, 19, 41, 61, 83, 107, 137, 163, 191, 223, 241, 271, 307, 337, 367, 397, 431, 457, 487, 521]
sage: P = lazy_list(iter([1,2,3]))
sage: P.list()
[1, 2, 3]
sage: P[:100000].list()
[1, 2, 3]
sage: P[1:7:2].list()
[2]
```

TESTS:

Check that the cache is immutable:

```
sage: lazy = lazy_list(iter(Primes()))[:5]
sage: l = lazy.list(); l
[2, 3, 5, 7, 11]
sage: l[0] = -1; l
[-1, 3, 5, 7, 11]
sage: lazy.list()
[2, 3, 5, 7, 11]
```

start_stop_step()

Return the triple (start, stop, step) of reference points of the original lazy list.

EXAMPLES:

```
sage: from sage.misc.lazy_list import lazy_list
sage: p = lazy_list(iter(Primes()))[:2147483647]
sage: p.start_stop_step()
(0, 2147483647, 1)
sage: q = p[100:1042233:12]
sage: q.start_stop_step()
(100, 1042240, 12)
sage: r = q[233::3]
sage: r.start_stop_step()
(2896, 1042252, 36)
sage: 1042241%3 == 233%3
True
```

class sage.misc.lazy_list.**lazy_list_iterator**

Bases: object

Iterator for a lazy list.

INPUT:

- l – a lazy list
- pos – (Default: None) None or a non-negative integer specifying the starting position

next ()

x.next() -> the next value, or raise StopIteration

class sage.misc.lazy_list.**stopped_lazy_list_iterator**

Bases: object

A lazy list iterator which eventually stops.

INPUT:

- `l` – a lazy list
- `pos` – (Default: None) None or a non-negative integer specifying the starting position

next()

`x.next()` -> the next value, or raise `StopIteration`

2.10.5 Lazy import cache

This is a pure Python file with no dependencies so it can be used in `setup.py`.

`sage.misc.lazy_import_cache.get_cache_file()`

Returns a per-branch file for caching names of lazily imported modules.

EXAMPLES:

```
sage: from sage.misc.lazy_import_cache import get_cache_file
sage: get_cache_file()
'...-lazy_import_cache.pickle'
sage: get_cache_file().startswith(DOT_SAGE)
True
sage: 'src' in get_cache_file()
True
```

It shouldn't matter whether `DOT_SAGE` ends with a slash:

```
sage: OLD = DOT_SAGE
sage: sage.misc.lazy_import_cache.DOT_SAGE = '/tmp/'
sage: get_cache_file().startswith('/tmp/')
True
sage: sage.misc.lazy_import_cache.DOT_SAGE = OLD
```

2.10.6 Lazy strings

Based on `speaklater`: <https://github.com/mitsuhiko/speaklater>.

A lazy string is an object that behaves almost exactly like a string but where the value is not computed until needed. To define a lazy string you specify a function that produces a string together with the appropriate arguments for that function. Sage uses lazy strings in `sage.misc.misc` so that the filenames for `SAGE_TMP` (which depends on the pid of the process running Sage) are not computed when importing the Sage library. This means that when the doctesting code imports the Sage library and then forks, the variable `SAGE_TMP` depends on the new pid rather than the old one.

EXAMPLES:

```
sage: from sage.misc.lazy_string import lazy_string
sage: L = []
sage: s = lazy_string(lambda x: str(len(x)), L)
sage: L.append(5)
sage: s
1'1'
```

Note that the function is recomputed each time:

```
sage: L.append(6)
sage: s
1'2'
```

`sage.misc.lazy_string.is_lazy_string(obj)`
Checks if the given object is a lazy string.

EXAMPLES:

```
sage: from sage.misc.lazy_string import lazy_string, is_lazy_string
sage: f = lambda: "laziness"
sage: s = lazy_string(f)
sage: is_lazy_string(s)
True
```

`sage.misc.lazy_string.lazy_string(func, *args, **kwargs)`
Creates a lazy string by invoking func with args.

EXAMPLES:

```
sage: from sage.misc.lazy_string import lazy_string
sage: f = lambda: "laziness"
sage: s = lazy_string(f); s
1'laziness'
```

2.11 Caching

2.11.1 A weakref cache factory

class `sage.misc.cache.Cache` (*factory_function*)

A weakref cache for arbitrary objects via an arbitrary function.

format_names (*names, n=1*)

Many objects have a names argument. This function formats it, i.e., if it is a list it turns it into a tuple, and if it is a string and the number of names needed is bigger than 1, it makes each letter of the string an element of a tuple.

has_object (**args, **kws*)

Returns true if cache has object defined by the given args.

key (**args, **kws*)

Return the key associated to the given args. Note that the values must be hashable.

2.11.2 Cached Functions and Methods

AUTHORS:

- William Stein: initial version, (inspired by conversation with Justin Walker)
- Mike Hansen: added doctests and made it work with class methods.
- Willem Jan Palenstijn: add CachedMethodCaller for binding cached methods to instances.
- Tom Boothby: added DiskCachedFunction.
- Simon King: improved performance, more doctests, cython version, CachedMethodCallerNoArgs, weak cached function, cached special methods.

- Julian Rueth (2014-03-19, 2014-05-09): added `key` parameter, allow caching for unhashable elements

EXAMPLES:

By [trac ticket #11115](#), cached functions and methods are now also available in Cython code. The following examples cover various ways of usage.

Python functions:

```
sage: @cached_function
....: def test_pfunc(x):
....:     '''
....:     Some documentation
....:     '''
....:     return -x
sage: test_pfunc(5) is test_pfunc(5)
True
```

In some cases, one would only want to keep the result in cache as long as there is any other reference to the result. By [trac ticket #12215](#), this is enabled for `UniqueRepresentation`, which is used to create unique parents: If an algebraic structure, such as a finite field, is only temporarily used, then it will not stay in cache forever. That behaviour is implemented using `weak_cached_function`, that behaves the same as `cached_function`, except that it uses a `WeakValueDictionary` for storing the results.

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: class A: pass
sage: @weak_cached_function
....: def f():
....:     print "doing a computation"
....:     return A()
sage: a = f()
doing a computation
```

The result is cached:

```
sage: b = f()
sage: a is b
True
```

However, if there are no strong references left, the result may be garbage collected, and thus a new computation would take place:

```
sage: del a
sage: del b
sage: import gc
sage: n = gc.collect()
sage: a = f()
doing a computation
```

Cython `cdef` functions do not allow arbitrary decorators. However, one can wrap a Cython function and turn it into a cached function, by [trac ticket #11115](#). We need to provide the name that the wrapped method or function should have, since otherwise the name of the original function would be used:

```
sage: cython('''cpdef test_func(x): return -x''')
sage: wrapped_func = cached_function(test_func, name='wrapped_func')
sage: wrapped_func
Cached version of <built-in function test_func>
sage: wrapped_func.__name__
'wrapped_func'
```

```
sage: wrapped_func(5)
-5
sage: wrapped_func(5) is wrapped_func(5)
True
```

We can proceed similarly for cached methods of Cython classes, provided that they allow attribute assignment or have a public attribute `__cached_methods` of type `<dict>`. Since [trac ticket #11115](#), this is the case for all classes inheriting from `Parent`. See below for a more explicit example. By [trac ticket #12951](#), cached methods of extension classes can be defined by simply using the decorator. However, an indirect approach is still needed for `cpdef` methods:

```
sage: cython_code = ['cpdef test_meth(self,x):',
....: '    "some doc for a wrapped cython method"',
....: '    return -x',
....: 'from sage.all import cached_method',
....: 'from sage.structure.parent cimport Parent',
....: 'cdef class MyClass(Parent):',
....: '    @cached_method',
....: '    def direct_method(self, x):',
....: '        "Some doc for direct method"',
....: '        return 2*x',
....: '    wrapped_method = cached_method(test_meth,name="wrapped_method")']
sage: cython(os.linesep.join(cython_code))
sage: O = MyClass()
sage: O.direct_method
Cached version of <method 'direct_method' of '...MyClass' objects>
sage: O.wrapped_method
Cached version of <built-in function test_meth>
sage: O.wrapped_method.__name__
'wrapped_method'
sage: O.wrapped_method(5)
-5
sage: O.wrapped_method(5) is O.wrapped_method(5)
True
sage: O.direct_method(5)
10
sage: O.direct_method(5) is O.direct_method(5)
True
```

In some cases, one would only want to keep the result in cache as long as there is any other reference to the result. By [trac ticket #12215](#), this is enabled for `UniqueRepresentation`, which is used to create unique parents: If an algebraic structure, such as a finite field, is only temporarily used, then it will not stay in cache forever. That behaviour is implemented using `weak_cached_function`, that behaves the same as `cached_function`, except that it uses a `WeakValueDictionary` for storing the results.

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: class A: pass
sage: @weak_cached_function
....: def f():
....:     print "doing a computation"
....:     return A()
sage: a = f()
doing a computation
```

The result is cached:

```
sage: b = f()
sage: a is b
True
```


However, if there are no strong references left, the result may be garbage collected, and thus a new computation would take place:

```
sage: del a
sage: del b
sage: import gc
sage: n = gc.collect()
sage: a = f()
doing a computation
```

By [trac ticket #11115](#), even if a parent does not allow attribute assignment, it can inherit a cached method from the parent class of a category (previously, the cache would have been broken):

```
sage: cython_code = ["from sage.all import cached_method, cached_in_parent_method, Category, Objects",
....: "class MyCategory(Category):",
....: "    @cached_method",
....: "    def super_categories(self):",
....: "        return [Objects()]",
....: "    class ElementMethods:",
....: "        @cached_method",
....: "        def element_cache_test(self):",
....: "            return -self",
....: "        @cached_in_parent_method",
....: "        def element_via_parent_test(self):",
....: "            return -self",
....: "    class ParentMethods:",
....: "        @cached_method",
....: "        def one(self):",
....: "            return self.element_class(self, 1)",
....: "        @cached_method",
....: "        def invert(self, x):",
....: "            return -x"]
sage: cython('\n'.join(cython_code))
sage: C = MyCategory()
```

In order to keep the memory footprint of elements small, it was decided to not support the same freedom of using cached methods for elements: If an instance of a class derived from `Element` does not allow attribute assignment, then a cached method inherited from the category of its parent will break, as in the class `MyBrokenElement` below.

However, there is a class `ElementWithCachedMethod` that has generally a slower attribute access, but fully supports cached methods. We remark, however, that cached methods are *much* faster if attribute access works. So, we expect that `ElementWithCachedMethod` will hardly be used.

```
sage: cython_code = ["from sage.structure.element cimport Element, ElementWithCachedMethod",
....: "cdef class MyBrokenElement(Element):",
....: "    cdef public object x",
....: "    def __init__(self, P, x):",
....: "        self.x=x",
....: "        Element.__init__(self, P)",
....: "    def __neg__(self):",
....: "        return MyBrokenElement(self.parent(), -self.x)",
....: "    def _repr_(self):",
....: "        return '<%s>' % self.x",
....: "    def __hash__(self):",
....: "        return hash(self.x)",
....: "    cpdef int _cmp_(left, Element right) except -2:",
....: "        return cmp(left.x, right.x)",
```

```
.....: "    def raw_test(self):",
.....: "        return -self",
.....: "cdef class MyElement(ElementWithCachedMethod):",
.....: "    cdef public object x",
.....: "    def __init__(self,P,x):",
.....: "        self.x=x",
.....: "        ElementWithCachedMethod.__init__(self,P)",
.....: "    def __neg__(self):",
.....: "        return MyElement(self.parent(),-self.x)",
.....: "    def _repr_(self):",
.....: "        return '<%s>' %self.x",
.....: "    def __hash__(self):",
.....: "        return hash(self.x)",
.....: "    cpdef int _cmp_(left, Element right) except -2:",
.....: "        return cmp(left.x,right.x)",
.....: "    def raw_test(self):",
.....: "        return -self",
.....: "from sage.structure.parent cimport Parent",
.....: "cdef class MyParent(Parent):",
.....: "    Element = MyElement"]
sage: cython('\n'.join(cython_code))
sage: P = MyParent(category=C)
sage: ebroken = MyBrokenElement(P,5)
sage: e = MyElement(P,5)
```

The cached methods inherited by the parent works:

```
sage: P.one()
<1>
sage: P.one() is P.one()
True
sage: P.invert(e)
<-5>
sage: P.invert(e) is P.invert(e)
True
```

The cached methods inherited by MyElement works:

```
sage: e.element_cache_test()
<-5>
sage: e.element_cache_test() is e.element_cache_test()
True
sage: e.element_via_parent_test()
<-5>
sage: e.element_via_parent_test() is e.element_via_parent_test()
True
```

The other element class can only inherit a `cached_in_parent_method`, since the cache is stored in the parent. In fact, equal elements share the cache, even if they are of different types:

```
sage: e == ebroken
True
sage: type(e) == type(ebroken)
False
sage: ebroken.element_via_parent_test() is e.element_via_parent_test()
True
```

However, the cache of the other inherited method breaks, although the method as such works:

```
sage: ebroken.element_cache_test()
<-5>
sage: ebroken.element_cache_test() is ebroken.element_cache_test()
False
```

The cache can be emptied:

```
sage: a = test_pfunc(5)
sage: test_pfunc.clear_cache()
sage: a is test_pfunc(5)
False
sage: a = P.one()
sage: P.one.clear_cache()
sage: a is P.one()
False
```

Since `e` and `ebroken` share the cache, when we empty it for one element it is empty for the other as well:

```
sage: b = ebroken.element_via_parent_test()
sage: e.element_via_parent_test.clear_cache()
sage: b is ebroken.element_via_parent_test()
False
```

Introspection works:

```
sage: from sage.misc.edit_module import file_and_line
sage: from sage.misc.sageinspect import sage_getdoc, sage_getfile, sage_getsource
sage: print sage_getdoc(test_pfunc)
Some documentation
sage: print sage_getdoc(O.wrapped_method)
some doc for a wrapped cython method

sage: print sage_getdoc(O.direct_method)
Some doc for direct method

sage: print sage_getsource(O.wrapped_method)
cpdef test_meth(self, x):
    "some doc for a wrapped cython method"
    return -x
sage: print sage_getsource(O.direct_method)
def direct_method(self, x):
    "Some doc for direct method"
    return 2*x
```

It is a very common special case to cache a method that has no arguments. In that special case, the time needed to access the cache can be drastically reduced by using a special implementation. The cached method decorator automatically determines which implementation ought to be chosen. A typical example is `sage.rings.polynomial.multi_polynomial_ideal.MPolynomialIdeal.gens()` (no arguments) versus `sage.rings.polynomial.multi_polynomial_ideal.MPolynomialIdeal.groebner_basis()` (several arguments):

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens()
[a, b]
sage: I.gens() is I.gens()
True
sage: I.groebner_basis()
```

```
[a, b]
sage: I.groebner_basis() is I.groebner_basis()
True
sage: type(I.gens)
<type 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: type(I.groebner_basis)
<type 'sage.misc.cachefunc.CachedMethodCaller'>
```

By [trac ticket #12951](#), the `cached_method` decorator is also supported on non-c(p)def methods of extension classes, as long as they either support attribute assignment or have a public attribute of type `<dict>` called `__cached_methods`. The latter is easy:

```
sage: cython_code = [
....: "from sage.misc.cachefunc import cached_method",
....: "cdef class MyClass:",
....: "    cdef public dict __cached_methods",
....: "    @cached_method",
....: "    def f(self, a,b):",
....: "        return a*b"]
sage: cython(os.linesep.join(cython_code))
sage: P = MyClass()
sage: P.f(2,3)
6
sage: P.f(2,3) is P.f(2,3)
True
```

Providing attribute access is a bit more tricky, since it is needed that an attribute inherited by the instance from its class can be overridden on the instance. That is why providing a `__getattr__` would not be enough in the following example:

```
sage: cython_code = [
....: "from sage.misc.cachefunc import cached_method",
....: "cdef class MyOtherClass:",
....: "    cdef dict D",
....: "    def __init__(self):",
....: "        self.D = {}",
....: "    def __setattr__(self, n,v):",
....: "        self.D[n] = v",
....: "    def __getattr__(self, n):",
....: "        try:",
....: "            return self.D[n]",
....: "        except KeyError:",
....: "            pass",
....: "        return getattr(type(self),n).__get__(self)",
....: "    @cached_method",
....: "    def f(self, a,b):",
....: "        return a+b"]
sage: cython(os.linesep.join(cython_code))
sage: Q = MyOtherClass()
sage: Q.f(2,3)
5
sage: Q.f(2,3) is Q.f(2,3)
True
```

Note that supporting attribute access is somehow faster than the easier method:

```
sage: timeit("a = P.f(2,3)") # random
625 loops, best of 3: 1.3 µs per loop
```

```
sage: timeit("a = Q.f(2,3)")    # random
625 loops, best of 3: 931 ns per loop
```

Some immutable objects (such as p -adic numbers) cannot implement a reasonable hash function because their `==` operator has been modified to return `True` for objects which might behave differently in some computations:

```
sage: K.<a> = Qq(9)
sage: b = a.add_bigoh(1)
sage: c = a + 3
sage: b
a + O(3)
sage: c
a + 3 + O(3^20)
sage: b == c
True
sage: b == a
True
sage: c == a
False
```

If such objects defined a non-trivial hash function, this would break caching in many places. However, such objects should still be usable in caches. This can be achieved by defining an appropriate method `_cache_key`:

```
sage: hash(b)
Traceback (most recent call last):
...
TypeError: unhashable type: 'sage.rings.padics.padic_ZZ_pX_CR_element.pAdicZZpXCRElement'
sage: @cached_method
....: def f(x): return x == a
sage: f(b)
True
sage: f(c) # if b and c were hashable, this would return True
False

sage: b._cache_key()
(..., ((0, 1),), 0, 1)
sage: c._cache_key()
(..., ((0, 1), (1,)), 0, 20)
```

Note: This attribute will only be accessed if the object itself is not hashable.

An implementation must make sure that for elements a and b , if $a \neq b$, then also $a._cache_key() \neq b._cache_key()$. In practice this means that the `_cache_key` should always include the parent as its first argument:

```
sage: S.<a> = Qq(4)
sage: d = a.add_bigoh(1)
sage: b._cache_key() == d._cache_key() # this would be True if the parents were not included
False
```

class `sage.misc.cachefunc.CachedFunction`

Bases: `object`

Create a cached version of a function, which only recomputes values it hasn't already computed. Synonyme: `cached_function`

INPUT:

- `f` – a function
- `name` – (optional string) name that the cached version of `f` should be provided with
- `key` – (optional callable) takes the input and returns a key for the cache, typically one would use this to normalize input

If `f` is a function, do either `g = CachedFunction(f)` or `g = cached_function(f)` to make a cached version of `f`, or put `@cached_function` right before the definition of `f` (i.e., use Python decorators):

```
@cached_function
def f(...):
    ....
```

The inputs to the function must be hashable or they must define `sage.structure.sage_object.SageObject._cache_key()`.

EXAMPLES:

```
sage: @cached_function
....: def mul(x, y=2):
....:     return x*y
sage: mul(3)
6
```

We demonstrate that the result is cached, and that, moreover, the cache takes into account the various ways of providing default arguments:

```
sage: mul(3) is mul(3, 2)
True
sage: mul(3, y=2) is mul(3, 2)
True
```

The user can clear the cache:

```
sage: a = mul(4)
sage: mul.clear_cache()
sage: a is mul(4)
False
```

It is also possible to explicitly override the cache with a different value:

```
sage: mul.set_cache('foo', 5)
sage: mul(5, 2)
'foo'
```

The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```
sage: @cached_function(key=lambda x,y,algorithm: (x,y))
....: def mul(x, y, algorithm="default"):
....:     return x*y
sage: mul(1,1,algorithm="default") is mul(1,1,algorithm="algorithm") is mul(1,1) is mul(1,1,'def
True
```

cache

`clear_cache()`

Clear the cache dictionary.

EXAMPLES:

```

sage: g = CachedFunction(number_of_partitions)
sage: a = g(5)
sage: g.get_cache()
{((5, 'default'), ()): 7}
sage: g.clear_cache()
sage: g.get_cache()
{}

```

f

get_cache()

Returns the cache dictionary.

EXAMPLES:

```

sage: g = CachedFunction(number_of_partitions)
sage: a = g(5)
sage: g.get_cache()
{((5, 'default'), ()): 7}

```

get_key(*args, **kws)

Return the key in the cache to be used when args and kws are passed in as parameters.

EXAMPLES:

```

sage: @cached_function
....: def foo(x):
....:     return x^2
sage: foo(2)
4
sage: foo.get_key(2)
((2,), ())
sage: foo.get_key(x=3)
((3,), ())

```

is_in_cache(*args, **kws)

Checks if the argument list is in the cache.

EXAMPLES:

```

sage: class Foo:
....:     def __init__(self, x):
....:         self._x = x
....:     @cached_method
....:     def f(self, z, y=0):
....:         return self._x*z+y
sage: a = Foo(2)
sage: a.f.is_in_cache(3)
False
sage: a.f(3)
6
sage: a.f.is_in_cache(3,y=0)
True

```

TESTS:

Check that [trac ticket #16316](#) has been fixed, i.e., caching works for immutable unhashable objects which define `sage.structure.sage_object.SageObject._cache_key()`:

```

sage: @cached_function
....: def f(x): return x

```

```
sage: K.<u> = Qq(4)
sage: x = K(1,1); x
1 + O(2)
sage: f.is_in_cache(x)
False
sage: f(x)
1 + O(2)
sage: f.is_in_cache(x)
True
```

precompute (*arglist*, *num_processes=1*)

Cache values for a number of inputs. Do the computation in parallel, and only bother to compute values that we haven't already cached.

INPUT:

- *arglist* – list (or iterables) of arguments for which the method shall be precomputed.
- *num_processes* – number of processes used by `parallel()`

EXAMPLES:

```
sage: @cached_function
....: def oddprime_factors(n):
....:     l = [p for p,e in factor(n) if p != 2]
....:     return len(l)
sage: oddprime_factors.precompute(range(1,100), 4)
sage: oddprime_factors.cache[(25,), ()]
1
```

set_cache (*value*, **args*, ***kws*)

Set the value for those args and keyword args. Mind the unintuitive syntax (*value* first). Any idea on how to improve that welcome!

EXAMPLES:

```
sage: g = CachedFunction(number_of_partitions)
sage: a = g(5)
sage: g.get_cache()
{((5, 'default'), ()): 7}
sage: g.set_cache(17, 5)
sage: g.get_cache()
{((5, 'default'), ()): 17}
sage: g(5)
17
```

TESTS:

Check that [trac ticket #16316](#) has been fixed, i.e., caching works for immutable unhashable objects which define `sage.structure.sage_object.SageObject._cache_key()`:

```
sage: @cached_function
....: def f(x): return x
sage: K.<u> = Qq(4)
sage: x = K(1,1); x
1 + O(2)
sage: f.set_cache(x,x)
sage: f.is_in_cache(x)
True
```

DEVELOPER NOTE:

Is there a way to use the following intuitive syntax?

```
sage: g(5) = 19      # todo: not implemented
sage: g(5)          # todo: not implemented
19
```

class `sage.misc.cachefunc.CachedInParentMethod`

Bases: `sage.misc.cachefunc.CachedMethod`

A decorator that creates a cached version of an instance method of a class.

In contrast to `CachedMethod`, the cache dictionary is an attribute of the parent of the instance to which the method belongs.

ASSUMPTION:

This way of caching works only if

- the instances *have* a parent, and
- the instances are hashable (they are part of the cache key) or they define `sage.structure.sage_object.SageObject._cache_key()`

NOTE:

For proper behavior, the method must be a pure function (no side effects). If this decorator is used on a method, it will have identical output on equal elements. This is since the element is part of the hash key. Arguments to the method must be hashable or define `sage.structure.sage_object.SageObject._cache_key()`. The instance it is assigned to must be hashable.

Examples can be found at `cachefunc`.

class `sage.misc.cachefunc.CachedMethod`

Bases: `object`

A decorator that creates a cached version of an instance method of a class.

Note: For proper behavior, the method must be a pure function (no side effects). Arguments to the method must be hashable or transformed into something hashable using `key` or they must define `sage.structure.sage_object.SageObject._cache_key()`.

EXAMPLES:

```
sage: class Foo(object):
....:     @cached_method
....:     def f(self, t, x=2):
....:         print 'computing'
....:         return t**x
sage: a = Foo()
```

The example shows that the actual computation takes place only once, and that the result is identical for equivalent input:

```
sage: res = a.f(3, 2); res
computing
9
sage: a.f(t = 3, x = 2) is res
True
sage: a.f(3) is res
True
```

Note, however, that the `CachedMethod` is replaced by a `CachedMethodCaller` or `CachedMethodCallerNoArgs` as soon as it is bound to an instance or class:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: type(I.__class__.gens)
<type 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
```

So, you would hardly ever see an instance of this class alive.

The parameter `key` can be used to pass a function which creates a custom cache key for inputs. In the following example, this parameter is used to ignore the `algorithm` keyword for caching:

```
sage: class A(object):
....:     def _f_normalize(self, x, algorithm): return x
....:     @cached_method(key=_f_normalize)
....:     def f(self, x, algorithm='default'): return x
sage: a = A()
sage: a.f(1, algorithm="default") is a.f(1) is a.f(1, algorithm="algorithm")
True
```

class `sage.misc.cachefunc.CachedMethodCaller`
Bases: `sage.misc.cachefunc.CachedFunction`

Utility class that is used by `CachedMethod` to bind a cached method to an instance.

Note: Since [trac ticket #11115](#), there is a special implementation `CachedMethodCallerNoArgs` for methods that do not take arguments.

EXAMPLE:

```
sage: class A:
....:     @cached_method
....:     def bar(self, x):
....:         return x^2
sage: a = A()
sage: a.bar
Cached version of <function bar at 0x...>
sage: type(a.bar)
<type 'sage.misc.cachefunc.CachedMethodCaller'>
sage: a.bar(2) is a.bar(x=2)
True
```

get_key (*args, **kws)

Convert arguments to the key for this instance's cache.

EXAMPLES:

```
sage: class Foo:
....:     def __init__(self, x):
....:         self._x = x
....:     @cached_method
....:     def f(self, y, z=0):
....:         return self._x * y + z
sage: a = Foo(2)
sage: z = a.f(37)
sage: k = a.f.get_key(37); k
((37, 0), ())
sage: a.f.get_cache()[k] is z
True
```

Note that the method does not test whether there are too many arguments, or wrong argument names:

```
sage: a.f.get_key(1,2,3,x=4,y=5,z=6)
((1, 2, 3), (('x', 4), ('y', 5), ('z', 6)))
```

It does, however, take into account the different ways of providing named arguments, possibly with a default value:

```
sage: a.f.get_key(5)
((5, 0), ())
sage: a.f.get_key(y=5)
((5, 0), ())
sage: a.f.get_key(5,0)
((5, 0), ())
sage: a.f.get_key(5,z=0)
((5, 0), ())
sage: a.f.get_key(y=5,z=0)
((5, 0), ())
```

precompute (*arglist*, *num_processes=1*)

Cache values for a number of inputs. Do the computation in parallel, and only bother to compute values that we haven't already cached.

INPUT:

- *arglist* – list (or iterables) of arguments for which the method shall be precomputed.
- *num_processes* – number of processes used by `parallel()`

EXAMPLES:

```
sage: class Foo(object):
....:     @cached_method
....:     def f(self, i):
....:         return i^2
sage: foo = Foo()
sage: foo.f(3)
9
sage: foo.f(1)
1
sage: foo.f.precompute(range(2), 2)
sage: foo.f.cache
{((0,), ()): 0, ((1,), ()): 1, ((3,), ()): 9}
```

class `sage.misc.cachefunc.CachedMethodCallerNoArgs`

Bases: `sage.misc.cachefunc.CachedFunction`

Utility class that is used by `CachedMethod` to bind a cached method to an instance, in the case of a method that does not accept any arguments except `self`.

Note: The return value `None` would not be cached. So, if you have a method that does not accept arguments and may return `None` after a lengthy computation, then `@cached_method` should not be used.

EXAMPLE:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens
Cached version of <function gens at 0x...>
sage: type(I.gens)
<type 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: I.gens is I.gens
```

```
True
sage: I.gens() is I.gens()
True
```

AUTHOR:

•Simon King (2011-04)

clear_cache()

Clear the cache dictionary.

EXAMPLES:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens()
[a, b]
sage: I.gens.set_cache('bar')
sage: I.gens()
'bar'
```

The cache can be emptied and thus the original value will be reconstructed:

```
sage: I.gens.clear_cache()
sage: I.gens()
[a, b]
```

is_in_cache()

Answers whether the return value is already in the cache.

Note: Recall that a cached method without arguments can not cache the return value `None`.

EXAMPLE:

```
sage: P.<x,y> = QQ[]
sage: I = P*[x,y]
sage: I.gens.is_in_cache()
False
sage: I.gens()
[x, y]
sage: I.gens.is_in_cache()
True
```

set_cache(value)

Override the cache with a specific value.

Note: `None` is not suitable for a cached value. It would be interpreted as an empty cache, forcing a new computation.

EXAMPLES:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: I.gens()
[a, b]
sage: I.gens.set_cache('bar')
sage: I.gens()
'bar'
```

The cache can be emptied and thus the original value will be reconstructed:

```
sage: I.gens.clear_cache()
sage: I.gens()
[a, b]
```

The attempt to assign None to the cache fails:

```
sage: I.gens.set_cache(None)
sage: I.gens()
[a, b]
```

class `sage.misc.cachefunc.CachedMethodPickle` (*inst, name, cache=None*)

Bases: object

This class helps to unpickle cached methods.

Note: Since [trac ticket #8611](#), a cached method is an attribute of the instance (provided that it has a `__dict__`). Hence, when pickling the instance, it would be attempted to pickle that attribute as well, but this is a problem, since functions can not be pickled, currently. Therefore, we replace the actual cached method by a place holder, that kills itself as soon as any attribute is requested. Then, the original cached attribute is reinstated. But the cached values are in fact saved.

EXAMPLES:

```
sage: R.<x, y, z> = PolynomialRing(QQ, 3)
sage: I = R*(x^3 + y^3 + z^3, x^4-y^4)
sage: I.groebner_basis()
[y^5*z^3 - 1/4*x^2*z^6 + 1/2*x*y*z^6 + 1/4*y^2*z^6,
 x^2*y*z^3 - x*y^2*z^3 + 2*y^3*z^3 + z^6,
 x*y^3 + y^4 + x*z^3, x^3 + y^3 + z^3]
sage: I.groebner_basis
Cached version of <function groebner_basis at 0x...>
```

We now pickle and unpickle the ideal. The cached method `groebner_basis` is replaced by a placeholder:

```
sage: J = loads(dumps(I))
sage: J.groebner_basis
Pickle of the cached method "groebner_basis"
```

But as soon as any other attribute is requested from the placeholder, it replaces itself by the cached method, and the entries of the cache are actually preserved:

```
sage: J.groebner_basis.is_in_cache()
True
sage: J.groebner_basis
Cached version of <function groebner_basis at 0x...>
sage: J.groebner_basis() == I.groebner_basis()
True
```

TESTS:

Since [trac ticket #11115](#), there is a special implementation for cached methods that don't take arguments:

```
sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: type(I.gens)
<type 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: type(I.groebner_basis)
<type 'sage.misc.cachefunc.CachedMethodCaller'>
```

We demonstrate that both implementations can be pickled and preserve the cache. For that purpose, we assign nonsense to the cache. Of course, it is a very bad idea to override the cache in that way. So, please don't try this at home:

```
sage: I.groebner_basis.set_cache('foo', algorithm='singular')
sage: I.groebner_basis(algorithm='singular')
'foo'
sage: I.gens.set_cache('bar')
sage: I.gens()
'bar'
sage: J = loads(dumps(I))
sage: J.gens()
'bar'
sage: J.groebner_basis(algorithm='singular')
'foo'
```

Anyway, the cache will be automatically reconstructed after clearing it:

```
sage: J.gens.clear_cache()
sage: J.gens()
[a, b]
sage: J.groebner_basis.clear_cache()
sage: J.groebner_basis(algorithm='singular')
[a, b]
```

AUTHOR:

•Simon King (2011-01)

class `sage.misc.cachefunc.CachedSpecialMethod`

Bases: `sage.misc.cachefunc.CachedMethod`

Cached version of *special* python methods.

IMPLEMENTATION:

For new style classes `C`, it is not possible to override a special method, such as `__hash__`, in the `__dict__` of an instance `c` of `C`, because Python will for efficiency reasons always use what is provided by the class, not by the instance.

By consequence, if `__hash__` would be wrapped by using `CachedMethod`, then `hash(c)` will access `C.__hash__` and bind it to `c`, which means that the `__get__` method of `CachedMethod` will be called. But there, we assume that Python has already inspected `__dict__`, and thus a `CachedMethodCaller` will be created over and over again.

Here, the `__get__` method will explicitly access the `__dict__`, so that `hash(c)` will rely on a single `CachedMethodCaller` stored in the `__dict__`.

EXAMPLES:

```
sage: class C:
.....:     @cached_method
.....:     def __hash__(self):
.....:         print "compute hash"
.....:         return int(5)
.....:
sage: c = C()
sage: type(C.__hash__)
<type 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
```

The hash is computed only once, subsequent calls will use the value from the cache. This was implemented in [trac ticket #12601](#).

```

sage: hash(c)           # indirect doctest
compute hash
5
sage: hash(c)
5

```

class sage.misc.cachefunc.**ClearCacheOnPickle**

Bases: object

This class implements an appropriate `__getstate__` method that clears the cache of the methods (see `@cached_method`) before passing them on to the caller, typically the pickle and copy modules.

The implemented `__getstate__` method calls the `__getstate__` methods of classes later in the method resolution order. Therefore, classes which want this behaviour should inherit first from this one.

EXAMPLE:

In the following example, we create a Python class that inherits from multivariate polynomial ideals, but does not pickle cached values. We provide the definition in Cython, however, since interactive Cython definitions provide introspection by [trac ticket #9976](#), whereas Python definitions don't.

```

sage: P.<a,b,c,d> = QQ[]
sage: I = P*[a,b]
sage: classdef = ['from sage.misc.cachefunc import ClearCacheOnPickle',
....:   'from sage.all import QQ',
....:   'P = QQ["a","b","c","d"]; I = P*[P.gen(0),P.gen(1)]',
....:   'class MyClass(ClearCacheOnPickle,I.__class__):',
....:   '    def __init__(self,ring,gens):',
....:   '        I.__class__.__init__(self,ring,gens)',
....:   '    def __getnewargs__(self):',
....:   '        return (self._Ideal_generic__ring,self._Ideal_generic__gens)']
sage: cython('\n'.join(classdef))

```

We destroy the cache of two methods of `I` on purpose (demonstrating that the two different implementations of cached methods are correctly dealt with). Pickling `I` preserves the cache:

```

sage: I.gens.set_cache('bar')
sage: I.groebner_basis.set_cache('foo',algorithm='singular')
sage: J = loads(dumps(I))
sage: J.gens()
'bar'
sage: J.groebner_basis(algorithm='singular')
'foo'

```

However, if we have an ideal that additionally descends from `ClearCacheOnPickle`, the carefully corrupted cache is not pickled:

```

sage: A = MyClass(P,[a,b])
sage: A
Ideal (a, b) of Multivariate Polynomial Ring in a, b, c, d over Rational Field
sage: A.gens.set_cache('foo')
sage: A.groebner_basis.set_cache('bar',algorithm='singular')
sage: A.gens()
'foo'
sage: A.groebner_basis(algorithm='singular')
'bar'
sage: B = loads(dumps(A))
sage: B.gens()
[a, b]
sage: B.groebner_basis(algorithm='singular')

```

```
[a, b]
sage: A.gens()
'foo'
```

class `sage.misc.cachefunc.DiskCachedFunction` (*f*, *dir*, *memory_cache=False*, *key=None*)
Bases: `sage.misc.cachefunc.CachedFunction`

Works similar to `CachedFunction`, but instead, we keep the cache on disk (optionally, we keep it in memory too).

EXAMPLES:

```
sage: from sage.misc.cachefunc import DiskCachedFunction
sage: dir = tmp_dir()
sage: factor = DiskCachedFunction(factor, dir, memory_cache=True)
sage: f = factor(2775); f
3 * 5^2 * 37
sage: f is factor(2775)
True
```

class `sage.misc.cachefunc.FileCache` (*dir*, *prefix=''*, *memory_cache=False*)
`FileCache` is a dictionary-like class which stores keys and values on disk. The keys take the form of a tuple (*A*, *K*)

- *A* is a tuple of objects *t* where each *t* is an exact object which is uniquely identified by a short string.
- *K* is a tuple of tuples (*s*, *v*) where *s* is a valid variable name and *v* is an exact object which is uniquely identified by a short string with letters [a-zA-Z0-9-._]

The primary use case is the `DiskCachedFunction`. If `memory_cache == True`, we maintain a cache of objects seen during this session in memory – but we don't load them from disk until necessary. The keys and values are stored in a pair of files:

- `prefix-argstring.key.sobj` contains the key only,
- `prefix-argstring.sobj` contains the tuple (*key*, *val*)

where `self[key] == val`.

Note: We assume that each `FileCache` lives in its own directory. Use **extreme** caution if you wish to break that assumption.

file_list()

Return the list of files corresponding to `self`.

EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = True, prefix='t')
sage: FC[(), ()] = 1
sage: FC[(1, 2), ()] = 2
sage: FC[(1,), (('a', 1),)] = 3
sage: for f in sorted(FC.file_list()): print f[len(dir):]
t-.key.sobj
t-.sobj
t-1_2.key.sobj
t-1_2.sobj
t-a-1.1.key.sobj
t-a-1.1.sobj
```


items()

Return a list of tuples (k, v) where $\text{self}[k] = v$.

EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = False)
sage: FC[(), ()] = 1
sage: FC[(1, 2), ()] = 2
sage: FC[(1,), (('a', 1),)] = 3
sage: I = FC.items()
sage: I.sort(); print I
[((), (), 1), ((1,), (('a', 1),)), 3), ((1, 2), (), 2)]
```

keys()

Return a list of keys k where $\text{self}[k]$ is defined.

EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = False)
sage: FC[(), ()] = 1
sage: FC[(1, 2), ()] = 2
sage: FC[(1,), (('a', 1),)] = 3
sage: K = FC.keys()
sage: K.sort(); print K
[((), (),), ((1,), (('a', 1),)), ((1, 2), ())]
```

values()

Return a list of values that are stored in `self`.

EXAMPLES:

```
sage: from sage.misc.cachefunc import FileCache
sage: dir = tmp_dir()
sage: FC = FileCache(dir, memory_cache = False)
sage: FC[(), ()] = 1
sage: FC[(1, 2), ()] = 2
sage: FC[(1,), (('a', 1),)] = 3
sage: FC[(), (('a', 1),)] = 4
sage: v = FC.values()
sage: v.sort(); print v
[1, 2, 3, 4]
```

class sage.misc.cachefunc.WeakCachedFunction

Bases: `sage.misc.cachefunc.CachedFunction`

A version of `CachedFunction` using weak references on the values.

If f is a function, do either $g = \text{weak_cached_function}(f)$ to make a cached version of f , or put `@weak_cached_function` right before the definition of f (i.e., use Python decorators):

```
@weak_cached_function
def f(...):
    ...
```

EXAMPLES:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: class A: pass
```

```
sage: @weak_cached_function
....: def f():
....:     print "doing a computation"
....:     return A()
sage: a = f()
doing a computation
```

The result is cached:

```
sage: b = f()
sage: a is b
True
```

However, if there are no strong references left, the result may be garbage collected, and thus a new computation would take place:

```
sage: del a
sage: del b
sage: import gc
sage: n = gc.collect()
sage: a = f()
doing a computation
```

The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```
sage: @weak_cached_function(key=lambda x, algorithm: x)
....: def mod_ring(x, algorithm="default"):
....:     return IntegerModRing(x)
sage: mod_ring(1, algorithm="default") is mod_ring(1, algorithm="algorithm") is mod_ring(1) is mod
True
```

is_in_cache (*args, **kws)

Check if the argument list is in the cache.

EXAMPLES:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: class A:
....:     def __init__(self, x):
....:         self.x = x
sage: @weak_cached_function
....: def f(n):
....:     return A(n)
sage: a = f(5)
```

The key 5 is in the cache, as long as there is a strong reference to the corresponding value:

```
sage: f.is_in_cache(5)
True
```

However, if there are no strong references left, the cached item is removed from cache after garbage collection:

```
sage: del a
sage: import gc
sage: n = gc.collect()
sage: f.is_in_cache(5)
False
```

TESTS:

Check that [trac ticket #16316](#) has been fixed, i.e., caching works for immutable unhashable objects which define `sage.structure.sage_object.SageObject._cache_key()`:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: @weak_cached_function
....: def f(x): return x
sage: K.<u> = QQ(4)
sage: R.<t> = K[]
sage: f.is_in_cache(t)
False
sage: f(t)
(1 + O(2^20))*t
sage: f.is_in_cache(t)
True
```

set_cache (*value*, *args, **kwds)

Set the value for those args and keyword args. Mind the unintuitive syntax (value first). Any idea on how to improve that welcome!

It is required that the given value is weak referenceable. The item will be removed from cache if the value is garbage collected.

EXAMPLES:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: @weak_cached_function
....: def f(n):
....:     raise RuntimeError
sage: f.set_cache(ZZ, 5)
sage: f(5)
Integer Ring
```

TESTS:

Check that [trac ticket #16316](#) has been fixed, i.e., caching works for immutable unhashable objects which define `sage.structure.sage_object.SageObject._cache_key()`:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: @weak_cached_function
....: def f(x): return x
sage: K.<u> = QQ(4)
sage: R.<t> = K[]
sage: f.set_cache(t,t)
sage: f.is_in_cache(t)
True
```

`sage.misc.cachefunc.cached_function` (*self*, *args, **kwds)

Create a cached version of a function, which only recomputes values it hasn't already computed. Synonyme: `cached_function`

INPUT:

- *f* – a function
- *name* – (optional string) name that the cached version of *f* should be provided with
- *key* – (optional callable) takes the input and returns a key for the cache, typically one would use this to normalize input

If *f* is a function, do either `g = CachedFunction(f)` or `g = cached_function(f)` to make a

cached version of `f`, or put `@cached_function` right before the definition of `f` (i.e., use Python decorators):

```
@cached_function
def f(...):
    ....
```

The inputs to the function must be hashable or they must define `sage.structure.sage_object.SageObject._cache_key()`.

EXAMPLES:

```
sage: @cached_function
....: def mul(x, y=2):
....:     return x*y
sage: mul(3)
6
```

We demonstrate that the result is cached, and that, moreover, the cache takes into account the various ways of providing default arguments:

```
sage: mul(3) is mul(3, 2)
True
sage: mul(3, y=2) is mul(3, 2)
True
```

The user can clear the cache:

```
sage: a = mul(4)
sage: mul.clear_cache()
sage: a is mul(4)
False
```

It is also possible to explicitly override the cache with a different value:

```
sage: mul.set_cache('foo', 5)
sage: mul(5, 2)
'foo'
```

The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```
sage: @cached_function(key=lambda x, y, algorithm: (x, y))
....: def mul(x, y, algorithm="default"):
....:     return x*y
sage: mul(1, 1, algorithm="default") is mul(1, 1, algorithm="algorithm") is mul(1, 1) is mul(1, 1, 'def
True
```

`sage.misc.cachefunc.cached_in_parent_method(self, inst, *args, **kws)`

A decorator that creates a cached version of an instance method of a class.

In contrast to `CachedMethod`, the cache dictionary is an attribute of the parent of the instance to which the method belongs.

ASSUMPTION:

This way of caching works only if

- the instances *have* a parent, and
- the instances are hashable (they are part of the cache key) or they define `sage.structure.sage_object.SageObject._cache_key()`

NOTE:

For proper behavior, the method must be a pure function (no side effects). If this decorator is used on a method, it will have identical output on equal elements. This is since the element is part of the hash key. Arguments to the method must be hashable or define `sage.structure.sage_object.SageObject._cache_key()`. The instance it is assigned to must be hashable.

Examples can be found at [cachefunc](#).

```
sage.misc.cachefunc.cached_method(f, name=None, key=None)
```

A decorator for cached methods.

EXAMPLES:

In the following examples, one can see how a cached method works in application. Below, we demonstrate what is done behind the scenes:

```
sage: class C:
....:     @cached_method
....:     def __hash__(self):
....:         print "compute hash"
....:         return int(5)
....:     @cached_method
....:     def f(self, x):
....:         print "computing cached method"
....:         return x*2
sage: c = C()
sage: type(C.__hash__)
<type 'sage.misc.cachefunc.CachedMethodCallerNoArgs'>
sage: hash(c)
compute hash
5
```

When calling a cached method for the second time with the same arguments, the value is gotten from the cache, so that a new computation is not needed:

```
sage: hash(c)
5
sage: c.f(4)
computing cached method
8
sage: c.f(4) is c.f(4)
True
```

Different instances have distinct caches:

```
sage: d = C()
sage: d.f(4) is c.f(4)
computing cached method
False
sage: d.f.clear_cache()
sage: c.f(4)
8
sage: d.f(4)
computing cached method
8
```

Using cached methods for the hash and other special methods was implemented in [trac ticket #12601](#), by means of `CachedSpecialMethod`. We show that it is used behind the scenes:

```
sage: cached_method(c.__hash__)
<sage.misc.cachefunc.CachedSpecialMethod object at ...>
```

```
sage: cached_method(c.f)
<sage.misc.cachefunc.CachedMethod object at ...>
```

class `sage.misc.cachefunc.disk_cached_function` (*dir*, *memory_cache=False*, *key=None*)
Decorator for `DiskCachedFunction`.

EXAMPLES:

```
sage: dir = tmp_dir()
sage: @disk_cached_function(dir)
....: def foo(x): return next_prime(2^x)%x
sage: x = foo(200);x
11
sage: @disk_cached_function(dir)
....: def foo(x): return 1/x
sage: foo(200)
11
sage: foo.clear_cache()
sage: foo(200)
1/200
```

`sage.misc.cachefunc.weak_cached_function` (*self*, **args*, ***kws*)
A version of `CachedFunction` using weak references on the values.

If *f* is a function, do either `g = weak_cached_function(f)` to make a cached version of *f*, or put `@weak_cached_function` right before the definition of *f* (i.e., use Python decorators):

```
@weak_cached_function
def f(...):
    ...
```

EXAMPLES:

```
sage: from sage.misc.cachefunc import weak_cached_function
sage: class A: pass
sage: @weak_cached_function
....: def f():
....:     print "doing a computation"
....:     return A()
sage: a = f()
doing a computation
```

The result is cached:

```
sage: b = f()
sage: a is b
True
```

However, if there are no strong references left, the result may be garbage collected, and thus a new computation would take place:

```
sage: del a
sage: del b
sage: import gc
sage: n = gc.collect()
sage: a = f()
doing a computation
```

The parameter `key` can be used to ignore parameters for caching. In this example we ignore the parameter `algorithm`:

```

sage: @weak_cached_function(key=lambda x,algorithm: x)
....: def mod_ring(x, algorithm="default"):
....:     return IntegerModRing(x)
sage: mod_ring(1,algorithm="default") is mod_ring(1,algorithm="algorithm") is mod_ring(1) is mod
True

```

2.11.3 Fast and safe weak value dictionary

AUTHORS:

- Simon King (2013-10)
- Nils Bruin (2013-10)
- Julian Rueth (2014-03-16): improved handling of unhashable objects

Python's `weakref` module provides `WeakValueDictionary`. This behaves similar to a dictionary, but it does not prevent its values from garbage collection. Hence, it stores the values by weak references with callback functions: The callback function deletes a key-value pair from the dictionary, as soon as the value becomes subject to garbage collection.

However, a problem arises if hash and comparison of the key depend on the value that is being garbage collected:

```

sage: import weakref
sage: class Vals(object): pass
sage: class Keys:
....:     def __init__(self, val):
....:         self.val = weakref.ref(val)
....:     def __hash__(self):
....:         return hash(self.val())
....:     def __eq__(self, other):
....:         return self.val() == other.val()
sage: ValList = [Vals() for _ in range(10)]
sage: D = weakref.WeakValueDictionary()
sage: for v in ValList:
....:     D[Keys(v)] = v
sage: len(D)
10
sage: del ValList, v
Exception KeyError: (<__main__.Keys instance at ...>,) in <function remove at ...> ignored
Exception KeyError: (<__main__.Keys instance at ...>,) in <function remove at ...> ignored
Exception KeyError: (<__main__.Keys instance at ...>,) in <function remove at ...> ignored
Exception KeyError: (<__main__.Keys instance at ...>,) in <function remove at ...> ignored
...
sage: len(D) > 1
True

```

Hence, there are scary error messages, and moreover the defunct items have not been removed from the dictionary.

Therefore, Sage provides an alternative implementation `sage.misc.weak_dict.WeakValueDictionary`, using a callback that removes the defunct item not based on hash and equality check of the key (this is what fails in the example above), but based on comparison by identity. This is possible, since references with callback function are distinct even if they point to the same object. Hence, even if the same object `O` occurs as value for several keys, each reference to `O` corresponds to a unique key. We see no error messages, and the items get correctly removed:

```

sage: ValList = [Vals() for _ in range(10)]
sage: import sage.misc.weak_dict
sage: D = sage.misc.weak_dict.WeakValueDictionary()

```

```
sage: for v in ValList:
....:     D[Keys(v)] = v
sage: len(D)
10
sage: del ValList
sage: len(D)
1
sage: del v
sage: len(D)
0
```

Another problem arises when iterating over the items of a dictionary: If garbage collection occurs during iteration, then the content of the dictionary changes, and the iteration breaks for `weakref.WeakValueDictionary`:

```
sage: class Cycle:
....:     def __init__(self):
....:         self.selfref = self
sage: C = [Cycle() for n in range(10)]
sage: D = weakref.WeakValueDictionary(enumerate(C))
sage: import gc
sage: gc.disable()
sage: del C[:5]
sage: len(D)
10
```

With `WeakValueDictionary`, the behaviour is safer. Note that iteration over a `WeakValueDictionary` is non-deterministic, since the lifetime of values (and hence the presence of keys) in the dictionary may depend on when garbage collection occurs. The method implemented here will at least postpone dictionary mutations due to garbage collection callbacks. This means that as long as there is at least one iterator active on a dictionary, none of its keys will be deallocated (which could have side-effects). Which entries are returned is of course still dependent on when garbage collection occurs. Note that when a key gets returned as “present” in the dictionary, there is no guarantee one can actually retrieve its value: it may have been garbage collected in the mean time.

Note that Sage’s weak value dictionary is actually an instance of `dict`, in contrast to `weakref`’s weak value dictionary:

```
sage: isinstance(weakref.WeakValueDictionary, dict)
False
sage: isinstance(sage.misc.weak_dict.WeakValueDictionary, dict)
True
```

See [trac ticket #13394](#) for a discussion of some of the design considerations.

class `sage.misc.weak_dict.WeakValueDictEraser`

Bases: `object`

Erases items from a `sage.misc.weak_dict.WeakValueDictionary` when a weak reference becomes invalid.

This is of internal use only. Instances of this class will be passed as a callback function when creating a weak reference.

EXAMPLES:

```
sage: from sage.misc.weak_dict import WeakValueDictionary
sage: v = frozenset([1])
sage: D = WeakValueDictionary({1 : v})
sage: len(D)
1
sage: del v
```



```
sage: len(D)
0
```

AUTHOR:

•Nils Bruin (2013-11)

```
class sage.misc.weak_dict.WeakValueDictionary
    Bases: dict
```

IMPLEMENTATION:

The `WeakValueDictionary` inherits from `dict`. In its implementation, it stores weakrefs to the actual values under the keys. All access routines are wrapped to transparently place and remove these weakrefs.

NOTE:

In contrast to `weakref.WeakValueDictionary` in Python's `weakref` module, the callback does not need to assume that the dictionary key is a valid Python object when it is called. There is no need to compute the hash or compare the dictionary keys. This is why the example below would not work with `weakref.WeakValueDictionary`, but does work with `sage.misc.weak_dict.WeakValueDictionary`.

EXAMPLES:

```
sage: import weakref
sage: class Vals(object): pass
sage: class Keys:
....:     def __init__(self, val):
....:         self.val = weakref.ref(val)
....:     def __hash__(self):
....:         return hash(self.val())
....:     def __eq__(self, other):
....:         return self.val() == other.val()
sage: ValList = [Vals() for _ in range(10)]
sage: import sage.misc.weak_dict
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: for v in ValList:
....:     D[Keys(v)] = v
sage: len(D)
10
sage: del ValList
sage: len(D)
1
sage: del v
sage: len(D)
0
```

TESTS:

The following reflects the behaviour of the callback on weak dict values, as discussed on [trac ticket #13394](#).

```
sage: from sage.misc.weak_dict import WeakValueDictionary
sage: V = [set(range(n)) for n in range(5)]
sage: D = WeakValueDictionary(enumerate(V))
```

The line `V[k] = None` triggers execution of the callback functions of the dictionary values. However, the actual deletion is postponed till after the iteration over the dictionary has finished. Hence, when the callbacks are executed, the values which the callback belongs to has already been overridden by a new value. Therefore, the callback does not delete the item:

```
sage: for k in D.iterkeys():      # indirect doctest
....:     V[k] = None
....:     D[k] = ZZ
sage: len(D)
5
sage: D[1]
Integer Ring
```

The following is a stress test for weak value dictionaries:

```
sage: class C(object):
....:     def __init__(self, n):
....:         self.n = n
....:     def __cmp__(self, other):
....:         return cmp(type(self), type(other)) or cmp(self.n, other.n)
sage: B = 100
sage: L = [None]*B
sage: D1 = WeakValueDictionary()
sage: D2 = WeakValueDictionary()
sage: for i in range(10000):
....:     ki = floor(random()*B)
....:     vi = C(floor(random()*B))
....:     D1[ki] = vi
....:     D2[ki] = vi
....:     L[ki] = vi
....:     del vi
....:     ko = floor(random()*B)
....:     if ko in D1:
....:         del D1[ko]
....:         L[ko] = None
....:     assert D1 == D2
```

get (*k*, *d=None*)

Return the stored value for a key, or a default value for unknown keys.

The default value defaults to None.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: L = [GF(p) for p in prime_range(10^3)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: 100 in D
True
sage: 200 in D
False
sage: D.get(100, "not found")
Finite Field of size 547
sage: D.get(200, "not found")
'not found'
sage: D.get(200) is None
True
```

TESTS:

Check that [trac ticket #15956](#) has been fixed, i.e., a `TypeError` is raised for unhashable objects:

```
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: D.get(matrix([]))
Traceback (most recent call last):
```

```
...
TypeError: mutable matrices are unhashable
```

items()

The key-value pairs of this dictionary.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>"%self.n
....:     def __cmp__(self, other):
....:         c = cmp(type(self), type(other))
....:         if c:
....:             return c
....:         return cmp(self.n, other.n)
sage: class Keys(object):
....:     def __init__(self, n):
....:         self.n = n
....:     def __hash__(self):
....:         if self.n%2:
....:             return 5
....:         return 3
....:     def __repr__(self):
....:         return "[%s]"%self.n
....:     def __cmp__(self, other):
....:         c = cmp(type(self), type(other))
....:         if c:
....:             return c
....:         return cmp(self.n, other.n)
sage: L = [(Keys(n), Vals(n)) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(L)
```

We remove one dictionary item directly. Another item is removed by means of garbage collection. By consequence, there remain eight items in the dictionary:

```
sage: del D[Keys(2)]
sage: del L[5]
sage: sorted(D.items())
[([0], <0>),
 ([1], <1>),
 ([3], <3>),
 ([4], <4>),
 ([6], <6>),
 ([7], <7>),
 ([8], <8>),
 ([9], <9>)]
```

iteritems()

Iterate over the items of this dictionary.

Warning: Iteration is unsafe, if the length of the dictionary changes during the iteration! This can also happen by garbage collection.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>"%self.n
....:     def __cmp__(self, other):
....:         c = cmp(type(self),type(other))
....:         if c:
....:             return c
....:         return cmp(self.n,other.n)
sage: class Keys(object):
....:     def __init__(self, n):
....:         self.n = n
....:     def __hash__(self):
....:         if self.n%2:
....:             return 5
....:         return 3
....:     def __repr__(self):
....:         return "[%s]"%self.n
....:     def __cmp__(self, other):
....:         c = cmp(type(self),type(other))
....:         if c:
....:             return c
....:         return cmp(self.n,other.n)
sage: L = [(Keys(n), Vals(n)) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(L)
```

We remove one dictionary item directly. Another item is removed by means of garbage collection. By consequence, there remain eight items in the dictionary:

```
sage: del D[Keys(2)]
sage: del L[5]
sage: for k,v in sorted(D.iteritems()):
....:     print k, v
[0] <0>
[1] <1>
[3] <3>
[4] <4>
[6] <6>
[7] <7>
[8] <8>
[9] <9>
```

iterkeys()

Iterate over the keys of this dictionary.

Warning: Iteration is unsafe, if the length of the dictionary changes during the iteration! This can also happen by garbage collection.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals(object): pass
sage: L = [Vals() for _ in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: del L[4]
```

One item got deleted from the list `L` and hence the corresponding item in the dictionary got deleted as well. Therefore, the corresponding key 4 is missing in the list of keys:

```
sage: list(sorted(D.iterkeys()))
[0, 1, 2, 3, 5, 6, 7, 8, 9]
```

intervalvalues()

Iterate over the values of this dictionary.

Warning: Iteration is unsafe, if the length of the dictionary changes during the iteration! This can also happen by garbage collection.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>"%self.n
....:     def __cmp__(self, other):
....:         c = cmp(type(self), type(other))
....:         if c:
....:             return c
....:         return cmp(self.n, other.n)
sage: L = [Vals(n) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
```

We delete one item from `D` and we delete one item from the list `L`. The latter implies that the corresponding item from `D` gets deleted as well. Hence, there remain eight values:

```
sage: del D[2]
sage: del L[5]
sage: for v in sorted(D.intervalvalues()):
....:     print v
<0>
<1>
<3>
<4>
<6>
<7>
<8>
<9>
```

keys()

The list of keys.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals(object): pass
sage: L = [Vals() for _ in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: del L[4]
```

One item got deleted from the list `L` and hence the corresponding item in the dictionary got deleted as well. Therefore, the corresponding key 4 is missing in the list of keys:

```
sage: sorted(D.keys())
[0, 1, 2, 3, 5, 6, 7, 8, 9]
```

pop(*k*)

Return the value for a given key, and delete it from the dictionary.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: L = [GF(p) for p in prime_range(10^3)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
sage: 20 in D
True
sage: D.pop(20)
Finite Field of size 73
sage: 20 in D
False
sage: D.pop(20)
Traceback (most recent call last):
...
KeyError: 20
```

TESTS:

Check that [trac ticket #15956](#) has been fixed, i.e., a `TypeError` is raised for unhashable objects:

```
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: D.pop(matrix([]))
Traceback (most recent call last):
...
TypeError: mutable matrices are unhashable
```

popitem()

Return and delete some item from the dictionary.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: D[1] = ZZ
```

The dictionary only contains a single item, hence, it is clear which one will be returned:

```
sage: D.popitem()
(1, Integer Ring)
```

Now, the dictionary is empty, and hence the next attempt to pop an item will fail with a `KeyError`:

```
sage: D.popitem()
Traceback (most recent call last):
...
KeyError: 'popitem(): weak value dictionary is empty'
```

setdefault(*k*, *default=None*)

Return the stored value for a given key; return and store a default value if no previous value is stored.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: L = [(p, GF(p)) for p in prime_range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(L)
sage: len(D)
4
```

The value for an existing key is returned and not overridden:

```
sage: D.setdefault(5, ZZ)
Finite Field of size 5
sage: D[5]
Finite Field of size 5
```

For a non-existing key, the default value is stored and returned:

```
sage: 4 in D
False
sage: D.setdefault(4, ZZ)
Integer Ring
sage: 4 in D
True
sage: D[4]
Integer Ring
sage: len(D)
5
```

TESTS:

Check that [trac ticket #15956](#) has been fixed, i.e., a `TypeError` is raised for unhashable objects:

```
sage: D = sage.misc.weak_dict.WeakValueDictionary()
sage: D.setdefault(matrix([]), ZZ)
Traceback (most recent call last):
...
TypeError: mutable matrices are unhashable
```

`values()`

Return the list of values.

EXAMPLES:

```
sage: import sage.misc.weak_dict
sage: class Vals:
....:     def __init__(self, n):
....:         self.n = n
....:     def __repr__(self):
....:         return "<%s>" % self.n
....:     def __cmp__(self, other):
....:         c = cmp(type(self), type(other))
....:         if c:
....:             return c
....:         return cmp(self.n, other.n)
sage: L = [Vals(n) for n in range(10)]
sage: D = sage.misc.weak_dict.WeakValueDictionary(enumerate(L))
```

We delete one item from `D` and we delete one item from the list `L`. The latter implies that the corresponding item from `D` gets deleted as well. Hence, there remain eight values:

```
sage: del D[2]
sage: del L[5]
sage: sorted(D.values())
[<0>, <1>, <3>, <4>, <6>, <7>, <8>, <9>]
```

`sage.misc.weak_dict.test_del_dictitem_by_exact_value(D, value, h)`

This function helps testing some cdef function used to delete dictionary items.

INPUT:

- `D` – a Python `<dict>`.
- `value` – an object that is value `D`.
- `h` – the hash of the key under which to find `value` in `D`.

The underlying `cdef` function deletes an item from `D` that is in the hash bucket determined by `h` and whose value is identical with `value`. Of course, this only makes sense if the pairs `(h, value)` corresponding to items in `D` are pair-wise distinct.

If a matching item can not be found, the function does nothing and silently returns.

TESTS:

See [trac ticket #13394](#) for a discussion.

```
sage: from sage.misc.weak_dict import test_del_dictitem_by_exact_value
sage: B=1000
sage: L=list(range(B))
sage: D1=dict()
sage: D2=dict()
sage: for i in range(100000):          # long time
....:     ki=L[floor(random()*B)]
....:     vi=L[floor(random()*B)]
....:     D1[ki]=vi
....:     D2[ki]=vi
....:     ko=L[floor(random()*B)]
....:     if ko in D1:
....:         vo=D1[ko]
....:         del D1[ko]
....:         test_del_dictitem_by_exact_value(D2, vo, hash(ko))
....:     assert D1 == D2
```

No action is taken if the item prescribed by key hash and value does not exist in the dictionary:

```
sage: D = {1: ZZ}
sage: test_del_dictitem_by_exact_value(D, ZZ, 2)
sage: D
{1: Integer Ring}
sage: test_del_dictitem_by_exact_value(D, QQ, 1)
sage: D
{1: Integer Ring}
```

2.12 Fast Expression Evaluation

2.12.1 Fast Expression Evaluation

For many applications such as numerical integration, differential equation approximation, plotting a 3d surface, optimization problems, Monte-Carlo simulations, etc., one wishes to pass around and evaluate a single algebraic expression many, many times at various floating point values. Other applications may need to evaluate an expression many times in interval arithmetic, or in a finite field. Doing this via recursive calls over a python representation of the object (even if Maxima or other outside packages are not involved) is extremely inefficient.

