Lecture 3: Stacks and Queues

Stack

Warmup

API

Stack of *String* data type

```java
public class StackOfStrings {

    // create an empty stack
    public StackOfStrings() {
        ...
    }

    // insert a new string onto stack
    public void push(String item) {
        ...
    }

    // remove and return the string most recently added
    public String pop() {
        ...
    }

    // is the stack empty?
    public boolean isEmpty() {
        ...
    }

    // number of strings on the stack
    public int size() {
        ...
    }
}
```

Test Client

Read *String* from standard input:

- if *String* equals "-", `pop()` *String* from stack and `print()`
- Otherwise, `push()` *String* onto stack

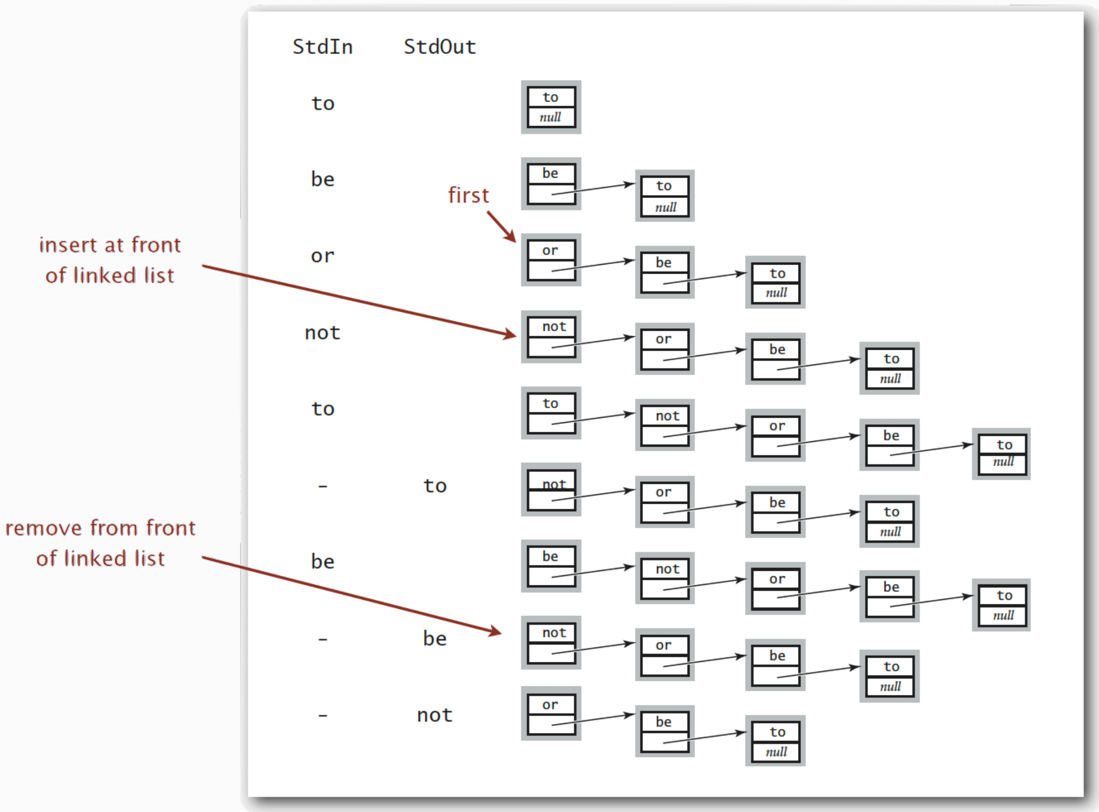
```java
public static void main (Strings[] args) {
    StackOfStrings stack = new StackOfStrings();
    while(!StdIn.isEmpty()) {
        String s = StdIn.readString();
        if (s.equals("-")) StdOut.print(stack.pop());
        else stack.push(s);
    }
}
```

Example

```
% more tobe.txt
to be or not to - be - - that - - - is

% java StackOfStrings < tobe.txt
to be not that or be
```

