- **BOOLEAN ALGEBRA**

Boolean algebra is a set of rules with digital variables to develop, manipulate and simplify logic expressions.

Axioms / postulates :-

AND :-

0	1	0
0	0	0
1	1	1
1	0	0

OR :

0	0	0
0	1	1
1	0	1
1	1	1

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NOT:-

0	1
1	0

1. COMMUTATIVE:

a) $A \cdot B = B \cdot A$

b) $A + B = B + A$

2) ASSOCIATIVE:

a) $(A \cdot B) \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$

b) $(A+B)+C = A+(B+C)$

3) DISTRIBUTIVE:

a) $A \cdot (B+C) = A \cdot B + A \cdot C$

b) ~~$(A+B)$~~

b) $A + (B \cdot C) = (A+B) \cdot (A+C)$

4) IDENPOTENT:

a) $A \cdot A \cdot A$

b) $A + A \cdot A$

5) IDENTITY:

$A \cdot 1 \cdot A$

$A + 1 \cdot 1$

6) NULL PROPERTY:

$A \cdot 0 = 0$

$A + 0 = A$

7) NEGATION:

$A \cdot \overline{A} = 0$

$A + \overline{A} \cdot 1$

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8) DOUBLE NEGATION:-

$\overline{\overline{A}} = A$

9) Absorption

a) $A + AB = A$

b) $A + \overline{A}B = A + B$

c) $A + BC = (A+B)(A+C)$

- PREFERENCE OF OPERATORS

* NOT

* AND

* OR

=> A + AB

Sol:-

=> A(1+B)

=> A(1)

= A

2) $A + \bar{A}B$

Sol:-

= $A + \bar{A}B$

= $(A+\bar{A})(A+B)$

= $1(A+B)$

= $(A+B)$ //

3) A + BC

Sol:-

= A + BC

= (A+B)(A+C)

8) $AB + A(B+C) + B(B+C)$

Sol:-

$\Rightarrow AB + AB + AC + BB + BC$

$\Rightarrow AB + AC + B \cdot B + BC$

$\Rightarrow AB + AC + B + BC$

$\Rightarrow AB + AC + B(1+C)$

$\Rightarrow AB + AC + B$

$\Rightarrow AB + B + AC$

$\Rightarrow B(1+A) + AC$

$\Rightarrow B + AC$

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(*) $(A+\bar{B})(A+C)$

Sol:-

$\Rightarrow (A+\bar{B})(A+C)$

$\Rightarrow (AA+AC+A\bar{B}+\bar{B}C)$

$\Rightarrow A+AC+A\bar{B}+\bar{B}C$ $\quad \because A \cdot A = A$

$\Rightarrow A(1+C)+A\bar{B}+\bar{B}C$

$\Rightarrow A(1)+A\bar{B}+\bar{B}C$ $\quad \because (1+C)=1$

$\Rightarrow A+A\bar{B}+\bar{B}C$

$\Rightarrow A(1+\bar{B})+\bar{B}C$

$\Rightarrow A(1)+\bar{B}C$ $\quad \because (1+\bar{B})=1$

$\Rightarrow A+\bar{B}C$

(*) $(A+\bar{A})(AB+AB\bar{C})$

Sol:-

$\Rightarrow AAB + AAB\bar{C} + \bar{A}AB + \bar{A}AB\bar{C}$

$\Rightarrow AB + AB\bar{C} + 0 + 0$

$\Rightarrow AB + AB\bar{C}$

$\Rightarrow AB(1+\bar{C})$

$\Rightarrow AB$

(*) $(B+BC)(B+\bar{B}C)(B+D)$

Sol:-

$= BB + B\bar{B}C + BBC + \bar{B}CBC\ (B+D)$

$= B + 0 + BC + 0\ (B+D)$

$\Rightarrow (B+BC)(B+D)$

$= BB + BD + BBC + BCD$

$\cdot\ B + BD + BC + BCD$

$\cdot\ B(1+D) + BC(1+D)$

$\Rightarrow B + BC$

$= B(1+C)$

$= B$ //

* $X(\bar{X}YZ + \bar{X}Y\bar{Z})$

Sol:

$\Rightarrow X(\bar{X}YZ + \bar{X}Y\bar{Z})$

$\Rightarrow X\bar{X}YZ + X\bar{X}Y\bar{Z}$

$\Rightarrow 0 + 0$

$\Rightarrow 0$

* $\bar{X}Y + \bar{X}$

Sol:

$\Rightarrow \bar{X}Y + \bar{X}$

$\Rightarrow \bar{X}(1+Y)$

$\Rightarrow \bar{X}$

3) $(X_1+X_2)(\overline{X}_1+X_1X_2)(\overline{X}_2+X_1\overline{X}_2)$

Sol:-

$\Rightarrow (X_1+X_2)(\overline{X}_1+X_1X_2)(\overline{X}_2+X_1\overline{X}_2)$

$\rightarrow (X_1\overline{X}_1+X_1X_1X_2+X_2\overline{X}_1+X_1X_2X_2)$
$(\overline{X}_2+X_1\overline{X}_2)$

