

Question 2)

(a) Given: $P(f(\text{left}) | f(\text{left})) = 0.5$

$$P(a(\text{right}) | f(\text{left})) = 0.2$$

$$P(a(\text{stay}) | f(\text{left})) = 0.3$$

$$P(a(\text{left}) | f(\text{right})) = 0.2$$

$$P(a(\text{right}) | f(\text{right})) = 0.5$$

$$P(a(\text{stay}) | f(\text{right})) = 0.3$$

$$P(a(\text{left}) | f(\text{same})) = 0.25$$

$$P(a(\text{right}) | f(\text{same})) = 0.25$$

$$P(a(\text{stay}) | f(\text{same})) = 0.5$$

initial

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
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0

ant location

$$P(f(\text{left of ant})) = \frac{1}{5} = 0.2$$

$$P(f(\text{right of ant})) = 3 \cdot \frac{1}{5} = \frac{3}{5} = 0.6$$

$$P(f(\text{same as ant})) = \frac{1}{5} = 0.2$$

$$P(a(\text{same}) | P(\text{same}))$$

or

$$P(a(\text{same})) = P(a(\text{same}) | f(\text{left})) \cdot P(f(\text{left})) + P(a(\text{same}) | f(\text{right})) \cdot P(f(\text{right})) + P(a(\text{same}) | f(\text{same})) \cdot P(f(\text{same}))$$

$$= (0.3)(0.2) + (0.3)(0.6) + (0.5)(0.2) = 0.34$$

$$P(a(R)) = P(a(R) | f(L)) P(f(L)) + P(a(R) | f(R)) P(f(R)) + P(a(R) | f(S)) P(f(S))$$

$$= (0.2)(0.2) + (0.5)(0.6) + (0.25)(0.2) = 0.39$$

$$P(a(L)) = P(a(L) | f(L)) P(f(L)) + P(a(L) | f(R)) P(f(R)) + P(a(L) | f(S)) P(f(S))$$

$$= (0.5)(0.2) + (0.2)(0.6) + (0.25)(0.2) = 0.27$$

$$\text{* Notice } P(a(S)) + P(a(R)) + P(a(L)) = 0.34 + 0.39 + 0.27 = 1.0$$

0.2	0.2	0.2	0.2	0.2
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$$P(f(c_1) | a(R)) = \frac{P(a(R) | f(L)) P(f(L))}{P(a(R))} = \frac{(0.2)(0.2)}{(0.39)} = 0.1025641026$$

$$P(f(c_2) | a(R)) = \frac{P(a(R) | f(S)) P(f(S))}{P(a(R))} = \frac{(0.25)(0.2)}{(0.39)} = 0.1282051282$$

$$P(f(c_3) | a(R)) = P(f(c_4) | a(R)) = P(f(c_5) | a(R)) = \frac{1}{3} \cdot P(f(R) | a(R)) = \frac{1}{3} \left[\frac{P(a(R) | f(R)) P(f(R))}{P(a(R))} \right]$$

$$= \frac{1}{3} \frac{(0.5)(0.6)}{(0.39)} = 0.2564102564$$

$$P(f(c_1) | a(R)) + P(f(c_2) | a(R)) + P(f(c_3) | a(R)) + P(f(c_4) | a(R)) + P(f(c_5) | a(R))$$

$$= 0.1025641026 + 0.1282051282 + 3(0.2564102564) = 1$$

0.1025641026	0.1282051282	0.2564102564	0.2564102564	0.2564102564
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0
ant location
after 1 step right.

$$P(f(\text{left})) = 0.1025641026 + 0.1282051282 = 0.2307692308$$

$$P(f(\text{same})) = 0.2564102564$$

$$P(f(\text{right})) = 0.5128205128$$

$$P(a(\text{same})) = P(a(S) | f(L)) P(f(L)) + P(a(S) | f(R)) P(f(R)) + P(a(S) | f(S)) P(f(S))$$

$$= (0.3)(0.2307692308) + (0.3)(0.5128205128) + (0.5)(0.2564102564)$$

$$= 0.3512820513$$

$$P(a(R)) = P(a(R) | f(L)) P(f(L)) + P(a(R) | f(R)) P(f(R)) + P(a(R) | f(S)) P(f(S))$$

$$= (0.2)(0.2307692308) + (0.5)(0.5128205128) + (0.25)(0.2564102564)$$

$$= 0.3666666667$$

$$P(a(L)) = P(a(L) | f(L)) P(f(L)) + P(a(L) | f(R)) P(f(R)) + P(a(L) | f(S)) P(f(S))$$

$$= (0.5)(0.2307692308) + (0.2)(0.5128205128) + (0.2564102564)(0.25)$$

$$= 0.2820512821$$

$$P(a(S)) + P(a(R)) + P(a(L)) = 1 \quad (\text{check})$$

0.102564026	0.1282051282	0.256102564	0.256402564	0.256402564
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0 - - - - - 0
current location →

