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CS440 Intro to AI

Assignment 3

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Question 3 [25 points]:

a) Prove that

$$P(X \,|\, MB(X)) = \alpha P(X \,|\, U_1, \ldots, U_m) \prod_{Y_i}^{n} P(Y_i \,|\, Z_{i1}, \ldots)$$

where $MB(X)$ is the Markov Blanket of variable X.

The probability of the Markov Blanket of variable X can be written as:

$P(U_1, \ldots, U_m, X, Y_1, \ldots, Y_n, Z_{11}, \ldots, Z_{nj})$

$$= P(U_1 \cap \ldots \cap U_m \cap X \cap Y_1 \cap \ldots \cap Y_n \cap Z_{11} \cap \ldots \cap Z_{nj})$$

$$= [\prod_{i=1}^{m} P(U_i)] P(X \,|\, U_1, \ldots, U_m) [\prod_{i=1}^{n} P(Y_i \,|\, X, Z_{i1}, \ldots, Z_{ij}) \prod_{k=1}^{j} P(Z_{ik})]$$

The Markov Blanket of variable X is the set:

$MB(X) = \{U_1, \ldots, U_i, Y_1, \ldots, Y_n, Z_{11}, \ldots, Z_{nj}\}$

The conditional probability of X given $MB(X)$ can be written as:

$$P(X \mid MB(X)) = \frac{P(X \cap MB(X))}{P(MB(X))}$$

The numerator $P(X \cap MB(X))$ is equivalent to the probability of the Markov Blanket of variable X:

$$[\prod_{i=1}^{m} P(U_i)]P(X \mid U_1, \ldots, U_m)[\prod_{i=1}^{n} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})\prod_{k=1}^{j} P(Z_{ik})]$$

The denominator $P(MB(X))$ can be written as:

$$P(U_1, \ldots, U_m, Y_1, \ldots, Y_n, Z_{11}, \ldots, Z_{nj})$$

$$= P(U_1 \cap \ldots \cap U_m \cap Y_1 \cap \ldots \cap Y_n \cap Z_{11} \cap \ldots \cap Z_{nj})$$

$$= [\prod_{i=1}^{m} P(U_i)][\prod_{i=1}^{n} \sum_{x} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})\prod_{k=1}^{j} P(Z_{ik})]$$

Thus we have:

$$P(X \mid MB(X)) = \frac{[\prod_{i=1}^{m} P(U_i)]P(X \mid U_1, \ldots, U_m)[\prod_{i=1}^{n} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})][\prod_{i=1}^{n}\prod_{k=1}^{j} P(Z_{ik})]}{[\prod_{i=1}^{m} P(U_i)][\prod_{i=1}^{n} \sum_{x} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})][\prod_{i=1}^{n}\prod_{k=1}^{j} P(Z_{ik})]}$$

$$= \frac{P(X \mid U_1, \ldots, U_m)[\prod_{i=1}^{n} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})]}{[\prod_{i=1}^{n} \sum_{x} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})]}$$

And if $\frac{1}{[\prod_{i=1}^{n} \sum_{x} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})]}$ equals the normalization constant α, then:

$$P(X \mid MB(X)) = \alpha P(X \mid U_1, \ldots, U_m)[\prod_{i=1}^{n} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij})]$$