

Question 2)

(a) Given: $P(f(\text{left}) | f(\text{left})) = 0.5$

$$P(a(\text{right}) | f(\text{left})) = 0.2$$

$$P(a(\text{stay}) | f(\text{left})) = 0.3$$

$$P(a(\text{left}) | f(\text{right})) = 0.2$$

$$P(a(\text{right}) | f(\text{right})) = 0.5$$

$$P(a(\text{stay}) | f(\text{right})) = 0.3$$

$$P(a(\text{left}) | f(\text{same})) = 0.25$$

$$P(a(\text{right}) | f(\text{same})) = 0.25$$

$$P(a(\text{stay}) | f(\text{same})) = 0.5$$

initial

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
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0

ant location

$$P(f(\text{left of ant})) = \frac{1}{5} = 0.2$$

$$P(f(\text{right of ant})) = 3 \cdot \frac{1}{5} = \frac{3}{5} = 0.6$$

$$P(f(\text{same as ant})) = \frac{1}{5} = 0.2$$

$$P(a(\text{same}) | f(\text{same}))$$

or

$$P(a(\text{same})) = P(a(\text{same}) | f(\text{left})) \cdot P(f(\text{left})) + P(a(\text{same}) | f(\text{right})) \cdot P(f(\text{right})) + P(a(\text{same}) | f(\text{same})) \cdot P(f(\text{same}))$$

$$= (0.3)(0.2) + (0.3)(0.6) + (0.5)(0.2) = 0.34$$

$$P(a(R)) = P(a(R) | f(L)) \cdot P(f(L)) + P(a(R) | f(R)) \cdot P(f(R)) + P(a(R) | f(S)) \cdot P(f(S))$$

$$= (0.2)(0.2) + (0.5)(0.6) + (0.25)(0.2) = 0.39$$

$$P(a(L)) = P(a(L) | f(L)) \cdot P(f(L)) + P(a(L) | f(R)) \cdot P(f(R)) + P(a(L) | f(S)) \cdot P(f(S))$$

$$= (0.5)(0.2) + (0.2)(0.6) + (0.25)(0.2) = 0.27$$

$$\text{* Notice } P(a(S)) + P(a(R)) + P(a(L)) = 0.34 + 0.39 + 0.27 = 1.0$$

0.2	0.2	0.2	0.2	0.2
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$$P(F(C_1) | a(R)) = \frac{P(a(R) | F(L)) P(F(L))}{P(a(R))} = \frac{(0.2)(0.2)}{(0.39)} = 0.1025641026$$

$$P(F(C_2) | a(R)) = \frac{P(a(R) | F(S)) P(F(S))}{P(a(R))} = \frac{(0.25)(0.2)}{(0.39)} = 0.1282051282$$

$$P(F(C_3) | a(R)) = P(F(C_4) | a(R)) = P(F(C_5) | a(R)) = \frac{1}{3} \cdot P(F(R) | a(R)) = \frac{1}{3} \left[\frac{P(a(R) | F(R)) P(F(R))}{P(a(R))} \right]$$

$$= \frac{1}{3} \frac{(0.5)(0.6)}{(0.39)} = 0.2564102564$$

$$P(F(C_1) | a(R)) + P(F(C_2) | a(R)) + P(F(C_3) | a(R)) + P(F(C_4) | a(R)) + P(F(C_5) | a(R))$$

$$= 0.1025641026 + 0.1282051282 + 3(0.2564102564) = 1$$

0.1025641026	0.1282051282	0.2564102564	0.2564102564	0.2564102564
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0
ant location
after 1 step right.

$$P(F(\text{left})) = 0.1025641026 + 0.1282051282 = 0.2307692308$$

$$P(F(\text{same})) = 0.2564102564$$

$$P(F(\text{right})) = 0.5128205128$$

$$P(a(\text{same})) = P(a(S) | F(L)) P(F(L)) + P(a(S) | F(R)) P(F(R)) + P(a(S) | F(S)) P(F(S))$$

$$= (0.3)(0.2307692308) + (0.3)(0.5128205128) + (0.5)(0.2564102564)$$

$$= 0.3512820513$$

$$P(a(R)) = P(a(R) | F(L)) P(F(L)) + P(a(R) | F(R)) P(F(R)) + P(a(R) | F(S)) P(F(S))$$

$$= (0.2)(0.2307692308) + (0.5)(0.5128205128) + (0.25)(0.2564102564)$$

$$= 0.3666666667$$

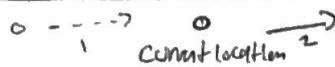
$$P(a(L)) = P(a(L) | F(L)) P(F(L)) + P(a(L) | F(R)) P(F(R)) + P(a(L) | F(S)) P(F(S))$$