Linked-List Implementation

Maintain pointer to first node in a linked list, insert/remove from front



Inner Class

```
private class Node {
	String item;
	Node next;
}
```

Java Implementation

```
public class LinkedStackOfStrings {
	private Node first = null;

	private class Node {
		String item;
		Node next;
	}

	public boolean isEmpty() {
		return first == null;
	}

	public void push(String item) {
		Node oldfirst = first;
		first = new Node();
		first.item = item;
		first.next = oldfirst;
	}

	public String pop() {
		String item = first.item;
		first = first.next;
		return item;
	}
}
```

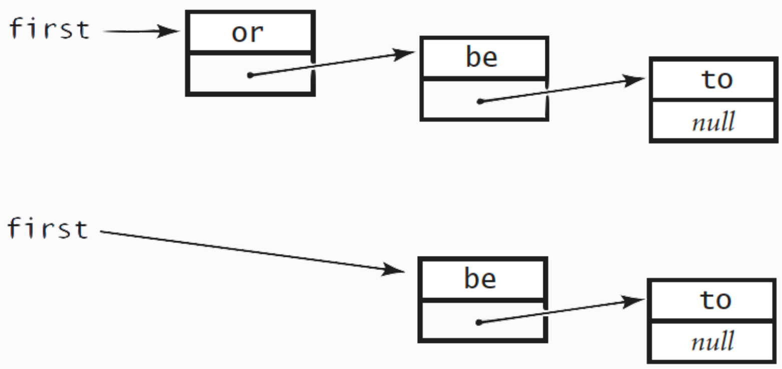
`pop()` *Implementation*

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



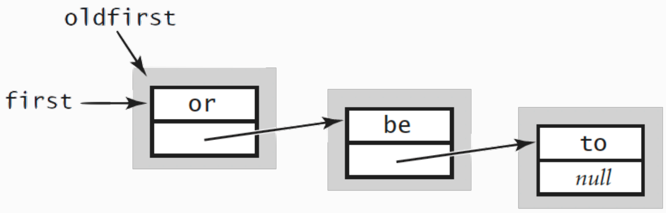
return saved item

```
return item;
```

`push()` *Implementation*

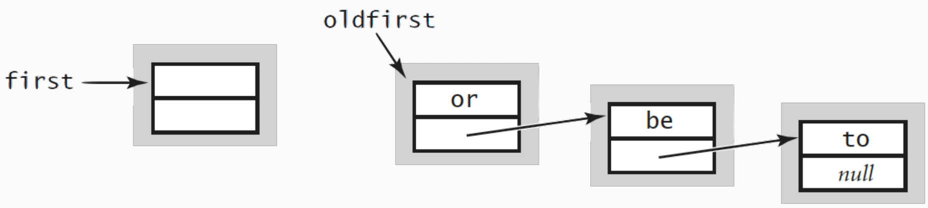
save a link to the list

```
Node oldfirst = first;
```



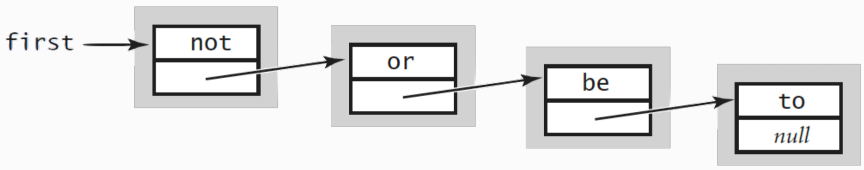
create a new node for the beginning

```
first = new Node();
```



set the instance variables in the new node

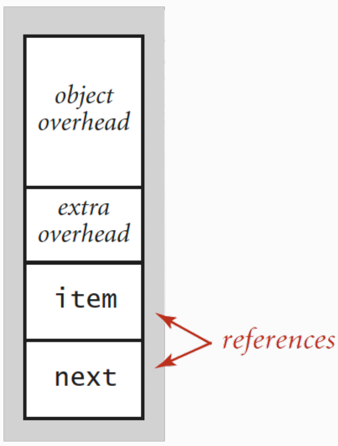
```
first.item = "not";
first.next = oldfirst;
```