$\Rightarrow (0+X_1X_2+\overline{X}_1X_2+X_1X_2)$
$(\overline{X}_2+X_1\overline{X}_2)$

$\Rightarrow X_1X_2\overline{X}_2+\overline{X}_1X_2\overline{X}_2+X_1X_2\overline{X}_2$

$+X_1X_2X_1\overline{X}_2+\overline{X}_1X_2X_1\overline{X}_2+X_1X_2+X_1\overline{X}_2$

$\rightarrow 0+0+0+0+0+0+0$

$\Rightarrow 0$

4) $abc(a\bar{b}+a\bar{c})+a\bar{b}c(\bar{a}\bar{b}+b\bar{c})$

Sol:-

$\Rightarrow abca\bar{b}+abca\bar{c}+a\bar{b}c\bar{a}\bar{b}+a\bar{b}cb\bar{c}$

$\Rightarrow 0+0+0+0$

$\Rightarrow 0$

5) $\bar{X}YZ+\bar{X}Y\bar{Z}+XZ$

Sol:-

$\Rightarrow \bar{X}YZ+\bar{X}Y\bar{Z}+XZ$

$\Rightarrow \bar{X}Y(Z+\bar{Z})+XZ$

$\Rightarrow \bar{X}Y(1)+XZ$

$\Rightarrow \bar{X}Y+XZ$

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6) $(wx + w\bar{y})(x+w) + wx(\bar{x}+\bar{y})$

Sol:-

$\Rightarrow wxx + wx\bar{y} + wwx + ww\bar{y} + wx\bar{x} + wx\bar{y}$

$\Rightarrow wx + wx\bar{y} + wx + w\bar{y} + 0 + wx\bar{y}$

$\Rightarrow wx + wx\bar{y} + w\bar{y} + wx\bar{y}$

$\Rightarrow wx + wx\bar{y} + w\bar{y}$

$\Rightarrow wx(1+\bar{y}) + w\bar{y}$

$\Rightarrow wx + w\bar{y}$

$\Rightarrow w(x+\bar{y})$

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7) $AB + AB'$

Sol:-

$\Rightarrow AB + AB'$

$\Rightarrow A(B + B')$

$\Rightarrow A(1)$

$\Rightarrow A$

8) $AB + AB'C + AB'C'$

Sol:

$= AB + AB'(C + C')$

$= AB + AB'(1)$

$\Rightarrow A(B + B')$

$\Rightarrow A(1)$

$\Rightarrow A$

Ex: 3 $F = (A+B+C)(A+B'+C)(A+B+C')$

Sol:

let, $X = A+B$

$\Rightarrow (X+C)(A+B'+C)(X+C')$

$\Rightarrow (X+C)(X+C')(A+B'+C)$

$\Rightarrow (X+C\cdot C')(A+B'+C)$

$\Rightarrow (X+0)(A+B'+C)$

$\Rightarrow (A+B)(A+B'+C)$

$\Rightarrow (A+B\cdot(B'+C))$

$\Rightarrow A+B\cdot B'+BC$

$\Rightarrow A+0+BC$

$\Rightarrow A+BC$

$\Rightarrow \boxed{(A+B)\cdot(A+C) = A+(B\cdot C)}$

Ex:4 $(A+B)(A+B')(A'+B)(A'+B')$

Sol:

$\Rightarrow (A+B)(A+B')(A'+B)(A'+B')$

$\Rightarrow (A+B\cdot B')(A'+B\cdot B')$

$\Rightarrow (A+0)(A'+0)$

$\Rightarrow A\cdot A'$

$\Rightarrow 0$

Ex:5 $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

Sol:-

$\Rightarrow \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

$\Rightarrow \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}(\bar{C}+C)$

$\Rightarrow \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}$

$\Rightarrow \bar{B}(A+\bar{A}C) + \bar{A}B\bar{C}$

$\Rightarrow \bar{B}(A+C) + \bar{A}B\bar{C}$

$\Rightarrow \bar{B}A + \bar{B}C + \bar{A}B\bar{C}$

⇒ Redundancy Theorem

i) Three variables

ii) Each variable is repeated twice

iii) One variable is complimented

iv) Take the complimented variable

=> Redundancy theorem is also called consensus theorem.

