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Optimal Microgrid Economic Operations under Progressive Second Price Auction Mechanisms

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Abstract: This paper studies the economic operation of microgrids in a distributed scenario such that the operational schedule of each of units, like generators, load units, storage units etc, in a microgrid system, is implemented by autonomous agents. We apply and generalize a so-called progressive second price auction mechanism which was firstly presented by Lazar and Semret for allocation of variable size shares of resources. In a microgrid system, (i) a storage unit, like plug-in electric vehicles, is a buyer when it charges or a seller when it discharges, and (ii) in connected mode with the main grid, besides the interactions among the individual units in the microgrid, units may sell (or buy) electricity resources from (or to) exogenous main grid with certain buying (or selling) price set by the utility. It makes the underlying auction problems distinct from most of the auction problems in the literature.

Key Words: Microgrids, Economic operation, Progressive second price auction, Nash equilibrium

1 Introduction

A microgrid system [1, 2] is a cluster of generators, storages, and loads which operates as a single controllable system that provides power and heat to its local area, and presents itself to main grid as a single controllable unit. Recently more and more works have been dedicated to studying the operation and management of microgrids which ranges from centralized control, e.g. [3–5], to partially decentralized control method, e.g. hierarchical control in [6, 7], to fully decentralized control dependent upon local information to improve the steady-state and transient response of microgrids, e.g. droop control in [8, 9], and PQ control discussed in [10, 11] and references therein.

We study economic operations of microgrids with dispatchable conventional (distributed) generators, controllable loads, and storages, like plug-in electric vehicles (PEVs), with coordinated charging and discharging behavior, see [11, 12]. In a microgrid system, traditional micro source units can be while renewable ones, e.g. photovoltaics and wind turbines etc., and these kinds of energy can be predicted in advance ([13] presents a power forecasting module) and shall be fully consumed . There are critical loads which are inelastic, and dispatched loads which are elastic and can be shed when necessary. Storage units can act as either elastic loads or dispatchable generators since they can charge or discharge. The optimal microgrid operation considered in this paper is to allocate resources, with available information, among the units in microgrids and power traded with main grids when the microgrid is connected with main grid.

Units in microgrids in a low-voltage distribution network may belong to distinct autonomous individual owners. It makes agent-based microgrid operations feasible, e.g. [14, 15] where the authors presented an auction-based distributed algorithm with fixed amount of energy, based on symmetric assignment problem for the optimal energy exchange. Actually the agent based methods had been widely applied to general resource allocation problems, like the economic dispatch in power systems [16] and network(bandwidth) resource sharing problems [17, 18], and so on.

The underlying operation problems for microgrids are in the context of divisible resource sharing problems which have been effectively solved with the so-called progressive second price (PSP) auctions, which is based upon the well-known VCG auction mechanics [19], by Lazar and Semret [17, 20] in single-side bid auction problems, and Jain and Walrand [21] in double-side bid auction problems, with the applications to the network resource sharing problems. In a double-side auction problems, each of agents submits a bid to the auctioneer: a seller submits a bid with a minimal selling price and a maximal power he would like to supply; a buyer submits a bid with a maximal buying price and a maximal power he asks.

In a microgrid system: (i) A storage unit is a buyer when it charges and becomes a seller when it discharges; and (ii) In connected mode, besides the trade among the individual units in the microgrid, these units may sell (or buy) electricity resources from (or to) exogenous main grid with certain buying (or selling) price set by the utility who does not actively participate in the auction. Due to these properties, to our knowledge, the associated auction games for microgrid systems are distinct from those studied in the literature.

As a key result of the paper, we show that under mild conditions, there exists an efficient (or socially optimal) Nash equilibrium for the game problems under PSP auction mechanisms.

The organization of the paper is as follows: In Section 2, the initial model and optimal objective of the microgrid operation is formulated. In Section 3, the auction based multi-agent system for microgrid operation is presented, with the optimal allocation of resources in microgrid supplement. Section 4 lists some of future works.

2 Formulation of Micrgrid Economic Operation Problems

Denote $\mathcal{I}$, $\mathcal{J}$ and $\mathcal{K}$ a set of dispatchable generators, elastic loads and storage units respectively in a microgrid.

We denote $y_{j,t}$ the power supplied by generator j at interval $t \in \mathcal{T}$ with $\mathcal{T}$ representing the whole operation interval of microgrid, and $x_{i,t}$ the demand of elastic load i at t, such that

$$0 \leq y_{j,t} \leq e_{j,max}, \quad \text{and} \quad 0 \leq x_{i,t} \leq e_{i,max} \tag{1}$$

where $e_{j,max}$ and $e_{i,max}$ represent the maximal generation

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capacity of generator j and the maximal required capacity of load i respectively.