This module provides a function, `fast_callable()`, to transform such expressions into a form where they can be evaluated quickly:

```
sage: f = sin(x) + 3*x^2
sage: ff = fast_callable(f, vars=[x])
```



```
sage: ff(3.5)
36.3992167723104
sage: ff(RIF(3.5))
36.39921677231038?
```

By default, `fast_callable()` only removes some interpretive overhead from the evaluation, but all of the individual arithmetic operations are done using standard Sage arithmetic. This is still a huge win over `sage.calculus`, which evidently has a lot of overhead. Compare the cost of evaluating Wilkinson’s polynomial (in unexpanded form) at $x=30$:

```
sage: wilk = prod((x-i) for i in [1 .. 20]); wilk
(x - 1)*(x - 2)*(x - 3)*(x - 4)*(x - 5)*(x - 6)*(x - 7)*(x - 8)*(x - 9)*(x - 10)*(x - 11)*(x - 12)*...
sage: timeit('wilk.subs(x=30)') # random, long time
625 loops, best of 3: 1.43 ms per loop
sage: fc_wilk = fast_callable(wilk, vars=[x])
sage: timeit('fc_wilk(30)') # random, long time
625 loops, best of 3: 9.72 us per loop
```

You can specify a particular domain for the evaluation using `domain=`:

```
sage: fc_wilk_zz = fast_callable(wilk, vars=[x], domain=ZZ)
```

The meaning of `domain=D` is that each intermediate and final result is converted to type `D`. For instance, the previous example of $\sin(x) + 3x^2$ with `domain=D` would be equivalent to $D(D(\sin(D(x))) + D(D(3)*D(D(x)^2)))$. (This example also demonstrates the one exception to the general rule: if an exponent is an integral constant, then it is not wrapped with `D()`.)

At first glance, this seems like a very bad idea if you want to compute quickly. And it is a bad idea, for types where we don’t have a special interpreter. It’s not too bad of a slowdown, though. To mitigate the costs, we check whether the value already has the correct parent before we call `D`.

We don’t yet have a special interpreter with domain `ZZ`, so we can see how that compares to the generic `fc_wilk` example above:

```
sage: timeit('fc_wilk_zz(30)') # random, long time
625 loops, best of 3: 15.4 us per loop
```

However, for other types, using `domain=D` will get a large speedup, because we have special-purpose interpreters for those types. One example is `RDF`. Since with `domain=RDF` we know that every single operation will be floating-point, we can just execute the floating-point operations directly and skip all the Python object creations that you would get from actually using `RDF` objects:

```
sage: fc_wilk_rdf = fast_callable(wilk, vars=[x], domain=RDF)
sage: timeit('fc_wilk_rdf(30.0)') # random, long time
625 loops, best of 3: 7 us per loop
```

The domain does not need to be a Sage type; for instance, `domain=float` also works. (We actually use the same fast interpreter for `domain=float` and `domain=RDF`; the only difference is that when `domain=RDF` is used, the return value is an `RDF` element, and when `domain=float` is used, the return value is a Python float.)

```
sage: fc_wilk_float = fast_callable(wilk, vars=[x], domain=float)
sage: timeit('fc_wilk_float(30.0)') # random, long time
625 loops, best of 3: 5.04 us per loop
```

We also have support for `RR`:

```
sage: fc_wilk_rr = fast_callable(wilk, vars=[x], domain=RR)
sage: timeit('fc_wilk_rr(30.0)') # random, long time
625 loops, best of 3: 13 us per loop
```

And support for CDF:

```
sage: fc_wilk_rr = fast_callable(wilk, vars=[x], domain=CDF)
sage: timeit('fc_wilk_rr(30.0)') # random, long time
625 loops, best of 3: 10.2 us per loop
```

Currently, `fast_callable()` can accept two kinds of objects: polynomials (univariate and multivariate) and symbolic expressions (elements of the Symbolic Ring). (This list is likely to grow significantly in the near future.) For polynomials, you can omit the ‘vars’ argument; the variables will default to the ring generators (in the order used when creating the ring).

```
sage: K.<x,y,z> = QQ[]
sage: p = 10*y + 100*z + x
sage: fp = fast_callable(p)
sage: fp(1,2,3)
321
```

But you can also specify the variable names to override the default ordering (you can include extra variable names here, too).

```
sage: fp = fast_callable(p, vars=('x','w','z','y'))
```

For symbolic expressions, you need to specify the variable names, so that `fast_callable()` knows what order to use.

```
sage: var('y,z,x')
(y, z, x)
sage: f = 10*y + 100*z + x
sage: ff = fast_callable(f, vars=(x,y,z))
sage: ff(1,2,3)
321
```

You can also specify extra variable names:

```
sage: ff = fast_callable(f, vars=('x','w','z','y'))
sage: ff(1,2,3,4)
341
```

This should be enough for normal use of `fast_callable()`; let’s discuss some more advanced topics.

Sometimes it may be useful to create a fast version of an expression without going through symbolic expressions or polynomials; perhaps because you want to describe to `fast_callable()` an expression with common subexpressions.

Internally, `fast_callable()` works in two stages: it constructs an expression tree from its argument, and then it builds a fast evaluator from that expression tree. You can bypass the first phase by building your own expression tree and passing that directly to `fast_callable()`, using an `ExpressionTreeBuilder`.

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=('x','y','z'))
```

An `ExpressionTreeBuilder` has three interesting methods: `constant()`, `var()`, and `call()`. All of these methods return `ExpressionTree` objects.

The `var()` method takes a string, and returns an expression tree for the corresponding variable.

```
sage: x = etb.var('x')
sage: y = etb.var('y')
sage: z = etb.var('y')
```

Expression trees support Python's numeric operators, so you can easily build expression trees representing arithmetic expressions.

```
sage: v1 = (x+y)*(y+z) + (y//z)
```

The `constant()` method takes a Sage value, and returns an expression tree representing that value.

```
sage: v2 = etb.constant(3.14159) * x + etb.constant(1729) * y
```

The `call()` method takes a sage/Python function and zero or more expression trees, and returns an expression tree representing the function call.

```
sage: v3 = etb.call(sin, v1+v2)
sage: v3
sin(add(add(mul(add(v_0, v_1), add(v_1, v_1)), floordiv(v_1, v_1)), add(mul(3.141590000000000, v_0), r
```

Many sage/Python built-in functions are specially handled; for instance, when evaluating an expression involving `sin()` over RDF, the C math library function `sin()` is called. Arbitrary functions are allowed, but will be much slower since they will call back to Python code on every call; for example, the following will work.

```
sage: def my_sqrt(x): return pow(x, 0.5)
sage: e = etb.call(my_sqrt, v1); e
{my_sqrt}(add(mul(add(v_0, v_1), add(v_1, v_1)), floordiv(v_1, v_1)))
sage: fast_callable(e)(1, 2, 3)
3.60555127546399
```

To provide `fast_callable()` for your own class (so that `fast_callable(x)` works when `x` is an instance of your class), implement a method `_fast_callable_(self, etb)` for your class. This method takes an `ExpressionTreeBuilder`, and returns an expression tree built up using the methods described above.

EXAMPLES:

```
sage: var('x')
x
sage: f = fast_callable(sqrt(x^7+1), vars=[x], domain=float)

sage: f(1)
1.4142135623730951
sage: f.op_list()
[('load_arg', 0), ('ipow', 7), ('load_const', 1.0), 'add', 'sqrt', 'return']
```

To interpret that last line, we load argument 0 ('x' in this case) onto the stack, push the constant 7.0 onto the stack, call the `pow` function (which takes 2 arguments from the stack), push the constant 1.0, add the top two arguments of the stack, and then call `sqrt`.

Here we take `sin` of the first argument and add it to `f`:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: f = etb.call(sqrt, x^7 + 1)
sage: g = etb.call(sin, x)
sage: fast_callable(f+g).op_list()
[('load_arg', 0), ('ipow', 7), ('load_const', 1), 'add', ('py_call', <function sqrt at ...>, 1), ('l
```

AUTHOR:

- Carl Witty (2009-02): initial version (heavily inspired by Robert Bradshaw's `fast_eval.pyx`)
-

Todo

The following bits of text were written for the module docstring. They are not true yet, but I hope they will be true someday, at which point I will move them into the main text.

The final interesting method of `ExpressionTreeBuilder` is `choice()`. This produces conditional expressions, like the C `COND ? T : F` expression or the Python `T if COND else F`. This lets you define piecewise functions using `fast_callable()`.

```
sage: v4 = etb.choice(v3 >= etb.constant(0), v1, v2)
```

The arguments are `(COND, T, F)` (the same order as in C), so the above means that if `v3` evaluates to a nonnegative number, then `v4` will evaluate to the result of `v1`; otherwise, `v4` will evaluate to the result of `v2`.

Let's see an example where we see that `fast_callable()` does not evaluate common subexpressions more than once. We'll make a `fast_callable()` expression that gives the result of 16 iterations of the Mandelbrot function.

```
sage: etb = ExpressionTreeBuilder('c')
sage: z = etb.constant(0)
sage: c = etb.var('c')
sage: for i in range(16):
....:     z = z*z + c
sage: mand = fast_callable(z, domain=CDF)
```

Now `ff` does 32 complex arithmetic operations on each call (16 additions and 16 multiplications). However, if `z*z` produced code that evaluated `z` twice, then this would do many thousands of arithmetic operations instead.

Note that the handling for common subexpressions only checks whether expression trees are the same Python object; for instance, the following code will evaluate `x+1` twice:

```
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: (x+1)*(x+1)
mul(add(v_0, 1), add(v_0, 1))
```

but this code will only evaluate `x+1` once:

```
sage: v = x+1; v*v
mul(add(v_0, 1), add(v_0, 1))
```

```
class sage.ext.fast_callable.CompilerInstrSpec(n_inputs, n_outputs, parameters)
    Bases: object
```

Describes a single instruction to the `fast_callable` code generator.

An instruction has a number of stack inputs, a number of stack outputs, and a parameter list describing extra arguments that must be passed to the `InstructionStream.instr` method (that end up as extra words in the code).

The parameter list is a list of strings. Each string is one of the following:

- 'args' - The instruction argument refers to an input argument of the wrapper class; it is just appended to the code.
- 'constants', 'py_constants' - The instruction argument is a value; the value is added to the corresponding list (if it's not already there) and the index is appended to the code.

- ‘n_inputs’, ‘n_outputs’ - The instruction actually takes a variable number of inputs or outputs (the n_inputs and n_outputs attributes of this instruction are ignored). The instruction argument specifies the number of inputs or outputs (respectively); it is just appended to the code.

class `sage.ext.fast_callable.Expression`

Bases: `object`

Represents an expression for fast_callable.

Supports the standard Python arithmetic operators; if arithmetic is attempted between an Expression and a non-Expression, the non-Expression is converted to an expression (using the `__call__` method of the Expression’s ExpressionTreeBuilder).

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb.var(x)
sage: etb(x)
v_0
sage: etb(3)
3
sage: etb.call(sin, x)
sin(v_0)
sage: (x+1)/(x-1)
div(add(v_0, 1), sub(v_0, 1))
sage: x//5
floordiv(v_0, 5)
sage: -abs(~x)
neg(abs(inv(v_0)))
```

abs()

Compute the absolute value of an Expression.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: abs(x)
abs(v_0)
sage: x.abs()
abs(v_0)
sage: x.__abs__()
abs(v_0)
```

class `sage.ext.fast_callable.ExpressionCall`

Bases: `sage.ext.fast_callable.Expression`

An Expression that represents a function call.

EXAMPLES: `sage: from sage.ext.fast_callable import ExpressionTreeBuilder` `sage: etb = ExpressionTreeBuilder(vars=(x,))` `sage: type(etb.call(sin, x)) <type 'sage.ext.fast_callable.ExpressionCall'>`

arguments()

Return the arguments from this ExpressionCall.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
```

```
sage: etb.call(sin, x).arguments()
[v_0]
```

function()

Return the function from this ExpressionCall.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.call(sin, x).function()
sin
```

class sage.ext.fast_callable.**ExpressionChoice**

Bases: sage.ext.fast_callable.Expression

A conditional expression.

(It's possible to create choice nodes, but they don't work yet.)

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.choice(etb.call(operator.eq, x, 0), 0, 1/x)
(0 if {eq}(v_0, 0) else div(1, v_0))
```

condition()

Return the condition of an ExpressionChoice.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: etb.choice(x, ~x, 0).condition()
v_0
```

if_false()

Return the false branch of an ExpressionChoice.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: etb.choice(x, ~x, 0).if_false()
0
```

if_true()

Return the true branch of an ExpressionChoice.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: x = etb(x)
sage: etb.choice(x, ~x, 0).if_true()
inv(v_0)
```

class sage.ext.fast_callable.**ExpressionConstant**

Bases: sage.ext.fast_callable.Expression

An Expression that represents an arbitrary constant.

EXAMPLES: `sage: from sage.ext.fast_callable import ExpressionTreeBuilder sage: etb = ExpressionTreeBuilder(vars=(x,)) sage: type(etb(3)) <type 'sage.ext.fast_callable.ExpressionConstant'>`

value()

Return the constant value of an ExpressionConstant.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb(3).value()
3
```

class `sage.ext.fast_callable.ExpressionIPow`
Bases: `sage.ext.fast_callable.Expression`

A power Expression with an integer exponent.

EXAMPLES: `sage: from sage.ext.fast_callable import ExpressionTreeBuilder sage: etb = ExpressionTreeBuilder(vars=(x,)) sage: type(etb.var('x')^17) <type 'sage.ext.fast_callable.ExpressionIPow'>`

base()

Return the base from this ExpressionIPow.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: (etb(33)^42).base()
33
```

exponent()

Return the exponent from this ExpressionIPow.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: (etb(x)^(-1)).exponent()
-1
```

class `sage.ext.fast_callable.ExpressionTreeBuilder`
Bases: `object`

A class with helper methods for building Expressions.

An instance of this class is passed to `_fast_callable_` methods; you can also instantiate it yourself to create your own expressions for `fast_callable`, bypassing `_fast_callable_`.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: (x+3)*5
mul(add(v_0, 3), 5)
```

call (*fn*, **args*)

Construct a call node, given a function and a list of arguments. The arguments will be converted to Expressions using `ExpressionTreeBuilder.__call__`.

As a special case, notices if the function is `operator.pow` and the second argument is integral, and constructs an `ExpressionIPow` instead.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.call(cos, x)
cos(v_0)
sage: etb.call(sin, 1)
sin(1)
sage: etb.call(sin, etb(1))
sin(1)
sage: etb.call(factorial, x+57)
{factorial}(add(v_0, 57))
sage: etb.call(operator.pow, x, 543)
ipow(v_0, 543)
```

choice (*cond, iftrue, iffalse*)

Construct a choice node (a conditional expression), given the condition, and the values for the true and false cases.

(It's possible to create choice nodes, but they don't work yet.)

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb.choice(etb.call(operator.eq, x, 0), 0, 1/x)
(0 if {eq}(v_0, 0) else div(1, v_0))
```

constant (*c*)

Turn the argument into an `ExpressionConstant`, converting it to our domain if we have one.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder('x')
sage: etb.constant(pi)
pi
sage: etb = ExpressionTreeBuilder('x', domain=RealField(200))
sage: etb.constant(pi)
3.1415926535897932384626433832795028841971693993751058209749
```

var (*v*)

Turn the argument into an `ExpressionVariable`. Looks it up in the list of variables. (Variables are matched by name.)

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: var('a,b,some_really_long_name')
(a, b, some_really_long_name)
sage: x = polygen(QQ)
sage: etb = ExpressionTreeBuilder(vars=('a','b',some_really_long_name, x))
sage: etb.var(some_really_long_name)
v_2
sage: etb.var('some_really_long_name')
v_2
sage: etb.var(x)
v_3
sage: etb.var('y')
```



```
Traceback (most recent call last):
...
ValueError: Variable 'y' not found
```

class `sage.ext.fast_callable.ExpressionVariable`

Bases: `sage.ext.fast_callable.Expression`

An Expression that represents a variable.

EXAMPLES: `sage: from sage.ext.fast_callable import ExpressionTreeBuilder sage: etb = ExpressionTreeBuilder(vars=(x,)) sage: type(etb.var(x)) <type 'sage.ext.fast_callable.ExpressionVariable'>`

variable_index()

Return the variable index of an ExpressionVariable.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=(x,))
sage: etb(x).variable_index()
0
```

class `sage.ext.fast_callable.InstructionStream`

Bases: `object`

An InstructionStream takes a sequence of instructions (passed in by a series of method calls) and computes the data structures needed by the interpreter. This is the stage where we switch from operating on Expression trees to a linear representation. If we had a peephole optimizer (we don't) it would go here.

Currently, this class is not very general; it only works for interpreters with a fixed set of memory chunks (with fixed names). Basically, it only works for stack-based expression interpreters. It should be generalized, so that the interpreter metadata includes a description of the memory chunks involved and the instruction stream can handle any interpreter.

Once you're done adding instructions, you call `get_current()` to retrieve the information needed by the interpreter (as a Python dictionary).

current_op_list()

Returns the list of instructions that have been added to this InstructionStream so far.

It's OK to call this, then add more instructions.

EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('py_call', math.sin, 1)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), ('py_call', <built-in function sin>, 1), 'abs', 'return']
```

get_current()

Return the current state of the InstructionStream, as a dictionary suitable for passing to a wrapper class.

NOTE: The dictionary includes internal data structures of the InstructionStream; you must not modify it.

EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.get_current()
{'args': 1,
 'code': [],
 'constants': [],
 'domain': None,
 'py_constants': [],
 'stack': 0}
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('py_call', math.sin, 1)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), ('py_call', <built-in function sin>, 1), 'abs', 'return']
sage: instr_stream.get_current()
{'args': 1,
 'code': [0, 0, 3, 0, 1, 12, 2],
 'constants': [],
 'domain': None,
 'py_constants': [<built-in function sin>],
 'stack': 1}
```

get_metadata()

Returns the interpreter metadata being used by the current InstructionStream.

The code generator sometimes uses this to decide which code to generate.

EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: md = instr_stream.get_metadata()
sage: type(md)
<type 'sage.ext.fast_callable.InterpreterMetadata'>
```

has_instr(opname)

Check whether this InstructionStream knows how to generate code for a given instruction.

EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.has_instr('return')
True
sage: instr_stream.has_instr('factorial')
False
sage: instr_stream.has_instr('abs')
True
```

instr(opname, *args)

Generate code in this InstructionStream for the given instruction and arguments.

The opname is used to look up a CompilerInstrSpec; the CompilerInstrSpec describes how to interpret the arguments. (This is documented in the class docstring for CompilerInstrSpec.)

EXAMPLES:

```

sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('sin')
sage: instr_stream.instr('py_call', math.sin, 1)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('factorial')
Traceback (most recent call last):
...
KeyError: 'factorial'
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), 'sin', ('py_call', <built-in function sin>, 1), 'abs', 'return']

```

load_arg(*n*)

Add a 'load_arg' instruction to this InstructionStream.

EXAMPLES:

```

sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream
sage: instr_stream = InstructionStream(metadata, 12)
sage: instr_stream.load_arg(5)
sage: instr_stream.current_op_list()
[('load_arg', 5)]
sage: instr_stream.load_arg(3)
sage: instr_stream.current_op_list()
[('load_arg', 5), ('load_arg', 3)]

```

load_const(*c*)

Add a 'load_const' instruction to this InstructionStream.

EXAMPLES:

```

sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream, op_list
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.load_const(5)
sage: instr_stream.current_op_list()
[('load_const', 5)]
sage: instr_stream.load_const(7)
sage: instr_stream.load_const(5)
sage: instr_stream.current_op_list()
[('load_const', 5), ('load_const', 7), ('load_const', 5)]

```

Note that constants are shared: even though we load 5 twice, it only appears once in the constant table.

```

sage: instr_stream.get_current()['constants']
[5, 7]

```

class sage.ext.fast_callable.IntegerPowerFunction(*n*)

Bases: object

This class represents the function x^n for an arbitrary integral power n . That is, IntegerPowerFunction(2) is the squaring function; IntegerPowerFunction(-1) is the reciprocal function.

EXAMPLES:

```
sage: from sage.ext.fast_callable import IntegerPowerFunction
sage: square = IntegerPowerFunction(2)
sage: square
(^2)
sage: square(pi)
pi^2
sage: square(I)
-1
sage: square(RIF(-1, 1)).str(style='brackets')
'[0.000000000000000000 .. 1.0000000000000000]'
sage: IntegerPowerFunction(-1)
(^(-1))
sage: IntegerPowerFunction(-1) (22/7)
7/22
sage: v = Integers(123456789) (54321)
sage: v^9876543210
79745229
sage: IntegerPowerFunction(9876543210) (v)
79745229
```

class `sage.ext.fast_callable.InterpreterMetadata`

Bases: `object`

The interpreter metadata for a `fast_callable` interpreter. Currently consists of a dictionary mapping instruction names to (`CompilerInstrSpec`, `opcode`) pairs, a list mapping opcodes to (instruction name, `CompilerInstrSpec`) pairs, and a range of exponents for which the `ipow` instruction can be used. This range can be `False` (if the `ipow` instruction should never be used), a pair of two integers (`a`, `b`), if `ipow` should be used for $a \leq n \leq b$, or `True`, if `ipow` should always be used. When `ipow` cannot be used, then we fall back on calling `IntegerPowerFunction`.

See the class docstring for `CompilerInstrSpec` for more information.

NOTE: You must not modify the metadata.

by_opcode

by_opname

ipow_range

class `sage.ext.fast_callable.Wrapper`

Bases: `object`

The parent class for all `fast_callable` wrappers. Implements shared behavior (currently only debugging).

get_orig_args()

Get the original arguments used when initializing this wrapper.

(Probably only useful when writing doctests.)

EXAMPLES:

```
sage: fast_callable(sin(x)/x, vars=[x], domain=RDF).get_orig_args()
{'args': 1,
 'code': [0, 0, 16, 0, 0, 8, 2],
 'constants': [],
 'domain': Real Double Field,
 'py_constants': [],
 'stack': 2}
```

op_list()

Return the list of instructions in this wrapper.

EXAMPLES:

```
sage: fast_callable(cos(x)*x, vars=[x], domain=RDF).op_list()
[('load_arg', 0), ('load_arg', 0), 'cos', 'mul', 'return']
```

python_calls()

List the Python functions that are called in this wrapper.

(Python function calls are slow, so ideally this list would be empty. If it is not empty, then perhaps there is an optimization opportunity where a Sage developer could speed this up by adding a new instruction to the interpreter.)

EXAMPLES:

```
sage: fast_callable(abs(sin(x)), vars=[x], domain=RDF).python_calls()
[]
sage: fast_callable(abs(sin(factorial(x))), vars=[x]).python_calls()
[factorial, sin]
```

```
sage.ext.fast_callable.fast_callable(x, domain=None, vars=None, _autocom-
                                         pute_vars_for_backward_compatibility_with_deprecated_fast_float_functionali-
                                         expect_one_var=False)
```

Given an expression x , compiles it into a form that can be quickly evaluated, given values for the variables in x .

Currently, x can be an expression object, an element of SR, or a (univariate or multivariate) polynomial; this list will probably be extended soon.

By default, x is evaluated the same way that a Python function would evaluate it – addition maps to `PyNumber_Add`, etc. However, you can specify `domain=D` where D is some Sage parent or Python type; in this case, all arithmetic is done in that domain. If we have a special-purpose interpreter for that parent (like RDF or float), `domain=...` will trigger the use of that interpreter.

If `vars` is `None` and x is a polynomial, then we will use the generators of `parent(x)` as the variables; otherwise, `vars` must be specified (unless x is a symbolic expression with only one variable, and `expect_one_var` is `True`, in which case we will use that variable).

EXAMPLES:

```
sage: var('x')
x
sage: expr = sin(x) + 3*x^2
sage: f = fast_callable(expr, vars=[x])
sage: f(2)
sin(2) + 12
sage: f(2.0)
12.9092974268257
```

We have special fast interpreters for `domain=float` and `domain=RDF`. (Actually it's the same interpreter; only the return type varies.) Note that the float interpreter is not actually more accurate than the RDF interpreter; elements of RDF just don't display all their digits. We have special fast interpreter for `domain=CDF`:

```
sage: f_float = fast_callable(expr, vars=[x], domain=float)
sage: f_float(2)
12.909297426825681
sage: f_rdf = fast_callable(expr, vars=[x], domain=RDF)
sage: f_rdf(2)
12.909297426825681
sage: f_cdf = fast_callable(expr, vars=[x], domain=CDF)
sage: f_cdf(2)
12.909297426825681
sage: f_cdf(2+I)
10.40311925062204 + 11.510943740958707*I
```

```

sage: f = fast_callable(expr, vars=('z', 'x', 'y'))
sage: f(1, 2, 3)
sin(2) + 12
sage: K.<x> = QQ[]
sage: p = K.random_element(6); p
-x^6 - 12*x^5 + 1/2*x^4 - 1/95*x^3 - 1/2*x^2 - 4
sage: fp = fast_callable(p, domain=RDF)
sage: fp.op_list()
[('load_arg', 0), ('load_const', -1.0), 'mul', ('load_const', -12.0), 'add', ('load_arg', 0), 'm
sage: fp(3.14159)
-4594.161823640176
sage: K.<x,y,z> = QQ[]
sage: p = K.random_element(degree=3, terms=5); p
-x*y^2 - x*z^2 - 6*x^2 - y^2 - 3*x*z
sage: fp = fast_callable(p, domain=RDF)
sage: fp.op_list()
[('load_const', 0.0), ('load_const', -3.0), ('load_arg', 0), ('ipow', 1), ('load_arg', 2), ('ipo
sage: fp(e, pi, sqrt(2)) # abs tol 3e-14
-98.00156403362932
sage: symbolic_result = p(e, pi, sqrt(2)); symbolic_result
-pi^2*e - pi^2 - 3*sqrt(2)*e - 6*e^2 - 2*e
sage: n(symbolic_result)
-98.0015640336293

sage: from sage.ext.fast_callable import ExpressionTreeBuilder
sage: etb = ExpressionTreeBuilder(vars=('x', 'y'), domain=float)
sage: x = etb.var('x')
sage: y = etb.var('y')
sage: expr = etb.call(sin, x^2 + y); expr
sin(add(ipow(v_0, 2), v_1))
sage: fc = fast_callable(expr, domain=float)
sage: fc(5, 7)
0.5514266812416906

```

Check that `fast_callable` also works for symbolic functions with evaluation functions:

```

sage: def evalf_func(self, x, y, parent): return parent(x*y) if parent != None else x*y
sage: x,y = var('x,y')
sage: f = function('f', evalf_func=evalf_func)
sage: fc = fast_callable(f(x, y), vars=[x, y])
sage: fc(3, 4)
f(3, 4)

```

And also when there are complex values involved:

```

sage: def evalf_func(self, x, y, parent): return parent(I*x*y) if parent != None else I*x*y
sage: g = function('g', evalf_func=evalf_func)
sage: fc = fast_callable(g(x, y), vars=[x, y])
sage: fc(3, 4)
g(3, 4)
sage: fc2 = fast_callable(g(x, y), domain=complex, vars=[x, y])
sage: fc2(3, 4)
12j
sage: fc3 = fast_callable(g(x, y), domain=float, vars=[x, y])
sage: fc3(3, 4)
Traceback (most recent call last):
...
TypeError: unable to simplify to float approximation

```

`sage.ext.fast_callable.function_name(fn)`

Given a function, returns a string giving a name for the function.

For functions we recognize, we use our standard opcode name for the function (so `operator.add` becomes ‘add’, and `sage.all.sin` becomes ‘sin’).

For functions we don’t recognize, we try to come up with a name, but the name will be wrapped in braces; this is a signal that we’ll definitely use a slow Python call to call this function. (We may use a slow Python call even for functions we do recognize, if we’re targeting an interpreter without an opcode for the function.)

Only used when printing Expressions.

EXAMPLES:

```
sage: from sage.ext.fast_callable import function_name
sage: function_name(operator.pow)
'pow'
sage: function_name(cos)
'cos'
sage: function_name(factorial)
'{factorial}'
```

`sage.ext.fast_callable.generate_code(expr, stream)`

Generate code from an Expression tree; write the result into an InstructionStream.

In `fast_callable`, first we create an Expression, either directly with an `ExpressionTreeBuilder` or with `_fast_callable_` methods. Then we optimize the Expression in tree form. (Unfortunately, this step is currently missing – we do no optimizations.)

Then we linearize the Expression into a sequence of instructions, by walking the Expression and sending the corresponding stack instructions to an `InstructionStream`.

EXAMPLES:

```
sage: from sage.ext.fast_callable import ExpressionTreeBuilder, generate_code, InstructionStream
sage: etb = ExpressionTreeBuilder('x')
sage: x = etb.var('x')
sage: expr = ((x+pi)*(x+1))
sage: from sage.ext.interpreters.wrapper_py import metadata, Wrapper_py
sage: instr_stream = InstructionStream(metadata, 1)
sage: generate_code(expr, instr_stream)
sage: instr_stream.instr('return')
sage: v = Wrapper_py(instr_stream.get_current())
sage: type(v)
<type 'sage.ext.interpreters.wrapper_py.Wrapper_py'>
sage: v(7)
8*pi + 56
```

TESTS:

```
sage: def my_sin(x): return sin(x)
sage: def my_norm(x, y): return x*x + y*y
sage: def my_sqrt(x):
...     if x < 0: raise ValueError, "sqrt of negative number"
...     return sqrt(x, extend=False)
sage: fc = fast_callable(expr, domain=RealField(130))
sage: fc(0)
3.1415926535897932384626433832795028842
sage: fc(1)
8.2831853071795864769252867665590057684
sage: fc = fast_callable(expr, domain=RDF)
sage: fc(0)
```

```
3.141592653589793
sage: fc(1)
8.283185307179586
sage: fc.op_list()
[('load_arg', 0), ('load_const', pi), 'add', ('load_arg', 0), ('load_const', 1), 'add', 'mul', 'return']
sage: fc = fast_callable(etb.call(sin, x) + etb.call(sqrt, x), domain=RDF)
sage: fc(1)
1.8414709848078965
sage: fc.op_list()
[('load_arg', 0), 'sin', ('load_arg', 0), 'sqrt', 'add', 'return']
sage: fc = fast_callable(etb.call(sin, x) + etb.call(sqrt, x))
sage: fc(1)
sin(1) + 1
sage: fc.op_list()
[('load_arg', 0), ('py_call', sin, 1), ('load_arg', 0), ('py_call', <function sqrt at ...>, 1), 'return']
sage: fc = fast_callable(etb.call(my_sin, x), domain=RDF)
sage: fc(3)
0.1411200080598672
sage: fc = fast_callable(etb.call(my_sin, x), domain=RealField(100))
sage: fc(3)
0.14112000805986722210074480281
sage: fc.op_list()
[('load_arg', 0), ('py_call', <function my_sin at 0x...>, 1), 'return']
sage: fc = fast_callable(etb.call(my_sqrt, x), domain=RDF)
sage: fc(3)
1.7320508075688772
sage: parent(fc(3))
Real Double Field
sage: fc(-3)
Traceback (most recent call last):
...
ValueError: sqrt of negative number
sage: fc = fast_callable(etb.call(my_sqrt, x), domain=RR)
sage: fc(3)
1.73205080756888
sage: fc(-3)
Traceback (most recent call last):
...
ValueError: sqrt of negative number
sage: etb2 = ExpressionTreeBuilder(('y', 'z'))
sage: y = etb2.var('y')
sage: z = etb2.var('z')
sage: fc = fast_callable(etb2.call(sqrt, etb2.call(my_norm, y, z)), domain=RDF)
sage: fc(3, 4)
5.0
sage: fc.op_list()
[('load_arg', 0), ('load_arg', 1), ('py_call', <function my_norm at 0x...>, 2), 'sqrt', 'return']
sage: fc.python_calls()
[<function my_norm at 0x...>]
sage: fc = fast_callable(etb2.call(sqrt, etb2.call(my_norm, y, z)), domain=RR)
sage: fc(3, 4)
5.000000000000000
sage: fc = fast_callable(etb2.call(my_norm, y, z), domain=ZZ)
sage: fc(3, 4)
25
sage: fc.op_list()
[('load_arg', 0), ('load_arg', 1), ('py_call', <function my_norm at 0x...>, 2), 'return']
sage: fc = fast_callable(expr)
```



```

sage: fc(3.0r)
4.0*pi + 12.0
sage: fc = fast_callable(x+3, domain=ZZ)
sage: fc(4)
7
sage: fc = fast_callable(x/3, domain=ZZ)
sage: fc(4)
Traceback (most recent call last):
...
TypeError: no conversion of this rational to integer
sage: fc(6)
2
sage: fc = fast_callable(etb.call(sin, x), domain=ZZ)
sage: fc(0)
0
sage: fc(3)
Traceback (most recent call last):
...
TypeError: unable to convert sin(3) to an integer

sage: fc = fast_callable(etb(x)^100)
sage: fc(pi)
pi^100
sage: fc = fast_callable(etb(x)^100, domain=ZZ)
sage: fc(2)
1267650600228229401496703205376
sage: fc = fast_callable(etb(x)^100, domain=RIF)
sage: fc(RIF(-2))
1.2676506002282295?e30
sage: fc = fast_callable(etb(x)^100, domain=RDF)
sage: fc.op_list()
[('load_arg', 0), ('ipow', 100), 'return']
sage: fc(1.1)
13780.61233982...
sage: fc = fast_callable(etb(x)^100, domain=RR)
sage: fc.op_list()
[('load_arg', 0), ('ipow', 100), 'return']
sage: fc(1.1)
13780.6123398224
sage: fc = fast_callable(etb(x)^(-100), domain=RDF)
sage: fc.op_list()
[('load_arg', 0), ('ipow', -100), 'return']
sage: fc(1.1)
7.25657159014...e-05
sage: fc = fast_callable(etb(x)^(-100), domain=RR)
sage: fc(1.1)
0.0000725657159014814
sage: expo = 2^32
sage: base = (1.0).nextabove()
sage: fc = fast_callable(etb(x)^expo, domain=RDF)
sage: fc.op_list()
[('load_arg', 0), ('py_call', (^4294967296), 1), 'return']
sage: fc(base)          # rel tol 1e-15
1.0000009536747712
sage: RDF(base)^expo
1.0000009536747712
sage: fc = fast_callable(etb(x)^expo, domain=RR)
sage: fc.op_list()

```

```
[('load_arg', 0), ('py_call', (^4294967296), 1), 'return']
sage: fc(base)
1.00000095367477
sage: base^expo
1.00000095367477
```

Make sure we don't overflow the stack with highly nested expressions (#11766):

```
sage: R.<x> = CC[]
sage: f = R(range(100000))
sage: ff = fast_callable(f)
sage: f(0.5)
2.0000000000000000
sage: ff(0.5)
2.0000000000000000
sage: f(0.9), ff(0.9)
(90.00000000000000, 90.00000000000000)
```

`sage.ext.fast_callable.get_builtin_functions()`

To handle ExpressionCall, we need to map from Sage and Python functions to opcode names.

This returns a dictionary which is that map.

We delay building builtin_functions to break a circular import between sage.calculus and this file.

EXAMPLES:

```
sage: from sage.ext.fast_callable import get_builtin_functions
sage: builtins = get_builtin_functions()
sage: sorted(list(builtins.values()))
['abs', 'abs', 'acos', 'acosh', 'add', 'asin', 'asinh', 'atan', 'atanh', 'ceil', 'cos', 'cosh',
sage: builtins[sin]
'sin'
sage: builtins[ln]
'log'
```

`sage.ext.fast_callable.op_list(args, metadata)`

Given a dictionary with the result of calling `get_current` on an `InstructionStream`, and the corresponding interpreter metadata, return a list of the instructions, in a simple somewhat human-readable format.

For debugging only. (That is, it's probably not a good idea to try to programmatically manipulate the result of this function; the expected use is just to print the returned list to the screen.)

There's probably no reason to call this directly; if you have a wrapper object, call `op_list` on it; if you have an `InstructionStream` object, call `current_op_list` on it.

EXAMPLES:

```
sage: from sage.ext.interpreters.wrapper_rdf import metadata
sage: from sage.ext.fast_callable import InstructionStream, op_list
sage: instr_stream = InstructionStream(metadata, 1)
sage: instr_stream.instr('load_arg', 0)
sage: instr_stream.instr('abs')
sage: instr_stream.instr('return')
sage: instr_stream.current_op_list()
[('load_arg', 0), 'abs', 'return']
sage: op_list(instr_stream.get_current(), metadata)
[('load_arg', 0), 'abs', 'return']
```

2.12.2 Fast Numerical Evaluation

For many applications such as numerical integration, differential equation approximation, plotting a 3d surface, optimization problems, monte-carlo simulations, etc., one wishes to pass around and evaluate a single algebraic expression many, many times at various floating point values. Doing this via recursive calls over a python representation of the object (even if Maxima or other outside packages are not involved) is extremely inefficient.

Up until now the solution has been to use lambda expressions, but this is neither intuitive, Sage-like, nor efficient (compared to operating on raw C doubles). This module provides a representation of algebraic expression in Reverse Polish Notation, and provides an efficient interpreter on C double values as a callable python object. It does what it can in C, and will call out to Python if necessary.

Essential to the understanding of this class is the distinction between symbolic expressions and callable symbolic expressions (where the latter binds argument names to argument positions). The `*vars` parameter passed around encapsulates this information.

See the function `fast_float(f, *vars)` to create a fast-callable version of `f`.

Note: Sage temporarily has two implementations of this functionality ; one in this file, which will probably be deprecated soon, and one in `fast_callable.pyx`. The following instructions are for the old implementation; you probably want to be looking at `fast_callable.pyx` instead.

To provide this interface for a class, implement `fast_float_(self, *vars)`. The basic building blocks are provided by the functions `fast_float_constant` (returns a constant function), `fast_float_arg` (selects the `n`-th value when called with `\geq n` arguments), and `fast_float_func` which wraps a callable Python function. These may be combined with the standard Python arithmetic operators, and support many of the basic math functions such `sqrt`, `exp`, and `trig` functions.

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float
sage: f = fast_float(sqrt(x^7+1), 'x', old=True)
sage: f(1)
1.4142135623730951
sage: f.op_list()
['load 0', 'push 7.0', 'pow', 'push 1.0', 'add', 'call sqrt(1)']
```

To interpret that last line, we load argument 0 (`x` in this case) onto the stack, push the constant 2.0 onto the stack, call the `pow` function (which takes 2 arguments from the stack), push the constant 1.0, add the top two arguments of the stack, and then call `sqrt`.

Here we take `sin` of the first argument and add it to `f`:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: g = fast_float_arg(0).sin()
sage: (f+g).op_list()
['load 0', 'push 7.0', 'pow', 'push 1.0', 'add', 'call sqrt(1)', 'load 0', 'call sin(1)', 'add']
```

TESTS:

This used to segfault because of an assumption that assigning `None` to a variable would raise a `TypeError`:

```
sage: from sage.ext.fast_eval import fast_float_arg, fast_float
sage: fast_float_arg(0)+None
Traceback (most recent call last):
...
TypeError
```

AUTHORS:

- Robert Bradshaw (2008-10): Initial version

class `sage.ext.fast_eval.FastDoubleFunc`

Bases: `object`

This class is for fast evaluation of algebraic expressions over the real numbers (e.g. for plotting). It represents an expression as a stack-based series of operations.

EXAMPLES:

```
sage: from sage.ext.fast_eval import FastDoubleFunc
sage: f = FastDoubleFunc('const', 1.5) # the constant function
sage: f()
1.5
sage: g = FastDoubleFunc('arg', 0) # the first argument
sage: g(5)
5.0
sage: h = f+g
sage: h(17)
18.5
sage: h = h.sin()
sage: h(pi/2-1.5)
1.0
sage: h.is_pure_c()
True
sage: list(h)
['push 1.5', 'load 0', 'add', 'call sin(1)']
```

We can wrap Python functions too:

```
sage: h = FastDoubleFunc('callable', lambda x,y: x*x*x - y, g, f)
sage: h(10)
998.5
sage: h.is_pure_c()
False
sage: list(h)
['load 0', 'push 1.5', 'py_call <function <lambda> at 0x...>(2)']
```

Here's a more complicated expression:

```
sage: from sage.ext.fast_eval import fast_float_constant, fast_float_arg
sage: a = fast_float_constant(1.5)
sage: b = fast_float_constant(3.14)
sage: c = fast_float_constant(7)
sage: x = fast_float_arg(0)
sage: y = fast_float_arg(1)
sage: f = a*x^2 + b*x + c - y/sqrt(sin(y)^2+a)
sage: f(2,3)
16.846610528508116
sage: f.max_height
4
sage: f.is_pure_c()
True
sage: list(f)
['push 1.5', 'load 0', 'dup', 'mul', 'mul', 'push 3.14', 'load 0', 'mul', 'add', 'push 7.0', 'ad
```

AUTHORS:

- Robert Bradshaw

abs ()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).abs()
sage: f(3)
3.0
```

arccos()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).arccos()
sage: f(sqrt(3)/2)
0.5235987755982989...
```

arccosh()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).arccosh()
sage: f(cosh(5))
5.0
```

arcsin()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).arcsin()
sage: f(0.5)
0.523598775598298...
```

arcsinh()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).arcsinh()
sage: f(sinh(5))
5.0
```

arctan()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).arctan()
sage: f(1)
0.785398163397448...
```

arctanh()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).arctanh()
sage: abs(f(tanh(0.5)) - 0.5) < 0.0000001
True
```

ceil()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).ceil()
sage: f(1.5)
2.0
```

```
sage: f(-1.5)
-1.0
```

cos()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).cos()
sage: f(0)
1.0
```

cosh()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).cosh()
sage: f(log(2))
1.25
```

cot()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).cot()
sage: f(pi/4)
1.0...
```

csc()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).csc()
sage: f(pi/2)
1.0
```

exp()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).exp()
sage: f(1)
2.718281828459045...
sage: f(100)
2.6881171418161356e+43
```

floor()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).floor()
sage: f(11.5)
11.0
sage: f(-11.5)
-12.0
```

is_pure_c()

Returns True if this function can be evaluated without any python calls (at any level).

EXAMPLES:

```

sage: from sage.ext.fast_eval import fast_float_constant, fast_float_arg, fast_float_func
sage: fast_float_constant(2).is_pure_c()
True
sage: fast_float_arg(2).sqrt().sin().is_pure_c()
True
sage: fast_float_func(lambda _: 2).is_pure_c()
False

```

log(base=None)

EXAMPLES:

```

sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).log()
sage: f(2)
0.693147180559945...
sage: f = fast_float_arg(0).log(2)
sage: f(2)
1.0
sage: f = fast_float_arg(0).log(3)
sage: f(9)
2.0...

```

max_height

nargs

nops

op_list()

Returns a list of string representations of the operations that make up this expression.

Python and C function calls may be only available by function pointer addresses.

EXAMPLES:

```

sage: from sage.ext.fast_eval import fast_float_constant, fast_float_arg
sage: a = fast_float_constant(17)
sage: x = fast_float_arg(0)
sage: a.op_list()
['push 17.0']
sage: x.op_list()
['load 0']
sage: (a*x).op_list()
['push 17.0', 'load 0', 'mul']
sage: (a+a*x^2).sqrt().op_list()
['push 17.0', 'push 17.0', 'load 0', 'dup', 'mul', 'mul', 'add', 'call sqrt(1)']

```

python_calls()

Returns a list of all python calls used by function.

EXAMPLES:

```

sage: from sage.ext.fast_eval import fast_float_func, fast_float_arg
sage: x = fast_float_arg(0)
sage: f = fast_float_func(hash, sqrt(x))
sage: f.op_list()
['load 0', 'call sqrt(1)', 'py_call <built-in function hash>(1)']
sage: f.python_calls()
[<built-in function hash>]

```

sec()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).sec()
sage: f(pi)
-1.0
```

sin()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).sin()
sage: f(pi/2)
1.0
```

sinh()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).sinh()
sage: f(log(2))
0.75
```

sqrt()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).sqrt()
sage: f(4)
2.0
```

tan()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).tan()
sage: f(pi/3)
1.73205080756887...
```

tanh()

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0).tanh()
sage: f(0)
0.0
```

`sage.ext.fast_eval.fast_float` (*f*, *old=None*, *expect_one_var=False*, **vars*)

Tries to create a function that evaluates *f* quickly using floating-point numbers, if possible. There are two implementations of `fast_float` in Sage; by default we use the newer, which is slightly faster on most tests.

On failure, returns the input unchanged.

INPUT:

- *f* – an expression
- *vars* – the names of the arguments
- *old* – use the original algorithm for `fast_float`

- `expect_one_var` – don't give deprecation warning if `vars` is omitted, as long as expression has only one var

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float
sage: x, y = var('x, y')
sage: f = fast_float(sqrt(x^2+y^2), 'x', 'y')
sage: f(3, 4)
5.0
```

Specifying the argument names is essential, as `fast_float` objects only distinguish between arguments by order.

```
sage: f = fast_float(x-y, 'x', 'y')
sage: f(1, 2)
-1.0
sage: f = fast_float(x-y, 'y', 'x')
sage: f(1, 2)
1.0
```

`sage.ext.fast_eval.fast_float_arg(n)`
Return a fast-to-evaluate argument selector.

INPUT:

- `n` – the (zero-indexed) argument to select

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_arg
sage: f = fast_float_arg(0)
sage: f(1, 2)
1.0
sage: f = fast_float_arg(1)
sage: f(1, 2)
2.0
```

This is all that goes on under the hood:

```
sage: fast_float_arg(10).op_list()
['load 10']
```

`sage.ext.fast_eval.fast_float_constant(x)`
Return a fast-to-evaluate constant function.

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_constant
sage: f = fast_float_constant(-2.75)
sage: f()
-2.75
```

This is all that goes on under the hood:

```
sage: fast_float_constant(pi).op_list()
['push 3.14159265359']
```

`sage.ext.fast_eval.fast_float_func(f, *args)`
Returns a wrapper around a python function.

INPUT:

- `f` – a callable python object

- args – a list of FastDoubleFunc inputs

EXAMPLES:

```
sage: from sage.ext.fast_eval import fast_float_func, fast_float_arg
sage: f = fast_float_arg(0)
sage: g = fast_float_arg(1)
sage: h = fast_float_func(lambda x,y: x-y, f, g)
sage: h(5, 10)
-5.0
```

This is all that goes on under the hood:

```
sage: h.op_list()
['load 0', 'load 1', 'py_call <function <lambda> at 0x...>(2)']
```

```
sage.ext.fast_eval.is_fast_float(x)
```

CODE EVALUATION

3.1 On-the-fly generation of compiled extensions

`sage.misc.cython_c.cython_compile` (*code*, *verbose=False*, *compile_message=False*,
make_c_file_nice=False, *use_cache=False*)

Given a block of Cython code (as a text string), this function compiles it using a C compiler, and includes it into the global scope of the module that called this function.

The following pragmas are available:

- `clang` - may be either `c` or `c++` (or `C` or `C++`) indicating whether a C or C++ compiler should be used.
- `clib` - additional libraries to be linked in, the space separated list is split and passed to `distutils`.
- `cinclude` - additional directories to search for header files. The space separated list is split and passed to `distutils`.
- `cfile` - additional C or C++ files to be compiled. Also, `$SAGE_ROOT` is expanded, but other environment variables are not.
- `cargs` - additional parameters passed to the compiler

For example:

```
#clang C++
#clib givaro
#cinclude /usr/local/include/
#cargs -ggdb
#cfile foo.c
```

AUTHOR: William Stein, 2006-10-31

3.2 Cython support functions

AUTHORS:

- William Stein (2006-01-18): initial version
- William Stein (2007-07-28): update from `sagex` to `cython`
- Martin Albrecht & William Stein (2011-08): `cfile` & `cargs`

`sage.misc.cython.atlas()`

Returns the name of the ATLAS library to use. On Darwin or Cygwin, this is `'blas'`, and otherwise it is `'atlas'`.

EXAMPLES:

```
sage: sage.misc.cython.atlas() # random -- depends on OS
'atlas'
```

```
sage.misc.cython.cblas()
```

Return the name of the cblas library on this system. If the environment variable `$SAGE_CBLAS` is set, just return its value. If not, return `'cblas'` if `/usr/lib/libcblas.so` or `/usr/lib/libcblas.dylib` exists, return `'blas'` if `/usr/lib/libblas.dll.a` exists, and return `'gslcblas'` otherwise.

EXAMPLES:

```
sage: sage.misc.cython.cblas() # random -- depends on OS, etc.
'cblas'
```

```
sage.misc.cython.compile_and_load(code)
```

INPUT:

- `code` – string containing code that could be in a `.pyx` file that is attached or put in a `%cython` block in the notebook.

OUTPUT: a module, which results from compiling the given code and importing it

EXAMPLES:

```
sage: module = sage.misc.cython.compile_and_load("def f(int n):\n    return n*n")
sage: module.f(10)
100
```

```
sage.misc.cython.cython(filename, verbose=False, compile_message=False, use_cache=False,
                        create_local_c_file=False, annotate=True, sage_namespace=True,
                        create_local_so_file=False)
```

Compile a Cython file. This converts a Cython file to a C (or C++ file), and then compiles that. The `.c` file and the `.so` file are created in a temporary directory.

INPUTS:

- `filename` - the name of the file to be compiled. Should end with `'pyx'`.
- `verbose` (bool, default False) - if True, print debugging information.
- `compile_message` (bool, default False) - if True, print `'Compiling <filename>...'` to the standard error.
- `use_cache` (bool, default False) - if True, check the temporary build directory to see if there is already a corresponding `.so` file. If so, and if the `.so` file is newer than the Cython file, don't recompile, just reuse the `.so` file.
- `create_local_c_file` (bool, default False) - if True, save a copy of the `.c` file in the current directory.
- `annotate` (bool, default True) - if True, create an html file which annotates the conversion from `.pyx` to `.c`. By default this is only created in the temporary directory, but if `create_local_c_file` is also True, then save a copy of the `.html` file in the current directory.
- `sage_namespace` (bool, default True) - if True, import `sage.all`.
- `create_local_so_file` (bool, default False) - if True, save a copy of the compiled `.so` file in the current directory.

TESTS:

Before [trac ticket #12975](#), it would have been needed to write `#clang c++`, but upper case C++ has resulted in an error:

```

sage: code = [
...     "#clang C++",
...     "#include %s/include/singular %s/include/factory"%(SAGE_LOCAL, SAGE_LOCAL),
...     "#clib m readline singular givaro ntl gmpxx gmp",
...     "from sage.rings.polynomial.multi_polynomial_libsingular cimport MPolynomial_libsingular",
...     "from sage.libs.singular.polynomial cimport singular_polynomial_pow",
...     "def test(MPolynomial_libsingular p):",
...     "    singular_polynomial_pow(&p._poly, p._poly, 2, p._parent_ring)"
sage: cython(os.linesep.join(code))

```

The function `test` now manipulates internal C data of polynomials, squaring them:

```

sage: P.<x,y>=QQ[]
sage: test(x)
sage: x
x^2

```

`sage.misc.cython.cython_create_local_so(filename)`

Compile filename and make it available as a loadable shared object file.

INPUT:

- filename - string: a Cython (.pyx) file

OUTPUT: None

EFFECT: A compiled, python “importable” loadable shared object file is created.

Note: Shared object files are *not* reloadable. The intent is for imports in other scripts. A possible development cycle might go thus:

- Attach a .pyx file
 - Interactively test and edit it to your satisfaction
 - Use `cython_create_local_so` to create the shared object file
 - Import the .so file in other scripts
-

EXAMPLES:

```

sage: curdir = os.path.abspath(os.curdir)
sage: dir = tmp_dir(); os.chdir(dir)
sage: f = open('hello.pyx', 'w')
sage: s = "def hello():\n    print 'hello'\n"
sage: f.write(s)
sage: f.close()
sage: cython_create_local_so('hello.pyx')
Compiling hello.pyx...
sage: sys.path.append('.')
sage: import hello
sage: hello.hello()
hello
sage: os.chdir(curdir)

```

AUTHORS:

- David Fu (2008-04-09): initial version

```

sage.misc.cython.cython_import(filename, verbose=False, compile_message=False,
                               use_cache=False, create_local_c_file=True, **kwargs)

```

Compile a file containing Cython code, then import and return the module. Raises an `ImportError` if anything goes wrong.

INPUT:

- `filename` - a string; name of a file that contains Cython code

See the function `sage.misc.cython.cython()` for documentation for the other inputs.

OUTPUT:

- the module that contains the compiled Cython code.

```
sage.misc.cython.cython_import_all(filename,      globals,      verbose=False,      com-  
                                pile_message=False,      use_cache=False,      cre-  
                                ate_local_c_file=True)
```

Imports all non-private (i.e., not beginning with an underscore) attributes of the specified Cython module into the given context. This is similar to:

```
from module import *
```

Raises an `ImportError` exception if anything goes wrong.

INPUT:

- `filename` - a string; name of a file that contains Cython code

```
sage.misc.cython.cython_lambda(vars,      expr,      verbose=False,      compile_message=False,  
                                use_cache=False)
```

Create a compiled function which evaluates `expr` assuming machine values for `vars`.

INPUT:

- `vars` - list of pairs (variable name, c-data type), where the variable names and data types are strings, OR a string such as `'double x, int y, int z'`
- `expr` - an expression involving the `vars` and constants; you can access objects defined in the current module scope `globals()` using `sage.object_name`.

Warning: Accessing `globals()` doesn't actually work, see [trac ticket #12446](#).

EXAMPLES:

We create a Lambda function in pure Python (using the `r` to make sure the 3.2 is viewed as a Python float):

```
sage: f = lambda x,y: x*x + y*y + x + y + 17r*x + 3.2r
```

We make the same Lambda function, but in a compiled form.

```
sage: g = cython_lambda('double x, double y', 'x*x + y*y + x + y + 17*x + 3.2')  
sage: g(2,3)  
55.2  
sage: g(0,0)  
3.2
```

In order to access Sage globals, prefix them with `sage.:`

```
sage: f = cython_lambda('double x', 'sage.sin(x) + sage.a')  
sage: f(0)  
Traceback (most recent call last):  
...  
NameError: global 'a' is not defined  
sage: a = 25  
sage: f(10)
```

```
24.45597888911063
sage: a = 50
sage: f(10)
49.45597888911063
```

`sage.misc.cython.environ_parse(s)`

Given a string `s`, find each substring of the form `'$ABC'`. If the environment variable `$ABC` is set, replace `'$ABC'` with its value and move on to the next such substring. If it is not set, stop parsing there.

EXAMPLES:

```
sage: from sage.misc.cython import environ_parse
sage: environ_parse('$SAGE_LOCAL') == SAGE_LOCAL
True
sage: environ_parse('$THIS_IS_NOT_DEFINED_ANYWHERE')
'$THIS_IS_NOT_DEFINED_ANYWHERE'
sage: os.environ['DEFINE_THIS'] = 'hello'
sage: environ_parse('$DEFINE_THIS/$THIS_IS_NOT_DEFINED_ANYWHERE/$DEFINE_THIS')
'hello/$THIS_IS_NOT_DEFINED_ANYWHERE/$DEFINE_THIS'
```

`sage.misc.cython.import_test(name)`

This is used by the testing infrastructure to test building Cython programs.

INPUT:

- `name` – string; name of a key to the TESTS dictionary above

OUTPUT: a module, which results from compiling the given code and importing it

EXAMPLES:

```
sage: module = sage.misc.cython.import_test("trac11680b")
sage: module.f(2,3,4)
9
```

`sage.misc.cython.parse_keywords(kwd, s)`

Given a keyword `kwd` and a string `s`, return a list of all arguments on the same line as that keyword in `s`, as well as a new copy of `s` in which each occurrence of `kwd` is in a comment. If a comment already occurs on the line containing `kwd`, no words after the `#` are added to the list.

EXAMPLES:

```
sage: sage.misc.cython.parse_keywords('clib', " clib foo bar baz\n #cinclude bar\n")
(['foo', 'bar', 'baz'], ' #clib foo bar baz\n #cinclude bar\n')

sage: sage.misc.cython.parse_keywords('clib', "# qux clib foo bar baz\n #cinclude bar\n")
(['foo', 'bar', 'baz'], '# qux clib foo bar baz\n #cinclude bar\n')

sage: sage.misc.cython.parse_keywords('clib', "# clib foo bar # baz\n #cinclude bar\n")
(['foo', 'bar'], '# clib foo bar # baz\n #cinclude bar\n')
```

`sage.misc.cython.pyx_preparse(s)`

Preparse a pyx file:

- include `cdefs.pxi`, `interrupt.pxi`, `stdsage.pxi`
- parse clang pragma (c or c++)
- parse `clib` pragma (additional libraries to link in)
- parse `cinclude` (additional include directories)
- parse `cfile` (additional files to be included)

- `parse_cargs` (additional parameters passed to the compiler)

The pragmas:

- `clang` - may be either `'c'` or `'c++'` indicating whether a C or C++ compiler should be used
- `clib` - additional libraries to be linked in, the space separated list is split and passed to `distutils`.
- `cinclude` - additional directories to search for header files. The space separated list is split and passed to `distutils`.
- `cfile` - additional C or C++ files to be compiled. Also, `$SAGE_SRC` and `$SAGE_LOCAL` are expanded, but other environment variables are not.
- `cargs` - additional parameters passed to the compiler

OUTPUT: preamble, libs, includes, language, files, args

EXAMPLES:

```
sage: from sage.misc.cython import pyx_preparse
sage: pyx_preparse("")
('\n#include "interrupt.pxi" # ctrl-c interrupt block support\n#include "stdsage.pxi"\n\n#include
['mpfr',
'gmp',
'gmpxx',
'stdc++',
'pari',
'm',
'ec',
'gsl',
'...blas',
...,
'ntl'],
['.../include',
'.../include/python...',
'.../lib/python.../site-packages/numpy/core/include',
'...',
'.../sage/ext'],
'c',
[], ['-w', '-O2'])
sage: s, libs, inc, lang, f, args = pyx_preparse("# clang c++\n #clib foo\n # cinclude bar\n")
sage: lang
'c++'

sage: libs
['foo', 'mpfr',
'gmp', 'gmpxx',
'stdc++',
'pari',
'm',
'ec',
'gsl', '...blas', ...,
'ntl']
sage: libs[1:] == sage.misc.cython.standard_libs
True

sage: inc
['bar',
'.../include',
'.../include/python...',
'.../lib/python.../site-packages/numpy/core/include',
```



```
'...',
'.../sage/ext']
```

```
sage: s, libs, inc, lang, f, args = pyx_preparse("# cargs -O3 -ggdb\n")
sage: args
['-w', '-O2', '-O3', '-ggdb']
```

TESTS:

```
sage: module = sage.misc.cython.import_test("trac11680") # long time (7s on sage.math, 2012)
sage: R.<x> = QQ[]
sage: module.evaluate_at_power_of_gen(x^3 + x - 7, 5) # long time
x^15 + x^5 - 7
```

`sage.misc.cython.sanitize(f)`

Given a filename `f`, replace it by a filename that is a valid Python module name.

This means that the characters are all alphanumeric or `_`'s and doesn't begin with a numeral.

EXAMPLES:

```
sage: from sage.misc.cython import sanitize
sage: sanitize('abc')
'abc'
sage: sanitize('abc/def')
'abc_def'
sage: sanitize('123/def-hij/file.py')
'_123_def_hij_file_py'
```

`sage.misc.cython.subtract_from_line_numbers(s, n)`

Given a string `s` and an integer `n`, for any line of `s` which has the form `'text:NUM:text'` subtract `n` from `NUM` and return `'text:(NUM-n):text'`. Return other lines of `s` without change.

EXAMPLES:

```
sage: from sage.misc.cython import subtract_from_line_numbers
sage: subtract_from_line_numbers('hello:1234:hello', 3)
'hello:1231:hello\n'
sage: subtract_from_line_numbers('text:123\nhello:1234:', 3)
'text:123\nhello:1231:\n'
```

3.3 Fortran compiler

`class sage.misc.inline_fortran.InlineFortran(globals=None)`

add_library(s)

add_library_path(s)

eval(x, globals=None, locals=None)

Compile fortran code `x` and adds the functions in it to `globals`.

INPUT:

- `x` – Fortran code
- `globals` – a dict to which to add the functions from the fortran module
- `locals` – ignored

EXAMPLES:

```
sage: code = '''
....: C FILE: FIB1.F
....:      SUBROUTINE FIB(A,N)
....: C
....: C      CALCULATE FIRST N FIBONACCI NUMBERS
....: C
....:      INTEGER N
....:      REAL*8 A(N)
....:      DO I=1,N
....:          IF (I.EQ.1) THEN
....:              A(I) = 0.0D0
....:          ELSEIF (I.EQ.2) THEN
....:              A(I) = 1.0D0
....:          ELSE
....:              A(I) = A(I-1) + A(I-2)
....:          ENDIF
....:      ENDDO
....:      END
....: C END FILE FIB1.F
....: '''
sage: fortran(code, globals())
sage: import numpy
sage: a = numpy.array(range(10), dtype=float)
sage: fib(a, 10)
sage: a
array([ 0.,  1.,  1.,  2.,  3.,  5.,  8., 13., 21., 34.] )
```

TESTS:

```
sage: os.chdir(SAGE_ROOT)
sage: fortran.eval("SYNTAX ERROR !@#$")
Traceback (most recent call last):
...
RuntimeError: failed to compile Fortran code:...
sage: os.getcwd() == SAGE_ROOT
True
```

3.4 A parser for symbolic equations and expressions

It is both safer and more powerful than using Python's `eval`, as one has complete control over what names are used (including dynamically creating variables) and how integer and floating point literals are created.

AUTHOR:

- Robert Bradshaw 2008-04 (initial version)

class `sage.misc.parser.LookupNameMaker`

Bases: `object`

This class wraps a dictionary as a callable for use in creating names. It takes a dictionary of names, and an (optional) callable to use when the given name is not found in the dictionary.

EXAMPLES:

```
sage: from sage.misc.parser import LookupNameMaker
sage: maker = LookupNameMaker({'pi': pi}, var)
sage: maker('pi')
```

```

pi
sage: maker('pi') is pi
True
sage: maker('a')
a

```

class `sage.misc.parser.Parser`
 Bases: `object`

Create a symbolic expression parser.

INPUT:

- `make_int` – callable object to construct integers from strings (default `int`)
- `make_float` – callable object to construct real numbers from strings (default `float`)
- `make_var` – callable object to construct variables from strings (default `str`) this may also be a dictionary of variable names
- `make_function` – callable object to construct callable functions from strings this may also be a dictionary
- `implicit_multiplication` – whether or not to accept implicit multiplication

OUTPUT:

The evaluated expression tree given by the string, where the above functions are used to create the leaves of this tree.

EXAMPLES:

```

sage: from sage.misc.parser import Parser
sage: p = Parser()
sage: p.parse("1+2")
3
sage: p.parse("1+2 == 3")
True

sage: p = Parser(make_var=var)
sage: p.parse("a*b^c - 3a")
a*b^c - 3*a

sage: R.<x> = QQ[]
sage: p = Parser(make_var = {'x': x })
sage: p.parse("(x+1)^5-x")
x^5 + 5*x^4 + 10*x^3 + 10*x^2 + 4*x + 1
sage: p.parse("(x+1)^5-x").parent() is R
True

sage: p = Parser(make_float=RR, make_var=var, make_function={'foo': (lambda x: x*x+x)})
sage: p.parse("1.5 + foo(b)")
b^2 + b + 1.500000000000000
sage: p.parse("1.9").parent()
Real Field with 53 bits of precision

```

p_arg (*tokens*)

Returns an `expr`, or a `(name, expr)` tuple corresponding to a single function call argument.