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Q. $Y = AB + A'C + BC$

Sol:

$\Rightarrow AB + A'C + BC$ → redundant term

$\Rightarrow AB + A'C$

$\Rightarrow$ Proof:

Y. $AB + A'C + BC$

Sol:-

$= AB + A'C + BC \cdot 1$

$= AB + A'C + BC \cdot (A + A')$

$= AB + A'C + ABC + A'BC$

$\Rightarrow AB + ABC + A'C + A'BC$

$\Rightarrow AB(1 + C) + A'C(1 + B)$

$\Rightarrow AB + A'C$ → proved!

→ extra work.

Q. $F = AB + B\bar{C} + AC$

Sol:-

redundant term

$\Rightarrow$ $AB + B\bar{C} + AC$

$\Rightarrow$ $AC + B\bar{C}$

PROOF:-

$= AB + B\bar{C} + AC$

$\Rightarrow AB \cdot 1 + B\bar{C} + AC$

$\Rightarrow AB(A + \bar{A}) + B\bar{C} + AC$

$\Rightarrow A \cdot AB + \bar{A}AB + B\bar{C} + AC$

$\Rightarrow 0 + 0 + B\bar{C} + AC$

$\Rightarrow AC + B\bar{C}$ //

→ extra work

Q. $F = A\bar{B} + BC + AC$

Sol:-

$\Rightarrow A\bar{B} + BC + (\bar{A}C)$ → redundant term

$\Rightarrow A\bar{B} + BC$

PROOF:-

$\Rightarrow A\bar{B} + BC + AC \cdot 1$

$\Rightarrow A\bar{B} + BC + AC(A + \bar{A})$

$\Rightarrow A\bar{B} + BC + AAC + A\bar{A}C$

$\Rightarrow$

Q. $(A+B) \cdot (\bar{A}+C) \cdot (B+C)$

Sol:

$\Rightarrow (A+B)(\bar{A}+C)(B+C)$ → redundant term

$\Rightarrow (A+B)(\bar{A}+C)$

Q. $(A+B)\cdot(\bar{B}+C)\cdot(A+C)$

Sol:

$\Rightarrow (A+B)\cdot(\bar{B}+C)\cdot(A+C)$ → redundant term

$\Rightarrow (A+B)\cdot(\bar{B}+C)$

Q. $F\cdot\bar{A}\bar{B}+A\bar{C}+\bar{B}\bar{C}$

Sol:

$F\cdot\bar{A}\bar{B}+A\bar{C}+\bar{B}\bar{C}$ → redundant term

$F\cdot\bar{A}\bar{B}+A\bar{C}$

DEMORGAN'S THEOREM:

$\overline{(A+B)} = \bar{A} \cdot \bar{B}$

First solve long bar

$\overline{A \cdot B} = \bar{A} + \bar{B}$

Q.1 $\overline{A + \bar{B}CD}$

Sol:-

$\Rightarrow \overline{A + \bar{B}CD}$

$\Rightarrow \bar{A} \cdot \overline{\bar{B}CD}$

$\Rightarrow \bar{A} \cdot (\bar{\bar{B}} + \bar{C} + \bar{D})$

$\Rightarrow \bar{A} \cdot (B + \bar{C} + \bar{D})$

Q.2 $\overline{A + \overline{BC}}$

Sol:-

$\Rightarrow \overline{A} \cdot \overline{\overline{BC}}$

$\Rightarrow \overline{A} \cdot (BC)$ //

Q.3 $\overline{(\overline{A}\overline{B})(\overline{C}+\overline{D})}$

Sol:

$\Rightarrow \overline{(\overline{A}\overline{B}) \cdot (\overline{C}+\overline{D})}$

$\Rightarrow \overline{(\overline{A}\overline{B})} + \overline{(\overline{C}+\overline{D})}$

$\Rightarrow \overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}} \cdot \overline{\overline{D}}$

$\Rightarrow A + B + (CD)$ //

4) $\overline{(x+\bar{y}\bar{z})(\bar{x}yz)}$

Sol:-

$\Rightarrow \overline{(x+\bar{y}\bar{z})}\,\overline{(\bar{x}yz)}$

$\Rightarrow \overline{(x+\bar{y}\bar{z})} + \overline{(\bar{x}yz)}$

$\Rightarrow x+\bar{y}\bar{z}+\bar{\bar{x}}+\bar{y}+\bar{z}$

$\Rightarrow x+\bar{y}\bar{z}+x+\bar{y}+\bar{z}$

$\Rightarrow x+\bar{y}(1+\bar{z})+\bar{z}$

$\Rightarrow x+\bar{y}+\bar{z}$

6. $\overline{\overline{A}(C+\overline{D})} + \overline{\overline{C}(A+\overline{B})}$

Sol:-

$\Rightarrow \overline{\overline{A}(C+\overline{D})} + \overline{\overline{C}(A+\overline{B})}$

$\Rightarrow \overline{\overline{A}} + \overline{(C+\overline{D})} + \overline{\overline{C}} \cdot \overline{(A+\overline{B})}$

