

## Question 2)

(a) Given:  $P(f(\text{left}) | f(\text{left})) = 0.5$

$$P(a(\text{right}) | f(\text{left})) = 0.2$$

$$P(a(\text{stay}) | f(\text{left})) = 0.3$$

$$P(a(\text{left}) | f(\text{right})) = 0.2$$

$$P(a(\text{right}) | f(\text{right})) = 0.5$$

$$P(a(\text{stay}) | f(\text{right})) = 0.3$$

$$P(a(\text{left}) | f(\text{same})) = 0.25$$

$$P(a(\text{right}) | f(\text{same})) = 0.25$$

$$P(a(\text{stay}) | f(\text{same})) = 0.5$$

initial

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
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0

ant location

$$P(f(\text{left of ant})) = \frac{1}{5} = 0.2$$

$$P(f(\text{right of ant})) = 3 \cdot \frac{1}{5} = \frac{3}{5} = 0.6$$

$$P(f(\text{same as ant})) = \frac{1}{5} = 0.2$$

$$P(a(\text{same}) | f(\text{same}))$$

or

$$\begin{aligned} P(a(\text{same})) &= P(a(\text{same}) | f(\text{left})) \cdot P(f(\text{left})) + P(a(\text{same}) | f(\text{right})) \cdot P(f(\text{right})) + P(a(\text{same}) | f(\text{same})) \cdot P(f(\text{same})) \\ &= (0.3)(0.2) + (0.3)(0.6) + (0.5)(0.2) = 0.34 \end{aligned}$$

$$\begin{aligned} P(a(R)) &= P(a(R) | f(L)) P(f(L)) + P(a(R) | f(R)) P(f(R)) + P(a(R) | f(S)) P(f(S)) \\ &= (0.2)(0.2) + (0.5)(0.6) + (0.25)(0.2) = 0.39 \end{aligned}$$

$$\begin{aligned} P(a(L)) &= P(a(L) | f(L)) P(f(L)) + P(a(L) | f(R)) P(f(R)) + P(a(L) | f(S)) P(f(S)) \\ &= (0.5)(0.2) + (0.2)(0.6) + (0.25)(0.2) = 0.27 \end{aligned}$$

\* Notice  $P(a(S)) + P(a(R)) + P(a(L)) = 0.34 + 0.39 + 0.27 = 1.0$

0.2	0.2	0.2	0.2	0.2
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$$P(F(C_1) | a(R)) = \frac{P(a(R) | F(L)) P(F(L))}{P(a(R))} = \frac{(0.2)(0.2)}{(0.39)} = 0.1025641026$$

$$P(F(C_2) | a(R)) = \frac{P(a(R) | F(S)) P(F(S))}{P(a(R))} = \frac{(0.25)(0.2)}{(0.39)} = 0.1282051282$$

$$P(F(C_3) | a(R)) = P(F(C_4) | a(R)) = P(F(C_5) | a(R)) = \frac{1}{3} \cdot P(F(R) | a(R)) = \frac{1}{3} \left[ \frac{P(a(R) | F(R)) P(F(R))}{P(a(R))} \right]$$

$$= \frac{1}{3} \frac{(0.5)(0.6)}{(0.39)} = 0.2564102564$$

$$P(F(C_1) | a(R)) + P(F(C_2) | a(R)) + P(F(C_3) | a(R)) + P(F(C_4) | a(R)) + P(F(C_5) | a(R))$$

$$= 0.1025641026 + 0.1282051282 + 3(0.2564102564) = 1$$

0.1025641026	0.1282051282	0.2564102564	0.2564102564	0.2564102564
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0  
ant location  
after 1 step right.

$$P(F(\text{left})) = 0.1025641026 + 0.1282051282 = 0.2307692308$$

$$P(F(\text{same})) = 0.2564102564$$

$$P(F(\text{right})) = 0.5128205128$$

$$P(a(\text{same})) = P(a(S) | F(L)) P(F(L)) + P(a(S) | F(R)) P(F(R)) + P(a(S) | F(S)) P(F(S))$$

$$= (0.3)(0.2307692308) + (0.3)(0.5128205128) + (0.5)(0.2564102564)$$

$$= 0.3512820513$$

$$P(a(R)) = P(a(R) | F(L)) P(F(L)) + P(a(R) | F(R)) P(F(R)) + P(a(R) | F(S)) P(F(S))$$

$$= (0.2)(0.2307692308) + (0.5)(0.5128205128) + (0.25)(0.2564102564)$$

$$= 0.3666666667$$

$$P(a(L)) = P(a(L) | F(L)) P(F(L)) + P(a(L) | F(R)) P(F(R)) + P(a(L) | F(S)) P(F(S))$$

$$= (0.5)(0.2307692308) + (0.2)(0.5128205128) + (0.2564102564)(0.25)$$

$$= 0.279512821$$

$$P(a(S)) + P(a(R)) + P(a(L)) = 1 \quad (\text{check})$$

0.102564026	0.1282051282	0.256102564	0.256402564	0.256402564
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0 - - - - - 0  
1 current location 2