$$P(F(C_1) | a(R)) = P(F(C_2) | a(R)) = \frac{1}{2} \left[\frac{P(a(R) | F(L)) \cdot P(F(L))}{P(a(R))} \right] = \frac{(0.2)(0.2307692308)}{0.3666666667(2)}$$

$$= 0.0629370629$$

$$P(F(C_3) | a(R)) = P(F(\text{same}) | a(Right)) = \frac{P(a(R) | F(S)) P(F(S))}{P(a(R))} = \frac{(0.25)(0.256402564)}{(0.3666666667)}$$

$$= 0.1748251748$$

$$P(F(C_4) | a(R)) = P(F(C_5) | a(R)) = \frac{1}{2} P(F(R) | a(R)) = \frac{P(a(R) | F(R)) P(F(R))}{2(P(a(R)))} = \frac{(0.5)(0.5128205128)}{2(0.3666666667)}$$

$$= 0.3496503496$$

$$\sum_{i=1}^5 P(F(C_i) | a(R)) = 2(0.0629370629) + 0.1748251748 + 2(0.3496503496) = 0.9999999998 \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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← 3 0
P(F(L)) = 0.1258741258
P(F(R)) = 0.6993006992
P(F(S)) = 0.1748251748

$$P(a(S)) = P(a(S) | F(L)) P(F(L)) + P(a(S) | F(R)) P(F(R)) + P(a(S) | F(S)) P(F(S))$$

$$= (0.3)(0.1258741258) + (0.3)(0.6993006992) + (0.5)(0.1748251748)$$

$$= 0.3349650349$$

$$P(a(R)) = P(a(R) | F(L)) P(F(L)) + P(a(R) | F(R)) P(F(R)) + P(a(R) | F(S)) P(F(S))$$

$$= (0.2)(0.1258741258) + (0.5)(0.6993006992) + (0.25)(0.1748251748)$$

$$= 0.4185314685$$

$$P(a(L)) = P(a(L) | F(L)) P(F(L)) + P(a(L) | F(R)) P(F(R)) + P(a(L) | F(S)) P(F(S))$$

$$= (0.5)(0.1258741258) + (0.2)(0.6993006992) + (0.25)(0.1748251748)$$

$$= 0.2465034964$$

$$P(a(S)) + P(a(R)) + P(a(L)) = 0.9999999998 \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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0
ant location
after 1st step
right,
west step

$$P(f(c_1) \vee f(c_2) \vee f(c_3) | a(L)) = \frac{1}{3} P(f(L) | a(L)) = \frac{P(a(L) | f(L)) \cdot P(f(L))}{3 \cdot P(a(L))} = \frac{(0.5)(0.1258741258)}{3(0.2465034964)}$$

$$= 0.085106383$$

$$P(f(c_4) | a(L)) = P(f(S) | a(L)) = \frac{P(a(L) | f(S)) P(f(S))}{P(a(L))} = \frac{(0.25)(0.3348251748)}{(0.2465034964)}$$

$$= 0.1773049646$$

$$P(f(c_5) | a(L)) = P(f(R) | a(L)) = \frac{P(a(L) | f(R)) P(f(R))}{P(a(L))} = \frac{(0.2)(0.6993006992)}{(0.2465034964)}$$

$$= 0.5664567002$$

$$\sum_{i=1}^5 P(f(c_i) | a(L)) = 0.9990808138 \approx 1$$

0.085106383	0.085106383	0.085106383	0.1773049646	0.5664567002
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0
final
location
after 3rd step.

Question 2

$$(b) P(F(L)) = 0.255319149$$

$$P(F(S)) = 0.1773049646$$

$$P(F(R)) = 0.5664567002$$

$$\begin{aligned} P(a(S)) &= P(a(S)|F(L))P(F(L)) + P(a(S)|F(R))P(F(R)) + P(a(S)|F(S))P(F(S)) \\ &= (0.3)(0.255319149) + (0.3)(0.5664567002) + (0.5)(0.1773049646) \\ &= 0.3351852371 \end{aligned}$$

$$\begin{aligned} P(a(R)) &= P(a(R)|F(L))P(F(L)) + P(a(R)|F(R))P(F(R)) + P(a(R)|F(S))P(F(S)) \\ &= (0.2)(0.255319149) + (0.8)(0.5664567002) + (0.25)(0.1773049646) \\ &= 0.3786184211 \end{aligned}$$

$$\begin{aligned} P(a(L)) &= P(a(L)|F(L))P(F(L)) + P(a(L)|F(R))P(F(R)) + P(a(L)|F(S))P(F(S)) \\ &= (0.5)(0.255319149) + (0.2)(0.5664567002) + (0.25)(0.1773049646) \\ &= 0.285271557 \end{aligned}$$

$$P(a(S)) + P(a(R)) + P(a(L)) = 0.999 \approx 1$$