A New Strategy for Bidding in the Network-Wide Progressive Second Price Auction for Bandwidth *

Marina Bitsaki
Department of Computer Science
University of Crete
P.O.Box 2208, Heraklion
Crete, GR-71409, Greece
marina@csd.uch.gr

George D. Stamoulis
Department of Informatics
Athens University of Economics and Business
76 Patision Str. Athens
GR-10434, Greece
gstamoul@aueb.gr

Costas Courcoubetis
Department of Informatics
Athens University of Economics and Business
76 Patision Str. Athens
GR-10434, Greece
courcou@aueb.gr

ABSTRACT

We revisit the network-wide Progressive Second Price auction (PSP) proposed by Lazar and Semret in [3]. In this mechanism, each bidder submits the same bid in each link of the path he is interested in, taking into account the overall competition in the path. We propose a new strategy in which each bidder apportions his total bid-price in the various links, while taking into account the competition in each link separately. We have carried out a wide variety of experiments and compare our strategy with the original one with respect to efficiency and bidders' net benefit. We show that our strategy yields an outcome closer to the optimal social welfare as well as higher expected net benefit for bidders in most of the cases.

Categories and Subject Descriptors

C.2 [**Computer Systems Organization**]: Computer - Communication Networks

General Terms

Algorithms, Economics, Experimentation

Keywords

Auctions, bandwidth allocation, efficiency, network

1. INTRODUCTION

Considerable research has been conducted recently on the development of new pricing and trading mechanisms for allocating network resources. There have been published several studies of auction-based resource allocation mechanisms in the literature. Lazar and Semret propose in [2] the progressive second price (PSP) auction for the allocation of a divisible resource in a single link without any a priori knowledge of demand. In [3] these authors extend PSP to perform bandwidth allocation in a network of arbitrary topology by running independent PSP auctions in each link. Maillé and Tuffin propose in [4] a one-shot multi-bid auction scheme that is related to PSP for the allocation of a divisible resource in a single link. They extend the multi-bid auction to a tree topology network [5]. Courcoubetis et al. present in [1] an entirely different approach, namely a descending auction mechanism for bandwidth allocation over paths, where bids are placed simultaneously and independently in each link. This is shown to be nearly efficient in a widely variety of cases, yet under the assumption of truthful bidding.

In this paper, we consider the problem of allocating bandwidth efficiently to a set of bidders in a communication *network* of arbitrary topology. Each bidder should obtain the *same* quantity of bandwidth in each link that belongs to the path he is interested in. This requirement is henceforth referred to as *consistent* bandwidth allocation to the bidder, and plays a significant role in designing the appropriate mechanism in this model. In the case where a bidder would obtain more bandwidth in a link than in the others, this excess quantity would be useless to him, while, in general, he would have paid for it some amount of money. Also, this bandwidth quantity could have been given to another bidder, thus increasing both the social welfare and the net benefit of both bidders. Therefore, consistent bandwidth allocation is necessary for attaining efficiency. Consistent bandwidth allocation can be attained by means of a combinatorial auction mechanism that would allow bids for bandwidth over paths rather than over individual links. However, since, in general, users are interested in a multitude of different paths, the winner determination process of a combinatorial auction is computationally very complicated, if not intractable. Lazar and Semret propose in [3] the network-wide PSP auction that allows for independent bids in each link. The authors propose a strategy under which each bidder submits the same bid in each link. This bid is derived on the basis of information on all links that is revealed to the bidder during the process. An important feature of this strategy is that the bidder exploits the fact that the auction is not of the pay-your-bid type, thus performing some kind of overbidding: the sum of the bids placed on all links

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exceeds considerably the bidder's willingness-to-pay for this bandwidth quantity in the path.

In this paper, we propose a new strategy for path bidders taking into account the competition that arises in each link separately: the bid-price is now defined to be the minimum price at which the bidder obtains the demanded quantity in a specific link. Thus, the bid-prices differ in the various links, reflecting the demand of each link, as opposed to reflecting the overall demand for the path, which is the case with the strategy of [3]. Essentially, the bidder *apportions* his willingness-to-pay over the links of the path and bids accordingly. We have studied the two strategies assuming that each bidder has a privately known valuation. We have carried out several experiments showing clearly that: The outcome of the proposed strategy is always much closer to the optimal social welfare, than that attained under the strategy of [3]; also, bidders obtain higher expected net benefit by adopting the proposed strategy, with the only exception arising in the case where a bidder with high valuation bids the first and drives the auction to immediate termination.

The PSP auction has played a significant role in the field of auction-based allocation of network resources and has attracted the attention of other researchers too; e.g. see [4, 5, 7]. These works mostly present one-shot (i.e. non-iterative) versions of the PSP auction.

The remainder of this paper is organized as follows: We briefly overview the network-wide PSP auction in Section 2. We then define a new strategy for the PSP auction in Section 3. We describe the set of experiments we have carried out to comparatively assess the two strategies, and present the corresponding results in Section 4. Finally, in Section 5, we discuss directions for future work.

2. PSP IN A NETWORK

Lazar and Semret extend in [3] the PSP auction in the network case. They consider a set of bidders $\mathcal{N} = \{1, \dots, N\}$ and a set of resources $\mathcal{L} = \{1, \dots, L\}$ corresponding to communication links of a network, with bandwidth capacity $C_1, \dots, C_L$ respectively. For simplicity reasons, we consider throughout the paper that all links have the same capacity C.

Each bidder desires bandwidth in a combination of links that form his path, which is fixed and known to him before the auction starts. Bids at any combination of links are allowed. Each bidder is assumed to have a privately known valuation. Bidder's i valuation is given by the function $\theta_i : [0, C] \to \Re$ and satisfies the following assumptions:

- θ_i is differentiable and $\theta_i(0) = 0$;
- $\theta_i' \geq 0$ is non-increasing and continuous;
- $\exists \gamma_i > 0 : \forall z \geq 0, \theta_i'(z) > 0 \Rightarrow \forall \eta < z, \theta_i'(z) \leq \theta_i'(\eta) - \gamma_i(z - \eta)$.

