**J Chen Lecture Notes**

# General Linear Model

We will study inference problem for GLM into the chapter.

General Linear Model(GLM)

Let be an observable random vector, be an x matrix of known fixed numbers. be a x vector of unknown parameter; be an x unobservable random (error) vector with and ; and let those quantities be related by:

These specification define a GLM

We will discuss estimation and hypothesis testing mostly for Case 1 and estimate Case 2 in the following sections of this chapter.

# Point Estimation of and Linear Function of : Case 1

Let as specified in Defn 1.1.1  
Assume Then the following results follow:

is the MLE for where

is the MLE for , where

, where

and are independent

and are sufficient statistics for and

and are complete statistics

**Proof:**   
 The likelihood function is:

where the parameter space is:

$$\begin{aligned}
\Longrightarrow \Big\{\_{(y-x\beta)'(y-x\beta)-n\alpha^{2} = 0}^{x'y-X'X\beta = 0} "normal equations"\end{aligned}$$

$$\begin{aligned}
\Longrightarrow \bigg\{\_{(\alpha^{2} = \dfrac{1}{n}(y-X\beta)'(y-x\beta)}^{X'X\beta = X'y} "are solutions"\end{aligned}$$

for the above normal equations.  
As X has rank P, is of full rank. That is exists. Then the MLEs are obtained as:

where

$$\begin{aligned}
\Longrightarrow \hat{\beta} = X'\underline{Y} is the MLE of \underline{\beta} \hspace{1cm}is proved\end{aligned}$$

also

$$\begin{aligned}
\Longrightarrow \hat{\beta} = X'\underline{Y} is the MLE of \underline{\beta} \hspace{1cm}(1) is proved\end{aligned}$$

is a function(a constant multiple) of the MLE of (2) is proved  
AS is a linear form of y.

(3) is proved  
As is a quadratic function of with  
 and