Submitted by Tony Trotter
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FINAN 6510
FINANCIAL APPLICATIONS OF STATISTICAL MODELS
ASSIGNMENT # 1

Exercise 1. Baskin-Robbins was once famous for boasting 31 unique flavors of ice cream. For this question, assume that Baskin-Robbins has unlimited quantities of exactly 31 distinct flavors available for purchase. Assuming, correctly, that the order of the scoops on an ice cream cone is of vital importance, how many different three-scoop ice cream cones can be ordered at Baskin-Robbins?

Order matters and with replacement

$n = 31, r = 3$

$n^r = 31^3 = 29{,}791$

Exercise 2. How many different 5-card hands can be dealt from a standard deck of 52 playing cards?

Order does not matter and without replacement

$n = 52, r = 5$

$$\frac{n!}{r!\,(n-r)!} = \frac{52!}{5!\,(52-5)!} = \frac{52*51*50*49*48*47!}{5*4*3*2*1*47!} = \frac{52*51*50*49*48*\cancel{47!}}{5*4*3*2*1*\cancel{47!}} =$$

$$= \frac{311{,}875{,}200}{120} = 2{,}598{,}960$$

Exercise 3. A certain company's board of directors has 11 members. How many ways are there to choose a president, vice president, and secretary of the board, where no member of the board can hold more than one office?

Order matters and without replacement

$n = 11, r = 3$

$$\frac{n!}{(n-r)!} = \frac{11!}{(11-3)!} = \frac{11*10*9*8!}{8!} = \frac{11*10*9*\cancel{8!}}{\cancel{8!}} = 990$$

Exercise 4. The G20 (or Group of Twenty) is an international economic forum for the governments of 19 countries and the European Union. The European Union is itself a block of 28 nations that are located primarily in Europe. How many ways are there to select 9 countries in the G20 to serve on an economic council if 7 are selected from among the 19 member nations not belonging to the European Union and the others are selected from the remaining 28 member nations (those belonging to the European Union)?

$$\binom{19}{7} * \binom{28}{2} = \frac{19!}{7!\,(19-7)!} * \frac{28!}{2!\,(28-2)!} = \frac{19!}{7!\,12!} * \frac{28!}{2!\,26!} =$$

$$\frac{19*18*17*16*15!*14!*13!*12!}{7!\,12!}*\frac{28*27*26*25!*26!}{2!\,26!}$$

$$=\frac{19*18*17*16*15*14*13*\cancel{12!}}{7*6*5*4*3*2*1*\cancel{12!}}*\frac{28*27*\cancel{26!}}{2*1*\cancel{26!}}=$$

$$\frac{19*18*17*16*15*14*13}{7*6*5*4*3*2*1}*\frac{28*27}{2*1}=19{,}046{,}664$$

Exercise 5. The Dow Jones Industrial Average (DJIA) consists of 30 stocks that are meant to reflect U.S. market performance. An analyst is planning to post the names of these stocks on two web pages, one marked "Buy" and the other marked "Sell". No stock name may appear on both pages, but every stock name will appear on one or the other. Conceivably, one of the pages may be empty. In how many ways can the analyst post the stock names to the web pages if the order in which the names appear on those pages matters?

Order does matter and without replacement

For "Buy"

$$\frac{n!}{(n-r)!}=\frac{k!}{(k-k)!}=\frac{k!}{0!}=k!$$

For "Sell"

$$\frac{(30-k)!}{\big(30-k-(30-k)\big)!}=\frac{(30-k)!}{0!}=(30-k)!$$

$$\sum_{k=0}^{30}\binom{30}{k}*k!\,(30-k)!=\sum_{k=0}^{30}\frac{30!}{\cancel{k!\,(30-k)!}}*\cancel{k!\,(30-k)!}=\sum_{k=0}^{30}30!=31*30!=31!$$

Exercise 6. The Dow Jones Industrial Average (DJIA) consists of 30 stocks that are meant to reflect U.S. market performance. An analyst is planning to post the names of these stocks on two web pages, one marked "Buy" and the other marked "Sell". No stock name may appear on both pages, but every stock name will appear on one or the other. Conceivably, one of the pages may be empty. In how many ways can the analyst post the stock names to the web pages if the order in which the names appear on those pages does not matter?

Order doesn't matter and without replacement

For "Buy"

$$\frac{n!}{r!\,(n-r)!}=\frac{k!}{k!\,(k-k)!}=\frac{k!}{k!*0!}=1$$

For "Sell"

$$\frac{n!}{r!\,(n-r)!}=\frac{(30-k)!}{(30-k)!\,\big(30-k-(30-k)\big)!}=\frac{(30-k)!}{(30-k)!\,(30-k-30+k)!}=1$$

$$=\sum_{k=0}^{30}\binom{30}{k}*1*1=\sum_{k=0}^{30}\frac{30!}{k!\,(30-k)!}$$

Exercise 7. How many ways are there to assign 5 tasks to 7 employees if each employee can be given more than one task?

Order does matter and with replacement

$n = 7, r = 5$

$n^r = 7^5 = 16{,}807$

Exercise 8. How many different anagrams (including nonsensical words) can be made from the letters in the word MONEY, using all the letters?

$n = 5, r = 5$

$$\frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5! = 5 * 4 * 3 * 2 * 1 = 120$$

Exercise 9. How many different anagrams (including nonsensical words) can be made from the letters in the word FINANCE, using all the letters?

$n = 7, r = 7$

$$\frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{7!}{1} = 7! = 7 * 6 * 5 * 4 * 3 * 2 * 1 = 5040$$

Exercise 10. Sonic Corp. once advertised that its drive-ins offer over 168, 000 drink combinations. Assuming that its drive-ins offer customers a choice of any of the 12 base drinks on its menu to which they are given the option of adding any collection of 6 or fewer flavor add-ins from among Sonic's 16 choices of flavor add-ins (chocolate, cherry, strawberry, diet cherry, pineapple, lemon, lime, blue coconut, grape, orange, Powerade, cranberry, apple juice, vanilla, Hi-C, and watermelon), how many different drink combinations can be ordered at Sonic's drive-ins?

Order does not matter, without replacement

$k(0 \le k \le 6)$ and $n = 16; r = k$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$\sum_{k=0}^{6} \binom{16}{k} = \frac{16!}{k!\,(16-k)!} = \frac{16!}{0!\,(16-0)!} + \frac{16!}{1!\,(16-1)!} + \frac{16!}{2!\,(16-2)!} + \frac{16!}{3!\,(16-3)!} + \frac{16!}{4!\,(16-4)!} + \frac{16!}{5!\,(16-5)!} + \frac{16!}{6!\,(16-6)!}$$

Total choices would be:

$$12 * \left[\frac{16!}{0!\,(16-0)!} + \frac{16!}{1!\,(16-1)!} + \cdots + \frac{16!}{6!\,(16-6)!}\right]$$

Exercise 11 (BONUS). Every November, each of my 5 older siblings and I draw names out of a hat (containing the names of all 6 of us) to determine a Christmas gifting exchange pattern between our families. An exchange pattern is admissible if and only if each one of us ends up with a name different from his / her own. How many distinct admissible Christmas gifting exchange patterns can my siblings and I exhibit between our families?

Good grief…