CS325 Winter 2013: HW1

Due Jan 18th in class

Instructions You are highly encouraged to work in groups of up to three (no more than three). Each group only need to submit one copy of the homework, with all group member names listed. See the course description page on the class webpage for grading information.

1. 0.2 from text book.
2. 0.3(a) from text book.
3. 2.3 from text book.
4. 2.4 from text book.
5. 2.17 from text book.

Sample questions The following questions are provided with solutions to help you review the contents.

0.1 from textbook

	$f(n)$	$g(n)$	$f = O(g)$?	$f = \Omega(g)$?	$f = \theta(g)$?	
a	$n - 100$	$n - 200$	yes	yes	yes	$=$
b	$n^{1/2}$	$n^{2/3}$	yes	no	no	$<$
c	$100n + \log n$	$n + (\log n)^2$	yes	yes	yes	$=$
d	$n \log n$	$10n \log 10n$	yes	yes	yes	$=$
e	$\log 2n$	$\log 3n$	yes	yes	yes	$=$
f	$10 \log n$	$\log(n^2)$	yes	yes	yes	$=$
g	$n^{1.01}$	$n(\log n)^2$	no	yes	no	$>$
h	$n^2 / \log n$	$n(\log n)^2$	no	yes	no	$>$
i	$n^{0.1}$	$(\log n)^{10}$	no	yes	no	$>$
j	$(\log n)^{\log n}$	$n / \log n$	no	yes	no	$>$
k	$\sqrt{n}$	$(\log n)^3$	no	yes	no	$>$
l	$n^{1/2}$	$5^{\log_2 n}$	yes	no	no	$<$
m	$n2^n$	3^n	yes	no	no	$<$
n	2^n	2^{n+1}	yes	yes	yes	$=$
o	$n!$	2^n	no	yes	no	$>$
p	$(\log n)^{\log n}$	$2^{(\log_2 n)^2}$	yes	no	no	$<$
q	$\sum_{i=1}^{n} i^k$	n^{k+1}	yes	yes	yes	$=$

2.5 from textbook

a. $T(n) = O(n^{\log_3 2})$
b. $T(n) = O(n^{\log_4 5})$
c. $T(n) = O(n \log_7 n)$
d. $T(n) = O(n^2 \log_3 n)$
e. $T(n) = O(n^3 \log_2 n)$
f. $T(n) = O(n^{3/2} \log n)$
g. $T(n) = O(n)$
h. $T(n) = n^c + (n-1)^c + ... + 1^c = \sum_{i=1}^{n} i^c = O(n^{c+1})$
i. $T(n) = \sum_{i=1}^{n} c^i = O(c^n)$
j. $T(n) = O(2^n)$
k. $T(n) = T(n^{1/2}) + 1 = T(n^{1/4}) + 2 = T(n^{(1/2)^k}) + k$ Let's assume that it stops the recursive call if the input size is a small constant b. Solving $n^{1/2^k} = b$ leads to $k = O(\log \log n)$ and $T(n) = O(\log \log n)$.

2.12 from textbook.

The recurrence relation is: $T(n) = 2T(n/2) + c$. Solving this recurrence relation using the master theorem, we have case 3 and $T(n) = \Theta(n^{\log_2 2}) = \Theta(n)$