EXAMPLES:

Parsing a normal expression:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_arg(Tokenizer("a+b"))
a + b
```

A keyword expression argument:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_arg(Tokenizer("val=a+b"))
('val', a + b)
```

A lone list::

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_arg(Tokenizer("[x]"))
[x]
```

p_args (*tokens*)

Returns a list, dict pair.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser()
sage: p.p_args(Tokenizer("1,2,a=3"))
([1, 2], {'a': 3})
sage: p.p_args(Tokenizer("1, 2, a = 1+5^2"))
([1, 2], {'a': 26})
```

p_atom (*tokens*)

Parse an atom. This is either a parenthesized expression, a function call, or a literal name/int/float.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var, make_function={'sin': sin})
sage: p.p_atom(Tokenizer("1"))
1
sage: p.p_atom(Tokenizer("12"))
12
sage: p.p_atom(Tokenizer("12.5"))
12.5
sage: p.p_atom(Tokenizer("(1+a)"))
a + 1
sage: p.p_atom(Tokenizer("(1+a)^2"))
a + 1
sage: p.p_atom(Tokenizer("sin(1+a)"))
sin(a + 1)
sage: p = Parser(make_var=var, make_function={'foo': sage.misc.parser.foo})
sage: p.p_atom(Tokenizer("foo(a, b, key=value)"))
((a, b), {'key': value})
sage: p.p_atom(Tokenizer("foo()"))
((), {})
```

p_eqn (*tokens*)

Parse an equation or expression.

This is the top-level node called by the code{parse} function.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_eqn(Tokenizer("1+a"))
a + 1

sage: p.p_eqn(Tokenizer("a == b"))
a == b
sage: p.p_eqn(Tokenizer("a < b"))
a < b
sage: p.p_eqn(Tokenizer("a > b"))
a > b
sage: p.p_eqn(Tokenizer("a <= b"))
a <= b
sage: p.p_eqn(Tokenizer("a >= b"))
a >= b
sage: p.p_eqn(Tokenizer("a != b"))
a != b
```

p_expr (*tokens*)

Parse a list of one or more terms.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_expr(Tokenizer("a+b"))
a + b
sage: p.p_expr(Tokenizer("a"))
a
sage: p.p_expr(Tokenizer("a - b + 4*c - d^2"))
-d^2 + a - b + 4*c
sage: p.p_expr(Tokenizer("a - -3"))
a + 3
sage: p.p_expr(Tokenizer("a + 1 == b"))
a + 1
```

p_factor (*tokens*)

Parse a single factor, which consists of any number of unary +/- and a power.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: R.<t> = ZZ[['t']]
sage: p = Parser(make_var={'t': t})
sage: p.p_factor(Tokenizer("- -t"))
t
sage: p.p_factor(Tokenizer("- + - -t^2"))
-t^2
sage: p.p_factor(Tokenizer("t^11 * x"))
t^11
```

p_list (*tokens*)

Parse a list of items.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
```

```
sage: p.p_list(Tokenizer("[1+2, 1e3]"))
[3, 1000.0]
sage: p.p_list(Tokenizer("[]"))
[]
```

p_matrix (*tokens*)

Parse a matrix

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_matrix(Tokenizer("([a,0],[0,a])"))
[a 0]
[0 a]
```

p_power (*tokens*)

Parses a power. Note that exponentiation groups right to left.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: R.<t> = ZZ[['t']]
sage: p = Parser(make_var={'t': t})
sage: p.p_factor(Tokenizer("-(1+t)^-1"))
-1 + t - t^2 + t^3 - t^4 + t^5 - t^6 + t^7 - t^8 + t^9 - t^10 + t^11 - t^12 + t^13 - t^14 +
sage: p.p_factor(Tokenizer("t**2"))
t^2
sage: p.p_power(Tokenizer("2^3^2")) == 2^9
True

sage: p = Parser(make_var=var)
sage: p.p_factor(Tokenizer('x!'))
factorial(x)
sage: p.p_factor(Tokenizer('(x^2)!'))
factorial(x^2)
sage: p.p_factor(Tokenizer('x!^2'))
factorial(x)^2
```

p_sequence (*tokens*)

Parse a (possibly nested) set of lists and tuples.

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_sequence(Tokenizer("[1+2,0]"))
[[3, 0]]
sage: p.p_sequence(Tokenizer("(1,2,3) , [1+a, 2+b, (3+c), (4+d,)]"))
[(1, 2, 3), [a + 1, b + 2, c + 3, (d + 4,)]]
```

p_term (*tokens*)

Parse a single term (consisting of one or more factors).

EXAMPLES:

```
sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_term(Tokenizer("a*b"))
a*b
sage: p.p_term(Tokenizer("a * b / c * d"))
```

```

a*b*d/c
sage: p.p_term(Tokenizer("-a * b + c"))
-a*b
sage: p.p_term(Tokenizer("a*(b-c)^2"))
a*(b - c)^2
sage: p.p_term(Tokenizer("-3a"))
-3*a

```

p_tuple (*tokens*)

Parse a tuple of items.

EXAMPLES:

```

sage: from sage.misc.parser import Parser, Tokenizer
sage: p = Parser(make_var=var)
sage: p.p_tuple(Tokenizer("( (), (1), (1,), (1,2), (1,2,3), (1+2)^2, )"))
((), 1, (1,), (1, 2), (1, 2, 3), 9)

```

parse (*s*, *accept_eqn=True*)

Parse the given string.

EXAMPLES:

```

sage: from sage.misc.parser import Parser
sage: p = Parser(make_var=var)
sage: p.parse("E = m c^2")
E == c^2*m

```

parse_expression (*s*)

Parse an expression.

EXAMPLES:

```

sage: from sage.misc.parser import Parser
sage: p = Parser(make_var=var)
sage: p.parse_expression('a-3b^2')
-3*b^2 + a

```

parse_sequence (*s*)

Parse a (possibly nested) set of lists and tuples.

EXAMPLES:

```

sage: from sage.misc.parser import Parser
sage: p = Parser(make_var=var)
sage: p.parse_sequence("1,2,3")
[1, 2, 3]
sage: p.parse_sequence("[1,2,(a,b,c+d)]")
[1, 2, (a, b, c + d)]
sage: p.parse_sequence("13")
13

```

class sage.misc.parser.**Tokenizer**

Bases: object

This class takes a string and turns it into a list of tokens for use by the parser.

The tokenizer wraps a string object, to tokenize a different string create a new tokenizer.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer
sage: Tokenizer("1.5+2*3^4-sin(x)").test()
['FLOAT(1.5)', '+', 'INT(2)', '*', 'INT(3)', '^', 'INT(4)', '-', 'NAME(sin)', '(', 'NAME(x)', ')']
```

The single character tokens are given by:

```
sage: Tokenizer("+-*/^(),=<>[]{}").test()
['+', '-', '*', '/', '^', '(', ')', ',', '=', '<', '>', '[', ']', '{', '}']
```

Two-character comparisons accepted are:

```
sage: Tokenizer("<= >= != == **").test()
['LESS_EQ', 'GREATER_EQ', 'NOT_EQ', '=', '^']
```

Integers are strings of 0-9:

```
sage: Tokenizer("1 123 9879834759873452908375013").test()
['INT(1)', 'INT(123)', 'INT(9879834759873452908375013)']
```

Floating point numbers can contain a single decimal point and possibly exponential notation:

```
sage: Tokenizer("1. .01 1e3 1.e-3").test()
['FLOAT(1.)', 'FLOAT(.01)', 'FLOAT(1e3)', 'FLOAT(1.e-3)']
```

Note that negative signs are not attached to the token:

```
sage: Tokenizer("-1 -1.2").test()
['-', 'INT(1)', '-', 'FLOAT(1.2)']
```

Names are alphanumeric sequences not starting with a digit:

```
sage: Tokenizer("a a1 _a_24").test()
['NAME(a)', 'NAME(a1)', 'NAME(_a_24)']
```

Anything else is an error:

```
sage: Tokenizer("&@~").test()
['ERROR', 'ERROR', 'ERROR']
```

No attempt for correctness is made at this stage:

```
sage: Tokenizer(") ( 5e5e5").test()
[')', '(', 'FLOAT(5e5)', 'NAME(e5)']
sage: Tokenizer("?$%").test()
['ERROR', 'ERROR', 'ERROR']
```

backtrack()

Put self in such a state that the subsequent call to next() will return the same as if next() had not been called.

Currently, one can only backtrack once.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a+b")
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.next())
'+'
sage: t.backtrack()      # the return type is bint for performance reasons
False
```



```
sage: token_to_str(t.next())
'+'
```

last()

Returns the last token seen.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("3a")
sage: token_to_str(t.next())
'INT'
sage: token_to_str(t.last())
'INT'
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.last())
'NAME'
```

last_token_string()

Return the actual contents of the last token.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a - 1e5")
sage: token_to_str(t.next())
'NAME'
sage: t.last_token_string()
'a'
sage: token_to_str(t.next())
'_'
sage: token_to_str(t.next())
'FLOAT'
sage: t.last_token_string()
'1e5'
```

next()

Returns the next token in the string.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a+3")
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.next())
'+'
sage: token_to_str(t.next())
'INT'
sage: token_to_str(t.next())
'EOS'
```

peek()

Returns the next token that will be encountered, without changing the state of self.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("a+b")
sage: token_to_str(t.peek())
```

```
'NAME'
sage: token_to_str(t.next())
'NAME'
sage: token_to_str(t.peak())
'+'
sage: token_to_str(t.peak())
'+'
sage: token_to_str(t.next())
'+'
```

reset (*pos=0*)

Reset the tokenizer to a given position.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer
sage: t = Tokenizer("a+b*c")
sage: t.test()
['NAME (a)', '+', 'NAME (b)', '*', 'NAME (c)']
sage: t.test()
[]
sage: t.reset()
sage: t.test()
['NAME (a)', '+', 'NAME (b)', '*', 'NAME (c)']
sage: t.reset(3)
sage: t.test()
['*', 'NAME (c)']
```

No care is taken to make sure we don't jump in the middle of a token:

```
sage: t = Tokenizer("12345+a")
sage: t.test()
['INT (12345)', '+', 'NAME (a)']
sage: t.reset(2)
sage: t.test()
['INT (345)', '+', 'NAME (a)']
```

test ()

This is a utility function for easy testing of the tokenizer.

Destructively read off the tokens in self, returning a list of string representations of the tokens.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer
sage: t = Tokenizer("a b 3")
sage: t.test()
['NAME (a)', 'NAME (b)', 'INT (3)']
sage: t.test()
[]
```

`sage.misc.parser.foo(*args, **kws)`

This is a function for testing that simply returns the arguments and keywords passed into it.

EXAMPLES:

```
sage: from sage.misc.parser import foo
sage: foo(1, 2, a=3)
((1, 2), {'a': 3})
```

`sage.misc.parser.token_to_str(token)`

For speed reasons, tokens are integers. This function returns a string representation of a given token.

EXAMPLES:

```
sage: from sage.misc.parser import Tokenizer, token_to_str
sage: t = Tokenizer("+ 2")
sage: token_to_str(t.next())
'+'
sage: token_to_str(t.next())
'INT'
```

3.5 Evaluating Python code without any preparsing

class `sage.misc.python.Python`

Allows for evaluating a chunk of code without any preparsing.

eval (*x*, *globals*, *locals=None*)

Evaluate *x* with given *globals*; *locals* is completely ignored. This is specifically meant for evaluating code blocks with `%python` in the notebook.

INPUT:

x – a string *globals* – a dictionary *locals* – completely IGNORED

EXAMPLES:

```
sage: from sage.misc.python import Python
sage: python = Python()
sage: python.eval('2+2', globals())
4
''
```

Any variables that are set during evaluation of the block will propagate to the *globals* dictionary.

```
sage: python.eval('a=5\nb=7\na+b', globals())
12
''
sage: b
7
```

The *locals* variable is ignored – it is there only for completeness. It is ignored since otherwise the following won't work:

```
sage: python.eval("def foo():\n return 'foo'\nprint foo()\ndef mumble():\n print 'mumble',fo
foo
mumble foo
''
sage: mumble
<function mumble at ...>
```

3.6 Evaluating a String in Sage

`sage.misc.sage_eval.sage_eval(source, locals=None, cmds='', preparse=True)`

Obtain a Sage object from the input string by evaluating it using Sage. This means calling `eval` after preparsing and with *globals* equal to everything included in the scope of `from sage.all import *`.

INPUT:

- source - a string or object with a `_sage_` method
- locals - evaluate in namespace of `sage.all` plus the locals dictionary
- cmds - string; sequence of commands to be run before source is evaluated.
- preparse - (default: True) if True, preparse the string expression.

EXAMPLES: This example illustrates that preparsing is applied.

```
sage: eval('2^3')
1
sage: sage_eval('2^3')
8
```

However, preparsing can be turned off.

```
sage: sage_eval('2^3', preparse=False)
1
```

Note that you can explicitly define variables and pass them as the second option:

```
sage: x = PolynomialRing(RationalField(), "x").gen()
sage: sage_eval('x^2+1', locals={'x':x})
x^2 + 1
```

This example illustrates that evaluation occurs in the context of `from sage.all import *`. Even though `bernoulli` has been redefined in the local scope, when calling `sage_eval` the default value meaning of `bernoulli` is used. Likewise for `QQ` below.

```
sage: bernoulli = lambda x : x^2
sage: bernoulli(6)
36
sage: eval('bernoulli(6)')
36
sage: sage_eval('bernoulli(6)')
1/42
```

```
sage: QQ = lambda x : x^2
sage: QQ(2)
4
sage: sage_eval('QQ(2)')
2
sage: parent(sage_eval('QQ(2)'))
Rational Field
```

This example illustrates setting a variable for use in evaluation.

```
sage: x = 5
sage: eval('4/3 + x', {'x':25})
26
sage: sage_eval('4/3 + x', locals={'x':25})
79/3
```

You can also specify a sequence of commands to be run before the expression is evaluated:

```
sage: sage_eval('p', cmds='K.<x> = QQ[]\np = x^2 + 1')
x^2 + 1
```

If you give commands to execute and a dictionary of variables, then the dictionary will be modified by assignments in the commands:

```
sage: vars = {}
sage: sage_eval('None', cmds='y = 3', locals=vars)
sage: vars['y'], parent(vars['y'])
(3, Integer Ring)
```

You can also specify the object to evaluate as a tuple. A 2-tuple is assumed to be a pair of a command sequence and an expression; a 3-tuple is assumed to be a triple of a command sequence, an expression, and a dictionary holding local variables. (In this case, the given dictionary will not be modified by assignments in the commands.)

```
sage: sage_eval(('f(x) = x^2', 'f(3)'))
9
sage: vars = {'rt2': sqrt(2.0)}
sage: sage_eval(('rt2 += 1', 'rt2', vars))
2.41421356237309
sage: vars['rt2']
1.41421356237310
```

This example illustrates how `sage_eval` can be useful when evaluating the output of other computer algebra systems.

```
sage: R.<x> = PolynomialRing(RationalField())
sage: gap.eval('R:=PolynomialRing(Rationals,["x"]);')
'Rationals[x]'
sage: ff = gap.eval('x:=IndeterminatesOfPolynomialRing(R);; f:=x^2+1;'); ff
'x^2+1'
sage: sage_eval(ff, locals={'x':x})
x^2 + 1
sage: eval(ff)
Traceback (most recent call last):
...
RuntimeError: Use ** for exponentiation, not '^', which means xor
in Python, and has the wrong precedence.
```

Here you can see `eval` simply will not work but `sage_eval` will.

TESTS:

We get a nice minimal error message for syntax errors, that still points to the location of the error (in the input string):

```
sage: sage_eval('RR(22/7)')
Traceback (most recent call last):
...
File "<string>", line 1
  RR(Integer(22)/Integer(7))
                        ^
SyntaxError: unexpected EOF while parsing

sage: sage_eval('None', cmds='$x = $y[3] # Does Perl syntax work?')
Traceback (most recent call last):
...
File "<string>", line 1
  $x = $y[Integer(3)] # Does Perl syntax work?
      ^
SyntaxError: invalid syntax
```

`sage.misc.sage_eval.sageobj(x, vars=None)`

Return a native Sage object associated to `x`, if possible and implemented.

If the object has an `_sage_` method it is called and the value is returned. Otherwise `str` is called on the object, and

all preparsing is applied and the resulting expression is evaluated in the context of `from sage.all import *`. To evaluate the expression with certain variables set, use the `vars` argument, which should be a dictionary.

EXAMPLES:

```
sage: type(sageobj(gp('34/56')))  
<type 'sage.rings.rational.Rational'>  
sage: n = 5/2  
sage: sageobj(n) is n  
True  
sage: k = sageobj('Z(8^3/1)', {'Z':ZZ}); k  
512  
sage: type(k)  
<type 'sage.rings.integer.Integer'>
```

This illustrates interfaces:

```
sage: f = gp('2/3')  
sage: type(f)  
<class 'sage.interfaces.gp.GpElement'>  
sage: f._sage_()  
2/3  
sage: type(f._sage_())  
<type 'sage.rings.rational.Rational'>  
sage: a = gap(939393/2433)  
sage: a._sage_()  
313131/811  
sage: type(a._sage_())  
<type 'sage.rings.rational.Rational'>
```

3.7 Evaluating shell scripts

class `sage.misc.sh.Sh`

Evaluates a shell script and returns the output.

To use this from the notebook type `sh` at the beginning of the input cell. The working directory is then the (usually temporary) directory where the Sage worksheet process is executing.

eval (*code*, *globals*=None, *locals*=None)

This is difficult to test because the output goes to the screen rather than being captured by the doctest program, so the following really only tests that the command doesn't bomb, not that it gives the right output:

```
sage: sh.eval('''echo "Hello there"\nif [ $? -eq 0 ]; then\nnecho "good"\nfi''') # random out
```

FORMATTED OUTPUT

4.1 Formatted Output

4.1.1 Symbols for Character Art

This module defines single- and multi-line symbols.

EXAMPLES:

```
sage: from sage.typeset.symbols import *

sage: symbols = ascii_art(u'')
sage: for i in range(1, 5):
....:     symbols += ascii_left_parenthesis.character_art(i)
....:     symbols += ascii_art(u' ')
....:     symbols += ascii_right_parenthesis.character_art(i)
....:     symbols += ascii_art(u' ')
sage: for i in range(1, 5):
....:     symbols += ascii_left_square_bracket.character_art(i)
....:     symbols += ascii_art(u' ')
....:     symbols += ascii_right_square_bracket.character_art(i)
....:     symbols += ascii_art(u' ')
sage: for i in range(1, 5):
....:     symbols += ascii_left_curly_brace.character_art(i)
....:     symbols += ascii_art(u' ')
....:     symbols += ascii_right_curly_brace.character_art(i)
....:     symbols += ascii_art(u' ')
sage: symbols
( )          [ ]          { }
( ) ( )      [ ] [ ]      { } { }
( ) ( ) ( )  [ ] [ ] [ ]  { } { } { }
( ) ( ) ( ) ( ) [ ] [ ] [ ] { } { } { } { }

sage: symbols = unicode_art(u'')
sage: for i in range(1, 5):
....:     symbols += unicode_left_parenthesis.character_art(i)
....:     symbols += unicode_art(u' ')
....:     symbols += unicode_right_parenthesis.character_art(i)
....:     symbols += unicode_art(u' ')
sage: for i in range(1, 5):
....:     symbols += unicode_left_square_bracket.character_art(i)
....:     symbols += unicode_art(u' ')
....:     symbols += unicode_right_square_bracket.character_art(i)
....:     symbols += unicode_art(u' ')
```

```

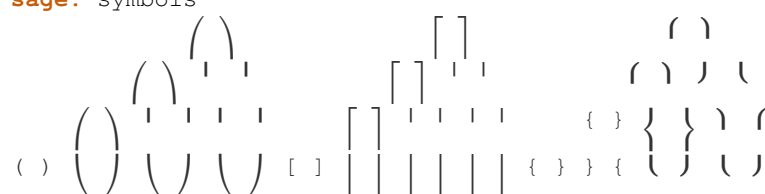
sage: for i in range(1, 5):
....:     symbols += unicode_left_curly_brace.character_art(i)
....:     symbols += unicode_art(u' ')
....:     symbols += unicode_right_curly_brace.character_art(i)
....:     symbols += unicode_art(u' ')

```

```

sage: symbols

```



```

class sage.typeset.symbols.CompoundAsciiSymbol(character, top, extension, bottom, middle=None, middle_top=None, middle_bottom=None, top_2=None, bottom_2=None)

```

Bases: `sage.typeset.symbols.CompoundSymbol`

A multi-character (ascii/unicode art) symbol

INPUT:

Instead of string, each of these can be unicode in Python 2:

- `character` – string. The single-line version of the symbol.
- `top` – string. The top line of a multi-line symbol.
- `extension` – string. The extension line of a multi-line symbol (will be repeated).
- `bottom` – string. The bottom line of a multi-line symbol.
- `middle` – optional string. The middle part, for example in curly braces. Will be used only once for the symbol, and only if its height is odd.
- `middle_top` – optional string. The upper half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.
- `middle_bottom` – optional string. The lower half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.
- `top_2` – optional string. The upper half of a 2-line symbol.
- `bottom_2` – optional string. The lower half of a 2-line symbol.

EXAMPLES:

```

sage: from sage.typeset.symbols import CompoundSymbol
sage: i = CompoundSymbol('I', '+', '|', '+', '|')
sage: i.print_to_stdout(1)
I
sage: i.print_to_stdout(3)
+
|
+

```

character_art (*num_lines*)

Return the ASCII art of the symbol

EXAMPLES:


```

sage: from sage.typeset.symbols import *
sage: ascii_left_curly_brace.character_art(3)
{
{
{

```

```

class sage.typeset.symbols.CompoundSymbol(character, top, extension, bottom, middle=None,
                                           middle_top=None, middle_bottom=None,
                                           top_2=None, bottom_2=None)

```

Bases: `sage.structure.sage_object.SageObject`

A multi-character (ascii/unicode art) symbol

INPUT:

Instead of string, each of these can be unicode in Python 2:

- `character` – string. The single-line version of the symbol.
- `top` – string. The top line of a multi-line symbol.
- `extension` – string. The extension line of a multi-line symbol (will be repeated).
- `bottom` – string. The bottom line of a multi-line symbol.
- `middle` – optional string. The middle part, for example in curly braces. Will be used only once for the symbol, and only if its height is odd.
- `middle_top` – optional string. The upper half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.
- `middle_bottom` – optional string. The lower half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.
- `top_2` – optional string. The upper half of a 2-line symbol.
- `bottom_2` – optional string. The lower half of a 2-line symbol.

EXAMPLES:

```

sage: from sage.typeset.symbols import CompoundSymbol
sage: i = CompoundSymbol('I', '+', '|', '+', '|')
sage: i.print_to_stdout(1)
I
sage: i.print_to_stdout(3)
+
|
+

```

`print_to_stdout(num_lines)`

Print the multi-line symbol

This method is for testing purposes.

INPUT:

- `num_lines` – integer. The total number of lines.

EXAMPLES:

```

sage: from sage.typeset.symbols import *
sage: unicode_integral.print_to_stdout(1)
∫
sage: unicode_integral.print_to_stdout(2)
∫

```

```

 $\int$ 
sage: unicode_integral.print_to_stdout(3)
 $\int$ 
 $\parallel$ 
 $\int$ 
sage: unicode_integral.print_to_stdout(4)
 $\int$ 
 $\parallel$ 
 $\parallel$ 
 $\int$ 
```

```
class sage.typeset.symbols.CompoundUnicodeSymbol(character, top, extension, bottom,
                                                    middle=None, middle_top=None,
                                                    middle_bottom=None, top_2=None,
                                                    bottom_2=None)
```

Bases: `sage.typeset.symbols.CompoundSymbol`

A multi-character (ascii/unicode art) symbol

INPUT:

Instead of string, each of these can be unicode in Python 2:

- `character` – string. The single-line version of the symbol.
- `top` – string. The top line of a multi-line symbol.
- `extension` – string. The extension line of a multi-line symbol (will be repeated).
- `bottom` – string. The bottom line of a multi-line symbol.
- `middle` – optional string. The middle part, for example in curly braces. Will be used only once for the symbol, and only if its height is odd.
- `middle_top` – optional string. The upper half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.
- `middle_bottom` – optional string. The lower half of the 2-line middle part if the height of the symbol is even. Will be used only once for the symbol.
- `top_2` – optional string. The upper half of a 2-line symbol.
- `bottom_2` – optional string. The lower half of a 2-line symbol.

EXAMPLES:

```
sage: from sage.typeset.symbols import CompoundSymbol
sage: i = CompoundSymbol('I', '+', '|', '+', '|')
sage: i.print_to_stdout(1)
I
sage: i.print_to_stdout(3)
+
|
+
```

character_art (*num_lines*)

Return the unicode art of the symbol

EXAMPLES:

```
sage: from sage.typeset.symbols import *
sage: unicode_left_curly_brace.character_art(3)
(
```

}

4.1.2 Base Class for Character-Based Art

This is the common base class for `sage.typeset.ascii_art.AsciiArt` and `sage.typeset.ascii_art.UnicodeArt`. They implement simple graphics by placing characters on a rectangular grid, in other words, using monospace fonts. The difference is that one is restricted to 7-bit ascii, the other uses all unicode code points.

```
class sage.typeset.character_art.CharacterArt (lines=[], breakpoints=[], baseline=None,
                                              atomic=None)
```

Bases: `sage.structure.sage_object.SageObject`

Abstract base class for character art

INPUT:

- `lines` – the list of lines of the representation of the character art object
- `breakpoints` – the list of points where the representation can be split
- `baseline` – the reference line (from the bottom)

EXAMPLES:

```
sage: i = var('i')
sage: ascii_art(sum(pi^i/factorial(i)*x^i, i, 0, oo))
pi*x
e
```

TESTS:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: aao = AsciiArt()
sage: aao

sage: aa = AsciiArt([" * ", " * * ", "*****"]); aa
 *
 * *
*****
```

classmethod `empty()`

Return the empty character art object

EXAMPLES:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: AsciiArt.empty()
```

get_baseline()

Return the line where the baseline is, for example:

```
      5      4
14*x  + 5*x
```

the baseline has at line 0 and

```
{ o      }
{ \  : 4 }
{ o      }
```

has at line 1.

TESTS:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: aa = AsciiArt([" * ", " * * ", " * * * ", "*****"], baseline=1);aa
 *
 * *
 *  *
*****
sage: aa.get_baseline()
1
sage: b = AsciiArt(["<-"])
sage: aa+b
 *
 * *
 *  * <-
*****
```

get_breakpoints()

Return an iterator of breakpoints where the object can be split.

For example the expression:

```
      5      4
14x + 5x
```

can be split on position 4 (on the +).

EXAMPLES:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: p3 = AsciiArt([" * ", "***"])
sage: p5 = AsciiArt([" * ", " * * ", "*****"])
sage: aa = ascii_art([p3, p5])
sage: aa.get_breakpoints()
[6]
```

height()

Return the height of the ASCII art object.

OUTPUT:

Integer. The number of lines.

TESTS:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: p3 = AsciiArt([" * ", "***"])
sage: p3.height()
2
sage: p5 = AsciiArt([" * ", " * * ", "*****"])
sage: p5.height()
3
```

split(pos)

Split the representation at the position pos.

EXAMPLES:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: p3 = AsciiArt([" * ", "***"])
sage: p5 = AsciiArt([" * ", " * * ", "*****"])
sage: aa = ascii_art([p3, p5])
sage: a,b= aa.split(6)
sage: a
[
[ *
[ ***,
sage: b
    *   ]
    * * ]
***** ]
```

width()

Return the length (width) of the ASCII art object.

OUTPUT:

Integer. The number of characters in each line.

TESTS:

```
sage: from sage.typeset.ascii_art import AsciiArt
sage: p3 = AsciiArt([" * ", "***"])
sage: len(p3), p3.width(), p3.height()
(3, 3, 2)
sage: p5 = AsciiArt([" * ", " * * ", "*****"])
sage: len(p5), p5.width(), p5.height()
(5, 5, 3)
```

4.1.3 Factory for Character-Based Art

```
class sage.typeset.character_art_factory.CharacterArtFactory(art_type, string_type,
                                                            magic_method_name,
                                                            parenthesis,
                                                            square_bracet,
                                                            curly_brace)
```

Bases: `sage.structure.sage_object.SageObject`

Abstract base class for character art factory

This class is the common implementation behind `ascii_art()` and `unicode_art()`.

INPUT:

- `art_type` – type of the character art (i.e. a subclass of `CharacterArt`)
- `string_type` – type of strings (the lines in the character art, e.g. `str` or `unicode`).
- `magic_method_name` – name of the Sage magic method (e.g. `'_ascii_art_'` or `'_unicode_art_'`).
- `parenthesis` – left/right pair of two multi-line symbols. The parenthesis, a.k.a. round brackets (used for printing tuples).
- `square_bracket` – left/right pair of two multi-line symbols. The square brackets (used for printing lists).
- `curly_brace` – left/right pair of two multi-line symbols. The curly braces (used for printing sets).

EXAMPLES:

```
sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
sage: type(factory)
<class 'sage.typeset.character_art_factory.CharacterArtFactory'>
```

build(*obj*)

Construct a character art representation

INPUT:

- *obj* – anything. The object whose ascii art representation we want.

OUTPUT:

Character art object.

EXAMPLES:

```
sage: ascii_art(integral(exp(x+x^2)/(x+1), x))
/
|
|      2
|     x  + x
|      e
|  ----- dx
|     x + 1
|
/
```

TESTS:

```
sage: n = var('n')
sage: ascii_art(sum(binomial(2 * n, n + 1) * x^n, n, 0, oo))
/
|      _____ \
| -\2*x + \ / -4*x + 1 - 1/
| -----
|
|      2*x*\ / -4*x + 1
sage: ascii_art(list(DyckWords(3)))
[
|      /\      ]
|      /\      /\      /\      /\      ]
| /\ /\ /\, /\ /\, /\ /\, /\ /\, /\ /\ ]
sage: ascii_art(1)
1
```

build_container(*content*, *left_border*, *right_border*)

Return character art for a container

INPUT:

- *content* – `CharacterArt`. The content of the container, usually comma-separated entries.
- *left_border* – `CompoundSymbol`. The left border of the container.
- *right_border* – `CompoundSymbol`. The right border of the container.

TESTS:

```
sage: l = ascii_art(list(DyckWords(3))); l
[
|      /\      ]
|      /\      /\      /\      /\      ]
| /\ /\ /\, /\ /\, /\ /\, /\ /\, /\ /\ ]
```

```
sage: l.get_breakpoints()
[9, 17, 25, 33]
```

build_dict(*d*)

Return an character art output of a dictionary.

TESTS:

```
sage: d = ascii_art({i:dw for i,dw in enumerate(DyckWords(3))})
sage: d
{
  {
    { 0:/\//\/, 1:/\// \, 2:/ \//\, 3:/ \// \, 4:/ \// \ }
  }
}
sage: d.get_breakpoints()
[11, 21, 31, 41]
```

build_empty()

Return the empty character art object

OUTPUT:

Character art instance.

build_from_magic_method(*obj*)

Return the character art object created by the object's magic method

OUTPUT:

Character art instance.

EXAMPLES:

```
sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
sage: out = factory.build_from_magic_method(identity_matrix(2)); out
[1 0]
[0 1]
sage: type(out)
<class 'sage.typeset.ascii_art.AsciiArt'>
```

build_from_string(*obj*)

Return the character art object created from splitting the object's string representation

INPUT:

- obj* – utf-8 encoded byte string or unicode.

OUTPUT:

Character art instance.

EXAMPLES:

```
sage: from sage.typeset.ascii_art import _ascii_art_factory as factory
sage: out = factory.build_from_string('a\nbb\nccc')
sage: out + out + out
a  a  a
bb bb bb
cccccccc
sage: type(out)
<class 'sage.typeset.ascii_art.AsciiArt'>
```

TESTS:

```
sage: factory.build_from_string(u'a\nbb\nccc') # same with unicode
a
bb
ccc
```

build_list(l)

Return an character art output of a list.

TESTS:

```
sage: l = ascii_art(list(DyckWords(3))); l
[
      /\
[      /\      /\      /\      /\
[ /\ /\ /\, /\ /\, /\ /\, /\ /\
sage: l.get_breakpoints()
[9, 17, 25, 33]
```

The breakpoints of the object are used as breakpoints:

```
sage: l = ascii_art([DyckWords(2).list(), DyckWords(2).list()])
sage: l.get_breakpoints()
[9, 17, 25]
```

build_set(s)

Return an character art output of a set.

TESTS:

```
sage: ascii_art(set(DyckWords(3)))
{
      /\
{ /\      /\      /\      /\
{ /\ /\ /\, /\ /\, /\ /\, /\ /\
{ /\ /\ /\, /\ /\, /\ /\, /\ /\
```

build_tuple(t)

Return an character art output of a tuple.

TESTS:

```
sage: ascii_art(tuple(DyckWords(3)))
(
      /\
(      /\      /\      /\      /\
( /\ /\ /\, /\ /\, /\ /\, /\ /\
```

concatenate(iterable, separator, empty=None)

Concatenate multiple character art instances

The breakpoints are set as the breakpoints of the `separator` together with the breakpoints of the objects in `iterable`. If there is `None`, the end of the separator is used.

INPUT:

- `iterable` – iterable of character art.
- `separator` – character art. The separator in-between the iterable.
- `empty` – an optional character art which is returned if `iterable` is empty

EXAMPLES:

```
sage: i2 = identity_matrix(2)
sage: ascii_art(i2, i2, i2, sep=ascii_art(1/x))
1      1
```



```

[ *, * , * , * , * , * , * , * , * , ** , ** , ** , ** , *** , *** , **** ,
]
]
]
]
]
**** ]
sage: shell.run_cell('%display simple')
sage: shell.quit()

.
.? .~?$NNO.II7. .GOGG
., ~7NNI7NDG$ ...~~~MGG
...IG... ..G7:DDGNND0... ..7:~~~GNN
..~GGGGG... ..O .=$+OD7GI$ ...GG:~+~~~MG?M.7?
..~~~D~~GGGGDGOM$..~+IN=NDG.G::D~~~?~~~7?NDI:??G
..~~~G:~~G~D:~NONGMMOGD$ND~~::I77~~$?++MN7G GI?D.
7+I~~DG~~~N:GNGGOD$7OIOOMII::~:I7?G+OGN7G $III?.
.7==I~~~+IN?+++$7ND$==G:G????N$OG$ ~III?7
.$$+++GG+=G?GDM?GOG:NGMG7?????D$D~GODIII???.
D++++NG+DD$+=GGONGI$DNGDN?????, IN=DOGODN???.
, +++++N?D$I+I=GNMDGDGNIINN?D+::O~$GGGDIG?7
:DD:::OG?G=I~~G7GNM7777NIGODDNG:::I$GGGGG7??O.
~GDDM:::GGMG+?+~$NNN$7IG7NMNMMN, =::I7GGGGGO??~
:GDND7?::OOM.D+O~GGDI77777DMGNM$. ~:, $IGGGGO??DO.
OGDDNN.D77OO. $?7==GG777$7GNMGO. NOIGGGGO??G$.
.OODNNN,DIGDM$GGGGG==GGGGIGNDMDMG, IGIGGGDON??GG:
.GODNDM.G$I$I$IOIOMG$G$?GGGIO,7OD7GG. ,GDIGGG?????GGG.
.DGDNI7I7MDI+OODN$O,$7DMIN,,IOO77O. G?$DIGGG?ID??OGG
GGDNNMMO7GD+OOOGIMOG7::NN====:?MMNGIDD,..IINIGGG?I?DIORG?
.7ODMMN.G7IOGOOODIMG,,:::$~==:7OGG~IGOMDGMNIGGG????OG$G.
?ODDMNNNO,IISOGODMGDM?:DMG==~MDINNM$.7$IONDGI?GGG????$GGG.
.$MDMNNNN..:7GDDDG,GMG?~:GGGDND.GIM7D+GI$ON.:?GGG????$OGG.
.7DNDNNDMDOGG.=IGGND=7II+N??:$GII,O,IIGMG?7I7G$ON?,IGGG????7IGG.
~GGGDNMMOOGG$MGMMGDGMDGM?,G:GNG,:IIGDG7IGGGGG$+NMIGGO????IGGG.
.GGGDDMM7OGNGMODMNDDDO.MII?GI$7IIIIING7GGDMM.IGDG.G7GGGG??$7GGG
.IGDDDDOINNGMGMDMNDGDGO...$OI+??7OIIIDDGN+$==I=GD ?,NGGGGN7??G
7GGNDM$GONDGDD$MMGNNDN. G.:$G$?$7IIG$GO,O=+7O7O~N?OM?GGGGGD+??$GG
OOGDOI7DGMG..~DG$DD. $,,,$$D??7ODOOOODG$777$G OGMM:GGGGG??GGG.
.$GDDG?7DMOGDNNMGDDG ..=O7$GG$+O$OG=+O:GI77G$. ...DIGGGGI?$GGG.
.OGGGGO7OGDGGNGGGDGG. O:,77$O$D $GN:7777GD ..$GGG??GGG$.
.GGGGDI GD.~NOGDGG MG,777G77D7 +$~D7777= ..7GGG??GGO.
GGD. .~NODN... ..D::77O$7G77 .OG:G??G7. 7$NOM??GG=.
. . . .+~DD~77$G7OD7 .?GGO?$OM$D ?????DGG...
7I77DN$II7$M.G$G . .:G?==G$ .????D??~
. .:IIIGO7O$GN..O$ 7I=+~?I. =???7? GGIG
$,III7NGGNNG, .O. O====7I. .DG??$. GG?G
.$7IIII$7777$~ . +====D$GG ~$???: .GIG.
:..+IIII$77$G$O,+ OI==+7I$7=.:??? .GG$.
,,IIII$7?I7$OG7D ?+,==+77G?$,???. GG?
?IIIII=+$I7777 .:++=++O77:,??: GG.
.IIIIG+III7777. 7,=,++?II7$,?G .GO .
$OII7?OIIIIIO .I:++??IIIII???. I$ .
.DNGDMOIG777. +~++?I$IIMN?, .G,
.G.I?IIDGII~ .OG??$MIMGI. G,
N+?II7OGI7. DD?$GGDGID. :
=?IIOOI$. ?~~GGG$II .+.
,7=IINGI ?=IGGGI?.

```

```

      G,+IGOI
      .,:?IGOI
      .:N+=?IMGI7=.
      .$:IGO?$IIII7+.
      .::=IIGIIIIII.
      .+:$IIIMII7G$?.
      .SI$+7IIIMMG7I7
      .S~:$IIGMNGND77I:
      ?=?IIIGIIIIINI7777?
      ONGNDG??IG?IIISN7I777
      .:$??7IIIIII.....,....
      ./.

```

```

?+II7??+
?+GIOII
.~:$I?IO.
O::I7I?.
.~, $IG?IO
I::I7IIG7
?G,DGGNII.
+I$GOD=?=
.7~===??.
D~====I
O::==~$D.
..M:I.==$7G.
I?::IIII7.
.~:G:IIIIIG.
.$:,O7IIIS$O
::~DOGGNNO
.:, IOODI,
.7????$.
...

```

```

class sage.typeset.ascii_art.AsciiArt (lines=[ ], breakpoints=[ ], baseline=None,
                                           atomic=None)
Bases: sage.typeset.character_art.CharacterArt

```

An Ascii art object is an object with some specific representation for *printing*.

INPUT:

- `lines` – the list of lines of the representation of the ascii art object
- `breakpoints` – the list of points where the representation can be split
- `baseline` – the reference line (from the bottom)

EXAMPLES:

```

sage: i = var('i')
sage: ascii_art(sum(pi^i/factorial(i)*x^i, i, 0, oo))
pi*x
e

```

```

sage.typeset.ascii_art.ascii_art (*obj, **kws)

```

Return an ASCII art representation

INPUT:

- `*obj` – any number of positional arguments, of arbitrary type. The objects whose ascii art representation we want.
- `sep` – optional '`sep=...`' keyword argument. Anything that can be converted to ascii art (default: empty ascii art). The separator in-between a list of objects. Only used if more than one object given.

OUTPUT:

`AsciiArt` instance.

EXAMPLES:

```

sage: ascii_art(integral(exp(x+x^2)/(x+1), x))
/
|
|      2
|  x  + x

```

```

      | e
      | ----- dx
      | x + 1
      |
      /

sage: ident = lambda n: identity_matrix(ZZ, n)
sage: ascii_art(ident(1), ident(2), ident(3), sep=' : ')
      [1 0 0]
      [1 0]  [0 1 0]
[1] : [0 1] : [0 0 1]

TESTS:
sage: n = var('n')
sage: ascii_art(sum(binomial(2 * n, n + 1) * x^n, n, 0, oo))
      /
      -----
      -\2*x + \ / -4*x + 1 - 1/

      2*x*\ / -4*x + 1
sage: ascii_art(list(DyckWords(3)))
[
[
[ /\ /\ /\, /\ /\, /\ /\, /\ /\, /\ /\ ]
[ /\ /\ /\, /\ /\, /\ /\, /\ /\, /\ /\ ]
]
]
sage: ascii_art(1)
1

```

4.1.5 Unicode Art

This module implements ascii art using unicode characters. It is a strict superset of `ascii_art`.

```
class sage.typeset.unicode_art.UnicodeArt (lines=[], breakpoints=[], baseline=None,
                                           atomic=None)
```

Bases: `sage.typeset.character_art.CharacterArt`

An Ascii art object is an object with some specific representation for *printing*.

INPUT:

- `lines` – the list of lines of the representation of the ascii art object
- `breakpoints` – the list of points where the representation can be split
- `baseline` – the reference line (from the bottom)

EXAMPLES:

```

sage: i = var('i')
sage: unicode_art(sum(pi^i/factorial(i)*x^i, i, 0, oo))
      π·x
      e

```

```
sage.typeset.unicode_art.unicode_art (*obj, **kws)
```

Return a unicode art representation

INPUT:

- `*obj` – any number of positional arguments, of arbitrary type. The objects whose ascii art representation we want.

- `sep` – optional '`sep=...`' keyword argument. Anything that can be converted to unicode art (default: empty unicode art). The separator in-between a list of objects. Only used if more than one object given.

OUTPUT:

UnicodeArt instance.

EXAMPLES:

```
sage: unicode_art(integral(exp(sqrt(x))/(x+pi), x))
```

$$\int \frac{e^{\sqrt{x}}}{x + \pi} dx$$

```
sage: ident = lambda n: identity_matrix(ZZ, n)
```

```
sage: unicode_art(ident(1), ident(2), ident(3), sep=' : ')
```

$$(1) : \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} : \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

TESTS:

```
sage: n = var('n')
```

```
sage: unicode_art(sum(binomial(2 * n, n + 1) * x^n, n, 0, oo))
```

$$-\left(2 \cdot x + \sqrt{-4 \cdot x + 1} - 1\right)$$

$$\frac{2 \cdot x \cdot \sqrt{-4 \cdot x + 1}}{}$$

```
sage: unicode_art(list(DyckWords(3)))
```

$$\left[\begin{array}{c} \diagup \diagdown \\ \diagup \diagdown \diagup \diagdown \\ \diagup \diagdown \diagup \diagdown \diagup \diagdown \end{array} \right]$$

```
sage: unicode_art(1)
```

1

4.1.6 Sage Input Formatting

This module provides the function `sage_input()` that takes an arbitrary sage value and produces a sequence of commands that, if typed at the `sage:` prompt, will recreate the value. If this is not implemented for a particular value, then an exception is raised instead. This might be useful in understanding a part of Sage, or for debugging. For instance, if you have a value produced in a complicated way in the middle of a debugging session, you could use `sage_input()` to find a simple way to produce the same value. We attempt to produce commands that are readable and idiomatic.:

```
sage: sage_input(3)
```

3

```
sage: sage_input((polygen(RR) + RR(pi))^2, verify=True)
```

```
# Verified
```

```
R.<x> = RR[]
```

```
x^2 + 6.2831853071795862*x + 9.869604401089358
```

With `verify=True`, `sage_input()` also verifies the results, by calling `sage_eval()` on the result and verifying that it is equal to the input.:

```
sage: sage_input(GF(2)(1), verify=True)
# Verified
GF(2)(1)
```

We can generate code that works without the preparer, with `preparse=False`; or we can generate code that will work whether or not the preparer is enabled, with `preparse=None`. Generating code with `preparse=False` may be useful to see how to create a certain value in a Python or Cython source file.:

```
sage: sage_input(5, verify=True)
# Verified
5
sage: sage_input(5, preparse=False)
ZZ(5)
sage: sage_input(5, preparse=None)
ZZ(5)
sage: sage_input(5r, verify=True)
# Verified
5r
sage: sage_input(5r, preparse=False)
5
sage: sage_input(5r, preparse=None)
int(5)
```

Adding `sage_input()` support to your own classes is straightforward. You need to add a `_sage_input_()` method which returns a `SageInputExpression` (henceforth abbreviated as SIE) which will reconstruct this instance of your class.

A `_sage_input_` method takes two parameters, conventionally named `sib` and `coerced`. The first argument is a `SageInputBuilder`; it has methods to build SIEs. The second argument, `coerced`, is a boolean. This is only useful if your class is a subclass of `Element` (although it is always present). If `coerced` is `False`, then your method must generate an expression which will evaluate to a value of the correct type with the correct parent. If `coerced` is `True`, then your method may generate an expression of a type that has a canonical coercion to your type; and if `coerced` is `2`, then your method may generate an expression of a type that has a conversion to your type.

Let's work through some examples. We'll build a sequence of functions that would be acceptable as `_sage_input_` methods for the `Rational` class.

Here's the first and simplest version.:

```
sage: def qq_sage_input_v1(self, sib, coerced):
....:     return sib(self.numerator())/sib(self.denominator())
```

We see that given a `SageInputBuilder` `sib`, you can construct a SIE for a value `v` simply with `sib(v)`, and you can construct a SIE for a quotient with the division operator. Of course, the other operators also work, and so do function calls, method calls, subscripts, etc.

We'll test with the following code, which you don't need to understand. (It produces a list of 8 results, showing the formatted versions of $-5/7$ and 3 , with the preparer either enabled or disabled and either with or without an automatic coercion to `QQ`.):

```
sage: from sage.misc.sage_input import SageInputBuilder
sage: def test_qq_formatter(fmt):
....:     results = []
....:     for v in [-5/7, QQ(3)]:
....:         for pp in [False, True]:
....:             for coerced in [False, True]:
```

```

.....:         sib = SageInputBuilder(preparse=pp)
.....:         results.append(sib.result(fmt(v, sib, coerced)))
.....:     return results

sage: test_qq_formatter(qq_sage_input_v1)
[-ZZ(5)/ZZ(7), -ZZ(5)/ZZ(7), -5/7, -5/7, ZZ(3)/ZZ(1), ZZ(3)/ZZ(1), 3/1, 3/1]

```

Let's try for some shorter, perhaps nicer-looking output. We'll start by getting rid of the ZZ in the denominators; even without the preparser, $-ZZ(5)/7 == -ZZ(5)/ZZ(7)$..

```

sage: def qq_sage_input_v2(self, sib, coerced):
.....:     return sib(self.numerator())/sib.int(self.denominator())

```

The `int` method on `SageInputBuilder` returns a SIE for an integer that is always represented in the simple way, without coercions. (So, depending on the preparser mode, it might read in as an `Integer`, an `int`, or a `long`.)

```

sage: test_qq_formatter(qq_sage_input_v2)
[-ZZ(5)/7, -ZZ(5)/7, -5/7, -5/7, ZZ(3)/1, ZZ(3)/1, 3/1, 3/1]

```

Next let's get rid of the divisions by 1. These are more complicated, since if we're not careful we'll get results in ZZ instead of QQ:

```

sage: def qq_sage_input_v3(self, sib, coerced):
.....:     if self.denominator() == 1:
.....:         if coerced:
.....:             return sib.int(self.numerator())
.....:         else:
.....:             return sib.name('QQ')(sib.int(self.numerator()))
.....:     return sib(self.numerator())/sib.int(self.denominator())

```

We see that the `method{name}` method gives an SIE representing a sage constant or function.:

```

sage: test_qq_formatter(qq_sage_input_v3)
[-ZZ(5)/7, -ZZ(5)/7, -5/7, -5/7, QQ(3), 3, QQ(3), 3]

```

This is the prettiest output we're going to get, but let's make one further refinement. Other `_sage_input_` methods, like the one for polynomials, analyze the structure of SIEs; they work better (give prettier output) if negations are at the outside. If the above code were used for rationals, then `sage_input(polygen(QQ) - 2/3)` would produce `x + (-2/3)`; if we change to the following code, then we would get `x - 2/3` instead.:

```

sage: def qq_sage_input_v4(self, sib, coerced):
.....:     num = self.numerator()
.....:     neg = (num < 0)
.....:     if neg: num = -num
.....:     if self.denominator() == 1:
.....:         if coerced:
.....:             v = sib.int(num)
.....:         else:
.....:             v = sib.name('QQ')(sib.int(num))
.....:     else:
.....:         v = sib(num)/sib.int(self.denominator())
.....:     if neg: v = -v
.....:     return v

sage: test_qq_formatter(qq_sage_input_v4)
[-ZZ(5)/7, -ZZ(5)/7, -5/7, -5/7, QQ(3), 3, QQ(3), 3]

```

AUTHORS:

- Carl Witty (2008-04): new file
- Vincent Delecroix (2015-02): documentation formatting

class `sage.misc.sage_input.SIE_assign(sib, lhs, rhs)`
Bases: `sage.misc.sage_input.SageInputExpression`

This class represents an assignment command.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.assign(sib.name('foo').x, sib.name('pi'))
{assign: {getattr: {atomic:foo}.x} {atomic:pi}}
```

class `sage.misc.sage_input.SIE_binary(sib, op, lhs, rhs)`
Bases: `sage.misc.sage_input.SageInputExpression`

This class represents an arithmetic expression with a binary operator and its two arguments, in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib(3)+5
{binop:+ {atomic:3} {atomic:5}}
```

class `sage.misc.sage_input.SIE_call(sib, func, args, kwargs)`
Bases: `sage.misc.sage_input.SageInputExpression`

This class represents a function-call node in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie = sib.name('GF')
sage: sie(49)
{call: {atomic:GF}({atomic:49})}
```

class `sage.misc.sage_input.SIE_dict(sib, entries)`
Bases: `sage.misc.sage_input.SageInputExpression`

This class represents a dict node in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.dict([('TeX', RR(pi)), ('Metafont', RR(e))])
{dict: {{atomic:'TeX'}:{call: {atomic:RR}({atomic:3.1415926535897931})}, {atomic:'Metafont'}:{ca
sage: sib.dict({-40:-40, 0:32, 100:212})
{dict: {{unop:- {atomic:40}}:{unop:- {atomic:40}}, {atomic:0}:{atomic:32}, {atomic:100}:{atomic:
```

class `sage.misc.sage_input.SIE_gen(sib, parent, name)`
Bases: `sage.misc.sage_input.SageInputExpression`

This class represents a named generator of a parent with named generators.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.gen(ZZ['x'])
{gen:x {constr_parent: {subscr: {atomic:ZZ}{{atomic:'x'}}} with gens: ('x',)}}
```

```
class sage.misc.sage_input.SIE_gens_constructor(sib, constr, gen_names,
                                                gens_syntax=None)
Bases: sage.misc.sage_input.SageInputExpression
```

This class represents an expression that can create a sage parent with named generators, optionally using the sage preparser generators syntax (like $K.\langle x \rangle = \mathbb{Q}\mathbb{Q}[]$).

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: qq = sib.name('QQ')
sage: sib.parent_with_gens("some parent", qq['x'],
...                        ('x',), 'QQx',
...                        gens_syntax=sib.empty_subscript(qq))
{constr_parent: {subscr: {atomic:QQ}{{atomic:'x'}}} with gens: ('x',)}}
```

```
class sage.misc.sage_input.SIE_getattr(sib, obj, attr)
Bases: sage.misc.sage_input.SageInputExpression
```

This class represents a getattr node in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie = sib.name('CC').gen()
sage: sie
{call: {getattr: {atomic:CC}.gen()}}
```

```
class sage.misc.sage_input.SIE_import_name(sib, module, name, alt_name=None)
Bases: sage.misc.sage_input.SageInputExpression
```

This class represents a name which has been imported from a module.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.import_name('sage.rings.integer', 'make_integer')
{import:sage.rings.integer/make_integer}
sage: sib.import_name('sage.foo', 'happy', 'sad')
{import:sage.foo/happy as sad}
```

```
class sage.misc.sage_input.SIE_literal(sib)
Bases: sage.misc.sage_input.SageInputExpression
```

An abstract base class for literals (basically, values which consist of a single token).

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder, SIE_literal

sage: sib = SageInputBuilder()
sage: sie = sib(3)
sage: sie
{atomic:3}
sage: isinstance(sie, SIE_literal)
True
```

```
class sage.misc.sage_input.SIE_literal_stringrep(sib, n)
Bases: sage.misc.sage_input.SIE_literal
```

Values in this class are leaves in a `sage_input()` expression tree. Typically they represent a single token, and consist of the string representation of that token. They are used for integer, floating-point, and string literals, and for name expressions.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder, SIE_literal_stringrep

sage: sib = SageInputBuilder()
sage: isinstance(sib(3), SIE_literal_stringrep)
True
sage: isinstance(sib(3.14159, True), SIE_literal_stringrep)
True
sage: isinstance(sib.name('pi'), SIE_literal_stringrep)
True
sage: isinstance(sib(False), SIE_literal_stringrep)
True
sage: sib(False)
{atomic:False}
```

```
class sage.misc.sage_input.SIE_subscript(sib, coll, key)
Bases: sage.misc.sage_input.SageInputExpression
```

This class represents a subscript node in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie = sib.name('QQ')['x,y']
sage: sie
{subscr: {atomic:QQ} [{atomic:'x,y'}]}
```

```
class sage.misc.sage_input.SIE_tuple(sib, values, is_list)
Bases: sage.misc.sage_input.SageInputExpression
```

This class represents a tuple or list node in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib((1, 'howdy'))
{tuple: ({atomic:1}, {atomic:'howdy'})}
sage: sib(["lists"])
{list: ({atomic:'lists'})}
```

```
class sage.misc.sage_input.SIE_unary(sib, op, operand)
    Bases: sage.misc.sage_input.SageInputExpression
```

This class represents an arithmetic expression with a unary operator and its argument, in a `sage_input()` expression tree.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: -sib(256)
{unop:- {atomic:256}}
```

```
class sage.misc.sage_input.SageInputAnswer
    Bases: tuple
```

This class inherits from tuple, so it acts like a tuple when passed to `sage_eval()`; but it prints as a sequence of commands.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputAnswer
sage: v = SageInputAnswer('x = 22\n', 'x/7'); v
x = 22
x/7
sage: isinstance(v, tuple)
True
sage: v[0]
'x = 22\n'
sage: v[1]
'x/7'
sage: len(v)
2
sage: v = SageInputAnswer('', 'sin(3.14)', {'sin': math.sin}); v
LOCALS:
  sin: <built-in function sin>
sin(3.14)
sage: v[0]
''
sage: v[1]
'sin(3.14)'
sage: v[2]
{'sin': <built-in function sin>}
```

```
class sage.misc.sage_input.SageInputBuilder(allow_locals=False, preparse=True)
```

An instance of this class is passed to `_sage_input_` methods. It keeps track of the current state of the `_sage_input_` process, and contains many utility methods for building `SageInputExpression` objects.

In normal use, instances of `SageInputBuilder` are created internally by `sage_input()`, but it may be useful to create an instance directly for testing or doctesting.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
```

We can create a `SageInputBuilder`, use it to create some `SageInputExpression`s, and get a result. (As mentioned above, this is only useful for testing or doctesting; normally you would just use `sage_input()`):

```
sage: sib = SageInputBuilder()
sage: sib.result((sib(3) + sib(4)) * (sib(5) + sib(6)))
(3 + 4)*(5 + 6)
```

assign (*e, val*)

Constructs a command that performs the assignment `e=val`.

Can only be used as an argument to the `command` method.

INPUT:

- *e, val* – `SageInputExpression`

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: circular = sib([None])
sage: sib.command(circular, sib.assign(circular[0], circular))
sage: sib.result(circular)
si = [None]
si[0] = si
si
```

cache (*x, sie, name*)

INPUT:

- *x* - an arbitrary value
- *sie* - a `SageInputExpression`
- *name* - a requested variable name

Enters *x* and *sie* in a cache, so that subsequent calls `self(x)` will directly return *sie*. Also, marks the requested name of this *sie* to be *name*.

This should almost always be called as part of the `method_{sage_input_}` method of a parent. It may also be called on values of an arbitrary type, which may be useful if the values are both large and likely to be used multiple times in a single expression.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sie42 = sib(GF(101)(42))
sage: sib.cache(GF(101)(42), sie42, 'the_ultimate_answer')
sage: sib.result(sib(GF(101)(42)) + sib(GF(101)(42)))
the_ultimate_answer = GF(101)(42)
the_ultimate_answer + the_ultimate_answer
```

Note that we don't assign the result to a variable if the value is only used once.:

```
sage: sib = SageInputBuilder()
sage: sie42 = sib(GF(101)(42))
sage: sib.cache(GF(101)(42), sie42, 'the_ultimate_answer')
sage: sib.result(sib(GF(101)(42)) + sib(GF(101)(43)))
GF_101 = GF(101)
GF_101(42) + GF_101(43)
```

command(*v, cmd*)

INPUT:

- *v, cmd* – `SageInputExpression`

Attaches a command to *v*, which will be executed before *v* is used. Multiple commands will be executed in the order added.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: incr_list = sib([])
sage: sib.command(incr_list, incr_list.append(1))
sage: sib.command(incr_list, incr_list.extend([2, 3]))
sage: sib.result(incr_list)
si = []
si.append(1)
si.extend([2, 3])
si
```

dict(*entries*)

Given a dictionary, or a list of (key, value) pairs, produces a `SageInputExpression` representing the dictionary.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.dict({1:1, 2:5/2, 3:100/3}))
{1:1, 2:5/2, 3:100/3}
sage: sib.result(sib.dict([('hello', 'sunshine'), ('goodbye', 'rain')]))
{'hello': 'sunshine', 'goodbye': 'rain'}
```

empty_subscript(*parent*)

Given a `SageInputExpression` representing *foo*, produces a `SageInputExpression` representing *foo*[]. Since this is not legal Python syntax, it is useful only for producing the sage generator syntax for a polynomial ring.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.empty_subscript(sib(2) + sib(3)))
(2 + 3)[]
```

The following calls this method indirectly.:

```
sage: sage_input(polygen(ZZ['y']))
R.<x> = ZZ['y'][]
x
```

float_str(*n*)

Given a string representing a floating-point number, produces a `SageInputExpression` that formats as that string.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.float_str(repr(RR(e))))
2.71828182845905
```

gen (*parent*, *n=0*)

Given a parent, returns a `SageInputExpression` for the *n*-th (default 0) generator of the parent.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.gen(ZZ['y']))
R.<y> = ZZ[]
y
```

getattr (*sie*, *attr*)

Given a `SageInputExpression` representing *foo* and an attribute name *bar*, produce a `SageInputExpression` representing *foo.bar*. Normally, you could just use attribute-access syntax, but that doesn't work if *bar* is some attribute that bypasses `__getattr__` (such as if *bar* is `'__getattr__'` itself).

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib getattr(ZZ, '__getattr__')
{getattr: {atomic:ZZ}.__getattr__}
sage: sib getattr(sib.name('foo'), '__new__')
{getattr: {atomic:foo}.__new__}
```

id_cache (*x*, *sie*, *name*)

INPUT:

- *x* - an arbitrary value
- *sie* - a `SageInputExpression`
- *name* - a requested variable name

Enters *x* and *sie* in a cache, so that subsequent calls `self(x)` will directly return *sie*. Also, marks the requested name of this *sie* to be *name*. Differs from the method{cache} method in that the cache is keyed by `id(x)` instead of by *x*.

This may be called on values of an arbitrary type, which may be useful if the values are both large and likely to be used multiple times in a single expression; it should be preferred to method{cache} if equality on the values is difficult or impossible to compute.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: x = polygen(ZZ)
sage: sib = SageInputBuilder()
sage: my_42 = 42*x
sage: sie42 = sib(my_42)
sage: sib.id_cache(my_42, sie42, 'the_ultimate_answer')
sage: sib.result(sib(my_42) + sib(my_42))
```

```
R.<x> = ZZ[]
the_ultimate_answer = 42*x
the_ultimate_answer + the_ultimate_answer
```

Since `id_cache` keys off of object identity (“is”), the following does not trigger the cache.:

```
sage: sib.result(sib(42*x) + sib(42*x))
42*x + 42*x
```

Note that we don’t assign the result to a variable if the value is only used once.:

```
sage: sib = SageInputBuilder()
sage: my_42 = 42*x
sage: sie42 = sib(my_42)
sage: sib.id_cache(my_42, sie42, 'the_ultimate_answer')
sage: sib.result(sib(my_42) + sib(43*x))
R.<x> = ZZ[]
42*x + 43*x
```

import_name (*module, name, alt_name=None*)

INPUT:

- *module, name, alt_name* – strings

Creates an expression that will import a name from a module and then use that name.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: v1 = sib.import_name('sage.foo.bar', 'baz')
sage: v2 = sib.import_name('sage.foo.bar', 'ZZ', 'not_the_real_ZZ')
sage: sib.result(v1+v2)
from sage.foo.bar import baz
from sage.foo.bar import ZZ as not_the_real_ZZ
baz + not_the_real_ZZ
```

We adjust the names if there is a conflict.:

```
sage: sib = SageInputBuilder()
sage: v1 = sib.import_name('sage.foo', 'poly')
sage: v2 = sib.import_name('sage.bar', 'poly')
sage: sib.result(v1+v2)
from sage.foo import poly as poly1
from sage.bar import poly as poly2
poly1 + poly2
```

int (*n*)

Return a raw SIE from the integer *n*

As it is raw, it may read back as a Sage Integer, a Python int or a Python long, depending on its size and whether the preparser is enabled.

INPUT:

- *n* - a Sage Integer, a Python int or a Python long

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
```

```
sage: sib = SageInputBuilder()
sage: sib.result(sib.int(-3^50))
-717897987691852588770249

sage: sib = SageInputBuilder()
sage: sib.result(sib.int(long(2^65)))
36893488147419103232

sage: sib = SageInputBuilder()
sage: sib.result(sib.int(-42r))
-42
```

name (*n*)

Given a string representing a Python name, produces a `SageInputExpression` for that name.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.name('pi') + sib.name('e'))
pi + e
```

parent_with_gens (*parent, sie, gen_names, name, gens_syntax=None*)

This method is used for parents with generators, to manage the sage parser generator syntax (like `K.<x> = QQ[]`).

The method `{_sage_input_}` method of a parent class with generators should construct a `SageInputExpression` for the parent, and then call this method with the parent itself, the constructed SIE, a sequence containing the names of the generators, and (optionally) another SIE to use if the sage generator syntax is used; typically this will be the same as the first SIE except omitting a `names` parameter.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: def test_setup(use_gens=True, preparse=True):
...     sib = SageInputBuilder(preparse=preparse)
...     gen_names=('foo', 'bar')
...     parent = "some parent"
...     normal_sie = sib.name('make_a_parent')(names=gen_names)
...     if use_gens:
...         gens_sie = sib.name('make_a_parent')()
...     else:
...         gens_sie = None
...     name = 'the_thing'
...     result = sib.parent_with_gens(parent, normal_sie,
...                                   gen_names, name,
...                                   gens_syntax=gens_sie)
...     return sib, result

sage: sib, par_sie = test_setup()
sage: sib.result(par_sie)
make_a_parent(names=('foo', 'bar'))

sage: sib, par_sie = test_setup()
sage: sib.result(sib(3) * sib.gen("some parent", 0))
```



```

the_thing.<foo,bar> = make_a_parent()
3*foo

sage: sib, par_sie = test_setup(preparse=False)
sage: sib.result(par_sie)
make_a_parent(names=('foo', 'bar'))

sage: sib, par_sie = test_setup(preparse=False)
sage: sib.result(sib(3) * sib.gen("some parent", 0))
the_thing = make_a_parent(names=('foo', 'bar'))
foo,bar = the_thing.gens()
ZZ(3)*foo

sage: sib, par_sie = test_setup(use_gens=False)
sage: sib.result(par_sie)
make_a_parent(names=('foo', 'bar'))

sage: sib, par_sie = test_setup(use_gens=False)
sage: sib.result(sib(3) * sib.gen("some parent", 0))
the_thing = make_a_parent(names=('foo', 'bar'))
foo,bar = the_thing.gens()
3*foo

sage: sib, par_sie = test_setup()
sage: sib.result(par_sie - sib.gen("some parent", 1))
the_thing.<foo,bar> = make_a_parent()
the_thing - bar

```

preparse()

Checks the preparse status.