$$P(F(C_1) | a(R)) = P(F(C_2) | a(R)) = \frac{1}{2} \left[ \frac{P(a(R) | F(L)) \cdot P(F(L))}{P(a(R))} \right] = \frac{(0.2)(0.2307692308)}{0.3666666667(2)}$$

$$= 0.0629370629$$

$$P(F(C_3) | a(R)) = P(F(\text{same}) | a(R_{\text{right}})) = \frac{P(a(R) | F(S)) P(F(S))}{P(a(R))} = \frac{(0.25)(0.256402564)}{(0.3666666667)}$$

$$= 0.1748251748$$

$$P(F(C_4) | a(R)) = P(F(C_5) | a(R)) = \frac{1}{2} P(F(R) | a(R)) = \frac{P(a(R) | F(R)) P(F(R))}{2(P(a(R)))} = \frac{(0.5)(0.5128205128)}{2(0.3666666667)}$$

$$= 0.3496503496$$

$$\sum_{i=1}^5 P(F(C_i) | a(R)) = 2(0.0629370629) + 0.1748251748 + 2(0.3496503496) = 0.9999999998 \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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← 3 0  
P(F(L)) = ~~0.1258741258~~ 0.3006993006  
P(F(R)) = ~~0.6993006992~~ 0.3496503496  
P(F(S)) = ~~0.1748251748~~ 0.3496503496

$$P(a(S)) = P(a(S) | F(L)) P(F(L)) + P(a(S) | F(R)) P(F(R)) + P(a(S) | F(S)) P(F(S))$$

$$= (0.3) \left( \frac{0.3006993006}{0.1258741258} \right) + (0.3) \left( \frac{0.3496503496}{0.6993006992} \right) + (0.5) \left( \frac{0.3496503496}{0.1748251748} \right)$$

$$= \cancel{0.3349656349} 0.3699301431$$

$$P(a(R)) = P(a(R) | F(L)) P(F(L)) + P(a(R) | F(R)) P(F(R)) + P(a(R) | F(S)) P(F(S))$$

$$= (0.2) \left( \frac{0.3006993006}{0.1258741258} \right) + (0.5) \left( \frac{0.3496503496}{0.6993006992} \right) + (0.25) \left( \frac{0.3496503496}{0.1748251748} \right)$$

$$= \cancel{0.4185314685} 0.3223776223$$

$$P(a(L)) = P(a(L) | F(L)) P(F(L)) + P(a(L) | F(R)) P(F(R)) + P(a(L) | F(S)) P(F(S))$$

$$= (0.5) \left( \frac{0.3006993006}{0.1258741258} \right) + (0.2) \left( \frac{0.3496503496}{0.6993006992} \right) + (0.25) \left( \frac{0.3496503496}{0.1748251748} \right)$$

$$= \cancel{0.2465034964} 0.3076923076$$

$$P(a(S)) + P(a(R)) + P(a(L)) = \frac{1.000000073}{0.9999999998} \approx 1$$

0.0629370629	0.0629370629	0.1748251748	0.3496503496	0.3496503496
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0  
ant location  
after 2<sup>nd</sup> step  
right,  
next step

$$P(f(c_1) | v(f(c_2) | v(f(c_3) | a(L))) = \frac{1}{3} P(f(L) | a(L)) = \frac{P(a(L) | f(L)) \cdot P(f(L))}{3 \cdot P(a(L))} = \frac{(0.5) (0.3006993006)}{3 (0.2465034964)} = 0.3076923076$$

$$= 0.4886363636 \left(\frac{1}{3}\right) = 0.1628787879$$

$$P(f(c_4) | a(L)) = P(f(S) | a(L)) = \frac{P(a(L) | f(S)) P(f(S))}{P(a(L))} = \frac{(0.25) (0.3496503496)}{(0.2465034964)} = 0.3076923076$$

$$= 0.2840909091$$

$$P(f(c_5) | a(L)) = P(f(R) | a(L)) = \frac{P(a(L) | f(R)) P(f(R))}{P(a(L))} = \frac{(0.2) (0.3496503496)}{(0.2465034964)} = 0.3076923076$$

$$= 0.2276325375$$

$$\sum_{i=1}^5 P(f(c_i) | a(L)) = 0.4496808138 + 1.00035981 \approx 1$$

<del>0.085106383</del>	<del>0.085106383</del>	<del>0.085106383</del>	<del>0.1773649646</del>	<del>0.5664567002</del>
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0  
final  
location  
after 3<sup>rd</sup> step.

0.1628787879	0.1628787879	0.1628787879	0.2840909091	0.2276325375
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$$P(f(L)) = 0.3257575758$$

$$P(f(S)) = 0.1628787879$$

$$P(f(R)) = 0.5661818182$$

0  
current  
location

$$(b) P(a(S)) = (0.5) (0.325) + (0.3) (0.163) + (0.5) (0.568) = 0.4304$$

$$P(a(R)) = (0.2) (0.325) + (0.5) (0.163) + (0.25) (0.568) = 0.2885$$

$$P(a(L)) = (0.5) (0.325) + (0.2) (0.163) + (0.25) (0.568) = 0.3371$$

$$1.056$$

$$\approx 1$$

determining.