

# MERGE PROCEDURE AND MERGE SORT

Merge (A, B)

1.  $n_A = A.length$
2.  $n_B = B.length$
3. let  $L[n_A+n_B]$  be a new array
4.  $A[n_A+1] = \infty$
5.  $B[n_B+1] = \infty$
6.  $i = 1$
7.  $j = 1$
8. for  $k = 1$  to  $n_A+n_B$
9.     if  $A[i] \leq B[j]$
10.          $L[k] = A[i]$
11.          $i = i + 1$
12.     else
13.          $L[k] = B[j]$
14.          $j = j + 1$
15. return  $L$

• DRY RUNNING:-

$A = [1, 4]$

$B = [3, 5]$

1.  $n_A = 2$

2.  $n_B = 2$



3.  $L = []$

4.  $A = [1, 4, \infty]$

5.  $B = [3, 5, \infty]$

6.  $i = 1$

7.  $j = 1$

8. **1ST ITERATION OF FOR-LOOP:-**

8.  $K = 1$

9.  $\text{if } 1 < 3$   
10.  $L[i] = 1 \rightarrow L = [1]$   
11.  $i = 2$

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

**2ND ITERATION OF FOR LOOP:-**

8.  $K = 2$

9.  $\text{if } 4 < 3$

10. \_\_\_\_\_

11. \_\_\_\_\_

12.  $\text{else}$

13.  $L[2] = 3 \rightarrow L = [1, 3]$

14.  $j = 2$



## 3<sup>RD</sup> ITERATION OF FOR-LOOP:-

8.  $K=3$

9.

10.

11.

12.

13.

14.

If

$4 < 5$

$L[3] = 4 \rightarrow L = [1, 3, 4]$

$i = 3$

## 4<sup>TH</sup> ITERATION OF FOR-LOOP:-

8.  $K=4$

9.

10.

11.

12.

13.

14.

If

$\infty < 5$

else

$L[4] = 5 \rightarrow$

$L = [1, 3, 4, 5]$

$j = 3$

## 5<sup>TH</sup> ITERATION OF FOR-LOOP:-

8.  $K=5$

$\Rightarrow$  Loop terminates

x

x

x

15.

Return  $[1, 3, 4, 5]$



Line no of code	Time/ instruction	frequency
1	C	1
2	C	1
3	C	1
4	C	1
5	C	1
6	C	1
7	C	1
8	C	$\frac{(n_A + n_B) + 1}{n}$
9	C	$\frac{n_A + n_B}{n}$
10	C	$n_A$
11	C	$n_A$
12	C	
13	C	$n_B$
14	C	$n_B$
15	C	1



$$T(n) = c[7 + n + 1 + n + 2(n_A + n_B) + 1]$$

$$T(n) = c[9 + n + n + 2n]$$

$$T(n) = c[4n + 9]$$

$$T(n) = 4cn + 9c$$

$$T(n) = 4K_1n + K_2$$

$K_1$  and  $K_2$  are constants

$\Rightarrow$  Linear growth

## • Discussion:-

$\Rightarrow$  Merge procedure grows linearly

$\Rightarrow$  Time complexity of merge procedure is  $O(n)$



# PYTHON CODE

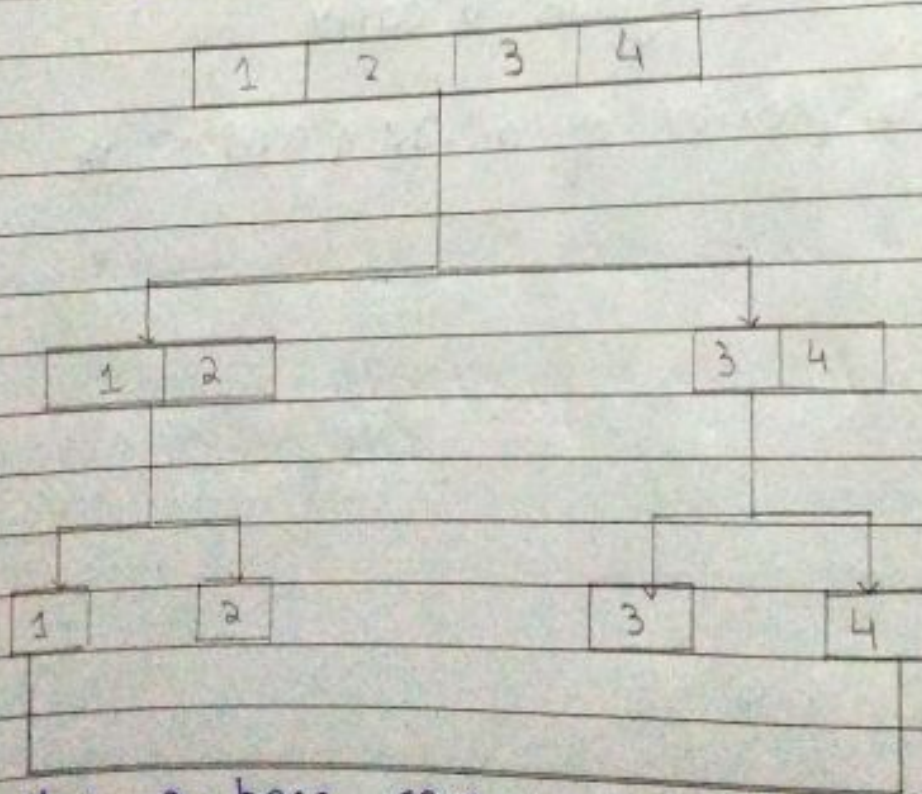
Date: \_\_\_\_\_

```
def mergeSort(A):  
    n = len(A)  
    S = List()  
    if n == 1:  
        S = A  
  
    else:  
        a = (n//2);  
        S1 = mergeSort(A[0:a]);  
        S2 = mergeSort(A[a:n]);  
        S = merge(S1, S2)  
    return S
```

• FOR BEST CASE :-

A = [1, 2, 3, 4]

• DRY RUNNING :- (DIVIDE)



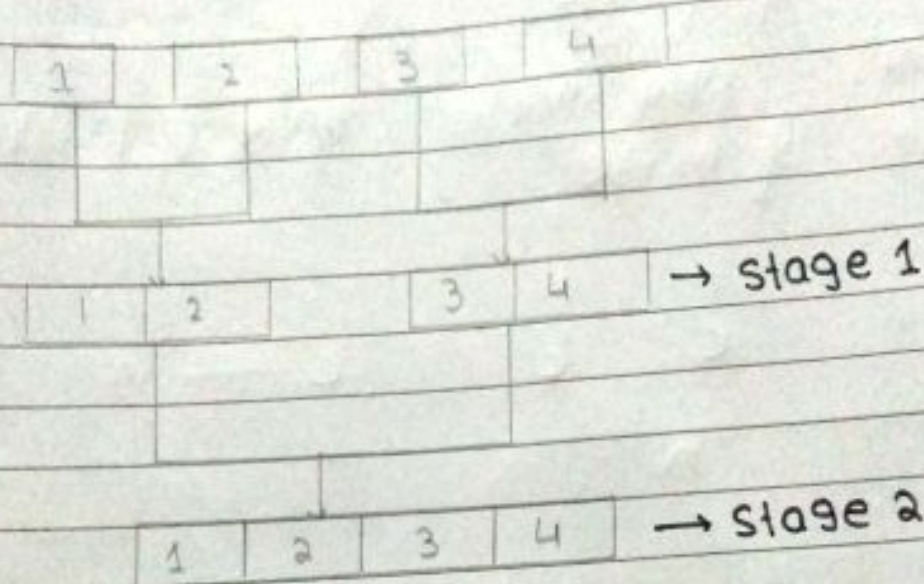
• for a base case we have 'c' second.





- For 'n' number of elements we will call base case for n-times
- Time for all base cases:  $nc$

## • MERGE:-

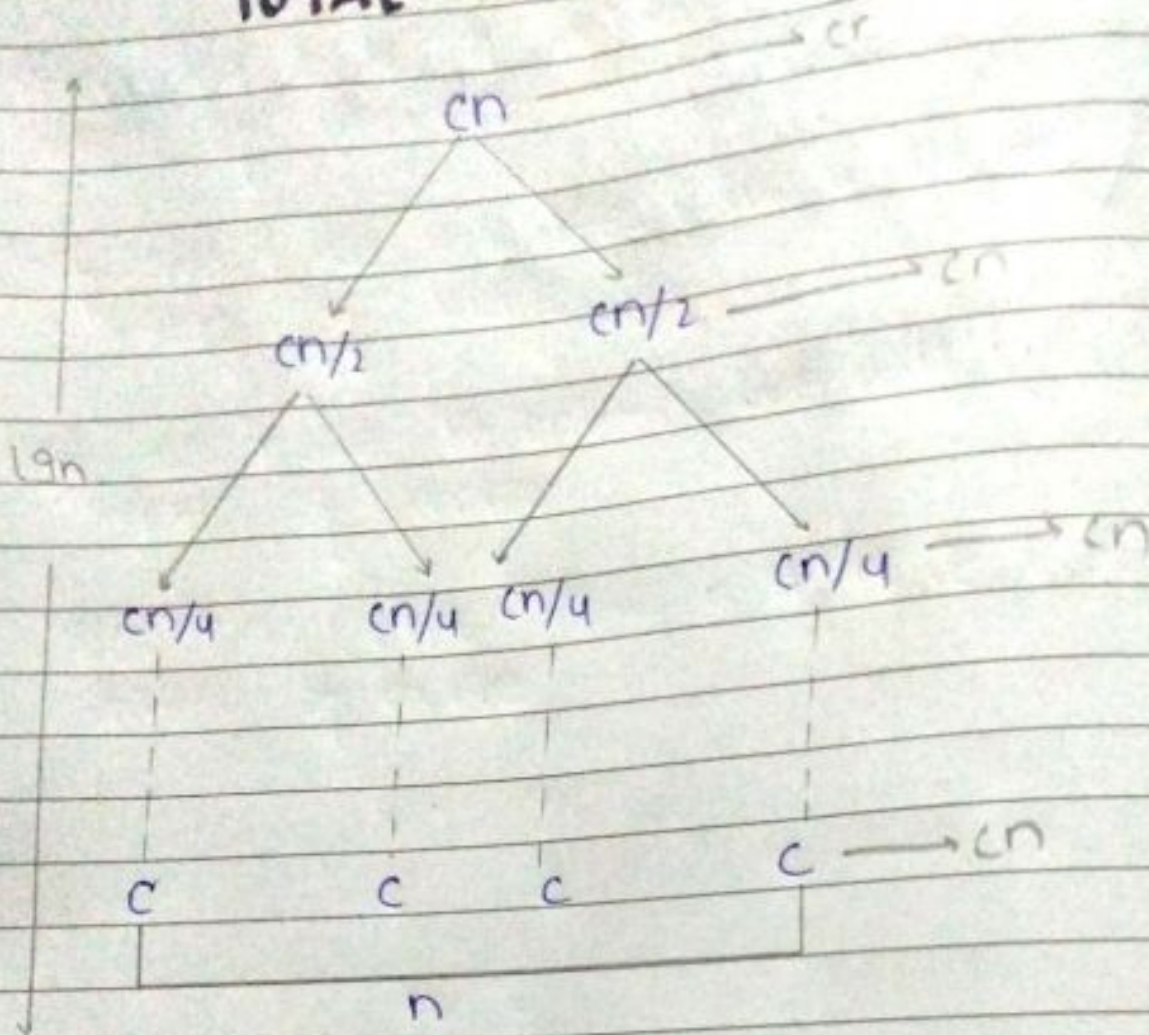


## • COUNT STAGES OF MERGE:-

- When we have 4 entries, we got two stages of merge
- If we have 8 entries, we got three stages of merge
- If we have 16 entries, we will have 4 stages of merge
- For n entries we will have  $\log_2(n)$  stages of merge



## TOTAL TIME :-



$$\text{Total time} = cn \lg n + cn = cn(1 + \lg n)$$

• logarithmic growth

### • DISCUSSION:-

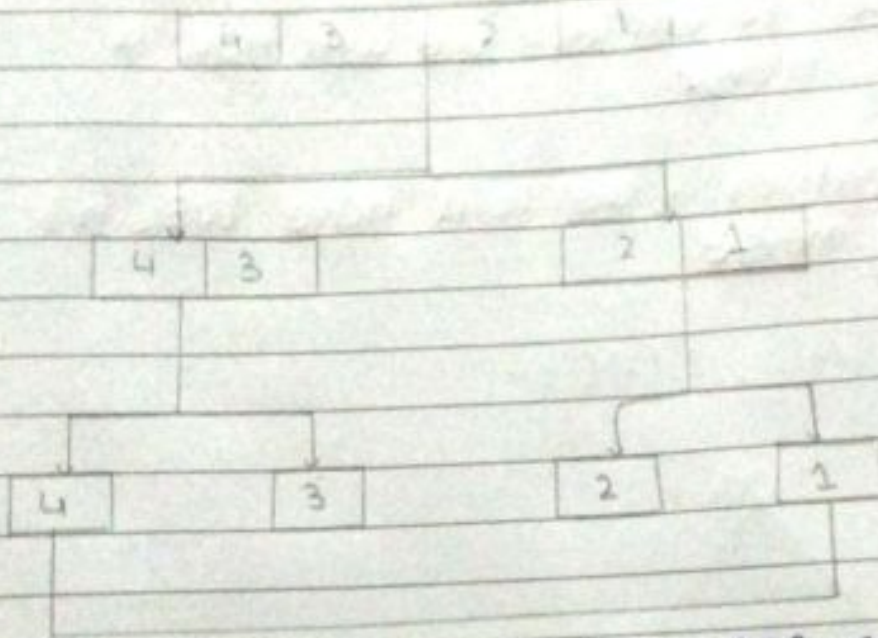
- Merge sort in best case grows logarithmic.
- Time complexity of merge sort in best case is  $O(n \lg n)$



## • FOR WORST CASE:-

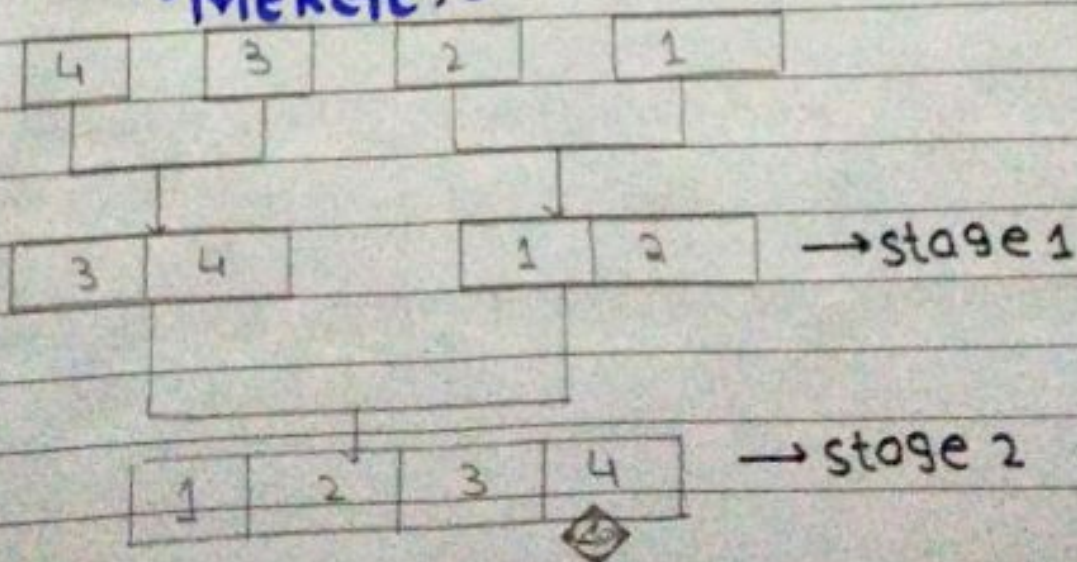
A = [4, 3, 2, 1]