$\Rightarrow A + (C+\overline{D}) + \overline{C} \cdot (\overline{A} \cdot \overline{\overline{B}})$

$\Rightarrow A + (\overline{C} \cdot \overline{\overline{D}}) + \overline{C} \cdot (\overline{A} \cdot B)$

$\Rightarrow A + (\overline{C} \cdot D) + \overline{C} \cdot (\overline{A} \cdot B)$

$\Rightarrow A + \overline{C} \cdot D + \overline{A}\overline{C} \cdot B$

$\Rightarrow A + \overline{A}\overline{C} \cdot B + \overline{C} \cdot D$

$\Rightarrow (A+\overline{A})(A+\overline{C}B) + \overline{C} \cdot D$

$\Rightarrow A + B\overline{C} + \overline{C}D$

$\Rightarrow A + \overline{C}(B+D)$ //

6) $\overline{A + (B \cdot C)} + \overline{A \cdot \bar{B}}$

Sol:-

$\Rightarrow \overline{A + (B \cdot C)} + \overline{A \cdot \bar{B}}$

$\Rightarrow \overline{A + (B \cdot C)} \cdot \overline{A \cdot \bar{B}}$

$\Rightarrow (\underset{a}{A} + \underset{b}{B \cdot C}) \cdot (\underset{a}{A} \cdot \underline{\bar{B}})$

$\Rightarrow A A \bar{B} + A \bar{B} B C$

$\Rightarrow A\bar{B} + 0$

$\Rightarrow A\bar{B}$ //

7) $\overline{\overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{B \cdot C}} = Y$

Sol:-

$\Rightarrow Y = \overline{\overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{B \cdot C}}$

$\Rightarrow \overline{(\overline{A} \cdot \overline{B} \cdot C)} \cdot \overline{(A \cdot \overline{B \cdot C})}$

$\Rightarrow (\overline{\overline{A}} + \overline{\overline{B}} + \overline{C}) \cdot (\overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}})$

$\Rightarrow (A + B + \overline{C}) \cdot (\overline{A} + B + C)$

$\Rightarrow$

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8) $\overline{\overline{\bar{A}C} + A\bar{B}} + \overline{\bar{B}+C}$

Sol:-

$\Rightarrow \overline{\overline{\bar{A}C} + A\bar{B}} + \overline{\bar{B}+C}$

$\Rightarrow \overline{\bar{A}C} \cdot \overline{A\bar{B}} \cdot \overline{(\bar{B}+C)}$

$\Rightarrow (\bar{A}C) \cdot \overline{(A\bar{B})} \cdot (\bar{B}+C)$

$\Rightarrow (\bar{A}C) \cdot (\bar{A}+\bar{\bar{B}}) \cdot (\bar{B}+C)$

$\Rightarrow (\bar{A}C) \cdot (\bar{A}+B) \cdot (\bar{B}+C)$

$\Rightarrow (\bar{A}\bar{A}C + \bar{A}CB) \cdot (\bar{B}+C)$

$\cdot (\bar{A}C + \bar{A}CB) \cdot (\bar{B}+C)$

$\cdot \bar{A}C(1+B) \cdot (\bar{B}+C)$

$\cdot \bar{A}C \cdot (\bar{B}+C)$

$\cdot \bar{A}BC + \bar{A}CC$

$\cdot \bar{A}BC + \bar{A}C$

$= \bar{A}C(1+\bar{B})$

$= \bar{A}C$ //

ii) a) $\overline{\bar{A}\bar{B} + \bar{A} + AB}$

Sol:-

$\Rightarrow \overline{\bar{A}\bar{B} + \bar{A} + AB}$

$\Rightarrow \bar{\bar{A}}\bar{B} \cdot \bar{\bar{A}} \cdot \overline{AB}$

$\Rightarrow (AB) \cdot (A) \cdot (\bar{A}\bar{B})$

$AB \cdot (\bar{A} + \bar{B})$

$\Rightarrow A\bar{A}B + AB\bar{B}$

$\Rightarrow 0 + 0$

$\Rightarrow 0$

10) $\overline{\bar{A} \cdot (A+C)}$

Sol:-

$\Rightarrow \overline{\bar{A} \cdot (A+C)}$

$\Rightarrow \bar{\bar{A}} + \overline{(A+C)}$

$\Rightarrow A + (\bar{A} \cdot \bar{C})$

$\Rightarrow (A+\bar{A})(A+\bar{C})$

$\Rightarrow A(A+\bar{C})$

$\Rightarrow A\bar{A} + A\bar{C}$

$\Rightarrow A + A\bar{C}$

$\Rightarrow A \cdot (1+\bar{C})$

$\Rightarrow A \cdot 1$

$\Rightarrow A$ //

Standard Forms Of Boolean Expressions

=> SOP FORM

=> POS FORM

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THE SUM OF PRODUCTS (SOP FORM)

When two or more products are summed by Boolean addition, the resulting expression is sum of products (SOP)

DOMAIN OF A BOOLEAN EXPRESSION:

The domain of a general Boolean expression is the set of variables contained in the expression in either complemented or uncomplemented form

For example: the domain of the expression $\bar{A}B + A\bar{B}C$ is the set of variables A, B, C.