A bidder whose valuation satisfies the above assumptions is characterized to have elastic demand. As discussed in Section 1, it is assumed that each bidder benefits only from the *minimum* allocated quantity of bandwidth among all links of his path. The PSP auction is performed in each link independently, so that the whole mechanism is decentralized: the outcome at any link can be extracted without knowledge of other links' state. Each bidder combines information released from all links he is interested in, and determines his strategy such that his net benefit be maximized.

A repeated game of incomplete information is formed, where each bidder places a bid at each link independently and asynchronously to his opponents, after observing their bids, thus replacing his previous bids. New responses from other bidders follow, and the procedure terminates when none of them is willing to renew his bids. Next we describe the PSP auction in one link that is of interest to bidder i, say link l. Bidder's i bid in link l is the pair $s_i = (q_i, p_i)$, where q_i is the desired quantity at a price p_i per bandwidth unit. (We do not show explicitly the dependence of s_i on l for simplicity.) The allocation is derived according to the following rule: for a fixed opponent profile (i.e. given the opponents' bids), a bidder gains the minimum of his bid quantity and the maximum available quantity at his bid-price. He pays the social opportunity cost; that is the amount offered for this quantity by the bidders that are excluded by this bid of him. Formally, the allocation and payment rules for bidder i are given by:

$$\text{Allocation: } a_i(s) = \min\{q_i, \underline{Q}_i(p_i, s_{-i})\},$$

$$\text{Payment: } c_i(s) = \sum_{j \neq i} p_j [a_j(0; s_{-i}) - a_j(s_i; s_{-i})],$$

where $s = (s_1, \dots, s_N)$ is the bid profile of all bidders, $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N)$ is the bid profile of bidder's i opponents, and

$$\underline{Q}_i(y, s_{-i}) = \max\left\{0, C - \sum_{p_k \geq y,\ k \neq i} q_k\right\}, \text{ for } y \geq 0,$$

is the maximum available quantity for bidder i for a fixed opponent profile, if those of his opponents that have bidded at or above price y, obtain their demanded quantity. When depicted as a demand function, s_{-i} is a piece-wise constant and increasing function of price, and thus it will be referred to as bidder's i "staircase" in the specific link.

Lazar and Semret introduce the notion of the ϵ-Nash equilibrium. For a fixed opponent profile s_{-i}, they define the ϵ-best reply s_i^ϵ to be the bid that *maximizes* bidder's i net benefit within ϵ: $NB_i(s_i^\epsilon; s_{-i}) \geq NB_i(s_i'; s_{-i}) - \epsilon$, $\forall s_i' \in S_i(s_{-i})$, where $S_i(s_{-i})$ is the set of all feasible bids of bidder i for a fixed opponent profile. An ϵ-Nash equilibrium is a bid profile s^ϵ, if for each i, s_i^ϵ is i's best reply given s_{-i}^ϵ.

Lazar and Semret in [3], prove that a bidder cannot do better than place the *same* bid at all links on his path (consistent bidding) and that there exists a truthful ϵ-best reply, which leads the game at a truthful ϵ-Nash equilibrium if reserve prices for each link are introduced. The ϵ-best reply $s_i^\epsilon = (q_i^\epsilon, p_i^\epsilon)$ of bidder i that is submitted in each link separately is derived as follows: the best reply $s_i = (q_i, p_i)$ is the intersection of bidder i's marginal valuation function and the "staircase" that depicts the market price at each quantity. For bidder i, the market price at quantity q equals the *sum* of the prices at quantity q offered by the opponents at the auctions of the various links that form bidder's i path. That is, the "market staircase" of bidder i equals the sum of the "staircases" of each link in his path: for each quantity prices are added. In other words, q_i is the largest quantity such that his marginal value is just greater than the market price, whereas p_i equals his marginal valuation at q_i. That is, $p_i = \theta_i'(q_i)$. The ϵ-best reply is then calculated by decreasing q_i by $\epsilon/\theta'(0)$ so that $q_i^\epsilon = q_i - \epsilon/\theta'(0)$, and adjusting p_i such that $p_i^\epsilon = \theta_i'(q_i^\epsilon)$. The bidder then bids $(q_i^\epsilon, p_i^\epsilon)$

in each link of his path. Figure 1 illustrates the derivation of the ϵ-best reply of bidder i whose path comprises two links, given his opponents' profiles in the two links. Efficiency in the network-wide PSP auction is analyzed in [6]. It is proved in [6] that the social welfare is within a bound from its maximum value provided that the second derivatives of bidders' valuation functions are bounded. However, as discussed below, this bound is loose in certain cases.

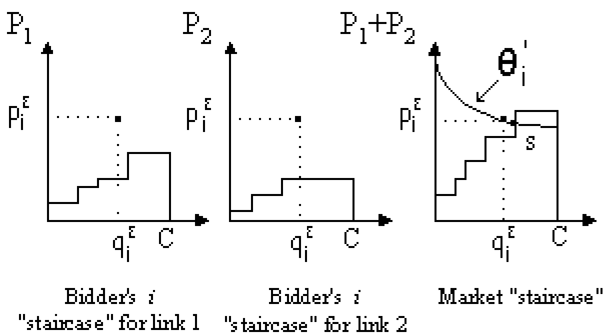

Figure 1: Bidder's i ϵ-best reply $(q_i^\epsilon, p_i^\epsilon)$ proposed in [3] for a network of two links with the same capacity.

2.1 Efficiency and Related Aspects in the Network-Wide PSP Auction

The aforementioned bidding strategy for the network case suggests that path bidders submit a high bid, the same in each link, for the demanded quantity, despite the fact that, in this step, they could obtain this same quantity by submitting a lower price in each link. Thus, path bidders perform some kind of overbidding. This is apparently harmless to the path bidder himself, due to the fact that charging is performed according to social opportunity cost. By construction of the market "staircase", it follows that the total charge in all links never exceeds p_i^ϵ. Nevertheless, we claim that efficiency is not very close to maximum in many cases, due to this overbidding approach. A competitive path bidder is reasonably expected to have higher valuations in general than single-link bidders, because he has to be a winner in the various auctions for the constituent links of his path. Thus, a bidder that is interested in a path with many links has an advantage over other bidders: he can submit a high bid and exclude them from winning. We have carried out several experiments that verify our assertion. These will be described in Section 4. Next, we provide an example in which the approach of [3] leads to an inefficient allocation.

Example 1.

Consider two communication links A and B with capacities $C_A = C_B = 8$ and three bidders with marginal valuations $\theta_1'(x) = -x + 9$ for link A, $\theta_2'(x) = -x + 9$ for link B, $\theta_3'(x) = -x + 17$ for the path of links A and B respectively. Their respective valuations are $\theta_1(x) = -x^2/2 + 9x$, $\theta_2(x) = -x^2/2 + 9x$ and $\theta_3(x) = -x^2/2 + 17x$. We take ϵ equal to 1 and the reserve price p_0 in each link equal to 1 too. The efficient allocation in this example is the solution of the following social welfare maximization problem:

$$\max_{\vec{x}} \{\theta_1(x_1) + \theta_2(x_2) + \theta_3(x_3)\}$$
$$\text{s.t. } x_1 + x_3 \leq 8 \text{ and } x_2 + x_3 \leq 8$$

The solution of this problem is the vector of quantities $\vec{x} = (3, 3, 5)$ and the maximum value of the social welfare is $\mathrm{SW}_{\mathrm{opt}} = 117.5$. If we apply the network-wide PSP auction with the bidding strategy of [3], then the game terminates immediately after the first bid of the path bidder 3, independently of the order the bidders may submit their bids. For example, assume for the sake of simplicity that bidder 3 plays first; that is, he observes no demand in neither of the two links. His bid $s_3^\epsilon = (q_3^\epsilon, p_3^\epsilon)$ for each link according to the strategy is calculated as follows: $q_3^\epsilon = 8 - \epsilon/\theta_3'(0) = 7.94$ and $p_3^\epsilon = \theta_3'(7.94) = 9.06$. The other single-link bidders cannot compete against bidder 3, since their marginal valuation is less than 9.06 for any quantity of bandwidth. Thus, each of them wins the remaining quantity of bandwidth which equals 0.06 units in each link at the reserve price. Thus, the auction terminates at the allocation $\vec{x} = (0.06, 0.06, 7.94)$. The resulting social welfare equals 104.5346, which is significantly below (namely, 11%) the optimal value $\mathrm{SW}_{\mathrm{opt}}$. ▲

Next, we justify why the social welfare of the network-wide PSP auction often deviates considerably from the corresponding optimal value. This is due to the fact that the ϵ-best reply is *not truthful* in the following sense: The ϵ-best reply is a point of the overall marginal valuation function of bidder i. However, on a per link basis, the ϵ-best reply submitted is not a truthful one, since the sum of the prices p_i^ϵ offered for the quantity q_i^ϵ in each link is higher than the bidder's marginal valuation at q_i^ϵ which equals p_i^ϵ. Actually, the price p_i^ϵ offered in each link is considerably higher than the price that would suffice to obtain the quantity q_i^ϵ. Since the auction in each link is a PSP auction itself, this strategy amounts to overbidding. However, overbidding forces prices increase more aggressively in each link. Thus, prices do not converge to those resulting in the efficient outcome, as in the single-link PSP auction. Next, we provide an example in which the deviation of the social welfare from its maximum value can be very high for a large number of links.

Example 2.

Consider a linear network consisting of L links, each of capacity C. Moreover, there is one path bidder interested in the entire path of the L links and one single-link bidder in each link. We take that the valuation functions of the various single-link bidders are identical, say θ. Let θ_p be the valuation function of the path bidder, and x the quantity of bandwidth for each single-link bidder. We further assume the following:

- $\theta_p'(C) = \theta'(0)$ and,
- $L \cdot [\theta(C) - \theta(x)] > \theta_p(C - x)\ \forall\ x < C$.

For example, $\theta(x) = -ax^2/2 + bx$ and $\theta_p(x) = -ax^2/2 + (b + aC)x$ s.t. $a > 0$ and $C \leq b(L-1)/[a(L+1)]$ satisfy the above assumptions $\forall\ x < C$. Then, the efficient allocation is $x = C$; i.e. to award the entire capacity C of each link to the corresponding single-link bidder. The optimal social welfare is $\mathrm{SW}_{\mathrm{opt}} = L\theta(C)$. We assume that the path bidder adopts the strategy of Lazar and Semret and plays first. Then the path bidder obtains the whole capacity at once and the single-link bidders cannot top this offer. Indeed, due to the equality $\theta_p'(C) = \theta'(0)$, the demand function of each single-link bidder is below the submitted price $\theta_p'(C)$. The resulting social welfare equals $\mathrm{SW} = \theta(C)$. Thus, the percentage loss of the social welfare equals $[L\theta(C) - \theta(C)]/L\theta(C) = (L-1)/L$.

Note that the loss increases and approaches 100% as the number of links increases. ▲

3. A NEW BIDDING STRATEGY FOR PATH BIDDERS

As already mentioned, a key question regarding the strategy for path bidders of the network-wide PSP auction is whether it is fair or optimal (for the bidder himself and/or for the society) to apply overbidding in the various links. It is not obvious whether this approach leads to optimal price discovery and to optimal final bandwidth allocation and/or to optimal net benefits for individual bidders. We propose a new strategy for path bidders of the network-wide PSP auction, which we consider to be more intuitive. The optimal quantity for the bid is derived similarly as in the strategy of [3]. However, our strategy suggests that instead of bidding the market clearing price in each link, to *apportion* this over the constituent links of the path; then, set the prices -differently in each link- to be the smallest ones that assure winning in the present round of the quantity bidded for. Recall that, by assumption, each bidder knows his valuation for the whole path of his interest. If he could split his valuation and determine the corresponding portion for each link, then he could find the right price for this link's bid. However, notice that, for each link, this price corresponds to the intersection of the vertical line through the optimal quantity with *this* link's "staircase". In general, these prices *sum* to the market clearing price, and in certain cases even less than that. Therefore, a path bidder employing this strategy reveals his real demand, thus *avoiding* overbidding.

