Problem 4

a) $(P_1 \wedge \dots \wedge P_m) \Rightarrow Q \equiv \neg(P_1 \wedge \dots \wedge P_m) \vee Q$ (Implication elimination: $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$)

$\equiv (\neg P_1 \vee \dots \vee \neg P_m \vee Q)$ (associativity of $\vee$)

$\equiv (\neg P_1 \vee \dots \vee \neg P_m \vee Q)$

As a result, we can conclude $(P_1 \wedge \dots \wedge P_m) \Rightarrow Q$ is equivalent to $(\neg P_1 \vee \dots \vee \neg P_m \vee Q)$

b) $(P_1 \wedge \dots \wedge P_m) \Rightarrow (Q_1 \vee \dots Q_n) \equiv \neg(P_1 \wedge \dots \wedge P_m) \vee (Q_1 \vee \dots \vee Q_n)$

$\equiv (\neg P_1 \vee \dots \vee \neg P_m) \vee (Q_1 \vee \dots \vee Q_n)$

$\equiv (\neg P_1 \vee \dots \neg P_m \vee Q_1 \vee \dots \vee Q_n)$ (associativity of $\vee$: $(\alpha \vee \beta) \vee (\gamma) \equiv (\alpha) \vee (\beta \vee \gamma)$)

With help from the derivation of 4a and the associativity of $\vee$, we see that $(P_1 \wedge \dots \wedge P_m) \Rightarrow (Q_1 \vee \dots \vee Q_n)$ is equivalent.

c) $Lother \equiv (l_1 \lor \dots \lor l_{i-1} \lor l_{i+1} \lor \dots \lor l_k)$

$Mother \equiv (m_1 \lor \dots \lor m_{j-1} \lor m_{j+1} \lor \dots \lor m_n)$

$l_i \equiv \neg m_j$

$(l_1 \lor \dots \lor l_k) \equiv (l_i \lor Lother) \equiv true$

$(m_1 \lor \dots \lor m_n) \equiv (m_j \lor Mother) \equiv (\neg l_i \lor Mother) \equiv true$

$$\frac{(l_i \lor Lother),\ (m_j \lor Mother)}{Lother \lor Mother} \longrightarrow \quad (l_i \lor Lother) \land (\neg l_i \lor Mother) \Rightarrow (Lother \lor Mother)$$

Since we know that $(l_i \lor Lother)$ is true and given, and $(\neg l_i \lor Mother)$ is also true and given. If l_i is true, then m_j is false. This would mean that ~~Lother or~~ Mother needs to be true for the statement $(\underset{T}{l_i} \lor \underset{\text{unknown}}{Lother}) \land (\underset{\text{false}}{\neg l_i} \lor \underset{\text{must be true}}{Mother})$ to be true.

This would mean that $(Lother \lor Mother)$ is true.

If m_j is true, l_i must be false. This would mean that Lother needs to be true for the statement $(\underset{\text{false}}{l_i} \lor \underset{\text{true}}{Lother}) \land (\underset{\text{true}}{\neg l_i} \lor \underset{\text{unknown}}{Mother})$ to be true. This would mean that

$(Lother \lor Mother)$ would be true.