Thomas Fiorilla (trf40)

Srikanth Kundeti (sk1799)

Anthony Tiongson (ast119)

Ethan Wang (ew360)

Professor McMahon

CS440 Intro to AI

Assignment 3

April 24, 2020

Question 3 [25 points]:

a) Prove that

$$P(X \mid MB(X)) = \alpha P(X \mid U_1, \ldots, U_m) \prod_{Y_i}^{n} P(Y_i \mid Z_{i1}, \ldots)$$

where $MB(X)$ is the Markov Blanket of variable X.

The probability of the Markov Blanket of variable X can be written as:

$$P(U_1, \ldots, U_m, X, Y_1, \ldots, Y_n, Z_{11}, \ldots, Z_{nj})$$

$$= P(U_1 \cap \ldots \cap U_m \cap X \cap Y_1 \cap \ldots \cap Y_n \cap Z_{11} \cap \ldots \cap Z_{nj})$$

$$= [\prod_{i=1}^{m} P(U_i)] P(X \mid U_1, \ldots, U_m) [\prod_{i=1}^{n} P(Y_i \mid X, Z_{i1}, \ldots, Z_{ij}) \prod_{k=1}^{j} P(Z_{ik})]$$

The Markov Blanket of variable X is the set:

$$MB(X) = \{U_1, \ldots, U_i, Y_1, \ldots, Y_n, Z_{11}, \ldots, Z_{nj}\}$$

The conditional probability of X given $MB(X)$ can be written as:

$$P(x \mid MB(x)) = \frac{P(x \cap MB(x))}{P(MB(x))}$$

The numerator $P(x \cap MB(x))$ can be equivalently expressed as the probability of the Markov Blanket of variable x:

$$[\prod_{i=1}^{m} P(u_i)] P(x \mid u_1, \ldots, u_m) [\prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij}) \prod_{k=1}^{j} P(z_{ik})]$$

The denominator $P(MB(x))$ can be written as:

$$P(u_1, \ldots, u_m, y_1, \ldots, y_n, z_{11}, \ldots, z_{nj})$$

$$= P(u_1 \cap \ldots \cap u_m \cap y_1 \cap \ldots \cap y_n \cap z_{11} \cap \ldots \cap z_{nj})$$

$$= [\prod_{i=1}^{m} P(u_i)][\sum_{x} \prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij}) \prod_{k=1}^{j} P(z_{ik})]$$

Thus we have:

$$P(x \mid MB(x)) = \frac{[\prod_{i=1}^{m} P(u_i)] P(x \mid u_1, \ldots, u_m) [\prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij}) \prod_{k=1}^{j} P(z_{ik})]}{[\prod_{i=1}^{m} P(u_i)][\sum_{x} \prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij}) \prod_{k=1}^{j} P(z_{ik})]}$$

$$= \frac{[\prod_{i=1}^{m} P(u_i)] P(x \mid u_1, \ldots, u_m) [\prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij})][\prod_{i=1}^{n} \prod_{k=1}^{j} P(z_{ik})]}{[\prod_{i=1}^{m} P(u_i)][\sum_{x} \prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij})][\prod_{i=1}^{n} \prod_{k=1}^{j} P(z_{ik})]}$$

$$= \frac{P(x \mid u_1, \ldots, u_m) \prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij})}{\sum_{x} \prod_{i=1}^{n} P(y_i \mid x, z_{i1}, \ldots, z_{ij})}$$

And if $\frac{1}{\sum_x \prod_{i=1}^{n} P(y_i | x, z_{i1}, \ldots, z_{ij})}$ equals the normalization constant α, then:

$$P(x | MB(x)) = \alpha P(x | u_1, \ldots, u_m)[\prod_{i=1}^{n} P(y_i | x, z_{i1}, \ldots, z_{ij})]$$

b) Consider the query

$$P(Rain | Sprinkler = true, WetGrass = true)$$

in the Rain/Sprinkler network and how MCMC would answer it. How many possible states are there for the approach to consider given the network and the available evidence variables?

c) Calculate the transition matrix Q that stores the probabilities $P(y \rightarrow y')$ for all the states y, y'. If the Markov Chain has n states, then the transition matrix has size $n \times n$ and you should compute n^2 probabilities.