For each of storages k, with $k \in \mathcal{K}$, we denote $x_{k,t}$ and $y_{k,t}$ the charging and discharging rate at interval t respectively, and denote $soc_{k,t}$, such that

$$soc_{min} \leq soc_{k,t} \leq soc_{max}, \qquad \text{for all } t, \tag{2}$$

the value of state of charge at the end of subinterval t, i.e. the ratio of the stored energy of a storage to its maximum attainable energy capacity; then subject to a coordination behavior $(x_{k,t}, y_{k,t})$ during interval t, we have

$$soc_{k,t+1} = soc_{k,t} + \frac{\Delta T}{\Gamma_k}(\gamma_k^+ x_{k,t} - y_{k,t}), \tag{3}$$

where Γ_k and γ_k^+, with $0 < \gamma_k^+ < 1$, represent the maximum attainable energy capacity and the charging efficiency of storage k respectively, and ΔT denotes the uniform length of any subinterval t.

We say a pair of $(x_{k,t}, y_{k,t})$ *admissible* if

$$x_{k,t}\, y_{k,t} = 0, \tag{4a}$$

$$0 \leq x_{k,t} \leq A_n^+ \equiv \frac{\Gamma_k}{\Delta T \gamma_k^+}(soc_{max} - soc_{k,t}), \tag{4b}$$

$$0 \leq y_{k,t} \leq A_n^- \equiv \frac{\Gamma_k}{\Delta T}(soc_{k,t} - soc_{min}), \tag{4c}$$

Besides the dispatchable units specified above, there are intermittent renewable energy units and loads with inelastic demand in the microgrid, such that prior to a subinterval t, (i) the aggregated power generated by renewable resource units during the subinterval t, denoted $e_{r,t}$, is predicted, and (ii) the total inelastic demand of those loads, denoted $e_{d,t}$, is given. For analytical simplicity $e_{r,t}$ and $e_{d,t}$ are assumed to be constant during interval t. Moreover in order to maximize the social welfare and become as environmentally friendly as possible, we assume that the power generated by the renewable generators shall be fully utilized.

In isolated mode, the units in the microgrid system interact with each other to minimize operational cost, while in connection mode we suppose that the microgrid is permitted to exchange the electricity with main gird bidirectionally, i.e. the power flow can be from the main grid toward the microgrid or vice versa/the other way around. Denote $p_{s,t}$ and $e_{s,t}$ the selling price and power flow toward main grid from microgrid, and $p_{b,t}$ and $e_{b,t}$ the buying price and power flow toward microgrid from main grid respectively. To avoid microgrid systems to make money by selling the bought electricity from main grid, we suppose that $p_{s,t} < p_{b,t}$ for all t.

A collection of strategies of microgrid system $\mathfrak{a}_t \equiv (x_t, y_t, z_t, e_{b,t}, e_{s,t})$, during the interval t, is called an *admissible strategy* with respect to $e_{r,t}$, $e_{d,t}$ and $soc_{k,t}$, for all k, if $\mathfrak{a}_t$ satisfies the constraints (1, 4) and the *power conservation law* below:

$$\begin{aligned} e_{r,t} &+ \sum_j y_{j,t} + \sum_k \gamma_k^- y_{k,t} + e_{b,t} \\ &= e_{d,t} + \sum_i x_{i,t} + \sum_k x_{k,t} + e_{s,t} \end{aligned} \tag{5}$$

where γ_k^-, with $0 < \gamma_k^- \leq 1$, is the discharging efficiency of storage k. The set of admissible strategies is denoted $\mathcal{S}$.

2.1 Economic Operations of Microgrid Systems

We specify an operation cost of microgrid, denoted J subject to a collection of admissible strategies $\mathfrak{a}_t$, as below:

$$\begin{aligned} J(\mathfrak{a}_t) = &\sum_{j \in \mathcal{J}} c_j(y_{j,t}) - \sum_{i \in \mathcal{I}} v_i(x_{i,t}) + \sum_{k \in \mathcal{K}} w_k(z_{k,t}) \\ &+ p_{b,t} e_{b,t} - p_{s,t} e_{s,t} \end{aligned} \tag{6}$$

where v_i, c_j and w_k represent a utility function of load i, a cost function of generator j and a cost function of storage k respectively.

The objective of the economic operation of microgrids is to assign an optimal allocation to minimize the operation cost (6) over the set of admissible strategies $\mathcal{S}$. We call the optimal allocation profile *efficient*.

The underlying economic operation problems are optimization problems with inequality and equality constraints, and can be solved by the *Lagrange multiplier* methods.

Let $\lambda \equiv (\lambda_x, \lambda_y, \lambda_z)$ be the Lagrange multiplier corresponding to the inequality constraints (1), (2) and (4), with $\lambda_x \equiv (\lambda_i; i \in \mathcal{I})$, $\mu_y \equiv (\mu_j; j \in \mathcal{J})$, $(\lambda, \mu)_z \equiv ((\lambda_k, \mu_k); k \in \mathcal{K})$, and ν is the Lagrange multiplier corresponding to the equality constraint of (5); then it can be verified that the associated *KKT conditions* are (5) together with (7) specified below:

In the paper we consider that:

- (A1) Utility function $v_i(x_i)$ of load i is increasing and strictly concave on x_i;
- (A2) Cost function $c_j(y_j)$ of generator j is increasing and strictly convex on y_j.

