Question 1

a) Gain (q^+) = \$4,000 gain − \$3,000 cost = \$1,000 Profit

Gain (q^-) = \$4,000 gain − (\$3,000 cost + \$1,400 repair) = −\$400 Profit

* Since not given a test, mechanic fee is not substituted into cost assessment

b) $P(\text{Pass}) = P(\text{Pass}|q^+)\,P(q^+) + P(\text{Pass}|q^-)\,P(q^-) = (0.8)(0.7) + (0.35)(0.3) = 0.665$

$P(\neg\text{Pass}) = P(\text{fail}) = (1 - P(\text{Pass}|q^+))\,P(q^+) + (1 - P(\text{Pass}|q^-))\,P(q^-) = (0.2)(0.7) + (0.65)(0.3) = 0.335$

Observe $P(\text{Pass}) + P(\text{fail}) = 0.665 + 0.335 = 1$

$$P(q^+|\text{Pass}) = \frac{P(\text{Pass}|q^+)\,P(q^+)}{P(\text{Pass})} = \frac{(0.8)(0.7)}{(0.665)} = 0.842$$

$$P(q^-|\text{Pass}) = \frac{P(\text{Pass}|q^-)\,P(q^-)}{P(\text{Pass})} = \frac{(0.35)(0.3)}{(0.665)} = 0.157894737$$

$$P(q^+|\neg\text{Pass}) = \frac{(1 - P(\text{Pass}|q^+))\,P(q^+)}{P(\neg\text{Pass})} = \frac{(0.2)(0.7)}{0.335} = 0.417910447$$

$$P(q^-|\neg\text{Pass}) = \frac{(1 - P(\text{Pass}|q^-))\,P(q^-)}{P(\neg\text{Pass})} = \frac{(0.65)(0.3)}{0.335} = 0.582089552$$

Question 1

(c) It seems the best decision would be the amount of money received if the test is passed since there would not need to be repairs done on the car.

$$E[\text{Pass}] = P(q^+ \mid \text{pass})\, \text{cost}(q^+) + P(q^- \mid \text{pass})\, \text{cost}(q^-)$$

$$= (\$1000)(0.842) + (-\$400)(0.157)$$

$$= \$779.20$$

$$E[\text{Fail}] = P(q^+ \mid \text{fail})\, \text{cost}(q^+) + P(q^- \mid \text{fail})\, \text{cost}(q^-)$$

$$= (\$1000)(0.4179) + (-\$400)(0.582) = \$185.10$$

clearly, the expected utility of passing is greater.

(d) since $100 is charged by the mechanic. $\text{cost}(q^+) = \$1000 - \$100 = \$100$ and $\text{cost}(q^-) = -\$400 - \$100 = -\$500$

$$E[\text{Pass}] = (\$900)(0.842) + (-\$500)(0.157) = \$679.30$$

$$E[\text{Fail}] = (\$900)(0.418) + (-\$500)(0.582) = \$85.20$$

Since the expected utility values of both pass and fail given a mechanic is consulted is lower than when he is not consulted, C_1 will not be taken to the mechanic.