Formally, consider a bidder i that is interested in a path consisting of K links, which are taken to be links $1, \ldots, K$. Let $s_i = (q_i, p_i)$ and $s_i^\epsilon = (q_i^\epsilon, p_i^\epsilon)$ be the best reply and the ϵ-best reply respectively, as defined in the previous section. For $j = 1, \ldots, K$, let I_j be the set of all intersection points of the vertical line through q_i with the "staircase" of link j. For the set of points I_j there are two possible cases: a) I_j is a single point that lies on the interior of a horizontal segment of the "staircase" of link j; this case arises more frequently. b) I_j is a set of points that lie on a vertical segment of the "staircase" of link j. This means that the quantity q_i coincides with another previously submitted quantity in link j. Essentially, path bidder i first derives the best quantity and (total) price for the entire path and then *apportions the price* among the various links so that he can win the desired quantity in all links. Obviously, for single-link bidders, this strategy reduces to the one proposed by Lazar and Semret in [2] in the single-link case. In order to define the price that bidder i will submit in each link j, we distinguish among the following three cases:

1. For each $j = 1, \ldots, K$, I_j is a single point. Then, define $p_{i,j}$ to be the price that corresponds to the intersection point I_j. Obviously, prices $p_{i,j}$ are uniquely defined in this case. Figure 2 depicts the prices $p_{i,1}$ and $p_{i,2}$ for bidder i in a two-link path in this case.

2. For each $j = 1, \ldots, K$, $j \neq k$, I_j is a single point, while for a unique link k, I_k is a set of points. Then, for each $j = 1, \ldots, K$, $j \neq k$ define $p_{i,j}$ as previously and let $p_{i,k}$ equal $\theta'(q_i) - (\sum_{j \neq k} p_{i,j})$. Again, prices $p_{i,j}$ are uniquely defined. Figure 3 depicts the prices $p_{i,1}$ and $p_{i,2}$ in a two-link path in this case, with $k = 1$.

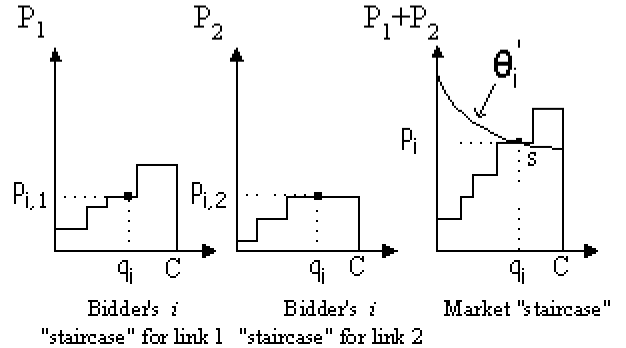

Figure 2: The vertical line through q_i intersects the "staircase" of both links in a horizontal segment (Case 1).

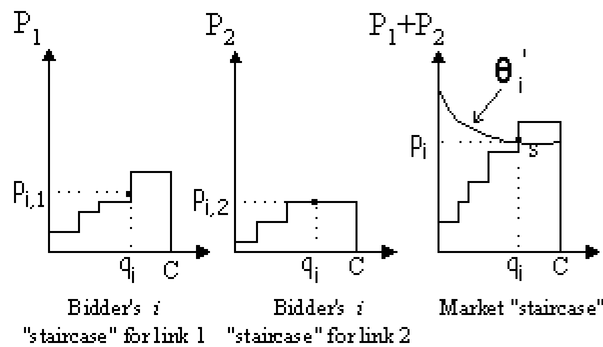

Figure 3: The vertical line through q_i intersects the "staircase" of link 1 in a vertical segment and the "staircase" of link 2 in a horizontal segment (Case 2).

3. For two or more links, the respective I_j's are intervals rather than single points. Then, for each such link j, define $p_{i,j}$ to be the lowest price of all prices that correspond to the intersection points of set I_j. For the remaining links define $p_{i,j}$ as in case 1. Prices $p_{i,j}$ are uniquely defined in this case too. Figure 4 depicts prices $p_{i,1}$ and $p_{i,2}$ in a two-link path in this case.

Recall that the market price p_i at quantity q_i equals the sum of the prices at quantity q_i offered by i's opponents at the auctions of the various links that form the path of bidder i. By definition of $p_{i,j}$'s we have the following:

$$p_{i,1} + \cdots + p_{i,K} = p_i, \text{ for cases 1-2 and,} \tag{1}$$

$$p_{i,1} + \cdots + p_{i,K} \leq p_i, \text{ for case 3.} \tag{2}$$

Similarly with the strategy of [3], the quantity to be demanded in all links is slightly less than q_i. To keep in line with [3] as much as possible, this decrease is taken equal to $\epsilon/\theta_i'(0)$. Since price increases as quantity decreases, we have that $p_i^\epsilon = p_i + \Delta p$, where p_i^ϵ is such that $p_i^\epsilon = \theta_i'(q_i^\epsilon)$ and $\Delta p = p_i^\epsilon - p_i > 0$. The prices of the bids of the various links should also be increased. For simplicity, we take that each