In the paper, we consider that $w_k(x_k, y_k)$ is to measure the deviation cost of the SOC value from a preferred energy level soc_{ref}, such that

$$w_k(x_{k,t}, y_{k,t}) = \delta_k(soc_{k,t+1} - soc_{ref})^2, \tag{8}$$

with $soc_{k,t+1}$ specified in (3), where δ_k, with $\delta_k > 0$, is a parameter.

Remark: [22] It is noted that the underlying economic operation problems are convex optimization problems, and the necessary KKT conditions (7) are also sufficient conditions for the optimality of the cost function (6), since (i) we can verify that the constraints specified in (6) determine a convex domain; (ii) this is a convex optimization problem under Assumptions (A1) and (A2). Thus there exists a unique solution and it is characterized by the KKT conditions *specified in (7).*

Lemma 2.1 Suppose (e_b^*, e_s^*) is the power flow between microgrid and main grid subject to the optimal allocation (6) under Assumptions (A1, A2); then $e_s^* e_b^* = 0$ in case $p_s < p_b$. ■

2.2 A simulation example

In this section we will illustrate the performance of the auction-based algorithm with a few of simulation examples. For the purpose of demonstration we consider a microgrid composed of a wind turbine, a collection of inelastic loads, a diesel generator with generation capacity of 30kw with cost function $c(y_t) = 0.07y_t^{1.17}$, see [23], an elastic load with utility function specified as $v(x_t) = 0.115x_t^{0.9}$, a storage possessing a maximum capacity of 30kwh, and it cost

$$\begin{cases} v_i'(x_{i,t}) - \nu - \lambda_i & \leq 0, \quad x_{i,t} & \geq 0, \quad (v_i'(x_{i,t}) - \nu - \lambda_i)x_{i,t} & = 0 \\ -c_j'(y_{j,t}) + \nu - \mu_j & \leq 0, \quad y_{j,t} & \geq 0, \quad (-c_j'(y_{j,t}) + \nu - \mu_j)y_{j,t} & = 0 \\ -\dfrac{\partial w_k}{\partial x_{k,t}} - \nu - \lambda_k & \leq 0, \quad x_{k,t} & \geq 0, \quad \left(-\dfrac{\partial w_k}{\partial x_{k,t}} - \nu - \lambda_k\right) x_{k,t} & = 0 \\ -\dfrac{\partial w_k}{\partial y_{k,t}} + \nu - \mu_k & \leq 0, \quad y_{k,t} & \geq 0, \quad \left(-\dfrac{\partial w_k}{\partial y_{k,t}} + \nu - \mu_k\right) y_{k,t} & = 0 \end{cases}, \tag{7a}$$

$$\begin{cases} -p_{b,t} + \nu & \leq 0, \quad e_{b,t} & \geq 0, \quad (-p_{b,t} + \nu)e_{b,t} & = 0 \\ p_{s,t} - \nu & \leq 0, \quad e_{s,t} & \geq 0, \quad (p_{s,t} - \nu)e_{s,t} & = 0 \end{cases}, \tag{7b}$$

function $w(x_t, y_t) = \delta(soc_{ref} - soc_{t+1})^2$, with an initial SOC value as $soc_0 = 0.45$, $\delta = 5$, $soc_{ref} = 0.5$. We also consider a battery capacity of $\Gamma = 30$kwh, $soc_{min} = 0.1$, $soc_{max} = 0.9$, and charging and discharging efficiencies $\gamma^+ = \gamma^- = 98\%$.

Ahead of any interval t the generation of wind turbine $P_{r,t}$ and the aggregated demand of the collection of inelastic loads are given as illustrated in Fig. 1. We consider $p_{s,t} < p_{b,t}$ for all t, such that $p_{s,t} = 0.09\$/\text{kwh}$ and $p_{b,t} = 0.095\$/\text{kwh}$ for all t.

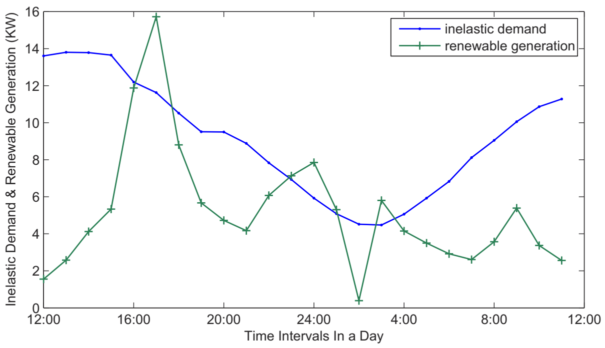

Fig. 1: Generations of wind turbine and demands of inelastic loads.

Fig. 2 illustrates the optimal allocation profile for economic operation problems with respect to the specification given above. The optimal allocation is determined by the interaction of the electricity trade price and the marginal valuation (or cost).

