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CS440 Intro to AI

Assignment 3

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Question 3 [25 points]:

a) Prove that

$$P(X | MB(X)) = \alpha P(X | U_1, \dots, U_m) \prod_{Y_i}^n P(Y_i | Z_{i1}, \dots)$$

where $MB(X)$ is the Markov Blanket of variable X .

The probability of the Markov Blanket of variable X can be written as:

$$\begin{aligned} P(U_1, \dots, U_m, X, Y_1, \dots, Y_n, Z_{11}, \dots, Z_{nj}) \\ &= P(U_1 \cap \dots \cap U_m \cap X \cap Y_1 \cap \dots \cap Y_n \cap Z_{11} \cap \dots \cap Z_{nj}) \\ &= [\prod_{i=1}^m P(U_i)] P(X | U_1, \dots, U_m) [\prod_{i=1}^n P(Y_i | X, Z_{i1}, \dots, Z_{ij}) \prod_{k=1}^j P(Z_{ik})] \end{aligned}$$

The Markov Blanket of variable X is the set:

$$MB(X) = \{U_1, \dots, U_i, Y_1, \dots, Y_n, Z_{11}, \dots, Z_{nj}\}$$

The conditional probability of X given $MB(X)$ can be written as:

$$P(x | MB(x)) = \frac{P(x \cap MB(x))}{P(MB(x))}$$

The numerator $P(x \cap MB(x))$ can be equivalently expressed as the probability of the Markov Blanket of variable x :

$$[\prod_{i=1}^m P(u_i)]P(x | u_1, \dots, u_m)[\prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij}) \prod_{k=1}^j P(z_{ik})]$$

The denominator $P(MB(x))$ can be written as:

$$\begin{aligned} P(u_1, \dots, u_m, y_1, \dots, y_n, z_{11}, \dots, z_{nj}) \\ &= P(u_1 \cap \dots \cap u_m \cap y_1 \cap \dots \cap y_n \cap z_{11} \cap \dots \cap z_{nj}) \\ &= [\prod_{i=1}^m P(u_i)][\sum_x \prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij}) \prod_{k=1}^j P(z_{ik})] \end{aligned}$$

Thus we have:

$$\begin{aligned} P(x | MB(x)) &= \frac{[\prod_{i=1}^m P(u_i)]P(x | u_1, \dots, u_m)[\prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij}) \prod_{k=1}^j P(z_{ik})]}{[\prod_{i=1}^m P(u_i)][\sum_x \prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij}) \prod_{k=1}^j P(z_{ik})]} \\ &= \frac{[\prod_{i=1}^m P(u_i)]P(x | u_1, \dots, u_m)[\prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij})][\prod_{i=1}^n \prod_{k=1}^j P(z_{ik})]}{[\prod_{i=1}^m P(u_i)][\sum_x \prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij})][\prod_{i=1}^n \prod_{k=1}^j P(z_{ik})]} \\ &= \frac{P(x | u_1, \dots, u_m) \prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij})}{\sum_x \prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij})} \end{aligned}$$

The ratio $\frac{1}{\sum_x \prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij})}$ represents the normalization constant α ,

thus we have proven:

$$P(x | MB(x)) = \alpha P(x | u_1, \dots, u_m) \left[\prod_{i=1}^n P(y_i | x, z_{i1}, \dots, z_{ij}) \right]$$

b) Consider the query

$$P(Rain | Sprinkler = true, WetGrass = true)$$

in the Rain/Sprinkler network and how MCMC would answer it. How many possible states are there for the approach to consider given the network and the available evidence variables?

c) Using the query in 3b, calculate the transition matrix Q that stores the probabilities

$P(y \rightarrow y')$ for all the states y, y' . If the Markov Chain has n states, then the transition matrix has size $n \times n$ and you should compute n^2 probabilities.