It returns `True` if the parser will be enabled, `False` if it will be disabled, and `None` if the result must work whether or not the parser is enabled.

For example, this is useful in the `method{ _sage_input_ }` methods of `Integer` and `RealNumber`; but most `method{ _sage_input_ }` methods will not need to examine this.

EXAMPLES:

```

sage: from sage.misc.sage_input import SageInputBuilder
sage: SageInputBuilder().preparse()
True
sage: SageInputBuilder(preparse=False).preparse()
False

```

prod(factors, simplify=False)

Given a sequence, returns a `SageInputExpression` for the product of the elements.

With `simplify=True`, performs some simplifications first. If any element is formatted as a string `'0'`, then that element is returned directly. If any element is formatted as a string `'1'`, then it is removed from the sequence (unless it is the only element in the sequence). And any negations are removed from the elements and moved to the outside of the product.

EXAMPLES:

```

sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 0, 1, -2]))
-1*0*1*-2

```

```
sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 0, 1, 2], simplify=True))
0

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 2, -3, -4], simplify=True))
-2*3*4

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([-1, 1, -1, -1], simplify=True))
-1

sage: sib = SageInputBuilder()
sage: sib.result(sib.prod([1, 1, 1], simplify=True))
1
```

result (*e*)

Given a `SageInputExpression` constructed using `self`, returns a tuple of a list of commands and an expression (and possibly a dictionary of local variables) suitable for `sage_eval()`.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: r = sib.result(sib(6) * sib(7)); r
6*7
sage: tuple(r)
(' ', '6*7')
```

share (*sie*)

Mark the given expression as sharable, so that it will be replaced by a variable if it occurs multiple times in the expression. (Most non-single-token expressions are already sharable.)

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder
```

Without explicitly using `.share()`, string literals are not shared:

```
sage: sib = SageInputBuilder()
sage: e = sib('hello')
sage: sib.result(sib((e, e)))
('hello', 'hello')
```

See the difference if we use `.share()`:

```
sage: sib = SageInputBuilder()
sage: e = sib('hello')
sage: sib.share(e)
sage: sib.result(sib((e, e)))
si = 'hello'
(si, si)
```

sum (*terms*, *simplify=False*)

Given a sequence, returns a `SageInputExpression` for the product of the elements.

With `simplify=True`, performs some simplifications first. If any element is formatted as a string `'0'`, then it is removed from the sequence (unless it is the only element in the sequence); and any instances of

$a + -b$ are changed to $a - b$.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: sib.result(sib.sum([-1, 0, 1, 0, -1]))
-1 + 0 + 1 + 0 + -1

sage: sib = SageInputBuilder()
sage: sib.result(sib.sum([-1, 0, 1, 0, -1], simplify=True))
-1 + 1 - 1

sage: sib = SageInputBuilder()
sage: sib.result(sib.sum([0, 0, 0], simplify=True))
0
```

use_variable (*sie*, *name*)

Marks the `SageInputExpression` *sie* to use a variable even if it is only referenced once. (If *sie* is the final top-level expression, though, it will not use a variable.)

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder

sage: sib = SageInputBuilder()
sage: e = sib.name('MatrixSpace')(ZZ, 10, 10)
sage: sib.use_variable(e, 'MS')
sage: sib.result(e.zero_matrix())
MS = MatrixSpace(ZZ, 10, 10)
MS.zero_matrix()
```

Without the call to `use_variable`, we get this instead:

```
sage: sib = SageInputBuilder()
sage: e = sib.name('MatrixSpace')(ZZ, 10, 10)
sage: sib.result(e.zero_matrix())
MatrixSpace(ZZ, 10, 10).zero_matrix()
```

And even with the call to `use_variable`, we don't use a variable here:

```
sage: sib = SageInputBuilder()
sage: e = sib.name('MatrixSpace')(ZZ, 10, 10)
sage: sib.use_variable(e, 'MS')
sage: sib.result(e)
MatrixSpace(ZZ, 10, 10)
```

class `sage.misc.sage_input.SageInputExpression` (*sib*)

Bases: object

Subclasses of this class represent expressions for `sage_input()`. `sage` classes should define a method `{_sage_input_}` method, which will return an instance of `SageInputExpression`, created using methods of `SageInputBuilder`.

To the extent possible, operations on `SageInputExpression` objects construct a new `SageInputExpression` representing that operation. That is, if *a* is a `SageInputExpression`, then *a* + *b* constructs a `SageInputExpression` representing this sum. This also works for attribute access, function calls, subscripts, etc. Since arbitrary attribute accesses might be used to construct a new attribute-access expression, all internal attributes and methods have names that begin with `_sie_` to reduce the chance of collisions.

It is expected that instances of this class will not be directly created outside this module; instead, instances will be created using methods of `SageInputBuilder` and `SageInputExpression`.

Values of type `SageInputExpression` print in a fairly ugly way, that reveals the internal structure of the expression tree.

class `sage.misc.sage_input.SageInputFormatter`

An instance of this class is used to keep track of variable names and a sequence of generated commands during the `sage_input()` formatting process.

format (*e*, *prec*)

Format a Sage input expression into a string.

INPUT:

- *e* - a `SageInputExpression`
- *prec* - an integer representing a precedence level

First, we check to see if *e* should be replaced by a variable. If so, we generate the command to assign the variable, and return the name of the variable.

Otherwise, we format the expression by calling its method `{_sie_format}` method, and add parentheses if necessary.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputBuilder, SageInputFormatter

sage: sib = SageInputBuilder()
sage: sif = SageInputFormatter()
sage: sie = sib(GF(5))
```

Here we cheat by calling method `{_sie_prepare}` twice, to make it use a variable.:

```
sage: sie._sie_prepare(sif)
sage: sie._sie_prepare(sif)
sage: sif._commands
''

sage: sif.format(sie, 0)
'GF_5'

sage: sif._commands
'GF_5 = GF(5)\n'
```

We demonstrate the use of commands, by showing how to construct code that will produce a random matrix:

```
sage: sib = SageInputBuilder()
sage: sif = SageInputFormatter()
sage: sie = sib.name('matrix')(sib.name('ZZ'), 10, 10)
sage: sib.command(sie, sie.randomize())
sage: sie._sie_prepare(sif)
sage: sif._commands
''

sage: sif.format(sie, 0)
'si'

sage: sif._commands
'si = matrix(ZZ, 10, 10)\nsi.randomize()\n'
```

get_name (*name*)

Return a name corresponding to a given requested name. If only one request for a name is received, then we will use the requested name; otherwise, we will add numbers to the end of the name to make it unique.

If the input name is `None`, then it is treated as a name of 'si'.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputFormatter

sage: sif = SageInputFormatter()
sage: names = ('x', 'x', 'y', 'z')
sage: for n in names: sif.register_name(n)
sage: for n in names: sif.get_name(n)
'x1'
'x2'
'y'
'z'
```

register_name(*name*)

Register that some value would like to use a given name. If only one request for a name is received, then we will use the requested name; otherwise, we will add numbers to the end of the name to make it unique.

If the input name is `None`, then it is treated as a name of 'si'.

EXAMPLES:

```
sage: from sage.misc.sage_input import SageInputFormatter

sage: sif = SageInputFormatter()
sage: sif._names, sif._dup_names
(set(), {})
sage: sif.register_name('x')
sage: sif.register_name('y')
sage: sif._names, sif._dup_names
({'x', 'y'}, {})
sage: sif.register_name('x')
sage: sif._names, sif._dup_names
({'x', 'y'}, {'x': 0})
```

`sage.misc.sage_input.sage_input`(*x*, *preparse*=`True`, *verify*=`False`, *allow_locals*=`False`)

Return a sequence of commands that can be used to rebuild the object *x*.

INPUT:

- *x* - the value we want to find an input form for
- *preparse* - (default `True`) Whether to generate code that requires the preparer. With `True`, generated code requires the preparer. With `False`, generated code requires that the preparer not be used. With `None`, generated code will work whether or not the preparer is used.
- *verify* - (default `False`) If `True`, then the answer will be evaluated with `sage_eval()`, and an exception will be raised if the result is not equal to the original value. (In fact, for `verify=True`, `sage_input()` is effectively run three times, with `preparse` set to `True`, `False`, and `None`, and all three results are checked.) This is particularly useful for doctests.
- *allow_locals* - (default `False`) If `True`, then values that `sage_input()` cannot handle are returned in a dictionary, and the returned code assumes that this dictionary is passed as the `locals` parameter of `sage_eval()`. (Otherwise, if `sage_input()` cannot handle a value, an exception is raised.)

EXAMPLES:

```
sage: sage_input(GF(2)(1))
GF(2)(1)
sage: sage_input((GF(2)(0), GF(2)(1)), verify=True)
# Verified
```

```
GF_2 = GF(2)
      (GF_2(0), GF_2(1))
```

When the preparser is enabled, we use the sage generator syntax.:

```
sage: K.<x> = GF(5) []
sage: sage_input(x^3 + 2*x, verify=True)
# Verified
R.<x> = GF(5) []
x^3 + 2*x
sage: sage_input(x^3 + 2*x, preparse=False)
R = GF(5) ['x']
x = R.gen()
x**3 + 2*x
```

The result of `sage_input()` is actually a pair of strings with a special `__repr__` method to print nicely.:

```
sage: r = sage_input(RealField(20)(pi), verify=True)
sage: r
# Verified
RealField(20) (3.1415939)
sage: isinstance(r, tuple)
True
sage: len(r)
2
sage: tuple(r)
('# Verified\n', 'RealField(20) (3.1415939)')
```

We cannot find an input form for a function.:

```
sage: sage_input((3, lambda x: x))
Traceback (most recent call last):
...
ValueError: Can't convert <function <lambda> at 0x...> to sage_input form
```

But we can have `sage_input()` continue anyway, and return an input form for the rest of the expression, with `allow_locals=True`.:

```
sage: r = sage_input((3, lambda x: x), verify=True, allow_locals=True)
sage: r
LOCALS:
  _sill: <function <lambda> at 0x...>
# Verified
(3, _sill)
sage: tuple(r)
('# Verified\n', '(3, _sill)', {'_sill': <function <lambda> at 0x...>})
```

`sage.misc.sage_input.verify_same(a, b)`

Verify that two Sage values are the same. This is an extended equality test; it checks that the values are equal and that their parents are equal. (For values which are not Elements, the types are checked instead.)

If the values are the same, we return `None`; otherwise, we raise an exception.

EXAMPLES:

```
sage: from sage.misc.sage_input import verify_same
sage: verify_same(1, 1)
sage: verify_same(1, 2)
Traceback (most recent call last):
...
```

```

AssertionError: Expected 1 == 2
sage: verify_same(1, 1r)
Traceback (most recent call last):
...
AttributeError: 'int' object has no attribute 'parent'
sage: verify_same(1r, 1)
Traceback (most recent call last):
...
    assert(type(a) == type(b))
AssertionError
sage: verify_same(5, GF(7)(5))
Traceback (most recent call last):
...
    assert(a.parent() == b.parent())
AssertionError

```

`sage.misc.sage_input.verify_si_answer(x, answer, preparse)`

Verify that evaluating `answer` gives a value equal to `x` (with the same parent/type). If `preparse` is `True` or `False`, then we evaluate `answer` with the parser enabled or disabled, respectively; if `preparse` is `None`, then we evaluate `answer` both with the parser enabled and disabled and check both results.

On success, we return `None`; on failure, we raise an exception.

INPUT:

- `x` - an arbitrary Sage value
- `answer` - a string, or a `SageInputAnswer`
- `preparse` - `True`, `False`, or `None`

EXAMPLES:

```

sage: from sage.misc.sage_input import verify_si_answer
sage: verify_si_answer(1, '1', True)
sage: verify_si_answer(1, '1', False)
Traceback (most recent call last):
...
AttributeError: 'int' object has no attribute 'parent'
sage: verify_si_answer(1, 'ZZ(1)', None)

```

4.1.7 Tables

Display a rectangular array as a table, either in plain text, LaTeX, or html. See the documentation for `table` for details and examples.

AUTHORS:

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```

class sage.misc.table.table(rows=None, columns=None, header_row=False,
                             header_column=False, frame=False, align='left')
    Bases: sage.structure.sage_object.SageObject

```

Display a rectangular array as a table, either in plain text, LaTeX, or html.

INPUTS:

- `rows` (default `None`) - a list of lists (or list of tuples, etc.), containing the data to be displayed.

- `columns` (default `None`) - a list of lists (etc.), containing the data to be displayed, but stored as columns. Set either `rows` or `columns`, but not both.
- `header_row` (default `False`) - if `True`, first row is highlighted.
- `header_column` (default `False`) - if `True`, first column is highlighted.
- `frame` (default `False`) - if `True`, put a box around each cell.
- `align` (default `'left'`) - the alignment of each entry: either `'left'`, `'center'`, or `'right'`

EXAMPLES:

```
sage: rows = [['a', 'b', 'c'], [100, 2, 3], [4, 5, 60]]
```

```
sage: table(rows)
```

```
  a    b    c
100    2    3
  4     5   60
```

```
sage: latex(table(rows))
```

```
\begin{tabular}{lll}
a & b & c \\
$100$ & $2$ & $3$ \\
$4$ & $5$ & $60$ \\
\end{tabular}
```

If `header_row` is `True`, then the first row is highlighted. If `header_column` is `True`, then the first column is highlighted. If `frame` is `True`, then print a box around every “cell”.

```
sage: table(rows, header_row=True)
```

```
  a    b    c
+-----+-----+
100    2    3
  4     5   60
```

```
sage: latex(table(rows, header_row=True))
```

```
\begin{tabular}{lll}
a & b & c \\ \hline
$100$ & $2$ & $3$ \\
$4$ & $5$ & $60$ \\
\end{tabular}
```

```
sage: table(rows=rows, frame=True)
```

```
+-----+-----+
| a | b | c |
+-----+-----+
| 100 | 2 | 3 |
+-----+-----+
| 4 | 5 | 60 |
+-----+-----+
```

```
sage: latex(table(rows=rows, frame=True))
```

```
\begin{tabular}{|lll|} \hline
a & b & c \\ \hline
$100$ & $2$ & $3$ \\ \hline
$4$ & $5$ & $60$ \\ \hline
\end{tabular}
```

```
sage: table(rows, header_column=True, frame=True)
```

```
+-----+-----+
| a | | b | c |
+-----+-----+
| 100 | | 2 | 3 |
+-----+-----+
| 4 | | 5 | 60 |
+-----+-----+
```



```

sage: latex(table(rows, header_row=True, frame=True))
\begin{tabular}{|l|l|l|} \hline
a & b & c \\ \hline
$100$ & $2$ & $3$ \\ \hline
$4$ & $5$ & $60$ \\ \hline
\end{tabular}
sage: table(rows, header_column=True)
  a | b | c
100 | 2 | 3
 4 | 5 | 60

```

The argument `header_row` can, instead of being `True` or `False`, be the contents of the header row, so that `rows` consists of the data, while `header_row` is the header information. The same goes for `header_column`. Passing lists for both arguments simultaneously is not supported.

```

sage: table([(x,n(sin(x), digits=2)) for x in [0..3]], header_row=["$x$", "$\sin(x)$"], frame=True)
+-----+-----+
| $x$ | $\sin(x)$ |
+=====+=====+
| 0 | 0.00 |
+-----+-----+
| 1 | 0.84 |
+-----+-----+
| 2 | 0.91 |
+-----+-----+
| 3 | 0.14 |
+-----+-----+

```

You can create the transpose of this table in several ways, for example, “by hand,” that is, changing the data defining the table:

```

sage: table(rows=[[x for x in [0..3]], [n(sin(x), digits=2) for x in [0..3]]], header_column=['$x$', '$\sin(x)$'], frame=True)
+-----+-----+-----+-----+
| $x$ | 0 | 1 | 2 | 3 |
+-----+-----+-----+-----+
| $\sin(x)$ | 0.00 | 0.84 | 0.91 | 0.14 |
+-----+-----+-----+-----+

```

or by passing the original data as the columns of the table and using `header_column` instead of `header_row`:

```

sage: table(columns=[(x,n(sin(x), digits=2)) for x in [0..3]], header_column=['$x$', '$\sin(x)$'], frame=True)
+-----+-----+-----+-----+
| $x$ | 0 | 1 | 2 | 3 |
+-----+-----+-----+-----+
| $\sin(x)$ | 0.00 | 0.84 | 0.91 | 0.14 |
+-----+-----+-----+-----+

```

or by taking the `transpose()` of the original table:

```

sage: table(rows=[(x,n(sin(x), digits=2)) for x in [0..3]], header_row=['$x$', '$\sin(x)$'], frame=True)
+-----+-----+-----+-----+
| $x$ | 0 | 1 | 2 | 3 |
+-----+-----+-----+-----+
| $\sin(x)$ | 0.00 | 0.84 | 0.91 | 0.14 |
+-----+-----+-----+-----+

```

In either plain text or LaTeX, entries in tables can be aligned to the left (default), center, or right:

```

sage: table(rows, align='left')
  a    b    c
100   2    3
  4    5   60
sage: table(rows, align='center')
  a    b    c
100   2    3
  4    5   60
sage: table(rows, align='right', frame=True)
+-----+-----+
|  a  |  b  |  c  |
+-----+-----+
| 100 |  2  |  3  |
+-----+-----+
|   4 |  5  | 60  |
+-----+-----+

```

To print HTML, use either `table(...)._html_()` or `html(table(...))`:

```

sage: html(table(["$x$", "$\sin(x)$"] + [(x,n(sin(x), digits=2)) for x in [0..3]], header_row=
<html>
<div class="nottruncate">
<table border="1" class="table_form">
<tbody>
<tr>
<th><script type="math/tex">x</script></th>
<th><script type="math/tex">\sin(x)</script></th>
</tr>
<tr class="row-a">
<td><script type="math/tex">0</script></td>
<td><script type="math/tex">0.00</script></td>
</tr>
<tr class="row-b">
<td><script type="math/tex">1</script></td>
<td><script type="math/tex">0.84</script></td>
</tr>
<tr class="row-a">
<td><script type="math/tex">2</script></td>
<td><script type="math/tex">0.91</script></td>
</tr>
<tr class="row-b">
<td><script type="math/tex">3</script></td>
<td><script type="math/tex">0.14</script></td>
</tr>
</tbody>
</table>
</div>
</html>

```

It is an error to specify both rows and columns:

```

sage: table(rows=[[1,2,3], [4,5,6]], columns=[[0,0,0], [0,0,1024]])
Traceback (most recent call last):
...
ValueError: Don't set both 'rows' and 'columns' when defining a table.

sage: table(columns=[[0,0,0], [0,0,1024]])
0 0
0 0

```

```
0 1024
```

Note that if `rows` is just a list or tuple, not nested, then it is treated as a single row:

```
sage: table([1,2,3])
1  2  3
```

Also, if you pass a non-rectangular array, the longer rows or columns get truncated:

```
sage: table([[1,2,3,7,12], [4,5]])
1  2
4  5
sage: table(columns=[[1,2,3], [4,5,6,7]])
1  4
2  5
3  6
```

TESTS:

```
sage: TestSuite(table(["$x$", "$\sin(x)$"] + [(x,n(sin(x), digits=2)) for x in [0..3]], header=
```

options (***kws*)

With no arguments, return the dictionary of options for this table. With arguments, modify options.

INPUTS:

- `header_row` - if `True`, first row is highlighted.
- `header_column` - if `True`, first column is highlighted.
- `frame` - if `True`, put a box around each cell.
- `align` - the alignment of each entry: either 'left', 'center', or 'right'

EXAMPLES:

```
sage: T = table(['a', 'b', 'c'], [1,2,3])
sage: T.options()['align'], T.options()['frame']
('left', False)
sage: T.options(align='right', frame=True)
sage: T.options()['align'], T.options()['frame']
('right', True)
```

Note that when first initializing a table, `header_row` or `header_column` can be a list. In this case, during the initialization process, the header is merged with the rest of the data, so changing the header option later using `table.options(...)` doesn't affect the contents of the table, just whether the row or column is highlighted. When using this `options()` method, no merging of data occurs, so here `header_row` and `header_column` should just be `True` or `False`, not a list.

```
sage: T = table([1,2,3], [4,5,6], header_row=['a', 'b', 'c'], frame=True)
sage: T
+---+---+---+
| a | b | c |
+---+---+---+
| 1 | 2 | 3 |
+---+---+---+
| 4 | 5 | 6 |
+---+---+---+
sage: T.options(header_row=False)
sage: T
+---+---+---+
| a | b | c |
```

```
+---+---+---+
| 1 | 2 | 3 |
+---+---+---+
| 4 | 5 | 6 |
+---+---+---+
```

If you do specify a list for `header_row`, an error is raised:

```
sage: T.options(header_row=['x', 'y', 'z'])
Traceback (most recent call last):
...
TypeError: header_row should be either True or False.
```

transpose()

Return a table which is the transpose of this one: rows and columns have been interchanged. Several of the properties of the original table are preserved: whether a frame is present and any alignment setting. On the other hand, header rows are converted to header columns, and vice versa.

EXAMPLES:

```
sage: T = table([[1,2,3], [4,5,6]])
sage: T.transpose()
1 4
2 5
3 6
sage: T = table([[1,2,3], [4,5,6]], header_row=['x', 'y', 'z'], frame=True)
sage: T.transpose()
+---+---+---+
| x || 1 | 4 |
+---+---+---+
| y || 2 | 5 |
+---+---+---+
| z || 3 | 6 |
+---+---+---+
```

4.2 HTML and MathML

4.2.1 HTML typesetting for the notebook

class `sage.misc.html.HTML`

eval (*s*, *globals=None*, *locals=None*)

Return an html representation for an object *s*.

If *s* has a method `_html_()`, call that. Otherwise, call `math_parse()` on `str(s)`, evaluate any variables in the result, and add some html preamble and postamble.

In any case, *print* the resulting html string. This method always *returns* an empty string.

EXAMPLES:

```
sage: html.eval('<hr>')
<html><font color='black'><hr></font></html>
''
```

iframe (*url*, *height=400*, *width=800*)

Put an existing web page into a worksheet.

INPUT:

- `url` – a url string, either with or without URI scheme (defaults to “http”).
- `height` – the number of pixels for the page height. Defaults to 400.
- `width` – the number of pixels for the page width. Defaults to 800.

OUTPUT:

Opens the url in a worksheet. If the url is a regular web page it will appear in the worksheet. This was originally intended to bring GeoGebra worksheets into Sage, but it can be used for many other purposes.

EXAMPLES:

```
sage: html.iframe("sagemath.org")
<html><font color='black'><iframe height="400" width="800"
src="http://sagemath.org"></iframe></font></html>
sage: html.iframe("http://sagemath.org",30,40)
<html><font color='black'><iframe height="30" width="40"
src="http://sagemath.org"></iframe></font></html>
sage: html.iframe("https://sagemath.org",30)
<html><font color='black'><iframe height="30" width="800"
src="https://sagemath.org"></iframe></font></html>
sage: html.iframe("/home/admin/0/data/filename")
<html><font color='black'><iframe height="400" width="800"
src="/home/admin/0/data/filename"></iframe></font></html>
sage: html.iframe('data:image/png;base64,iVBORw0KGgoAAAANSUheEUgAAA'
... 'AUAAAAFCAYAAACNbyblAAAAHElEQVQI12P4//8/w38GIAXDIBKE0DHxgljNBA'
... 'AO9TXLOY40HwAAAABJRU5ErkJggg==')
<html><font color='black'><iframe height="400" width="800"
src="data:image/png;base64,iVBORw0KGgoAAAANSUheEUgAAAAUAAAAFCAYAAACNbyblAAAAHElEQVQI12P4//8/w38GIAXDIBKE0DHxgljNBA"></iframe></font></html>
```

AUTHOR:

- Bruce Cohen (2011-06-14)

table (*x*, *header*=False)

Print a nested list as a HTML table. Strings of html will be parsed for math inside dollar and double-dollar signs. 2D graphics will be displayed in the cells. Expressions will be latexed.

INPUT:

- `x` – a list of lists (i.e., a list of table rows)
- `header` – a row of headers. If True, then the first row of the table is taken to be the header.

EXAMPLES:

```
sage: html.table([(i, j, i == j) for i in [0..1] for j in [0..1]])
<html>
<div class="nottruncate">
<table class="table_form">
<tbody>
<tr class="row-a">
<td><script type="math/tex">0</script></td>
<td><script type="math/tex">0</script></td>
<td><script type="math/tex">\mathrm{True}</script></td>
</tr>
<tr class="row-b">
<td><script type="math/tex">0</script></td>
<td><script type="math/tex">1</script></td>
<td><script type="math/tex">\mathrm{False}</script></td>
```

```
</tr>
<tr class ="row-a">
<td><script type="math/tex">1</script></td>
<td><script type="math/tex">0</script></td>
<td><script type="math/tex">\mathrm{False}</script></td>
</tr>
<tr class ="row-b">
<td><script type="math/tex">1</script></td>
<td><script type="math/tex">1</script></td>
<td><script type="math/tex">\mathrm{True}</script></td>
</tr>
</tbody>
</table>
</div>
</html>
```

```
sage: html.table([(x,n(sin(x), digits=2)) for x in [0..3]], header = ["$x$", "$\sin(x)$"])
<html>
<div class="nottruncate">
<table class="table_form">
<tbody>
<tr>
<th><script type="math/tex">x</script></th>
<th><script type="math/tex">\sin(x)</script></th>
</tr>
<tr class ="row-a">
<td><script type="math/tex">0</script></td>
<td><script type="math/tex">0.00</script></td>
</tr>
<tr class ="row-b">
<td><script type="math/tex">1</script></td>
<td><script type="math/tex">0.84</script></td>
</tr>
<tr class ="row-a">
<td><script type="math/tex">2</script></td>
<td><script type="math/tex">0.91</script></td>
</tr>
<tr class ="row-b">
<td><script type="math/tex">3</script></td>
<td><script type="math/tex">0.14</script></td>
</tr>
</tbody>
</table>
</div>
</html>
```

class sage.misc.html.**HTMLExpr**

Bases: str

A class for HTML expression

sage.misc.html.**html** (*s*, *globals=None*, *locals=None*)

Display the given HTML expression in the notebook.

INPUT:

- *s* – a string

OUTPUT:

- prints a code that embeds HTML in the output.

By default in the notebook an output cell has two parts, first a plain text preformat part, then second a general HTML part (not pre). If you call `html(s)` at any point then that adds something that will be displayed in the preformatted part in html.

EXAMPLES:

```
sage: html('<a href="http://sagemath.org">sagemath</a>')
<html><font color='black'><a href="http://sagemath.org">sagemath</a></font></html>
sage: html('<hr>')
<html><font color='black'><hr></font></html>
```

`sage.misc.html.math_parse(s)`

Turn the HTML-ish string `s` that can have `$$` and `$'s` in it into pure HTML. See below for a precise definition of what this means.

INPUT:

- `s` – a string

OUTPUT:

- a string.

Do the following:

- Replace all `$ text $'s` by `<script type="math/tex"> text </script>`
- Replace all `$$ text $$'s` by `<script type="math/tex; mode=display"> text </script>`
- Replace all `\ $'s` by `$'s`. Note that in the above two cases nothing is done if the `$` is preceeded by a backslash.
- Replace all `\[text \]'s` by `<script type="math/tex; mode=display"> text </script>`

EXAMPLES:

```
sage: sage.misc.html.math_parse('This is $2+2$.')
'This is <script type="math/tex">2+2</script>.'
sage: sage.misc.html.math_parse('This is $$2+2$$.')
'This is <script type="math/tex; mode=display">2+2</script>.'
sage: sage.misc.html.math_parse('This is \[2+2\].')
'This is <script type="math/tex; mode=display">2+2</script>.'
sage: sage.misc.html.math_parse(r'This is \[2+2\].')
'This is <script type="math/tex; mode=display">2+2</script>.'
```

TESTS:

```
sage: sage.misc.html.math_parse(r'This $$is $2+2$.')
'This $$is <script type="math/tex">2+2</script>.'
```

4.2.2 MathML output support

In order to support MathML formatting, an object should define a special method `_mathml_(self)` that returns its MathML representation.

```
class sage.misc.mathml.MathML
    Bases: str
```

```
sage.misc.mathml.bool_function(x)
sage.misc.mathml.list_function(x)
sage.misc.mathml.mathml(x)
    Output x formatted for inclusion in a MathML document.
sage.misc.mathml.str_function(x)
sage.misc.mathml.tuple_function(x)
```

4.3 LaTeX

4.3.1 Installing and using SageTeX

SageTeX is a system for embedding computations and plots from Sage into LaTeX documents. It is included by default with Sage, so if you have installed Sage, you already have SageTeX. However, to get it to work, you need to make TeX aware of SageTeX. Instructions for that are in the “Make SageTeX known to TeX” section of the [Sage installation guide](#) (this link should take you to a local copy of the installation guide).

4.3.2 LaTeX printing support

In order to support latex formatting, an object should define a special method `_latex_(self)` that returns a string, which will be typeset in a mathematical mode (the exact mode depends on circumstances).

AUTHORS:

- William Stein: original implementation
- Joel B. Mohler: `latex_variable_name()` drastic rewrite and many doc-tests

```
class sage.misc.latex.Latex(debug=False, slide=False, density=150, pdflatex=None, engine=None)
    Bases: sage.misc.latex.LatexCall
```

Enter, e.g.,

```
%latex
```

The equation $y^2 = x^3 + x$ defines an elliptic curve.

We have $2006 = \text{sage}\{\text{factor}(2006)\}$.

in an input cell in the notebook to get a typeset version. Use `%latex_debug` to get debugging output.

Use `latex(...)` to typeset a Sage object. Use `LatexExpr` to typeset LaTeX code that you create by hand.

Use `%slide` instead to typeset slides.

Warning: You must have dvipng (or dvips and convert) installed on your operating system, or this command won't work.

EXAMPLES:

```
sage: latex(x^20 + 1)
x^{20} + 1
sage: latex(FiniteField(25, 'a'))
\Bold{F}_{5^{2}}
sage: latex("hello")
\text{\texttt{hello}}
sage: LatexExpr(r"\frac{x^2 - 1}{x + 1} = x - 1")
\frac{x^2 - 1}{x + 1} = x - 1
```


LaTeX expressions can be added; note that a space is automatically inserted:

```
sage: LatexExpr(r"y \neq") + latex(x^20 + 1)
y \neq x^{20} + 1
```

add_macro (*macro*)

Append to the string of extra LaTeX macros, for use with %latex, %html, and %mathjax.

INPUT:

- macro – string

EXAMPLES:

```
sage: latex.extra_macros()
''
sage: latex.add_macro("\\newcommand{\\foo}{bar}")
sage: latex.extra_macros()
'\\newcommand{\\foo}{bar}'
sage: latex.extra_macros("") # restore to default
```

add_package_to_preamble_if_available (*package_name*)

Adds a `\usepackage{package_name}` instruction to the latex preamble if not yet present there, and if `package_name.sty` is available in the LaTeX installation.

INPUT:

- package_name – a string

See also:

- add_to_preamble()
- has_file().

TESTS:

```
sage: latex.add_package_to_preamble_if_available("xypic")
sage: latex.add_package_to_preamble_if_available("nonexistent_package")
sage: latex.extra_preamble() # optional - latex
'\\usepackage{xypic}\\n'
sage: latex.extra_preamble('')
```

add_to_mathjax_avoid_list (*s*)

Add to the list of strings which signal that MathJax should not be used when ‘view’ing.

INPUT:

- s – string; add s to the list of ‘MathJax avoid’ strings

If you want to replace the current list instead of adding to it, use `latex.mathjax_avoid_list`.

EXAMPLES:

```
sage: latex.add_to_mathjax_avoid_list("\\mathsf")
sage: latex.mathjax_avoid_list() # display current setting
['\\mathsf']
sage: latex.add_to_mathjax_avoid_list("tkz-graph")
sage: latex.mathjax_avoid_list() # display current setting
['\\mathsf', 'tkz-graph']
sage: latex.mathjax_avoid_list([]) # reset to default
```

```
sage: latex.mathjax_avoid_list()
[]
```

add_to_preamble(*s*)

Append to the string *s* of extra LaTeX macros, for use with %latex. Anything in this string won't be processed by %mathjax.

EXAMPLES:

```
sage: latex.extra_preamble()
''
sage: latex.add_to_preamble("\\DeclareMathOperator{\\Ext}{Ext}")
```

At this point, a notebook cell containing

```
%latex
 $\Ext_A^*(\GF{2}, \GF{2}) \rightarrow \pi_*(S^0)$ 
```

will be typeset correctly.

```
sage: latex.add_to_preamble("\\usepackage{xypic}")
sage: latex.extra_preamble()
'\\DeclareMathOperator{\\Ext}{Ext}\\usepackage{xypic}'
```

Now one can put various xypic diagrams into a %latex cell, such as

```
%latex

$$\begin{array}{ccccccc} \circ & \ar & r[d]^a & \rightarrow & r[r]^b & \rightarrow & r[r]^c \\ & \downarrow & & & & & \\ & d[dr]^e & & & & & \\ & & & & \circ & \circ & \circ \end{array}$$

```

Reset the preamble to its default, the empty string:

```
sage: latex.extra_preamble('')
sage: latex.extra_preamble()
''
```

blackboard_bold(*t=None*)

Controls whether Sage uses blackboard bold or ordinary bold face for typesetting ZZ, RR, etc.

INPUT:

- *t* – boolean or None

OUTPUT:

If *t* is None, return the current setting (True or False).

If *t* is True, use blackboard bold (`\mathbb`); otherwise use boldface (`\mathbf`).

EXAMPLES:

```
sage: latex.blackboard_bold()
False
sage: latex.blackboard_bold(True)
sage: latex.blackboard_bold()
True
sage: latex.blackboard_bold(False)
```

check_file(*file_name*, *more_info*='')

INPUT:

- *file_name* – a string

- `more_info` – a string (default: “”)

Emit a warning if the local LaTeX installation does not include `file_name`. The string `more_info` is appended to the warning message. The warning is only emitted the first time this method is called.

EXAMPLES:

```
sage: latex.check_file("article.cls")           # optional - latex
sage: latex.check_file("some_inexistent_file.sty")
Warning: 'some_inexistent_file.sty' is not part of this computer's TeX installation.
sage: latex.check_file("some_inexistent_file.sty")
sage: latex.check_file("some_inexistent_file.sty", "This file is required for blah. It can be downloaded from: http://blah.org/")
Warning: 'some_inexistent_file.sty' is not part of this computer's TeX installation.
This file is required for blah. It can be downloaded from: http://blah.org/
```

This test checks that the bug in [trac ticket #9091](#) is fixed:

```
sage: latex.check_file("article.cls", "The article class is really critical.") # optional
```

engine (*e=None*)

Set Sage to use *e* as latex engine when typesetting with `view()`, in %latex cells, etc.

INPUT:

- *e* – ‘latex’, ‘pdflatex’, ‘xelatex’ or None

If *e* is None, return the current engine.

If using the XeLaTeX engine, it will almost always be necessary to set the proper preamble with `extra_preamble()` or `add_to_preamble()`. For example:

```
latex.extra_preamble(r'\usepackage{fontspec,xunicode,xltxtra}
\setmainfont[Mapping=tex-text]{some font here}
\setmonofont[Mapping=tex-text]{another font here}')
```

EXAMPLES:

```
sage: latex.engine()
'pdflatex'
sage: latex.engine("latex")
sage: latex.engine()
'latex'
sage: latex.engine("xelatex")
sage: latex.engine()
'xelatex'
```

eval (*x*, *globals*, *strip=False*, *filename=None*, *debug=None*, *density=None*, *pdflatex=None*, *engine=None*, *locals={}*)

Compiles the formatted tex given by *x* as a png and writes the output file to the directory given by *filename*.

INPUT:

- *globals* – a globals dictionary
- *x* – string to evaluate.
- *strip* – ignored
- *filename* – output filename
- *debug* – whether to print verbose debugging output
- *density* – how big output image is.

- `pdflatex` – whether to use `pdflatex`. This is deprecated. Use `engine` option instead.
- `engine` – latex engine to use. Currently `latex`, `pdflatex`, and `xelatex` are supported.
- `locals` – extra local variables used when evaluating Sage code in `x`.

Warning: When using `latex` (the default), you must have ‘`dvipng`’ (or ‘`dvips`’ and ‘`convert`’) installed on your operating system, or this command won’t work. When using `pdflatex` or `xelatex`, you must have ‘`convert`’ installed.

OUTPUT:

If it compiled successfully, this returns an empty string `''`, otherwise it returns `None`.

EXAMPLES:

```
# This would generate a file named "test.png"
sage: latex.eval("\\ZZ[x]", locals(), filename="test") # not tested
''

# This would generate a file named "/path/to/test.png"
sage: latex.eval("\\ZZ[x]", locals(), filename="/path/to/test") # not tested
''

sage: latex.eval("\\ThisIsAnInvalidCommand", {}) # optional -- ImageMagick
An error occurred...
No pages of output...
```

extra_macros (*macros=None*)

String containing extra LaTeX macros to use with `%latex`, `%html`, and `%mathjax`.

INPUT:

- `macros` – string (default: `None`)

If `macros` is `None`, return the current string. Otherwise, set it to `macros`. If you want to *append* to the string of macros instead of replacing it, use `latex.add_macro`.

EXAMPLES:

```
sage: latex.extra_macros("\\newcommand{\\foo}{bar}")
sage: latex.extra_macros()
'\\newcommand{\\foo}{bar}'
sage: latex.extra_macros("")
sage: latex.extra_macros()
''
```

extra_preamble (*s=None*)

String containing extra preamble to be used with `%latex`. Anything in this string won’t be processed by `%mathjax`.

INPUT:

- `s` – string or `None`

If `s` is `None`, return the current preamble. Otherwise, set it to `s`. If you want to *append* to the current extra preamble instead of replacing it, use `latex.add_to_preamble`.

You will almost certainly need to use this when using the XeLaTeX engine; see below or the documentation for `engine()` for a suggested preamble.

EXAMPLES:

```
sage: latex.extra_preamble("\\DeclareMathOperator{\\Ext}{Ext}")
sage: latex.extra_preamble()
'\\DeclareMathOperator{\\Ext}{Ext}'
```

```

sage: latex.extra_preamble("\\"+r"usepackage{fontspec,xunicode,xltxtra}\setmainfont[Mapping=
sage: latex.extra_preamble()
'\usepackage{fontspec,xunicode,xltxtra}\setmainfont[Mapping=tex-text]{UnBatang}\setmonof
sage: latex.extra_preamble("")
sage: latex.extra_preamble()
''

```

has_file(*file_name*)

INPUT:

- *file_name* – a string

Tests whether the local LaTeX installation includes *file_name*.

EXAMPLES:

```

sage: latex.has_file("article.cls")          # optional - latex
True
sage: latex.has_file("some_inexistent_file.sty")
False

```

mathjax_avoid_list(*L=None*)

List of strings which signal that MathJax should not be used when ‘view’ing.

INPUT:

- *L* – A list or None

If *L* is None, then return the current list. Otherwise, set it to *L*. If you want to *append* to the current list instead of replacing it, use `latex.add_to_mathjax_avoid_list`.

EXAMPLES:

```

sage: latex.mathjax_avoid_list(["\\mathsf", "pspicture"])
sage: latex.mathjax_avoid_list()          # display current setting
['\\mathsf', 'pspicture']
sage: latex.mathjax_avoid_list([])        # reset to default
sage: latex.mathjax_avoid_list()
[]

```

matrix_column_alignment(*align=None*)

Changes the column-alignment of the LaTeX representation of matrices.

INPUT:

- *align* – a string ('r' for right, 'c' for center, 'l' for left) or None.

OUTPUT:

If *align* is None, then returns the current alignment-string. Otherwise, set this alignment.

The input *align* can be any string which the LaTeX `array`-environment understands as a parameter for aligning a column.

EXAMPLES:

```

sage: a = matrix(1, 1, [42])
sage: latex(a)
\left(\begin{array}{r}
42
\end{array}\right)
sage: latex.matrix_column_alignment('c')
sage: latex(a)
\left(\begin{array}{c}

```

```
42
\end{array}\right)
sage: latex.matrix_column_alignment('l')
sage: latex(a)
\left(\begin{array}{l}
42
\end{array}\right)
```

Restore defaults:

```
sage: latex.matrix_column_alignment('r')
```

matrix_delimiters (*left=None, right=None*)

Change the left and right delimiters for the LaTeX representation of matrices

INPUT:

- *left, right* - strings or None

If both *left* and *right* are None, then return the current delimiters. Otherwise, set the left and/or right delimiters, whichever are specified.

Good choices for *left* and *right* are any delimiters which LaTeX understands and knows how to resize; some examples are:

- parentheses: `'('`, `')'`
- brackets: `'['`, `']'`
- braces: `'{'`, `'\{'`
- vertical lines: `'|'`
- angle brackets: `'\angle'`, `'\rangle'`

Note: Putting aside aesthetics, you may combine these in any way imaginable; for example, you could set *left* to be a right-hand bracket `']'` and *right* to be a right-hand brace `'\{'`, and it will be typeset correctly.

EXAMPLES:

```
sage: a = matrix(1, 1, [17])
sage: latex(a)
\left(\begin{array}{r}
17
\end{array}\right)
sage: latex.matrix_delimiters("[", "]")
sage: latex(a)
\left[\begin{array}{r}
17
\end{array}\right]
sage: latex.matrix_delimiters(left="\{"")
sage: latex(a)
\left\{\begin{array}{r}
17
\end{array}\right\}
sage: latex.matrix_delimiters()
['\\{', '\\}']
```

Restore defaults:

```
sage: latex.matrix_delimiters("(", ")")
```

vector_delimiters (*left=None, right=None*)

Change the left and right delimiters for the LaTeX representation of vectors

INPUT:

- *left, right* – strings or None

If both *left* and *right* are None, then return the current delimiters. Otherwise, set the left and/or right delimiters, whichever are specified.

Good choices for *left* and *right* are any delimiters which LaTeX understands and knows how to resize; some examples are:

- parentheses: ‘(, ’)
- brackets: ‘[, ’]
- braces: ‘{, \}
- vertical lines: ‘|’
- angle brackets: ‘\angle’, ‘\rangle’

Note: Putting aside aesthetics, you may combine these in any way imaginable; for example, you could set *left* to be a right-hand bracket ‘]’ and *right* to be a right-hand brace ‘\}’, and it will be typeset correctly.

EXAMPLES:

```
sage: a = vector(QQ, [1,2,3])
sage: latex(a)
\left(1,\,2,\,3\right)
sage: latex.vector_delimiters("[", "]")
sage: latex(a)
\left[1,\,2,\,3\right]
sage: latex.vector_delimiters(right="\\\")
sage: latex(a)
\left[1,\,2,\,3\right\\]
sage: latex.vector_delimiters()
['[', '\\\\']
```

Restore defaults:

```
sage: latex.vector_delimiters("(", ")")
```

class sage.misc.latex.**LatexCall**

Typeset Sage objects via a `__call__` method to this class, typically by calling those objects’ `_latex_` methods. The class `Latex` inherits from this. This class is used in `latex_macros`, while functions from `latex_macros` are used in `Latex`, so this is here primarily to avoid circular imports.

EXAMPLES:

```
sage: from sage.misc.latex import LatexCall
sage: LatexCall()(ZZ)
\Bold{Z}
sage: LatexCall().__call__(ZZ)
\Bold{Z}
```

This returns an instance of the class `LatexExpr`:

```
sage: type(LatexCall()(ZZ))
<class 'sage.misc.latex.LatexExpr'>
```

class sage.misc.latex.**LatexExamples**

A catalogue of Sage objects with complicated `_latex_` methods. Use these for testing `latex()`, `view()`, the Typeset button in the notebook, etc.

The classes here only have `__init__`, `_repr_`, and `_latex_` methods.

EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: K = latex_examples.knot()
sage: K
LaTeX example for testing display of a knot produced by xypic...
sage: latex(K)
\vtop{\vbox{\xygraph{!{0;/r1.5pc/:}
[u] !{\vloop<(-.005)\khole||\vcrossneg \vunder- }
[] !{\ar @{-}@' {p-(1,0)@+}+(-1,1)}
[ul] !{\vcap[3]>\khole}
[rrr] !{\ar @{-}@' {p-(0,1)@+}-(1,1)}
}}}
```

class **diagram**

Bases: `sage.structure.sage_object.SageObject`

LaTeX example for testing display of commutative diagrams. See its string representation for details.

EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: CD = latex_examples.diagram()
sage: CD
LaTeX example for testing display of a commutative diagram...
```

class `LatexExamples.graph`

Bases: `sage.structure.sage_object.SageObject`

LaTeX example for testing display of graphs. See its string representation for details.

EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: G = latex_examples.graph()
sage: G
LaTeX example for testing display of graphs...
```

class `LatexExamples.knot`

Bases: `sage.structure.sage_object.SageObject`

LaTeX example for testing display of knots. See its string representation for details.

EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: K = latex_examples.knot()
sage: K
LaTeX example for testing display of a knot...
```

class `LatexExamples.pstricks`

Bases: `sage.structure.sage_object.SageObject`

LaTeX example for testing display of pstricks output. See its string representation for details.

EXAMPLES:

```
sage: from sage.misc.latex import latex_examples
sage: PS = latex_examples.pstricks()
sage: PS
LaTeX example for testing display of pstricks...
```

class sage.misc.latex.**LatexExpr**

Bases: str

A class for LaTeX expressions.

Normally, objects of this class are created by a `latex()` call. It is also possible to generate `LatexExpr` directly from a string, which must contain valid LaTeX code for typesetting in math mode (without dollar signs). In the Sage notebook, use `pretty_print()` or the “Typeset” checkbox to actually see the typeset LaTeX code; alternatively, from either the command-line or the notebook, use the `view()` function.

INPUT:

- `str` – a string with valid math mode LaTeX code (or something which can be converted to such a string).

OUTPUT:

- `LatexExpr` wrapping the string representation of the input.

EXAMPLES:

```
sage: latex(x^20 + 1)
x^{20} + 1
sage: LatexExpr(r"\frac{x^2 + 1}{x - 2}")
\frac{x^2 + 1}{x - 2}
```

`LatexExpr` simply converts to string without doing anything extra, it does *not* call `latex()`:

```
sage: latex(ZZ)
\Bold{Z}
sage: LatexExpr(ZZ)
Integer Ring
```

The result of `latex()` is of type `LatexExpr`:

```
sage: L = latex(x^20 + 1)
sage: L
x^{20} + 1
sage: type(L)
<class 'sage.misc.latex.LatexExpr'>
```

A `LatexExpr` can be converted to a plain string:

```
sage: str(latex(x^20 + 1))
'x^{20} + 1'
```

class sage.misc.latex.**MathJax**

Render LaTeX input using MathJax. This returns a `MathJaxExpr`.

EXAMPLES:

```
sage: from sage.misc.latex import MathJax
sage: MathJax()(3)
<html><script type="math/tex; mode=display">\newcommand{\Bold}[1]{\mathbf{#1}}3</script></html>
sage: MathJax()(ZZ)
<html><script type="math/tex; mode=display">\newcommand{\Bold}[1]{\mathbf{#1}}\Bold{Z}</script></html>
```

eval (*x*, *globals=None*, *locals=None*, *mode='display'*, *combine_all=False*)

Render LaTeX input using MathJax. This returns a `MathJaxExpr`.

INPUT:

- *x* - a Sage object
- *globals* - a globals dictionary
- *locals* - extra local variables used when evaluating Sage code in *x*.
- **mode - string (optional, default 'display')**: 'display' for displaymath, 'inline' for inline math, or 'plain' for just the LaTeX code without the surrounding html and script tags.
- *combine_all* - boolean (Default: False): If *combine_all* is True and the input is a tuple, then it does not return a tuple and instead returns a string with all the elements separated by a single space.

OUTPUT:

A `MathJaxExpr`

EXAMPLES:

```
sage: from sage.misc.latex import MathJax
sage: MathJax().eval(3, mode='display')
<html><script type="math/tex; mode=display">\newcommand{\Bold}[1]{\mathbf{#1}}3</script></html>
sage: MathJax().eval(3, mode='inline')
<html><script type="math/tex">\newcommand{\Bold}[1]{\mathbf{#1}}3</script></html>
sage: MathJax().eval(type(3), mode='inline')
<html><script type="math/tex">\newcommand{\Bold}[1]{\mathbf{#1}}\verb|<type|\phantom{\verb|
```

class `sage.misc.latex.MathJaxExpr` (*y*)

An arbitrary MathJax expression that can be nicely concatenated.

EXAMPLES:

```
sage: from sage.misc.latex import MathJaxExpr
sage: MathJaxExpr("a^{2}") + MathJaxExpr("x^{-1}")
a^{2}x^{-1}
```

`sage.misc.latex.None_function` (*x*)

Returns the LaTeX code for None.

INPUT:

- *x* - None

EXAMPLES:

```
sage: from sage.misc.latex import None_function
sage: print None_function(None)
\mathrm{None}
```

`sage.misc.latex.bool_function` (*x*)

Returns the LaTeX code for a boolean *x*.

INPUT:

- *x* - boolean

EXAMPLES:

```

sage: from sage.misc.latex import bool_function
sage: print bool_function(2==3)
\mathrm{False}
sage: print bool_function(3==(2+1))
\mathrm{True}

```

`sage.misc.latex.builtin_constant_function(x)`

Returns the LaTeX code for a builtin constant x .

INPUT:

• x – builtin constant

See also:

Python built-in Constants <http://docs.python.org/library/constants.html>

EXAMPLES:

```

sage: from sage.misc.latex import builtin_constant_function
sage: builtin_constant_function(True)
'\mbox{\rm True}'
sage: builtin_constant_function(None)
'\mbox{\rm None}'
sage: builtin_constant_function(NotImplemented)
'\mbox{\rm NotImplemented}'
sage: builtin_constant_function(Ellipsis)
'\mbox{\rm Ellipsis}'

```

TESTS:

```

sage: sage.misc.latex.EMBEDDED_MODE = True
sage: builtin_constant_function(True)
'\rm True'
sage: sage.misc.latex.EMBEDDED_MODE = False

```

`sage.misc.latex.coeff_repr(c)`

LaTeX string representing coefficients in a linear combination.

INPUT:

• c – a coefficient (i.e., an element of a ring)

OUTPUT:

A string

EXAMPLES:

```

sage: from sage.misc.latex import coeff_repr
sage: coeff_repr(QQ(1/2))
'\frac{1}{2}'
sage: coeff_repr(-x^2)
'\left(-x^{2}\right)'

```

`sage.misc.latex.dict_function(x)`

Returns the LaTeX code for a dictionary x .

INPUT:

• x – a dictionary

EXAMPLES:

```
sage: from sage.misc.latex import dict_function
sage: x,y,z = var('x,y,z')
sage: print dict_function({x/2: y^2})
\left\{\frac{1}{2} \, x : y^2\right\}
sage: d = {(1,2,x^2): [sin(z^2), y/2]}
sage: latex(d)
\left\{\left(1, 2, x^2\right) : \left[\sin\left(z^2\right), \frac{1}{2} \, y\right]\right\}
```

`sage.misc.latex.float_function(x)`

Returns the LaTeX code for a python float `x`.

INPUT:

- `x` – a python float

EXAMPLES:

```
sage: from sage.misc.latex import float_function
sage: float_function(float(3.14))
3.14
sage: float_function(float(1e-10))
1 \times 10^{-10}
sage: float_function(float(2e10))
20000000000.0
```

TESTS:

Check that [trac ticket #7356](#) is fixed:

```
sage: latex(float(2e-13))
2 \times 10^{-13}
```

`sage.misc.latex.has_latex_attr(x)`

Return True if `x` has a `_latex_` attribute, except if `x` is a type, in which case return False.

EXAMPLES:

```
sage: from sage.misc.latex import has_latex_attr
sage: has_latex_attr(identity_matrix(3))
True
sage: has_latex_attr("abc") # strings have no _latex_ method
False
```

Types inherit the `_latex_` method of the class to which they refer, but calling it is broken:

```
sage: T = type(identity_matrix(3)); T
<type 'sage.matrix.matrix_integer_dense.Matrix_integer_dense'>
sage: hasattr(T, '_latex_')
True
sage: T._latex_()
Traceback (most recent call last):
...
TypeError: descriptor '_latex_' of 'sage.matrix.matrix0.Matrix' object needs an argument
sage: has_latex_attr(T)
False
```

`sage.misc.latex.have_convert()`

Return True if this computer has the program `convert`.

If this computer doesn't have `convert` installed, you may obtain it (along with the rest of the ImageMagick suite)

from <http://www.imagemagick.org>

EXAMPLES:

```
sage: from sage.misc.latex import have_convert
sage: have_convert() # random
True
```

`sage.misc.latex.have_dvipng()`

Return True if this computer has the program dvipng.

If this computer doesn't have dvipng installed, you may obtain it from <http://sourceforge.net/projects/dvipng/>

EXAMPLES:

```
sage: from sage.misc.latex import have_dvipng
sage: have_dvipng() # random
True
```

`sage.misc.latex.have_latex()`

Return True if this computer has the program latex.

If this computer doesn't have LaTeX installed, you may obtain it from <http://ctan.org/>.

EXAMPLES:

```
sage: from sage.misc.latex import have_latex
sage: have_latex() # random
True
```

`sage.misc.latex.have_pdflatex()`

Return True if this computer has the program pdflatex.

If this computer doesn't have pdflatex installed, you may obtain it from <http://ctan.org/>.

EXAMPLES:

```
sage: from sage.misc.latex import have_pdflatex
sage: have_pdflatex() # random
True
```

`sage.misc.latex.have_xelatex()`

Return True if this computer has the program xelatex.

If this computer doesn't have xelatex installed, you may obtain it from <http://ctan.org/>.

EXAMPLES:

```
sage: from sage.misc.latex import have_xelatex
sage: have_xelatex() # random
True
```

`sage.misc.latex.latex(x, combine_all=False)`

Return a `LatexExpr` built out of the argument `x`.

INPUT:

- `x` – a Sage object
- `combine_all` – boolean (Default: False) If `combine_all` is True and the input is a tuple, then it does not return a tuple and instead returns a string with all the elements separated by a single space.

OUTPUT:

A `LatexExpr` built from `x`

EXAMPLES:

```
sage: latex(Integer(3)) # indirect doctest
3
sage: latex(1==0)
\mathrm{False}
sage: print latex([x, 2])
\left[x, 2\right]
```

Check that [trac ticket #11775](#) is fixed:

```
sage: latex((x, 2), combine_all=True)
x 2
```

`sage.misc.latex.latex_extra_preamble()`

Return the string containing the user-configured preamble, `sage_latex_macros`, and any user-configured macros. This is used in the `eval()` method for the `Latex` class, and in `_latex_file_()`; it follows either `LATEX_HEADER` or `SLIDE_HEADER` (defined at the top of this file) which is a string containing the documentclass and standard usepackage commands.

EXAMPLES:

```
sage: from sage.misc.latex import latex_extra_preamble
sage: print latex_extra_preamble()
...
```

```
\newcommand{\ZZ}{\Bold{Z}}
\newcommand{\NN}{\Bold{N}}
\newcommand{\RR}{\Bold{R}}
\newcommand{\CC}{\Bold{C}}
\newcommand{\QQ}{\Bold{Q}}
\newcommand{\QQbar}{\overline{\QQ}}
\newcommand{\GF}[1]{\Bold{F}_{#1}}
\newcommand{\Zp}[1]{\ZZ_{#1}}
\newcommand{\Qp}[1]{\QQ_{#1}}
\newcommand{\Zmod}[1]{\ZZ/#1\ZZ}
\newcommand{\CDF}{\Bold{C}}
\newcommand{\CIF}{\Bold{C}}
\newcommand{\CLF}{\Bold{C}}
\newcommand{\RDF}{\Bold{R}}
\newcommand{\RIF}{\Bold{I} \Bold{R}}
\newcommand{\RLF}{\Bold{R}}
\newcommand{\CFF}{\Bold{CFF}}
\newcommand{\Bold}[1]{\mathbf{#1}}
```

`sage.misc.latex.latex_variable_name(x, is_fname=False)`

Return latex version of a variable name.

Here are some guiding principles for usage of this function:

- 1.If the variable is a single letter, that is the latex version.
- 2.If the variable name is suffixed by a number, we put the number in the subscript.
- 3.If the variable name contains an ‘_’ we start the subscript at the underscore. Note that #3 trumps rule #2.
- 4.If a component of the variable is a Greek letter, escape it properly.
- 5.Recurse nicely with subscripts.

Refer to the examples section for how these rules might play out in practice.

EXAMPLES:

```

sage: from sage.misc.latex import latex_variable_name
sage: latex_variable_name('a')
'a'
sage: latex_variable_name('abc')
'\mathit{abc}'
sage: latex_variable_name('sigma')
'\sigma'
sage: latex_variable_name('sigma_k')
'\sigma_{k}'
sage: latex_variable_name('sigma389')
'\sigma_{389}'
sage: latex_variable_name('beta_00')
'\beta_{00}'
sage: latex_variable_name('Omega84')
'\Omega_{84}'
sage: latex_variable_name('sigma_alpha')
'\sigma_{\alpha}'
sage: latex_variable_name('nothing1')
'\mathit{nothing}_{1}'
sage: latex_variable_name('nothing1', is_fname=True)
'\rm nothing_{1}'
sage: latex_variable_name('nothing_abc')
'\mathit{nothing}_{\mathit{abc}}'
sage: latex_variable_name('nothing_abc', is_fname=True)
'\rm nothing_{\rm abc}'
sage: latex_variable_name('alpha_beta_gamma12')
'\alpha_{\beta_{\gamma_{12}}}'
sage: latex_variable_name('x_ast')
'x_{\ast}'

```

TESTS:

```

sage: latex_variable_name('_C') # :trac: 16007
'C'
sage: latex_variable_name('_K1')
'K_{1}'

```

`sage.misc.latex.latex_varify(a, is_fname=False)`

Convert a string `a` to a LaTeX string: if it's an element of `common_varnames`, then prepend a backslash. If `a` consists of a single letter, then return it. Otherwise, return either `"{\rm a}"` or `"\mbox{a}"` if `"is_fname"` flag is True or False.

INPUT:

- `a` – string

OUTPUT:

A string

EXAMPLES:

```

sage: from sage.misc.latex import latex_varify
sage: latex_varify('w')
'w'
sage: latex_varify('aleph')
'\mathit{aleph}'
sage: latex_varify('aleph', is_fname=True)
'\rm aleph'
sage: latex_varify('alpha')
'\alpha'

```

```
'\\alpha'
sage: latex_varify('ast')
'\\ast'
```

TESTS:

```
sage: abc = var('abc') sage: latex((abc/(abc+1)+42)/(abc-1)) # trac #15870
frac{frac{mathit{abc}}{mathit{abc} + 1} + 42}{mathit{abc} - 1}
```

```
sage.misc.latex.list_function(x)
```

Returns the LaTeX code for a list x .

INPUT: x - a list

EXAMPLES:

```
sage: from sage.misc.latex import list_function
sage: list_function([1,2,3])
'\\left[1, 2, 3\\right]'
sage: latex([1,2,3]) # indirect doctest
\\left[1, 2, 3\\right]
sage: latex([Matrix(ZZ,3,range(9)), Matrix(ZZ,3,range(9))]) # indirect doctest
\\left[\\left(\\begin{array}{rrr}
0 & 1 & 2 \\\\
3 & 4 & 5 \\\\
6 & 7 & 8
\\end{array}\\right), \\left(\\begin{array}{rrr}
0 & 1 & 2 \\\\
3 & 4 & 5 \\\\
6 & 7 & 8
\\end{array}\\right)\\right]
```

```
sage.misc.latex.png(x, filename, density=150, debug=False, do_in_background=False, tiny=False,
                    pdflatex=True, engine='pdflatex')
```

Create a png image representation of x and save to the given filename.

INPUT:

- x – object to be displayed
- filename – file in which to save the image
- density – integer (default: 150)
- debug – bool (default: False): print verbose output
- do_in_background – bool (default: False): Unused, kept for backwards compatibility
- tiny – bool (default: False): use ‘tiny’ font
- pdflatex – bool (default: True): use pdflatex. This option is deprecated. Use engine option instead. See below.
- engine – (default: ‘pdflatex’) ‘latex’, ‘pdflatex’, or ‘xelatex’

EXAMPLES:

```
sage: from sage.misc.latex import png
sage: png(ZZ[x], os.path.join(SAGE_TMP, "zz.png")) # random - error if no latex
```

```
sage.misc.latex.pretty_print_default(enable=True)
```

Enable or disable default pretty printing. Pretty printing means rendering things so that MathJax or some other latex-aware front end can render real math.

This function is pretty useless without the notebook, it shouldn't be in the global namespace.

INPUT:

- `enable` – bool (optional, default `True`). If `True`, turn on pretty printing; if `False`, turn it off.

EXAMPLES:

```
sage: pretty_print_default(True)
sage: 'foo'
\newcommand{\Bold}[1]{\mathbf{#1}}\verb|foo|
sage: pretty_print_default(False)
sage: 'foo'
'foo'
```

`sage.misc.latex.repr_lincomb(symbols, coeffs)`

Compute a latex representation of a linear combination of some formal symbols.

INPUT:

- `symbols` – list of symbols
- `coeffs` – list of coefficients of the symbols

OUTPUT:

A string

EXAMPLES:

```
sage: t = PolynomialRing(QQ, 't').0
sage: from sage.misc.latex import repr_lincomb
sage: repr_lincomb(['a', 's', ''], [-t, t - 2, t^12 + 2])
'-t\\text{\\texttt{a}} + \\left(t - 2\\right)\\text{\\texttt{s}} + \\left(t^{12} + 2\\right)'
sage: repr_lincomb(['a', 'b'], [1,1])
'\\text{\\texttt{a}} + \\text{\\texttt{b}}'
```

Verify that a certain corner case works (see [trac ticket #5707](#) and [trac ticket #5766](#)):

```
sage: repr_lincomb([1,5,-3],[2,8/9,7])
'2\\cdot 1 + \\frac{8}{9}\\cdot 5 + 7\\cdot -3'
```

`sage.misc.latex.str_function(x)`

Return a LaTeX representation of the string `x`.

The main purpose of this function is to generate LaTeX representation for classes that do not provide a customized method.

If `x` contains only digits with, possibly, a single decimal point and/or a sign in front, it is considered to be its own representation. Otherwise each line of `x` is wrapped in a `\texttt` command and these lines are assembled in a left-justified array. This gives to complicated strings the closest look to their “terminal representation”.

Warning: Such wrappers **cannot** be used as arguments of LaTeX commands or in command definitions. If this causes you any problems, they probably can be solved by implementing a suitable `_latex_` method for an appropriate class.

INPUT:

- `x` – a string.

OUTPUT:

A string

EXAMPLES:

```
sage: from sage.misc.latex import str_function
sage: str_function('34')
'34'
sage: str_function('34.5')
'34.5'
sage: str_function('-34.5')
'-34.5'
sage: str_function('+34.5')
'+34.5'
sage: str_function('hello_world')
'\\text{\\texttt{hello{\\char\\_}world}}'
sage: str_function('-1.00000?') # trac 12178
'-1.00000?'
```

`sage.misc.latex.tuple_function(x, combine_all=False)`

Returns the LaTeX code for a tuple `x`.

INPUT:

- `x` – a tuple
- `combine_all` – boolean (Default: `False`) If `combine_all` is `True`, then it does not return a tuple and instead returns a string with all the elements separated by a single space. It does not collapse tuples which are inside tuples.

EXAMPLES:

```
sage: from sage.misc.latex import tuple_function
sage: tuple_function((1,2,3))
'\\left(1, 2, 3\\right)'
```

Check that [trac ticket #11775](#) is fixed:

```
sage: tuple_function((1,2,3), combine_all=True)
'1 2 3'
sage: tuple_function(((1,2),3), combine_all=True)
'\\left(1, 2\\right) 3'
```

`sage.misc.latex.view(objects, title='Sage', debug=False, sep='', tiny=False, pdflatex=None, engine=None, viewer=None, tightpage=None, mode='inline', combine_all=False, **kws)`

Compute a latex representation of each object in `objects`, compile, and display typeset. If used from the command line, this requires that latex be installed.

INPUT:

- `objects` – list (or object)
- `title` – string (default: `'Sage'`): title for the document
- `debug` – bool (default: `False`): print verbose output
- `sep` – string (default: `''`): separator between math objects
- `tiny` – bool (default: `False`): use tiny font.
- `pdflatex` – bool (default: `False`): use pdflatex. This is deprecated. Use `'engine'` option instead.
- `engine` – string or `None` (default: `None`). Can take the following values:
 - `None` – the value defined in the LaTeX global preferences `latex.engine()` is used.

- 'pdflatex' – compilation does `tex -> pdf`
- 'xelatex' – compilation does `tex -> pdf`
- 'latex' – compilation first tries `tex -> dvi -> png` and if an error occurs then tries `dvi -> ps -> pdf`. This is slower than 'pdflatex' and known to be broken when overfull hbox are detected.
- `viewer` – string or `None` (default: `None`): specify a viewer to use; currently the only options are `None` and 'pdf'.
- `tightpage` – bool (default: `False`): use the LaTeX package 'preview' with the 'tightpage' option.
- `mode` – string (default: 'inline'): 'display' for displaymath or 'inline' for inline math
- `combine_all` – bool (default: `False`): If `combine_all` is `True` and the input is a tuple, then it does not return a tuple and instead returns a string with all the elements separated by a single space.

OUTPUT:

Display typeset objects.

This function behaves differently depending on whether in notebook mode or not.

If not in notebook mode, the output is displayed in a separate viewer displaying a dvi (or pdf) file, with the following: the title string is printed, centered, at the top. Beneath that, each object in `objects` is typeset on its own line, with the string `sep` inserted between these lines.

The value of `sep` is inserted between each element of the list `objects`; you can, for example, add vertical space between objects with `sep='\\vspace{15mm}'`, while `sep='\\hrule'` adds a horizontal line between objects, and `sep='\\newpage'` inserts a page break between objects.

If `pdflatex` is `True`, then the latex engine is set to `pdflatex`.

If the engine is either `pdflatex` or `xelatex`, it produces a pdf file. Otherwise, it produces a dvi file, and if the program `dvipng` is installed, it checks the dvi file by trying to convert it to a png file. If this conversion fails, the dvi file probably contains some postscript special commands or it has other issues which might make displaying it a problem; in this case, the file is converted to a pdf file, which is then displayed.

Setting `viewer` to 'pdf' forces the use of a separate viewer, even in notebook mode. This also sets the latex engine to be `pdflatex` if the current engine is `latex`.

Setting the option `tightpage` to `True` tells LaTeX to use the package 'preview' with the 'tightpage' option. Then, each object is typeset in its own page, and that page is cropped to exactly the size of the object. This is typically useful for very large pictures (like graphs) generated with `tikz`. This only works when using a separate viewer. Note that the object are currently typeset in plain math mode rather than displaymath, because the latter imposes a limit on the width of the picture. Technically, `tightpage` adds

```
\\usepackage[tightpage,active]{preview}
\\PreviewEnvironment{page}
```

to the LaTeX preamble, and replaces the `\\[` and `\\]` around each object by `\\begin{page}$` and `$\\end{page}`.

If in notebook mode with `viewer` equal to `None`, this usually uses MathJax – see the next paragraph for the exception – to display the output in the notebook. Only the first argument, `objects`, is relevant; the others are ignored. If `objects` is a list, each object is printed on its own line.

In the notebook, this *does not* use MathJax if the LaTeX code for `objects` contains a string in `latex.mathjax_avoid_list()`. In this case, it creates and displays a png file.

EXAMPLES:

```
sage: sage.misc.latex.EMBEDDED_MODE = True
sage: view(3)
```

```
<html><script type="math/tex">\newcommand{\Bold}[1]{\mathbf{#1}}3</script></html>
sage: view(3, mode='display')
<html><script type="math/tex; mode=display">\newcommand{\Bold}[1]{\mathbf{#1}}3</script></html>
sage: view((x,2), combine_all=True) # trac 11775
<html><script type="math/tex">\newcommand{\Bold}[1]{\mathbf{#1}}x 2</script></html>
sage: sage.misc.latex.EMBEDDED_MODE = False
```

TESTS:

```
sage: from sage.misc.latex import _run_latex_, _latex_file_
sage: g = sage.misc.latex.latex_examples.graph()
sage: latex.add_to_preamble(r"\usepackage{tkz-graph}")
sage: file = os.path.join(SAGE_TMP, "temp.tex")
sage: O = open(file, 'w'); O.write(_latex_file_(g)); O.close()
sage: _run_latex_(file, engine="pdflatex") # optional - latex
'pdf'

sage: latex.extra_preamble('') # reset the preamble

sage: view(4, engine="garbage")
Traceback (most recent call last):
...
ValueError: Unsupported LaTeX engine.
sage: sage.misc.latex.EMBEDDED_MODE = True
sage: view(4, engine="garbage", viewer="pdf")
Traceback (most recent call last):
...
ValueError: Unsupported LaTeX engine.
```

4.3.3 LaTeX macros

AUTHORS:

- John H. Palmieri (2009-03)

The code here sets up LaTeX macro definitions for use in the documentation. To add a macro, modify the list `macros`, near the end of this file, and then run `'sage -b'`. The entries in this list are used to produce `sage_latex_macros`, a list of strings of the form `'\newcommand{...}'`, and `sage_mathjax_macros`, a list of strings suitable for parsing by MathJax. The LaTeX macros are produced using the `_latex_` method for each Sage object listed in `macros`, and the MathJax macros are produced from the LaTeX macros. The list of LaTeX macros is used in the file `SAGE_DOC/common/conf.py` to add to the preambles of both the LaTeX file used to build the PDF version of the documentation and the LaTeX file used to build the HTML version. The list of MathJax macros is used in the file `sagenb/notebook/tutorial.py` to define MathJax macros for use in the live documentation (and also in the notebook).

Any macro defined here may be used in docstrings or in the tutorial (or other pieces of documentation). In a docstring, for example, `"ZZ"` in backquotes (demarking math mode) will appear as `"ZZ"` in interactive help, but will be typeset as `"\Bold{Z}"` in the reference manual.

More details on the list `macros`: the entries are lists or tuples of the form `[name]` or `[name, arguments]`, where `name` is a string and `arguments` consists of valid arguments for the Sage object named `name`. For example, `["ZZ"]` and `["GF", 2]` produce the LaTeX macros `'\newcommand{\ZZ}{\Bold{Z}}'` and `'\newcommand{\GF}[1]{\Bold{F}_{#1}}'`, respectively. (For the second of these, `latex(GF(2))` is called and the string `'2'` gets replaced by `'#1'`, so `["GF", 17]` would have worked just as well. `["GF", p]` would have raised an error, though, because `p` is not defined, and `["GF", 4]` would have raised an error, because to define the field with four elements in Sage, you also need to specify the name of a generator.)

To see evidence of the results of the code here, run `sage -docbuild tutorial latex` (for example), and look at the resulting LaTeX file in `SAGE_DOC/output/latex/en/tutorial/`. The preamble should contain ‘newcommand’ lines for each of the entries in macros.

`sage.misc.latex_macros.convert_latex_macro_to_mathjax(macro)`

This converts a LaTeX macro definition (newcommand...) to a MathJax macro definition (MathJax.Macro...).

INPUT:

- macro* - LaTeX macro definition

See the web page <http://www.mathjax.org/docs/1.1/options/TeX.html> for a description of the format for MathJax macros.

EXAMPLES:

```
sage: from sage.misc.latex_macros import convert_latex_macro_to_mathjax
sage: convert_latex_macro_to_mathjax('\newcommand{\ZZ}{\Bold{Z}}')
'ZZ: "\\\Bold{Z}"'
sage: convert_latex_macro_to_mathjax('\newcommand{\GF}[1]{\Bold{F}_{#1}}')
'GF: ["\\Bold{F}_{#1}", 1]'
```

`sage.misc.latex_macros.produce_latex_macro(name, *sample_args)`

Produce a string defining a LaTeX macro.

INPUT:

- name* - name of macro to be defined, also name of corresponding Sage object
- sample_args* - (optional) sample arguments for this Sage object

EXAMPLES:

```
sage: from sage.misc.latex_macros import produce_latex_macro
sage: produce_latex_macro('ZZ')
'\newcommand{\ZZ}{\Bold{Z}}'
```

If the Sage object takes arguments, then the LaTeX macro will accept arguments as well. You must pass valid arguments, which will then be converted to #1, #2, etc. in the macro definition. The following allows the use of “GF{pⁿ}”, for example:

```
sage: produce_latex_macro('GF', 37)
'\newcommand{\GF}[1]{\Bold{F}_{#1}}'
```

If the Sage object is not in the global name space, describe it like so:

```
sage: produce_latex_macro('sage.rings.finite_rings.constructor.FiniteField', 3)
'\newcommand{\FiniteField}[1]{\Bold{F}_{#1}}'
```

`sage.misc.latex_macros.sage_latex_macros()`

Return list of LaTeX macros for Sage. This just runs the function `produce_latex_macro()` on the list macros defined in this file, and appends `sage_configurable_latex_macros`. To add a new macro for permanent use in Sage, modify macros.

EXAMPLES:

```
sage: from sage.misc.latex_macros import sage_latex_macros
sage: sage_latex_macros()
['\newcommand{\ZZ}{\Bold{Z}}', '\newcommand{\NN}{\Bold{N}}', ...]
```

`sage.misc.latex_macros.sage_mathjax_macros()`

Return list of MathJax macro definitions for Sage as JavaScript. This feeds each item output by `sage_latex_macros()` to `convert_latex_macro_to_mathjax()`.

```
sage: from sage.misc.latex_macros import sage_mathjax_macros
sage: sage_mathjax_macros()
['ZZ: "\\|\\|\\|\\|Bold{Z}"', 'NN: "\\|\\|\\|\\|Bold{N}"', ...]
```

SAVING AND LOADING SAGE OBJECTS

5.1 Support for persistent functions in .sage files

Persistent functions are functions whose values are stored on disk so they do not have to be recomputed.

The inputs to the function must be hashable (so lists are not allowed). Though a hash is used, in the incredibly unlikely event that a hash collision occurs, your function will not return an incorrect result because of this (though the cache might not be used either).

This is meant to be used from .sage files, not from library .py files.

To use this disk caching mechanism, just put `@func_persist` right before your function definition. For example,

```
@func_persist
def bern(n):
    "Return the n-th Bernoulli number, caching the result to disk."
    return bernoulli(n)
```

You can then use the function `bern` as usual, except it will almost instantly return values that have already been computed, even if you quit and restart.

The disk cache files are stored by default in the subdirectory `func_persist` of the current working directory, with one file for each evaluation of the function.

```
class sage.misc.func_persist.func_persist(f, dir='func_persist')
    Put @func_persist right before your function definition to cache values it computes to disk.
```

5.2 Object persistence

You can load and save most Sage object to disk using the `load` and `save` member functions and commands.

Note: It is impossible to save certain Sage objects to disk. For example, if x is a MAGMA object, i.e., a wrapper around an object that is defined in MAGMA, there is no way to save x to disk, since MAGMA doesn't support saving of individual objects to disk.

- **Versions:** Loading and saving of objects is guaranteed to work even if the version of Python changes. Saved objects can be loaded in future versions of Python. However, if the data structure that defines the object, e.g., in Sage code, changes drastically (or changes name or disappears), then the object might not load correctly or work correctly.
- Objects are zlib compressed for space efficiency.

```
sage.misc.persist.db(name)
```

Load object with given name from the Sage database. Use `x.db(name)` or `db_save(x, name)` to save objects to the database.

The database directory is `$HOME/.sage/db`.

```
sage.misc.persist.db_save(x, name=None)
```

Save `x` to the Sage database.

The database directory is `$HOME/.sage/db`.

```
sage.misc.persist.load_sage_element(cls, parent, dic_pic)
```

```
sage.misc.persist.load_sage_object(cls, dic)
```

5.3 Function pickling

REFERENCE: The python cookbook.

```
sage.misc.fpickle.call_pickled_function(fpargs)
```

```
sage.misc.fpickle.code_ctor(*args)
```

EXAMPLES:

This indirectly tests this function.

```
sage: def foo(a,b,c=10): return a+b+c
sage: sage.misc.fpickle.reduce_code(foo.__code__)
(<built-in function code_ctor>, ...)
sage: unpickle_function(pickle_function(foo))
<function foo at ...>
```

```
sage.misc.fpickle.pickleMethod(method)
```

support function for `copy_reg` to pickle method refs

```
sage.misc.fpickle.pickleModule(module)
```

support function for `copy_reg` to pickle module refs

```
sage.misc.fpickle.pickle_function(func)
```

Pickle the Python function `func`. This is not a normal pickle; you must use the `unpickle_function` method to unpickle the pickled function.

NOTE: This does not work on all functions, but does work on ‘surprisingly’ many functions. In particular, it does not work on functions that includes nested functions.

INPUT:

`func` – a Python function

OUTPUT:

a string

EXAMPLES:

```
sage: def f(N): return N+1
...
sage: g = pickle_function(f)
sage: h = unpickle_function(g)
sage: h(10)
11
```



```
sage.misc.fpickle.reduce_code(co)
```

EXAMPLES:

```
sage: def foo(N): return N+1
sage: sage.misc.fpickle.reduce_code(foo.__code__)
(<built-in function code_ctor>, ...)
```

```
sage.misc.fpickle.unpickleMethod(im_name, __self__, im_class)
support function for copy_reg to unpickle method refs
```

```
sage.misc.fpickle.unpickleModule(name)
support function for copy_reg to unpickle module refs
```

```
sage.misc.fpickle.unpickle_function(pickled)
Unpickle a pickled function.
```

EXAMPLES:

```
sage: def f(N,M): return N*M ... sage: unpickle_function(pickle_function(f))(3,5) 15
```

5.4 A tool for inspecting Python pickles

AUTHORS:

- Carl Witty (2009-03)

The `explain_pickle` function takes a pickle and produces Sage code that will evaluate to the contents of the pickle. Ideally, the combination of `explain_pickle` to produce Sage code and `sage_eval` to evaluate the code would be a 100% compatible implementation of `cPickle`'s unpickler; this is almost the case now.

EXAMPLES:

```
sage: explain_pickle(dumps(12345))
pg_make_integer = unpickle_global('sage.rings.integer', 'make_integer')
pg_make_integer('clp')
sage: explain_pickle(dumps(polygen(QQ)))
pg_Polynomial_rational_flint = unpickle_global('sage.rings.polynomial.polynomial_rational_flint', 'P
pg_PolynomialRing = unpickle_global('sage.rings.polynomial.polynomial_ring_constructor', 'PolynomialR
pg_RationalField = unpickle_global('sage.rings.rational_field', 'RationalField')
pg = unpickle_instantiate(pg_RationalField, ())
pg_make_rational = unpickle_global('sage.rings.rational', 'make_rational')
pg_Polynomial_rational_flint(pg_PolynomialRing(pg, 'x', None, False), [pg_make_rational('0'), pg_make
sage: sage_eval(explain_pickle(dumps(polygen(QQ)))) == polygen(QQ)
True
```

By default (as above) the code produced contains calls to several utility functions (`unpickle_global`, etc.); this is done so that the code is truly equivalent to the pickle. If the pickle can be loaded into a future version of Sage, then the code that `explain_pickle` produces today should work in that future Sage as well.

It is also possible to produce simpler code, that is tied to the current version of Sage; here are the above two examples again:

```
sage: explain_pickle(dumps(12345), in_current_sage=True)
from sage.rings.integer import make_integer
make_integer('clp')
sage: explain_pickle(dumps(polygen(QQ)), in_current_sage=True)
from sage.rings.polynomial.polynomial_rational_flint import Polynomial_rational_flint
from sage.rings.rational import make_rational
Polynomial_rational_flint(PolynomialRing(RationalField(), 'x', None, False), [make_rational('0'), ma
```

The `explain_pickle` function has several use cases.

- Write pickling support for your classes

You can use `explain_pickle` to see what will happen when a pickle is unpickled. Consider: is this sequence of commands something that can be easily supported in all future Sage versions, or does it expose internal design decisions that are subject to change?

- Debug old pickles

If you have a pickle from an old version of Sage that no longer unpickles, you can use `explain_pickle` to see what it is trying to do, to figure out how to fix it.

- Use `explain_pickle` in doctests to help maintenance

If you have a `loads(dumps(S))` doctest, you could also add an `explain_pickle(dumps(S))` doctest. Then if something changes in a way that would invalidate old pickles, the output of `explain_pickle` will also change. At that point, you can add the previous output of `explain_pickle` as a new set of doctests (and then update the `:obj'explain_pickle'` doctest to use the new output), to ensure that old pickles will continue to work. (These problems will also be caught using the `picklejar`, but having the tests directly in the relevant module is clearer.)

As mentioned above, there are several output modes for `explain_pickle`, that control fidelity versus simplicity of the output. For example, the GLOBAL instruction takes a module name and a class name and produces the corresponding class. So GLOBAL of `sage.rings.integer`, `Integer` is approximately equivalent to `sage.rings.integer.Integer`.

However, this class lookup process can be customized (using `sage.structure.sage_object.register_unpickle_override`). For instance, if some future version of Sage renamed `sage/rings/integer.pyx` to `sage/rings/knuth_was_here.pyx`, old pickles would no longer work unless `register_unpickle_override` was used; in that case, GLOBAL of `'sage.rings.integer'`, `'integer'` would mean `sage.rings.knuth_was_here.integer`.

By default, `explain_pickle` will map this GLOBAL instruction to `unpickle_global('sage.rings.integer', 'integer')`. Then when this code is evaluated, `unpickle_global` will look up the current mapping in the `register_unpickle_override` table, so the generated code will continue to work even in hypothetical future versions of Sage where `integer.pyx` has been renamed.

If you pass the flag `in_current_sage=True`, then `explain_pickle` will generate code that may only work in the current version of Sage, not in future versions. In this case, it would generate:

```
from sage.rings.integer import integer
```

and if you ran `explain_pickle` in hypothetical future sage, it would generate:

```
from sage.rings.knuth_was_here import integer
```

but the current code wouldn't work in the future sage.

If you pass the flag `default_assumptions=True`, then `explain_pickle` will generate code that would work in the absence of any special unpickling information. That is, in either current Sage or hypothetical future Sage, it would generate:

```
from sage.rings.integer import integer
```

The intention is that `default_assumptions` output is prettier (more human-readable), but may not actually work; so it is only intended for human reading.

There are several functions used in the output of `explain_pickle`. Here I give a brief description of what they usually do, as well as how to modify their operation (for instance, if you're trying to get old pickles to work).

- `unpickle_global(module, classname)`: `unpickle_global('sage.foo.bar', 'baz')` is usually equivalent to `sage.foo.bar.baz`, but this can be customized with `register_unpickle_override`.
- `unpickle_newobj(klass, args)`: Usually equivalent to `klass.__new__(klass, *args)`. If `klass` is a Python class, then you can define `__new__()` to control the result (this result actually need not be an instance of `klass`). (This doesn't work for Cython classes.)
- `unpickle_build(obj, state)`: If `obj` has a `__setstate__()` method, then this is equivalent to `obj.__setstate__(state)`. Otherwise uses `state` to set the attributes of `obj`. Customize by defining `__setstate__()`.
- `unpickle_instantiate(klass, args)`: Usually equivalent to `klass(*args)`. Cannot be customized.
- `unpickle_appends(lst, vals)`: Appends the values in `vals` to `lst`. If not `isinstance(lst, list)`, can be customized by defining a `append()` method.

class `sage.misc.explain_pickle.EmptyNewstyleClass`

Bases: `object`

A featureless new-style class (inherits from `object`); used for testing `explain_pickle`.

class `sage.misc.explain_pickle.EmptyOldstyleClass`

A featureless old-style class (does not inherit from `object`); used for testing `explain_pickle`.

class `sage.misc.explain_pickle.PickleDict` (*items*)

Bases: `object`

An object which can be used as the value of a `PickleObject`. The `items` is a list of key-value pairs, where the keys and values are `SageInputExpressions`. We use this to help construct dictionary literals, instead of always starting with an empty dictionary and assigning to it.

class `sage.misc.explain_pickle.PickleExplainer` (*sib*, *in_current_sage=False*, *default_assumptions=False*, *pedantic=False*)

Bases: `object`

An interpreter for the pickle virtual machine, that executes symbolically and constructs `SageInputExpressions` instead of directly constructing values.

APPEND ()

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(['a'])
0: \x80 PROTO      2
2: ]      EMPTY_LIST
3: q      BININPUT  1
5: U      SHORT_BINSTRING 'a'
8: a      APPEND
9: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
['a']
result: ['a']
```

As shown above, we prefer to create a list literal. This is not possible if the list is recursive:

```
sage: v = []
sage: v.append(v)
sage: test_pickle(v)
0: \x80 PROTO      2
2: ]      EMPTY_LIST
3: q      BININPUT  1
```

```
5: h      BINGET      1
7: a      APPEND
8: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si = []
list.append(si, si)
si
result: [[...]]
```

APPENDS()

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(['a', 'b'])
0: \x80 PROTO      2
2: ]      EMPTY_LIST
3: q      BINPUT    1
5: (      MARK
6: U      SHORT_BINSTRING 'a'
9: U      SHORT_BINSTRING 'b'
12: e      APPENDS    (MARK at 5)
13: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
['a', 'b']
result: ['a', 'b']
```

As shown above, we prefer to create a list literal. This is not possible if the list is recursive:

```
sage: v = []
sage: v.append(v)
sage: v.append(v)
sage: test_pickle(v)
0: \x80 PROTO      2
2: ]      EMPTY_LIST
3: q      BINPUT    1
5: (      MARK
6: h      BINGET      1
8: h      BINGET      1
10: e      APPENDS    (MARK at 5)
11: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si = []
list.extend(si, [si, si])
si
result: [[...], [...]]
```

BINFLOAT(*f*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(float(pi))
0: \x80 PROTO      2
2: G      BINFLOAT    3.141592653589793
11: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
```

```
float(RR(3.1415926535897931))
result: 3.141592653589793
```

BINGET (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + BINPUT + 'x' + POP + BINGET + 'x' + '.')
0: ]      EMPTY_LIST
1: q      BINPUT      120
3: 0      POP
4: h      BINGET      120
6: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []
```

BININT (*n*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(dumps(100000r, compress=False))
0: \x80 PROTO      2
2: J      BININT      100000
7: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
100000
result: 100000
```

BININT1 (*n*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(dumps(100r, compress=False))
0: \x80 PROTO      2
2: K      BININT1      100
4: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
100
result: 100
```

BININT2 (*n*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(dumps(1000r, compress=False))
0: \x80 PROTO      2
2: M      BININT2      1000
5: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
1000
result: 1000
```

BINPERSID ()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(INT + "\n" + BINPERSID + '.', args=('Yo!'))
0: I      INT      0
3: Q      BINPERSID
4: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
unpickle_persistent(0)
result: 'Yo!'
```

BINPUT (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + BINPUT + 'x' + POP + BINGET + 'x')
0: ]      EMPTY_LIST
1: q      BINPUT    120
3: 0      POP
4: h      BINGET    120
6: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []
```

BINSTRING (*s*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle('T\5\0\0\0hello.')
0: T      BINSTRING 'hello'
10: .     STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
'hello'
result: 'hello'
```

BINUNICODE (*s*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(u'hi\u1234\u00012345')
0: \x80   PROTO      2
2: X      BINUNICODE u'hi\u1234\u00012345'
16: q     BINPUT      1
18: .     STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
u'hi\u1234\u00012345'
result: u'hi\u1234\u00012345'
```

BUILD ()

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(TestBuild())
0: \x80   PROTO      2
```

```

2: c    GLOBAL      'sage.misc.explain_pickle TestBuild'
38: q    BINPUT      1
40: )    EMPTY_TUPLE
41: \x81 NEWOBJ
42: q    BINPUT      2
44: }    EMPTY_DICT
45: q    BINPUT      3
47: U    SHORT_BINSTRING 'x'
50: K    BININT1      3
52: s    SETITEM
53: }    EMPTY_DICT
54: q    BINPUT      4
56: U    SHORT_BINSTRING 'y'
59: K    BININT1      4
61: s    SETITEM
62: \x86 TUPLE2
63: b    BUILD
64: .    STOP

highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from sage.misc.explain_pickle import TestBuild
si = unpickle_newobj(TestBuild, ())
si.__dict__['x'] = 3
si.y = 4
si
explain_pickle in_current_sage=False:
pg_TestBuild = unpickle_global('sage.misc.explain_pickle', 'TestBuild')
si = unpickle_newobj(pg_TestBuild, ())
unpickle_build(si, ({'x':3}, {'y':4}))
si
result: TestBuild: x=3; y=4

sage: test_pickle(TestBuildSetstate(), verbose_eval=True)
0: \x80 PROTO      2
2: c    GLOBAL      'sage.misc.explain_pickle TestBuildSetstate'
46: q    BINPUT      1
48: )    EMPTY_TUPLE
49: \x81 NEWOBJ
50: q    BINPUT      2
52: }    EMPTY_DICT
53: q    BINPUT      3
55: U    SHORT_BINSTRING 'x'
58: K    BININT1      3
60: s    SETITEM
61: }    EMPTY_DICT
62: q    BINPUT      4
64: U    SHORT_BINSTRING 'y'
67: K    BININT1      4
69: s    SETITEM
70: \x86 TUPLE2
71: b    BUILD
72: .    STOP

highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from sage.misc.explain_pickle import TestBuildSetstate
si = unpickle_newobj(TestBuildSetstate, ())
si.__setstate__(({ 'x':3}, { 'y':4}))
si

```

```
explain_pickle in_current_sage=False:
pg_TestBuildSetstate = unpickle_global('sage.misc.explain_pickle', 'TestBuildSetstate')
si = unpickle_newobj(pg_TestBuildSetstate, ())
unpickle_build(si, ({'x':3}, {'y':4}))
si
evaluating explain_pickle in_current_sage=True:
setting state from ({'x': 3}, {'y': 4})
evaluating explain_pickle in_current_sage=False:
setting state from ({'x': 3}, {'y': 4})
loading pickle with cPickle:
setting state from ({'x': 3}, {'y': 4})
result: TestBuild: x=4; y=3
```

DICT()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(DICT, args=('mark', 'a', 1, 2, 'b'))
0: (      MARK
1: P      PERSID      '1'
4: P      PERSID      '2'
7: P      PERSID      '3'
10: P     PERSID      '4'
13: d      DICT      (MARK at 0)
14: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
{unpickle_persistent('1'):unpickle_persistent('2'), unpickle_persistent('3'):unpickle_persistent('4')}
result: {'a': 1, 2: 'b'}
```

DUP()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + DUP + TUPLE2 + STOP)
0: ]      EMPTY_LIST
1: 2      DUP
2: \x86   TUPLE2
3: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si = []
(si, si)
result: ([], [])
```

EMPTY_DICT()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_DICT)
0: }      EMPTY_DICT
1: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
{}
result: {}
```


EMPTY_LIST()

TESTS:

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST)
0: ]      EMPTY_LIST
1: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []

```

EMPTY_TUPLE()

TESTS:

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_TUPLE)
0: )      EMPTY_TUPLE
1: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
()
result: ()

```

EXT1 (n)

TESTS:

```

sage: from copy_reg import *
sage: from sage.misc.explain_pickle import *
sage: add_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 42)
sage: test_pickle(EmptyNewstyleClass())
0: \x80 PROTO      2
2: \x82 EXT1       42
4: )      EMPTY_TUPLE
5: \x81 NEWOBJ
6: q      BINPUT    1
8: }      EMPTY_DICT
9: q      BINPUT    2
11: b     BUILD
12: .     STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si = unpickle_newobj(unpickle_extension(42), ())
unpickle_build(si, {})
si
result: EmptyNewstyleClass
sage: remove_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 42)

```

EXT2 (n)

TESTS:

```

sage: from copy_reg import *
sage: from sage.misc.explain_pickle import *
sage: add_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 31415)
sage: test_pickle(EmptyNewstyleClass())
0: \x80 PROTO      2

```

```
2: \x83 EXT2      31415
5: )      EMPTY_TUPLE
6: \x81 NEWOBJ
7: q      BINPUT   1
9: }      EMPTY_DICT
10: q      BINPUT   2
12: b      BUILD
13: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si = unpickle_newobj(unpickle_extension(31415), ())
unpickle_build(si, {})
si
result: EmptyNewstyleClass
sage: remove_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 31415)
```

EXT4 (*n*)

TESTS:

```
sage: from copy_reg import *
sage: from sage.misc.explain_pickle import *
sage: add_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 27182818)
sage: test_pickle(EmptyNewstyleClass())
0: \x80 PROTO      2
2: \x84 EXT4      27182818
7: )      EMPTY_TUPLE
8: \x81 NEWOBJ
9: q      BINPUT   1
11: }     EMPTY_DICT
12: q      BINPUT   2
14: b      BUILD
15: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si = unpickle_newobj(unpickle_extension(27182818), ())
unpickle_build(si, {})
si
result: EmptyNewstyleClass
sage: remove_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 27182818)
```

FLOAT (*f*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(FLOAT + '2.71828\n')
0: F      FLOAT      2.71828
9: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
2.71828
result: 2.71828
```

GET (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + PUT + '1\n' + POP + GET + '1\n' + '.')
```

```

0: ]      EMPTY_LIST
1: p      PUT          1
4: 0      POP
5: g      GET          1
8: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []

```

GLOBAL (*name*)

TESTS:

```
sage: from sage.misc.explain_pickle import *
```

We've used `register_unpickle_override` so that `unpickle_global` will map `TestGlobalOldName` to `TestGlobalNewName`.

```

sage: test_pickle(TestGlobalOldName())
0: \x80 PROTO          2
2: c      GLOBAL        'sage.misc.explain_pickle TestGlobalOldName'
46: q      BINPUT        1
48: )      EMPTY_TUPLE
49: \x81 NEWOBJ
50: q      BINPUT        2
52: }      EMPTY_DICT
53: q      BINPUT        3
55: b      BUILD
56: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from sage.misc.explain_pickle import TestGlobalNewName
unpickle_newobj(TestGlobalNewName, ())
explain_pickle in_current_sage=False:
pg_TestGlobalOldName = unpickle_global('sage.misc.explain_pickle', 'TestGlobalOldName')
si = unpickle_newobj(pg_TestGlobalOldName, ())
unpickle_build(si, {})
si
result: TestGlobalNewName

```

Note that `default_assumptions` blithely assumes that it should use the old name, giving code that doesn't actually work as desired:

```

sage: explain_pickle(dumps(TestGlobalOldName()), default_assumptions=True)
from sage.misc.explain_pickle import TestGlobalOldName
unpickle_newobj(TestGlobalOldName, ())

```

A class name need not be a valid identifier:

```

sage: sage.misc.explain_pickle.__dict__['funny$name'] = TestGlobalFunnyName # see comment at
sage: test_pickle((TestGlobalFunnyName(), TestGlobalFunnyName()))
0: \x80 PROTO          2
2: c      GLOBAL        'sage.misc.explain_pickle funny$name'
39: q      BINPUT        1
41: )      EMPTY_TUPLE
42: \x81 NEWOBJ
43: q      BINPUT        2
45: }      EMPTY_DICT
46: q      BINPUT        3

```

```
48: b      BUILD
49: h      BINGET      1
51: )      EMPTY_TUPLE
52: \x81   NEWOBJ
53: q      BINPUT      4
55: }      EMPTY_DICT
56: q      BINPUT      5
58: b      BUILD
59: \x86   TUPLE2
60: q      BINPUT      6
62: .      STOP

highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
si1 = unpickle_global('sage.misc.explain_pickle', 'funny$name')
si2 = unpickle_newobj(si1, ())
unpickle_build(si2, {})
si3 = unpickle_newobj(si1, ())
unpickle_build(si3, {})
(si2, si3)
result: (TestGlobalFunnyName, TestGlobalFunnyName)
```

INST (*name*)

TESTS:

```
sage: import pickle
sage: from sage.misc.explain_pickle import *
sage: test_pickle(pickle.dumps(EmptyOldstyleClass(), protocol=0))
0: (      MARK
1: i      INST      'sage.misc.explain_pickle EmptyOldstyleClass' (MARK at 0)
46: p     PUT      0
49: (      MARK
50: d      DICT      (MARK at 49)
51: p     PUT      1
54: b     BUILD
55: .     STOP

highest protocol among opcodes = 0
explain_pickle in_current_sage=True:
from types import InstanceType
from sage.misc.explain_pickle import EmptyOldstyleClass
InstanceType(EmptyOldstyleClass)
explain_pickle in_current_sage=False:
pg_EmptyOldstyleClass = unpickle_global('sage.misc.explain_pickle', 'EmptyOldstyleClass')
pg = unpickle_instantiate(pg_EmptyOldstyleClass, ())
unpickle_build(pg, {})
pg
result: EmptyOldstyleClass
```

INT (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(INT + "-12345\n")
0: I      INT      -12345
8: .      STOP

highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
-12345
result: -12345
```

INT can also be used to record True and False:

```
sage: test_pickle(INT + "00\n")
0: I      INT      False
4: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
False
result: False
sage: test_pickle(INT + "01\n")
0: I      INT      True
4: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
True
result: True
```

LIST()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(MARK + NONE + NEWFALSE + LIST)
0: (      MARK
1: N      NONE
2: \x89   NEWFALSE
3: l      LIST      (MARK at 0)
4: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
[None, False]
result: [None, False]
```

LONG(n)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(LONG + "12345678909876543210123456789L\n")
0: L      LONG      12345678909876543210123456789L
32: .     STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
12345678909876543210123456789
result: 12345678909876543210123456789L
```

LONG1(n)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(1L)
0: \x80  PROTO      2
2: \x8a  LONG1      1L
5: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
1L
result: 1L
```

LONG4 (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(LONG4 + '\014\0\0\0' + 'hello, world')
0: \x8b LONG4      31079605376604435891501163880L
17: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
31079605376604435891501163880
result: 31079605376604435891501163880L
```

LONG_BINGET (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + LONG_BINPUT + 'Sage' + POP + LONG_BINGET + 'Sage')
0: ]      EMPTY_LIST
1: r      LONG_BINPUT 1701273939
6: 0      POP
7: j      LONG_BINGET 1701273939
12: .     STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []
```

LONG_BINPUT (*n*)

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + LONG_BINPUT + 'Sage' + POP + LONG_BINGET + 'Sage')
0: ]      EMPTY_LIST
1: r      LONG_BINPUT 1701273939
6: 0      POP
7: j      LONG_BINGET 1701273939
12: .     STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []
```

MARK ()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(MARK + TUPLE)
0: (      MARK
1: t      TUPLE      (MARK at 0)
2: .     STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
()
result: ()
```

NEWFALSE ()

TESTS:

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(NEWFALSE)
0: \x89 NEWFALSE
1: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
False
result: False

```

NEWOBJ()**TESTS:**

```

sage: from sage.misc.explain_pickle import *
sage: test_pickle(EmptyNewstyleClass())
0: \x80 PROTO      2
2: c      GLOBAL    'sage.misc.explain_pickle EmptyNewstyleClass'
47: q     BINPUT    1
49: )     EMPTY_TUPLE
50: \x81 NEWOBJ
51: q     BINPUT    2
53: }     EMPTY_DICT
54: q     BINPUT    3
56: b     BUILD
57: .     STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from sage.misc.explain_pickle import EmptyNewstyleClass
unpickle_newobj(EmptyNewstyleClass, ())
explain_pickle in_current_sage=False:
pg_EmptyNewstyleClass = unpickle_global('sage.misc.explain_pickle', 'EmptyNewstyleClass')
si = unpickle_newobj(pg_EmptyNewstyleClass, ())
unpickle_build(si, {})
si
result: EmptyNewstyleClass

```

NEWTRUE()**TESTS:**

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(NEWTRUE)
0: \x88 NEWTRUE
1: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
True
result: True

```

NONE()**TESTS:**

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(NONE)
0: N      NONE
1: .      STOP
highest protocol among opcodes = 0

```

```
explain_pickle in_current_sage=True/False:
None
result: None
```

OBJ()**TESTS:**

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EmptyOldstyleClass())
0: \x80 PROTO      2
2: (      MARK
3: c      GLOBAL    'sage.misc.explain_pickle EmptyOldstyleClass'
48: q      BINPUT    1
50: o      OBJ      (MARK at 2)
51: q      BINPUT    2
53: }      EMPTY_DICT
54: q      BINPUT    3
56: b      BUILD
57: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from types import InstanceType
from sage.misc.explain_pickle import EmptyOldstyleClass
InstanceType(EmptyOldstyleClass)
explain_pickle in_current_sage=False:
pg_EmptyOldstyleClass = unpickle_global('sage.misc.explain_pickle', 'EmptyOldstyleClass')
pg = unpickle_instantiate(pg_EmptyOldstyleClass, ())
unpickle_build(pg, {})
pg
result: EmptyOldstyleClass
```

PERSID(*id*)**TESTS:**

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(PERSID + "0\n" + '.', args=('Yo!',))
0: P      PERSID    '0'
3: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
unpickle_persistent('0')
result: 'Yo!'
```

POP()**TESTS:**

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(INT + "0\n" + POP + INT + "42\n")
0: I      INT      0
3: 0      POP
4: I      INT      42
8: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
42
result: 42
```


POP_MARK()

TESTS:

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(MARK + NONE + NEWFALSE + POP_MARK + NEWTRUE)
0: (      MARK
1: N      NONE
2: \x89   NEWFALSE
3: 1      POP_MARK      (MARK at 0)
4: \x88   NEWTRUE
5: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
True
result: True

```

PROTO (proto)

TESTS:

```

sage: from sage.misc.explain_pickle import *
sage: test_pickle(0r)
0: \x80 PROTO      2
2: K      BININT1   0
4: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
0
result: 0

```

PUT (n)

TESTS:

```

sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_LIST + PUT + '1\n' + POP + GET + '1\n' + '.')
0: ]      EMPTY_LIST
1: p      PUT      1
4: 0      POP
5: g      GET      1
8: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
[]
result: []

```

REDUCE ()

TESTS:

```

sage: import pickle
sage: from sage.misc.explain_pickle import *
sage: test_pickle(pickle.dumps(EmptyNewstyleClass(), protocol=1))
0: c      GLOBAL      'copy_reg _reconstructor'
25: q     BINPUT      0
27: (     MARK
28: c      GLOBAL      'sage.misc.explain_pickle EmptyNewstyleClass'
73: q     BINPUT      1
75: c      GLOBAL      '__builtin__ object'
95: q     BINPUT      2
97: N     NONE

```

```

98: t      TUPLE      (MARK at 27)
99: q      BINPUT      3
101: R      REDUCE
102: q      BINPUT      4
104: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True:
from copy_reg import _reconstructor
from sage.misc.explain_pickle import EmptyNewstyleClass
from __builtin__ import object
_reconstructor(EmptyNewstyleClass, object, None)
explain_pickle in_current_sage=False:
pg__reconstructor = unpickle_global('copy_reg', '_reconstructor')
pg_EmptyNewstyleClass = unpickle_global('sage.misc.explain_pickle', 'EmptyNewstyleClass')
pg_object = unpickle_global('__builtin__', 'object')
pg__reconstructor(pg_EmptyNewstyleClass, pg_object, None)
result: EmptyNewstyleClass

sage: test_pickle(TestReduceGetinitargs(), verbose_eval=True)
Running __init__ for TestReduceGetinitargs
  0: \x80 PROTO      2
  2: (      MARK
  3: c      GLOBAL      'sage.misc.explain_pickle TestReduceGetinitargs'
51: q      BINPUT      1
53: o      OBJ      (MARK at 2)
54: q      BINPUT      2
56: }      EMPTY_DICT
57: q      BINPUT      3
59: b      BUILD
60: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from sage.misc.explain_pickle import TestReduceGetinitargs
TestReduceGetinitargs()
explain_pickle in_current_sage=False:
pg_TestReduceGetinitargs = unpickle_global('sage.misc.explain_pickle', 'TestReduceGetinitargs')
pg = unpickle_instantiate(pg_TestReduceGetinitargs, ())
unpickle_build(pg, {})
pg
evaluating explain_pickle in_current_sage=True:
Running __init__ for TestReduceGetinitargs
evaluating explain_pickle in_current_sage=False:
Running __init__ for TestReduceGetinitargs
loading pickle with cPickle:
Running __init__ for TestReduceGetinitargs
result: TestReduceGetinitargs

sage: test_pickle(TestReduceNoGetinitargs(), verbose_eval=True)
Running __init__ for TestReduceNoGetinitargs
  0: \x80 PROTO      2
  2: (      MARK
  3: c      GLOBAL      'sage.misc.explain_pickle TestReduceNoGetinitargs'
53: q      BINPUT      1
55: o      OBJ      (MARK at 2)
56: q      BINPUT      2
58: }      EMPTY_DICT
59: q      BINPUT      3
61: b      BUILD
```

```

62: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
from types import InstanceType
from sage.misc.explain_pickle import TestReduceNoGetinitargs
InstanceType(TestReduceNoGetinitargs)
explain_pickle in_current_sage=False:
pg_TestReduceNoGetinitargs = unpickle_global('sage.misc.explain_pickle', 'TestReduceNoGetini
pg = unpickle_instantiate(pg_TestReduceNoGetinitargs, ())
unpickle_build(pg, {})
pg
evaluating explain_pickle in_current_sage=True:
evaluating explain_pickle in_current_sage=False:
loading pickle with cPickle:
result: TestReduceNoGetinitargs

```

SETITEM()

```

TESTS:
sage: import pickle
sage: from sage.misc.explain_pickle import *
sage: test_pickle(pickle.dumps({'a': 'b'}))
0: (      MARK
1: d      DICT      (MARK at 0)
2: p      PUT      0
5: S      STRING   'a'
10: p     PUT      1
13: S     STRING   'b'
18: p     PUT      2
21: s     SETITEM
22: .     STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
{'a': 'b'}
result: {'a': 'b'}

```

We see above that we output the result as a dictionary literal, when possible. This is impossible when a key or value is recursive. First we test recursive values:

```

sage: value_rec = dict()
sage: value_rec['circular'] = value_rec
sage: test_pickle(pickle.dumps(value_rec))
0: (      MARK
1: d      DICT      (MARK at 0)
2: p      PUT      0
5: S      STRING   'circular'
17: p     PUT      1
20: g     GET      0
23: s     SETITEM
24: .     STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
si = {}
si['circular'] = si
si
result: {'circular': {...}}

```

Then we test recursive keys:

```
sage: key_rec = dict()
sage: key = EmptyNewstyleClass()
sage: key.circular = key_rec
sage: key_rec[key] = 'circular'
sage: test_pickle(pickle.dumps(key_rec))
0: (      MARK
1: d      DICT      (MARK at 0)
2: p      PUT      0
5: c      GLOBAL    'copy_reg _reconstructor'
30: p     PUT      1
33: (      MARK
34: c      GLOBAL    'sage.misc.explain_pickle EmptyNewstyleClass'
79: p      PUT      2
82: c      GLOBAL    '__builtin__ object'
102: p     PUT      3
105: N     NONE
106: t      TUPLE     (MARK at 33)
107: p     PUT      4
110: R     REDUCE
111: p     PUT      5
114: (      MARK
115: d      DICT      (MARK at 114)
116: p     PUT      6
119: S     STRING    'circular'
131: p     PUT      7
134: g     GET      0
137: s     SETITEM
138: b     BUILD
139: g     GET      7
142: s     SETITEM
143: .     STOP

highest protocol among opcodes = 0
explain_pickle in_current_sage=True:
sil = {}
from copy_reg import _reconstructor
from sage.misc.explain_pickle import EmptyNewstyleClass
from __builtin__ import object
si2 = _reconstructor(EmptyNewstyleClass, object, None)
si2.__dict__['circular'] = sil
sil[si2] = 'circular'
sil
explain_pickle in_current_sage=False:
sil = {}
pg__reconstructor = unpickle_global('copy_reg', '_reconstructor')
pg_EmptyNewstyleClass = unpickle_global('sage.misc.explain_pickle', 'EmptyNewstyleClass')
pg_object = unpickle_global('__builtin__', 'object')
si2 = pg__reconstructor(pg_EmptyNewstyleClass, pg_object, None)
unpickle_build(si2, {'circular':sil})
sil[si2] = 'circular'
sil
result: {EmptyNewstyleClass: 'circular'}
```

SETITEMS()

TESTS:

```
sage: import pickle
sage: from sage.misc.explain_pickle import *
sage: test_pickle(pickle.dumps({'a': 'b', 1r : 2r}, protocol=2))
```

```

0: \x80 PROTO      2
2: }      EMPTY_DICT
3: q      BINPUT    0
5: (      MARK
6: U      SHORT_BINSTRING 'a'
9: q      BINPUT    1
11: U     SHORT_BINSTRING 'b'
14: q      BINPUT    2
16: K      BININT1    1
18: K      BININT1    2
20: u      SETITEMS   (MARK at 5)
21: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
{'a':'b', 1:2}
result: {'a': 'b', 1: 2}

```

Similar to the tests for SETITEM, we test recursive keys and values:

```

sage: recdict = {}
sage: recdict['Circular value'] = recdict
sage: key = EmptyOldstyleClass()
sage: key.recdict = recdict
sage: recdict[key] = 'circular_key'
sage: test_pickle(pickle.dumps(recdict, protocol=2))
0: \x80 PROTO      2
2: }      EMPTY_DICT
3: q      BINPUT    0
5: (      MARK
6: (      MARK
7: c      GLOBAL      'sage.misc.explain_pickle EmptyOldstyleClass'
52: q      BINPUT      1
54: o      OBJ          (MARK at 6)
55: q      BINPUT      2
57: }      EMPTY_DICT
58: q      BINPUT      3
60: U      SHORT_BINSTRING 'recdict'
69: q      BINPUT      4
71: h      BINGET      0
73: s      SETITEM
74: b      BUILD
75: U      SHORT_BINSTRING 'circular_key'
89: q      BINPUT      5
91: U      SHORT_BINSTRING 'Circular value'
107: q     BINPUT      6
109: h     BINGET      0
111: u     SETITEMS   (MARK at 5)
112: .     STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True:
sil = {}
from types import InstanceType
from sage.misc.explain_pickle import EmptyOldstyleClass
si2 = InstanceType(EmptyOldstyleClass)
si2.__dict__['recdict'] = sil
sil[si2] = 'circular_key'
sil['Circular value'] = sil
sil
explain_pickle in_current_sage=False:

```

```
si = {}
pg_EmptyOldstyleClass = unpickle_global('sage.misc.explain_pickle', 'EmptyOldstyleClass')
pg = unpickle_instantiate(pg_EmptyOldstyleClass, ())
unpickle_build(pg, {'recdict':si})
si[pg] = 'circular_key'
si['Circular value'] = si
si
result: {EmptyOldstyleClass: 'circular_key', 'Circular value': {...}}
```

SHORT_BINSTRING(s)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(dumps('hello', compress=False))
0: \x80 PROTO      2
2: U      SHORT_BINSTRING 'hello'
9: q      BININPUT  1
11: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
'hello'
result: 'hello'
```

STOP()

TESTS:

```
sage: from pickle import *
sage: from sage.misc.explain_pickle import *
sage: test_pickle(EMPTY_TUPLE)
0: )      EMPTY_TUPLE
1: .      STOP
highest protocol among opcodes = 1
explain_pickle in_current_sage=True/False:
()
result: ()
```

STRING(s)

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle("S'Testing...'\n.")
0: S      STRING    'Testing...'
14: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
'Testing...'
result: 'Testing...'
```

TUPLE()

TESTS:

```
sage: import pickle
sage: from sage.misc.explain_pickle import *
sage: test_pickle(pickle.dumps(('a',)))
0: (      MARK
1: S      STRING    'a'
6: p      PUT      0
9: t      TUPLE    (MARK at 0)
10: p     PUT      1
```

```

13: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
('a',)
result: ('a',)

```

We prefer to produce tuple literals, as above; but if the tuple is recursive, we need a more complicated construction. It used to be the case that the cPickle unpickler couldn't handle this case, but that's no longer true (see <http://bugs.python.org/issue5794>):

```

sage: v = ([],)
sage: v[0].append(v)
sage: test_pickle(pickle.dumps(v))
0: (      MARK
1: (      MARK
2: l      LIST      (MARK at 1)
3: p      PUT      0
6: (      MARK
7: g      GET      0
10: t     TUPLE     (MARK at 6)
11: p      PUT      1
14: a      APPEND
15: 0      POP
16: 0      POP      (MARK at 0)
17: g      GET      1
20: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
si1 = []
si2 = (si1,)
list.append(si1, si2)
si2
result: ([(...)],)

```

TUPLE1()

TESTS:

```

sage: from sage.misc.explain_pickle import *
sage: test_pickle(('a',))
0: \x80 PROTO      2
2: U   SHORT_BINSTRING 'a'
5: \x85 TUPLE1
6: q   BININPUT     1
8: .   STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
('a',)
result: ('a',)

```

TUPLE2()

TESTS:

```

sage: from sage.misc.explain_pickle import *
sage: test_pickle(('a','b'))
0: \x80 PROTO      2
2: U   SHORT_BINSTRING 'a'
5: U   SHORT_BINSTRING 'b'
8: \x86 TUPLE2
9: q   BININPUT     1

```

```
11: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
('a', 'b')
result: ('a', 'b')
```

TUPLE3 ()

TESTS:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(('a','b','c'))
0: \x80 PROTO      2
2: U      SHORT_BINSTRING 'a'
5: U      SHORT_BINSTRING 'b'
8: U      SHORT_BINSTRING 'c'
11: \x87 TUPLE3
12: q      BININPUT    1
14: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
('a', 'b', 'c')
result: ('a', 'b', 'c')
```

UNICODE (s)

TESTS:

```
sage: import pickle
sage: from sage.misc.explain_pickle import *
sage: test_pickle(pickle.dumps(u'hi\u1234\u00012345'))
0: V      UNICODE      u'hi\u1234\u00012345'
20: p      PUT          0
23: .      STOP
highest protocol among opcodes = 0
explain_pickle in_current_sage=True/False:
u'hi\u1234\u00012345'
result: u'hi\u1234\u00012345'
```

check_value (v)

Check that the given value is either a SageInputExpression or a PickleObject. Used for internal sanity checking.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: pe.check_value(7)
Traceback (most recent call last):
...
AssertionError
sage: pe.check_value(sib(7))
```

is_mutable_pickle_object (v)

Test whether a PickleObject is mutable (has never been converted to a SageInputExpression).

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
```



```

sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: v = PickleObject(1, sib(1))
sage: pe.is_mutable_pickle_object(v)
True
sage: sib(v)
{atomic:1}
sage: pe.is_mutable_pickle_object(v)
False

```

pop()

Pop a value from the virtual machine's stack, and return it.

EXAMPLES:

```

sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: pe.push(sib(7))
sage: pe.pop()
{atomic:7}

```

pop_to_mark()

Pop all values down to the 'mark' from the virtual machine's stack, and return the values as a list.

EXAMPLES:

```

sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: pe.push_mark()
sage: pe.push(sib(7))
sage: pe.push(sib('hello'))
sage: pe.pop_to_mark()
[{atomic:7}, {atomic:'hello'}]

```

push(v)

Push a value onto the virtual machine's stack.

EXAMPLES:

```

sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: pe.push(sib(7))
sage: pe.stack[-1]
{atomic:7}

```

push_and_share(v)

Push a value onto the virtual machine's stack; also mark it as shared for sage_input if we are in pedantic mode.

EXAMPLES:

```

sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)

```

```
sage: pe.push_and_share(sib(7))
sage: pe.stack[-1]
{atomic:7}
sage: pe.stack[-1]._sie_share
True
```

push_mark()

Push a ‘mark’ onto the virtual machine’s stack.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: pe.push_mark()
sage: pe.stack[-1]
'mark'
sage: pe.stack[-1] is the_mark
True
```

run_pickle(p)

Given an (uncompressed) pickle as a string, run the pickle in this virtual machine. Once a STOP has been executed, return the result (a SageInputExpression representing code which, when evaluated, will give the value of the pickle).

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: sib(pe.run_pickle('T\5\0\0\0hello.'))
{atomic:'hello'}
```

share(v)

Mark a sage_input value as shared, if we are in pedantic mode.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: from sage.misc.sage_input import SageInputBuilder
sage: sib = SageInputBuilder()
sage: pe = PickleExplainer(sib, in_current_sage=True, default_assumptions=False, pedantic=True)
sage: v = sib(7)
sage: v._sie_share
False
sage: pe.share(v)
{atomic:7}
sage: v._sie_share
True
```

```
class sage.misc.explain_pickle.PickleInstance(klass)
```

Bases: object

An object which can be used as the value of a PickleObject. Unlike other possible values of a PickleObject, a PickleInstance doesn’t represent an exact value; instead, it gives the class (type) of the object.

```
class sage.misc.explain_pickle.PickleObject(value, expression)
```

Bases: object

Pickles have a stack-based virtual machine. The `explain_pickle` pickle interpreter mostly uses `SageInputExpressions`, from `sage_input`, as the stack values. However, sometimes we want some more information about the value on the stack, so that we can generate better (prettier, less confusing) code. In such cases, we push a `PickleObject` instead of a `SageInputExpression`. A `PickleObject` contains a value (which may be a standard Python value, or a `PickleDict` or `PickleInstance`), an expression (a `SageInputExpression`), and an “immutable” flag (which checks whether this object has been converted to a `SageInputExpression`; if it has, then we must not mutate the object, since the `SageInputExpression` would not reflect the changes).

```
class sage.misc.explain_pickle.TestAppendList
```

Bases: list

A subclass of list, with deliberately-broken append and extend methods. Used for testing `explain_pickle`.

```
append()
```

A deliberately broken append method.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: v = TestAppendList()
sage: v.append(7)
Traceback (most recent call last):
...
TypeError: append() takes exactly 1 argument (2 given)
```

We can still append by directly using the list method: `sage: list.append(v, 7)` `sage: v [7]`

```
extend()
```

A deliberately broken extend method.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: v = TestAppendList()
sage: v.extend([3,1,4,1,5,9])
Traceback (most recent call last):
...
TypeError: extend() takes exactly 1 argument (2 given)
```

We can still extend by directly using the list method: `sage: list.extend(v, (3,1,4,1,5,9))` `sage: v [3, 1, 4, 1, 5, 9]`

```
class sage.misc.explain_pickle.TestAppendNonlist
```

Bases: object

A list-like class, carefully designed to test exact unpickling behavior. Used for testing `explain_pickle`.

```
class sage.misc.explain_pickle.TestBuild
```

Bases: object

A simple class with a `__getstate__` but no `__setstate__`. Used for testing `explain_pickle`.

```
class sage.misc.explain_pickle.TestBuildSetstate
```

Bases: `sage.misc.explain_pickle.TestBuild`

A simple class with a `__getstate__` and a `__setstate__`. Used for testing `explain_pickle`.

```
class sage.misc.explain_pickle.TestGlobalFunnyName
```

Bases: object

A featureless new-style class which has a name that’s not a legal Python identifier.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: globals()['funny$name'] = TestGlobalFunnyName # see comment at end of file
sage: TestGlobalFunnyName.__name__
'funny$name'
sage: globals()['funny$name'] is TestGlobalFunnyName
True
```

```
class sage.misc.explain_pickle.TestGlobalNewName
Bases: object
```

A featureless new-style class. When you try to unpickle an instance of TestGlobalOldName, it is redirected to create an instance of this class instead. Used for testing explain_pickle.

EXAMPLES: sage: from sage.misc.explain_pickle import * sage: loads(dumps(TestGlobalOldName())) TestGlobalNewName

```
class sage.misc.explain_pickle.TestGlobalOldName
Bases: object
```

A featureless new-style class. When you try to unpickle an instance of this class, it is redirected to create a TestGlobalNewName instead. Used for testing explain_pickle.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: loads(dumps(TestGlobalOldName()))
TestGlobalNewName
```

```
class sage.misc.explain_pickle.TestReduceGetinitargs
An old-style class with a __getinitargs__ method. Used for testing explain_pickle.
```

```
class sage.misc.explain_pickle.TestReduceNoGetinitargs
An old-style class with no __getinitargs__ method. Used for testing explain_pickle.
```

```
sage.misc.explain_pickle.explain_pickle(pickle=None, file=None, compress=True,
                                         **kwargs)
```

Explain a pickle. That is, produce source code such that evaluating the code is equivalent to loading the pickle. Feeding the result of explain_pickle to sage_eval should be totally equivalent to loading the pickle with cPickle.

INPUTS:

- **pickle** – the pickle to explain, as a string (default: None)
- **file** – a filename of a pickle (default: None)
- **compress** – if False, don't attempt to decompress the pickle (default: True)
- **in_current_sage** – if True, produce potentially simpler code that is tied to the current version of Sage. (default: False)
- **default_assumptions** – if True, produce potentially simpler code that assumes that generic unpickling code will be used. This code may not actually work. (default: False)
- **eval** – if True, then evaluate the resulting code and return the evaluated result. (default: False)
- **preparse** – if True, then produce code to be evaluated with Sage's preparser; if False, then produce standard Python code; if None, then produce code that will work either with or without the preparser. (default: True)
- **pedantic** – if True, then carefully ensures that the result has at least as much sharing as the result of cPickle (it may have more, for immutable objects). (default: False)

Exactly one of `pickle` (a string containing a pickle) or `file` (the filename of a pickle) must be provided.

EXAMPLES:

```
sage: explain_pickle(dumps({'a', 'b'): [1r, 2r]}))
{'a', 'b': [1r, 2r]}
sage: explain_pickle(dumps(RR(pi)), in_current_sage=True)
from sage.rings.real_mpfr import __create__RealNumber_version0
from sage.rings.real_mpfr import __create__RealField_version0
__create__RealNumber_version0(__create__RealField_version0(53r, False, 'RNDN'), '3.4gvm1245kc0@0
sage: s = 'hi'
sage: explain_pickle(dumps((s, s)))
('hi', 'hi')
sage: explain_pickle(dumps((s, s)), pedantic=True)
si = 'hi'
(si, si)
sage: explain_pickle(dumps(5r))
5r
sage: explain_pickle(dumps(5r), preparse=False)
5
sage: explain_pickle(dumps(5r), preparse=None)
int(5)
sage: explain_pickle(dumps(22/7))
pg_make_rational = unpickle_global('sage.rings.rational', 'make_rational')
pg_make_rational('m/7')
sage: explain_pickle(dumps(22/7), in_current_sage=True)
from sage.rings.rational import make_rational
make_rational('m/7')
sage: explain_pickle(dumps(22/7), default_assumptions=True)
from sage.rings.rational import make_rational
make_rational('m/7')
```

```
sage.misc.explain_pickle.explain_pickle_string(pickle, in_current_sage=False, de-
fault_assumptions=False, eval=False,
preparse=True, pedantic=False)
```

This is a helper function for `explain_pickle`. It takes a decompressed pickle string as input; other than that, its options are all the same as `explain_pickle`.

EXAMPLES:

```
sage: sage.misc.explain_pickle.explain_pickle_string(dumps("Hello, world", compress=False))
'Hello, world'
```

(See the documentation for `explain_pickle` for many more examples.)

```
sage.misc.explain_pickle.name_is_valid(name)
```

Test whether a string is a valid Python identifier. (We use a conservative test, that only allows ASCII identifiers.)

EXAMPLES:

```
sage: from sage.misc.explain_pickle import name_is_valid
sage: name_is_valid('fred')
True
sage: name_is_valid('Yes!ValidName')
False
sage: name_is_valid('_happy_1234')
True
```

```
sage.misc.explain_pickle.test_pickle(p, verbose_eval=False, pedantic=False, args=())
```

Tests `explain_pickle` on a given pickle `p`. `p` can be:

- a string containing an uncompressed pickle (which will always end with a '.')
- a string containing a pickle fragment (not ending with '.') `test_pickle` will synthesize a pickle that will push args onto the stack (using persistent IDs), run the pickle fragment, and then STOP (if the string 'mark' occurs in args, then a mark will be pushed)
- an arbitrary object; `test_pickle` will pickle the object

Once it has a pickle, `test_pickle` will print the pickle's disassembly, run `explain_pickle` with `in_current_sage=True` and `False`, print the results, evaluate the results, unpickle the object with `cPickle`, and compare all three results.

If `verbose_eval` is `True`, then `test_pickle` will print messages before evaluating the pickles; this is to allow for tests where the unpickling prints messages (to verify that the same operations occur in all cases).

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: test_pickle(['a'])
0: \x80 PROTO      2
2: ]      EMPTY_LIST
3: q      BININPUT  1
5: U      SHORT_BINSTRING 'a'
8: a      APPEND
9: .      STOP
highest protocol among opcodes = 2
explain_pickle in_current_sage=True/False:
['a']
result: ['a']
```

`sage.misc.explain_pickle.unpickle_appends(lst, vals)`

Given a list (or list-like object) and a sequence of values, appends the values to the end of the list. This is careful to do so using the exact same technique that `cPickle` would use. Used by `explain_pickle`.

EXAMPLES:

```
sage: v = []
sage: unpickle_appends(v, (1, 2, 3))
sage: v
[1, 2, 3]
```

`sage.misc.explain_pickle.unpickle_build(obj, state)`

Set the state of an object. Used by `explain_pickle`.

EXAMPLES:

```
sage: from sage.misc.explain_pickle import *
sage: v = EmptyNewstyleClass()
sage: unpickle_build(v, {'hello': 42})
sage: v.hello
42
```

`sage.misc.explain_pickle.unpickle_extension(code)`

Takes an integer index and returns the extension object with that index. Used by `explain_pickle`.

EXAMPLES:

```
sage: from copy_reg import *
sage: add_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 42)
sage: unpickle_extension(42)
<class 'sage.misc.explain_pickle.EmptyNewstyleClass'>
sage: remove_extension('sage.misc.explain_pickle', 'EmptyNewstyleClass', 42)
```

`sage.misc.explain_pickle.unpickle_instantiate` (*fn, args*)

Instantiate a new object of class *fn* with arguments *args*. Almost always equivalent to `fn(*args)`. Used by `explain_pickle`.

EXAMPLES:

```
sage: unpickle_instantiate(Integer, ('42',))
42
```

`sage.misc.explain_pickle.unpickle_newobj` (*klass, args*)

Create a new object; this corresponds to the C code `klass->tp_new(klass, args, NULL)`. Used by `explain_pickle`.

EXAMPLES: `sage: unpickle_newobj(tuple, ([1, 2, 3],)) (1, 2, 3)`

`sage.misc.explain_pickle.unpickle_persistent` (*s*)

Takes an integer index and returns the persistent object with that index; works by calling whatever callable is stored in `unpickle_persistent_loader`. Used by `explain_pickle`.