### DIVIDE:-



- For a base case we have 'c' second.
- For 'n' number of elements we will call base case for n times
- Time for all base cases =  $n \times c$

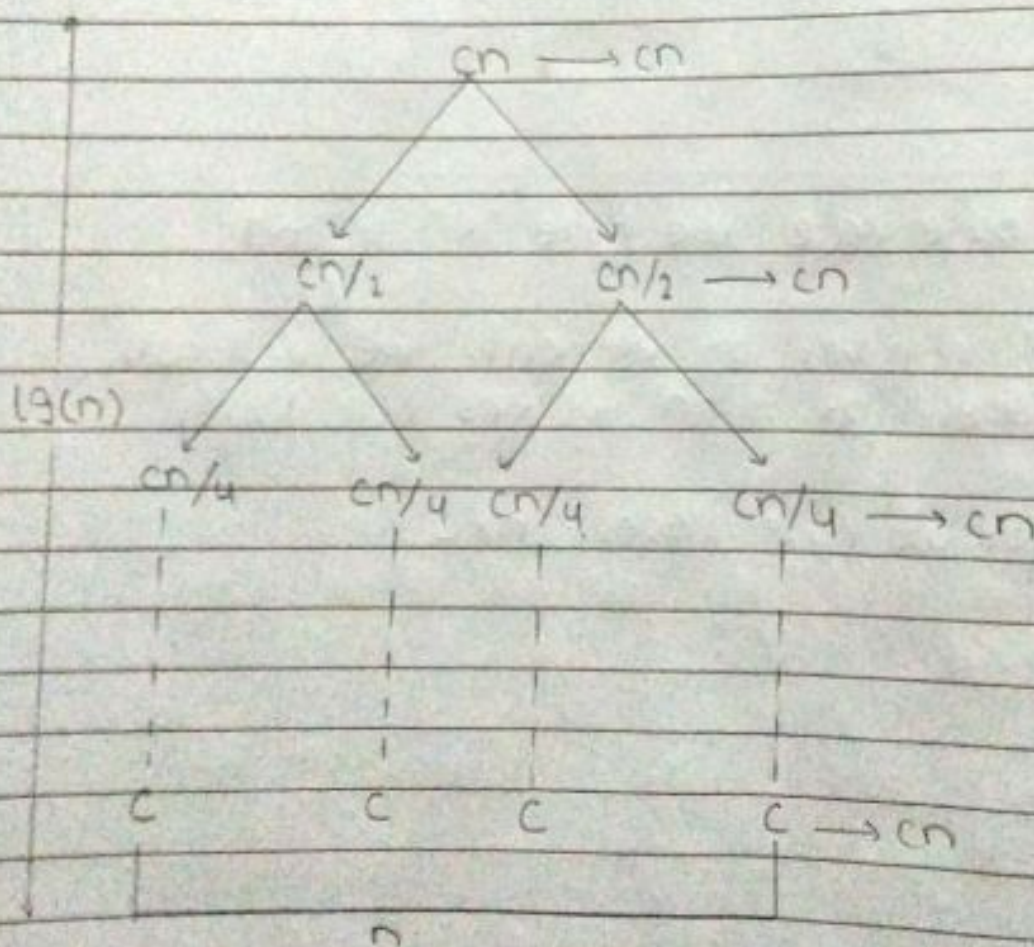
### • MERGE:-





- When we have 4 entries, we got 2 stages of merge
- If we have 8 entries, we will have 3 stages of merge
- If we have 16 entries, we will have 4 stages of merge.
- For 'n' entries, we will have  $\log_2(n)$  stages of merge

### TOTAL TIME:-





Total time:  $n \lg n + cn = cn(1 + \lg n)$

$\Rightarrow$  logarithmic growth

### • Discussion:-

$\Rightarrow$  Merge sort in worst case also grows logarithmic

$\Rightarrow$  Time complexity of merge sort for worst case is  $O(n \lg n)$