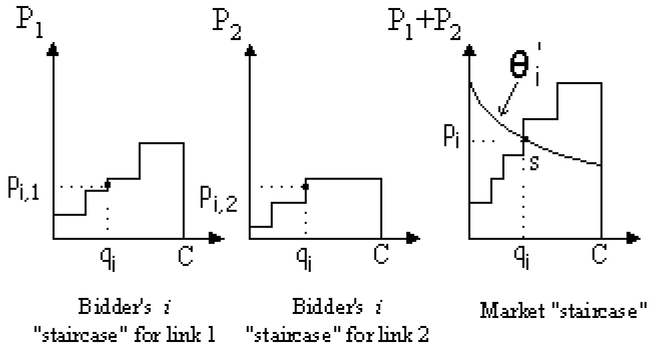

Figure 4: The vertical line through q_i intersects the "staircase" of both links in a vertical segment (Case 3).

of them is increased by the same quantity. Thus,

$$p^{\epsilon}_{i,j} = p_{i,j} + \frac{\Delta p}{K},$$

so that their sum $\sum_{j=1}^{K} p^{\epsilon}_{i,j}$ now equals p^{ϵ}_i rather than p_i. (Alternatively, Δp could have been split proportionally with respect to the $p_{i,j}$.) Combining this with equations (1) and (2) we obtain the following:

$$p^{\epsilon}_{i,1} + \cdots + p^{\epsilon}_{i,K} = p_{i,1} + \cdots + p_{i,K} + \Delta p = p_i + \Delta p = p^{\epsilon}_i,$$

$$p^{\epsilon}_{i,1} + \cdots + p^{\epsilon}_{i,K} \leq p_{i,1} + \cdots + p_{i,K} + \Delta p \leq p_i + \Delta p = p^{\epsilon}_i,$$

for cases 1-2 and 3 respectively.

Bidder's i bid for link j is $(q^{\epsilon}_i, p^{\epsilon}_{i,j})$ for $j = 1, \ldots, K$. Figure 5 depicts the derivation of bidder's i bids according to our approach for a two-link network.

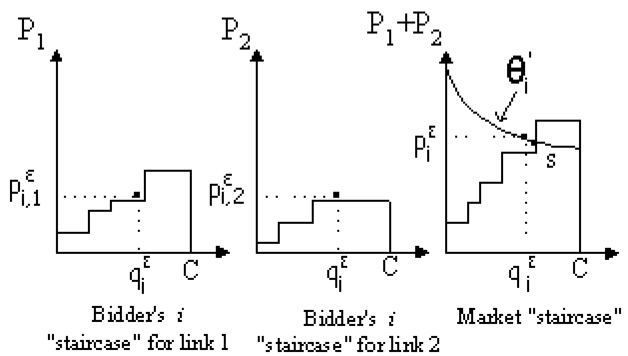

Figure 5: Bidder i submits the bid (q^{ϵ}_i, p^1_i) in link 1 and (q^{ϵ}_i, p^2_i) in link 2 according to the proposed strategy in a network of two links. This corresponds to case 1.

3.1 Discussion

Lazar and Semret have showed in [3], that the strategy they propose is the ϵ-best reply for each bidder in the *short run*; that is, without taking into consideration subsequent actions of his opponents. Our strategy is an ϵ-best reply in the short run too in the following sense: given a particular "staircase", if the path bidder applies our strategy, then he obtains the same quantity and pays the same total charge as he would with the overbid strategy. Indeed, the quantity bidded for in our strategy is the same and even though the price in each link is lower than that of the overbid strategy, it is high enough so that the desired quantity be won. Since the charge in each link is the social opportunity cost, it is the same in both cases had the auction terminated with this bid. Thus, the path bidder's net benefit is the same with that under the overbid strategy of [3], which would be optimal, had this bid caused termination of the auction.

However, the two strategies *differ* in how the auction evolves in subsequent steps and of course in the final allocation. In fact, under the overbid strategy the price in each link increases fast, thus excluding many bidders from submitting new bids. On the contrary, under our strategy the price increases gradually and allows for bidders to compete fairly for the bandwidth. We expect that, since our strategy amounts to less aggressive bidding, it yields outcomes that are closer to the maximum social welfare. It is also reasonable that path bidders with high valuation prefer our strategy, since they will win anyway their share (as in the efficient allocation), but without increasing prices that would result in a higher payment and a lower net benefit for them. On the contrary, path bidders (particularly those with low valuation) may prefer the overbid strategy, since they win more capacity than under the efficient allocation. These assertions are justified in the experiments that follow.

4. EXPERIMENTAL EVALUATION OF THE STRATEGIES

In this section, we compare the strategy of Lazar and Semret (overbid strategy, also denoted as "LS") with our proposed strategy (minimum bid strategy, also denoted as "new") in terms of individual net benefit for path bidders, social welfare, and equilibrium properties.

4.1 Specification of Experiments

We carried out several experiments in a network consisting of 2 links, each having capacity $C = 5$ units of bandwidth. This constitutes the most interesting case of comparison, as explained in detail at the end of this section. Bidders are assumed to have elastic demand of the form $\theta'(q) = aq + b$, $a < 0$ and $C \leq -b/a$. Three categories of bidders take part in the game: single-link bidders interested in link 1, single-link bidders interested in link 2 and path bidders. We carried out experiments with the following mixes of bidders: a) many single-link bidders and one path bidder, b) many single-link bidders and a few path bidders, and c) a few single-link bidders and many path bidders. The order of bidders is predefined and is applied periodically throughout an experiment. We consider a reserve price $p_0 = 0.1$ in each link, which is set by the corresponding seller. We take the parameter ϵ to be equal to 1. (Recall that ϵ is the bound within which a bidder maximizes his net benefit.) When applying the new strategy for a path bidder i, we split the extra price $\epsilon/\theta'_i(0)$ equally between the two links, so that $p^{\epsilon}_{i,1} = p_{i,1} + 0.5\epsilon/\theta'_i(0)$ and $p^{\epsilon}_{i,2} = p_{i,2} + 0.5\epsilon/\theta'_i(0)$. (We employ the notation defined in Section 3.) Each experiment involves two runs of the network-wide PSP auction. In the first run the LS strategy is applied by every bidder. In the second run the new strategy is applied by every bidder. Additionally, we have carried out several experiments in which one bidder applies one of the two strategies and all the other bidders apply the other strategy.