EXAMPLES:

```
sage: import sage.misc.explain_pickle
sage: sage.misc.explain_pickle.unpickle_persistent_loader = lambda n: n+7
sage: unpickle_persistent(35)
42
```

5.5 Fixing Pickle for Nested Classes

As of Python 2.6, names for nested classes are set by Python in a way which is incompatible with the pickling of such classes (pickling by name):

```
sage: class A:
...     class B:
...         pass
sage: A.B.__name__
'B'
```

instead of the a priori more natural `"A.B"`.

Furthermore, upon pickling (here in `save_global`) and unpickling (in `load_global`) a class with name `"A.B"` in a module `mod`, the standard `cPickle` module searches for `"A.B"` in `mod.__dict__` instead of looking up `"A"` and then `"B"` in the result.

See: http://groups.google.com/group/sage-devel/browse_thread/thread/6c7055f4a580b7ae/

This module provides two utilities to workaround this issue:

- `nested_pickle()` “fixes” recursively the name of the subclasses of a class and inserts their fullname `"A.B"` in `mod.__dict__`
- `NestedClassMetaclass` is a metaclass ensuring that `nested_pickle()` is called on a class upon creation.

See also `sage.misc.nested_class_test`.

EXAMPLES:

```
sage: from sage.misc.nested_class import A1, nested_pickle
sage: A1.A2.A3.__name__
```

```
'A3'
sage: A1.A2.A3
<class sage.misc.nested_class.A3 at ...>

sage: nested_pickle(A1)
<class sage.misc.nested_class.A1 at ...>

sage: A1.A2
<class sage.misc.nested_class.A1.A2 at ...>

sage: A1.A2.A3
<class sage.misc.nested_class.A1.A2.A3 at ...>
sage: A1.A2.A3.__name__
'A1.A2.A3'

sage: sage.misc.nested_class.__dict__['A1.A2'] is A1.A2
True
sage: sage.misc.nested_class.__dict__['A1.A2.A3'] is A1.A2.A3
True
```

All of this is not perfect. In the following scenario:

```
sage: class A1:
...     class A2:
...         pass
sage: class B1:
...     A2 = A1.A2
...
```

The name for "A1.A2" could potentially be set to "B1.A2". But that will work anyway.

```
sage.misc.nested_class.modify_for_nested_pickle(cls, name_prefix, module,
                                                first_run=True)
```

Modify the subclasses of the given class to be picklable, by giving them a mangled name and putting the mangled name in the module namespace.

INPUTS:

- `cls` - The class to modify.
- `name_prefix` - The prefix to prepend to the class name.
- `module` - The module object to modify with the mangled name.
- `first_run` - optional bool (default True): Whether or not this function is run for the first time on `cls`.

NOTE:

This function would usually not be directly called. It is internally used in `NestedClassMetaclass`.

EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: class A(object):
...     class B(object):
...         pass
...
sage: module = sys.modules['__main__']
sage: A.B.__name__
'B'
sage: getattr(module, 'A.B', 'Not found')
'Not found'
```



```

sage: modify_for_nested_pickle(A, 'A', sys.modules['__main__'])
sage: A.B.__name__
'A.B'
sage: getattr(module, 'A.B', 'Not found')
<class '__main__.A.B'>

```

Here we demonstrate the effect of the `first_run` argument:

```

sage: modify_for_nested_pickle(A, 'X', sys.modules['__main__'])
sage: A.B.__name__ # nothing changed
'A.B'
sage: modify_for_nested_pickle(A, 'X', sys.modules['__main__'], first_run=False)
sage: A.B.__name__
'X.A.B'

```

Note that the class is now found in the module under both its old and its new name:

```

sage: getattr(module, 'A.B', 'Not found')
<class '__main__.X.A.B'>
sage: getattr(module, 'X.A.B', 'Not found')
<class '__main__.X.A.B'>

```

TESTS:

The following is a real life example, that was enabled by the internal use of the “`first_run`” in [trac ticket #9107](#):

```

sage: cython_code = [
....: "from sage.structure.unique_representation import UniqueRepresentation",
....: "class A1(UniqueRepresentation):",
....: "    class B1(UniqueRepresentation):",
....: "        class C1: pass",
....: "    class B2:",
....: "        class C2: pass"]
sage: import os
sage: cython(os.linesep.join(cython_code))

```

Before [trac ticket #9107](#), the name of `A1.B1.C1` would have been wrong:

```

sage: A1.B1.C1.__name__
'A1.B1.C1'
sage: A1.B2.C2.__name__
'A1.B2.C2'
sage: A_module = sys.modules[A1.__module__]
sage: getattr(A_module, 'A1.B1.C1', 'Not found').__name__
'A1.B1.C1'
sage: getattr(A_module, 'A1.B2.C2', 'Not found').__name__
'A1.B2.C2'

```

`sage.misc.nested_class.nested_pickle(cls)`

This decorator takes a class that potentially contains nested classes. For each such nested class, its name is modified to a new illegal identifier, and that name is set in the module. For example, if you have:

```

sage: from sage.misc.nested_class import nested_pickle
sage: module = sys.modules['__main__']
sage: class A(object):
...     class B:
...         pass
sage: nested_pickle(A)
<class '__main__.A'>

```

then the name of class "B" will be modified to "A.B", and the "A.B" attribute of the module will be set to class "B":

```
sage: A.B.__name__
'A.B'
sage: getattr(module, 'A.B', 'Not found')
<class __main__.A.B at ...>
```

In Python 2.6, decorators work with classes; then `@nested_pickle` should work as a decorator:

```
sage: @nested_pickle      # todo: not implemented
...   class A2(object):
...       class B:
...           pass
sage: A2.B.__name__      # todo: not implemented
'A2.B'
sage: getattr(module, 'A2.B', 'Not found')      # todo: not implemented
<class __main__.A2.B at ...>
```

EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: loads(dumps(MainClass.NestedClass())) # indirect doctest
<sage.misc.nested_class.MainClass.NestedClass object at 0x...>
```

class `sage.misc.nested_class.NestedClassMetaclass`

Bases: `type`

A metaclass for nested pickling.

Check that one can use a metaclass to ensure `nested_pickle` is called on any derived subclass:

```
sage: from sage.misc.nested_class import NestedClassMetaclass
sage: class ASuperClass(object):
...     __metaclass__ = NestedClassMetaclass
...
sage: class A3(ASuperClass):
...     class B(object):
...         pass
...
sage: A3.B.__name__
'A3.B'
sage: getattr(sys.modules['__main__'], 'A3.B', 'Not found')
<class '__main__.A3.B'>
```

class `sage.misc.nested_class.MainClass`

Bases: `object`

A simple class to test `nested_pickle`.

EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: loads(dumps(MainClass()))
<sage.misc.nested_class.MainClass object at 0x...>
```

class `NestedClass`

Bases: `object`

EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: loads(dumps(MainClass.NestedClass()))
<sage.misc.nested_class.MainClass.NestedClass object at 0x...>
```

class NestedSubClass

Bases: object

EXAMPLES:

```
sage: from sage.misc.nested_class import *
sage: loads(dumps(MainClass.NestedClass.NestedSubClass()))
<sage.misc.nested_class.MainClass.NestedClass.NestedSubClass object at 0x...>
sage: getattr(sage.misc.nested_class, 'MainClass.NestedClass.NestedSubClass')
<class 'sage.misc.nested_class.MainClass.NestedClass.NestedSubClass'>
sage: MainClass.NestedClass.NestedSubClass.__name__
'MainClass.NestedClass.NestedSubClass'
```

dummy (x , $r=(1, 2, 3.4)$, $*args$, $**kwds$)

A dummy method to demonstrate the embedding of method signature for nested classes.

TESTS:

```
sage: from sage.misc.nested_class import MainClass
sage: print MainClass.NestedClass.NestedSubClass.dummy.__doc__
NestedSubClass.dummy(self, x, *args, r=(1, 2, 3.4), **kwds)
File: sage/misc/nested_class.pyx (starting at line 314)
```

```
A dummy method to demonstrate the embedding of
method signature for nested classes.
```

```
...
```

5.6 Loading and saving sessions and listing all variables

EXAMPLES:

We reset the current session, then define a rational number $2/3$, and verify that it is listed as a newly defined variable:

```
sage: reset()
sage: w = 2/3; w
2/3
sage: show_identifiers()
['w']
```

We next save this session. We are using a file in `SAGE_TMP`. We do this *for testing* only — please do not do this, when you want to save your session permanently, since `SAGE_TMP` will be removed when leaving Sage!

```
sage: save_session(os.path.join(SAGE_TMP, 'session'))
```

This saves a dictionary with `w` as one of the keys:

```
sage: z = load(os.path.join(SAGE_TMP, 'session'))
sage: z.keys()
['w']
sage: z['w']
2/3
```

Next we reset the session, verify this, and load the session back.:

```
sage: reset()
sage: show_identifiers()
[]
sage: load_session(os.path.join(SAGE_TMP, 'session'))
```

Indeed `w` is now defined again.:

```
sage: show_identifiers()
['w']
sage: w
2/3
```

It is not needed to clean up the file created in the above code, since it resides in the directory `SAGE_TMP`.

AUTHOR:

- William Stein

`sage.misc.session.init(state=None)`

Initialize some dictionaries needed by the `show_identifiers()`, `save_session()`, and `load_session()` functions.

INPUT:

- `state` – a dictionary or `None`; if `None` the `locals()` of the caller is used.

EXAMPLES:

```
sage: reset()
sage: w = 10
sage: show_identifiers()
['w']
```

When we call `init()` below it reinitializes the internal table, so the `w` we just defined doesn't count as a new identifier:

```
sage: sage.misc.session.init()
sage: show_identifiers()
[]
```

`sage.misc.session.load_session(name='sage_session', verbose=False)`

Load a saved session.

This merges in all variables from a previously saved session. It does not clear out the variables in the current sessions, unless they are overwritten. You can thus merge multiple sessions, and don't necessarily loose all your current work when you use this command.

Note: In the Sage notebook the session name is searched for both in the current working cell and the `DATA` directory.

EXAMPLES:

```
sage: a = 5
sage: f = lambda x: x^2
```

For testing, we use a temporary file, that will be removed as soon as Sage is left. Of course, for permanently saving your session, you should choose a permanent file.

```
sage: tmp_f = tmp_filename()
sage: save_session(tmp_f)
sage: del a; del f
```

```
sage: load_session(tmp_f)
sage: print a
5
```

Note that `f` does not come back, since it is a function, hence couldn't be saved:

```
sage: print f
Traceback (most recent call last):
...
NameError: name 'f' is not defined
```

`sage.misc.session.save_session(name='sage_session', verbose=False)`

Save all variables that can be saved to the given filename. The variables will be saved to a dictionary, which can be loaded using `load(name)` or `load_session()`.

Note:

1. Function and anything else that can't be pickled is not saved. This failure is silent unless you set `verbose=True`.
2. In the Sage notebook the session is saved both to the current working cell and to the DATA directory.
3. One can still make sessions that can't be reloaded. E.g., define a class with:

```
class Foo: pass
```

and make an instance with:

```
f = Foo()
```

Then `save_session()` followed by `quit` and `load_session()` fails. I doubt there is any good way to deal with this. Fortunately, one can simply re-evaluate the code to define `Foo`, and suddenly `load_session()` works fine.

INPUT:

- `name` – string (default: 'sage_session') name of `sobj` to save the session to.
- `verbose` – bool (default: False) if True, print info about why certain variables can't be saved.

OUTPUT:

- Creates a file and returns silently.

EXAMPLES:

For testing, we use a temporary file that will be removed as soon as Sage is left. Of course, for permanently saving your session, you should choose a permanent file.

```
sage: a = 5
sage: tmp_f = tmp_filename()
sage: save_session(tmp_f)
sage: del a
sage: load_session(tmp_f)
sage: print a
5
```

We illustrate what happens when one of the variables is a function:

```
sage: f = lambda x : x^2
sage: save_session(tmp_f)
sage: save_session(tmp_f, verbose=True)
Saving...
Not saving f: f is a function, method, class or type
...
```

Something similar happens for cython-defined functions:

```
sage: g = cython_lambda('double x', 'x*x + 1.5')
sage: save_session(tmp_f, verbose=True)
Not saving g: g is a function, method, class or type
...
```

`sage.misc.session.show_identifiers(hidden=False)`

Returns a list of all variable names that have been defined during this session. By default, this returns only those identifiers that don't start with an underscore.

INPUT:

- `hidden` – bool (Default: False); If True, also return identifiers that start with an underscore.

OUTPUT:

A list of variable names

EXAMPLES:

We reset the state of all variables, and see that none are defined:

```
sage: reset()
sage: show_identifiers()
[]
```

We then define two variables, one which overwrites the default factor function; both are shown by `show_identifiers()`:

```
sage: a = 10
sage: factor = 20
sage: show_identifiers()
['a', 'factor']
```

To get the actual value of a variable from the list, use the `globals()` function.:

```
sage: globals()['factor']
20
```

By default `show_identifiers()` only returns variables that don't start with an underscore. There is an option `hidden` that allows one to list those as well:

```
sage: _hello = 10
sage: show_identifiers()
['a', 'factor']
sage: '_hello' in show_identifiers(hidden=True)
True
```

Many of the hidden variables are part of the IPython command history, at least in command line mode.:

```
sage: show_identifiers(hidden=True) # random output
['_', '_i', '_6', '_4', '_3', '_1', '_ii', '__doc__', '__builtins__', '___', '_9', '__name__',
```

INTERACTIVE USAGE SUPPORT

6.1 Interactive Sage Sessions

6.1.1 Logging of Sage sessions

TODO: Pressing “control-D” can mess up the I/O sequence because of a known bug.

You can create a log of your Sage session as a web page and/or as a latex document. Just type `log_html()` to create an HTML log, or `log_dvi()` to create a dvi (LaTeX) log. Your complete session so far up until when you type the above command will be logged, along with any future input. Thus you can view the log system as a way to print or view your entire session so far, along with a way to see nicely typeset incremental updates as you work.

If `L=log_dvi()` or `L=log_html()` is a logger, you can type `L.stop()` and `L.start()` to stop and start logging.

The environment variables `BROWSER` and `DVI_VIEWER` determine which web browser or dvi viewer is used to display your running log.

For both log systems you must have a TeX system installed on your computer. For HTML logging, you must have the `convert` command, which comes with the free ImageMagick tools.

Note: The HTML output is done via LaTeX and PNG images right now, sort of like how `latex2html` works. Obviously it would be interesting to do something using MathML in the long run.

AUTHORS:

- William Stein (2006-02): initial version
- William Stein (2006-02-27): changed html generation so log directory is relocatable (no hardcoded paths).
- William Stein (2006-03-04): changed environment variable to `BROWSER`.
- Didier Deshommes (2006-05-06): added MathML support; refactored code.
- Dan Drake (2008-03-27): fix bit rotting so that optional directories work, dvi logging works, `viewer()` command works, remove no-longer-working MathML logger; fix off-by-one problems with IPython history; add text logger; improve documentation about viewers.

class `sage.misc.log.Log` (*dir=None, debug=False, viewer=None*)

This is the base logger class. The two classes that you actually instantiate are derived from this one.

dir ()

Return the directory that contains the log files.

start ()

Start the logger. To stop use the stop function.

stop()

Stop the logger. To restart use the start function.

class `sage.misc.log.log_dvi` (*dir=None, debug=False, viewer=None*)

Bases: `sage.misc.log.Log`

Create a running log of your Sage session as a nicely typeset dvi file.

Easy usage: `log_dvi()`

TODO: Pressing “control-D” can mess up the I/O sequence because of a known bug.

Use `L=log_dvi([optional directory])` to create a dvi log. Your complete session so far up until when you type the above command will be logged, along with any future input. Thus you can view the log system as a way to print or view your entire session so far, along with a way to see nicely typeset incremental updates as you work.

If `L` is a logger, you can type `L.stop()` and `L.start()` to stop and start logging.

The environment variable `DVI_VIEWER` determines which web browser or dvi viewer is used to display your running log. You can also specify a viewer when you start the logger with something like `log_dvi([opt.dir], viewer='xdvi')`.

You must have a LaTeX system installed on your computer and a dvi viewer.

view()

class `sage.misc.log.log_html` (*dir=None, debug=False, viewer=None*)

Bases: `sage.misc.log.Log`

Create a running log of your Sage session as a web page.

Easy usage: `log_html()`

TODO: Pressing “control-D” can mess up the I/O sequence because of a known bug.

Use `L=log_html([optional directory])` to create an HTML log. Your complete session so far up until when you type the above command will be logged, along with any future input. Thus you can view the log system as a way to print or view your entire session so far, along with a way to see nicely typeset incremental updates as you work.

If `L` is a logger, you can type `L.stop()` and `L.start()` to stop and start logging.

The environment variable `WEB_BROWSER` determines which web browser or dvi viewer is used to display your running log. You can also specify a viewer when you start the logger with something like `log_html([opt.dir], viewer='firefox')`.

You must have a TeX system installed on your computer, and you must have the convert command, which comes with the free ImageMagick tools.

view()

class `sage.misc.log.log_text` (*dir=None, debug=False, viewer=None*)

Bases: `sage.misc.log.Log`

Create a running log of your Sage session as a plain text file.

Easy usage: `log_text()`

TODO: Pressing “control-D” can mess up the I/O sequence because of a known bug.

Use `L=log_text([optional directory])` to create a text log. Your complete session so far up until when you type the above command will be logged, along with any future input. Thus you can view the log system as a way to print or view your entire session so far, along with a way to see incremental updates as you work.

Unlike the `html` and `dvi` loggers, this one does not automatically start a viewer unless you specify one; you can do that when you start the logger with something like `log_text([opt. dir], viewer='xterm -e tail -f')`.

If `L` is a logger, you can type `L.stop()` and `L.start()` to stop and start logging.

view()

`sage.misc.log.update()`

6.1.2 SageMath version and banner info

`sage.misc.banner.banner()`

Print the Sage banner.

INPUT:

None

OUTPUT:

None

EXAMPLES:

sage: `banner()`

```
+-----+
| SageMath Version ..., Release Date: ...
| Type "notebook()" for the browser-based notebook interface.
| Type "help()" for help.
|
...

```

`sage.misc.banner.banner_text()`

Text for the Sage banner.

INPUT:

None

OUTPUT:

A string containing the banner message.

EXAMPLES:

sage: `print sage.misc.banner.banner_text()`

```
+-----+
| SageMath Version ...

```

`sage.misc.banner.require_version(major, minor=0, tiny=0, prerelease=False, print_message=False)`

True if Sage version is at least `major.minor.tiny`.

INPUT:

- `major` – integer
- `minor` – integer (optional, default = 0)
- `tiny` – float (optional, default = 0)
- `prerelease` – boolean (optional, default = False)
- `print_message` – boolean (optional, default = False)

OUTPUT:

True if major.minor.tiny is \leq version of Sage, False otherwise

For example, if the Sage version number is 3.1.2, then `require_version(3, 1, 3)` will return False, while `require_version(3, 1, 2)` will return True. If the Sage version is 3.1.2.alpha0, then `require_version(3, 1, 1)` will return True, while, by default, `require_version(3, 1, 2)` will return False. Note, though, that `require_version(3, 1, 2, prerelease=True)` will return True: if the optional argument `prerelease` is True, then a prerelease version of Sage counts as if it were the released version.

If optional argument `print_message` is True and this function is returning False, print a warning message.

EXAMPLES:

```
sage: from sage.misc.banner import require_version
sage: require_version(2, 1, 3)
True
sage: require_version(821, 4)
False
sage: require_version(821, 4, print_message=True)
This code requires at least version 821.4 of SageMath to run correctly.
You are running version ...
False
```

`sage.misc.banner.version(clone=False)`

Return the version of Sage.

INPUT:

nothing

OUTPUT:

str

EXAMPLES:

```
sage: version()
'SageMath Version ..., Release Date: ...'
sage: version(clone=True)
('SageMath Version ..., Release Date: ...',
 'Mercurial clone branch: ...')
```

`sage.misc.banner.version_dict()`

A dictionary describing the version of Sage.

INPUT:

nothing

OUTPUT:

dictionary with keys 'major', 'minor', 'tiny', 'prerelease'

This process the Sage version string and produces a dictionary. It expects the Sage version to be in one of these forms:

```
N.N
N.N.N
N.N.N.N
N.N.str
N.N.N.str
N.N.N.N.str
```

where ‘N’ stands for an integer and ‘str’ stands for a string. The first integer is stored under the ‘major’ key and the second integer under ‘minor’. If there is one more integer, it is stored under ‘tiny’; if there are two more integers, then they are stored together as a float N.N under ‘tiny’. If there is a string, then the key ‘prerelease’ returns True.

For example, if the Sage version is ‘3.2.1’, then the dictionary is {‘major’: 3, ‘minor’: 2, ‘tiny’: 1, ‘prerelease’: False}. If the Sage version is ‘3.2.1.2’, then the dictionary is {‘major’: 3, ‘minor’: 2, ‘tiny’: 1.200..., ‘prerelease’: False}. If the Sage version is ‘3.2.alpha0’, then the dictionary is {‘major’: 3, ‘minor’: 2, ‘tiny’: 0, ‘prerelease’: True}.

EXAMPLES:

```
sage: from sage.misc.banner import version_dict
sage: print "SageMath major version is %s" % version_dict()['major']
SageMath major version is ...
sage: version_dict()['major'] == int(sage.version.version.split('.')[0])
True
```

6.1.3 Interpreter reset

`sage.misc.reset.reset` (*vars=None, attached=False*)

Delete all user-defined variables, reset all global variables back to their default states, and reset all interfaces to other computer algebra systems.

If *vars* is specified, just restore the value of *vars* and leave all other variables alone (i.e., call `restore`).

Note that the variables in the set `sage.misc.reset.EXCLUDE` are excluded from being reset.

INPUT:

- *vars* - a list, or space or comma separated string (default: None), variables to restore
- *attached* - boolean (default: False), if *vars* is not None, whether to detach all attached files

EXAMPLES:

```
sage: x = 5
sage: reset()
sage: x
x

sage: fn = tmp_filename(ext='foo.py')
sage: sage.misc.reset.EXCLUDE.add('fn')
sage: open(fn, 'w').write('a = 111')
sage: attach(fn)
sage: [fn] == attached_files()
True
sage: reset()
sage: [fn] == attached_files()
True
sage: reset(attached=True)
sage: [fn] == attached_files()
False
sage: sage.misc.reset.EXCLUDE.remove('fn')
```

TESTS:

Confirm that assumptions don’t survive a reset (trac #10855):

```
sage: assume(x > 3)
sage: assumptions()
```

```
[x > 3]
sage: bool(x > 3)
True
sage: reset()
sage: assumptions()
[]
sage: bool(x > 3)
False
```

```
sage.misc.reset.reset_interfaces()
```

```
sage.misc.reset.restore(vars=None)
Restore predefined global variables to their default values.
```

INPUT:

- vars - string or list (default: None), if not None, restores just the given variables to the default value.

EXAMPLES:

```
sage: x = 10; y = 15/3; QQ='red'
sage: QQ
'red'
sage: restore('QQ')
sage: QQ
Rational Field
sage: x
10
sage: y = var('y')
sage: restore('x y')
sage: x
x
sage: y
Traceback (most recent call last):
...
NameError: name 'y' is not defined
sage: x = 10; y = 15/3; QQ='red'
sage: ww = 15
sage: restore()
sage: x, QQ, ww
(x, Rational Field, 15)
sage: restore('ww')
sage: ww
Traceback (most recent call last):
...
NameError: name 'ww' is not defined
```

6.1.4 Determination of programs for viewing web pages, etc.

The function `default_viewer()` defines reasonable defaults for these programs. To use something else, use `viewer`. First import it:

```
sage: from sage.misc.viewer import viewer
```

On OS X, PDFs are opened by default using the ‘open’ command, which runs whatever has been designated as the PDF viewer in the OS. To change this to use ‘Adobe Reader’:

```
sage: viewer.pdf_viewer('open -a /Applications/Adobe\ Reader.app') # not tested
```

Similarly, you can set `viewer.browser(...)`, `viewer.dvi_viewer(...)`, and `viewer.png_viewer(...)`. You can make this change permanent by adding lines like these to your `SAGE_STARTUP_FILE` (which is `$HOME/.sage/init.sage` by default):

```
from sage.misc.viewer import viewer
viewer.pdf_viewer('open -a /Applications/Adobe\ Reader.app')
```

Functions and classes

class `sage.misc.viewer.Viewer`

Bases: `sage.structure.sage_object.SageObject`

Set defaults for various viewing applications: a web browser, a dvi viewer, a pdf viewer, and a png viewer.

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_browser = viewer.browser() # indirect doctest
sage: viewer.browser('open -a /Applications/Firefox.app')
sage: viewer.browser()
'open -a /Applications/Firefox.app'
sage: viewer.browser(old_browser) # restore old value
```

browser (*app=None*)

Change the default browser. Return the current setting if *arg* is `None`, which is the default.

INPUTS:

- *app* – `None` or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_browser = viewer.browser()
sage: viewer.browser('open -a /Applications/Firefox.app') # indirect doctest
sage: viewer.browser()
'open -a /Applications/Firefox.app'
sage: viewer.browser(old_browser) # restore old value
```

dvi_viewer (*app=None*)

Change the default dvi viewer. Return the current setting if *arg* is `None`, which is the default.

INPUTS:

- *app* – `None` or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_dvi_app = viewer.dvi_viewer()
sage: viewer.dvi_viewer('/usr/bin/xdvi') # indirect doctest
sage: viewer.dvi_viewer()
'/usr/bin/xdvi'
sage: viewer.dvi_viewer(old_dvi_app) # restore old value
```

pdf_viewer (*app=None*)

Change the default pdf viewer. Return the current setting if *arg* is `None`, which is the default.

INPUTS:

- app – None or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_pdf_app = viewer.pdf_viewer()
sage: viewer.pdf_viewer('/usr/bin/pdfoopen') # indirect doctest
sage: viewer.pdf_viewer()
'/usr/bin/pdfoopen'
sage: viewer.pdf_viewer(old_pdf_app) # restore old value
```

png_viewer(app=None)

Change the default png viewer. Return the current setting if arg is None, which is the default.

INPUTS:

- app – None or a string, the program to use

EXAMPLES:

```
sage: from sage.misc.viewer import viewer
sage: old_png_app = viewer.png_viewer()
sage: viewer.png_viewer('display') # indirect doctest
sage: viewer.png_viewer()
'display'
sage: viewer.png_viewer(old_png_app) # restore old value
```

sage.misc.viewer.browser()

Return the program used to open a web page. By default, the program used depends on the platform and other factors, like settings of certain environment variables. To use a different program, call `viewer.browser('PROG')`, where 'PROG' is the desired program.

This will start with 'sage-native-execute', which sets the environment appropriately.

EXAMPLES:

```
sage: from sage.misc.viewer import browser
sage: browser() # random -- depends on OS, etc.
'sage-native-execute sage-open'
sage: browser().startswith('sage-native-execute')
True
```

sage.misc.viewer.default_viewer(viewer=None)

Set up default programs for opening web pages, PDFs, PNGs, and DVI files.

INPUT:

- viewer: None or a string: one of 'browser', 'pdf', 'png', 'dvi' – return the name of the corresponding program. None is treated the same as 'browser'.

EXAMPLES:

```
sage: from sage.misc.viewer import default_viewer
sage: default_viewer(None) # random -- depends on OS, etc.
'sage-open'
sage: default_viewer('pdf') # random -- depends on OS, etc.
'xdg-open'
sage: default_viewer('jpg')
Traceback (most recent call last):
...
ValueError: Unknown type of viewer: jpg.
```

```
sage.misc.viewer.dvi_viewer()
```

Return the program used to display a dvi file. By default, the program used depends on the platform and other factors, like settings of certain environment variables. To use a different program, call `viewer.dvi_viewer('PROG')`, where 'PROG' is the desired program.

This will start with 'sage-native-execute', which sets the environment appropriately.

EXAMPLES:

```
sage: from sage.misc.viewer import dvi_viewer
sage: dvi_viewer() # random -- depends on OS, etc.
'sage-native-execute sage-open'
sage: dvi_viewer().startswith('sage-native-execute')
True
```

```
sage.misc.viewer.pdf_viewer()
```

Return the program used to display a pdf file. By default, the program used depends on the platform and other factors, like settings of certain environment variables. To use a different program, call `viewer.pdf_viewer('PROG')`, where 'PROG' is the desired program.

This will start with 'sage-native-execute', which sets the environment appropriately.

EXAMPLES:

```
sage: from sage.misc.viewer import pdf_viewer, viewer
sage: old_pdf_app = viewer.pdf_viewer()
sage: viewer.pdf_viewer('acroread')
sage: pdf_viewer()
'sage-native-execute acroread'
sage: viewer.pdf_viewer('old_pdf_app')
```

```
sage.misc.viewer.png_viewer()
```

Return the program used to display a png file. By default, the program used depends on the platform and other factors, like settings of certain environment variables. To use a different program, call `viewer.png_viewer('PROG')`, where 'PROG' is the desired program.

This will start with 'sage-native-execute', which sets the environment appropriately.

EXAMPLES:

```
sage: from sage.misc.viewer import png_viewer
sage: png_viewer() # random -- depends on OS, etc.
'sage-native-execute xdg-open'
sage: png_viewer().startswith('sage-native-execute')
True
```

6.1.5 Pager for showing strings

```
sage.misc.pager.cat(x)
```

```
sage.misc.pager.pager()
```

6.1.6 Format Sage documentation for viewing with IPython and the notebook

AUTHORS:

- William Stein (2005): initial version.
- Nick Alexander (2007): nodetex functions

- Nick Alexander (2008): `search_src`, `search_def` improvements
- Martin Albrecht (2008-03-21): parse LaTeX description environments in `sagedoc`
- John Palmieri (2009-04-11): fix for #5754 plus doctests
- Dan Drake (2009-05-21): refactor `search_*` functions, use system ‘find’ instead of `sage -grep`
- John Palmieri (2009-06-28): don’t use ‘find’ – use Python (`os.walk`, `re.search`) instead.
- Simon King (2011-09): Use `os.linesep`, avoid destruction of embedding information, enable `nodetex` in a docstring. Consequently use `sage getdoc`.

TESTS:

Check that argspecs of extension function/methods appear correctly, see [trac ticket #12849](#):

```
sage: docfilename = os.path.join(SAGE_DOC, 'output', 'html', 'en', 'reference', 'calculus', 'sage', '
sage: for line in open(docfilename):
...     if "#sage.symbolic.expression.Expression.N" in line:
...         print line
<tt class="descname">N</tt><big>(</big><em>prec=None</em>, <em>digits=None</em>, <em>algorithm=None</em>
```

sage.misc.sagedoc.constructions

Sage constructions. Attempts to answer the question “How do I construct ... in Sage?”

EXAMPLES:

```
sage: constructions() # indirect doctest, not tested
```

```
sage.misc.sagedoc.detex(s, embedded=False)
```

This strips LaTeX commands from a string; it is used by the `format` function to process docstrings for display from the command line interface.

INPUT:

- s - string
- embedded - boolean (optional, default False)

If `embedded` is `False`, then do the replacements in both `math_substitutes` and `nonmath_substitutes`. If `True`, then only do `nonmath_substitutes`.

OUTPUT:

string

EXAMPLES:

```
sage: from sage.misc.sagedoc import detex
sage: detex(r'Some math: '\n\geq k'. A website: \url{sagemath.org}.')
'Some math: n >= k. A website: sagemath.org.\n'
sage: detex(r'More math: 'x \mapsto y'. {\bf Bold face}.)
'More math: x |-> y. { Bold face}.\n'
sage: detex(r'\a, b, c, \ldots, z')
'a, b, c, ..., z\n'
sage: detex(r'\a, b, c, \ldots, z', embedded=True)
'a, b, c, \\ldots, z'
sage: detex(r'\left(\lvert x\rast y \rvert\right)')
'| x * y ||\n'
sage: detex(r'\left(\leq\le\leftarrow\rightarrow\to')
'(<=<=leftarrow rightarrow-->\n'
```


`sage.misc.sagedoc.developer`

The Sage developer's guide. Learn to develop programs for Sage.

EXAMPLES:

```
sage: developer() # indirect doctest, not tested
```

`sage.misc.sagedoc.format(s, embedded=False)`

noreplace Format Sage documentation *s* for viewing with IPython.

This calls `detex` on *s* to convert LaTeX commands to plain text, unless the directive `nodetex` is given in the first line of the string.

Also, if *s* contains a string of the form `<<<obj>>>`, then it replaces it with the docstring for *obj*, unless the directive `noreplace` is given in the first line. If an error occurs under the attempt to find the docstring for *obj*, then the substring `<<<obj>>>` is preserved.

Directives must be separated by a comma.

INPUT:

- *s* - string
- *embedded* - boolean (optional, default False)

OUTPUT: string

Set *embedded* equal to True if formatting for use in the notebook; this just gets passed as an argument to `detex`.

See also:

`sage.misc.sageinspect.sage_getdoc()` to get the formatted documentation of a given object.

EXAMPLES:

```
sage: from sage.misc.sagedoc import format
sage: identity_matrix(2).rook_vector.__doc__[201:273]
'Let `A` be an `m` by `n` (0,1)-matrix. We identify `A` with a chessboard'

sage: format(identity_matrix(2).rook_vector.__doc__[201:273])
'Let A be an m by n (0,1)-matrix. We identify A with a chessboard\n'
```

If the first line of the string is `'nodetex'`, remove `'nodetex'` but don't modify any TeX commands:

```
sage: format("nodetex\n`x` \\geq y`")
'`x` \\geq y`'
```

Testing a string enclosed in triple angle brackets:

```
sage: format('<<<identity_matrix')
'<<<identity_matrix\n'
sage: format('identity_matrix>>>')
'identity_matrix>>>\n'
sage: format('<<<identity_matrix>>>')[:28]
'Definition: identity_matrix('
```

TESTS:

We check that the todo Sphinx extension is correctly activated:

```
sage: sage.misc.sagedoc.format(sage.combinat.ranker.on_fly.__doc__)
" Returns ... Todo: add tests as in combinat::rankers\n"
```

In the following use case, the `nodetex` directive would have been ignored prior to #11815:

```
sage: cython_code = ["def testfunc(x):",
... "    """",
... "    nodetex",
... "    This is a doc string with raw latex",
... "    """,
... "    `x \\geq y`",
... "    """,
... "    return -x"]
sage: cython('`n`.join(cython_code))
sage: from sage.misc.sageinspect import sage_getdoc
sage: print sage_getdoc(testfunc)
```

This is a doc string with raw latex

 $'x \ \backslashgeq \ y'$

We check that the `noreplace` directive works, even combined with `nodetex` (see [trac ticket #11817](#)):

```
sage: print format(''''nodetex, noreplace\n<<<identity_matrix>>>\n\n'''  
<<<identity_matrix>>>\n\n')
<<<identity_matrix>>>\n\n
```

If replacement is impossible, then no error is raised:

```
sage: print format('<<<bla\n<<<bla>>>\n<<<identity_matrix>>>')
<<<bla <<<bla>>>
```

Definition: `identity_matrix(ring, n=0, sparse=False)`

This function is available as `identity_matrix(...)` and `matrix.identity(...)`.

Return the $n \times n$ identity matrix over the given ring.

...

```
sage.misc.sagedoc.format search as html (what, r, search)
```

Format the output from `search src`, `search def`, or `search doc` as `html`, for use in the notebook.

INPUT:

- `what` - (string) what was searched (source code or documentation)
- `r` - (string) the results of the search
- `search` - (string) what was being searched for

This function parses `r`: it should have the form `FILENAME: string` where `FILENAME` is the file in which the string that matched the search was found. Everything following the first colon is ignored; we just use the filename. If `FILENAME` ends in `‘.html’`, then this is part of the documentation; otherwise, it is in the source code. In either case, an appropriate link is created.

EXAMPLES:

[illegible]

```
sage.misc.sagedoc.format_src(s)
```

Format Sage source code `s` for viewing with IPython.

If `s` contains a string of the form “<<<obj>>>”, then it replaces it with the source code for “obj”.

INPUT: `s` - string

OUTPUT: string

EXAMPLES:

```
sage: from sage.misc.sagedoc import format_src
sage: format_src('unladen swallow')
'unladen swallow'
sage: format_src('<<<Sq>>>') [5:15]
'Sq(*nums):'
```

`sage.misc.sagedoc.help` (*module=None*)

If there is an argument `module`, print the Python help message for `module`. With no argument, print a help message about getting help in Sage.

EXAMPLES:

```
sage: help()
Welcome to Sage ...
```

`sage.misc.sagedoc.manual`

The Sage reference manual.

EXAMPLES:

```
sage: reference() # indirect doctest, not tested
sage: manual() # indirect doctest, not tested
```

`sage.misc.sagedoc.my_getsource` (*obj*, *oname=''*)

Retrieve the source code for `obj`.

INPUT:

- `obj` – a Sage object, function, etc.
- `oname` – str (optional). A name under which the object is known. Currently ignored by Sage.

OUTPUT:

Its documentation (string)

EXAMPLES:

```
sage: from sage.misc.sagedoc import my_getsource
sage: s = my_getsource(identity_matrix)
sage: s[15:34]
'def identity_matrix'
```

`sage.misc.sagedoc.process_dollars` (*s*)

Replace dollar signs with backticks.

More precisely, do a regular expression search. Replace a plain dollar sign (\$) by a backtick (`). Replace an escaped dollar sign (\\$) by a dollar sign (\$). Don't change a dollar sign preceded or followed by a backtick (`\$ or \$`), because of strings like “\$HOME”. Don't make any changes on lines starting with more spaces than the first nonempty line in `s`, because those are indented and hence part of a block of code or examples.

This also doesn't replace dollar signs enclosed in curly braces, to avoid nested math environments.

EXAMPLES:

```
sage: from sage.misc.sagedoc import process_dollars
sage: process_dollars('hello')
'hello'
sage: process_dollars('some math: $x=y$')
'some math: `x=y`'
```

Replace `\$` with `$`, and don't do anything when backticks are involved:

```
sage: process_dollars(r'a ``$REAL`` dollar sign: \$')
'a ``$REAL`` dollar sign: $'
```

Don't make any changes on lines indented more than the first nonempty line:

```
sage: s = '\n first line\n      indented $x=y$'
sage: s == process_dollars(s)
True
```

Don't replace dollar signs enclosed in curly braces:

```
sage: process_dollars(r'f(n) = 0 \text{ if $n$ is prime}')
```

`'f(n) = 0 \text{ if n is prime}'`

This is not perfect:

```
sage: process_dollars(r'$f(n) = 0 \text{ if $n$ is prime}$')
'$f(n) = 0 \text{ if $n$ is prime}$'
```

The regular expression search doesn't find the last `$`. Fortunately, there don't seem to be any instances of this kind of expression in the Sage library, as of this writing.

`sage.misc.sagedoc.process_extlinks(s, embedded=False)`

In docstrings at the command line, process markup related to the Sphinx extlinks extension. For example, replace `:trac:NUM` with `http://trac.sagemath.org/NUM`, and similarly with `:python:TEXT` and `:wikipedia:TEXT`, looking up the url from the dictionary `extlinks` in `SAGE_DOC/common/conf.py`. If `TEXT` is of the form `blah <LINK>`, then it uses `LINK` rather than `TEXT` to construct the url.

In the notebook, don't do anything: let sphinxify take care of it.

INPUT:

- `s` – string, in practice a docstring
- `embedded` – boolean (optional, default `False`)

This function is called by `format()`, and if in the notebook, it sets `embedded` to be `True`, otherwise `False`.

EXAMPLES:

```
sage: from sage.misc.sagedoc import process_extlinks
sage: process_extlinks('See :trac:1234', :wikipedia:'Wikipedia <Sage_(mathematics_software)>', a
'See http://trac.sagemath.org/1234, https://en.wikipedia.org/wiki/Sage_(mathematics_software), a
sage: process_extlinks('See :trac:1234 for more information.', embedded=True)
'See :trac:1234 for more information.'
sage: process_extlinks('see :python:Implementing Descriptors <reference/datamodel.html#implemen
'see https://docs.python.org/release/.../reference/datamodel.html#implementing-descriptors ...'
```

`sage.misc.sagedoc.process_mathtt(s)`

Replace `\mathtt{BLAH}` with `BLAH` in the command line.

INPUT:

- `s` - string, in practice a docstring

This function is called by `format()`.

EXAMPLES:

```
sage: from sage.misc.sagedoc import process_mathtt
sage: process_mathtt(r'e^\mathtt{self}')
'e^self'
```

`sage.misc.sagedoc.reference`

The Sage reference manual.

EXAMPLES:

```
sage: reference() # indirect doctest, not tested
sage: manual() # indirect doctest, not tested
```

`sage.misc.sagedoc.search_def(name, extra1='', extra2='', extra3='', extra4='', extra5='', **kws)`

Search Sage library source code for function definitions containing name. The search is case sensitive.

INPUT: same as for `search_src()`.

OUTPUT: same as for `search_src()`.

Note: The regular expression used by this function only finds function definitions that are preceded by spaces, so if you use tabs on a “def” line, this function will not find it. As tabs are not allowed in Sage library code, this should not be a problem.

EXAMPLES:

See the documentation for `search_src()` for more examples.

```
sage: print search_def("fetch", interact=False) # random # long time
matrix/matrix0.pyx: cdef fetch(self, key):
matrix/matrix0.pxd: cdef fetch(self, key)
```

```
sage: print search_def("fetch", path_re="pyx", interact=False) # random # long time
matrix/matrix0.pyx: cdef fetch(self, key):
```

`sage.misc.sagedoc.search_doc(string, extra1='', extra2='', extra3='', extra4='', extra5='', **kws)`

Search Sage HTML documentation for lines containing string. The search is case-sensitive.

The file paths in the output are relative to `$SAGE_DOC/output`.

INPUT: same as for `search_src()`.

OUTPUT: same as for `search_src()`.

EXAMPLES:

See the documentation for `search_src()` for more examples.

```
sage: search_doc('creates a polynomial', path_re='tutorial', interact=False) # random
html/en/tutorial/tour_polynomial.html:<p>This creates a polynomial ring and tells Sage to use (t
```

If you search the documentation for ‘tree’, then you will get too many results, because many lines in the documentation contain the word ‘toctree’. If you use the `whole_word` option, though, you can search for ‘tree’ without returning all of the instances of ‘toctree’. In the following, since `search_doc('tree', interact=False)` returns a string with one line for each match, counting the length of `search_doc('tree', interact=False).splitlines()` gives the number of matches.

```
sage: len(search_doc('tree', interact=False).splitlines()) > 4000 # long time
True
sage: len(search_doc('tree', whole_word=True, interact=False).splitlines()) < 2000 # long time
True
```

```
sage.misc.sagedoc.search_src(string, extra1='', extra2='', extra3='', extra4='', extra5='',
                             **kws)
```

Search Sage library source code for lines containing *string*. The search is case-sensitive.

INPUT:

- *string* - a string to find in the Sage source code.
- *extra1*, ..., *extra5* - additional strings to require when searching. Lines must match all of these, as well as *string*.
- *whole_word* (optional, default False) - if True, search for *string* and *extra1* (etc.) as whole words only. This assumes that each of these arguments is a single word, not a regular expression, and it might have unexpected results if used with regular expressions.
- *ignore_case* (optional, default False) - if True, perform a case-insensitive search
- *multiline* (optional, default False) - if True, search more than one line at a time. In this case, print any matching file names, but don't print line numbers.
- *interact* (optional, default True) - if False, return a string with all the matches. Otherwise, this function returns None, and the results are displayed appropriately, according to whether you are using the notebook or the command-line interface. You should not ordinarily need to use this.
- *path_re* (optional, default '') - regular expression which the filename (including the path) must match.
- *module* (optional, default 'sage') - the module in which to search. The default is 'sage', the entire Sage library. If *module* doesn't start with "sage", then the links in the notebook output may not function.

OUTPUT: If *interact* is False, then return a string with all of the matches, separated by newlines. On the other hand, if *interact* is True (the default), there is no output. Instead: at the command line, the search results are printed on the screen in the form *filename:line_number:line of text*, showing the filename in which each match occurs, the line number where it occurs, and the actual matching line. (If *multiline* is True, then only the filename is printed for each match.) The file paths in the output are relative to \$SAGE_SRC. In the notebook, each match produces a link to the actual file in which it occurs.

The *string* and *extraN* arguments are treated as regular expressions, as is *path_re*, and errors will be raised if they are invalid. The matches will be case-sensitive unless *ignore_case* is True.

Note: The *extraN* parameters are present only because `search_src(string, *extras, interact=False)` is not parsed correctly by Python 2.6; see <http://bugs.python.org/issue1909>.

EXAMPLES:

First note that without using *interact=False*, this function produces no output, while with *interact=False*, the output is a string. These examples almost all use this option, so that they have something to which to compare their output.

You can search for "matrix" by typing `search_src("matrix")`. This particular search will produce many results:

```
sage: len(search_src("matrix", interact=False).splitlines()) # random # long time
9522
```

You can restrict to the Sage calculus code with `search_src("matrix", module="sage.calculus")`, and this produces many fewer results:

```
sage: len(search_src("matrix", module="sage.calculus", interact=False).splitlines()) # random
26
```

Note that you can do tab completion on the module string. Another way to accomplish a similar search:

```
sage: len(search_src("matrix", path_re="calc", interact=False).splitlines()) > 15
True
```

The following produces an error because the string ‘fetch(‘ is a malformed regular expression:

```
sage: print search_src(" fetch(", "def", interact=False)
Traceback (most recent call last):
...
error: unbalanced parenthesis
```

To fix this, *escape* the parenthesis with a backslash:

```
sage: print search_src(" fetch\\(", "def", interact=False) # random # long time
matrix/matrix0.pyx:      cdef fetch(self, key):
matrix/matrix0.pxd:      cdef fetch(self, key)
```

```
sage: print search_src(" fetch\\(", "def", "pyx", interact=False) # random # long time
matrix/matrix0.pyx:      cdef fetch(self, key):
```

As noted above, the search is case-sensitive, but you can make it case-insensitive with the ‘ignore_case’ key word:

```
sage: s = search_src('Matrix', path_re='matrix', interact=False); s.find('x') > 0
True
```

```
sage: s = search_src('MatRiX', path_re='matrix', interact=False); s.find('x') > 0
False
```

```
sage: s = search_src('MatRiX', path_re='matrix', interact=False, ignore_case=True); s.find('x')
True
```

Searches are by default restricted to single lines, but this can be changed by setting `multiline` to be `True`. In the following, since `search_src(string, interact=False)` returns a string with one line for each match, counting the length of `search_src(string, interact=False).splitlines()` gives the number of matches.

```
sage: len(search_src('log', 'derivative', interact=False).splitlines()) < 40
True
sage: len(search_src('log', 'derivative', interact=False, multiline=True).splitlines()) > 70
True
```

A little recursive narcissism: let’s do a doctest that searches for this function’s doctests. Note that you can’t put “sage:” in the doctest string because it will get replaced by the Python “>>>” prompt.

```
sage: print search_src('^ *sage[:] .*search_src\\(', interact=False) # long time
misc/sagedoc.py:... len(search_src("matrix", interact=False).splitlines()) # random # long time
misc/sagedoc.py:... len(search_src("matrix", module="sage.calculus", interact=False).splitlines()
misc/sagedoc.py:... len(search_src("matrix", path_re="calc", interact=False).splitlines()) > 15
misc/sagedoc.py:... print search_src(" fetch(", "def", interact=False)
misc/sagedoc.py:... print search_src(" fetch\\(", "def", interact=False) # random # long time
misc/sagedoc.py:... print search_src(" fetch\\(", "def", "pyx", interact=False) # random # long t
misc/sagedoc.py:... s = search_src('Matrix', path_re='matrix', interact=False); s.find('x') > 0
misc/sagedoc.py:... s = search_src('MatRiX', path_re='matrix', interact=False); s.find('x') > 0
misc/sagedoc.py:... s = search_src('MatRiX', path_re='matrix', interact=False, ignore_case=True)
misc/sagedoc.py:... len(search_src('log', 'derivative', interact=False).splitlines()) < 40
```

```
misc/sagedoc.py:... len(search_src('log', 'derivative', interact=False, multiline=True).splitlines())
misc/sagedoc.py:... print search_src('^ *sage[:] .*search_src\(', interact=False) # long time
misc/sagedoc.py:... len(search_src("matrix", interact=False).splitlines()) > 9000 # long time
misc/sagedoc.py:... print search_src('matrix', 'column', 'row', 'sub', 'start', 'index', interact=False)
```

TESTS:

As of this writing, there are about 9500 lines in the Sage library that contain “matrix”; it seems safe to assume we’ll continue to have over 9000 such lines:

```
sage: len(search_src("matrix", interact=False).splitlines()) > 9000 # long time
True
```

Check that you can pass 5 parameters:

```
sage: print search_src('matrix', 'column', 'row', 'sub', 'start', 'index', interact=False) # ran
matrix/matrix0.pyx:598:         Get The 2 x 2 submatrix of M, starting at row index and column
matrix/matrix0.pyx:607:         Get the 2 x 3 submatrix of M starting at row index and column ind
matrix/matrix0.pyx:924:         Set the 2 x 2 submatrix of M, starting at row index and column
matrix/matrix0.pyx:933:         Set the 2 x 3 submatrix of M starting at row index and column
```

sage.misc.sagedoc.**tutorial**

The Sage tutorial. To get started with Sage, start here.

EXAMPLES:

```
sage: tutorial() # indirect doctest, not tested
```

6.2 Distribution

6.2.1 Sage package management commands

A Sage package has the extension .spkg. It is a tarball that is (usually) bzip2 compressed that contains arbitrary data and an spkg-install file. An Sage package typically has the following components:

- spkg-install - shell script that is run to install the package
- Sage.txt - file that describes how the package was made, who maintains it, etc.
- sage - directory with extra patched version of files that needed during the install

Use the `install_package` command to install a new package, and use `optional_packages` to list all optional packages available on the central Sage server. The `upgrade` command upgrades all *standard* packages - there is no auto-upgrade command for optional packages.

All package management can also be done via the Sage command line.

sage.misc.package.**experimental_packages**()

Return two lists. The first contains the installed and the second contains the not-installed experimental packages that are available from the Sage repository. You must have an internet connection.

OUTPUT:

- installed experimental packages (as a list)
- NOT installed experimental packages (as a list)

Use `install_package(package_name)` to install or re-install a given package.

See also:


```
install_package(), upgrade()
```

EXAMPLE:

```
sage: from sage.misc.package import experimental_packages
sage: installed, not_installed = experimental_packages() # optional internet
sage: min(installed+not_installed)                    # optional internet
'PyQt4'
sage: max(installed+not_installed)                    # optional internet
'yassl'
```

`sage.misc.package.install_all_optional_packages` (*force=True, dry_run=False*)

Install all available optional spkg's in the official Sage spkg repository. Returns a list of all spkg's that *fail* to install.

INPUT:

- *force* – bool (default: True); whether to force reinstall of spkg's that are already installed.
- *dry_run* – bool (default: False); if True, just list the packages that would be installed in order, but don't actually install them.

OUTPUT:

list of strings

Note: This is designed mainly for testing purposes. This also doesn't do anything with respect to dependencies – the packages are installed in alphabetical order. Dependency issues will be dealt with in a future version.

AUTHOR:

– William Stein (2008-12)

EXAMPLES:

```
sage: sage.misc.package.install_all_optional_packages(dry_run=True) # optional - internet
Installing ...
[]
```

`sage.misc.package.install_package` (*package=None, force=False*)

Install a package or return a list of all packages that have been installed into this Sage install.

You must have an internet connection. Also, you will have to restart Sage for the changes to take affect.

It is not needed to provide the version number.

INPUT:

- *package* – optional; if specified, install the given package. If not, list all installed packages.
- *force* – boolean (default: False); if True, reinstall the package given if it is already installed. (Otherwise, an already installed package doesn't get reinstalled, as with 'sage -i ...')

EXAMPLES:

With no arguments, list the installed packages:

```
sage: install_package()
[...'atlas...'python...]
```

With an argument, install the named package:

```
sage: install_package('chomp') # not tested
Attempting to download package chomp-20100213.p2
...
```

IMPLEMENTATION:

Calls 'sage -f ...' to (re)install the package if a package name is given. If no package name is given, simply list the contents of spkg/installed.

See also:

`optional_packages()`, `upgrade()`

```
sage.misc.package.is_package_installed(package)
```

Return true if package is installed.

EXAMPLES:

```
sage: is_package_installed('pari')
True
```

Giving just the beginning of the package name is not good enough:

```
sage: is_package_installed('matplotlib')
False
```

Otherwise, installing “pillow” will cause this function to think that “pil” is installed, for example.

```
sage.misc.package.optional_packages()
```

Return two lists. The first contains the installed and the second contains the not-installed optional packages that are available from the Sage repository. You must have an internet connection.

OUTPUT:

- installed optional packages (as a list)
- NOT installed optional packages (as a list)

Use `install_package(package_name)` to install or re-install a given package.

See also:

`install_package()`, `upgrade()`

EXAMPLE:

```
sage: from sage.misc.package import optional_packages
sage: installed, not_installed = optional_packages() # optional internet
sage: min(installed+not_installed)                 # optional internet
'4ti2'
sage: max(installed+not_installed)                   # optional internet
'zeromq'
```

```
sage.misc.package.package_mesg(package_name)
```

```
sage.misc.package.package_versions(package_type, local=False)
```

Return version information for each Sage package.

INPUT:

- `package_type` (string) – one of “*standard*”, “*optional*” or “*experimental*”
- `local` (boolean) – only query local data (no internet needed)

For packages of the given type, return a dictionary whose entries are of the form ‘package’: (installed, latest), where `installed` is the installed version (or `None` if not installed) and `latest` is the latest available version. If the package has a directory in `SAGE_ROOT/build/pkgs/`, then `latest` is

determined by the file `package-version.txt` in that directory. If `local` is `False`, then Sage's servers are queried for package information.

EXAMPLES:

```
sage: std = package_versions('standard', local=True)
sage: 'gap' in std
True
sage: std['zn_poly']
('0.9.p11', '0.9.p11')
```

`sage.misc.package.standard_packages()`

Return two lists. The first contains the installed and the second contains the not-installed standard packages that are available from the Sage repository. You must have an internet connection.

OUTPUT:

- installed standard packages (as a list)
- NOT installed standard packages (as a list)

Use `install_package(package_name)` to install or re-install a given package.

See also:

`install_package()`, `upgrade()`

EXAMPLE:

```
sage: from sage.misc.package import standard_packages
sage: installed, not_installed = standard_packages() # optional internet
sage: installed[0], installed[-1]                 # optional internet
('atlas', 'zn_poly')
sage: 'mercurial' in not_installed                 # optional internet
True
```

`sage.misc.package.upgrade()`

Download and build the latest version of Sage.

You must have an internet connection. Also, you will have to restart Sage for the changes to take affect.

This upgrades to the latest version of core packages (optional packages are not automatically upgraded).

This will not work on systems that don't have a C compiler.

See also:

`install_package()`, `optional_packages()`

6.2.2 Installing shortcut scripts

`sage.misc.dist.install_scripts(directory=None, ignore_existing=False)`

Running `install_scripts(directory)` creates scripts in the given directory that run various software components included with Sage. Each of these scripts essentially just runs `sage --CMD` where `CMD` is also the name of the script:

- 'gap' runs GAP
- 'gp' runs the PARI/GP interpreter
- 'hg' runs Mercurial
- 'ipython' runs IPython

- ‘maxima’ runs Maxima
- ‘mwrnk’ runs mwrnk
- ‘R’ runs R
- ‘singular’ runs Singular
- ‘sqlite3’ runs SQLite version 3
- ‘kash’ runs Kash if it is installed (Kash is an optional Sage package)
- ‘M2’ runs Macaulay2 if it is installed (Macaulay2 is an experimental Sage package)

This command:

- verbosely tells you which scripts it adds, and
- will *not* overwrite any scripts you already have in the given directory.

INPUT:

- `directory` - string; the directory into which to put the scripts. This directory must exist and the user must have write and execute permissions.
- `ignore_existing` - bool (optional, default False): if True, install script even if another version of the program is in your path.

OUTPUT: Verbosely prints what it is doing and creates files in `directory` that are world executable and readable.

Note: You may need to run `sage` as `root` in order to run `install_scripts` successfully, since the user running `sage` needs write permissions on `directory`. Note that one good candidate for `directory` is `‘/usr/local/bin’`, so from the shell prompt, you could run

```
sudo sage -c "install_scripts(' /usr/local/bin' )"
```

Note: Running `install_scripts(directory)` will be most helpful if `directory` is in your path.

AUTHORS:

- William Stein: code / design
- Arthur Gaer: design
- John Palmieri: revision, 2011-07 (trac ticket #11602)

EXAMPLES:

```
sage: install_scripts(str(SAGE_TMP), ignore_existing=True)
Checking that Sage has the command 'gap' installed
...
```

6.3 Credits

6.3.1 Dependency usage tracking for citations

`sage.misc.citation.get_systems(cmd)`

Returns a list of the systems used in running the command `cmd`. Note that the results can sometimes include

systems that did not actually contribute to the computation. Due to caching and the inability to follow all C calls, it could miss some dependencies as well.

INPUT:

- cmd - a string to run

EXAMPLES:

```
sage: from sage.misc.citation import get_systems
sage: s = get_systems('integrate(x^2, x)'); #priming coercion model
sage: get_systems('integrate(x^2, x)')
['ginac', 'Maxima']
sage: R.<x,y,z> = QQ[]
sage: I = R.ideal(x^2+y^2, z^2+y)
sage: get_systems('I.primary_decomposition()')
['Singular']

sage: a = var('a')
sage: get_systems('((a+1)^2).expand()')
['ginac', 'GMP']
```

6.3.2 License

```
class sage.misc.copying.License
```


DEVELOPMENT TOOLS

7.1 Testing

7.1.1 Unit testing for Sage objects

class sage.misc.sage_unittest.**InstanceTester** (*instance, elements=None, verbose=False, prefix='', max_runs=4096, **options*)

Bases: unittest.case.TestCase

A gadget attached to an instance providing it with testing utilities.

EXAMPLES:

```
sage: from sage.misc.sage_unittest import InstanceTester
sage: InstanceTester(instance = ZZ, verbose = True, elements = [1,2,3])
Testing utilities for Integer Ring
```

This is used by SageObject._tester, which see:

```
sage: QQ._tester()
Testing utilities for Rational Field
```

info (*message, newline=True*)

Displays user information

EXAMPLES:

```
sage: from sage.misc.sage_unittest import InstanceTester
sage: tester = InstanceTester(ZZ, verbose = True)

sage: tester.info("hello"); tester.info("world")
hello
world

sage: tester = InstanceTester(ZZ, verbose = False)
sage: tester.info("hello"); tester.info("world")

sage: tester = InstanceTester(ZZ, verbose = True)
sage: tester.info("hello", newline = False); tester.info(" world")
hello world
```

runTest ()

Trivial implementation of unittest.TestCase.runTest () to please the super class TestCase. That's the price to pay for abusively inheriting from it.

EXAMPLES:

```
sage: from sage.misc.sage_unittest import InstanceTester
sage: tester = InstanceTester(ZZ, verbose = True)
sage: tester.runTest()
```

some_elements (*S=None*)

Returns a list (or iterable) of elements of `self` on which the tests should be run. This is only meaningful for container objects like parents.

INPUT:

- *S* – a set of elements to select from. By default this will use the elements passed to this tester at creation time, or the result of `some_elements()` if no elements were specified.

OUTPUT:

A list of at most `self._max_runs` elements of *S*.

EXAMPLES:

By default, this calls `some_elements()` on the instance:

```
sage: from sage.misc.sage_unittest import InstanceTester
sage: class MyParent(Parent):
...     def some_elements(self):
...         return [1,2,3,4,5]
...
sage: tester = InstanceTester(MyParent())
sage: list(tester.some_elements())
[1, 2, 3, 4, 5]

sage: tester = InstanceTester(MyParent(), max_runs=3)
sage: list(tester.some_elements())
[1, 2, 3]

sage: tester = InstanceTester(MyParent(), max_runs=7)
sage: list(tester.some_elements())
[1, 2, 3, 4, 5]

sage: tester = InstanceTester(MyParent(), elements=[1,3,5])
sage: list(tester.some_elements())
[1, 3, 5]

sage: tester = InstanceTester(MyParent(), elements=[1,3,5], max_runs=2)
sage: list(tester.some_elements())
[1, 3]

sage: tester = InstanceTester(FiniteEnumeratedSet(['a','b','c','d']), max_runs=3)
sage: tester.some_elements()
['a', 'b', 'c']

sage: tester = InstanceTester(FiniteEnumeratedSet([]))
sage: list(tester.some_elements())
[]

sage: tester = InstanceTester(ZZ)
sage: ZZ.some_elements()
# yikes, shamelessly trivial ...
<generator object _some_elements_from_iterator at 0x...>
sage: list(tester.some_elements())
[0, 1, -1, 2, -2, ..., 49, -49, 50]
```



```

sage: tester = InstanceTester(ZZ, elements = ZZ, max_runs=5)
sage: list(tester.some_elements())
[0, 1, -1, 2, -2]

sage: tester = InstanceTester(ZZ, elements = srange(100), max_runs=5)
sage: list(tester.some_elements())
[0, 1, 2, 3, 4]

sage: tester = InstanceTester(ZZ, elements = srange(3), max_runs=5)
sage: list(tester.some_elements())
[0, 1, 2]

```

Test for trac ticket #15919, trac ticket #16244:

```

sage: Z = IntegerModRing(25) # random.sample, which was used pre #16244, has a threshold at
sage: Z[1]                  # since #8389, indexed access is used for ring extensions
Traceback (most recent call last):
...
ValueError: first letter of variable name must be a letter
sage: tester = InstanceTester(Z, elements=Z, max_runs=5)
sage: list(tester.some_elements())
[0, 1, 2, 3, 4]

sage: C = CartesianProduct(Z, Z, Z, Z)
sage: len(C)
390625
sage: tester = InstanceTester(C, elements = C, max_runs=4)
sage: list(tester.some_elements())
[[0, 0, 0, 0], [0, 0, 0, 1], [0, 0, 0, 2], [0, 0, 0, 3]]

```

class sage.misc.sage_unittest.**PythonObjectWithTests** (*instance*)
 Bases: object

Utility class for running basis tests on a plain Python object (that is not in SageObject). More test methods can be added here.

EXAMPLES:

```
sage: TestSuite("bla").run()
```

class sage.misc.sage_unittest.**TestSuite** (*instance*)
 Bases: object

Test suites for Sage objects.

EXAMPLES:

```
sage: TestSuite(ZZ).run()
```

No output means that all tests passed. Which tests? In practice this calls all the methods `._test_*` of this object, in alphabetic order:

```

sage: TestSuite(1).run(verbose = True)
running ._test_category() . . . pass
running ._test_eq() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . pass

```

Those methods are typically implemented by abstract super classes, in particular via categories, in order to

enforce standard behavior and API, or provide mathematical sanity checks. For example if `self` is in the category of finite semigroups, this checks that the multiplication is associative (at least on some elements):

```
sage: S = FiniteSemigroups().example(alphabet = ('a', 'b'))
sage: TestSuite(S).run(verbose = True)
running ._test_an_element() . . . pass
running ._test_associativity() . . . pass
running ._test_cardinality() . . . pass
running ._test_category() . . . pass
running ._test_elements() . . .
  Running the test suite of self.an_element()
  running ._test_category() . . . pass
  running ._test_eq() . . . pass
  running ._test_not_implemented_methods() . . . pass
  running ._test_pickling() . . . pass
  pass
running ._test_elements_eq_reflexive() . . . pass
running ._test_elements_eq_symmetric() . . . pass
running ._test_elements_eq_transitive() . . . pass
running ._test_elements_neq() . . . pass
running ._test_enumerated_set_contains() . . . pass
running ._test_enumerated_set_iter_cardinality() . . . pass
running ._test_enumerated_set_iter_list() . . . pass
running ._test_eq() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . pass
running ._test_some_elements() . . . pass
```

The different test methods can be called independently:

```
sage: S._test_associativity()
```

Debugging tip: in case of failure of some test, use `%pdb` on to turn on automatic debugging on error. Run the failing test independently: the debugger will stop right where the first assertion fails. Then, introspection can be used to analyse what exactly the problem is. See also the `catch = False` option to `run()`.