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⇒ CONVERSION OF A GENERAL EXPRESSION TO SOP FORM:

Any logic expression can be changed into SOP form by applying boolean algebra techniques

For example: the expression A(B+CD) can be converted to SOP form by applying distributive law

$$A(B+CD) = AB + ACD$$

CONANICAL / STANDARD S·O·P FORM:

⇒ Each minterm is having all the variables in normal or complemented form

Ex: $Y = A\bar{B}C + \bar{A}BC$

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MINIMAL SOP FORM:-

Each minterm doesn't have all the variables in normal or complemented form.

EX: $Y = A + B\bar{C} + A\bar{B}$

IMP POINTS:

⇒ S.O.P is written when the function is high

⇒ Standard / conanical S.O.P form is written directly from truth table.

⇒ P.O.S is written when the function is low.

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EXAMPLE 4-12

convert each of the following Boolean expressions to SOP form:

(a) $AB + B(CD+EF)$

Sol:

$\Rightarrow AB + B(CD+EF)$

$\Rightarrow AB + BCD + BEF$

↳ minimal S·O·P form //

(b) $(A+B)(B+C+D)$

Sol:

$\Rightarrow (A+B)(B+C+D)$

$\Rightarrow AB + AC + AD + BB + BC + BD$ //

↳ minimal S·O·P form

(c) $\overline{\overline{(A+B)}+C}$

Sol:-

$\Rightarrow \overline{\overline{(A+B)}+C}$

$\Rightarrow \overline{\overline{(A+B)} \cdot \overline{C}}$

$\Rightarrow (A+B) \cdot \overline{C}$

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=> $A\bar{C} + B\bar{C}$ // → minimal S·O·P form

Related Problem

Q. convert $\bar{A}B\bar{C} + (A+\bar{B})(B+\bar{C}+A\bar{B})$ to SOP form.

Sol:

=> $\bar{A}B\bar{C} + (A+\bar{B})(B+\bar{C}+A\bar{B})$

=> $\bar{A}B\bar{C} + AB + A\bar{C} + AA\bar{B} + B\bar{B} + \bar{B}\bar{C} + \bar{B}\bar{B}A$

=> $\bar{A}B\bar{C} + AB + A\bar{C} + A\bar{B} + \bar{B}\bar{C} + A\bar{B}$

=> $\bar{A}B\bar{C} + AB + A\bar{C} + A\bar{B} + \bar{B}\bar{C}$

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CONVERSION OF MINIMAL SOP FORM TO STANDARD SOP FORM:

EXAMPLE: 4-13

convert the following Boolean expression into standard SOP form:

$$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$$

Sol:

Step: 01

There are four variables A, B, C, D in the expression

Step: 02

m_1	m_2	m_3
A ✓	A ✓	A ✓
B ✓	B ✓	B ✓
C ✓	C ✗	C ✓
D ✗	D ✗	D ✓

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Step: 03

$\Rightarrow A\bar{B}C \cdot 1 + \bar{A}\bar{B} \cdot 1 \cdot 1 + AB\bar{C}D$

$\Rightarrow A\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}(C+\bar{C})(D+\bar{D}) + AB\bar{C}D$

$\Rightarrow A\bar{B}CD + A\bar{B}C\bar{D} + (\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C})(D+\bar{D}) + AB\bar{C}D$

$\Rightarrow A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

$\Rightarrow A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$ //

↳ Standard/ conanical S·O·P expression

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RELATED PROBLEM

convert the expression $W\bar{X}Y + \bar{X}Y\bar{Z} + WX\bar{Y}$ to standard SOP form

Sol:

Step:01

There are four variables in the expression: W, X, Y, Z

Step:02

m_1	m_2	m_3
W ⌐	W X	W ⌐
X ⌐	X ⌐	X ⌐
Y ⌐	Y ⌐	Y ⌐
Z X	Z ⌐	Z X

Step:03

$\Rightarrow W\bar{X}Y \cdot 1 + 1 \cdot \bar{X}Y\bar{Z} + WX\bar{Y} \cdot 1$

$\Rightarrow W\bar{X}Y(Z+\bar{Z}) + (W+\bar{W})\bar{X}Y\bar{Z} + WX\bar{Y}(Z+\bar{Z})$

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$= W\bar{X}Y\bar{Z} + W\bar{X}YZ + W\bar{X}\bar{Y}\bar{Z} + \bar{W}\bar{X}YZ + WX\bar{Y}Z + WX\bar{Y}\bar{Z}$

$= W\bar{X}YZ + W\bar{X}Y\bar{Z} + \bar{W}\bar{X}YZ + WX\bar{Y}Z + WX\bar{Y}\bar{Z}$ //

↳ Standard S·O·P form

THE PRODUCT OF SUM : (POS FORM)

⇒ When two or more sum terms are multiplied are multiplied, then the resulting expression is Product of sum.