Note that the PSP auction algorithm that we implemented differs from that of [3] in two points. The ϵ-best reply is calculated, as in [3]. If a tie arises, then the price is reduced again by $\epsilon/\theta_i'(0)$ as in the first reduction. The quantity is further adjusted so that $p_i' = \theta_i'(q_i')$ is satisfied for the new bid (q_i', p_i'). We repeat this adjustment procedure until the tie is resolved: in [3] the quantity is adjusted only once. Second, we do *not* interpret ϵ as a fee per bid. That is, a bidder submits his next bid if he thus improves his net benefit by any positive amount. Recall that in [3] it is necessary that this improvement is at least ϵ; otherwise, the bid is not submitted. Of course, ϵ is used to derive the ϵ-best reply, which is further employed to calculate the next value of net benefit. In our opinion, there is no reason to include ϵ again in the criterion of improving one's net benefit. Thus, the algorithm bidder i employs is as follows:

1. Initially $s_i = 0$.
2. Take the updated s_{-i}.
3. Compute the ϵ-best reply s_i^ϵ as given by the corresponding strategy.
4. If necessary, adjust the bid s_i^ϵ so as to eliminate ties.
5. Let s_f be the bid after executing steps 3 and 4. If $NB_i(s_f) > NB_i(s_i)$, then replace the bid s_i with s_f.
6. In the next event of bidding by i go to step 2.

For the experimental evaluation of the two strategies we have developed a special software using the Java programming language. The optimal value of social welfare in each experiment was derived using Mathematica, on the basis of full information about bidders' valuation functions.

The experiments confirm our assertions for efficiency and bidding behavior in the network-wide PSP auction. In particular, the experiments reveal that the new strategy outperforms the LS strategy with respect to efficiency of the mechanism, while it is also individually beneficial for path bidders. We will discuss about these findings in further detail below. Table 1 shows a typical representation of a pair of experiments. Each row provides information and results for a specific bidder. The first column shows the identity of each bidder, the second column refers to the link(s) each bidder is interested in, and columns 3 and 4 indicate the parameters a and b of the marginal valuation of each bidder. The optimal allocation is indicated in column 5. Each bidder's allocation and net benefit under the LS strategy is indicated in column 6, marked "LS". Each bidder's allocation and net benefit under the new strategy is indicated in column 7, marked "New". The last row shows the optimal social welfare, the social welfare under the LS strategy and the social welfare under the new strategy. We observe that the social welfare as well as the net benefit of path bidder 11 are both higher under the new strategy.

4.2 Comparative Assessment of Efficiency in the Network-Wide PSP Auction

Running the same experiment with both strategies, the LS strategy results in *lower* social welfare than with the new strategy. This was the case for *every* experiment we carried out. In fact, the more the path bidders involved or the higher their valuation, the greater the observed loss in social welfare with the LS strategy. Below we present the results of five sets of experiments with different mixes of path and single-link bidders. Each set comprises eight experiments, with the same numbers of single-link and path bidders, but with different sets of bidders' valuations.

- Set A: 6 single-link bidders contend in link 1, 6 single-link bidders contend in link 2, while there is also 1 path bidder (interested in both links). In Table 2, we present the optimal social welfare, the social welfare under the LS strategy and the social welfare under the new strategy for a multitude of experiments. The two last columns show the percentage loss of social welfare under the LS strategy and the new strategy respectively. (All tables for Sets B to E of experiments have the same structure.)
- Set B: 7 single-link bidders contend in link 1, 5 single-link bidders contend in link 2, while there is also 1 path bidder. The results for a multitude of experiments are presented in Table 3.
- Set C: 7 single-link bidders contend in link 1, 5 single-link bidders contend in link 2, while there are also 2 path bidders. The results for a multitude of experiments are presented in Table 4.
- Set D: 4 single-link bidders contend in link 1, 4 single-link bidders contend in link 2, while there are also 4 path bidders. The results for a multitude of experiments are presented in Table 5.
- Set E: 2 single-link bidders contend in link 1, 2 single-link bidders contend in link 2, while there are also 6 path bidders. The results for a multitude of experiments are presented in Table 6.

The results of Tables 2 - 6, reveal the following: In each experiment the new strategy yields *higher* (and, rarely, equal) social welfare than the LS strategy. Moreover, social welfare under the new strategy is *always almost equal to the optimal.* The maximum social welfare loss observed was less than 0.22%. On the contrary, the social welfare under the LS strategy, in general is considerably lower than the optimal value. For example, in experiment 2 of set E, the loss observed was 31.82%, in experiment 5 of set D the loss observed was 8.94% and in the experiment 2 of set B the loss observed was 9.72%. In fact, the more the path bidders, the higher the social welfare loss under the LS strategy. In the experiments of set E, where there are 6 path bidders involved, the minimum loss of social welfare is 4.44% and the maximum loss of social welfare is 31.82% under the LS strategy. In most experiments of set A, the social welfare loss under the LS strategy is very close to that under the new strategy, since there is only one path bidder. Nevertheless, in experiment 4 of set A, the loss under the LS strategy is remarkably high (namely, 14.03%) because the path bidder in this experiment has a high valuation and, thus, overbidding is beneficial for him. Experiments 5, 6 and 7 of set E involve the same set of bidders and bidders' valuations, yet different bidding orders. We observe that the social welfare under the new strategy is the same for all these experiments, while the social welfare under the LS strategy varies with the bidding order.

Table 1: A pair of experiments that shows the allocation and net benefit per bidder under the two strategies as well as the optimal ones.

Bidder	Link	a	b	Optimal Allocation	LS		New	
					Bandw.	NB	Bandw.	NB
1	1	-1	13	2.2	0	0	2.13	24.54
3	1	-1	12	1.2	0	0	1.19	12.79
5	1	-1	11	0.2	0	0	0.18	1.32
7	1	-1	9	0	0	0	0	0
9	1	-1	10	0	0.05	0.09	0.05	0.1
2	2	-1	3	0	0	0	0	0
4	2	-1	6	0.2	0	0	0.15	0.29
6	2	-1	7	1.2	0	0	1.18	6.49
8	2	-1	8	2.2	0	0	2.2	13.65
10	2	-1	5.5	0	0.05	0.13	0.02	0.13
11	1-2	-1	18	1.4	4.94	19.96	1.42	22.63
SW				90.3	77.63		90.24	

Table 2: Social welfare comparison in set A.

Exp	SW_{opt}	SW_{LS}	SW_{new}	$loss_{LS}$	$loss_{new}(\%)$
1	187.5	186.97	187.12	0.28	0.2
2	137.5	137.28	137.39	0.16	0.08
3	110.58	109.47	110.58	1.003	0
4	90.3	77.63	90.24	14.03	0.06
5	98.45	96.92	98.44	1.55	0.01
6	247.5	247.36	247.49	0.05	0.004
7	337.5	337.14	337.24	0.1	0.07
8	493.81	487.53	493.36	1.27	0.09

Table 3: Social welfare comparison in set B.

Exp	SW_{opt}	SW_{LS}	SW_{new}	$loss_{LS}$	$loss_{new}(\%)$
1	215.67	193.73	215.60	10.17	0.03
2	215.70	194.73	215.64	9.72	0.02
3	215.76	195.72	215.68	9.28	0.03
4	215.85	196.71	215.77	8.86	0.03
5	215.96	197.71	215.87	8.45	0.04
6	220.30	212.63	219.99	3.48	0.14
7	237.70	237.51	237.70	0.07	0
8	262.5	262.42	262.42	0.03	0.03

Table 4: Social welfare comparison in set C.

Exp	SW_{opt}	SW_{LS}	SW_{new}	$loss_{LS}$	$loss_{new}(\%)$
1	262.5	262.39	262.42	0.04	0.03
2	237.75	216.62	237.48	8.88	0.11
3	223.09	207.65	222.97	6.92	0.05
4	223.95	212.63	223.75	5.05	0.08
5	225.45	217.60	225.15	3.48	0.13
6	243.75	237.51	243.74	2.56	0.004
7	233.36	232.54	233.33	0.35	0.01
8	247.5	247.48	247.48	0.008	0.008

Table 5: Social welfare comparison in set D.

Exp	SW_{opt}	SW_{LS}	SW_{new}	$loss_{LS}$	$loss_{new}(\%)$
1	204.16	186.92	203.78	8.44	0.18
2	204.3	186.92	204.25	8.5	0.02
3	205	186.92	204.95	8.81	0.02
4	208.7	191.93	208.68	8.03	0.009
5	210.8	191.94	210.75	8.94	0.02
6	207.2	191.94	206.74	7.36	0.22
7	208.7	191.96	208.51	8.02	0.09
8	210.57	199.42	210.35	5.29	0.1

4.3 Comparative Assessment of Equilibrium Properties in Network-Wide PSP Auction

We carried out a multitude of experiments in order to gain intuition on which of the two strategies is beneficial for bidders as well as whether each strategy constitutes or not an equilibrium. We ran four types of experiments: a) experiments in which all bidders employ the LS strategy, b) experiments in which all bidders employ the new strategy, c) experiments in which one bidder employs the new strategy and all the others employ the LS strategy and d) experiments in which one bidder employs the LS strategy and all the others employ the new strategy. Note that in each experiment, each bidder employs a specific strategy throughout the auction. We have observed the following:

1) We have identified certain cases in which a path bidder benefits by applying the new strategy rather than the LS strategy, when all the others follow the latter. In Table 7, we present the result of a pair of experiments (of set E) in which bidder 6 obtains a much higher net benefit if he deviates from the LS strategy to the new strategy (22.39 versus 2.20). (Recall that a and b are the valuation parameters of $\theta_i'(x) = ax + b$.) This outcome is due to the fact that path bidder 6 raises the price under the LS strategy without excluding other bidders from playing. Thus, all winners pay more under the LS strategy for almost the same quantity of bandwidth.

2) Similarly, we have identified certain cases in which a path bidder benefits by applying the LS strategy rather than the new one, when all the others follow the latter. The result of such a pair of experiments is presented in Table 8: bidder 8 obtains a much higher net benefit if he deviates from the new strategy to the LS strategy (80.02 versus 54.72). This is due to the fact that bidder 8 raises the price under the LS

Table 6: **Social welfare comparison in set E.**

Exp	SW_{opt}	SW_{LS}	SW_{new}	$loss_{LS}$	$loss_{new}(\%)$
1	97.91	87.34	97.91	10.79	0
2	142.96	97.47	142.96	31.82	0
3	142.99	127.14	142.98	11.08	0.006
4	90.3	77.63	90.24	14.03	0.06
5	143.43	137.06	143.42	4.44	0.006
6	143.43	135.57	143.42	5.48	0.006
7	143.43	128.63	143.42	10.31	0.006
8	167.5	127.29	167.46	24.005	0.02

strategy and manages to obtain almost the entire capacity by excluding the other bidders from playing.
Based on the previous two remarks, we conclude that *none* of the two strategies constitutes an equilibrium in the *iterated game*.

3) When all path bidders apply the LS strategy, the first path bidder to bid, obtains most of the capacity and all the others obtain almost no bandwidth. The underlying intuition is as follows: If his valuation is not low, then the path bidder raises the price very much and the others are unable to respond. Thus, when the LS strategy is applied the allocation is affected by the *order* bidders place their bids. An experiment that illustrates clearly this fact is given in Table 9. The order of players in this experiment is as follows: 1 2 10 4 7 8 5 6 9 3, which is then repeated periodically. Note that bidder 10 is the first path bidder to bid and obtains most of the capacity. The valuation of path bidder 7 is almost identical to that of path bidder 10. Nevertheless, since the former places a bid after the latter, he wins no bandwidth which is in disagreement with the optimal allocation.