When meaningful, one can further customize on which elements the tests are run. Here, we use it to *prove* that the multiplication is indeed associative, by running the test on all the elements:

```
sage: S._test_associativity(elements = S)
```

Adding a new test boils down to adding a new method in the class of the object or any super class (e.g. in a category). This method should use the utility `_tester()` to handle standard options and report test failures. See the code of `_test_an_element()` for an example. Note: Python's testunit convention is to look for methods called `.test*`; we use instead `._test_*` so as not to pollute the object's interface.

Eventually, every implementation of a `SageObject` should run a `TestSuite` on one of its instances in its `doctest` (replacing the current `loads(dumps(x))` tests).

Finally, running `TestSuite` on a standard Python object does some basic sanity checks:

```
sage: TestSuite(int(1)).run(verbose = True)
running ._test_pickling() . . . pass
```

TODO:

- Allow for customized behavior in case of failing assertion (warning, error, statistic accounting). This involves reimplementing the methods `fail / failIf / ...` of `unittest.TestCase` in `InstanceTester`
- Don't catch the exceptions if `TestSuite(...).run()` is called under the debugger, or with `%pdb` on (how to detect this? see `get_ipython()`, `IPython.Magic.shell.call_pdb`, ...) In the mean

time, see the `catch=False` option.

- Run the tests according to the inheritance order, from most generic to most specific, rather than alphabetically. Then, the first failure will be the most relevant, the others being usually consequences.
- Improve integration with doctests (statistics on failing/passing tests)
- Add proper support for nested testsuites.
- Integration with unittest: Make `TestSuite` inherit from `unittest.TestSuite`? Make `.run(...)` accept a result object
- Add some standard option `proof = True`, asking for the test method to choose appropriately the elements so as to prove the desired property. The test method may assume that a parent implements properly all the super categories. For example, the `_test_commutative` method of the category `CommutativeSemigroups()` may just check that the provided generators commute, implicitly assuming that generators indeed generate the semigroup (as required by `Semigroups()`).

run (*category=None, skip=[], catch=True, raise_on_failure=False, **options*)

Run all the tests from this test suite:

INPUT:

- `category` - a category; reserved for future use
- `skip` - a string or list (or iterable) of strings
- `raise_on_failure` - a boolean (default: `False`)
- `catch` - a boolean (default: `True`)

All other options are passed down to the individual tests.

EXAMPLES:

```
sage: TestSuite(ZZ).run()
```

We now use the verbose option:

```
sage: TestSuite(1).run(verbose = True)
running ._test_category() . . . pass
running ._test_eq() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . pass
```

Some tests may be skipped using the `skip` option:

```
sage: TestSuite(1).run(verbose = True, skip = "_test_pickling")
running ._test_category() . . . pass
running ._test_eq() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
sage: TestSuite(1).run(verbose = True, skip = ["_test_pickling", "_test_category"])
running ._test_eq() . . . pass
running ._test_nonzero_equal() . . . pass
running ._test_not_implemented_methods() . . . pass
```

We now show (and test) some standard error reports:

```
sage: class Blah(SageObject):
...     def _test_a(self, tester): pass
...     def _test_b(self, tester): tester.fail()
...     def _test_c(self, tester): pass
```

```
...         def _test_d(self, tester): tester.fail()

sage: TestSuite(Blah()).run()
Failure in _test_b:
Traceback (most recent call last):
...
AssertionError: None
-----
Failure in _test_d:
Traceback (most recent call last):
...
AssertionError: None
-----
Failure in _test_pickling:
Traceback (most recent call last):
...
PicklingError: Can't pickle <class '__main__.Blah'>: attribute lookup __main__.Blah failed
-----
The following tests failed: _test_b, _test_d, _test_pickling

sage: TestSuite(Blah()).run(verbose = True)
running ._test_a() . . . pass
running ._test_b() . . . fail
Traceback (most recent call last):
...
AssertionError: None
-----
running ._test_c() . . . pass
running ._test_category() . . . pass
running ._test_d() . . . fail
Traceback (most recent call last):
...
AssertionError: None
-----
running ._test_not_implemented_methods() . . . pass
running ._test_pickling() . . . fail
Traceback (most recent call last):
...
PicklingError: Can't pickle <class '__main__.Blah'>: attribute lookup __main__.Blah failed
-----
The following tests failed: _test_b, _test_d, _test_pickling

File "/opt/sage/local/lib/python/site-packages/sage/misc/sage_unittest.py", line 183, in run
test_method(tester = tester)
```

The `catch=False` option prevents `TestSuite` from catching exceptions:

```
sage: TestSuite(Blah()).run(catch=False)
Traceback (most recent call last):
...
File ..., in _test_b
    def _test_b(self, tester): tester.fail()
...
AssertionError: None
```

In conjunction with `%pdb` on, this allows for the debugger to jump directly to the first failure location.

exception `sage.misc.sage_unittest.TestSuiteFailure`

Bases: `exceptions.AssertionError`

`sage.misc.sage_unittest.instance_tester(instance, tester=None, **options)`

Returns a gadget attached to instance providing testing utilities.

EXAMPLES:

```
sage: from sage.misc.sage_unittest import instance_tester
sage: tester = instance_tester(ZZ)

sage: tester.assert_(1 == 1)
sage: tester.assert_(1 == 0)
Traceback (most recent call last):
...
AssertionError: False is not true
sage: tester.assert_(1 == 0, "this is expected to fail")
Traceback (most recent call last):
...
AssertionError: this is expected to fail

sage: tester.assertEquals(1, 1)
sage: tester.assertEquals(1, 0)
Traceback (most recent call last):
...
AssertionError: 1 != 0
```

The available assertion testing facilities are the same as in `unittest.TestCase` [UNITTEST], which see (actually, by a slight abuse, `tester` is currently an instance of this class).

TESTS:

```
sage: instance_tester(ZZ, tester = tester) is tester
True
```

REFERENCES:

7.1.2 Random testing

Some Sage modules do random testing in their doctests; that is, they construct test cases using a random number generator. To get the broadest possible test coverage, we want everybody who runs the doctests to use a different random seed; but we also want to be able to reproduce the problems when debugging. This module provides a decorator to help write random testers that meet these goals.

`sage.misc.random_testing.random_testing(fn)`

This decorator helps create random testers. These can be run as part of the standard Sage test suite; everybody who runs the test will use a different random number seed, so many different random tests will eventually be run.

INPUT:

- `fn` - The function that we are wrapping for random testing.

The resulting function will take two additional arguments, `seed` (default `None`) and `print_seed` (default `False`). The result will set the random number seed to the given seed value (or to a truly random value, if `seed` is not specified), then call the original function. If `print_seed` is true, then the seed will be printed before calling the original function. If the original function raises an exception, then the random seed that was used will be displayed, along with a message entreating the user to submit a bug report. All other arguments will be passed through to the original function.

Here is a set of recommendations for using this wrapper.

The function to be tested should take arguments specifying the difficulty of the test (size of the test cases, number of iterations, etc.), as well as an argument *verbose* (defaulting to false). With *verbose* true, it should print the values being tested. Suppose `test_foo()` takes an argument for number of iterations. Then the doctests could be:

```
test_foo(2, verbose=True, seed=0)
test_foo(10)
test_foo(100) # long time
```

The first doctest, with the specified seed and `verbose=True`, simply verifies that the tests really are reproducible (that `test_foo` is correctly using the `randstate` framework). The next two tests use truly random seeds, and will print out the seed used if the test fails (raises an exception).

If you want a very long-running test using this setup, you should do something like:

```
for _ in xrange(10^10): test_foo(100)
```

instead of:

```
test_foo(10^12)
```

If the test fails after several hours, the latter snippet would make you rerun the test for several hours while reproducing and debugging the problem. With the former snippet, you only need to rerun `test_foo(100)` with a known-failing random seed.

See `sage.misc.random_testing.test_add_commutes()` for a simple example using this decorator, and `sage.rings.tests` for realistic uses.

Setting `print_seed` to true is useless in doctests, because the random seed printed will never match the expected doctest result (and using `# random` means the doctest framework will never report an error even if one happens). However, it is useful if you have a random test that sometimes segfaults. The normal print-the-random-seed-on-exceptions won't work then, so you can run:

```
while True: test_foo(print_seed=True)
```

and look at the last seed that was printed before it crashed.

TESTS:

```
sage: from sage.misc.random_testing import random_testing
sage: def foo(verbose=False):
...     'oh look, a docstring'
...     n = ZZ.random_element(2^50)
...     if verbose:
...         print "Random value: %s" % n
...     assert(n == 49681376900427)
sage: foo = random_testing(foo)
sage: foo(seed=0, verbose=True)
Random value: 49681376900427
sage: foo(seed=15, verbose=True)
Random value: 1049538412064764
Random testing has revealed a problem in foo
Please report this bug! You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:
Random seed: 15
AssertionError()
sage: foo() # random
Random testing has revealed a problem in foo
Please report this bug! You may be the first
person in the world to have seen this problem.
```

```

Please include this random seed in your bug report:
Random seed: 272500700755151445506092479579811710040
AssertionError()
sage: foo.__doc__
'oh look, a docstring'
sage: foo.__name__
'foo'
sage: def bar(): pass
sage: bar = random_testing(bar)
sage: bar(print_seed=True) # random
Random seed: 262841091890156346923539765543814146051

```

`sage.misc.random_testing.test_add_commutes(*args, **kwargs)`

This is a simple demonstration of the `random_testing()` decorator and its recommended usage.

We test that addition is commutative over rationals.

EXAMPLES:

```

sage: from sage.misc.random_testing import test_add_commutes
sage: test_add_commutes(2, verbose=True, seed=0)
a == -4, b == 0 ...
Passes!
a == -1/2, b == -1/95 ...
Passes!
sage: test_add_commutes(10)
sage: test_add_commutes(1000) # long time

```

`sage.misc.random_testing.test_add_is_mul(*args, **kwargs)`

This example demonstrates a failing `random_testing()` test, and shows how to reproduce the error.

DO NOT USE THIS AS AN EXAMPLE OF HOW TO USE `random_testing()`! Instead, look at `sage.misc.random_testing.test_add_commutes()`.

We test that $a+b == a*b$, for a, b rational. This is of course false, so the test will almost always fail.

EXAMPLES:

```

sage: from sage.misc.random_testing import test_add_is_mul

```

We start by testing that we get reproducible results when setting *seed* to 0.

```

sage: test_add_is_mul(2, verbose=True, seed=0)
a == -4, b == 0 ...
Random testing has revealed a problem in test_add_is_mul
Please report this bug! You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:
Random seed: 0
AssertionError()

```

Normally in a `@random_testing` doctest, we would leave off the `verbose=True` and the `# random`. We put it in here so that we can verify that we are seeing the exact same error when we reproduce the error below.

```

sage: test_add_is_mul(10, verbose=True) # random
a == -2/7, b == 1 ...
Random testing has revealed a problem in test_add_is_mul
Please report this bug! You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:

```

```
Random seed: 216390410596009428782506007128692114173
AssertionError()
```

OK, now assume that some user has reported a `test_add_is_mul()` failure. We can specify the same *random_seed* that was found in the bug report, and we will get the exact same failure so that we can debug the “problem”.

```
sage: test_add_is_mul(10, verbose=True, seed=216390410596009428782506007128692114173)
a == -2/7, b == 1 ...
Random testing has revealed a problem in test_add_is_mul
Please report this bug! You may be the first
person in the world to have seen this problem.
Please include this random seed in your bug report:
Random seed: 216390410596009428782506007128692114173
AssertionError()
```

7.1.3 Test for nested class Parent

This file contains a discussion, examples, and tests about nested classes and parents. It is kept in a separate file to avoid import loops.

EXAMPLES:

Currently pickling fails for parents using nested classes (typically for categories), but deriving only from Parent:

```
sage: from sage.misc.nested_class_test import TestParent1, TestParent2, TestParent3, TestParent4
sage: P = TestParent1()
sage: TestSuite(P).run()
Failure ...
The following tests failed: _test_elements, _test_pickling
```

They actually need to be in the NestedClassMetaclass. However, due to a technical detail, this is currently not directly supported:

```
sage: P = TestParent2()
Traceback (most recent call last):
...
TypeError: metaclass conflict: the metaclass of a derived class must be a (non-strict) subclass of the
sage: TestSuite(P).run() # not tested
```

Instead, the easiest is to inherit from UniqueRepresentation, which is what you want to do anyway most of the time:

```
sage: P = TestParent3()
sage: TestSuite(P).run()
```

This is what all Sage’s parents using categories currently do. An alternative is to use ClasscallMetaclass as metaclass:

```
sage: P = TestParent4()
sage: TestSuite(P).run()
```


7.2 Benchmarking and Profiling

7.2.1 Benchmarks

`sage.misc.benchmark.SetPartitionsAk(k)`

Returns the combinatorial class of set partitions of type A_k .

EXAMPLES:

```
sage: A3 = SetPartitionsAk(3); A3
```

Set partitions of $\{1, \dots, 3, -1, \dots, -3\}$

```
sage: A3.first() #random
```

```
\{1, 2, 3, -1, -3, -2\}
```

```
sage: A3.last() #random
```

```
\{-1\}, \{-2\}, \{3\}, \{1\}, \{-3\}, \{2\}
```

```
sage: A3.random_element() #random
```

```
\{1, 3, -3, -1\}, \{2, -2\}
```

```
sage: A3.cardinality()
```

```
203
```

```
sage: A2p5 = SetPartitionsAk(2.5); A2p5
```

Set partitions of $\{1, \dots, 3, -1, \dots, -3\}$ with 3 and -3 in the same block

```
sage: A2p5.cardinality()
```

```
52
```

```
sage: A2p5.first() #random
```

```
\{1, 2, 3, -1, -3, -2\}
```

```
sage: A2p5.last() #random
```

```
\{-1\}, \{-2\}, \{2\}, \{3, -3\}, \{1\}
```

```
sage: A2p5.random_element() #random
```

```
\{-1\}, \{-2\}, \{3, -3\}, \{1, 2\}
```

`sage.misc.benchmark.SetPartitionsBk(k)`

Returns the combinatorial class of set partitions of type B_k . These are the set partitions where every block has size 2.

EXAMPLES:

```
sage: B3 = SetPartitionsBk(3); B3
```

Set partitions of $\{1, \dots, 3, -1, \dots, -3\}$ with block size 2

```
sage: B3.first() #random
```

```
\{2, -2\}, \{1, -3\}, \{3, -1\}
```

```
sage: B3.last() #random
```

```
\{1, 2\}, \{3, -2\}, \{-3, -1\}
```

```
sage: B3.random_element() #random
```

```
\{2, -1\}, \{1, -3\}, \{3, -2\}
```

```
sage: B3.cardinality()
```

```
15
```

```
sage: B2p5 = SetPartitionsBk(2.5); B2p5
```

Set partitions of $\{1, \dots, 3, -1, \dots, -3\}$ with 3 and -3 in the same block and with block size 2

```
sage: B2p5.first() #random
```

```
\{2, -1\}, \{3, -3\}, \{1, -2\}
```

```
sage: B2p5.last() #random
```

```
{{1, 2}, {3, -3}, {-1, -2}}
sage: B2p5.random_element() #random
{{2, -2}, {3, -3}, {1, -1}}

sage: B2p5.cardinality()
3
```

`sage.misc.benchmark.SetPartitionsIk(k)`

Returns the combinatorial class of set partitions of type I_k . These are set partitions with a propagating number of less than k . Note that the identity set partition $\{\{1, -1\}, \dots, \{k, -k\}\}$ is not in I_k .

EXAMPLES:

```
sage: I3 = SetPartitionsIk(3); I3
Set partitions of {1, ..., 3, -1, ..., -3} with propagating number < 3
sage: I3.cardinality()
197
```

```
sage: I3.first() #random
{{1, 2, 3, -1, -3, -2}}
sage: I3.last() #random
{{-1}, {-2}, {3}, {1}, {-3}, {2}}
sage: I3.random_element() #random
{{-1}, {-3, -2}, {2, 3}, {1}}
```

```
sage: I2p5 = SetPartitionsIk(2.5); I2p5
Set partitions of {1, ..., 3, -1, ..., -3} with 3 and -3 in the same block and propagating number < 2.5
sage: I2p5.cardinality()
50
```

```
sage: I2p5.first() #random
{{1, 2, 3, -1, -3, -2}}
sage: I2p5.last() #random
{{-1}, {-2}, {2}, {3, -3}, {1}}
sage: I2p5.random_element() #random
{{-1}, {-2}, {1, 3, -3}, {2}}
```

`sage.misc.benchmark.SetPartitionsPRk(k)`

`sage.misc.benchmark.SetPartitionsPk(k)`

Returns the combinatorial class of set partitions of type P_k . These are the planar set partitions.

EXAMPLES:

```
sage: P3 = SetPartitionsPk(3); P3
Set partitions of {1, ..., 3, -1, ..., -3} that are planar
sage: P3.cardinality()
132
```

```
sage: P3.first() #random
{{1, 2, 3, -1, -3, -2}}
sage: P3.last() #random
{{-1}, {-2}, {3}, {1}, {-3}, {2}}
sage: P3.random_element() #random
{{1, 2, -1}, {-3}, {3, -2}}
```

```
sage: P2p5 = SetPartitionsPk(2.5); P2p5
Set partitions of {1, ..., 3, -1, ..., -3} with 3 and -3 in the same block and that are planar
sage: P2p5.cardinality()
42
```

```

sage: P2p5.first() #random
{{1, 2, 3, -1, -3, -2}}
sage: P2p5.last() #random
{{-1}, {-2}, {2}, {3, -3}, {1}}
sage: P2p5.random_element() #random
{{1, 2, 3, -3}, {-1, -2}}

```

`sage.misc.benchmark.SetPartitionsRk(k)`

`sage.misc.benchmark.SetPartitionsSk(k)`

Returns the combinatorial class of set partitions of type S_k . There is a bijection between these set partitions and the permutations of $1, \dots, k$.

EXAMPLES:

```

sage: S3 = SetPartitionsSk(3); S3
Set partitions of {1, ..., 3, -1, ..., -3} with propagating number 3
sage: S3.cardinality()
6

```

```

sage: S3.list() #random
[{{2, -2}, {3, -3}, {1, -1}},
 {{1, -1}, {2, -3}, {3, -2}},
 {{2, -1}, {3, -3}, {1, -2}},
 {{1, -2}, {2, -3}, {3, -1}},
 {{1, -3}, {2, -1}, {3, -2}},
 {{1, -3}, {2, -2}, {3, -1}}]

```

```

sage: S3.first() #random
{{2, -2}, {3, -3}, {1, -1}}

```

```

sage: S3.last() #random
{{1, -3}, {2, -2}, {3, -1}}

```

```

sage: S3.random_element() #random
{{1, -3}, {2, -1}, {3, -2}}

```

```

sage: S3p5 = SetPartitionsSk(3.5); S3p5
Set partitions of {1, ..., 4, -1, ..., -4} with 4 and -4 in the same block and propagating number 3
sage: S3p5.cardinality()
6

```

```

sage: S3p5.list() #random
[{{2, -2}, {3, -3}, {1, -1}, {4, -4}},
 {{2, -3}, {1, -1}, {4, -4}, {3, -2}},
 {{2, -1}, {3, -3}, {1, -2}, {4, -4}},
 {{2, -3}, {1, -2}, {4, -4}, {3, -1}},
 {{1, -3}, {2, -1}, {4, -4}, {3, -2}},
 {{1, -3}, {2, -2}, {4, -4}, {3, -1}}]

```

```

sage: S3p5.first() #random
{{2, -2}, {3, -3}, {1, -1}, {4, -4}}

```

```

sage: S3p5.last() #random
{{1, -3}, {2, -2}, {4, -4}, {3, -1}}

```

```

sage: S3p5.random_element() #random
{{1, -3}, {2, -2}, {4, -4}, {3, -1}}

```

`sage.misc.benchmark.SetPartitionsTk(k)`

Returns the combinatorial class of set partitions of type T_k . These are planar set partitions where every block is of size 2.

EXAMPLES:

```
sage: T3 = SetPartitionsTk(3); T3
Set partitions of {1, ..., 3, -1, ..., -3} with block size 2 and that are planar
sage: T3.cardinality()
5
```

```
sage: T3.first() #random
{{1, -3}, {2, 3}, {-1, -2}}
sage: T3.last() #random
{{1, 2}, {3, -1}, {-3, -2}}
sage: T3.random_element() #random
{{1, -3}, {2, 3}, {-1, -2}}
```

```
sage: T2p5 = SetPartitionsTk(2.5); T2p5
Set partitions of {1, ..., 3, -1, ..., -3} with 3 and -3 in the same block and with block size 2
sage: T2p5.cardinality()
2
```

```
sage: T2p5.first() #random
{{2, -2}, {3, -3}, {1, -1}}
sage: T2p5.last() #random
{{1, 2}, {3, -3}, {-1, -2}}
```

```
sage.misc.benchmark.bench0()
Run a benchmark.
```

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench0()[0]
Benchmark 0: Factor the following polynomial over
the rational numbers: (x^97+19*x+1)*(x^103-19*x^97+14)*(x^100-1)
```

```
sage.misc.benchmark.bench1()
Run a benchmark.
```

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench1()[0]
Find the Mordell-Weil group of the elliptic curve 5077A using mwrank
```

```
sage.misc.benchmark.bench2()
Run a benchmark.
```

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench2()[0]
Some basic arithmetic with very large Integer numbers: '3^1000001 * 19^100001
```

```
sage.misc.benchmark.bench3()
Run a benchmark.
```

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench3()[0]
Some basic arithmetic with very large Rational numbers: '(2/3)^100001 * (17/19)^100001
```

```
sage.misc.benchmark.bench4()
```

Run a benchmark.

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench4()[0]
Rational polynomial arithmetic using Sage. Compute  $(x^{29}+17x-5)^{200}$ .
```

```
sage.misc.benchmark.bench5()
```

Run a benchmark.

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench5()[0]
Rational polynomial arithmetic using Sage. Compute  $(x^{19} - 18x + 1)^{50}$  one hundred times.
```

```
sage.misc.benchmark.bench6()
```

Run a benchmark.

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench6()[0]
Compute the p-division polynomials of  $y^2 = x^3 + 37x - 997$  for primes  $p < 40$ .
```

```
sage.misc.benchmark.bench7()
```

Run a benchmark.

BENCHMARK:

```
sage: from sage.misc.benchmark import *
sage: print bench7()[0]
Compute the Mordell-Weil group of  $y^2 = x^3 + 37x - 997$ .
```

```
sage.misc.benchmark.benchmark(n=-1)
```

Run a well-chosen range of Sage commands and record the time it takes for each to run.

INPUT:

n – int (default: -1) the benchmark number; the default of -1 runs all the benchmarks.

OUTPUT:

list – summary of timings for each benchmark. int – if $n == -1$, also return the total time

EXAMPLE:

```
sage: from sage.misc.benchmark import *
sage: _ = benchmark()
Running benchmark 0
Benchmark 0: Factor the following polynomial over
the rational numbers:  $(x^{97}+19x+1)*(x^{103}-19x^{97}+14)*(x^{100}-1)$ 
Time: ... seconds
Running benchmark 1
Find the Mordell-Weil group of the elliptic curve 5077A using mwrank
Time: ... seconds
Running benchmark 2
Some basic arithmetic with very large Integer numbers:  $3^{1000001} * 19^{100001}$ 
Time: ... seconds
Running benchmark 3
Some basic arithmetic with very large Rational numbers:  $(2/3)^{100001} * (17/19)^{100001}$ 
Time: ... seconds
```

```
Running benchmark 4
Rational polynomial arithmetic using Sage. Compute (x^29+17*x-5)^200.
Time: ... seconds
Running benchmark 5
Rational polynomial arithmetic using Sage. Compute (x^19 - 18*x + 1)^50 one hundred times.
Time: ... seconds
Running benchmark 6
Compute the p-division polynomials of  $y^2 = x^3 + 37x - 997$  for primes  $p < 40$ .
Time: ... seconds
Running benchmark 7
Compute the Mordell-Weil group of  $y^2 = x^3 + 37x - 997$ .
Time: ... seconds
Running benchmark 8
```

7.2.2 Accurate timing information for Sage commands

This is an implementation of nice `timeit` functionality, like the `%timeit` magic command in IPython. To use it, use the `timeit` command. This command then calls `sage_timeit()`, which you can find below.

EXAMPLES:

```
sage: timeit('1+1')      # random output
625 loops, best of 3: 314 ns per loop
```

AUTHOR:

– William Stein, based on code by Fernando Perez included in IPython

class `sage.misc.sage_timeit.SageTimeitResult` (*stats, series=None*)
Bases: object

Represent the statistics of a `timeit()` command.

Prints as a string so that it can be easily returned to a user.

INPUT:

- *stats* – tuple of length 5 containing the following information:
 - integer, number of loops
 - integer, repeat number
 - Python integer, number of digits to print
 - number, best timing result
 - str, time unit

EXAMPLES:

```
sage: from sage.misc.sage_timeit import SageTimeitResult
sage: SageTimeitResult( (3, 5, int(8), pi, 'ms') )
3 loops, best of 5: 3.1415927 ms per loop

sage: units = ["s", "ms", "\xc2\xbf5s", "ns"]
sage: scaling = [1, 1e3, 1e6, 1e9]
sage: number = 7
sage: repeat = 13
sage: precision = int(5)
sage: best = pi / 10 ^ 9
```

```
sage: order = 3
sage: stats = (number, repeat, precision, best * scaling[order], units[order])
sage: SageTimeitResult(stats)
7 loops, best of 13: 3.1416 ns per loop
```

If the third argument is not a Python integer, a `TypeError` is raised:

```
sage: SageTimeitResult( (1, 2, 3, 4, 's') )
<repr(<sage.misc.sage_timeit.SageTimeitResult at 0x...>) failed: TypeError: * wants int>
```

`sage.misc.sage_timeit.sage_timeit(stmt, globals_dict=None, preparse=None, number=0, repeat=3, precision=3, seconds=False)`
 Accurately measure the wall time required to execute `stmt`.

INPUT:

- `stmt` – a text string.
- `globals_dict` – a dictionary or `None` (default). Evaluate `stmt` in the context of the `globals` dictionary. If not set, the current `globals()` dictionary is used.
- `preparse` – (default: use `globals` preparser default) if `True` preparse `stmt` using the Sage preparser.
- `number` – integer, (optional, default: 0), number of loops.
- `repeat` – integer, (optional, default: 3), number of repetition.
- `precision` – integer, (optional, default: 3), precision of output time.
- `seconds` – boolean (default: `False`). Whether to just return time in seconds.

OUTPUT:

An instance of `SageTimeitResult` unless the optional parameter `seconds=True` is passed. In that case, the elapsed time in seconds is returned as a floating-point number.

EXAMPLES:

```
sage: from sage.misc.sage_timeit import sage_timeit
sage: sage_timeit('3^100000', globals(), preparse=True, number=50)      # random output
'50 loops, best of 3: 1.97 ms per loop'
sage: sage_timeit('3^100000', globals(), preparse=False, number=50)    # random output
'50 loops, best of 3: 67.1 ns per loop'
sage: a = 10
sage: sage_timeit('a^2', globals(), number=50)                        # random output
'50 loops, best of 3: 4.26 us per loop'
```

If you only want to see the timing and not have access to additional information, just use the `timeit` object:

```
sage: timeit('10^2', number=50)
50 loops, best of 3: ... per loop
```

Using `sage_timeit` gives you more information though:

```
sage: s = sage_timeit('10^2', globals(), repeat=1000)
sage: len(s.series)
1000
sage: mean(s.series)      # random output
3.1298141479492283e-07
sage: min(s.series)      # random output
2.9258728027343752e-07
sage: t = stats.TimeSeries(s.series)
```

```
sage: t.scale(10^6).plot_histogram(bins=20,figsize=[12,6], ymax=2)
Graphics object consisting of 20 graphics primitives
```

The input expression can contain newlines (but doctests cannot, so we use `os.linesep` here):

```
sage: from sage.misc.sage_timeit import sage_timeit
sage: from os import linesep as CR
sage: # sage_timeit(r'a = 2\nb=131\nfactor(a^b-1)')
sage: sage_timeit('a = 2' + CR + 'b=131' + CR + 'factor(a^b-1)',
...               globals(), number=10)
10 loops, best of 3: ... per loop
```

Test to make sure that `timeit` behaves well with output:

```
sage: timeit("print 'Hi'", number=50)
50 loops, best of 3: ... per loop
```

If you want a machine-readable output, use the `seconds=True` option:

```
sage: timeit("print 'Hi'", seconds=True) # random output
1.42555236816e-06
sage: t = timeit("print 'Hi'", seconds=True)
sage: t #r random output
3.6010742187499999e-07
```

TESTS:

Make sure that garbage collection is re-enabled after an exception occurs in `timeit`:

```
sage: def f(): raise ValueError
sage: import gc
sage: gc.isenabled()
True
sage: timeit("f()")
Traceback (most recent call last):
...
ValueError
sage: gc.isenabled()
True
```

7.2.3 The `timeit` command

This uses the function `sage_timeit()`.

class `sage.misc.sage_timeit_class.SageTimeit`

Time execution of a command or block of commands.

Displays the best WALL TIME for execution of the given code. This is based on the Python `timeit` module, which avoids a number of common traps for measuring execution times. It is also based on IPython's `%timeit` command.

TYPICAL INPUT FORMAT:

```
timeit(statement, preparse=None, number=0, repeat=3, precision=3)
```

EXAMPLES:

```
sage: timeit('2^10000')
625 loops, best of 3: ... per loop
```


We illustrate some options:

```
sage: timeit('2+2', precision=2, number=20, repeat=5)
20 loops, best of 5: ... per loop
```

The preparer is on by default (if it is on), but the preparse option allows us to override it:

```
sage: timeit('2^10000', preparse=False, number=50)
50 loops, best of 3: ... per loop
```

The input can contain newlines:

```
sage: timeit("a = 2\nb=131\nfactor(a^b-1)", number=25)
25 loops, best of 3: ... per loop
```

See also:

`runsnake()`

eval (*code*, *globs*=None, *locals*=None, ***kws*)

This eval function is called when doing %timit in the notebook.

INPUT:

- *code* – string of code to evaluate; may contain newlines.
- *globs* – global variables; if not given, uses module scope globals.
- *locals* – ignored completely.
- *kws* – passed onto `sage_timeit`. Common options are `preparse`, `number`, `repeat`, `precision`. See `sage_timeit()` for details.

OUTPUT: string – timing information as a string

EXAMPLES:

```
sage: timeit.eval("2+2") # random output
'625 loops, best of 3: 1.47 us per loop'
```

We emphasize that `timeit` times WALL TIME. This is good in the context of Sage where commands often call out to other subprocesses that don't appear in CPU time.

```
sage: timeit('sleep(0.5)', number=3) # long time (5s on sage.math, 2012)
3 loops, best of 3: ... ms per loop
```

7.2.4 Simple profiling tool

AUTHORS:

- David Harvey (August 2006)
- Martin Albrecht

class `sage.misc.profiler.Profiler` (*systems*=[], *verbose*=False)

Keeps track of CPU time used between a series of user-defined checkpoints.

It's probably not a good idea to use this class in an inner loop :-)

EXAMPLE:

```
sage: def f(): # not tested
...     p = Profiler() # not tested
```

Calling `p(message)` creates a checkpoint:

```
sage: p("try factoring 15")           # not tested
```

Do something time-consuming:

```
sage: x = factor(15)                  # not tested
```

You can create a checkpoints without a string; `Profiler` will use the source code instead:

```
sage: p()                             # not tested
sage: y = factor(25)                   # not tested
sage: p("last step")                 # not tested
sage: z = factor(35)                 # not tested
sage: p()                             # not tested
```

This will give a nice list of timings between checkpoints:

```
sage: print p                         # not tested
```

Let's try it out:

```
sage: f()                             # not tested
      3.020s -- try factoring 15
      15.240s -- line 17: y = factor(25)
      5000.190s -- last step
```

See also:

```
runsnake()
```

Todo

- Add Pyrex source code inspection (I assume it doesn't currently do this)
- Add ability to sort output by time
- Add option to constructor to print timing immediately when checkpoint is reached
- Migrate to Pyrex?
- Add ability to return timings in a more machine-friendly format

AUTHOR:

- David Harvey (August 2006)

`clear()`

`print_last()`

Prints the last profiler step

7.2.5 C Function Profiler Using Google Perftools

Note that the profiler samples 100x per second by default. In particular, you cannot profile anything shorter than 10ms. You can adjust the rate with the `CPUPROFILE_FREQUENCY` environment variable if you want to change it.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()           # optional - gperftools
```

```
sage: run_100ms()
sage: prof.stop()          # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
```

REFERENCE:

Uses the [Google performance analysis tools](#). Note that they are not included in Sage, you have to install them yourself on your system.

AUTHORS:

- Volker Braun (2014-03-31): initial version

```
class sage.misc.gperftools.Profiler(filename=None)
    Bases: sage.structure.sage_object.SageObject
```

Interface to the gperftools profiler

INPUT:

- `filename` – string or None (default). The file name to log to. By default, a new temporary file is created.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler
sage: Profiler()
Profiler logging to ...
```

`filename()`

Return the file name

OUTPUT:

String.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler
sage: prof = Profiler()
sage: prof.filename()
'.../tmp_...perf'
```

```
save(filename, cumulative=True, verbose=True)
```

Save report to disk.

INPUT:

- `filename` – string. The filename to save at. Must end with one of `.dot`, `.ps`, `.pdf`, `.svg`, `.gif`, or `.txt` to specify the output file format.
- `cumulative` – boolean (optional, default: `True`). Whether to return cumulative timings.
- `verbose` – boolean (optional, default: `True`). Whether to print informational messages.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()      # optional - gperftools
sage: run_100ms()       # optional - gperftools
sage: prof.stop()       # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
sage: f = tmp_filename(ext='.txt')      # optional - gperftools
sage: prof.save(f, verbose=False)       # optional - gperftools
```

start ()

Start profiling

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()      # optional - gperftools
sage: run_100ms()
sage: prof.stop()       # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
```

stop ()

Stop the CPU profiler

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler, run_100ms
sage: prof = Profiler()
sage: prof.start()      # optional - gperftools
sage: run_100ms()
sage: prof.stop()       # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
```

top (cumulative=True)

Print text report

OUTPUT:

Nothing. A textual report is printed to stdout.

EXAMPLES:

```
sage: from sage.misc.gperftools import Profiler
sage: prof = Profiler()
sage: prof.start()      # optional - gperftools
sage: # do something
sage: prof.stop()       # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
sage: prof.top()        # optional - gperftools
Using local file ...
Using local file ...
```

sage.misc.gperftools.crun (s, evaluator)

Profile single statement.

- s – string. Sage code to profile.
- evaluator – callable to evaluate.

EXAMPLES:

```
sage: import sage.misc.gperftools as gperf
sage: ev = lambda ex:eval(ex, globals(), locals())
sage: gperf.crun('gperf.run_100ms()', evaluator=ev)  # optional - gperftools
PROFILE: interrupts/evictions/bytes = ...
Using local file ...
Using local file ...
```

sage.misc.gperftools.run_100ms ()

Used for doctesting.

A function that performs some computation for more than (but not that much more than) 100ms.

EXAMPLES:

```
sage: from sage.misc.gperftools import run_100ms
sage: run_100ms()
```

7.3 Miscellaneous Inspection and Development Tools

7.3.1 Inspect Python, Sage, and Cython objects

This module extends parts of Python's inspect module to Cython objects.

AUTHORS:

- originally taken from Fernando Perez's IPython
- William Stein (extensive modifications)
- Nick Alexander (extensions)
- Nick Alexander (testing)
- Simon King (some extension for Cython, generalisation of SageArgSpecVisitor)

EXAMPLES:

```
sage: from sage.misc.sageinspect import *
```

Test introspection of modules defined in Python and Cython files:

Cython modules:

```
sage: sage_getfile(sage.rings.rational)
'.../rational.pyx'

sage: sage_getdoc(sage.rings.rational).lstrip()
'Rational Numbers...'

sage: sage_getsource(sage.rings.rational)[5:]
'Rational Numbers...'
```

Python modules:

```
sage: sage_getfile(sage.misc.sageinspect)
'.../sageinspect.py'

sage: print sage_getdoc(sage.misc.sageinspect).lstrip()[40:]
Inspect Python, Sage, and Cython objects

sage: sage_getsource(sage.misc.sageinspect).lstrip()[5:-1]
'Inspect Python, Sage, and Cython objects...'
```

Test introspection of classes defined in Python and Cython files:

Cython classes:

```
sage: sage_getfile(sage.rings.rational.Rational)
'.../rational.pyx'

sage: sage_getdoc(sage.rings.rational.Rational).lstrip()
```

```
'A rational number...'
```

```
sage: sage_getsource(sage.rings.rational.Rational)
'cdef class Rational...'
```

Python classes:

```
sage: sage_getfile(BlockFinder)
'.../sage/misc/sageinspect.py'
```

```
sage: sage_getdoc(BlockFinder).lstrip()
'Provide a tokeneater() method to detect the...'
```

```
sage: sage_getsource(BlockFinder)
'class BlockFinder:...'
```

Test introspection of functions defined in Python and Cython files:

Cython functions:

```
sage: sage_getdef(sage.rings.rational.make_rational, obj_name='mr')
'mr(s)'
```

```
sage: sage_getfile(sage.rings.rational.make_rational)
'.../rational.pyx'
```

```
sage: sage_getdoc(sage.rings.rational.make_rational).lstrip()
'Make a rational number ...'
```

```
sage: sage_getsource(sage.rings.rational.make_rational)[4:]
'make_rational(s):...'
```

Python functions:

```
sage: sage_getdef(sage.misc.sageinspect.sage_getfile, obj_name='sage_getfile')
'sage_getfile(obj)'
```

```
sage: sage_getfile(sage.misc.sageinspect.sage_getfile)
'.../sageinspect.py'
```

```
sage: sage_getdoc(sage.misc.sageinspect.sage_getfile).lstrip()
'Get the full file name associated to "obj" as a string...'
```

```
sage: sage_getsource(sage.misc.sageinspect.sage_getfile)[4:]
'sage_getfile(obj):...'
```

Unfortunately, no argspec is extractable from builtins. Hence, we use a generic argspec:

```
sage: sage_getdef(''.find, 'find')
'find(*args, **kwargs)'
```

```
sage: sage_getdef(str.find, 'find')
'find(*args, **kwargs)'
```

By [trac ticket #9976](#) and [trac ticket #14017](#), introspection also works for interactively defined Cython code, and with rather tricky argument lines:

```

sage: cython('def foo(unsigned int x=1, a=\'\')"\', b={not (2+1==3):\\'bar\'}, *args, **kwds): return')
sage: print sage_getsource(foo)
def foo(unsigned int x=1, a=\'\')"\', b={not (2+1==3):\\'bar\'}, *args, **kwds): return
sage: sage_getargspec(foo)
ArgSpec(args=['x', 'a', 'b'], varargs='args', keywords='kwds', defaults=(1, '\')"', {False: 'bar'})

```

class `sage.misc.sageinspect.BlockFinder`

Provide a `token eater()` method to detect the end of a code block.

This is the Python library's `inspect.BlockFinder` modified to recognize Cython definitions.

token eater (*type, token, srow_scol, erow_ecol, line*)

class `sage.misc.sageinspect.SageArgSpecVisitor`

Bases: `ast.NodeVisitor`

A simple visitor class that walks an abstract-syntax tree (AST) for a Python function's argspec. It returns the contents of nodes representing the basic Python types: None, booleans, numbers, strings, lists, tuples, and dictionaries. We use this class in `_sage_getargspec_from_ast()` to extract an argspec from a function's or method's source code.

EXAMPLES:

```

sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: visitor.visit(ast.parse('[1,2,3]').body[0].value)
[1, 2, 3]
sage: visitor.visit(ast.parse("{'a':('e',2,[None,({False:True},'pi')]), 37.0:'temp'}").body[0].value)
{37.0: 'temp', 'a': ('e', 2, [None, ({False: True}, 'pi')])}
sage: v = ast.parse("jc = ['veni', 'vidi', 'vici']").body[0]; v
<_ast.Assign object at ...>
sage: [x for x in dir(v) if not x.startswith('__')]
['_attributes', '_fields', 'col_offset', 'lineno', 'targets', 'value']
sage: visitor.visit(v.targets[0])
'jc'
sage: visitor.visit(v.value)
['veni', 'vidi', 'vici']

```

visit_BinOp (*node*)

Visit a Python AST `ast.BinOp` node.

INPUT:

- *node* - the node instance to visit

OUTPUT:

- The result that node represents

AUTHOR:

- Simon King

EXAMPLES:

```

sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['(3+(2*4))', '7|8', '5^3', '7/3', '7//3', '3<<4']] #indirect doctest
[11, 15, 6, 2, 2, 48]

```

visit_BoolOp (*node*)

Visit a Python AST `ast.BoolOp` node.

INPUT:

- `node` - the node instance to visit

OUTPUT:

- The result that `node` represents

AUTHOR:

- Simon King

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['True and 1', 'False or 3 or None', '3 and 4']] #indirect doctest
[1, 3, 4]
```

visit_Compare (*node*)

Visit a Python AST `ast.Compare` node.

INPUT:

- `node` - the node instance to visit

OUTPUT:

- The result that `node` represents

AUTHOR:

- Simon King

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Compare(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['1<2==2!=3', '1==1>2', '1<2>1', '1<3<2<4']]
[True, False, True, False]
```

visit_Dict (*node*)

Visit a Python AST `ast.Dict` node.

INPUT:

- `node` - the node instance to visit

OUTPUT:

- the dictionary the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Dict(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['{}', '{1:one, 'two':2, other:bother}']]
[{}, {1: 'one', 'other': 'bother', 'two': 2}]
```


visit_List (*node*)

Visit a Python AST `ast.List` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- the list the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_List(ast.parse(x).body[0].value)
sage: [vis(l) for l in ['[]', "['s', 't', 'u']", '[[e], [], [pi]]']]
[[], ['s', 't', 'u'], [['e'], [], ['pi']]]
```

visit_Name (*node*)

Visit a Python AST `ast.Name` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- None, True, False, or the node's name as a string.

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Name(ast.parse(x).body[0].value)
sage: [vis(n) for n in ['True', 'False', 'None', 'foo', 'bar']]
[True, False, None, 'foo', 'bar']
sage: [type(vis(n)) for n in ['True', 'False', 'None', 'foo', 'bar']]
[<type 'bool'>, <type 'bool'>, <type 'NoneType'>, <type 'str'>, <type 'str'>]
```

visit_Num (*node*)

Visit a Python AST `ast.Num` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- the number the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Num(ast.parse(x).body[0].value)
sage: [vis(n) for n in ['123', '0.0', str(-pi.n())]]
[123, 0.0, -3.14159265358979]
```

visit_Str (*node*)

Visit a Python AST `ast.Str` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- the string the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Str(ast.parse(x).body[0].value)
sage: [vis(s) for s in ['"abstract"', "u'syntax'", '''r"tr\ee"''']]
['abstract', u'syntax', 'tr\\ee']
```

visit_Tuple(*node*)

Visit a Python AST `ast.Tuple` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- the tuple the node represents

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_Tuple(ast.parse(x).body[0].value)
sage: [vis(t) for t in ['()', '(x,y)', '("Au", "Al", "Cu)']]
[(), ('x', 'y'), ('Au', 'Al', 'Cu')]
```

visit_UnaryOp(*node*)

Visit a Python AST `ast.BinOp` node.

INPUT:

- node - the node instance to visit

OUTPUT:

- The result that node represents

AUTHOR:

- Simon King

EXAMPLES:

```
sage: import ast, sage.misc.sageinspect as sms
sage: visitor = sms.SageArgSpecVisitor()
sage: vis = lambda x: visitor.visit_UnaryOp(ast.parse(x).body[0].value)
sage: [vis(d) for d in ['+(3*2)', '-(3*2)']]
[6, -6]
```

`sage.misc.sageinspect.isclassinstance`(*obj*)

Checks if argument is instance of non built-in class

INPUT: *obj* - object

EXAMPLES:

```
sage: from sage.misc.sageinspect import isclassinstance
sage: isclassinstance(int)
False
sage: isclassinstance(FreeModule)
True
```

```

sage: class myclass: pass
sage: isinstance(myclass)
False
sage: class mymetaclass(type): pass
sage: class myclass2:
...     __metaclass__ = mymetaclass
sage: isinstance(myclass2)
False

```

```
sage.misc.sageinspect.loadable_module_extension()
```

Return the filename extension of loadable modules, including the dot. It is '.dll' on cygwin, '.so' otherwise.

EXAMPLES:

```

sage: from sage.misc.sageinspect import loadable_module_extension
sage: sage.structure.sage_object.__file__.endswith(loadable_module_extension())
True

```

```
sage.misc.sageinspect.sage_getargspec(obj)
```

Return the names and default values of a function's arguments.

INPUT:

obj, any callable object

OUTPUT:

An ArgSpec is returned. This is a named tuple (args, varargs, keywords, defaults).

- args is a list of the argument names (it may contain nested lists).
- varargs and keywords are the names of the * and ** arguments or None.
- defaults is an n -tuple of the default values of the last n arguments.

NOTE:

If the object has a method `_sage_argspec_` then the output of that method is transformed into a named tuple and then returned.

If a class instance has a method `_sage_src_` then its output is studied to determine the argspec. This is because currently the `CachedMethod` decorator has no `_sage_argspec_` method.

EXAMPLES:

```

sage: from sage.misc.sageinspect import sage_getargspec
sage: def f(x, y, z=1, t=2, *args, **keywords):
...     pass
sage: sage_getargspec(f)
ArgSpec(args=['x', 'y', 'z', 't'], varargs='args', keywords='keywords', defaults=(1, 2))

```

We now run `sage_getargspec` on some functions from the Sage library:

```

sage: sage_getargspec(identity_matrix)
ArgSpec(args=['ring', 'n', 'sparse'], varargs=None, keywords=None, defaults=(0, False))
sage: sage_getargspec(factor)
ArgSpec(args=['n', 'proof', 'int_', 'algorithm', 'verbose'], varargs=None, keywords='kwds', defaults=())

```

In the case of a class or a class instance, the ArgSpec of the `__new__`, `__init__` or `__call__` method is returned:

```

sage: P.<x,y> = QQ[]
sage: sage_getargspec(P)

```

```
ArgSpec(args=['self', 'x'], varargs='args', keywords='kwds', defaults=(0,))
sage: sage_getargspec(P.__class__)
ArgSpec(args=['self', 'x'], varargs='args', keywords='kwds', defaults=(0,))
```

The following tests against various bugs that were fixed in [trac ticket #9976](#):

```
sage: from sage.rings.polynomial.real_roots import bernstein_polynomial_factory_ratlist
sage: sage_getargspec(bernstein_polynomial_factory_ratlist.coeffs_bitsize)
ArgSpec(args=['self'], varargs=None, keywords=None, defaults=None)
sage: from sage.rings.polynomial.pbori import BooleanMonomialMonoid
sage: sage_getargspec(BooleanMonomialMonoid.gen)
ArgSpec(args=['self', 'i'], varargs=None, keywords=None, defaults=(0,))
sage: I = P*[x,y]
sage: sage_getargspec(I.groebner_basis)
ArgSpec(args=['self', 'algorithm', 'deg_bound', 'mult_bound', 'prot'],
varargs='args', keywords='kwds', defaults=('', None, None, False))
sage: cython("cdef int foo(x,y) except -1: return 1")
sage: sage_getargspec(foo)
ArgSpec(args=['x', 'y'], varargs=None, keywords=None, defaults=None)
```

If a `functools.partial` instance is involved, we see no other meaningful solution than to return the `argspec` of the underlying function:

```
sage: def f(a,b,c,d=1): return a+b+c+d
...
sage: import functools
sage: f1 = functools.partial(f, 1,c=2)
sage: sage_getargspec(f1)
ArgSpec(args=['a', 'b', 'c', 'd'], varargs=None, keywords=None, defaults=(1,))
```

TESTS:

By [trac ticket #9976](#), rather complicated cases work. In the following example, we dynamically create an extension class that returns some source code, and the example shows that the source code is taken for granted, i.e., the `argspec` of an instance of that class does not coincide with the `argspec` of its call method. That behaviour is intended, since a decorated method appears to have the generic signature `*args, **kwds`, but in fact it is only supposed to be called with the arguments requested by the underlying undecorated method. We saw an easy example above, namely `I.groebner_basis`. Here is a more difficult one:

```
sage: cython_code = [
... 'cdef class MyClass:',
... '    def _sage_src_(self):',
... '        return "def foo(x, a=\\'\\'\\')\\'\\'\\'\\'\\'\\', b={ (2+1):\\'\\'bar\\'\\'\\', not 1:3, 3<<4:5}): return',
... '    def __call__(self, m,n): return "something"]
sage: cython('\\n'.join(cython_code))
sage: O = MyClass()
sage: print sage.misc.sageinspect.sage_getsource(O)
def foo(x, a='')", b={ (2+1):'bar', not 1:3, 3<<4:5}): return
sage: sage.misc.sageinspect.sage_getargspec(O)
ArgSpec(args=['x', 'a', 'b'], varargs=None, keywords=None, defaults=('', {False: 3, 48: 5, 3:
sage: sage.misc.sageinspect.sage_getargspec(O.__call__)
ArgSpec(args=['self', 'm', 'n'], varargs=None, keywords=None, defaults=None)

sage: cython('def foo(x, a=\\'\\'\\')\\'\\'\\', b={not (2+1==3):\\'bar\\'\\}): return')
sage: print sage.misc.sageinspect.sage_getsource(foo)
def foo(x, a=\\'\\'\\')\\'\\'\\', b={not (2+1==3):'bar\\'\\}): return

sage: sage.misc.sageinspect.sage_getargspec(foo)
ArgSpec(args=['x', 'a', 'b'], varargs=None, keywords=None, defaults=('\\'\\'\\')", {False: 'bar\\'\\}))
```

The following produced a syntax error before the patch at [trac ticket #11913](#):

```
sage: sage.misc.sageinspect.sage_getargspec(r.lm)
```

The following was fixed in [trac ticket #16309](#):

```
sage: cython('''
....: class Foo:
....:     @staticmethod
....:     def join(categories, bint as_list = False, tuple ignore_axioms=(), tuple axioms=()): p
....: cdef class Bar:
....:     @staticmethod
....:     def join(categories, bint as_list = False, tuple ignore_axioms=(), tuple axioms=()): p
....:     cpdef meet(categories, bint as_list = False, tuple ignore_axioms=(), tuple axioms=()):
....: ''')
sage: sage_getargspec(Foo.join)
ArgSpec(args=['categories', 'as_list', 'ignore_axioms', 'axioms'], varargs=None, keywords=None,
sage: sage_getargspec(Bar.join)
ArgSpec(args=['categories', 'as_list', 'ignore_axioms', 'axioms'], varargs=None, keywords=None,
sage: sage_getargspec(Bar.meet)
ArgSpec(args=['categories', 'as_list', 'ignore_axioms', 'axioms'], varargs=None, keywords=None,
```

Test that [trac ticket #17009](#) is fixed:

```
sage: sage_getargspec(gap)
ArgSpec(args=['self', 'x', 'name'], varargs=None, keywords=None, defaults=(None,))
```

By [trac ticket #17814](#), the following gives the correct answer (previously, the defaults would have been found None):

```
sage: from sage.misc.nested_class import MainClass
sage: sage_getargspec(MainClass.NestedClass.NestedSubClass.dummy)
ArgSpec(args=['self', 'x', 'r'], varargs='args', keywords='kwds', defaults=((1, 2, 3.4),))
```

In [trac ticket #18249](#) was decided to return a generic signature for Python builtin functions, rather than to raise an error (which is what Python's inspect module does):

```
sage: import inspect
sage: inspect.getargspec(range)
Traceback (most recent call last):
...
TypeError: <built-in function range> is not a Python function
sage: sage_getargspec(range)
ArgSpec(args=[], varargs='args', keywords='kwds', defaults=None)
```

AUTHORS:

- William Stein: a modified version of inspect.getargspec from the Python Standard Library, which was taken from IPython for use in Sage.
- Extensions by Nick Alexander
- Simon King: Return an ArgSpec, fix some bugs.

```
sage.misc.sageinspect.sage_getdef(obj, obj_name='')
```

Return the definition header for any callable object.

INPUT:

- obj - function

- obj_name - string (optional, default '')

obj_name is prepended to the output.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getdef
sage: sage_getdef(identity_matrix)
'(ring, n=0, sparse=False)'
sage: sage_getdef(identity_matrix, 'identity_matrix')
'identity_matrix(ring, n=0, sparse=False)'
```

Check that [trac ticket #6848](#) has been fixed:

```
sage: sage_getdef(RDF.random_element)
'(min=-1, max=1)'
```

If an exception is generated, None is returned instead and the exception is suppressed.

AUTHORS:

- William Stein
- extensions by Nick Alexander

`sage.misc.sageinspect.sage_getdoc(obj, obj_name='', embedded_override=False)`

Return the docstring associated to obj as a string.

If obj is a Cython object with an embedded position in its docstring, the embedded position is stripped.

If optional argument `embedded_override` is False (its default value), then the string is formatted according to the value of `EMBEDDED_MODE`. If this argument is True, then it is formatted as if `EMBEDDED_MODE` were True.

INPUT:

- obj – a function, module, etc.: something with a docstring.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getdoc
sage: sage_getdoc(identity_matrix)[87:124]
'Return the n x n identity matrix over'
sage: def f(a,b,c,d=1): return a+b+c+d
...
sage: import functools
sage: f1 = functools.partial(f, 1,c=2)
sage: f.__doc__ = "original documentation"
sage: f1.__doc__ = "specialised documentation"
sage: sage_getdoc(f)
'original documentation\n'
sage: sage_getdoc(f1)
'specialised documentation\n'
```

AUTHORS:

- William Stein
- extensions by Nick Alexander

`sage.misc.sageinspect.sage_getdoc_original(obj)`

Return the unformatted docstring associated to obj as a string.

If obj is a Cython object with an embedded position or signature in its docstring, the embedded information is stripped. If the stripped docstring is empty, then the stripped docstring of `obj.__init__` is returned instead.

Feed the results from this into the function `sage.misc.sagedoc.format()` for printing to the screen.

INPUT:

•obj – a function, module, etc.: something with a docstring.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getdoc_original
```

Here is a class that has its own docstring:

```
sage: print sage_getdoc_original(sage.rings.integer.Integer)
```

```
The ``Integer`` class represents arbitrary precision
integers. It derives from the ``Element`` class, so
integers can be used as ring elements anywhere in Sage.
```

```
...
```

Here is a class that does not have its own docstring, so that the docstring of the `__init__` method is used:

```
sage: print sage_getdoc_original(Parent)
```

```
Base class for all parents.
```

```
Parents are the Sage/mathematical analogues of container
objects in computer science.
```

```
...
```

Old-style classes are supported:

```
sage: class OldStyleClass:
....:     def __init__(self):
....:         '''The __init__ docstring'''
....:         pass
sage: print sage_getdoc_original(OldStyleClass)
The __init__ docstring
```

When there is no `__init__` method, we just get an empty string:

```
sage: class OldStyleClass:
....:     pass
sage: sage_getdoc_original(OldStyleClass)
''
```

If an instance of a class does not have its own docstring, the docstring of its class results:

```
sage: sage_getdoc_original(sage.plot.colors.aliceblue) == sage_getdoc_original(sage.plot.colors)
True
```

`sage.misc.sageinspect.sage_getfile(obj)`

Get the full file name associated to `obj` as a string.

INPUT: `obj`, a Sage object, module, etc.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getfile
sage: sage_getfile(sage.rings.rational)[-23:]
'sage/rings/rational.pyx'
sage: sage_getfile(Sq)[-42:]
'sage/algebras/steenrod/steenrod_algebra.py'
```

The following tests against some bugs fixed in [trac ticket #9976](#):

```
sage: obj = sage.combinat.partition_algebra.SetPartitionsAk
sage: obj = sage.combinat.partition_algebra.SetPartitionsAk
sage: sage_getfile(obj)
'...sage/combinat/partition_algebra.py'
```

And here is another bug, fixed in [trac ticket #11298](#):

```
sage: P.<x,y> = QQ[]
sage: sage_getfile(P)
'...sage/rings/polynomial/multi_polynomial_libsingular.pyx'
```

A problem fixed in [trac ticket #16309](#):

```
sage: cython('''
....: class Bar: pass
....: cdef class Foo: pass
....: ''')
sage: sage_getfile(Bar)
'...pyx'
sage: sage_getfile(Foo)
'...pyx'
```

By [trac ticket #18249](#), we return an empty string for Python builtins. In that way, there is no error when the user types, for example, `range?`:

```
sage: sage_getfile(range)
''
```

AUTHORS:

- Nick Alexander
- Simon King

`sage.misc.sageinspect.sage_getsource(obj)`

Return the source code associated to `obj` as a string, or `None`.

INPUT:

- `obj` – function, etc.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getsource
sage: sage_getsource(identity_matrix)[19:60]
'identity_matrix(ring, n=0, sparse=False):'
sage: sage_getsource(identity_matrix)[19:60]
'identity_matrix(ring, n=0, sparse=False):'
```

AUTHORS:

- William Stein
- extensions by Nick Alexander

`sage.misc.sageinspect.sage_getsourcelines(obj)`

Return a pair (`[source_lines]`, starting line number) of the source code associated to `obj`, or `None`.

INPUT:

- `obj` – function, etc.

OUTPUT:

(source_lines, lineno) or None: source_lines is a list of strings, and lineno is an integer.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getsourcelines
sage: sage_getsourcelines(matrix)[1]
732
sage: sage_getsourcelines(matrix)[0][0][6:]
'MatrixFactory(object):\n'
```

TESTS:

```
sage: cython("""cpdef test_func(x,y): return""")
sage: sage_getsourcelines(test_func)
(['cpdef test_func(x,y): return\n'], 6)
```

The following tests that an instance of `functools.partial` is correctly dealt with (see [trac ticket #9976](#)):

```
sage: obj = sage.combinat.partition_algebra.SetPartitionsAk
sage: sage_getsourcelines(obj)
(['def create_set_partition_function(letter, k):\n',
...
'    raise ValueError("k must be an integer or an integer + 1/2")\n'], 34)
```

Here are some cases that were covered in :trac`11298`; note that line numbers may easily change, and therefore we do not test them:

```
sage: P.<x,y> = QQ[]
sage: I = P*[x,y]
sage: sage_getsourcelines(P)
(['cdef class MPolynomialRing_libsingular(MPolynomialRing_generic):\n',
'\n',
'    def __cinit__(self):\n',
...
sage: sage_getsourcelines(I)
(['class MPolynomialIdeal( MPolynomialIdeal_singular_repr, \\n',
...
sage: x = var('x')
sage: sage_getsourcelines(x)
(['cdef class Expression(CommutativeRingElement):\n',
'    cpdef object pyobject(self):\n',
...
sage: sage_getsourcelines(x)[0][-1]      # last line
'    return S\n'
```

We show some enhancements provided by [trac ticket #11768](#). First, we use a dummy parent class that has defined an element class by a nested class definition:

```
sage: from sage.misc.nested_class_test import TestNestedParent
sage: from sage.misc.sageinspect import sage_getsource
sage: P = TestNestedParent()
sage: E = P.element_class
sage: E.__bases__
(<class sage.misc.nested_class_test.TestNestedParent.Element at ...>,
 <class 'sage.categories.sets_cat.Sets.element_class'>)
sage: print sage_getsource(E)
class Element:
    "This is a dummy element class"
    pass
```

```
sage: print sage_getsource(P)
class TestNestedParent(UniqueRepresentation, Parent):
    ...
    class Element:
        "This is a dummy element class"
        pass
```

Here is another example that relies on a nested class definition in the background:

```
sage: C = AdditiveMagmas()
sage: HC = C.Homsets()
sage: sage_getsourcelines(HC)
(['    class Homsets(HomsetsCategory):\n', ...], ...)
```

Testing against a bug that has occurred during work on #11768:

```
sage: P.<x,y> = QQ[]
sage: I = P*[x,y]
sage: sage_getsourcelines(I)
(['class MPolynomialIdeal( MPolynomialIdeal_singular_repr, \\n',
  '                        MPolynomialIdeal_macaulay2_repr, \\n',
  '                        MPolynomialIdeal_magma_repr, \\n',
  '                        Ideal_generic ):\n',
  '    def __init__(self, ring, gens, coerce=True):\n',
  ...])
```

AUTHORS:

- William Stein
- Extensions by Nick Alexander
- Extension to interactive Cython code by Simon King
- Simon King: If a class has no docstring then let the class definition be found starting from the `__init__` method.
- Simon King: Get source lines for dynamic classes.

`sage.misc.sageinspect.sage_getvariablename(self, omit_underscore_names=True)`
Attempt to get the name of a Sage object.

INPUT:

- `self` – any object.
- `omit_underscore_names` – boolean, default `True`.

OUTPUT:

If the user has assigned an object `obj` to a variable name, then return that variable name. If several variables point to `obj`, return a sorted list of those names. If `omit_underscore_names` is `True` (the default) then omit names starting with an underscore “_”.

This is a modified version of code taken from <http://pythonic.pocoo.org/2009/5/30/finding-objects-names>, written by Georg Brandl.

EXAMPLES:

```
sage: from sage.misc.sageinspect import sage_getvariablename
sage: A = random_matrix(ZZ, 100)
sage: sage_getvariablename(A)
'A'
```

```
sage: B = A
sage: sage_getvariablename(A)
['A', 'B']
```

If an object is not assigned to a variable, an empty list is returned:

```
sage: sage_getvariablename(random_matrix(ZZ, 60))
[]
```

7.3.2 Edit the source code of Sage interactively

AUTHORS:

- Nils Bruin
- William Stein – touch up for inclusion in Sage.
- Simon King: Make it usable on extension classes that do not have a docstring; include this module into the reference manual and fix some syntax errors in the doc strings.

This module provides a routine to open the source file of a python object in an editor of your choice, if the source file can be figured out. For files that appear to be from the sage library, the path name gets modified to the corresponding file in the current branch, i.e., the file that gets copied into the library upon ‘sage -br’.

The editor to be run, and the way it should be called to open the requested file at the right line number, can be supplied via a template. For a limited number of editors, templates are already known to the system. In those cases it suffices to give the editor name.

In fact, if the environment variable `EDITOR` is set to a known editor, then the system will use that if no template has been set explicitly.

```
sage.misc.edit_module.edit (obj, editor=None, bg=None)
Open source code of obj in editor of your choice.
```

INPUT:

- `editor` – str (default: None); If given, use specified editor. Choice is stored for next time.

AUTHOR:

- Nils Bruin (2007-10-03)

EXAMPLES:

This is a typical example of how to use this routine.

