

3a)

S = "Superman is defeatable"

A = "Superman is alone"

B = "Batman works with lex luthor"

P = "Batman acquires kryptonite"

W = "Wonderwoman intervenes and fights with superman"

$$S \Leftrightarrow (P \wedge A)$$

$$P \Leftrightarrow B$$

$$W \Leftrightarrow B$$

$$W \Rightarrow \neg S \equiv \neg W \vee \neg S$$

$$b) S \Leftrightarrow (P \wedge A) : (\neg P \vee \neg A \vee S) \wedge (\neg S \vee A) \wedge (\neg S \vee P)$$

$$P \Leftrightarrow B : (\neg B \vee P) \wedge (\neg P \vee B)$$

$$W \Leftrightarrow B : (\neg B \vee W) \wedge (B \vee \neg W) \equiv ((\neg B \vee W) \wedge B) \vee ((\neg B \vee W) \wedge \neg W) \\ \equiv ((\neg B \wedge B) \vee (W \wedge B)) \vee ((\neg B \vee W) \wedge \neg W) \\ \equiv (W \wedge B) \vee (\neg W \wedge \neg B)$$

$$(S \Rightarrow (P \wedge A)) \wedge ((P \wedge A) \Rightarrow S) \equiv (\neg S \vee (P \wedge A)) \wedge (\neg P \vee \neg A \vee S) \\ \equiv ((\neg S \vee P) \wedge (\neg S \vee A)) \wedge (\neg P \vee \neg A \vee S) \\ \equiv (P \vee \neg S) \wedge (A \vee \neg S) \wedge (\neg P \vee \neg A \vee S)$$

c) we know that for superman to be defeated, batman needs to work with lex luthor and wonderwoman needs to intervene and fight with superman.

$$A \equiv (B \wedge W) \Rightarrow \neg S \equiv (\neg B \vee \neg W) \vee \neg S$$

(If batman does not work with lex luthor or wonderwoman does not fight with superman, then superman will not be defeated)

$$(P \vee \neg S) \wedge (A \vee \neg S) \wedge (\neg P \vee \neg A \vee S) \wedge (\neg B \vee W) \wedge (B \vee \neg W) \wedge (\neg B \vee P) \wedge (\neg P \vee B)$$

And-Elimination: $\frac{\alpha \wedge \beta}{\alpha}$: $\frac{W \Leftrightarrow B, W \Rightarrow \neg S}{\neg S} \equiv \frac{(W \wedge B) \vee (\neg W \wedge \neg B), \neg W \vee \neg S}{\neg S}$

Modus Ponens: $\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$

Since we know if wonderwoman intervenes, superman will be alone, we can conclude that superman will be un-defeated.

$$W \Rightarrow A$$

$$A \Rightarrow \neg S$$