⇒ STANDARD / CONANICAL POS FORM:-

⇒ Each maxterm is having all the variables in normal or complemented form.

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⇒ MINIMAL POS FORM:

Each maxterm is not having all the variables in normal or complemented form.

⇒ CONVERSION OF MINIMAL POS FORM TO STANDARD/ CONANICAL POS FORM:

Example : 4-15

Q. convert the following boolean expression into standard pos form:

$(A+\bar{B}+C)(\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$

Sol:

Step: 01

There are four variables in the expression. A, B, C, D

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Step:02

M_1	M_2	M_3
A ✓	A ✗	A ✓
B ✓	B ✓	B ✓
C ✓	C ✓	C ✓
D ✗	D ✓	D ✓

Step:03

$\Rightarrow (A+\bar{B}+C+O)(O+\bar{B}+C+\bar{D})$
$(A+\bar{B}+\bar{C}+D)$

$\Rightarrow (A+\bar{B}+C+D\cdot\bar{D})(A\cdot\bar{A}+\bar{B}+C+\bar{D})$
$(A+\bar{B}+\bar{C}+D)$

$\Rightarrow (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$
$(A+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})$
$(A+\bar{B}+\bar{C}+D)$

$\Rightarrow (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$
$(\bar{A}+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$

↳ standard / conanical pos form

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RELATED PROBLEM

Convert the expression $(A+\bar{B})(B+C)$ to standard POS form.

Sol:-

=> Step:01

There are three variables in the expression : A, B, C

=> Step:02

M_1	M_2
A ✓	A X
B ✓	B ✓
C X	C ✓

Step:03

$\Rightarrow (A+\bar{B}+0)(0+B+C)$

$\Rightarrow (A+\bar{B}+C\cdot\bar{C})(A\cdot\bar{A}+B+C)$

$\Rightarrow (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C)$

↳ Standard/ conanical P.O.S form

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SOLUTION 4-6

CHECKUP

1) Identify each of the following expressions as SOP, standard SOP, POS or standard POS:

a) $AB + \bar{A}BD + \bar{A}C\bar{D}$

Sol:-

$AB + \bar{A}BD + \bar{A}C\bar{D}$ → minimal SOP form

b) $(A+\bar{B}+C)(A+B+\bar{C})$

Sol:

$(A+\bar{B}+C)(A+B+\bar{C})$ → standard/ conanical POS form

c) $\bar{A}BC + AB\bar{C}$

Sol:-

$\bar{A}BC + AB\bar{C}$ → standard / conanical SOP form

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d) $(A+\bar{C})(A+B)$

Sol:

$\Rightarrow$ $(A+\bar{C})(A+B)$ $\longrightarrow$ minimal POS form.

2. convert each SOP expression in Question 1 to Standard form.

a) $AB + \bar{A}BD + \bar{A}C\bar{D}$

Sol:

Step: 01

There are four variables in the expression : A, B, C, D

Step: 02

m_1	m_2	m_3
A ✓	A ✓	A ✓
B ✓	B ✓	B ✗
C ✗	C ✗	C ✓
D ✗	D ✓	D ✓

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Step: 03

$\Rightarrow AB \cdot 1 \cdot 1 + \bar{A}B \cdot 1 \cdot D + \bar{A} \cdot 1 \cdot C \cdot \bar{D}$

$\Rightarrow AB(C+\bar{C})(D+\bar{D}) + \bar{A}B(C+\bar{C})D$

$+ \bar{A} \cdot (B+\bar{B})C\bar{D}$

$\Rightarrow (ABC + AB\bar{C})(D+\bar{D}) + \bar{A}BCD$

$+ \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}\bar{B}C\bar{D}$

$\Rightarrow ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$

$+ \bar{A}BCD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} +$

$\bar{A}\bar{B}C\bar{D}$ //

↳ standard / conanical SOP form

$\Rightarrow$

b) $(\bar{A}BC) + (AB\bar{C})$

↳ already standard

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Q.3 convert each POS expression in Question 1 to Standard form.