In case the marginal valuation of load is relatively high, the system tends to satisfy the load's requirements.

In case the marginal cost of generator is relatively low, the generator tends to supply resources.

The storage unit tends to charge when the buying price is relatively low and the resources supplied by generators are more than those demanded by loads, and discharges vise versa. If the buying electricity price is low, the system tends to buy resources from main grid and if the selling electricity price is high, the system tends to sell resources to main grid.

For example, as illustrated in Fig. 2, during time intervals 12:00 – 15:00, the inelastic demand is more than the renewable generation, so the allocation to load is little, while the generation supply, storage discharge and the resources bought from main grid are large. During time intervals 16:00 – 17:00, the renewable generation increases and then the load demand increases and the storage charges while the resources bought from main grid decreases. At 17:00, there exist some resources to sell to main grid.

The optimal dispatch strategy of economic operations of

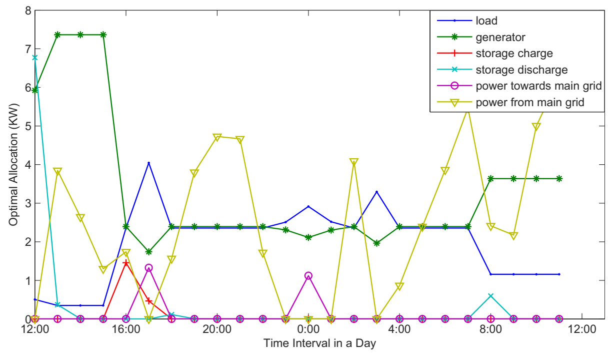

Fig. 2: Optimal allocation with respect to a retailed electricity price.

microgrid systems can be effectively implemented in case that the system controller has complete information and can directly schedule the behaviors of all individual units. However in practice the individual units may not want to share their private information with others or does not permit the system directly control their units. In this situation, distributed coordination methods with limited information can be adapted.

3 Microgrid Operations with PSP Auction Mechanism

A typical microgrid is composed of generators, loads and storage units which can be divided into three categories. The first category is buyers, i.e. (non-critical) loads. The second category is sellers, i.e. (non-renewable) generators. The last category is storages, because they can act as either buyer or seller, which depends on the operation condition of microgrid. Main grid creates extra resource to balance the flow in microgrid. This type of PSP mechanism is an extension of that studied in [21] where the units report a two-dimensional bid parameters instead of reporting their types or complete utility functions. The auction mechanism is illustrated in Fig. 3.

3.1 Bid profiles of individual units in microgrid systems

As a buyer, a load i submits a (2-dimension) bid profile $b_{i,t}$, such that

$$b_{i,t} = (\beta_{i,t}, d_{i,t}), \quad \text{with } 0 \leq d_{i,t} \leq e_{i,max}, \tag{9}$$

of the revealed utility function $\widehat{v}_i(x_{i,t}) = \beta_{i,t} \min(x_{i,t}, d_{i,t})$. where specifies the maximum per unit price $\beta_{i,t}$ that the load i is willing to pay and demands up to $d_{i,t}$ units of the electricity; then the corresponding admissible strategy $x_{i,t}$ of load i,

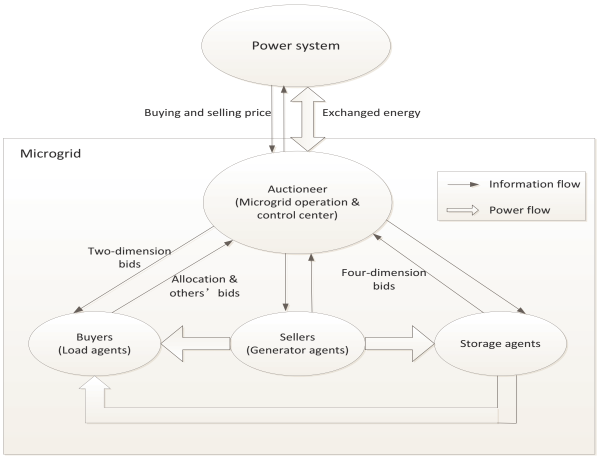

Fig. 3: PSP auction flowchart for a microgrid system

with respect to $b_{i,t}$, must satisfies:

$$0 \leq x_{i,t} \leq d_{i,t}$$

As a seller, a generator j specifies a (2-dimension) ask-bid $a_{j,t}$, such that

$$a_{j,t} = (\alpha_{j,t}, s_{j,t}), \text{ with } 0 \leq s_{j,t} \leq e_{j,max}, \tag{10}$$

of the revealed cost function $\widehat{c}_j(y_{j,t}) = \alpha_{j,t} \min(y_{j,t}, s_{j,t})$, where $\alpha_{j,t}$ is the minimum per unit price that generator j is willing to accept and can supply up to $s_{j,t}$ units of the electricity; then the corresponding admissible strategy $y_{j,t}$ of generator j, with respect to $a_{j,t}$, must satisfies:

$$0 \leq y_{j,t} \leq s_{j,t}$$

A storage can be either a buyer when it charges or a seller when it discharges. As a result, a storage k submits a (4-dimension) bid profile $r_{k,t}$ at interval t, with $r_{k,t} \equiv (b_{k,t}, a_{k,t})$, such that $b_{k,t}$ and $a_{k,t}$ are specified as follows:

$$b_{k,t} = (\beta_{k,t}, d_{k,t}), \text{ with } 0 \leq d_{k,t} \leq A_n^+, \tag{11a}$$
$$a_{k,t} = (\alpha_{k,t}, s_{k,t}), \text{ with } 0 \leq s_{k,t} \leq A_n^-, \tag{11b}$$

where (i) $\beta_{k,t}$ is the maximum per unit price that the storage k is willing to pay and can charge up to $d_{k,t}$ units of the electricity, and (ii) $\alpha_{k,t}$ is the minimum per unit price that the storage k is willing to accept and can discharge up to $s_{k,t}$ units of the electricity.

Hence, the revealed utility function of the storage k with respect to the bid profile specified in (11) is given as follows:

$$\widehat{w}_k(x_{k,t}, y_{k,t}) = \beta_{k,t} \min(x_{k,t}, d_{k,t}) - \alpha_{k,t} \min(y_{k,t}, s_{k,t}). \tag{12}$$

The corresponding admissible strategy $x_{i,t}$ of load i, with respect to $b_{i,t}$, must satisfies:

$$0 \leq x_{k,t} \leq d_{k,t}, \quad \text{and} \quad 0 \leq y_{k,t} \leq s_{k,t}$$

Remark: For notational simplicity, in the rest of the paper we skip the time index t in case no confusions are involved.

3.2 Resource allocation rule subject to bid profiles of units

Definition 3.1 Considering a collection of bid profiles (b, a, r), we call $\mathfrak{a} \equiv (x, y, z, e_b, e_s)$ is an *admissible allocation* with respect to (b, a, r), if the power conservation law (5) and the following constraints hold:

$$0 \leq x_i \leq d_i \tag{13a}$$
$$0 \leq y_j \leq s_j \tag{13b}$$
$$(0, 0) \leq (x_k, y_k) \leq (d_k, s_k) \tag{13c}$$
$$0 \leq e_s, e_b \tag{13d}$$

The set of admissible allocations with respect to (b, a, r) is denoted $\mathcal{A}(b, a, r)$. ■

We further define a function U on an admissible allocation $\mathfrak{a} \equiv (x, y, z, e_b, e_s)$ with respect to a bid profile (b, a, r) as the following:

$$U(\mathfrak{a}) = \sum_{i \in \mathcal{I}} \beta_i x_i - \sum_{j \in \mathcal{J}} \alpha_j y_j + \sum_{k \in \mathcal{K}} (\beta_k \gamma_k^+ x_k - \alpha_k y_k) + p_s e_s - p_b e_b. \tag{14}$$

Remark: $U(\mathfrak{a})$ can be interpreted as the total monetary income of the microgrid with allocation $\mathfrak{a} = (x, y, z, e_b, e_s)$.

The auctioneer will assign an optimal admissible allocation $\mathfrak{a}^* \equiv (x^*, y^*, z^*, e_b^*, e_s^*)$ with respect to a collection of a bid profile (b, a, r), such that

$$\mathfrak{a}^* = \underset{\mathfrak{a} \in \mathcal{A}(b,a,r)}{\text{argmax}} \{U(\mathfrak{a})\}. \tag{15}$$

and $\mathfrak{a}^*$ can be characterized by the KKT conditions in (**??**), (7b) and (16).

3.3 Transfer money of units subject to bid profiles

Considering any bid profile, we will specify the so-called *transfer money*, denoted τ, for each of loads, generators, and storages following the allocation way by the system auctioneer given in (15). Essentially, the payment/income of a unit can be expressed as, the summation of all users' utility functions when this unit didn't join the auction process, minus the summation of the all of the other units' utility functions when this unit joined the auction process. That is to say, the money transfer τ made by each of related units, is exactly the externality it imposes on others through its participation, just as in the VCG mechanism adapted in [19, 24, 25] etc.

3.3.1 Money transfer of loads

Let $\widehat{\mathfrak{a}}^*_{(i)}$ denote the solution to the allocation rule, defined in Section 3.2, but with, $0 \leq x_i \leq d_i$, the first constraint in the set of constraints (13) substituted with $d_i = 0$ for load i, i.e.

$$\widehat{\mathfrak{a}}^*_{(i)} = \underset{\mathfrak{a} \in \mathcal{A}((\widehat{b}_i, b_{-i}), a, r)}{\text{argmax}} \{U(\mathfrak{a})\} \tag{17}$$

where $\widehat{b}_i = (\widehat{\beta}_i, \widehat{d}_i)$ with $\widehat{d}_i = 0$.