Performance

Proposition: every operation takes constant time in the worst case

Proposition: A stack with N items uses $\sim 40N$ bytes



16 bytes (object overhead)

8 bytes (inner class extra overhead)

8 bytes (reference to String)

8 bytes (reference to Node)

40 bytes per stack node

> *Note that*:

> This accounts for the memory for the stack, but not the memory for *String* themselves, which the client owns

Array Implementation

- use array `s[]` to store N items on stack
- `push()` adds new item at `s[N]`
- `pop()` removes item from `s[N-1]`

s[]	to	be	or	not	to	be	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9
							N			capacity = 10

> *Defect*:
>
> *Stack* overflows when N exceeds capacity, but this will be addressed in following lectures

Java Implementation

```java
public class FixedCapacityStackOfStrings {
	private String[] s;
	private int N = 0;

	public FixedCapacityStackOfStrings (int capacity) {
		s = new String[capacity];
	}

	public boolean isEmpty() {
		return N  == 0;
	}

	public void push(String item) {
		// N++ used to index into array then increment N
		s[N++] = item;
	}

	public String pop() {
		// decrement N then use to index into array
		return s[--N]
	}
}
```

Consideration

Overflow: use resizing array for array implementation

Underflow: throw exception if pop from an empty stack

Null items: allow *null* items to be inserted

> ***Loitering***:
>
> holds a reference to an object when it is no longer needed
>
> **Solution**:
>
> ```
> public String pop() {
>     String item = s[--N];
>     s[N] = null;
>     return item;
> }
> ```
>
> Garbage collector can reclaim memory, but only if no outstanding references

Resizing-Arrays Implementation

First Try

- `pop()` : increase size of array `s[]` by 1
- `push()` : decrease size of array `s[]` by 1

Problems

- need to copy all items to a new array
- inserting first N items takes proportional to $1+2+\cdots+N \sim \frac{N^2}{2}$

> ***Challenge***: Ensure that array resizing happens infrequently

Growing Array Efficiently: Repeated Doubling

If array is full, create a new array of ***twice*** the size, and copy items

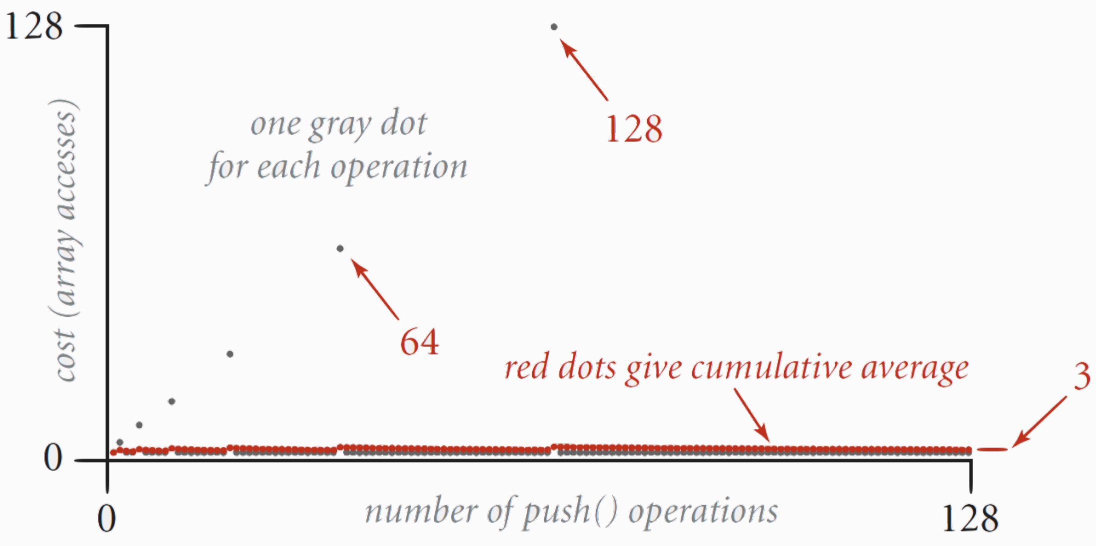
```
public class ResizingArrayStackOfStrings {
     private String[] s;
     private int N = 0;

     public ResizingArrayStackOfStrings() {
          s = new String[1]
     }

     public void push(String item) {
          if (N == s.length) resize(2 * s.length);
          s[N++] = item;
     }

     private void resize(int capacity) {
          String[] copy = new String[capacity];
          for (int i = 0; i < N; i++) {
               copy[i] = s[i];
          }
          s = copy;
     }
```

```
21  }
```

