
Blue Clue: A Mathematical View to Pursue the Lost in Blue

The tragedy of Titan's loss of life is heartbreaking. In order to formulate relevant safety procedures, we developed the **SLP model**, **RED model** and **FBR model** to achieve position prediction and efficient search and rescue of the submersible after an accident.

Location: We used the shallow water area of the eastern Ionian Sea as the sneaking location, and analyzed the two main uncertainties in the sea area: ocean currents and seawater density. Next, we use the "**Six Degrees of Freedom**" **Motion Model** to establish SLP model based on uncertainty. We divide the situation of the submersible's loss of contact into two categories: with and without power, and establish a position iteration equation. And use **Monte Carlo Simulation** to obtain its final position probability distribution. The results are shown in Figure (4) and Figure (5). In addition, we also selected two devices for MCMS's submersibles, **CTD and ADCP**, to detect and send **current speed, seawater temperature** and other information to reduce the uncertainty of prediction.

Preparation: We first divide the equipment into three categories: communication, positioning and rescue, use a **Multi-Objective Planning** algorithm, comprehensively consider factors such as availability, cost, readiness, etc., and construct a planning model with the goals of maximum availability, minimum preparation time, and minimum cost. When feasible Two sets of non-inferior solutions were obtained within the domain, and the kayak was included in the equipment list based on actual conditions. The specific information of the selected equipment is shown in Table (3).

Search: We first combined the location information provided by the SLP model and determined that the deployment points of the search equipment for both failure modes were at the maximum of their location probability distributions. Then we innovatively established the FBR Model, which constructs a "beam" based on the ocean current speed of the lost connection point and deployment point to limit the probability distribution area, and introducing grid search and **Bayesian search algorithms** for area division and probability update. Possible search times are further reduced. The function image of the probability of successful search and rescue with respect to time and cumulative search results is shown in Figure (12).

Extrapolation: We take the Caribbean Sea as an example, focus on the impact of climate and submarine geographical environment, and obtain the Jaccard coefficients after location migration of the built model. Next, we discussed the situation where multiple submersibles lost contact in the same area, and used the "**Dimension Reduction Method**" to convert the search for multiple targets into a search for a single target, which improved the practicality of the model. The result is shown in Figure (14).

In the Section 8, we completed a sensitivity analysis of the established model for different adjustment parameters of uncertain factors in different sea areas. We found that the calculation results of our model have little variation in different sea areas and have good generalizability. And a memo is sent to Greek Government for approval.

Keywords: Submersible SAR, Multi-Objective Planning, Monte Carlo simulation, Bayesian search algorithms

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Introduction

1.1 Problem Background

Submarine tours have become increasingly popular in recent years. However, deep oceans are very dangerous places where high pressure can easily crush hard metals. And just in recent years there have been some sad and unfortunate incidents.

"It's a very similar tragedy at the exact same site. It's astonishing and really quite surreal." James Cameron said in an interview. After a long search and rescue, the U.S. Coast Guard said on June 18, 2023 that the missing sightseeing submersible imploded near the wreckage of the Titanic, and all five people on board lost their lives.

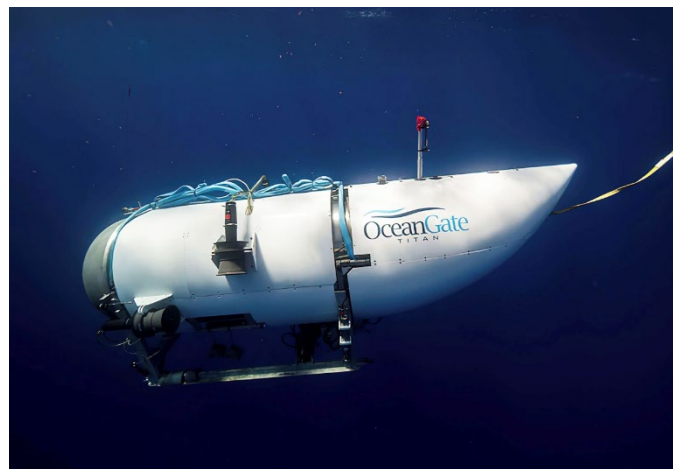


Figure 1: The submarine TITAN before the accident (Image source: NBC NEWS)

Therefore, the development of a strict set of safety procedures is necessary before a tourist submarine can be put into service. It will prevent the submarine from encountering dangerous situations as much as possible, and will enable it to be located and rescued in the first instance. More and more marine travel companies are focusing on the establishment and implementation of safety procedures that involve knowledge of marine physics, hydrology, and hydrodynamics.

1.2 Restatement of the Problem

Given the context of the problem and the associated constraints, we are required to address the following detailed questions:

- To locate, we are required to consider uncertainty in modeling location predictions and understand how to minimize their effects.
- To prepare, we need to think about the additional search equipment we need to carry on the host ship to deploy and consider the availability, maintenance and cost of use of the equipment, etc.
- To search, we are required to develop a model to determine the initial points of deployment, as well as the pattern in which the search and rescue would be conducted

to ensure that the location would be accomplished in the shortest possible time.

- To extrapolate, we need to apply our model to other sea areas and change it accordingly to accommodate multiple submersibles moving in the same sea area.
- Considering the results obtained above, we need to prepare two pages of memo submit to the Greek Government.

1.3 Literature Review

The topic is about the prediction of the position of a submersible and the consideration of external environmental factors. Underwater vehicles have been a hot topic of scientific and technological research in recent years, and we have reviewed a lot of related literature and made comparisons between different literatures.

- **Submarine movement:** Keke Wei et al.[1] used six degree of freedom floating motion to discuss the effect of the center of gravity axis on the motion of the submersible for different dive depths. However, most researchers tend to create simpler research systems to simplify the system[2].
- **Influence of currents:** It is the mainstream research method to establish a dynamic model of manned submersible under the action of sea current[3], and to simulate and analyze the hydrodynamic force of sea current on the submersible[4]. But this method has many steps and cumbersome procedures.
- **Influence of other factors:** A. V. Ochkin et al.[5] determined that the density of seawater is a function of temperature and salinity. Jianhua Zhang et al.[6] further used the method of computer simulation to analyze the effects of ortho and inverse leap layers on the submarine's depth change maneuvering respectively. However, the proposed solution measures are not detailed enough.

1.4 Our Work

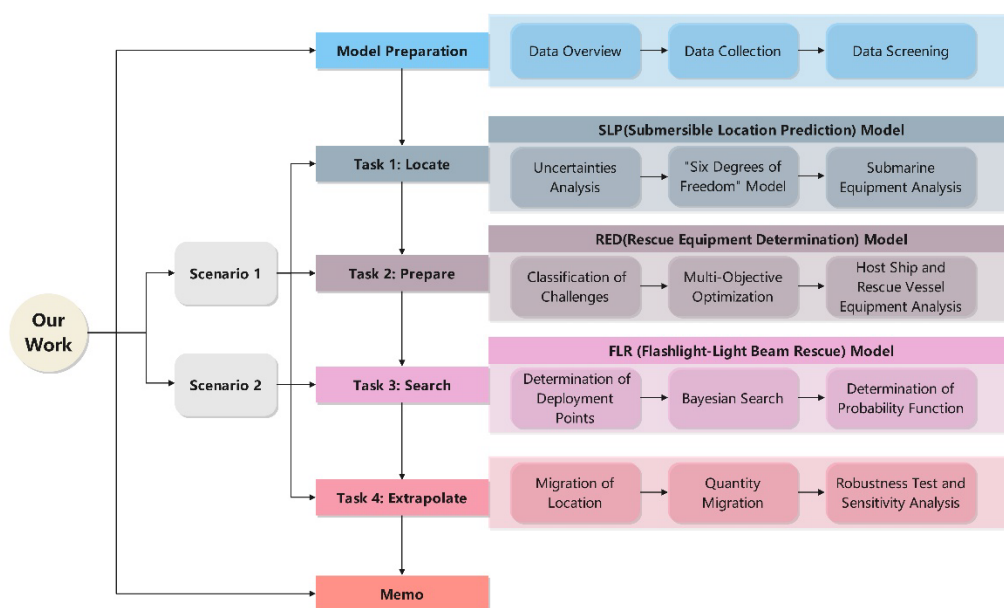


Figure 2: Flow chart of Our Work

2 Assumptions and Justifications

▼ Assumption 1: The data we use are accurate and valid.