4) If all path bidders adopt the new strategy, then they are all better off than when they all apply the LS strategy, except perhaps for the first path bidder; this, in certain cases, gains almost the entire capacity when all bidders employ the LS strategy. This exception does not apply if there are single-link bidders with competitive valuations that bid before the first path bidder. A pair of experiments that illustrates the superiority of the new strategy is presented in Table 10. Note that this is a pair of experiments of set C and the order of bidding is: 1 2 3 4 5 6 7 8 9 10 11 12.

From the above observations, we conclude that it is safer for a path bidder to choose the new strategy because, if he is not the first one to play, then he faces the risk of obtaining almost nothing if he did otherwise. In general, overbidding makes sense when one manages to prevent all the other bidders from playing. Otherwise, he raises prices that will cause him pay ultimately more for a lower quantity. However, the order of players is not determined by the bidders themselves. In the asynchronous implementation of the PSP auction without instant feedback of new bids, a bidder may not be the first one to play but may submit a bid as if he were. Other bidders may act similarly. Only one of them may benefit from this (the actual first one), but he is not in a position to know it prior to bidding. Thus, even if a bidder is willing to risk for being the actual first bidder, he may not benefit from overbidding. He will benefit only if he manages to displace all other bidders from playing.

Last, we have not considered networks with more than two links because the smaller the path lengths the better the LS strategy performs. Indeed, we have seen that the larger the number of path bidders, the higher the deviation from the optimal social welfare. In large networks the number of path bidders is expected to increase, thus giving a negative impact on social welfare. Additionally, we expect that the social welfare loss increases as the number of links increases. In fact, we have already seen this in Example 2. Thus, the negative effect of the LS strategy is magnified in the case of large networks. Therefore, it is more interesting to compare the two strategies in the case of two links. As already argued extensively, even this comparison is favorable for our strategy.

5. SUMMARY AND FUTURE WORK

In this paper, we propose a new strategy for the network-wide PSP auction. This strategy yields a more efficient outcome, its performance is independent of the bidders' order and is individually beneficial to them in most of the cases. As a direction for future research, one could investigate various adaptive strategies and how well they perform in PSP. One such strategy could be the following: one starts with the new strategy if he observes or believes low demand and switches to the LS strategy if demand is seen to increase. Or alternatively: one starts with the new strategy if he realizes or believes that he has a relatively high valuation (compared to his opponents) and switches to the LS strategy when he realizes that his valuation is relatively low.

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Table 7: A pair of experiments where it is beneficial for bidder 6 to deviate from the LS strategy.

Bidder	**Link**	a	b	**Optimal Allocation**	**LS**		**New Str. by Bidder 6**	
					bandw.	NB	Bandw.	NB
1	1	-0.08	30	4.62	4.24	126.16	4.32	128.44
2	2	-0.08	6	4.62	4.24	24.31	4.32	24.74
3	1-2	-1	28	0	0	0	0	0
4	1-2	-1	28.2	0	0	0	0	0
5	1-2	-1	28.3	0	0	0	0	0
6	1-2	-1	36	0.37	0.75	2.20	0.67	22.39
7	1-2	-1	26.8	0	0	0	0	0
8	1-2	-1	29.7	0	0	0	0	0
SW				179.07	178.27		178.27	

Table 8: A pair of experiments where it is beneficial for bidder 8 to deviate from the new strategy, although he is not the first to submit a bid.

Bidder	**Link**	a	b	**Optimal Allocation**	**New**		**LS by Bidder 8**	
					Bandw.	NB	Bandw.	NB
1	1	-0.08	6	0	0	0	0	0
2	2	-0.08	6	0	0	0	0	0
3	1-2	-1	28.3	0.5	0.51	6.40	0	0
4	1-2	-1	28.2	0.4	0.41	4.59	0	0
5	1-2	-1	29.7	1.9	1.88	45.79	0	0
6	1-2	-1	27.5	0	0	0	0	0
7	1-2	-1	26.8	0	0	0	0.03	0.51
8	1-2	-1	30	2.2	2.18	54.72	4.96	80.02
SW				143.43	143.42		137.55	

Table 9: A pair of experiments to compare bidders' net benefit in the LS strategy and the new strategy.

Bidder	**Link**	a	b	**Optimal Allocation**	**LS**		**New**	
					Bandw.	NB	Bandw.	NB
1	1	-0.08	6	0	0.25	0.14	0	0
3	1	-1	30	3.2	0	0	3.18	71.04
5	1	-1	11	0	0	0	0	0
2	2	-0.08	6	0	0.25	0.14	0.007	0.04
4	2	-1	15	3.2	0	0	3.17	24.76
6	2	-1	10	0	0	0	0	0
7	1-2	-1	39	0.4	0	0	0.39	10.71
8	1-2	-1	32	0	0	0	0	0
9	1-2	-1	25	0	0	0	0	0
10	1-2	-1	40	1.4	4.97	132.6	1.42	39.2
SW				204.3	186.92		204.25	

Table 10: **A pair of experiments to compare bidders' net benefit in the LS strategy and the new strategy.**

Bidder	**Link**	a	b	**Optimal Allocation**	**LS**		**New**	
					Bandw.	NB	Bandw.	NB
1	1	-1	6	0	0	0	0	0
3	1	-1	22	0	0	0	0	0
5	1	-1	19	0	0	0	0	0
7	1	-1	20	0	0	0	0	0
9	1	-1	25	0	0.02	0.15	0	0
11	1	-1	23	0	0	0	0	0
2	2	-1	6	0	0	0	0	0
4	2	-1	21	0	0	0	0	0
6	2	-1	21	0	0	0	0	0
8	2	-1	21	0	0.02	0.09	0	0
10	1-2	-1	50	2.5	4.98	56.53	2.49	70.40
12	1-2	-1	50	2.5	0	0	2.51	70.98
SW				243.75	237.51		243.74	