The money transfer to be made by load i (the payment) with bid profiles (b, a, r), denoted by $\tau_i(b, a, r)$, is given by (18) below,

$$\tau_i(b, a, r) = U(\widehat{\mathfrak{a}}^*_{(i)}) - (U(\mathfrak{a}^*) - \beta_i x_i^*). \tag{18}$$

with $\mathfrak{a}^* \equiv \mathfrak{a}^*(b, a, r)$ as specified in (15).

$$\begin{cases} \beta_l - \nu - \lambda_l & \le 0, \quad x_l \quad \ge 0, \quad (\beta_l - \nu - \lambda_l)x_l \quad = 0, \text{ with } l \in \mathcal{I} \cup \mathcal{K}, \\ -\alpha_l + \nu - \mu_l & \le 0, \quad y_l \quad \ge 0, \quad (-\alpha_l + \nu - \mu_l)y_l \quad = 0, \text{ with } l \in \mathcal{J} \cup \mathcal{K}, \end{cases} \tag{16a}$$

$$\begin{cases} d_l - x_l & \ge 0, \quad \lambda_l \quad \ge 0, \quad (d_l - x_l)\lambda_l \quad = 0, \text{ with } l \in \mathcal{I} \cup \mathcal{K}, \\ s_l - y_l & \ge 0, \quad \mu_l \quad \ge 0, \quad (s_l - y_l)\mu_l \quad = 0, \text{ with } l \in \mathcal{J} \cup \mathcal{K}. \end{cases} \tag{16b}$$

3.3.2 Money transfer of generators

Let $\widehat{\mathfrak{a}}^*_{(j)}$ denote the solution to the allocation rule, defined in Section 3.2, but with the 2nd constraint in the set of constraints (13) substituted with $s_j = 0$ for generator j, i.e.

$$\widehat{\mathfrak{a}}^*_{(j)} = \underset{\mathfrak{a} \in \mathcal{A}(b, (\widehat{a}_j, a_{-j}), r)}{\text{argmax}} \{U(\mathfrak{a})\} \tag{19}$$

where $\widehat{a}_j = (\widehat{\alpha}_j, \widehat{s}_j)$ with $\widehat{s}_j = 0$.

The money transfer to be made by generator j (the payment) with bid profiles (b, a, r), denoted by $\tau_j(b, a, r)$, is given (20) below,

$$\tau_j(b, a, r) \ = \ U(\widehat{\mathfrak{a}}^*_{(j)}) - (U(\mathfrak{a}^*) + \alpha_j y_j^*). \tag{20}$$

Remark: The negative value of τ_j means money transfer to the generator j.

3.3.3 Money transfer of storages

Let $\widehat{\mathfrak{a}}^*_{(k)}$ denote the solution to the allocation rule, defined in Section 3.2, but with the 3rd and 4th constraints in the set of constraints (13) substituted with $d_k = s_k = 0$ for storage k, i.e.

$$\widehat{\mathfrak{a}}^*_{(k)} = \underset{\mathfrak{a} \in \mathcal{A}(b, a, (\widehat{r}_k, r_{-k}))}{\text{argmax}} \{U(\mathfrak{a})\} \tag{21}$$

where $\widehat{r}_k = ((\widehat{\beta}_k, \widehat{d}_k), (\widehat{\alpha}_k, \widehat{s}_k))$ with $\widehat{d}_k = \widehat{s}_k = 0$.

The money transfer to be made by storage k with bid profiles (b, a, r), denoted by $\tau_k(b, a, r)$, is given by (**??**) below,

$$\tau_k(b, a, r) = U(\widehat{\mathfrak{a}}^*_{(k)}) - (U(\mathfrak{a}^*) - (\beta_k \gamma_k^+ x_k^* - \alpha_k y_k^*)).$$

Remark: Since a storage can either charge or discharge, the value of τ_k, the money transfer to the storage k, can be negative or positive.

3.4 Payoff functions of individual units

Subject to a collection of bid profiles, the *payoff functions* of individual units are specified by adapting the auctioneer's optimal resource allocation and the money transfer mechanism:

Load i has a payoff function $u_i(b, a, r)$, such that

$$u_i(b, a, r) = v_i(x_i^*) - \tau_i(b, a, r), \tag{22}$$

where $x_i^* \equiv x_i^*(b, a, r)$ represents the allocated demand for load i assigned by auctioneer with the bid profiles (b, a, r), and $\tau_i(b, a, r)$ is the money payed by the load i defined in (18).

Generator j has a payoff function $u_j(b, a, r)$, such that

$$u_j(b, a, r) = -\tau_j(b, a, r) - c_j(y_j^*), \tag{23}$$

where $y_j^* \equiv y_j^*(b, a, r)$ represents the allocated generation power for generator j assigned by auctioneer with (b, a, r), and $\tau_j(b, a, r)$ is the money transfer of the generator j defined in (20).