▲ Explanations: The data in this article comes directly from the latest results of the major online official databases and published literature.

▼ Assumption 2: The submersible can only suffer from communication or power failure.

▲ Explanations: The fault conditions of submersibles are complex. Here we only discuss the two most common faults.

▼ Assumption 3: Marine life does not affect submarine movement.

▲ Explanations: Marine organisms generally avoid submersibles on their own, and the probability of affecting their movements is small.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description
\mathbf{V}	velocity vector of the submarine relative to the ground
\mathbf{F}_{prop}	push force of the submarine
\mathbf{f}_D	drag force of the submarine
\mathbf{u}_c	difference in fluid velocity before and after the propeller
\mathbf{L}_n	location of the submersible at the n_{th} communication
$\mathbf{\Omega}$	angular velocity vector of the submersible
\mathbf{F}_B	buoyant force of the seawater in the vertical direction
ρ	density of the seawater
l	length of the submarine
C_d	drag coefficient of the submarine
A	effective drag area of the submarine

·**Note:** Some variables are not listed. Their specific meanings will be introduced below.

4 Model Preparation

4.1 Data Overview and Collection

Data collection is crucial to solving the modeling problem. Since the question does not give us the relevant data directly, we collected the data by ourselves, including sea depth of Ionian Sea, differing densities in the sea, current etc.

Due to the sheer volume of data, we were unable to give it directly in the text, so we took visualization measures to get the data in a more intuitive form.

We have gathered a lot of very useful information from the references. In addition, the

following websites provided us with sizable help in terms of data.

Table 2: Main Data Description and Source

Data Description	Data Source
UNESCO OCEAN	https://www.unesco.org/zh/ocean
Britannica Geology	https://www.britannica.com/place/Ionian-Sea
UNEP	https://www.unep.org/topics/ocean-seas-and-coasts
CODC	http://msdc.qdio.ac.cn/data/
World Ocean Database	https://www.ncei.noaa.gov/products/world-ocean-data-base

5 SLP(Submersible Location Prediction) Model

5.1 Selection of Regions

We collected data on the depth and distribution of shipwrecks in the Ionian Sea, and found that 33 shipwrecks are concentrated in the shallow eastern part of the sea, which is shown in Figure 3, with an average depth of 110 m and a maximum depth of 205 m. Therefore, the dive site for this trip was selected to be the shallow eastern part of the Ionian Sea. In this area, the surface elevation rises slowly in the near-shore direction and changes gently, and there are no raised seabed mountains, so it is considered that the influence of seabed geography on the movement of the submersible before and after the failure is smaller than that of other elements in this area, and will not be discussed here.

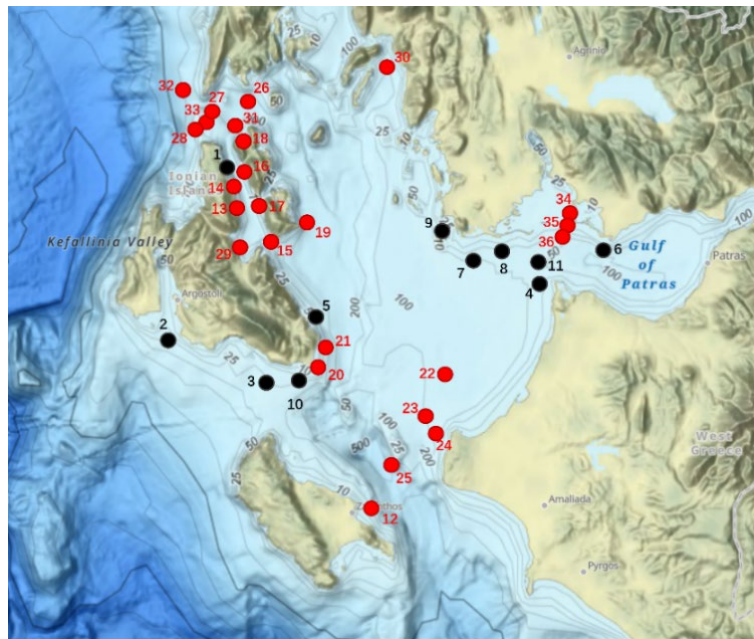


Figure 3: Eastern seas of Ionian Sea and the remains of shipwrecks

5.2 Construction of the SLP model

5.2.1 Introduction of Uncertainties

When a submarine loses power it is subjected to external forces to continue its motion.

Here the external force is provided directly by the sea water, and the uncertainty-type influences in this area are mainly in two aspects:

■ Current

After the submersible loses power, it continues to move forward under the action of inertia, and is subjected to the thrust of seawater in all directions, including buoyancy, which is approximated to be approximately equal and cancel each other out in the horizontal direction. However, when the place where it is located and the ocean current convergence, the thrust brought about by the sudden change so that the external force on the submersible changes can not be ignored.

$$F_{ext} = \frac{1}{2} \rho V_c^2 C_d' A \quad (1)$$

where F_{ext} is the current force on the submersible, V_c is the current velocity, and C_d' is the current drag coefficient. We consult the study of ocean currents and underwater vehicles[8] to know that the expression for V_c is:

$$V_c = 2.5v^* \ln(6.34 \times 10^6 v^* h) \quad (2)$$

where h is the height of the sea water from the bottom, $v^* \approx \sqrt{ghs}$, and s is the inclination of the sea water surface.

■ Variation in Seawater Density

The "abrupt change" in seawater density is the trigger for a large number of submersible accidents. Density of seawater has a decisive effect on the submarine sinking and floating, is an important uncertainty in the dive process that can not be ignored. The temperature, salinity and pressure of seawater are the three determining factors of seawater density. In a small range, it can be approximated that the effect of these three parameters on the density

of seawater is linear. We collect information[7] and get the international seawater equation of state $\rho(s, t, P)$, because it is too complex, beyond the required accuracy of this model, we use the sea temperature, salinity and pressure to bring into the equation, for the resultant curve for many times to fit, to get the simplified seawater density expression:

$$\rho(s, t, P) = \frac{\rho_w + \alpha s + \beta t}{1 - \frac{P}{\gamma}} \quad (3)$$

where s is the salinity of seawater, t is the temperature in Celsius, and P is the pressure, $\alpha=0.8$, $\beta=0.22$, $\gamma=19,652$, and the range of application:

$$0 \leq s \leq 42.000, \quad 0 \leq P \leq 986.9 \text{ atm}, \quad -2^\circ C \leq t \leq 42^\circ C \quad (4)$$

■ Data transmission error

In the external factors for the submersible's own position impact at the same time, due to the submersible sub own carry equipment precision limitations, the measured position, current speed, seawater temperature and other information there will be a certain degree of error, which will lead to the actual position and the predicted position of a certain degree of uncertainty.

5.2.2 Motion Model

The actual movement of the submersible in the water is carried out in three-dimensional space, and the effects of changes in its own dimensions and directions cannot be ignored, so a "six-degree-of-freedom" model is introduced to correct the movement state of the submersible. The so-called "six degrees of freedom", that is, in the Cartesian coordinate system along the x , y , z direction of the three translational degrees of freedom and three rotational degrees of freedom around the x -axis, y -axis, z -axis rotation. By synthesizing the effects of uncertainty factors on the motion and position of the submersible, the iterative equations for the motion state and position of the submersible diving in the sea are obtained:

$$m \left(\frac{d\mathbf{V}}{dt} + \boldsymbol{\Omega} \times \mathbf{V} \right) = \mathbf{F}_{prop} + \mathbf{F}_{ext} + \mathbf{F}_B - m\mathbf{g}\hat{z} - \mathbf{f}_D \quad (5)$$

$$\mathbf{L}_{n+1} = \mathbf{L}_n + \int_0^t \mathbf{V} dt \quad (6)$$

where $|\mathbf{f}_D| = \frac{1}{2} \rho \mathbf{V}^2 C_d A$ is the resistance to submersible travel in water, $\boldsymbol{\Omega}$ is the angular velocity vector of the submersible, $\mathbf{F}_B = \rho \mathbf{g} V_0$ is the buoyant force of the seawater in the vertical direction, ρ is the density of the seawater, V_0 is the volume of the submersible, \mathbf{g} is the local gravitational acceleration, and \mathbf{V} , $\boldsymbol{\Omega}$ can be expressed as:

$$\begin{cases} \mathbf{V} = \hat{u} + \hat{v} + \hat{w} \\ \boldsymbol{\Omega} = \hat{p} + \hat{q} + \hat{r} \end{cases} \quad (7)$$

\mathbf{u} , \mathbf{v} , and \mathbf{w} are the advective velocity components of the x , y , and z axes, respectively, and \mathbf{p} , \mathbf{q} , and \mathbf{r} are the angular velocity components around the x , y , and z axes, respectively, and they satisfy the relation:

$$\begin{cases} m(\hat{u} + \mathbf{q}\mathbf{w} - \mathbf{r}\mathbf{v}) = \mathbf{F}_x \\ m(\hat{v} + \mathbf{r}\mathbf{u} - \mathbf{p}\mathbf{w}) = \mathbf{F}_y \\ m(\hat{w} + \mathbf{p}\mathbf{v} - \mathbf{q}\mathbf{u}) = \mathbf{F}_z \end{cases} \quad (8)$$

where \mathbf{F}_x , \mathbf{F}_y , and \mathbf{F}_z are the combined forces on the x , y , and z axes, respectively.