```
# make some object obj
sage: edit(obj)      # not tested
```

Now for more details and customization:

```
sage: import sage.misc.edit_module as m
sage: m.set_edit_template("vi -c ${line} ${file}")
```

In fact, since `vi` is a well-known editor, you could also just use

```
sage: m.set_editor("vi")
```

To illustrate:

```
sage: m.edit_template.template
'vi -c ${line} ${file}'
```

And if your environment variable `EDITOR` is set to a recognised editor, you would not have to set anything.

To edit the source of an object, just type something like:

```
sage: edit(edit)           # not tested
```

`sage.misc.edit_module.edit_devel(self, filename, linenum)`

This function is for internal use and is called by IPython when you use the IPython commands `%edit` or `%ed`.

This hook calls the default implementation, but changes the filename for files that appear to be from the sage library: if the filename begins with ‘`SAGE_ROOT/local/lib/python.../site-packages`’, it replaces this by ‘`SAGE_ROOT/src`’.

EXAMPLES:

```
sage: %edit gcd           # indirect doctest, not tested
sage: %ed gcd             # indirect doctest, not tested
```

The above should open your favorite editor (as stored in the environment variable `EDITOR`) with the file in which `gcd` is defined, and when your editor supports it, also at the line in which `gcd` is defined.

`sage.misc.edit_module.file_and_line(obj)`

Look up source file and line number of `obj`.

If the file lies in the Sage library, the path name of the corresponding file in the current branch (i.e., the file that gets copied into the Sage library upon running ‘`sage -br`’). Note that the first line of a file is considered to be 1 rather than 0 because most editors think that this is the case.

AUTHORS:

- Nils Bruin (2007-10-03)
- Simon King (2011-05): Use `sageinspect` to get the file and the line.

EXAMPLES:

```
sage: import sage.misc.edit_module as edit_module
sage: edit_module.file_and_line(sage)
('...sage/__init__.py', 0)
```

The following tests against a bug that was fixed in trac ticket #11298:

```
sage: edit_module.file_and_line(x)
('...sage/symbolic/expression.pyx', ...)
```

`sage.misc.edit_module.set_edit_template(template_string)`

Sets default edit template string.

It should reference `${file}` and `${line}`. This routine normally needs to be called prior to using ‘`edit`’. However, if the editor set in the shell variable `EDITOR` is known, then the system will substitute an appropriate template for you. See `edit_module.template_defaults` for the recognised templates.

AUTHOR:

- Nils Bruin (2007-10-03)

EXAMPLES:

```
sage: from sage.misc.edit_module import set_edit_template
sage: set_edit_template("echo EDIT ${file}:${line}")
sage: edit(sage)           # not tested
EDIT /usr/local/sage/src/sage/__init__.py:1
```

`sage.misc.edit_module.set_editor(editor_name, opts='')`

Sets the editor to be used by the edit command by basic editor name.

Currently, the system only knows appropriate call strings for a limited number of editors. If you want to use another editor, you should set the whole edit template via `set_edit_template`.

AUTHOR:

- Nils Bruin (2007-10-05)

EXAMPLES:

```
sage: from sage.misc.edit_module import set_editor
sage: set_editor('vi')
sage: sage.misc.edit_module.edit_template.template
'vi -c ${line} ${file}'
```

`sage.misc.edit_module.template_fields(template)`

Given a `String.Template` object, returns the fields.

AUTHOR:

- Nils Bruin (2007-10-22)

EXAMPLES:

```
sage: from sage.misc.edit_module import template_fields
sage: from string import Template
sage: t=Template("Template ${one} with ${two} and ${three}")
sage: template_fields(t)
['three', 'two', 'one']
```

7.3.3 Get resource usage of process

AUTHORS:

- William Stein (2006-03-04): initial version

`sage.misc.getusage.VmB(VmKey)`

Function used internally by this module.

`sage.misc.getusage.get_memory_usage(t=None)`

Return memory usage.

INPUT:

- `t` - a float (default: `None`); output of an earlier call

OUTPUT:

- Linux - Returns float number (in megabytes)
- OS X - Returns float number (in megabytes) that matches `VSIZE` column of `top`
- Solaris or OpenSolaris - Returns float number (in megabytes) that matches `RSS` column of `prstat`. Depending on the memory usage, `prstat` will output the data in KB, MB or GB. In each case, the value returned by this function will always be in MB.
- FreeBSD - Returns float number (in megabytes) that matches `RSS` column of `ps -auxwww`
- other - not implemented for any other operating systems

EXAMPLES:

```
sage: t = get_memory_usage(); t # random
873.98046875
```

Note:

- Currently, `get_memory_usage()` calls `prstat` on Solaris and OpenSolaris to get the data it requires. In the long term, a better solution would be to use Solaris system calls.
 - In some instances, `top` may be used on OS X. This may break if the memory usage is greater than 9999 MB. However, normally `top` is not used on OS X.
-

```
sage.misc.getusage.linux_memory_usage()
```

Return memory usage in megabytes.

```
sage.misc.getusage.top()
```

Return the ‘top’ or ‘prstat’ line that contains this running Sage process. For FreeBSD, return the line containing this running Sage process from ‘ps -axwww -o pid,user,vsz,rss,state,pri,nice,time,cpu,comm’.

OUTPUT:

- a string

EXAMPLES:

```
sage: top() # random output
'72373 python      0.0%  0:01.36   1    14+  1197   39M+   34M+   55M+  130M+'

```

NOTES:

The external command ‘top’ (<http://www.unixtop.org/>) is called on Linux, and most other operating systems. The output format of ‘top’ is not consistent across all platforms and all versions of ‘top’. If the `top()` function does not work in Sage, you may need to install ‘top’.

The external command ‘prstat’ is called on the Solaris and OpenSolaris systems. That is part of Solaris, and will not need to be installed. The columns used in the ‘prstat’ output are:

PID	USERNAME	SIZE	RSS	STATE	PRI	NICE	TIME	CPU	PROCESS/NLWP
-----	----------	------	-----	-------	-----	------	------	-----	--------------

7.3.4 Class inheritance graphs

```
sage.misc.classgraph.class_graph(top,    depth=5,    name_filter=None,    classes=None,
                                   as_graph=True)
```

Returns the class inheritance graph of a module, class, or object

INPUT:

- `top` – the module, class, or object to start with (e.g. `sage`, `Integer`, `3`)
- `depth` – maximal recursion depth within submodules (default: `5`)
- `name_filter` – e.g. ‘`sage.rings`’ to only consider classes in `sage.rings`
- `classes` – optional dictionary to be filled in (it is also returned)
- `as_graph` – a boolean (default: `True`)

OUTPUT:

- An oriented graph, with class names as vertices, and an edge from each class to each of its bases.

EXAMPLES:

We construct the inheritance graph of the classes within a given module:

```
sage: from sage.rings.polynomial.padics import polynomial_padic_capped_relative_dense, polynomial_padic_flat
sage: G = class_graph(sage.rings.polynomial.padics); G
Digraph on 6 vertices
sage: G.vertices()
['Polynomial', 'Polynomial_generic_dense', 'Polynomial_generic_domain', 'Polynomial_padic', 'Polynomial_padic_capped_relative_dense', 'Polynomial_padic_flat']
sage: G.edges(labels=False)
[('Polynomial_padic', 'Polynomial'), ('Polynomial_padic_capped_relative_dense', 'Polynomial_padic'), ('Polynomial_padic_flat', 'Polynomial_padic'), ('Polynomial_generic_dense', 'Polynomial'), ('Polynomial_generic_domain', 'Polynomial')]
```

We construct the inheritance graph of a given class:

```
sage: class_graph(Parent).edges(labels=False)
[('CategoryObject', 'SageObject'), ('Parent', 'CategoryObject'), ('SageObject', 'object')]
```

We construct the inheritance graph of the class of an object:

```
sage: class_graph([1, 2, 3]).edges(labels=False)
[('list', 'object')]
```

Warning: the output of `class_graph` used to be a dictionary mapping each class name to the list of names of its bases. This can be emulated by setting the option `as_graph` to `False`:

```
sage: class_graph(sage.rings.polynomial.padics, depth=2, as_graph=False)
{'Polynomial_padic': ['Polynomial'],
 'Polynomial_padic_capped_relative_dense': ['Polynomial_generic_domain', 'Polynomial_padic'],
 'Polynomial_padic_flat': ['Polynomial_generic_dense', 'Polynomial_padic']}
```

Note: the `classes` and `as_graph` options are mostly intended for internal recursive use.

Note: `class_graph` does not yet handle nested classes

TESTS:

```
sage: G = class_graph(sage.rings.polynomial.padics, depth=2); G
Digraph on 6 vertices
```

7.3.5 Some tools for developers

AUTHORS:

- Nicolas M. Thiery: initial version
- Vincent Delecroix (2012 and 2013): improve `import_statements`

`sage.misc.dev_tools.find_object_modules(obj)`

Return a dictionary whose keys are the names of the modules where `obj` appear and the value at a given module name is the list of names that `obj` have in that module.

It is very unlikely that the output dictionary has several keys except when `obj` is an instance of a class.

EXAMPLES:

```
sage: from sage.misc.dev_tools import find_object_modules
sage: find_object_modules(RR)
```

```
{'sage.rings.real_mpfr': ['RR']}
sage: find_object_modules(ZZ)
{'sage.rings.integer_ring': ['Z', 'ZZ']}
```

Note: It might be a good idea to move this function in `sage.misc.sageinspect`.

`sage.misc.dev_tools.find_objects_from_name` (*name*, *module_name=None*)

Return the list of objects from *module_name* whose name is *name*.

If *name* is in the global namespace, the result is a list of length 1 that contains only this object. Otherwise, the function runs through all loaded modules and returns the list of objects whose name matches *name*.

If *module_name* is not `None`, then search only in submodules of *module_name*.

In order to search through more modules you might use the function `load_submodules()`.

EXAMPLES:

```
sage: import sage.misc.dev_tools as dt
sage: dt.find_objects_from_name('FareySymbol')
[<type 'sage.modular.arithgroup.farey_symbol.Farey'>]

sage: import sympy
sage: dt.find_objects_from_name('RR')
[Real Field with 53 bits of precision, RR]
sage: dt.find_objects_from_name('RR', 'sage')
[Real Field with 53 bits of precision]
sage: dt.find_objects_from_name('RR', 'sympy')
[RR]
```

Examples that do not belong to the global namespace but in a loaded module:

```
sage: 'find_objects_from_name' in globals()
False
sage: objs = dt.find_objects_from_name('find_objects_from_name')
sage: len(objs)
1
sage: dt.find_objects_from_name is dt.find_objects_from_name
True
```

`sage.misc.dev_tools.import_statement_string` (*module*, *names*, *lazy*)

Return a (lazy) import statement for *names* from *module*.

INPUT:

- *module* – the name of a module
- *names* – a list of 2-tuples containing names and alias to import
- *lazy* – a boolean: whether to return a lazy import statement

EXAMPLES:

```
sage: import sage.misc.dev_tools as dt
sage: modname = 'sage.misc.dev_tools'
sage: names_and_aliases = [('import_statement_string', 'iss')]
sage: dt.import_statement_string(modname, names_and_aliases, False)
'from sage.misc.dev_tools import import_statement_string as iss'
sage: dt.import_statement_string(modname, names_and_aliases, True)
"lazy_import('sage.misc.dev_tools', 'import_statement_string', 'iss')"
sage: dt.import_statement_string(modname, [('a', 'b'), ('c', 'c'), ('d', 'e')], False)
```



```
'from sage.misc.dev_tools import a as b, c, d as e'
sage: dt.import_statement_string(modname, [(None, None)], False)
'import sage.misc.dev_tools'
```

`sage.misc.dev_tools.import_statements(*objects, **kws)`

Print import statements for the given objects.

INPUT:

- `*objects` – a sequence of objects or names.
- `lazy` – a boolean (default: `False`) Whether to print a lazy import statement.
- `verbose` – a boolean (default: `True`) Whether to print information in case of ambiguity.
- `answer_as_str` – a boolean (default: `False`) If `True` return a string instead of printing the statement.

EXAMPLES:

```
sage: import_statements(WeylGroup, lazy_attribute)
from sage.combinat.root_system.weyl_group import WeylGroup
from sage.misc.lazy_attribute import lazy_attribute
```

```
sage: import_statements(IntegerRing)
from sage.rings.integer_ring import IntegerRing
```

If `lazy` is `True`, then `lazy_import()` statements are displayed instead:

```
sage: import_statements(WeylGroup, lazy_attribute, lazy=True)
from sage.misc.lazy_import import lazy_import
lazy_import('sage.combinat.root_system.weyl_group', 'WeylGroup')
lazy_import('sage.misc.lazy_attribute', 'lazy_attribute')
```

In principle, the function should also work on object which are instances. In case of ambiguity, one or two warning lines are printed:

```
sage: import_statements(RDF)
from sage.rings.real_double import RDF

sage: import_statements(ZZ)
# ** Warning **: several names for that object: Z, ZZ
from sage.rings.integer_ring import Z

sage: import_statements(euler_phi)
from sage.rings.arith import euler_phi

sage: import_statements(x)
from sage.calculus.predefined import x
```

If you don't like the warning you can disable them with the option `verbose`:

```
sage: import_statements(ZZ, verbose=False)
from sage.rings.integer_ring import Z

sage: import_statements(x, verbose=False)
from sage.calculus.predefined import x
```

If the object has several names, an other way to get the import statement you expect is to use a string instead of the object:

```
sage: import_statements(matrix)
# ** Warning **: several names for that object: Matrix, matrix
from sage.matrix.constructor import Matrix

sage: import_statements('cached_function')
from sage.misc.cachefunc import cached_function
sage: import_statements('Z')
# **Warning**: distinct objects with name 'Z' in:
#   - sage.calculus.predefined
#   - sage.rings.integer_ring
from sage.rings.integer_ring import Z
```

Specifying a string is also useful for objects that are not imported in the Sage interpreter namespace by default. In this case, an object with that name is looked up in all the modules that have been imported in this session:

```
sage: import_statement_string
Traceback (most recent call last):
...
NameError: name 'import_statement_string' is not defined

sage: import_statements("import_statement_string")
from sage.misc.dev_tools import import_statement_string
```

Sometimes objects are imported as an alias (from XXX import YYY as ZZZ) or are affected (XXX = YYY) and the function might detect it:

```
sage: import_statements('FareySymbol')
from sage.modular.arithgroup.farey_symbol import Farey as FareySymbol

sage: import_statements('sum')
from sage.misc.functional import symbolic_sum as sum

sage: import_statements('power')
from sage.structure.element import generic_power as power
```

In order to be able to detect functions that belong to a non-loaded module, you might call the helper `load_submodules()` as in the following:

```
sage: import_statements('EnumeratedSetFromIterator')
Traceback (most recent call last):
...
LookupError: no object named 'EnumeratedSetFromIterator'
sage: from sage.misc.dev_tools import load_submodules
sage: load_submodules(sage.sets)
load sage.sets.real_set... succeeded
load sage.sets.set_from_iterator... succeeded
sage: import_statements('EnumeratedSetFromIterator')
from sage.sets.set_from_iterator import EnumeratedSetFromIterator
```

We test different objects which have no appropriate answer:

```
sage: import_statements('my_tailor_is_rich')
Traceback (most recent call last):
...
LookupError: no object named 'my_tailor_is_rich'
sage: import_statements(5)
Traceback (most recent call last):
...
ValueError: no import statement found for '5'.
```

We test that it behaves well with lazy imported objects ([trac ticket #14767](#)):

```
sage: import_statements(NN)
from sage.rings.semirings.non_negative_integer_semiring import NN
sage: import_statements('NN')
from sage.rings.semirings.non_negative_integer_semiring import NN
```

Deprecated lazy imports are ignored (see [trac ticket #17458](#)):

```
sage: lazy_import('sage.all', 'RR', 'deprecated_RR', namespace=sage.__dict__, deprecation=17458)
sage: import_statements('deprecated_RR')
Traceback (most recent call last):
...
LookupError: object named 'deprecated_RR' is deprecated (see trac ticket 17458)
sage: lazy_import('sage.all', 'RR', namespace=sage.__dict__, deprecation=17458)
sage: import_statements('RR')
from sage.rings.real_mpfr import RR
```

The following were fixed with [trac ticket #15351](#):

```
sage: import_statements('Rationals')
from sage.rings.rational_field import RationalField as Rationals
sage: import_statements(sage.combinat.partition_algebra.SetPartitionsAk)
from sage.combinat.partition_algebra import SetPartitionsAk
sage: import_statements(CIF)
from sage.rings.all import CIF
sage: import_statements(NaN)
from sage.symbolic.constants import NaN
sage: import_statements(pi)
from sage.symbolic.constants import pi
sage: import_statements('SAGE_ENV')
from sage.env import SAGE_ENV
sage: import_statements('graph_decompositions')
import sage.graphs.graph_decompositions
```

Note: The programmers try to made this function as smart as possible. Nevertheless it is far from being perfect (for example it does not detect deprecated stuff). So, if you use it, double check the answer and report weird behaviors.

`sage.misc.dev_tools.load_submodules` (*module=None, exclude_pattern=None*)

Load all submodules of a given modules.

This method is intended to be used by developers and especially the one who uses `import_statements()`. By default it load the sage library and it takes around a minute.

INPUT:

- `module` - an optional module
- `exclude_pattern` - an optional regular expression pattern of module names that have to be excluded.

EXAMPLES:

```
sage: sage.misc.dev_tools.load_submodules(sage.combinat)
load sage.combinat.algebraic_combinatorics... succeeded
...
load sage.combinat.words.suffix_trees... succeeded
```

Calling a second time has no effect (since the function does not import modules already imported):

```
sage: sage.misc.dev_tools.load_submodules(sage.combinat)
```

The second argument allows to exclude a pattern:

```
sage: sage.misc.dev_tools.load_submodules(sage.geometry, "database$|lattice")
load sage.geometry.fan_isomorphism... succeeded
load sage.geometry.hyperplane_arrangement.affine_subspace... succeeded
...
load sage.geometry.riemannian_manifolds.surface3d_generators... succeeded

sage: sage.misc.dev_tools.load_submodules(sage.geometry)
load sage.geometry.polyhedron.lattice_euclidean_group_element... succeeded
load sage.geometry.polyhedron.palp_database... succeeded
load sage.geometry.polyhedron.ppl_lattice_polygon... succeeded
```

```
sage.misc.dev_tools.module_names_cmp(x, y)
```

A comparison function for module names.

This function first compares the depth of the modules and then breaks ties by alphabetical order.

See also:

This function is used in `import_statements()`.

TESTS:

```
sage: from sage.misc.dev_tools import module_names_cmp
sage: l = ['a', 'b', 'a.a', 'a.b', 'b.a', 'b.b']
sage: sorted(l, cmp=module_names_cmp)
['a', 'b', 'a.a', 'a.b', 'b.a', 'b.b']
```

```
sage.misc.dev_tools.runsnake(command)
```

Graphical profiling with runsnake

INPUT:

- `command` – the command to be run as a string.

EXAMPLES:

```
sage: runsnake("list(SymmetricGroup(3))") # optional - runsnake
```

`command` is first preparsed (see `preparse()`):

```
sage: runsnake('for x in range(1,4): print x^2') # optional - runsnake
1
4
9
```

`runsnake()` requires the program `runsnake`. Due to non trivial dependencies (`python-wxgtk`, ...), installing it within the Sage distribution is unpractical. Hence, we recommend installing it with the system wide Python. On Ubuntu 10.10, this can be done with:

```
> sudo apt-get install python-profiler python-wxgtk2.8 python-setuptools
> sudo easy_install RunSnakeRun
```

See the `runsnake` website for instructions for other platforms.

`runsnake()` further assumes that the system wide Python is installed in `/usr/bin/python`.

See also:

- The runsnake website
- %prun
- Profiler

7.3.6 Function Mangling

This module provides utilities for extracting information about python functions.

AUTHORS:

- Tom Boothby (2009): Original version in Python
- Simon King (2011): Use Cython. Speedup of `fix_to_pos`, cleaning documentation.

class `sage.misc.function_mangling.ArgumentFixer`
 Bases: `object`

This class provides functionality to normalize the arguments passed into a function. While the various ways of calling a function are perfectly equivalent from the perspective of the callee, they don't always look the same for an object watching the caller. For example,

```
sage: def f(x = 10):
...     return min(1, x)
```

the following calls are equivalent,

```
sage: f()
1
sage: f(10)
1
sage: f(x=10)
1
```

but from the perspective of a wrapper, they are different:

```
sage: def wrap(g):
...     def _g(*args, **kwargs):
...         print args, kwargs
...         return g(*args, **kwargs)
...     return _g
sage: h = wrap(f)
sage: t = h()
() {}
sage: t = h(10)
(10,) {}
sage: t = h(x=10)
() {'x': 10}
```

For the purpose of cached functions, it is important not to distinguish between these uses.

INPUTS:

- `f` – a function
- `classmethod` – boolean (default `False`) – `True` if the function is a classmethod and therefore the first argument is expected to be the class instance. In that case, we ignore the first argument.

EXAMPLES:

```

sage: from sage.misc.function_mangling import ArgumentFixer
sage: def wrap2(g):
...     af = ArgumentFixer(g)
...     def _g(*args, **kwargs):
...         print af.fix_to_pos()
...         return g(*args, **kwargs)
...     return _g
sage: h2 = wrap2(f)
sage: t = h2()
((10,), ())
sage: t = h2(10)
((10,), ())
sage: t = h2(x=10)
((10,), ())

sage: class one:
...     def __init__(self, x = 1):
...         self.x = x
sage: af = ArgumentFixer(one.__init__.__func__, classmethod=True)
sage: af.fix_to_pos(1, 2, 3, a=31, b=2, n=3)
((1, 2, 3), (('a', 31), ('b', 2), ('n', 3)))

```

defaults_to_pos (*Args*)

f

fix_to_named (*args, **kwargs)

Normalize the arguments with a preference for named arguments.

INPUT:

- any positional and named arguments.

OUTPUT:

We return a tuple

$$(e_1, e_2, \dots, e_k), ((n_1, v_1), \dots, (n_m, v_m))$$

where n_1, \dots, n_m are the names of the arguments and v_1, \dots, v_m are the values passed in; and e_1, \dots, e_k are the unnamed arguments. We minimize k .

The defaults are extracted from the function and filled into the list K of named arguments. The names n_1, \dots, n_t are in order of the function definition, where t is the number of named arguments. The remaining names, n_{t+1}, \dots, n_m are given in alphabetical order. This is useful to extract the names of arguments, but **does not** maintain equivalence of

```

A, K = self.fix_to_pos(...)
self.f(*A, **dict(K)) `

```

and

```
self.f(...)
```

in all cases.

EXAMPLE:

```

sage: from sage.misc.function_mangling import ArgumentFixer
sage: def sum3(a, b, c=3, *args, **kwargs):
...     return a+b+c
sage: AF = ArgumentFixer(sum3)

```

```

sage: AF.fix_to_named(1,2,3,4,5,6,f=14,e=16)
((4, 5, 6), (('a', 1), ('b', 2), ('c', 3), ('e', 16), ('f', 14)))
sage: AF.fix_to_named(1,2,f=14)
((), (('a', 1), ('b', 2), ('c', 3), ('f', 14)))

```

fix_to_pos (*args, **kws)

Normalize the arguments with a preference for positional arguments.

INPUT:

Any positional or named arguments

OUTPUT:

We return a tuple

$$(e_1, e_2, \dots, e_k), ((n_1, v_1), \dots, (n_m, v_m))$$

where n_1, \dots, n_m are the names of the arguments and v_1, \dots, v_m are the values passed in; and e_1, \dots, e_k are the unnamed arguments. We minimize m .

The commands

```

A,K = self.fix_to_pos(...)
self.f(*A,**dict(K))

```

are equivalent to

```
self.f(...)
```

though defaults are extracted from the function and appended to the tuple A of positional arguments. The names n_1, \dots, n_m are given in alphabetical order.

EXAMPLE:

```

sage: from sage.misc.function_mangling import ArgumentFixer
sage: def do_something(a,b,c=3,*args,**kwargs):
...     print a,b,c, args, kwargs
sage: AF = ArgumentFixer(do_something)
sage: A,K = AF.fix_to_pos(1,2,3,4,5,6,f=14,e=16); print A,K
(1, 2, 3, 4, 5, 6) (('e', 16), ('f', 14))
sage: do_something(*A,**dict(K))
1 2 3 (4, 5, 6) {'e': 16, 'f': 14}
sage: do_something(1,2,3,4,5,6,f=14,e=16)
1 2 3 (4, 5, 6) {'e': 16, 'f': 14}

```

7.3.7 Information about available RAM/swap

There is no portable way to figure these out, nor should you generally have to. But GAP currently needs to allocate a cache of fixed size upon startup, and we would like a certain fraction of the swap address space.

EXAMPLES:

```

sage: from sage.misc.memory_info import MemoryInfo, MemoryInfo_proc
sage: mem = MemoryInfo()
sage: mem.total_ram()           # random output
16708194304
sage: mem.available_ram()      # random output
1690738688
sage: mem.total_swap()         # random output

```

```
15728635904
sage: mem.available_swap()      # random output
15340593152
```

`sage.misc.memory_info.MemoryInfo()`
Provide information about memory

OUTPUT:

A class that encapsulates memory information. If no method for the particular host OS is provided, reasonable guesses are given.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo, MemoryInfo_proc
sage: mem = MemoryInfo()
sage: mem.total_ram()           # random output
16708194304
sage: mem.available_ram()      # random output
1690738688
sage: mem.total_swap()         # random output
15728635904
sage: mem.available_swap()     # random output
15340593152
```

class `sage.misc.memory_info.MemoryInfo_OSX`
Bases: `sage.misc.memory_info.MemoryInfo_base`

Memory info on OSX

TESTS:

```
sage: from sage.misc.memory_info import MemoryInfo_OSX
```

available_ram()
Return the available (free) RAM size

OUTPUT:

Integer. The free RAM size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().available_ram() > 0
True
```

available_swap()
Return the available (free) swap size
The OSX swap file is growing dynamically, so we just return twice the available ram.

OUTPUT:

Integer. The free swap size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().available_swap() in ZZ
True
```

total_ram()
Return the total RAM size

OUTPUT:

Integer. The RAM size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().total_ram() > 0
True
```

total_swap()

Return the total swap size

The OSX swap file is growing dynamically, so we just return twice the total ram.

OUTPUT:

Integer. The swap size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().total_swap() >= 0
True
```

class sage.misc.memory_info.**MemoryInfo_base**

Bases: sage.structure.sage_object.SageObject

Base class for memory info objects.

rlimit_address_space()

Return RLIMIT_AS.

OUTPUT:

Integer. The limit in bytes or -1 if no limit is set or cannot be found out.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: mem = MemoryInfo()
sage: mem.rlimit_address_space() in ZZ
True
```

virtual_memory_limit()

Return the upper limit for virtual memory usage

This is the value set by `ulimit -v` at the command line or a practical limit if no limit is set.

OUTPUT:

Integer. The virtual memory limit in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: mem = MemoryInfo()
sage: mem.virtual_memory_limit() > 0
True
```

class sage.misc.memory_info.**MemoryInfo_guess**

Bases: sage.misc.memory_info.MemoryInfo_base

Guess memory as a fallback.

TESTS:

```
sage: from sage.misc.memory_info import MemoryInfo_guess
sage: mem = MemoryInfo_guess()
sage: mem.total_ram()
4294967296      # 64-bit
4294967296L    # 32-bit
```

available_ram()

Return the available (free) RAM size

OUTPUT:

Integer. The free RAM size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().available_ram() > 0
True
```

available_swap()

Return the available (free) swap size

OUTPUT:

Integer. The free swap size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().available_swap() in ZZ
True
```

total_ram()

Return the total RAM size

OUTPUT:

Integer. The RAM size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().total_ram() > 0
True
```

total_swap()

Return the total swap size

OUTPUT:

Integer. The swap size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().total_swap() >= 0
True
```

class sage.misc.memory_info.**MemoryInfo_proc**

Bases: sage.misc.memory_info.MemoryInfo_base

Provide information from /proc/ pseudo-filesystem on most UNIXes

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: mem = MemoryInfo()
sage: mem.total_ram()      # random output
16708194304
```

available_ram()

Return the available (free) RAM size

OUTPUT:

Integer. The free RAM size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().available_ram() > 0
True
```

available_swap()

Return the available (free) swap size

OUTPUT:

Integer. The free swap size in bytes, excluding reserved swap space. Can be negative if the system is overcommitting memory.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().available_swap() in ZZ
True
```

total_ram()

Return the total RAM size

OUTPUT:

Integer. The RAM size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().total_ram() > 0
True
```

total_swap()

Return the total swap size

OUTPUT:

Integer. The swap size in bytes.

EXAMPLES:

```
sage: from sage.misc.memory_info import MemoryInfo
sage: MemoryInfo().total_swap() >= 0
True
```


LOW-LEVEL UTILITIES

8.1 Low-level memory allocation functions

TESTS:

Check that a `MemoryError` is raised if we try to allocate a ridiculously large integer, see [trac ticket #15363](#):

```
sage: 2^(2^63-2)
Traceback (most recent call last):
...
RuntimeError: exponent must be at most 2147483647          # 32-bit
MemoryError: failed to allocate 1152921504606847008 bytes # 64-bit
```

AUTHORS:

- Jeroen Demeyer (2011-01-13): initial version ([trac ticket #10258](#))
- Jeroen Demeyer (2014-12-14): add more functions ([trac ticket #10257](#))
- Jeroen Demeyer (2015-03-02): move from `c_lib` to Cython ([trac ticket #17881](#))

```
sage.ext.memory.init_memory_functions()
Set the MPIR/GMP memory functions to the above functions.
```

EXAMPLES:

```
sage: from sage.ext.memory import init_memory_functions
sage: init_memory_functions()
```

8.2 The C3 algorithm

The C3 algorithm is used as method resolution order for new style classes in Python. The implementation here is used to order the list of super categories of a category.

AUTHOR:

- Simon King (2011-11): initial version.

```
sage.misc.c3.C3_algorithm(start, bases, attribute, proper)
An implementation of the C3 algorithm.
```

C3 is the algorithm used by Python to construct the method resolution order for new style classes involving multiple inheritance.

After [trac ticket #11943](#) this implementation was used to compute the list of super categories of a category; see `all_super_categories()`. The purpose is to ensure that list of super categories matches with the method resolution order of the parent or element classes of a category.

Since [trac ticket #13589](#), this implementation is superseded by that in `sage.misc.c3_controlled`, that puts the C3 algorithm under control of some total order on categories. This guarantees that C3 always finds a consistent Method Resolution Order. For background, see `sage.misc.c3_controlled`.

INPUT:

- `start` – an object; the returned list is built upon data provided by certain attributes of `start`.
- `bases` – a string; the name of an attribute of `start` providing a list of objects.
- `attribute` – a string; the name of an attribute of the objects provided in `getattr(start, bases)`. That attribute is supposed to provide a list.

ASSUMPTIONS:

Our implementation of the algorithm only works on lists of objects that compare equal if and only if they are identical.

OUTPUT:

A list, the result of the C3 algorithm applied to the list `[getattr(X, attribute) for X in getattr(start, bases)]`.

EXAMPLES:

We create a class for elements in a hierarchy that uses the C3 algorithm to compute, for each element, a linear extension of the elements above it:

```
.. TODO:: Move back the __init__ at the beginning
```

```
sage: from sage.misc.c3 import C3_algorithm
sage: class HierarchyElement(UniqueRepresentation):
.....: @lazy_attribute .....: def _all_bases(self): .....: return C3_algorithm(self, '_bases', '_all_bases',
False) .....: def __repr__(self): .....: return self._name .....: def __init__(self, name, bases): .....:
self._name = name .....: self._bases = list(bases)
```

We construct a little hierarchy:

```
sage: T = HierarchyElement("T", ())
sage: X = HierarchyElement("X", (T,))
sage: Y = HierarchyElement("Y", (T,))
sage: A = HierarchyElement("A", (X, Y))
sage: B = HierarchyElement("B", (Y, X))
sage: Foo = HierarchyElement("Foo", (A, B))
```

And inspect the linear extensions associated to each element:

```
sage: T._all_bases
[T]
sage: X._all_bases
[X, T]
sage: Y._all_bases
[Y, T]
sage: A._all_bases
[A, X, Y, T]
sage: B._all_bases
[B, Y, X, T]
```

So far so good. However:

```

sage: Foo.__all__bases
Traceback (most recent call last):
...
ValueError: Can not merge the items X, Y.

```

The C3 algorithm is not able to create a consistent linear extension. Indeed, its specifications impose that, if X and Y appear in a certain order in the linear extension for an element of the hierarchy, then they should appear in the same order for any lower element. This is clearly not possible for Foo, since A and B impose incompatible orders. If the above was a hierarchy of classes, Python would complain that it cannot calculate a consistent Method Resolution Order.

TESTS:

Regression test for bug #1 of [trac ticket #13501](#):

```

sage: class C(object): pass
sage: class F(object): pass
sage: class G(object): pass
sage: class B(C,F):      pass
sage: class D(F,G):      pass
sage: class E(F):        pass
sage: class A(B,D,E):    pass
sage: [cls.__name__ for cls in A.mro()]
['A', 'B', 'C', 'D', 'E', 'F', 'G', 'object']

sage: C = HierarchyElement("C", ())
sage: F = HierarchyElement("F", ())
sage: G = HierarchyElement("G", ())
sage: B = HierarchyElement("B", (C, F))
sage: D = HierarchyElement("D", (F, G))
sage: E = HierarchyElement("E", (F,))
sage: A = HierarchyElement("A", (B, D, E))
sage: A.__all__bases
[A, B, C, D, E, F, G]

```

Regression test for bug #2 of [trac ticket #13501](#). The following should fail since A asks for B to come before C, where as B is a super class of C:

```

sage: class B(object): pass
sage: class C(B): pass
sage: class A(B, C): pass
Traceback (most recent call last):
...
TypeError: Error when calling the metaclass bases
    Cannot create a consistent method resolution
    order (MRO) for bases ...

sage: B = HierarchyElement("B", ())
sage: C = HierarchyElement("C", (B,))
sage: A = HierarchyElement("A", (B,C))
sage: A.__all__bases
Traceback (most recent call last):
...
ValueError: Can not merge the items B, C, B.

```

Since [trac ticket #11943](#), the following consistency tests are part of the test suites of categories (except for hom categories):

```
sage: C = Category.join([HopfAlgebrasWithBasis(QQ), FiniteEnumeratedSets()])
sage: C.parent_class.mro() == [x.parent_class for x in C.all_super_categories()]+[object]
True
sage: C.element_class.mro() == [x.element_class for x in C.all_super_categories()]+[object]
True
```

8.3 The C3 algorithm, under control of a total order

8.3.1 Abstract

Python handles multiple inheritance by computing, for each class, a linear extension of the poset of all its super classes (the Method Resolution Order, MRO). The MRO is calculated recursively from local information (the *ordered* list of the direct super classes), with the so-called C3 algorithm. This algorithm can fail if the local information is not consistent; worst, there exist hierarchies of classes with provably no consistent local information.

For large hierarchy of classes, like those derived from categories in Sage, maintaining consistent local information by hand does not scale and leads to unpredictable C3 failures (the dreaded “could not find a consistent method resolution order”); a maintenance nightmare.

This module implements a final solution to this problem. Namely, it allows for building automatically the local information from the bare class hierarchy in such a way that guarantees that the C3 algorithm will never fail.

Err, but you said that this was provably impossible? Well, not if one relaxes a bit the hypotheses; but that’s not something one would want to do by hand :-)

8.3.2 The problem

Consider the following hierarchy of classes:

```
sage: class A1(object): pass
sage: class A2(object):
....:     def foo(self): return 2
sage: class A3(object): pass
sage: class A4(object):
....:     def foo(self): return 4
sage: class A5(A2, A1):
....:     def foo(self): return 5
sage: class A6(A4, A3): pass
sage: class A7(A6, A5): pass
```

If `a` is an instance of `A7`, then Python needs to choose which implementation to use upon calling `a.foo()`: that of `A4` or `A5`, but obviously not that of `A2`. In Python, like in many other dynamic object oriented languages, this is achieved by calculating once for all a specific linear extension of the hierarchy of the super classes of each class, called its Method Resolution Order (MRO):

```
sage: [cls.__name__ for cls in A7.mro()]
['A7', 'A6', 'A4', 'A3', 'A5', 'A2', 'A1', 'object']
```

Thus, in our example, the implementation in `A4` is chosen:

```
sage: a = A7()
sage: a.foo()
4
```


Specifically, the MRO is calculated using the so-called C3 algorithm which guarantees that the MRO respects not only inheritance, but also the order in which the bases (direct super classes) are given for each class.

However, for large hierarchies of classes with lots of multiple inheritance, like those derived from categories in Sage, this algorithm easily fails if the order of the bases is not chosen consistently (here for A2 w.r.t. A1):

```
sage: class B6(A1,A2): pass
sage: class B7(B6,A5): pass
Traceback (most recent call last):
...
TypeError: Error when calling the metaclass bases
    Cannot create a consistent method resolution
    order (MRO) for bases ...
```

There actually exist hierarchies of classes for which C3 fails whatever order of the bases is chosen; the smallest such example, admittedly artificial, has ten classes (see below). Still, this highlights that this problem has to be tackled in a systematic way.

Fortunately, one can trick C3, without changing the inheritance semantic, by adding some super classes of A to the bases of A. In the following example, we completely force a given MRO by specifying *all* the super classes of A as bases:

```
sage: class A7(A6, A5, A4, A3, A2, A1): pass
sage: [cls.__name__ for cls in A7.mro()]
['A7', 'A6', 'A5', 'A4', 'A3', 'A2', 'A1', 'object']
```

Luckily this can be optimized; here it is sufficient to add a single base to enforce the same MRO:

```
sage: class A7(A6, A5, A4): pass
sage: [cls.__name__ for cls in A7.mro()]
['A7', 'A6', 'A5', 'A4', 'A3', 'A2', 'A1', 'object']
```

8.3.3 A strategy to solve the problem

We should recall at this point a design decision that we took for the hierarchy of classes derived from categories: *the semantic shall only depend on the inheritance order*, not on the specific MRO, and in particular not on the order of the bases (see the section *On the order of super categories in the category primer*). If a choice needs to be made (for example for efficiency reasons), then this should be done explicitly, on a method-by-method basis. In practice this design goal is not yet met.

Note: When managing large hierarchies of classes in other contexts this may be too strong a design decision.

The strategy we use for hierarchies of classes derived from categories is then:

1. To choose a global total order on the whole hierarchy of classes.
2. To control C3 to get it to return MROs that follow this total order.

A basic approach for point 1., that will work for any hierarchy of classes, is to enumerate the classes while they are constructed (making sure that the bases of each class are enumerated before that class), and to order the classes according to that enumeration. A more conceptual ordering may be desirable, in particular to get deterministic and reproducible results. In the context of Sage, this is mostly relevant for those doctests displaying all the categories or classes that an object inherits from.

8.3.4 Getting fine control on C3

This module is about point 2.

The natural approach would be to change the algorithm used by Python to compute the MRO. However, changing Python’s default algorithm just for our needs is obviously not an option, and there is currently no hook to customize specific classes to use a different algorithm. Pushing the addition of such a hook into stock Python would take too much time and effort.

Another approach would be to use the “adding bases” trick straightforwardly, putting the list of *all* the super classes of a class as its bases. However, this would have several drawbacks:

- It is not so elegant, in particular because it duplicates information: we already know through A5 that A7 is a subclass of A1. This duplication could be acceptable in our context because the hierarchy of classes is generated automatically from a conceptual hierarchy (the categories) which serves as single point of truth for calculating the bases of each class.
- It increases the complexity of the calculation of the MRO with C3. For example, for a linear hierarchy of classes, the complexity goes from $O(n^2)$ to $O(n^3)$ which is not acceptable.
- It increases the complexity of inspecting the classes. For example, the current implementation of the `dir` command in Python has no cache, and its complexity is linear in the number of maximal paths in the class hierarchy graph as defined by the bases. For a linear hierarchy, this is of complexity $O(p_n)$ where p_n is the number of integer partitions of n , which is exponential. And indeed, running `dir` for a typical class like `GradedHopfAlgebrasWithBasis(QQ).parent_class` with 37 super classes took 18 seconds with this approach.

Granted: this mostly affects the `dir` command and could be blamed on its current implementation. With appropriate caching, it could be reimplemented to have a complexity roughly linear in the number of classes in the hierarchy. But this won’t happen any time soon in a stock Python.

This module refines this approach to make it acceptable, if not seamless. Given a hierarchy and a total order on this hierarchy, it calculates for each element of the hierarchy the smallest list of additional bases that forces C3 to return the desired MRO. This is achieved by implementing an instrumented variant of the C3 algorithm (which we call *instrumented* “C3”) that detects when C3 is about to take a wrong decision and adds one base to force the right decision. Then, running the standard C3 algorithm with the updated list of bases (which we call *controlled* “C3”) yields the desired MRO.

EXAMPLES:

As an experimentation and testing tool, we use a class `HierarchyElement` whose instances can be constructed from a hierarchy described by a poset, a digraph, or more generally a successor relation. By default, the desired MRO is sorted decreasingly. Another total order can be specified using a sorting key.

We consider the smallest poset describing a class hierarchy admitting no MRO whatsoever:

```
sage: P = Poset({10: [9,8,7], 9:[6,1], 8:[5,2], 7:[4,3], 6: [3,2], 5:[3,1], 4: [2,1] }, linear_extensions=)
```

And build a `HierarchyElement` from it:

```
sage: from sage.misc.c3_controlled import HierarchyElement
sage: x = HierarchyElement(10, P)
```

Here are its bases:

```
sage: HierarchyElement(10, P)._bases
[9, 8, 7]
```

Using the standard C3 algorithm fails:

```
sage: x.mro_standard
Traceback (most recent call last):
...
ValueError: Can not merge the items 3, 3, 2.
```

We also get a failure when we relabel P according to another linear extension. For easy relabelling, we first need to set an appropriate default linear extension for P :

```
sage: linear_extension = list(reversed(IntegerRange(1,11)))
sage: P = P.with_linear_extension(linear_extension)
sage: list(P)
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
```

Now, we play with the fifth linear extension of P :

```
sage: L = P.linear_extensions()
sage: Q = L[5].to_poset()
sage: Q.cover_relations()
[[10, 9], [10, 8], [10, 7], [9, 6], [9, 3], [8, 5], [8, 2], [7, 4], [7, 1], [6, 2], [6, 1], [5, 3],
sage: x = HierarchyElement(10, Q)
sage: x.mro_standard
Traceback (most recent call last):
...
ValueError: Can not merge the items 2, 3, 3.
```

On the other hand, both the instrumented C3 algorithm, and the controlled C3 algorithm give the desired MRO:

```
sage: x.mro
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
sage: x.mro_controlled
[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
```

The above checks, and more, can be run with:

```
sage: x._test_mro()
```

In practice, the control was achieved by adding the following bases:

```
sage: x._bases
[9, 8, 7]
sage: x._bases_controlled
[9, 8, 7, 6, 5]
```

Altogether, four bases were added for control:

```
sage: sum(len(HierarchyElement(q, Q)._bases) for q in Q)
15
sage: sum(len(HierarchyElement(q, Q)._bases_controlled) for q in Q)
19
```

This information can also be recovered with:

```
sage: x.all_bases_len()
15
sage: x.all_bases_controlled_len()
19
```

We now check that the C3 algorithm fails for all linear extensions l of this poset, whereas both the instrumented and controlled C3 algorithms succeed; along the way, we collect some statistics:

```
sage: stats = []
sage: for l in L:
....:     x = HierarchyElement(10, l.to_poset())
....:     try: # Check that x.mro_standard always fails with a ValueError
....:         x.mro_standard
....:     except ValueError:
....:         pass
....:     else:
....:         assert False
....:     assert x.mro == list(P)
....:     assert x.mro_controlled == list(P)
....:     assert x.all_bases_len() == 15
....:     stats.append(x.all_bases_controlled_len()-x.all_bases_len())
```

Depending on the linear extension l it was necessary to add between one and five bases for control; for example, 216 linear extensions required the addition of four bases:

```
sage: Word(stats).evaluation_sparse()
[(1, 36), (2, 108), (3, 180), (4, 216), (5, 180)]
```

We now consider a hierarchy of categories:

```
sage: from operator import attrgetter
sage: x = HierarchyElement(Groups(), attrcall("super_categories"), attrgetter("_cmp_key"))
sage: x.mro
[Category of groups, Category of monoids, Category of semigroups,
Category of inverse unital magmas, Category of unital magmas, Category of magmas,
Category of sets, Category of sets with partial maps, Category of objects]
sage: x.mro_standard
[Category of groups, Category of monoids, Category of semigroups,
Category of inverse unital magmas, Category of unital magmas, Category of magmas,
Category of sets, Category of sets with partial maps, Category of objects]
```

For a typical category, few bases, if any, need to be added to force C3 to give the desired order:

```
sage: C = FiniteFields()
sage: x = HierarchyElement(C, attrcall("super_categories"), attrgetter("_cmp_key"))
sage: x.mro == x.mro_standard
False
sage: x.all_bases_len()
67
sage: x.all_bases_controlled_len()
70

sage: C = GradedHopfAlgebrasWithBasis(QQ)
sage: x = HierarchyElement(C, attrcall("super_categories"), attrgetter("_cmp_key"))
sage: x._test_mro()
sage: x.mro == x.mro_standard
False
sage: x.all_bases_len()
82
sage: x.all_bases_controlled_len()
89
```

The following can be used to search through the Sage named categories for any that requires the addition of some bases:

```

sage: from sage.categories.category import category_sample
sage: sorted([C for C in category_sample()
.....:         if len(C._super_categories_for_classes) != len(C.super_categories())],
.....:         key=str)
[Category of affine weyl groups,
Category of coxeter groups,
Category of fields,
Category of finite dimensional algebras with basis over Rational Field,
Category of finite dimensional hopf algebras with basis over Rational Field,
Category of finite permutation groups,
Category of graded hopf algebras with basis over Rational Field,
Category of hopf algebras with basis over Rational Field]

```

AUTHOR:

- Nicolas M. Thiery (2012-09): initial version.

sage.misc.c3_controlled.**C3_merge** (*lists*)
Return the input lists merged using the C3 algorithm.

EXAMPLES:

```

sage: from sage.misc.c3_controlled import C3_merge
sage: C3_merge([[3,2],[4,3,1]])
[4, 3, 2, 1]
sage: C3_merge([[3,2],[4,1]])
[3, 2, 4, 1]

```

This function is only used for testing and experimenting purposes, but exercised quite some by the other doctests in this file.

It is an extract of `sage.misc.c3.C3_algorithm()`; the latter could be possibly rewritten to use this one to avoid duplication.

sage.misc.c3_controlled.**C3_sorted_merge** (*lists*, *key='identity'*)
Return the sorted input lists merged using the C3 algorithm, with a twist.

INPUT:

- *lists* – a non empty list (or iterable) of lists (or iterables), each sorted strictly decreasingly according to *key*
- *key* – a function

OUTPUT: a pair (*result*, *suggestion*)

result is the sorted list obtained by merging the lists in *lists* while removing duplicates, and *suggestion* is a list such that applying C3 on *lists* with its last list replaced by *suggestion* would return *result*.

EXAMPLES:

With the following input, `C3_merge()` returns right away a sorted list:

```

sage: from sage.misc.c3_controlled import C3_merge
sage: C3_merge([[2],[1]])
[2, 1]

```

In that case, `C3_sorted_merge()` returns the same result, with the last line unchanged:

```

sage: from sage.misc.c3_controlled import C3_sorted_merge
sage: C3_sorted_merge([[2],[1]])
([2, 1], [1])

```

On the other hand, with the following input, `C3_merge()` returns a non sorted list:

```
sage: C3_merge([[1],[2]])
[1, 2]
```

Then, `C3_sorted_merge()` returns a sorted list, and suggests to replace the last line by `[2, 1]`:

```
sage: C3_sorted_merge([[1],[2]])
([2, 1], [2, 1])
```

And indeed `C3_merge` now returns the desired result:

```
sage: C3_merge([[1],[2,1]])
[2, 1]
```

From now on, we use this little wrapper that checks that `C3_merge`, with the suggestion of `C3_sorted_merge`, returns a sorted list:

```
sage: def C3_sorted_merge_check(lists):
.....:     result, suggestion = C3_sorted_merge(lists)
.....:     assert result == C3_merge(lists[:-1] + [suggestion])
.....:     return result, suggestion
```

Base cases:

```
sage: C3_sorted_merge_check([])
Traceback (most recent call last):
...
ValueError: The input should be a non empty list of lists (or iterables)
sage: C3_sorted_merge_check([[]])
([], [])
sage: C3_sorted_merge_check([[1]])
([1], [1])
sage: C3_sorted_merge_check([[3,2,1]])
([3, 2, 1], [3, 2, 1])
sage: C3_sorted_merge_check([[1],[1]])
([1], [1])
sage: C3_sorted_merge_check([[3,2,1],[3,2,1]])
([3, 2, 1], [3, 2, 1])
```

Exercise different states for the last line:

```
sage: C3_sorted_merge_check([[1],[2],[1]])
([2, 1], [2, 1])
sage: C3_sorted_merge_check([[1],[2],[1]])
([2, 1], [2, 1])
```

Explore (all?) the different execution branches:

```
sage: C3_sorted_merge_check([[3,1],[4,2]])
([4, 3, 2, 1], [4, 3, 2, 1])
sage: C3_sorted_merge_check([[4,1],[3,2]])
([4, 3, 2, 1], [3, 2, 1])
sage: C3_sorted_merge_check([[3,2],[4,1]])
([4, 3, 2, 1], [4, 3, 1])
sage: C3_sorted_merge_check([[1],[4,3,2]])
([4, 3, 2, 1], [4, 3, 2, 1])
sage: C3_sorted_merge_check([[1],[3,2],[1]])
([3, 2, 1], [2, 1])
sage: C3_sorted_merge_check([[1],[4,3,2],[1]])
([4, 3, 2, 1], [2, 1])
sage: C3_sorted_merge_check([[1],[4,3,2],[2]])
```

```

([4, 3, 2, 1], [2, 1])
sage: C3_sorted_merge_check([[2], [1], [4], [3]])
([4, 3, 2, 1], [3, 2, 1])
sage: C3_sorted_merge_check([[2], [1], [4], []])
([4, 2, 1], [4, 2, 1])
sage: C3_sorted_merge_check([[2], [1], [3], [4]])
([4, 3, 2, 1], [4, 3, 2, 1])
sage: C3_sorted_merge_check([[2], [1], [3, 2, 1], [3]])
([3, 2, 1], [3])
sage: C3_sorted_merge_check([[2], [1], [2, 1], [3]])
([3, 2, 1], [3, 2])

```

Exercises adding one item when the last list has a single element; the second example comes from an actual poset:

```

sage: C3_sorted_merge_check([[5, 4, 2], [4, 3], [5, 4, 1]])
([5, 4, 3, 2, 1], [5, 4, 3, 2, 1])
sage: C3_sorted_merge_check([[6, 4, 2], [5, 3], [6, 5, 1]])
([6, 5, 4, 3, 2, 1], [6, 5, 4, 3, 2, 1])

```

class sage.misc.c3_controlled.**CmpKey**

Bases: object

This class implements the lazy attribute `Category._cmp_key`.

The comparison key `A._cmp_key` of a category is used to define an (almost) total order on non-join categories by setting, for two categories A and B , $A < B$ if `A._cmp_key > B._cmp_key`. This order in turn is used to give a normal form to join's, and help toward having a consistent method resolution order for parent/element classes.

The comparison key should satisfy the following properties:

- If A is a subcategory of B , then $A < B$ (so that `A._cmp_key > B._cmp_key`). In particular, `Objects()` is the largest category.
- If $A \neq B$ and taking the join of A and B makes sense (e.g. taking the join of `Algebras(GF(5))` and `Algebras(QQ)` does not make sense), then $A < B$ or $B < A$.

The rationale for the inversion above between $A < B$ and `A._cmp_key > B._cmp_key` is that we want the order to be compatible with inclusion of categories, yet it's easier in practice to create keys that get bigger and bigger while we go down the category hierarchy.

This implementation applies to join-irreducible categories (i.e. categories that are not join categories). It returns a pair of integers `(flags, i)`, where `flags` is to be interpreted as a bit vector. The first bit is set if `self` is a facade set. The second bit is set if `self` is finite. And so on. The choice of the flags is adhoc and was primarily crafted so that the order between categories would not change too much upon integration of [trac ticket #13589](#) and would be reasonably session independent. The number `i` is there to resolve ambiguities; it is session dependent, and is assigned increasingly when new categories are created.

Note: This is currently not implemented using a `lazy_attribute` for speed reasons only (the code is in Cython and takes advantage of the fact that `Category` objects always have a `__dict__` dictionary)

Todo

- Handle nicely (covariant) functorial constructions and axioms
-

EXAMPLES:

```
sage: Objects()._cmp_key
(0, 0)
sage: SetsWithPartialMaps()._cmp_key
(0, 1)
sage: Sets()._cmp_key
(0, 2)
sage: Sets().Facade()._cmp_key
(1, ...)
sage: Sets().Finite()._cmp_key
(2, ...)
sage: Sets().Infinite()._cmp_key
(4, ...)
sage: EnumeratedSets()._cmp_key
(8, ...)
sage: FiniteEnumeratedSets()._cmp_key
(10, ...)
sage: SetsWithGrading()._cmp_key
(16, ...)
sage: Posets()._cmp_key
(32, ...)
sage: LatticePosets()._cmp_key
(96, ...)
sage: Crystals()._cmp_key
(136, ...)
sage: AdditiveMagmas()._cmp_key
(256, ...)
sage: Magmas()._cmp_key
(4096, ...)
sage: CommutativeAdditiveSemigroups()._cmp_key
(256, ...)
sage: Rings()._cmp_key
(225536, ...)
sage: Algebras(QQ)._cmp_key
(225536, ...)
sage: AlgebrasWithBasis(QQ)._cmp_key
(227584, ...)
sage: GradedAlgebras(QQ)._cmp_key
(226560, ...)
sage: GradedAlgebrasWithBasis(QQ)._cmp_key
(228608, ...)
```

For backward compatibility we currently want the following comparisons:

```
sage: EnumeratedSets()._cmp_key > Sets().Facade()._cmp_key
True
sage: AdditiveMagmas()._cmp_key > EnumeratedSets()._cmp_key
True

sage: Category.join([EnumeratedSets(), Sets().Facade()]).parent_class._an_element__.__module__
'sage.categories.enumerated_sets'

sage: CommutativeAdditiveSemigroups()._cmp_key < Magmas()._cmp_key
True
sage: VectorSpaces(QQ)._cmp_key < Rings()._cmp_key
True
sage: VectorSpaces(QQ)._cmp_key < Magmas()._cmp_key
True
```


class `sage.misc.c3_controlled.CmpKeyNamed`

Bases: `object`

This class implements the lazy attribute `CategoryWithParameters._cmp_key`.

See also:

- `CmpKey`
- `lazy_attribute`
- `sage.categories.category.CategoryWithParameters`.

Note:

- The value of the attribute depends only on the parameters of this category.
 - This is currently not implemented using a `lazy_attribute` for speed reasons only.
-

EXAMPLES:

```
sage: Algebras(GF(3))._cmp_key == Algebras(GF(5))._cmp_key # indirect doctest
True
sage: Algebras(ZZ)._cmp_key != Algebras(GF(5))._cmp_key
True
```

class `sage.misc.c3_controlled.HierarchyElement` (*value, bases, key, from_value*)

Bases: `object`

A class for elements in a hierarchy.

This class is for testing and experimenting with various variants of the C3 algorithm to compute a linear extension of the elements above an element in a hierarchy. Given the topic at hand, we use the following naming conventions. For x an element of the hierarchy, we call the elements just above x its *bases*, and the linear extension of all elements above x its *MRO*.

By convention, the bases are given as lists of `HierarchyElement`s, and MROs are given a list of the corresponding values.

INPUT:

- *value* – an object
- *succ* – a successor function, poset or digraph from which one can recover the successors of *value*
- *key* – a function taking values as input (default: the identity) this function is used to compute comparison keys for sorting elements of the hierarchy.

Note: Constructing a `HierarchyElement` immediately constructs the whole hierarchy above it.

EXAMPLES:

See the introduction of this module `sage.misc.c3_controlled` for many examples. Here we consider a large example, originally taken from the hierarchy of categories above `HopfAlgebrasWithBasis`:

```
sage: from sage.misc.c3_controlled import HierarchyElement
sage: G = DiGraph({
.....:     44 : [43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24,
.....:     43 : [42, 41, 40, 36, 35, 39, 38, 37, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20,
.....:     42 : [36, 35, 37, 30, 29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....:     41 : [40, 36, 35, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17, 16, 15,
.....:     40 : [36, 35, 32, 31, 30, 29, 28, 27, 26, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9,
```

```

.....: 39 : [38, 37, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14,
.....: 38 : [37, 33, 32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13,
.....: 37 : [30, 29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 36 : [35, 30, 29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 35 : [29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 34 : [33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14,
.....: 33 : [32, 31, 30, 29, 28, 27, 26, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11,
.....: 32 : [31, 30, 29, 28, 27, 26, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5,
.....: 31 : [30, 29, 28, 27, 26, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 30 : [29, 28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 29 : [28, 27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 28 : [27, 26, 15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 27 : [15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 26 : [15, 14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 25 : [24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3,
.....: 24 : [4, 2, 1, 0],
.....: 23 : [22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1,
.....: 22 : [21, 20, 18, 17, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 21 : [20, 17, 4, 2, 1, 0],
.....: 20 : [4, 2, 1, 0],
.....: 19 : [18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 18 : [17, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 17 : [4, 2, 1, 0],
.....: 16 : [15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 15 : [14, 12, 11, 9, 8, 5, 3, 2, 1, 0],
.....: 14 : [11, 3, 2, 1, 0],
.....: 13 : [12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 12 : [11, 9, 8, 5, 3, 2, 1, 0],
.....: 11 : [3, 2, 1, 0],
.....: 10 : [9, 8, 7, 6, 5, 4, 3, 2, 1, 0],
.....: 9 : [8, 5, 3, 2, 1, 0],
.....: 8 : [3, 2, 1, 0],
.....: 7 : [6, 5, 4, 3, 2, 1, 0],
.....: 6 : [4, 3, 2, 1, 0],
.....: 5 : [3, 2, 1, 0],
.....: 4 : [2, 1, 0],
.....: 3 : [2, 1, 0],
.....: 2 : [1, 0],
.....: 1 : [0],
.....: 0 : [],
.....: })

```

```
sage: x = HierarchyElement(44, G)
```

```
sage: x.mro
```

```
[44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21,
```

```
sage: x.cls
```

```
<class '44.cls'>
```

```
sage: x.cls.mro()
```

```
[<class '44.cls'>, <class '43.cls'>, <class '42.cls'>, <class '41.cls'>, <class '40.cls'>, <class
```

all_bases()

Return the set of all the HierarchyElement ``s above ``self, self included.

EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement
```

```
sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
```

```
sage: HierarchyElement(1, P).all_bases()
```

```
{1}
```

```

sage: HierarchyElement(10, P).all_bases() # random output
{10, 5, 2, 1}
sage: sorted([x.value for x in HierarchyElement(10, P).all_bases()])
[1, 2, 5, 10]

```

all_bases_controlled_len()

Return the cumulated size of the controlled bases of the elements above `self` in the hierarchy.

EXAMPLES:

```

sage: from sage.misc.c3_controlled import HierarchyElement
sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
sage: HierarchyElement(30, P).all_bases_controlled_len()
13

```

all_bases_len()

Return the cumulated size of the bases of the elements above `self` in the hierarchy.

EXAMPLES:

```

sage: from sage.misc.c3_controlled import HierarchyElement
sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
sage: HierarchyElement(30, P).all_bases_len()
12

```

bases()

The bases of `self`.

The bases are given as a list of `HierarchyElement`'s, sorted decreasingly according to the `key` function.

EXAMPLES:

```

sage: from sage.misc.c3_controlled import HierarchyElement
sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
sage: x = HierarchyElement(10, P)
sage: x.bases
[5, 2]
sage: type(x.bases[0])
<class 'sage.misc.c3_controlled.HierarchyElement'>
sage: x.mro
[10, 5, 2, 1]
sage: x._bases_controlled
[5, 2]

```

cls()

Return a Python class with inheritance graph parallel to the hierarchy above `self`.

EXAMPLES:

```

sage: from sage.misc.c3_controlled import HierarchyElement
sage: P = Poset((divisors(30), lambda x,y: y.divides(x)), facade=True)
sage: x = HierarchyElement(1, P)
sage: x.cls
<class '1.cls'>
sage: x.cls.mro()
[<class '1.cls'>, <type 'object'>]
sage: x = HierarchyElement(30, P)
sage: x.cls
<class '30.cls'>

```

```
sage: x.cls.mro()
[<class '30.cls'>, <class '15.cls'>, <class '10.cls'>, <class '6.cls'>, <class '5.cls'>, <cl
```

mro()

The MRO for this object, calculated with `C3_sorted_merge()`.

EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement, C3_sorted_merge, identity
sage: P = Poset({7: [5,6], 5:[1,2], 6: [3,4]}, facade = True)
sage: x = HierarchyElement(5, P)
sage: x.mro
[5, 2, 1]
sage: x = HierarchyElement(6, P)
sage: x.mro
[6, 4, 3]
sage: x = HierarchyElement(7, P)
sage: x.mro
[7, 6, 5, 4, 3, 2, 1]

sage: C3_sorted_merge([[6, 4, 3], [5, 2, 1], [6, 5]], identity)
([6, 5, 4, 3, 2, 1], [6, 5, 4])
```

TESTS:

```
sage: assert all(isinstance(v, Integer) for v in x.mro)
```

mro_controlled()

The MRO for this object, calculated with `C3_merge()`, under control of *C3_sorted_merge*

EXAMPLES:

```
sage: from sage.misc.c3_controlled import HierarchyElement, C3_merge
sage: P = Poset({7: [5,6], 5:[1,2], 6: [3,4]}, facade=True)
sage: x = HierarchyElement(5, P)
sage: x.mro_controlled
[5, 2, 1]
sage: x = HierarchyElement(6, P)
sage: x.mro_controlled
[6, 4, 3]
sage: x = HierarchyElement(7, P)
sage: x.mro_controlled
[7, 6, 5, 4, 3, 2, 1]
sage: x._bases
[6, 5]
sage: x._bases_controlled
[6, 5, 4]
sage: C3_merge([[6, 4, 3], [5, 2, 1], [6, 5]])
[6, 4, 3, 5, 2, 1]
sage: C3_merge([[6, 4, 3], [5, 2, 1], [6, 5, 4]])
[6, 5, 4, 3, 2, 1]
```

TESTS:

```
sage: assert all(isinstance(v, Integer) for v in x.mro_controlled)
```

mro_standard()

The MRO for this object, calculated with `C3_merge()`

EXAMPLES:

```

sage: from sage.misc.c3_controlled import HierarchyElement, C3_merge
sage: P = Poset({7: [5,6], 5:[1,2], 6: [3,4]}, facade=True)
sage: x = HierarchyElement(5, P)
sage: x.mro_standard
[5, 2, 1]
sage: x = HierarchyElement(6, P)
sage: x.mro_standard
[6, 4, 3]
sage: x = HierarchyElement(7, P)
sage: x.mro_standard
[7, 6, 4, 3, 5, 2, 1]
sage: C3_merge([[6, 4, 3], [5, 2, 1], [6, 5]])
[6, 4, 3, 5, 2, 1]

```

TESTS:

```

sage: assert all(isinstance(v, Integer) for v in x.mro_standard)

```

```

sage.misc.c3_controlled.identity(x)

```

EXAMPLES:

```

sage: from sage.misc.c3_controlled import identity
sage: identity(10)
10

```


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