Storage k has a payoff function $u_k(b, a, r)$, such that

$$u_k(b, a, r) = -w_k(x_k^*, y_k^*) - \tau_k(b, a, r), \tag{24}$$

where $x_k^* \equiv x_k^*(b, a, r)$ and $y_k^* \equiv y_k^*(b, a, r)$ represent respectively charging and discharging rates for storage k assigned by auctioneer with the bid profiles (b, a, r), and $\tau_k(b, a, r)$ is the money transfer of the storage k defined in Section 3.3.3.

3.5 Properties of allocation and equilibria: Efficiency

Definition 3.2 A collection of bid profiles $\mathbf{b}^0 \equiv (b^0, a^0, r^0)$ is a *Nash equilibrium* for the auction problems if the following holds:

$$b_i^0 = \underset{b_i \in \mathcal{B}_i}{\text{argmax}} \{u_i(b_i, \mathbf{b}^0_{-i})\}, \text{ for all } i \in \mathcal{I}, \tag{25a}$$

$$a_j^0 = \underset{a_j \in \mathcal{B}_j}{\text{argmax}} \{u_j(a_j, \mathbf{b}^0_{-j})\}, \text{ for all } j \in \mathcal{J}, \tag{25b}$$

$$r_k^0 = \underset{r_k \in \mathcal{B}_k}{\text{argmax}} \{u_k(r_k, \mathbf{b}^0_{-k})\}, \text{ for all } k \in \mathcal{K}. \tag{25c}$$

■

In Theorem 3.1, we will show that the bid profile with the efficient allocation is a Nash equilibrium; before that we first give Lemma 3.1 and Lemma **??** *below.*

Lemma 3.1 Suppose $\mathfrak{a}^{**} \equiv (x^{**}, y^{**}, z^{**}, e_b^{**}, e_s^{**})$ is the efficient allocation profile, and consider a collection of bid profiles $\mathbf{b}^*$ such that

$$b_i^* = (v_i'(d_i), d_i), \text{ with } d_i = x_i^{**}, \tag{26a}$$

$$a_j^* = (c_j'(s_j), s_j), \text{ with } s_j = y_j^{**}, \tag{26b}$$

$$r_k^* = \Big(-\frac{\partial w_k}{\partial d_k}, d_k, \frac{\partial w_k}{\partial s_k}, s_k\Big), \text{ with } (d_k, s_k) = (x_k^{**}, y_k^{**}), \tag{26c}$$

for all $i \in \mathcal{I}$, $j \in \mathcal{J}$ and $k \in \mathcal{K}$; then $\mathfrak{a}^*(\mathbf{b}^*) = \mathfrak{a}^{**}$, i.e. the allocation $\mathfrak{a}^*(\mathbf{b}^*)$ is efficient. ■

Lemma 3.2 Suppose $\mathfrak{a}^*$ is the efficient allocation with respect to (b^*, a^*, r^*) in (26); then

$$\beta_l^* \begin{cases} = \nu, & \text{in case } x_l^* > 0 \\ \le \nu, & \text{otherwise} \end{cases}, \quad \text{for all } l \in \mathcal{I} \cup \mathcal{K} \tag{27}$$

$$\alpha_l^* \begin{cases} = \nu, & \text{in case } y_l^* > 0 \\ \ge \nu, & \text{otherwise} \end{cases}, \quad \text{for all } l \in \mathcal{J} \cup \mathcal{K} \tag{28}$$

where the system price ν is specified below:

$$\nu \begin{cases} = p_b, & \text{in case } P_b^* > 0 \\ = p_s, & \text{in case } P_s^* > 0\ . \\ \in (p_s, p_b), & \text{otherwise} \end{cases} \tag{29}$$

■

Theorem 3.1 Under Assumptions (A1,A2), there exists an efficient Nash equilibrium (b^*, a^*, r^*) for auction allocation game (14), i.e. (b^*, a^*, r^*) is a Nash equilibrium and is optimal for the problems (15). ■

4 Conclusion and Future Works

An auction mechanism for the optimal allocation of power resource is proposed in this paper. The mechanism is VCG-style PSP auction but it has simpler calculation process just as in the context of network resource sharing studied by Jain and his collaborators in [21]. In the microgrid electricity economic operation problems in the paper, the generators or loads only need to report a two-dimensional bid: a price per unit and the maximum quantity demanded, as opposed to the VCG style which requires the full utility function; while the storages, which can charge or discharge, need to report a pair of two-dimensional bids. The auctioneer implements the optimal allocation based upon the collection of reported bids from all of the units. Under certain mild conditions, there exists efficient auction-based resource allocation profile.

As extensions of the current work, we are studying: (i) the bid strategy update algorithm under which the auction system can converge to the efficient bid profiles; (ii) the economic operations under the PSP auction mechanisms over multi time-slots and the bid profile update procedures subject to which the system can converge to the social optimal allocations.

References

[1] R. Lasseter, "Microgrids," in *IEEE Power Engineering Society Winter Meeting*, vol. 1, 27-31 January 2002, pp. 305–308.