5.3 Analysis of the Accident Situation

We categorize submersible accidents into two main types: (1) loss of communication with the host ship and damage to the power system (no autonomous motion in both the horizontal and vertical directions) (2) loss of communication with the host ship but no damage to the power system. The location prediction model is established for the above two cases.

(1) loss of communication with the host ship and damage to the power system

In this case, due to the loss of propulsion of the submersible, the position of the submersible depends on its own inertia and the effect of external factors such as current thrust on it, and we use the information that the submersible last reported to the main ship to predict its

position. The current velocity \mathbf{V}_C , the density of the surrounding seawater ρ_C , the salinity S , and the temperature t are substituted into the equation of motion and the equation of location change:

$$m\left(\frac{d\mathbf{V}}{dt} + \boldsymbol{\Omega} \times \mathbf{V}\right) = \mathbf{F}_{ext} + \mathbf{F}_B - m\mathbf{g}\hat{z} - \mathbf{f}_D \quad (9)$$

$$\mathbf{L}_{n+1} = \mathbf{L}_n + \int_0^{\Delta t} \mathbf{V} dt \quad (10)$$

Then we can obtain the submarine location \mathbf{L} as a function of $\mathbf{L}(t)$ with respect to t .

Finally, we conducted several simulations based on the relevant data in the Ionian Sea. Due to the high number of simulations and the requirement for clear visualization, we present only one major trajectory in the figure, as well as multiple endpoints, and the results are shown in Figure 4.

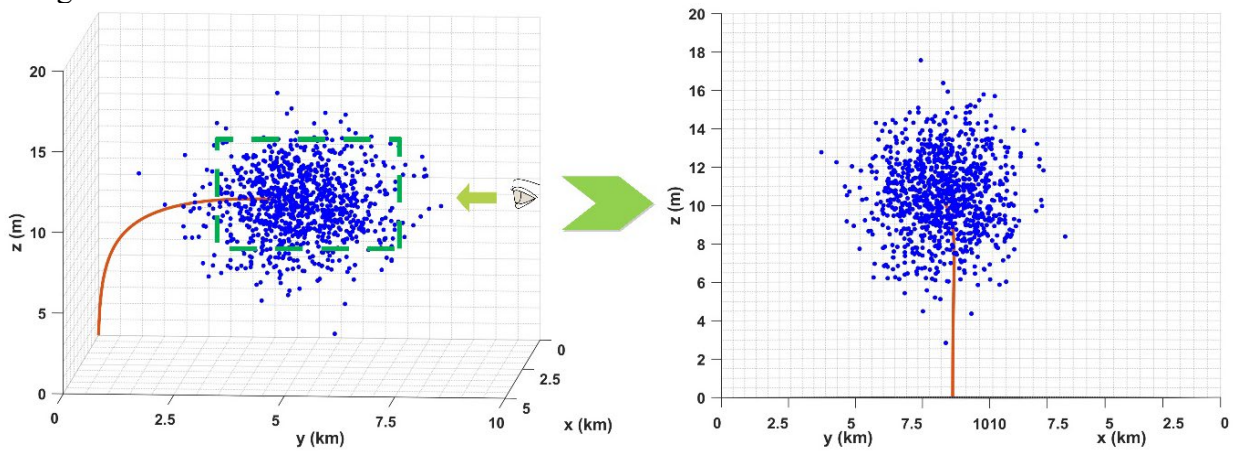


Figure 4: (a) Main view of the underwater trajectory (b) Side view of predicted endpoint

As seen from the figure, the submarine is easily influenced by uncertainties such as currents and seawater density, and the endpoint will change, but the probability of a large excursion is small.

(2) loss of communication with the host ship but no damage to the power system

In this case, in order to reduce the consumption of resources and ensure the safety of the personnel inside the submersible as much as possible, the captain will manipulate the power system to make it float to the surface within a short period of time, and the dive site is a shallow area with an average maximum depth of 110 m. Therefore, the horizontal displacement during this time period can be ignored, i.e., it is approximated that there is no change in the horizontal position of the submersible in the process of ascent.

Therefore, the positional uncertainty caused by the density of seawater is negligible compared to the effect of ocean currents on the surface of seawater. Combined with the drifting model, due to the short floating time and small size of the submersible, we ignore the effects of Coriolis force and direct wind push, and focus on the positional change of the submersible caused by the change of the velocity of the current due to the wind on the surface of the ocean after surfacing. We introduce the wind stress τ to estimate the effect of wind on the flow velocity of sea water at the sea surface:

$$|\boldsymbol{\tau}| = \rho_{air} \cdot C_D \cdot |\mathbf{U}|^2 \quad (11)$$

Where ρ_{air} is the air density, which is about 1.2 kg/m^3 at the sea level C_D is the coefficient of air resistance, which varies from 0.001 to 0.003, and is taken to be 0.002 here, and U is the wind speed. Next, we explore the specific transmission process of wind stress in the form of friction to transfer energy from Ekman's theory, which is simplified by defining the contact surface transmission coefficient k to denote the ratio of energy transmission, according to which corrections are made for the external forces on submersibles subjected to ocean currents underwater:

$$\mathbf{F}_{ext} = \frac{1}{2} \rho \mathbf{V}_c^2 C_d' A + k \boldsymbol{\tau} \quad (12)$$

where k is the contact surface transmission coefficient. After reviewing the literature[9] and substituting the average wind speed and sea surface condition in this sea area, $k = 0.39$. Next, through equations (9) and (10), we can get $L(t)$.

Lastly, we conducted Monte-Carlo simulation based on the relevant data of the Ionian Sea, and the results are shown in Figure 5.

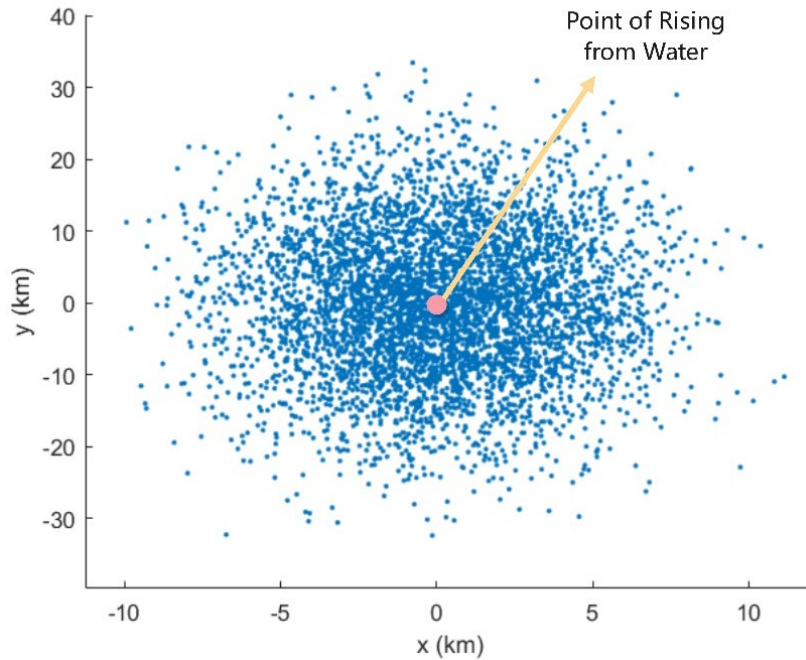


Figure 5: Endpoints on the water surface

As seen in the figure of the simulation results, there are more endpoints close to the point of rising from water, indicating that the probability of the submersible appearing near the point of rising from water is higher.