Cost of Inserting First N Items

$$N + (2 + 4 + 8 + \cdots + N) \sim 3N \tag{1}$$

- N : 1 array access per push
- $2 + 4 + 8 + \cdots + N$: k array accesses to double to size k (ignorering cost to create new array)



> ***Note that***:
>
> Performing n `push()` will call the `resize()` method to a logarithmic time because it will be called only when the array size is a power of 2, and there are $\sim \log_2 n$ powers of 2 between 1 and n.

Shrinking Array Efficiently

First Try

- `push()` : double size of array `s[]` when array is full
- `pop()` halve size of array `s[]` when array is ***one-half full***

> ***Thrashing***:
>
> - Consider `push()` - `pop()` - `push()` - `pop()` ... sequence when array is full
> - Each operation takes time proportional to N

N = 5	to	be	or	not	to	*null*	*null*	*null*
N = 4	to	be	or	not				
N = 5	to	be	or	not	to	*null*	*null*	*null*
N = 4	to	be	or	not				

Efficient Solution

- `push()` : double size of array `s[]` when array is full
- `pop()` : halve size of array `s[]` when array is ***one-quarter full***

```java
public String pop() {
     String item = s[--N];
     // N is not the same as index
     s[N] = null;
     if (N>0 && N == s.length/4) resize(s.length/2);
     return item;
}
```

> ***Invariant***:
>
> Array is between 25% and 100% full

push()	pop()	N	a.length	a[] 0	1	2	3	4	5	6	7
		0	1	*null*							
to		1	1	to							
be		2	2	to	be						
or		3	4	to	be	or	*null*				
not		4	4	to	be	or	not				
to		5	8	to	be	or	not	to	*null*	*null*	*null*
-	to	4	8	to	be	or	not	*null*	*null*	*null*	*null*
be		5	8	to	be	or	not	be	*null*	*null*	*null*
-	be	4	8	to	be	or	not	*null*	*null*	*null*	*null*
-	not	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
that		4	8	to	be	or	that	*null*	*null*	*null*	*null*
-	that	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
-	or	2	4	to	be	*null*	*null*				
-	be	1	2	to	*null*						
is		2		to	is						

Performance

> ***Amortised Analysis***: average running time per operation over a worst-case sequence of operations

Proposition: starting from an empty stack, any sequence of M `push()` and `pop()` operations takes time propotional to M

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and halving operations

order of growth of running time for resizing stack with N items

Memory Usage

Proposition: uses between $\sim 8N$ and $\sim 32N$ bytes to represent a stack with N items

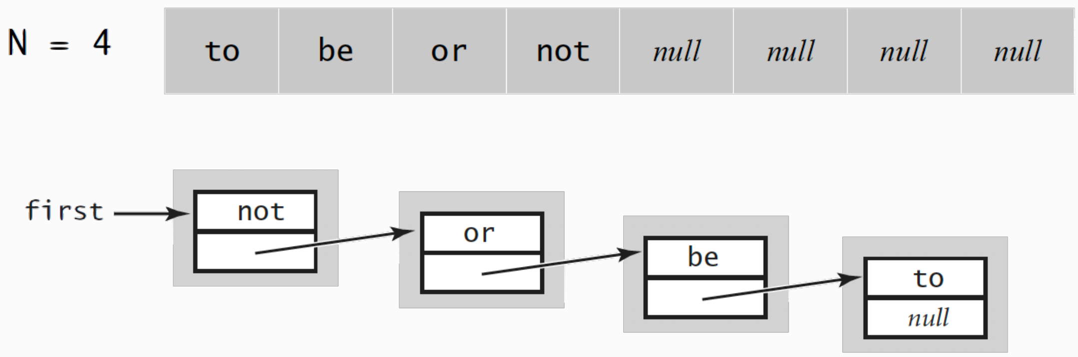
- $\sim 8N$ when full
- $\sim 32N$ when one-quarter full

```
public class ResizingArrayStackOfStrings
{
   private String[] s;
   private int N = 0;
   ...
}
```

