CS325 Winter 2013: HW 6

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1

Problem: Below is a definition of the graph isomorphism problem.

> **Input:** two graphs, $G_1 = (V_1, E_1)$, and $G_2 = (V_2, E_2)$.
>
> **Question:** Can the nodes of G_1 be renamed s.t. G_1 becomes G_2 ?
> In other words, is there a one-to-one function $f : V_1 \to V_2$ such that for any edge $(x, y) \in E_1$ if only if $(f(x), f(y)) \in E_2$. Show that the graph Isomorphism problem is in NP.

Solution

2

8.4 Consider the CLIQUE problem restricted to graphs in which every vertex has degree at most 3. Call this problem CLIQUE-3.

(a) Prove that CLIQUE -3 is in NP.

(b) What is wrong with the following proof of NP-completeness for CLIQUE -3? We know that the CLIQUE problem in general graphs is NP-complete, so it is enough to present a reduction from CLIQUE -3 to CLIQUE. Given a graph G with vertices of degree ≤ 3, and a parameter g, the reduction leaves the graph and the parameter unchanged: clearly the output of the reduction is a possible input for the CLIQUE problem. Furthermore, the answer to both problems is identical. This proves the correctness of the reduction and, therefore, the NP-completeness of CLIQUE -3.

(c) It is true that the VERTEX COVER problem remains NP-complete even when restricted to graphs in which every vertex has degree at most 3. Call this problem VC -3. What is wrong with the following proof of NP-completeness for CLIQUE -3? We present a reduction from VC -3 to CLIQUE -3. Given a graph G = (V, E) with node degrees bounded by 3, and a parameter b, we create an instance of CLIQUE -3 by leaving the graph unchanged and switching the parameter to $|V| - b$. Now, a subset $C \subset V$ is a vertex cover in G if and only if the complementary set $V - C$ is a clique in G. Therefore G has a vertex cover of size $\leq b$ if and only if it has a clique of size $\geq |V| - b$. This proves the correctness of the reduction and, consequently, the NP-completeness of CLIQUE-3.

(d) Describe an $O((—V—4))$ algorithm for CLIQUE-3.

Solution:

3

Problem: In the HITTING SET problem, we are given a family of sets S1 , S2 , . . . , Sn and a budget b, and we wish to find a set H of size $\leq b$ which intersects every S_i , if such an H exists. In other words, we want $H \cap S_i = \oslash$ for all i. Show that HITTING SET is NP-complete.

Solution:

4

6.26 from book: Proving NP-completeness by generalization. For each of the problems below, prove that it is NP- complete by showing that it is a generalization of some NP-complete problem we have seen in this chapter.

(a) SUBGRAPH ISOMORPHISM: Given as input two undirected graphs G and H, determine whether G is a subgraph of H (that is, whether by deleting certain vertices and edges of H we obtain a graph that is, up to renaming of vertices, identical to G), and if so, return the corresponding mapping of V (G) into V (H).

(b) LONGEST PATH : Given a graph G and an integer g, find in G a simple path of length g.

(c) DENSE SUBGRAPH: Given a graph and two integers a and b, find a set of a vertices of G such that there are at least b edges between them.

(d) SPARSE SUBGRAPH: Given a graph and two integers a and b, find a set of a vertices of G such that there are at most b edges between them.

Solution: