

SEC F TEST

Q. Consider $T(n)_{\text{best}} = K_1n + K_2$

$$T(n)_{\text{worst}} = K_1n^2 + K_2n + K_3$$

for a sorting algorithm 'A' and we have a function of A, titled 'Sort'

Div-solv (A)

$n = \text{length}(A)$

$S_1 = \text{Sort}(A[1 \text{ to } n/2])$

$S_2 = \text{Sort}(A[n/2+1 \text{ to } n])$

$S = \text{merge}(S_1, S_2)$

return S

* merge has running time $\propto n$

Find $T(n)$ for best and worst case

• DRY RUNNING:-

(BEST CASE)

$A = [1, 2, 3, 4, 5, 6]$

1. $n = 6$

2. $S_1 = \text{Sort}[1, 2, 3] = [1, 2, 3]$

3. $S_2 = \text{Sort}[4, 5, 6] = [4, 5, 6]$

4. $S = \text{merge}(S_1, S_2) = [1, 2, 3, 4, 5, 6]$

5. return $[1, 2, 3, 4, 5, 6]$

Lines of code	Frequency (Worst case) best
1	1
2	$K_1(n/2) + K_2$
3	$K_1(n/2) + K_2$
4	$K_1n + K_2$
5	1

$$T(n) = 1 + K_1 \frac{n}{2} + K_2 + K_1 \frac{n}{2} + K_2 + K_1n + K_2 + 1$$

$$T(n) = \frac{2K_1n}{2} + 3K_2 + 2 + K_1n$$

$$T(n) = 2K_1n + 3K_2 + 2$$

• Discussion:-

- Best case complexity of div-sol function is $O(n)$

Div-sol function in best case grows linearly.

- means that, minimum time which a code can take

• DRY RUNNING:-

(WORST CASE):-

$A = [6, 5, 4, 3, 2, 1]$

1. $n = 6$
2. $S_1 = \text{Sort}[6, 5, 4] = [4, 5, 6]$
3. $S_2 = \text{Sort}[3, 2, 1] = [1, 2, 3]$
4. $S = \text{merge}(S_1, S_2) = [1, 2, 3, 4, 5, 6]$
5. return $[1, 2, 3, 4, 5, 6]$

LINES OF CODE	FREQUENCY
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1	1
2	$K_1(n/2)^2 + K_2(n/2) + 1$
3	$K_1(n/2)^2 + K_2(n/2) + 1$
4	$K_1n + K_2$
5	1

$$T(n) = 1 + K_1 \frac{n^2}{4} + K_2 \left(\frac{n}{2} \right) + 1 + K_1 \frac{n^2}{4} + K_2 \frac{n}{2} + 1 + K_1n + K_2 + 1$$

$$T(n) = \frac{2K_1n^2}{4} + \frac{2K_2n}{2} + K_1n + K_2 + 4$$

$$T(n) = \frac{K_1n^2}{2} + K_2n + K_1n + K_2 + 4$$

$$T(n) = \frac{K_1n^2}{2} + (K_2 + K_1)n + K_2 + 4$$

• Discussion:-

- Worst case complexity of div-sol function is $O(n^2)$
- Div-sol function in worst case grows quadratically.
- means that, maximum time which a code can take