8 bytes (reference to array)
24 bytes (array overhead)
8 bytes × array size
4 bytes (int)
4 bytes (padding)

Resizing Array vs. Linked List

Algorithm	Performance	Memory Usage
Linked-List Implementation	Every operation takes constant time in the ***worst case***	Uses extra time and space to deal with the links
Resizing-Array Implementation	Every operation takes constant ***amortised*** time	Less wasted space



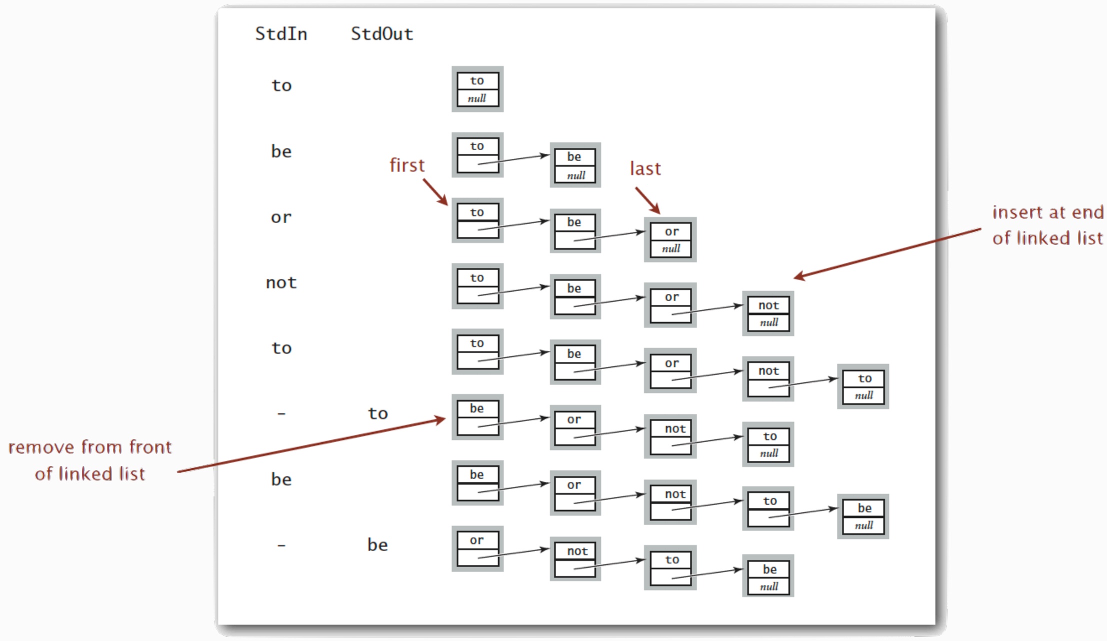
Queues

API

```java
public class QueueOfStrins {
    public QueueOfStings() {
        // create am empty queue
        ...
    }

    public void enqueue(String item) {
        // insert a new String onto queue
        ...
    }

    public String dequeue() {
        // remove and return the String least recently added
        ...
    }

    public boolean isEmpty() {
        // is the queue empty
        ...
    }

    public int size() {
        // number of String on the queue
    }
}
```

Linked-List Implementation

Maintain ***pointers*** to first and last nodes in a linked list, insert/remove from opposite ends