5.4 Equipment to Decrease Uncertainties

The motion of the submarine after the loss of power is determined by its own inertia and changes in the surrounding currents and seawater density. Therefore, the submarine should regularly send information to the main ship about the temperature, salinity, pressure of the seawater around it, as well as the speed and direction of the currents, in order to minimize the uncertainty in the prediction of the position after the accident. Uncertainty reduction procedure is shown in Figure 6.

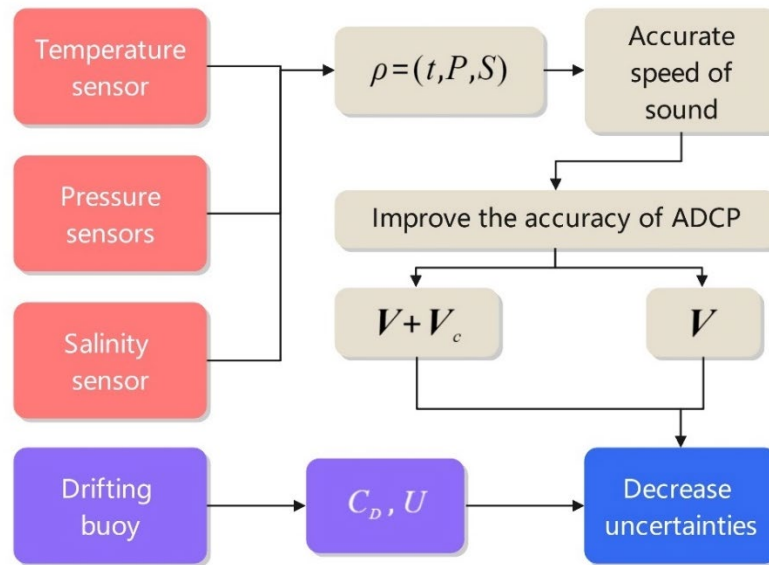


Figure 6: Uncertainties reduction procedures

The Conductivity Temperature Depth (CTD)

CTD is an oceanographic instrument used to measure the conductivity, temperature and pressure of seawater. These data allow submersibles to obtain accurate densities of the surrounding seawater $\rho(t, P, S)$ and also help us to obtain more accurate seawater sound velocities, improving the accuracy of other instruments that utilize sound velocity measurements.

The Acoustic Doppler Current Profiler (ADCP)

ADCP is a hydroacoustic flowmeter used to measure the speed of water. The ADCP transmits acoustic waves into the water, which are scattered by scatterers in the water; the ADCP receives the return signals from the scatterers, and analyzes the Doppler effect shift to measure its own speed relative to the bottom of the water and relative to the water current speed, in order to obtain more accurate velocity, location information compared to other detection devices such as sonar.

6 RED(Rescue Equipment Determination) Model

When selecting SAR equipment, it is firstly necessary to categorize them, and for each type of equipment, the equipment suitable to be carried by the main ship and the SAR ship is selected by considering factors such as availability, maintenance, purchase cost, and preparation time. In this regard, we use multi-objective optimization to select the best equipment selection scheme in the feasible domain.

Communication: Communication equipment, such as communication buoys, underwater telephones, hydroacoustic modems, that ensure good communication between each search and rescue vessel and the main vessel during the rescue.

Location: Equipment to quickly and accurately locate a submarine in the event of an accident, e.g., sonar positioning systems, underwater GPS, underwater locator beacons.

Rescue: Practical operational equipment for the rescue of submersible passengers, such

as self-help buoyancy devices, underwater drones, and life-saving equipment and rescue rubber boats.

6.1 Multi-Objective Optimization

Step 1: Definition of Objectives

1) Maximum availability goals:

The most direct purpose of equipping the main ship and search and rescue ship with relevant search and rescue equipment is to provide efficient rescue of trapped people. Therefore, the specific use effect of the equipment selected during actual operation must be optimal. Therefore, for the three types of equipment that solve different types of problems, we define specific normalized index factors respectively: Communication Range Factor

$$Q_{1i} = \frac{r_{1i} - \min\{r_{1i}\}}{\max\{r_{1i}\} - \min\{r_{1i}\}}, \text{ Positioning Range Factor } Q_{2i} = \frac{r_{2i} - \min\{r_{2i}\}}{\max\{r_{2i}\} - \min\{r_{2i}\}},$$

$$\text{Safety Utility Factor } Q_{3i} = \frac{\varepsilon_i - \min\{\varepsilon_i\}}{\max\{\varepsilon_i\} - \min\{\varepsilon_i\}}.$$

$$\max Q = \max \left(\sum_{i=1}^3 X_{1i} Q_{1i} + \sum_{i=1}^3 X_{2i} Q_{2i} + \sum_{i=1}^3 X_{3i} Q_{3i} \right) \quad (13)$$

where Q is the total utility factor, r_{1i} is the maximum communication radius of each communication device, r_{2i} is the maximum positioning radius of each positioning device, and ε_i is the safety factor of each rescue device.

2) Minimum preparation time goals:

In practical situations where a submersible is lost, search and rescue equipment needs to be deployed as quickly as possible to shorten the waiting time for rescuers. This requires the equipment to have the shortest preparation time when it is put into use, and the shortest time depends on the maximum preparation time of selected equipment:

$$\min T = \max \{x_i T_i\} \quad (14)$$

where T is the preparation time required to perform SAR, and T_i is the preparation time for each piece of equipment.

3) Minimum acquisition and maintenance cost goals:

The purchase cost of some search and rescue equipment is high, or they have high maintenance costs or high maintenance frequency. We should minimize the cost of purchasing and maintaining equipment without having a major impact on availability, allowing MCMS companies to reap the greatest economic benefits.

$$\min W = \sum_{i=1}^9 X_i w_i + \sum_{i=1}^9 X_i w_i' \quad (15)$$

W is the total economic benefit, w_i is the purchase cost of each equipment, and w_i' is the maintenance cost of the equipment per unit time.

In the above objective, x_i represents whether the device is selected or not, 1 if it is selected and 0 if it is not.

Step 2: Definition of Constraints

1) Constraints on the range of communications

Marine search and rescue operations often need to be carried out in a large sea space, so the communication range of the communication equipment used between each search and rescue ship and the main ship must not be too narrow, and its minimum communication range should not be less than the average depth of the sea area where it is located, that is:

$$r_{1i} \geq \bar{h} = 105 \quad (16)$$

Where \bar{h} is the average sea depth of the sea, taken as 110 m.

2) Constraints on rescue safety factors

In the search and rescue process, the life safety of the personnel to be rescued is always the first priority, so it must be ensured that the safety coefficient of the relevant rescue equipment is at a higher level, in general, the deeper the depth of the submersible is trapped, the greater the degree of risk of rescue, the higher the safety coefficient is required, and accordingly defines the lower limit of its safety coefficient:

$$\varepsilon_i \geq \beta_0 h_{\max} \quad (17)$$

where β_0 is the hazard coefficient, taken as 0.62, and h_{\max} is the maximum depth of the sea area, taken as 205 m.

3) Constraints on cost

MCMS, as a company providing submarine excursion services, has the main objective to obtain more economic benefits while providing the services, therefore the cost of equipment purchase and maintenance should not exceed a certain limit. we use a similar company providing similar services "Yellow Submarine", also located in Greece, as a reference. "As a reference, we deduce that the costs associated with the purchase and maintenance of equipment should be met:

$$W = \sum_{i=1}^9 X_i (w_i + w_i') \leq (1 - k_0) (\alpha_1 q_1 + \alpha_2 q_2) N \quad (18)$$

where k_0 is the annual net profit margin taken as 0.2, α_1 and α_2 are the selection coefficients taken as 0.7 and 0.3, respectively, and $(\alpha_1 q_1 + \alpha_2 q_2) N$ is the estimated annual turnover.