$$= (0.5)(0.2307692308) + (0.2)(0.5128205128) + (0.2564102564)(0.25)$$

$$= 0.2820512821$$

$$P(a(S)) + P(a(R)) + P(a(L)) = 1 \quad (\text{check})$$

0.102564026	0.1282051282	0.256102564	0.256402564	0.256402564
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$$P(F(C_1) | a(R)) = P(F(C_2) | a(R)) = \frac{1}{2} \left[\frac{P(a(R) | F(L)) \cdot P(F(L))}{P(a(R))} \right] = \frac{(0.2)(0.2307692308)}{0.3666666667(2)}$$

$$= 0.0629370629$$

$$P(F(C_3) | a(R)) = P(F(\text{same}) | a(R_{\text{right}})) = \frac{P(a(R) | F(S)) P(F(S))}{P(a(R))} = \frac{(0.25)(0.256402564)}{(0.3666666667)}$$

$$= 0.1748251748$$

$$P(F(C_4) | a(R)) = P(F(C_5) | a(R)) = \frac{1}{2} P(F(R) | a(R)) = \frac{P(a(R) | F(R)) P(F(R))}{2(P(a(R)))} = \frac{(0.5)(0.5128205128)}{2(0.3666666667)}$$

$$= 0.3496503496$$

$$\sum_{i=1}^5 P(F(C_i) | a(R)) = 2(0.0629370629) + 0.1748251748 + 2(0.3496503496) = 0.9999999998 \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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Diagram showing locations 0, 1, 2, 3 with arrows. Location 1 is labeled 'current location'.

$P(F(L)) = 0.3006993006$
 $P(F(R)) = 0.3496503496$
 $P(F(S)) = 0.3496503496$

$$P(a(S)) = P(a(S) | F(L)) P(F(L)) + P(a(S) | F(R)) P(F(R)) + P(a(S) | F(S)) P(F(S))$$

$$= (0.3)(0.3006993006) + (0.3)(0.3496503496) + (0.5)(0.3496503496)$$

$$= 0.3349650349$$

$$P(a(R)) = P(a(R) | F(L)) P(F(L)) + P(a(R) | F(R)) P(F(R)) + P(a(R) | F(S)) P(F(S))$$

$$= (0.2)(0.3006993006) + (0.5)(0.3496503496) + (0.25)(0.3496503496)$$

$$= 0.3223776223$$

$$P(a(L)) = P(a(L) | F(L)) P(F(L)) + P(a(L) | F(R)) P(F(R)) + P(a(L) | F(S)) P(F(S))$$

$$= (0.5)(0.3006993006) + (0.2)(0.3496503496) + (0.25)(0.3496503496)$$

$$= 0.3076923076$$

$$P(a(S)) + P(a(R)) + P(a(L)) = 0.9999999998 \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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0
ant location
after 25th
right,
next step left

$$P(f(c_1) | v(f(c_2) | v(f(c_3) | a(L))) = \frac{1}{3} P(f(L) | a(L)) = \frac{P(a(L) | f(L)) \cdot P(f(L))}{3 \cdot P(a(L))} = \frac{(0.5) (0.3006993006)}{3 (0.3076923076)} \\ = 0.1628787879$$

$$P(f(c_4) | a(L)) = P(f(S) | a(L)) = \frac{P(a(L) | f(S)) P(f(S))}{P(a(L))} = \frac{(0.25) (0.3496503496)}{(0.3076923076)} \\ = 0.2840909091$$

$$P(f(c_5) | a(L)) = P(f(R) | a(L)) = \frac{P(a(L) | f(R)) P(f(R))}{P(a(L))} = \frac{(0.2) (0.3496503496)}{(0.3076923076)} \\ = 0.2276325375$$

$$\sum_{i=1}^5 P(f(c_i) | a(L)) = 1.00035981 \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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0
ant location
after 25th step

0.1628787879	0.1628787879	0.1628787879	0.2840909091	0.2276325375
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$$P(f(L)) = 0.3257575758 \\ P(f(S)) = 0.1628787879 \\ P(f(R)) = 0.5681818182$$

0
current
location

$$(b) \begin{aligned} P(a(S)) &= (0.3) (0.325) + (0.3) (0.163) + (0.5) (0.568) = 0.4304 \\ P(a(R)) &= (0.2) (0.325) + (0.5) (0.163) + (0.25) (0.568) = 0.2885 \\ P(a(L)) &= (0.5) (0.325) + (0.2) (0.163) + (0.25) (0.568) = 0.3371 \end{aligned}$$

1.056

≈ 1

determining.