[2] R. Lasseter, A. Akhil, C. Marnay, J. Stephens, J. Dagle, R. Guttromson, A. Meliopoulos, R. Yinger, and J. Eto, "Integration of distributed energy resources. the CERTS microgrid concept," in *Consortium for Electric Reliability Technology Solutions (CERTS)*, April 2002.

[3] A. Tsikalakis and N. Hatziargyriou, "Centralized control for optimizing microgrids operation," *IEEE Transactions on Energy Conversion*, vol. 23, no. 1, pp. 241–248, March 2008.

[4] ——, "Centralized control for optimizing microgrids operation," in *Power and Energy Society General Meeting, 2011 IEEE*, San Diego, CA, 24-29 July 2011, p. 1C8.

[5] E. Mashhour and S. Moghaddas-Tafreshi, "Integration of distributed energy resources into low voltage grid a market-based multiperiod optimization model," *Electric Power Systems Research*, vol. 80, no. 4, p. 473C480, April 2010.

[6] N. Hatziargyriou, A. Dimeas, A. Tsikalakis, J. Lopes, G. Kariniotakis, and J. Oyarzabal, "Management of microgrids in market environment," in *International Conference on Future Power Systems*, November 18 2005.

[7] W. Tang and R. Jain, "Hierarchical auction mechanisms for network resource allocation," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 11, pp. 2117 – 2125, December 2012.

[8] K. Brabandere, B.Bolsens, J. V. den Keybus, A.Woyte, J.Driesen, and R.Belmans, "A voltage and frequency droop control method for parallel inverters," *Power Electronics, IEEE Transactions on*, vol. 22, no. 4, pp. 1107 – 1115, July 2007.

[9] R. Majumder, B. Chaudhuri, A. Ghosh, G. Ledwich, and F. Zare, "Improvement of stability and load sharing in an autonomous microgrid using supplementary droop control loop," *Power Systems, IEEE Transactions on*, vol. 25, no. 2, pp. 796 – 808, May 2010.

[10] J. Lopes, C. Moreira, and A. Madureira, "Defining control strategies for microgrids islanded operation," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 916–924, May 2006.

[11] J. Lopes, S. Polenz, C. Moreira, and R. Cherkaoui, "Identification of control and management strategies for lv unbalanced microgrids with plugged-in electric vehicles," *Electric Power Systems Research*, vol. 80, no. 8, p. 898C906, August 2010.

[12] C. H. Aramburo, T. Green, and N. Mugniot, "Fuel consumption minimization of a microgrid," *IEEE Transactions on Industry Applications*, vol. 41, no. 3, p. 673C681, January 2006.

[13] C. Chen, S. Duan, T. Cai, B. Liu, and G. Hu, "Smart energy management system for optimal microgrid economic operation," *Renewable Power Generation, IET*, vol. 5, no. 3, pp. 258 – 267, May 2011.

[14] A. Dimeas and N. Hatziargyriou, "Operation of a multiagent syetem for microgrid control," *IEEE Transactions on Power Systems*, vol. 20, no. 3, pp. 1447–1455, August 2005.

[15] T. Logenthiran, D. Srinivasan, and A. M. Khambadkone, "Multi-agent system for energy resource scheduling of integrated microgrids in a distributed system," *Electric Power Systems Research*, vol. 81, no. 1, p. 138C148, January 2011.

[16] M. Ilic, J.-Y. Joo, L. Xie, M. Prica, and N. Rotering, "A decision-making framework and simulator for sustainable electric energy systems," *Sustainable Energy, IEEE Transactions on*, vol. 2, no. 1, pp. 37 – 49, January 2011.

[17] A. Lazar and N. Semret, "Design and analysis of the progressive second price auction for network resource bandwith sharing," *Telecommunication System - Special issue on Network Economic*, 1999.

[18] A. Kakhbod and D. Teneketzis, "An efficient game form for unicast service provisioning," *Automatic Control, IEEE Transactions on*, vol. 57, no. 2, pp. 392 – 404, Febuary 2012.

[19] W. Vickrey, "Counterspeculation, auctions, and competitive sealed tenders," *The Journal of Finance*, vol. 16, no. 1, pp. 8–37, March 1961.

[20] A. Lazar and N. Semret, "The progressive second price auction mechanism for network resource sharing," in *8th International Symposium on Dynamic Games*, July 1998.

[21] R.Jain and J.Walrand, "An efficient Nash-implimentation mechanism for network resource allocation," *Automatica*, vol. 46, no. 8, p. 1276C1283, August 2010.

[22] S.Boyd and L.Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[23] F.Mohamed and H.Koivo, "Online management of microgrid with battery storage using multiobjective optimization," in *POWERENG*, Setubal, Portugal, April 12-14, 2007, p. 2501C2507.

[24] E. Clark, "Multipart pricing of public goods," *Public Choice*, vol. 11, no. 1, pp. 19–33, 1971.

[25] T. Groves, "Incentives in teams," *Econometrica*, vol. 41, no. 4, pp. 617–631, July 1973.