In summary, the whole Multi-Objective Optimization model is as follows:

$$\max Q = \max \left(\sum_{i=1}^3 X_{1i} Q_{1i} + \sum_{i=1}^3 X_{2i} Q_{2i} + \sum_{i=1}^3 X_{3i} Q_{3i} \right) \quad (13)$$

$$\min T = \max \{x_i T_i\} \quad (14)$$

$$\min W = \sum_{i=1}^9 X_i (w_i + w_i') \quad (15)$$

$$\begin{cases}
Q_{1i} = \frac{r_{1i} - \min\{r_{1i}\}}{\max\{r_{1i}\} - \min\{r_{1i}\}} \\
Q_{2i} = \frac{r_{2i} - \min\{r_{2i}\}}{\max\{r_{2i}\} - \min\{r_{2i}\}} \\
Q_{3i} = \frac{\varepsilon_i - \min\{\varepsilon_i\}}{\max\{\varepsilon_i\} - \min\{\varepsilon_i\}} \\
r_{1i} \geq \bar{h} = 105 \\
\varepsilon_i \geq \beta_0 h_{\max} \\
W = \sum_{i=1}^9 n_i X_i (w_i + w_i') \leq (1 - k_0) (\alpha_1 q_1 + \alpha_2 q_2) N
\end{cases} \quad i = 1, 2 \dots 9 \quad (19)$$

6.2 Solution of the Model

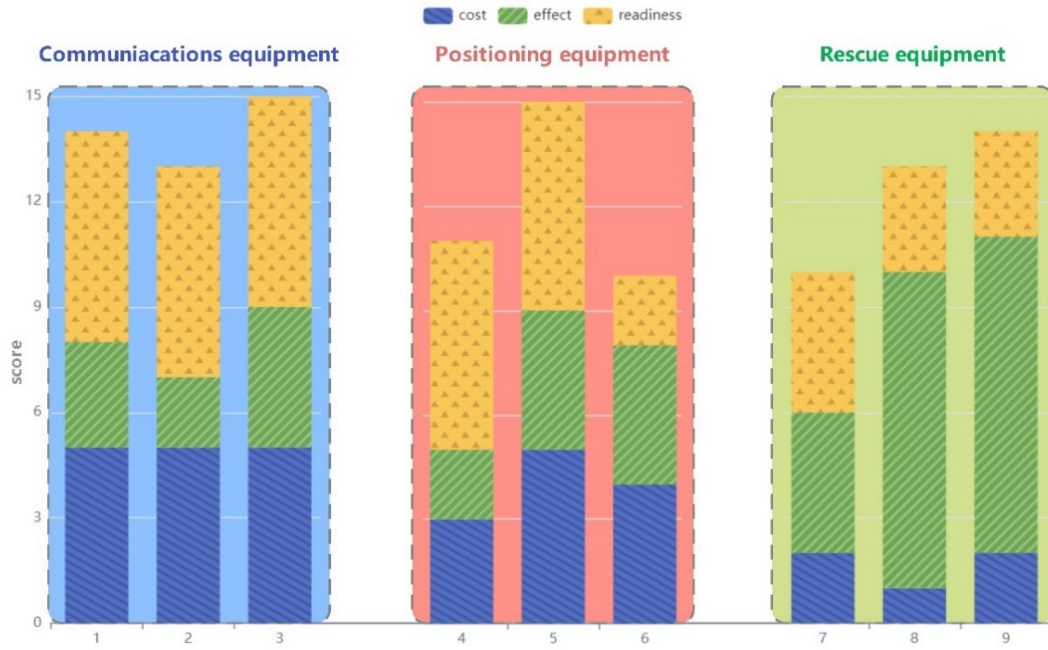


Figure 7: Comparison of 9 devices

We collected specific information of these 9 types of equipment, including availability, cost, preparation time, etc., and used Matlab to solve the model. After comparing the scores of the 9 devices, we can get the results shown in Figure 7. And after comprehensive consideration based on actual conditions, we obtained two effective solutions. Among the two valid solutions, we should choose UAM (3 in Figure 5) as the communication device and SPS (Sonar Positioning System) as the positioning device. In terms of rescue equipment, the two effective solutions jointly chose Remotely Operated Vehicle for underwater rescue. The difference was whether Fast Rescue Boats were chosen. However, considering the actual situation, if the crashed submersible can float without losing power, it is more reasonable to use Fast Rescue Boats, which is more economical for rescue. Therefore, we will select the following four types of equipment shown in Table 3.

Table 3: Equipment selected and related information

Name	Underwater Acoustic Modem	Sonar Positioning System	Remotely Operated Vehicle	Fast Rescue Boat
Model	LinkQuest TrackLink 1500	Edgetech 4200	Freedom ROV	Henshaw-400
Type	Communications equipment	Positioning equipment	Rescue equipment	Rescue equipment
Loaded Ship	rescue vessel	host ship	host ship	host ship
Purchase cost	7490.00\$	34405.00 \$	125400.00\$	11,360.68\$
Availability	high	high	high	medium
Maintenance cost	300.00\$/year	700.02\$/year	3540.00\$/year	908.85\$/year
Readiness	5-10 minutes	1-3 hours	2-4 hours	15-30 minutes

7 FBR (Flashlight-Beam Rescue) Model

For the two different accident scenarios of submersibles, determining their equipment deployment points and specific search and rescue modes need to be discussed in two different contexts. In this regard, we developed a FBR model based on Bayesian search and grid search algorithms, and discussed the equipment deployment points and search modes for the two scenarios.

7.1 Determination of Deployment Points

We have constructed SLP model to predict the location of the submersible after it is lost, obtaining a probability distribution of its location. Accordingly, the deployment point of the equipment is set as the place with the largest probability distribution in its location prediction model as a way to minimize the possible search time.

7.2 Determination of Search Patterns

Scenario 1: Floating to the surface while maintaining power

From the SLP model, the probability distribution of the submersible in this case is approximated as a circle centered at the probability maximum. Firstly, we take the direction of the current velocity V_{c1} at the time of loss of connection at the surface directly above the lost point as the upper (lower) limit of the search range, and the direction of the current velocity V_{c2} at the deployment point when the equipment is deployed as the lower (upper) limit of the search range, and thus the search range is constrained to the area surrounded by the upper and lower limits and the circle of probability distribution through the "light beam" formed by the upper and lower limits(Figure 8). The search range is then constrained to the area bounded by the upper and lower limits and the probability distribution circle.

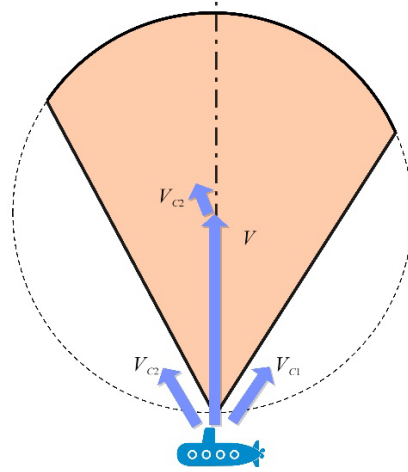


Figure 8: Flashlight-beam search range constraints of Scenario 1

Next, we introduce a Bayesian search with a grid search algorithm to grid the constrained search area and start the search from the grid with the highest probability. Let the priori probability of the n_{th} grid be P_n and its search success rate be U , then the posterior probability P_n' after no submersible is found in the region is:

$$P_n' = \frac{1 - U}{\frac{1}{P_n} - U} \quad (20)$$

On this basis, the posterior probability of the remaining grids becomes:

$$P_i' = \frac{P_i}{1 - P_n U} \quad (21)$$

The Bayesian grid search algorithm[10] is then used within the constraint range, and the search is carried out in the form of straight line search and stepwise extension. It should be noted that for grids that have been searched but failed in the initial stage of the search, the search success probability is still greater than some grids located at the edge of the constrained area. This will have a certain impact on the search order of the grids. Finally, we can get the shortest search route.