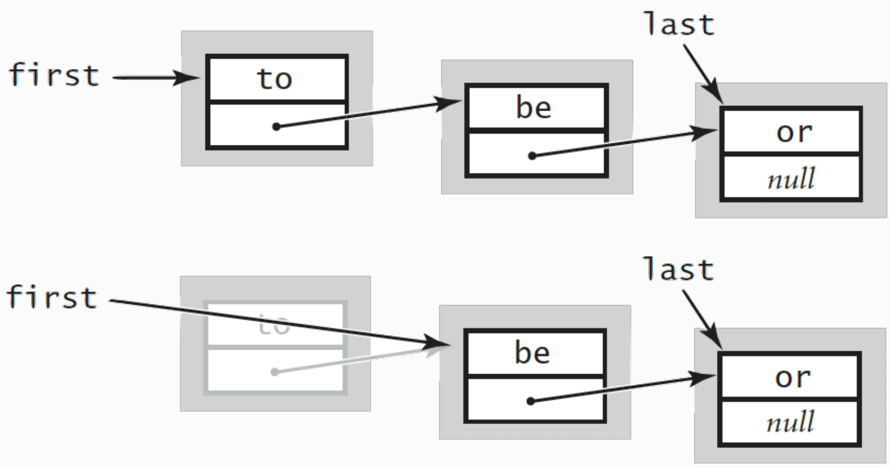
Dequeue

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



return saved item

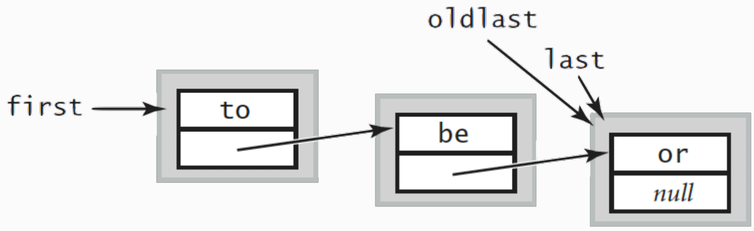
```
return item;
```

> *Remark*:
>
> Identical code to linked-list stack `pop()`

Enqueue

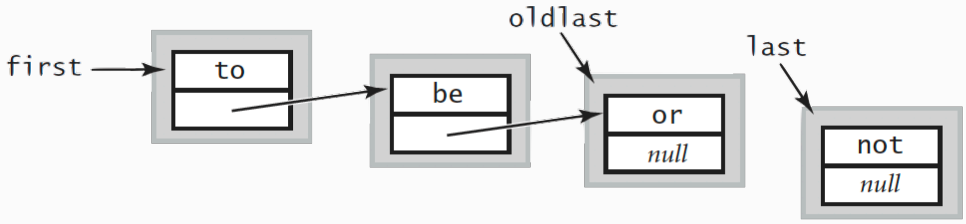
save a link to the last node

```
Node oldlast = last;
```



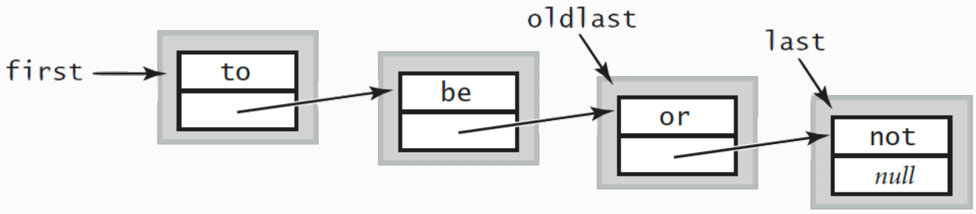
create a new node for the end

```
last = new Node();
last.item = "not";
```



link the new node to the end of the list

```
oldlast.next = last;
```



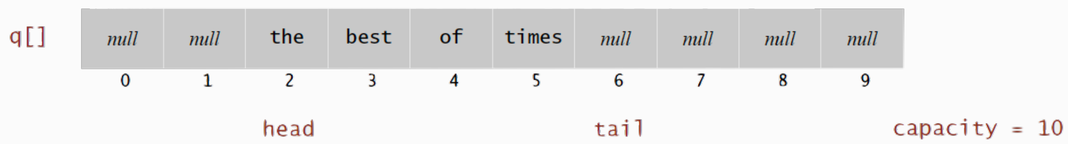
Java Implementation

```java
public class LinkedQueueOfStrings {
    private Node first, last;

    private class Node {
        /* same as in StackOfStrings*/
    }

    public boolean isEmpty() {
        return first == null;
    }

    public void enqueue(String item) {
```

```java
            Node oldlast = last;
            last = new Node();
            last.item = item;
            last.next = null;
            if (isEmpty()) first = last;
            else oldlast.next = last;
        }

        public String dequeue() {
            String item = first.item;
            first = first.next;
            if (isEmpty()) last = null;
            return item;
        }
    }
```

