Problem 4

a) $(P_1 \wedge \dots \wedge P_m) \Rightarrow Q \equiv \neg(P_1 \wedge \dots \wedge P_m) \vee Q$ (implication elimination: $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$)

$\equiv (\neg P_1 \vee \dots \vee \neg P_m \vee Q)$ (associativity of $\vee$)

$\equiv (\neg P_1 \vee \dots \vee \neg P_m \vee Q)$

b) $(P_1 \wedge \dots \wedge P_m) \Rightarrow (Q_1 \vee \dots Q_n) \equiv \neg(P_1 \wedge \dots \wedge P_m) \vee (Q_1 \vee \dots \vee Q_n)$

$\equiv (\neg P_1 \vee \dots \vee \neg P_m) \vee (Q_1 \vee \dots \vee Q_n)$

(associativity of $\vee$: $(\alpha \vee \beta) \vee (\gamma) \equiv (\alpha) \vee (\beta \vee \gamma)$)

$\equiv (\neg P_1 \vee \dots \neg P_m \vee Q_1 \vee \dots \vee Q_n)$

c) $Father \equiv (l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_K)$

$Mother \equiv (m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n)$

$l_i \equiv \neg m_j$

$(l_1 \vee \dots \vee l_K) \equiv (l_i \vee Father) \equiv true$

$(m_1 \vee \dots \vee m_n) \equiv (m_j \vee Mother) \equiv (\neg l_i \vee Mother) \equiv true$

$$\frac{(l_i \vee Father),\ (m_j \vee Mother)}{Father \vee Mother} \rightarrow (l_i \vee Father) \wedge (\neg l_i \vee Mother) \Rightarrow (Father \vee Mother)$$