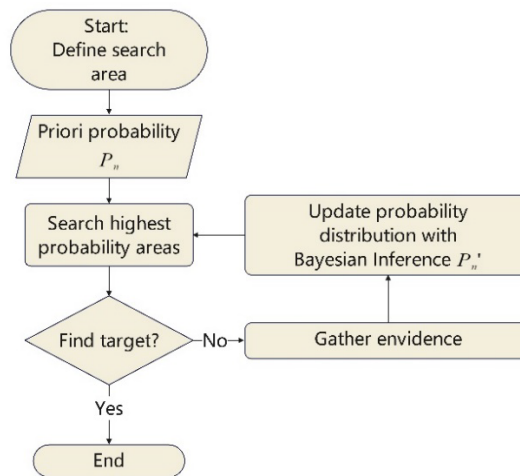


Figure 9: Diagram of the Bayesian search procedure

Scenario 2: Loss of power and movement under the sea

From the SLP model, the probability distribution of the submersible in this case is approximated as an ellipsoid with the center of mass at the probability maximum, and the larger value of the angle between the current velocity direction at the lost point and the current velocity direction at the deployment point of the equipment and the forward direction of the submersible at the lost point determines the size of the angle of the "beam" (shown in Figure 10 (a)), thus completing the constraints on the search range in the three-dimensional space, and constraining the "beam" to the ellipsoid of the probability distribution and the "beam" to the center of the ellipsoid. The search range is constrained to the volume surrounded by the "beam" and the ellipsoid of probability distribution.

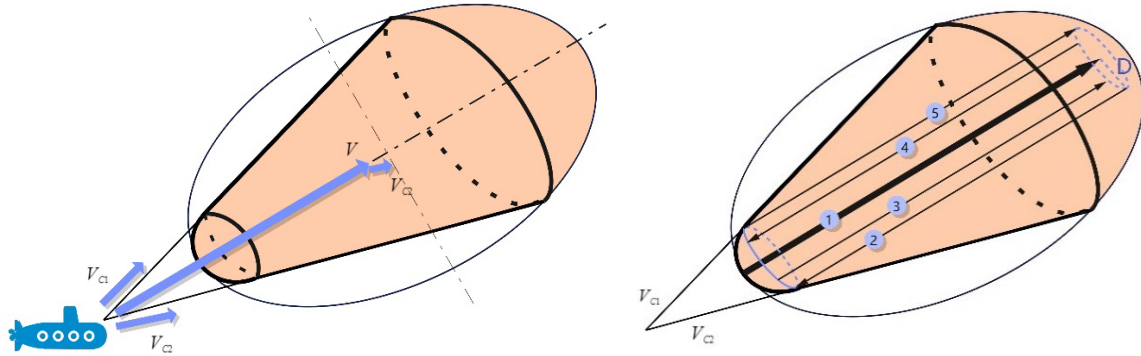


Figure 10: (a) Flashlight-beam search range constraints of Scenario 2
(b) Search pattern for scenario 2

Next, Bayesian search and grid search algorithms are introduced, and the constraint range is divided into multiple small cubes of equal volume in the three-dimensional space. In this case, the length of the ellipsoid is calculated from the length of the ellipsoid. Starting from the cube at the boundary of the axis, the search starts along the long axis. If unsuccessful, the next step is to explore on the cylinder spaced D from the long axis. Each exploration route is the busbar of the cylinder, and the distance is also D (shown in Figure 10 (b)). Similarly, the probability distribution of the long and short axis directions of the ellipsoid is explored, and it is believed that the probabilities of different cubes in the long axis direction are approximately equal. Its Bayesian probabilistic optimization is shown in the equation (20) and (21).

7.3 Determination of Probability Function

The change of probability over time and search results under the FBR model can be regarded as the probabilistic exploration results for the "straight line" and "square lattice" described in the previous section. Under the extensibility search model, it is determined that the probability distribution of the transverse direction within each constraint approximately follows a normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{x^2}{2\sigma^2}} \quad (22)$$

where σ^2 is the variance, taken as 16.

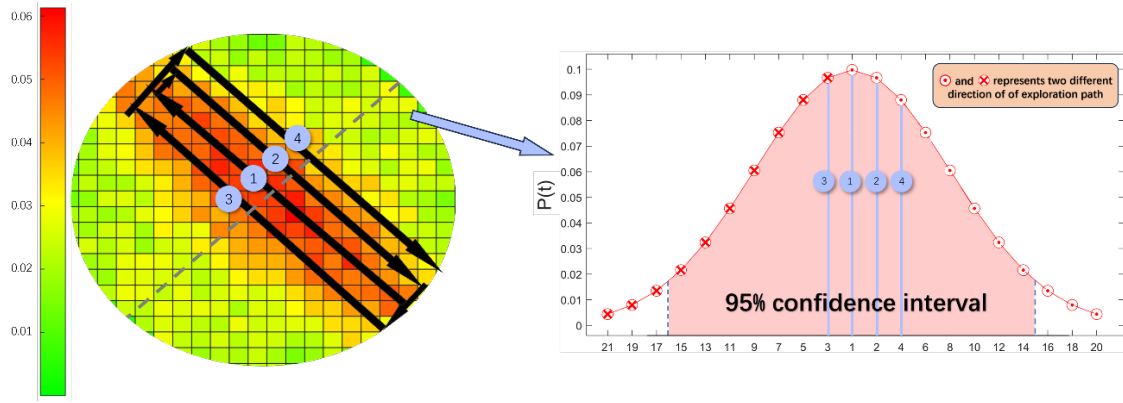


Figure 11: Search pattern and lateral probability distribution for Scenario 1

The probability of the i_{th} explored straight line under this model (Figure 11) is expressed as:

$$P_i = \begin{cases} f\left(-\frac{i-1}{2}\right), & i = 1, 3, 5 \dots \\ f\left(\frac{i}{2}\right), & i = 2, 4, 6 \dots \end{cases} \quad (23)$$

Therefore, the probability of the j_{th} grid or cube located in the i_{th} straight line within the grid search system is $P_{ij} = \frac{P_i}{N_i}$, and N_i is the number of grids or cubes contained in the i_{th} straight line. Let the search time for each grid or cube be approximately equal, then the resulting probability under the FBR model as a function of time versus search result is:

$$P(t) = \sum_{i=1}^{m-1} P_i + \frac{t - \sum_{i=1}^{m-1} N_i \Delta t}{\Delta t} \cdot \frac{P_m}{N_m} \quad (24)$$

$P(t)$ is the probability of successfully finding the submersible in the n_{th} grid or cube in the m_{th} straight line at time t , where N_m is the number of grids or cubes contained in the m_{th} straight line, and Δt is the search time for each grid or cube.

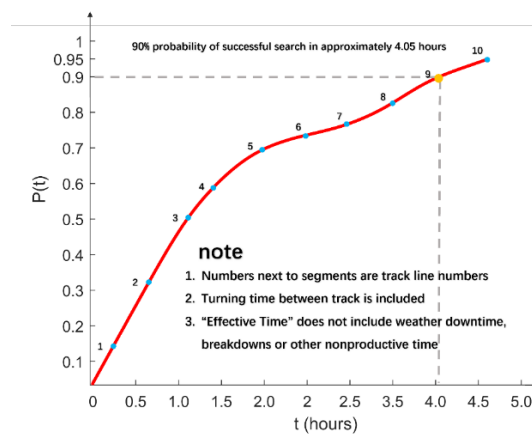


Figure 12: Uncertainties reduction procedures

8 Extrapolation of models

8.1 Migration of Location

When applying SLP Model to a different sea area, uncertainties such as currents, seawater density, and geography can vary, which can change the parameters in the SLP model and may have an impact on the results. We present an example of how the model may change when migrated to other regions, using the Caribbean Sea region as an example, and also complete an analysis of the sensitivity of the SLP model.

1) Climatic conditions

The Caribbean is located in the tropics and is affected by the northeast trade winds throughout the year. There are heavy rains in summer, which have a greater impact on the formation of ocean currents. In addition, the temperature, salinity and other indicators of the seawater are quite different from those in the Ionian Sea. These differences will cause Parameters in the original model's calculation of the impact of ocean currents and seawater density have changed, resulting in differences in predicted positions.

2) Undersea geographical environment

Different from the eastern Ionian sea, the Caribbean Sea has extremely diverse and complex landforms, including a series of faults, fissures and volcanoes, as well as numerous sea platforms and shoals. These complex geographical factors may cause changes in surrounding ocean currents or have more complex effects.