Resizing-Array Implementation

- Use array `q[]` to store items in queue
- `enqueue()` : add new item at `q[tail]`
- `dequeue()` : remove new item from `q[head]`
- Update `head` and `tail` modulo the `capacity`
- Add resizing array



Generics

Parameterised Stack

Implemented: `StackOfStrings`

Also wanted: `StackOfURLs`, `StackOfInts`, `StackOfVans`...

Attempt 1: Implement a separate stack class for each type

- Rewriting code is tedious and error-prone
- Maintaining cut-and-pasted code is tedious and error-prone

> *Fun Fact*:
>
> This is the most reasonable approach until Java 1.5.

Attempt 2: Implement a stack with items of type *Object*

- Casting is required in client
- Casting is error-prone: run-time error if types mismatch

```
StackOfObjects s = new StackOfObjects();
Apple a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);

a = (Apple) (s.pop());
// run-time error
```

***Attempt 3*: Java Generics**

- Avoid casting in client
- Discover type mismatch errors at compile-time instead of run-time

```
Stack<Apple> s = new Stack<Apple>();
Apple a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
// compile-time error
a = s.pop();
```

> ***Note that***:
>
> `<Apple>` is type parameter

Generic Stack: Linked-List Implementation

```
public class Stack<Item> {
     private Node first = null;

     private class Node {
          Item item;
          Node next;
     }

     public boolean isEmpty()
     { return first == null; }

     public void push(Item item) {
          Node oldfirst = first;
          first = new Node();
          first.item = item;
          first.next = oldfirst;
     }

     public Item pop() {
          Item item = first.item;
```

```java
            first = first.next;
            return item;
        }
}
```

> *Note that*: `Item` is generic type name

Generic Stack: Array Implementation

Intuitively:

```java
public class FixedCapacityStack<Item> {
     private Item[] s;
     private int N = 0;

     public FixedCapacityStack(int capacity) {
          s = new Item[capacity];
          // ILLEGAL
          // generic array creation not allowed in Java
     }

     public boolean isEmpty()
     { return N == 0; }

     public void push(Item item)
     { s[N++] = item; }

     public Item pop()
     { return s[--N]; }
}
```

Solution:

```java
public class FixedCapacityStack<Item> {
     private Item[] s;
     private int N = 0;

     public FixedCapacityStack(int capacity) {
          s = (Item[]) new Object[capacity];
          // the ugly cast
          // casting should be avoided if possible
     }

     public boolean isEmpty()
     { return N == 0; }

     public void push(Item item)
```

```java
        { s[N++] = item; }

        public Item pop()
        { return s[--N]; }
}
```

> ***Unchecked Cast***
>
> If compile `FixedCapacityStack.java`:
>
> ```
> % javac FixedCapacityStack.java
> Note: FixedCapacityStack.java uses unchecked or unsafe operations.
> Note: Recompile with -Xlint:unchecked for details.
>
> % javac -Xlint:unchecked FixedCapacityStack.java
> FixedCapacityStack.java:26: warning: [unchecked] unchecked cast
> found : java.lang.Object[]
> required: Item[]
> a = (Item[]) new Object[capacity];
>              ^
> 1 warning
> ```

Generic Data Types

> ***Wrapper Type***:
>
> - Each ***primitive*** type has a ***wrapper*** object type
> - Example: *Integer* is wrapper type for *int*