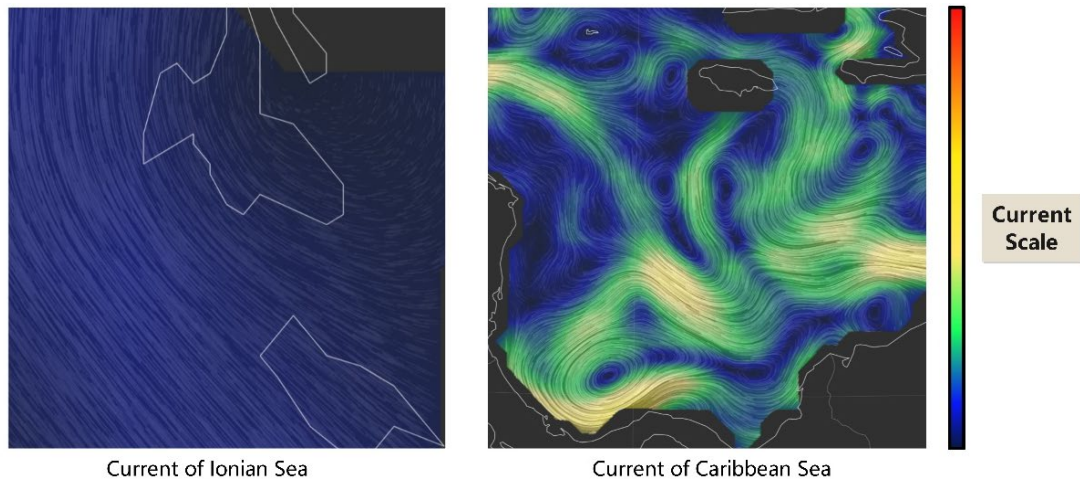


Figure 13: Comparison of current in the Ionian Sea and Caribbean Sea

In particular, we consider that the Caribbean Sea is one of the most complex circulation zones in the world (shown in Figure 13), which has a significant impact on the change of the location of the submersible after it loses contact[11]. Taking the case after loss of power as an example, we get the Monte- Carlo simulation of its position change and its location prediction. Intersecting with the Ionian Sea, its trajectory in three-dimensional space is greatly changed by the influence of the circulation, but the probability distribution of its predicted location is similar to that of the Ionian Sea. The probability distribution matrices A and B are constructed

for their predicted locations, and the Jaccard coefficients $J(A,B) = \frac{|A \cap B|}{|A \cup B|}$ are calculated,

and the value of 0.958 is obtained, which proves that the probability distributions are similar to one another, and further proves that the SLP model established in this paper has **strong robustness**.

8.2 Quantity migration

In the actual underwater diving of submersibles, they often do not dive alone, but there are multiple moving submersibles in the same area at the same time. In the event of an emergency, when all submersibles in a certain area lose communication with the main ship, in order to search and rescue each submersible as quickly as possible, the search and rescue mode will change accordingly. Below we discuss the situation where multiple submersibles lose communication and power in the same area.

When all submersibles lose communication and power and are moving in the sea, the goal of search and rescue is to rescue any submersible in the shortest time, and so on to search for the position of the next submersible in the shortest time, until all Search and rescue completed. Based on this, we expanded the search and rescue model based on the FBR model. Let the number of submersibles be N_0 . First, based on the SLP model, the predicted probability distribution of each submersible's position in the area after losing contact can be obtained, and further the prior probability p_{ij} of its position distribution can be obtained, which represents the probability of the i_{th} submersible. The prior probability of the j_{th} cube in the distribution space. Next, the prior probabilities of the position distributions of all submersibles are added together with equal weights, and the two probability heat maps are added (Figure 14) as follows:

$$P_{0j} = \frac{\sum_{i=1}^{N_0} P_{ij}}{N_0} \quad (25)$$

We obtain the total prior probability of each cube in the area, thereby converting "multi-objective" into "single-objective", using it as a new Bayesian search priori-end, and using the FBR model to further perform posteriori on the regional probability. Update to achieve the goal of searching for the first ship in the shortest time. By analogy, every time a submersible is searched, "probability update" and "target conversion" are performed to complete the search for multiple submersibles in the same area in the shortest time.

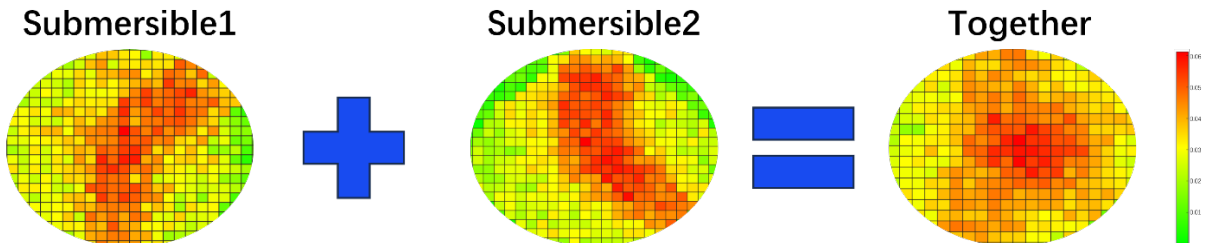


Figure 14: Prior probability superposition of location distributions

9 Sensitivity Analysis

In the Section 8, we completed a sensitivity analysis of the established model for different adjustment parameters of uncertain factors in different sea areas. We found that the calculation results of our model have little variation in different sea areas and have good generalizability.

10 Model Evaluation

10.1 Strengths

1. Highly generalizable. Our model can be applied to a wide range of sea areas beyond the Ionian Sea.
2. Relatively complete. We innovatively used flow dynamics principles and Bayesian search algorithms, and two scenarios were discussed.
3. Appropriate simplification. We are compatible with many factors and achieve the unification of theory and practice.

10.2 Weaknesses

1. The assumption is somewhat idealistic. We have some idealistic assumptions about the submarine rising in place while losing contact and maintaining power.
2. Quantification of seafloor geography could be done better. Due to data limitations, we are lacking in quantifying the impact of complex seafloor topography.

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Memorandum

From: Team # 2418161, MCM 2024

To: Greek Government

Date: February 5, 2024

Subject: Safety procedures for submersible rescue challenges

Dear Greek Government, we are honored to describe our safety procedures for submersible rescue challenges that MCMS company may encounter.

After carefully studying submersible movement and related SAR methods, we obtained the following safety procedures.

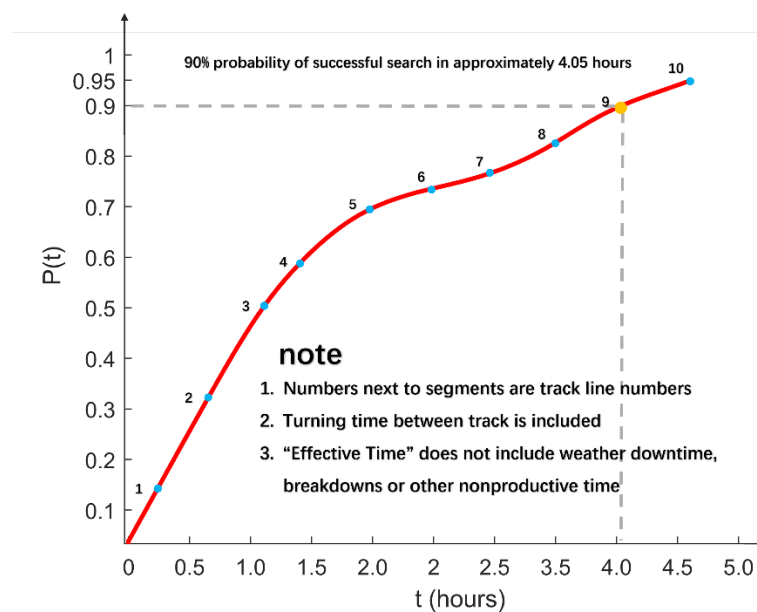
The MCMS Company has developed a project that uses submarines to guide tourists in exploring the underwater shipwreck sites in the Ionian Sea. To ensure the safety of tourists, the MCMS Company commissioned us to design safety response procedures for potential emergencies. We have divided the search and rescue mission for the MCMS Company's submarine, which encountered an accident in the Ionian Sea, into three phases: "Localization," "Preparation," and "Search and Rescue." Moreover, based on our evaluation, the relevant models of our safety procedures exhibit good robustness and adaptability, greatly ensuring rapid search and rescue after the MCMS Company's submarine loses contact.

1. Location: To precisely determine the movement trajectory and final position of the submarine after losing contact, we innovatively employed the "Six Degrees of Freedom model" and took into account uncertainties such as ocean currents, changes in seawater density, marine geography, and data transmission errors that could affect the submarine's movement trajectory and predicted endpoint. This led to the establishment of the SLP (Submersible Location Prediction) Model. To cover as many potential hazards as possible, we categorized the submarine's post-disconnection state into two scenarios: (1) loss of communication with the host ship but the power system remains functional (2) loss of communication and damaged power system. Detailed location prediction analyses were conducted for both scenarios, resulting in probability distributions for their respective endpoint locations. To mitigate the impact of uncertainties, we selected CTD (Conductivity Temperature Depth) and ADCP (Acoustic Doppler Current Profiler) devices for MCMS Company's submarines to reduce the impact of uncertainties and enhance prediction accuracy.

2. Preparation: Submarine search and rescue operations, unlike general surface water rescue, require additional special equipment to ensure the operation proceeds smoothly. Therefore, we developed the RED (Rescue Equipment Determination) Model for MCMS Company, synthesizing equipment functionality and availability to determine the necessary rescue equipment. We discussed in detail the extra equipment required for both the mother ship and the rescue vessels. We first categorized potential search and rescue issues into communication, location, and rescue. Then, using multi-objective optimization algorithms, we analyzed nine different pieces of equipment and ultimately selected four types to be equipped on the mother ship and rescue vessels to ensure the success of the rescue operation. The information about

the four additionally selected pieces of equipment is displayed in the table below:

3. Search and Rescue: Following the completion of the location and equipment preparation phases, to achieve the quickest possible rescue of MCMS Company's submarines, we devised a novel model based on Bayesian search theory: the FBR (Flashlight-Beam Rescue) Model. Initially, utilizing information from the location phase, we designate the highest probability point of the predicted endpoint distribution as the rescue deployment point. Then, according to the two potential post-disconnection scenarios, we divide the search area into grids and cubic blocks, respectively, and apply the flashlight-light beam extension shape to introduce Bayesian search theory specifically. The final result is a specific probability function related to time and cumulative search outcomes. The function's results indicate that, after 4.1 hours, our rescue method achieves a 90% success rate. Specific information is shown in following figure.



4. Program Verification: To account for a variety of scenarios as much as possible, we successfully transferred our model to the Caribbean Sea in the Atlantic Ocean to analyze the impact of different ocean currents and other environmental factors on our safety program. We found that our safety program exhibits excellent robustness. Moreover, we also conducted a detailed analysis and discussion on the cooperative work situation of multiple submarines, obtaining the best safety program for addressing multiple targets.

Our team believes that upon adopting our safety procedures, MCMS Company will possess the robust capability to conduct immediate search and rescue operations in the event of a submarine accident, thereby ensuring the safety of its submersible passengers.

The marine landscape is a precious treasure of human civilization, and we sincerely hope that the beautiful scenery of the Greek seas can become well-known to more people.

Yours Sincerely,

Team # 2418161

Report on Use of AI

1. Microsoft *NewBing* (Feb 2, 2024 version)

Query1: < Please find me some data about sunken shipwrecks in Ionian Sea. >

Output: < Certainly! The **Inner Ionian Sea** in Greece is home to a fascinating array of shipwrecks spanning various historical periods. Here are some notable findings:

1. **Ancient Shipwrecks**: These wrecks date back to ancient times and provide valuable insights into maritime history. Researchers have systematically studied 11 out of 36 known shipwrecks in the area using marine remote sensing techniques. These include side scan sonars, sub-bottom profilers, and multi-beam echo-sounders. The wrecks' conditions, debris fields, and man-made activities were assessed based on acoustic data. [The study sheds light on the rich maritime heritage of the Inner Ionian Sea 1](<https://www.mdpi.com/2571-9408/3/4/67>).

<https://www.bing.com/th?id=OSK.HEROIQwllwJABI5J86Kp7ctGLo6lQ1ro67YOB-dAPZYVcPh0&pid=cdx&w=200&h=128&c=7>

1. **World War I and II Shipwrecks**: The region also holds shipwrecks from both World War I (WWI) and World War II (WWII). These wrecks serve as historical artifacts, telling stories of conflict and human endeavors during those turbulent times.

<https://www.bing.com/th?id=OSK.73fa4740688be1cda3fbd6c752c273f4&pid=cdx&w=200&h=128&c=7&rs=1>

<https://www.bing.com/th?id=OSK.73fa4740688be1cda3fbd6c752c273f4&pid=cdx&w=170&h=128&c=7>

1. **Recent Shipwrecks**: Even more recent shipwrecks exist in the Inner Ionian Sea, reflecting the 20th-century maritime activity. These wrecks contribute to our understanding of modern shipping and navigation.

However, there's an urgent need for a **shipwreck protection framework** in this area. Man-made activities, such as fishing, pose a threat to the integrity of these underwater cultural heritage sites. [Preserving these shipwrecks ensures that their historical significance endures for generations to come 1](<https://www.mdpi.com/2571-9408/3/4/67>).

[If you're interested in exploring further, you can refer to the detailed research article titled "Atlas of Shipwrecks in Inner Ionian Sea (Greece): A Remote Sensing Approach" here](<https://doi.org/10.3390/heritage3040067>)[1](<https://www.mdpi.com/2571->

9408/3/4/67). It provides a comprehensive overview of the shipwrecks and their significance. >

2. OpenAI *ChatGPT* (Jan 10, 2024 version, ChatGPT-4)

Query1: < What is the approximate forward speed of a civilian submarine? >

Output: < Civilian submarines usually travel at lower speeds than military submarines, mainly because they are designed for different purposes, such as scientific research, sightseeing, exploration and commercial operations, rather than high-speed movement or tactical maneuvers. The speed of civilian submarines varies depending on design, size, and powertrain, but typical speed ranges are:

- **Small or private submarines**: The speed of these submarines is generally between 1 and 3 knots per hour (approximately 1.85 to 5.56 km/h). This speed is sufficient for close-range underwater exploration and sightseeing activities.
- **Medium Scientific Research Submarine**: Submarines used for scientific exploration and research may have slightly higher speeds, usually between 3 and 5 knots per hour (approximately 5.56 to 9.26 km/h). Such speed allows researchers to conduct efficient surveys and sampling over a wide ocean area.
- **Large commercial or exploration submarines**: These submarines are larger, sometimes equipped with more efficient power systems, and may reach speeds of more than 5 knots per hour (approximately 9.26 km/h). Certain submarines specially designed for deep-sea exploration or tourist diving may have higher maximum speeds to meet specific operational needs.

It is important to note that these speeds are approximate estimates and actual speeds will depend on the specific design of the submarine and the type of power used. Compared with military submarines, civilian submarines generally focus more on safety, reliability and comfort than high speed. In addition, because civilian submarines usually operate in shallow waters, high speed is not a primary design consideration in many cases. >

Query2: < The maximum buoyancy speed of a submarine?>

Output: < A submarine's maximum resurface speed depends on its design, size and whether emergency resurface measures are used. In general, the buoyancy speeds of conventionally powered submarines and nuclear-powered submarines will differ, mainly due to differences in their displacements, power systems and safety constraints.

The floating speed of **conventionally powered submarines** is usually between 1 and 2 meters per second, which is approximately 60 to 120 meters per minute. These submarines use a ballast water system to control buoyancy, and in an emergency the ballast water will be quickly discharged to achieve rapid buoyancy.

Nuclear-powered submarines can achieve similar or even faster buoyancy speeds,

which can theoretically reach 2 to 3 meters per second, especially when using emergency air blowing systems, this speed may be higher. Nuclear submarines are typically designed with more complex safety and operational standards in mind, and their emergency resurfacing measures are designed to ensure a safe return to the surface even in extreme circumstances.

It is important to note that it is highly unusual for a submarine to perform an emergency refloating operation (often referred to as a "blow-to-buoy" operation) as such an operation places extreme stress on the hull structure and may pose risks to the crew. In addition, the buoyancy rate will be affected by the submarine's current depth, water pressure and a series of other operational factors.

However, the specific maximum buoyancy speed data is very dependent on the design and operating guidelines of each type of submarine, and the above values mainly provide a general reference range. The exact speed can vary between different countries' submarines, and this information is often not made public to maintain military secrecy. >