Autoboxing

Automatic cast between a primitive type and its wrapper

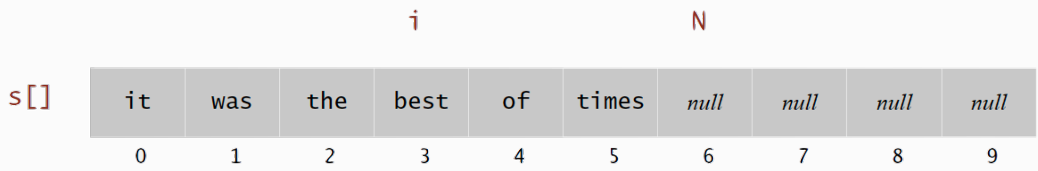
```java
Stack<Integer> s = new Stack<Integer>();
s.push(17); // s.push(Integer.valueOf(17));
int a = s.pop(); // int a = s.pop().intValue();
```

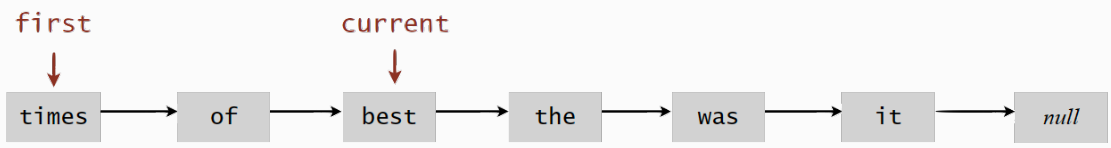
Conclusion: client code can use generic stack for ***any*** type of data

Iterators

Design Challenge

Support iteration over stack items by client, without revealing the internal representation of the stack





Java Solution:

Make stack implement the `java.util.Iterable` interface

> ***Iterable***:
>
> Has a method that returns an *Iterator*
>
> *Iterable Interface*:
>
> ```java
> public interface Iterable<Item> {
>       Iterator<Item> iterator();
> }
> ```

> ***Iterator***:
>
> Has methods `hasNext()` and `next()`
>
> *Iterator Interface*:
>
> ```java
> public interface Iterator<Item> {
>       boolean hasNext();
>       Item next();
>       void remove(); //optional and risky to use
> }
> ```

Why Make Data Structures Iterable?

Java supports elegant client code

***foreach* statement (shorthand)**

```java
for (String s : stack) {
      StdOut.println(s);
}
```

equivalent code (longhand)

```java
Iterator<String> i = stack.iterator();
while (i.hasNext()) {
    String s = i.next();
    StdOut.println(s);
}
```

Stack Iterator: Linked-List Implementation

```java
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item> {
    ...
    public Iterator<Item> iterator() {
        return new ListIterator();
    }

    private class ListIterator implements Iterator<Item> {
        private Node current = first;

        public boolean hasNext() {
            return current != null;
        }
        public void remove() {
            /* not supported */
        }
        public Item next() {
            Item item = current.item;
            current = current.next;
            return item;
        }
    }
}
```

Stack Iterator: Array Implementation

```java
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item> {
    ...
    public Iterator<Item> iterator() {
        return new ReverseArrayIterator();
    }

    private class ReverseArrayIterator implements Iterator<Item> {
        private int i = N;
```

```java

            public boolean hasNext() {
                return i > 0;
            }
            public void remove() {
                /* not supported*/
            }
            public Item next() {
                return s[--i];
            }
        }
}
```

Bag API

Adding items to a collection and iterating (when order doesn't matter)

```java
public class Bag<Item> implements Iterable<Item> {
        public Bag() {
            // create an empty bag
            ...
        }

        public void add(Item x) {
            // insert a new item onto bag
            ...
        }

        public int size() {
            // number of items in bag
            ...
        }

        public Iterable<Item> iterator() {
            // iterator for all items in bag
            ...
        }

        ...
}
```

Implementation

Stack with `pop()` or *Queue* without `dequeue()`

Question

Q: *Suppose that we copy the iterator code from our linked list and resizing array implementations of a stack to the corresponding implementations of a queue. Which queue iterator(s) will correctly return the items in FIFO order?*

A: linked-list iterator only

> ***Note that***:
>
> The linked-list iterator will work without modification because the items in the linked list are ordered in FIFO order (which is the main reason we dequeue from the front and enqueue to the back instead of vice versa).
>
> The array iterator will fail for two reasons. First, the the items should be iterated over in the opposite order. Second, the items won't typically be stored in the array as entries 0 to $n - 1$.