(b) $(A+\bar{B}+C)(A+B+\bar{C})$

Sol:-

⇒ already in standard POS form

(d) $(A+\bar{C})(A+B)$

Sol:-

⇒ **Step: 01**

There are three variables in the expression : A, B, C

⇒ **Step: 02**

M_1	M_2
A ✓	A ✓
B ✗	B ✓
C ✓	C ✗

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Step: 03

$\Rightarrow (A+0+\bar{C})(A+B+0)$

$\Rightarrow (A+B\cdot B'+\bar{C})(A+B+C\cdot\bar{C})$

$\Rightarrow (A+B+\bar{C})(A+\bar{B}+\bar{C})(A+B+C)(A+B+\bar{C})$

$\Rightarrow (A+B+\bar{C})(A+\bar{B}+\bar{C})(A+B+C)$

↳ Standard/ conanical POS form.

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CONVERTING STANDARD

Date:

SOP TO STANDARD POS:

Example 4-17

convert the following SOP expression to an equivalent POS expression.

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Sol:-

$\Rightarrow \bar{A}\bar{B}\bar{C} = \bar{0} \cdot \bar{0}$

$\Rightarrow 000 + 010 + 011 + 101 + 111$

TRUTH TABLE

A	B	C	Y	Sum term / product term
0	0	0	1	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	$(A+B+\bar{C})$
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	0	$(\bar{A}+B+C)$
1	0	1	1	$A\bar{B}C$
1	1	0	0	$(\bar{A}+\bar{B}+C)$
1	1	1	1	ABC

STANDARD POS EXPRESSION:-

$\Rightarrow (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+C)$

↳ standard/ conanical POS expression.

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EXAMPLE 4-18

Date:

Develop a truth table for the standard SOP expression $\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$.

Sol:- $\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

001 100 111

TRUTH TABLE:

A	B	C	Y	Product term
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

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RELATED PROBLEM

Date:

Q. Create a truth table for the standard SOP expression $\bar{A}B\bar{C} + A\bar{B}C$

SOL:-

$\Rightarrow \underset{010}{\bar{A}B\bar{C}} + \underset{101}{A\bar{B}C}$

TRUTH TABLE

A	B	C	Y	Product term
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	0	
1	1	1	0	

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CONVERTING POS EXPRESSION TO TRUTH TABLE FORMAT

Example 4-19

Determine the truth table for the following standard POS expression:

$$(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

Sol:-

$$\Rightarrow (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

(0 0 0) (0 1 0) (0 1 1) (1 0 1) (1 1 0)

⇒ TRUTH TABLE

A	B	C	Y	Sum term
0	0	0	0	$A+B+C$
0	0	1	1	
0	1	0	0	$A+\bar{B}+C$
0	1	1	0	$A+\bar{B}+\bar{C}$
1	0	0	1	
1	0	1	0	$\bar{A}+B+\bar{C}$
1	1	0	0	$\bar{A}+\bar{B}+C$
1	1	1	1	

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RELATED PROBLEM

Date:

Q. Develop a truth table for the following standard POS expression:

$(A+\bar{B}+C)(A+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

Sol:-

$\Rightarrow (A+\bar{B}+C)(A+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$\quad\quad 0\ 1\ 0\quad 0\ 0\ 1\quad 1\ 1\ 1$

=> TRUTH TABLE:-

A	B	C	Y	Sum term
0	0	0	1	
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	0	$\bar{A}+\bar{B}+\bar{C}$

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=> DETERMINING STANDARD EXPRESSION FROM TRUTH TABLE:

Example 4-20

From the truth table determine the standard SOP expression and equivalent standard POS expression.

A	B	C	O
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

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Date:

STANDARD SOP EXPRESSION:

$$O = \bar{A} \cdot B \cdot C + A \cdot \bar{B} \cdot \bar{C} + A \cdot B \cdot \bar{C} + A \cdot B \cdot C$$

STANDARD POS EXPRESSION:

$$O = (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+\bar{C})$$

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SECTION 4-6

Date:

STANDARD FORMS OF BOOLEAN EXPRESSIONS

Q.23 convert the following expressions to sum-of-product (SOP) form:

(a) $(A+B)(C+\bar{B})$

Sol:

$\Rightarrow (A+B)(C+\bar{B})$

$\Rightarrow AC + A\bar{B} + BC + B\bar{B}$

$\Rightarrow AC + A\bar{B} + BC \longrightarrow$ minimal S.O.P expression

(b) $(A+\bar{B}C)C$

Sol:

$\Rightarrow (A+\bar{B}C)C$

$\Rightarrow AC + \bar{B}CC$

$\Rightarrow AC + \bar{B}C \longrightarrow$ minimal S.O.P expression

c) $(A+C)(AB+AC)$

Sol:

$\Rightarrow (A+C)(AB+AC)$

$\Rightarrow AAB + AAC + ABC + ACC$

$\Rightarrow AB + AC + ABC + AC$

$\Rightarrow AB + AC + ABC$

$\Rightarrow AB + AC \longrightarrow$ minimal S.O.P expression

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Q.24 convert the following expressions to sum-of-product (SOP) forms:

(a) $AB + CD(A\bar{B} + CD)$

Sol:

$\Rightarrow AB + CD(A\bar{B} + CD)$

$\Rightarrow AB + A\bar{B}CD + CD$

$\Rightarrow AB + CD \longrightarrow$ minimal S.O.P expression

(b) $AB(\bar{B}\bar{C} + BD)$

Sol:-

$\Rightarrow AB(\bar{B}\bar{C} + BD)$

$\Rightarrow AB\bar{B}\bar{C} + ABBD$

$\Rightarrow ABD \longrightarrow$ minimal S.O.P expression

(c) $A + B[AC + (B + \bar{C})D]$

Sol:-

$\Rightarrow A + B[AC + BD + \bar{C}D]$

$\Rightarrow A + ABC + BD + B\bar{C}D$

$\Rightarrow A + BD \longrightarrow$ minimal S.O.P expression.

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Q.25

Date:

Define the domain of each SOP expression in Q.23 and convert the expression to standard SOP expression.

(a) $(A+B)(C+\bar{B})$

Sol:-

$\Rightarrow (A+B)(C+\bar{B})$

$\Rightarrow AC + A\bar{B} + BC + B\bar{B}$

$\Rightarrow AC + A\bar{B} + BC \rightarrow$ minimal S.O.P expression

STEP : 01

⇒ DOMAIN OF EXPRESSION:

There are three variables in the expression. ∴ A, B, C

STEP : 02

m_1	m_2	m_3
A ✓	A ✓	A ✗
B ✗	B ✓	B ✓
C ✓	C ✗	C ✓

STEP 03:

$\Rightarrow A \cdot 1 \cdot C + A\bar{B} \cdot 1 + 1 \cdot BC$

$\Rightarrow A(B+\bar{B})C + A\bar{B}(C+\bar{C}) + (A+\bar{A})BC$

$\Rightarrow ABC + A\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}BC$

$\Rightarrow ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$ ↳ standard S.O.P expression

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(b) $(A+\bar{B}C)C$

Sol:-

$\Rightarrow (A+\bar{B}C)C$

$\Rightarrow AC+\bar{B}CC$

$\Rightarrow AC+\bar{B}C \rightarrow$ minimal S·O·P expression.

STEP :01

=> DOMAIN OF EXPRESSION:-

There are three variables in the expression A, B, C

STEP : 02

m_1	m_2
A ✓	A ✗
B ✗	B ✓
C ✓	C ✓

STEP : 03

$\Rightarrow A\cdot 1\cdot C + 1\cdot \bar{B}C$

$\Rightarrow A(B+\bar{B})C + (A+\bar{A})\bar{B}C$

$\Rightarrow ABC + A\bar{B}C + A\bar{B}C + \bar{A}\bar{B}C$

$\Rightarrow ABC + A\bar{B}C + \bar{A}\bar{B}C$

↳ standard S·O·P expression.

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(C) (A+C)(AB+AC)

Sol:

⇒ (A+C)(AB+AC)
⇒ AAB + AAC + ABC + ACC
⇒ AB + AC + ABC + AC
⇒ AC + AB ⟶ minimal S·O·P expression

STEP: 01

⇒ DOMAIN OF EXPRESSION:-

There are three variables in the expression A, B, C

STEP: 02

m_1	m_2
A ✓	A ✓
B ✗	B ✓
C ✓	C ✗

STEP: 03

⇒ A·1·C + A·B·1
⇒ A·(B+$\bar{B}$)·C + AB(C+$\bar{C}$)
⇒ ABC + A$\bar{B}$C + ABC + AB$\bar{C}$
⇒ ABC + A$\bar{B}$C + AB$\bar{C}$
↳ standard SOP expression

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Q.26

Date:

convert each SOP expression in problem 24 to standard SOP form.

(a) $AB + CD(A\bar{B} + CD)$

Sol:-

$\Rightarrow AB + A\bar{B}CD + CD$

$\Rightarrow AB + CD \rightarrow$ minimal SOP expression

STEP : 01

$\Rightarrow$ DOMAIN OF EXPRESSION:-

There are four variables in the expression. A, B, C, D

STEP : 02

m_1	m_2
A ✓	A X
B ✓	B X
C X	C ✓
D X	D ✓

STEP : 03

$\Rightarrow AB \cdot 1 \cdot 1 + 1 \cdot 1 \cdot CD$

$\Rightarrow AB(C+\bar{C})(D+\bar{D}) + (A+\bar{A})(B+\bar{B})CD$

$\Rightarrow (ABC + AB\bar{C})(D+\bar{D}) + (ACD + \bar{A}CD)(B+\bar{B})$

$\Rightarrow ABCD + AB\bar{C}D + ABC\bar{D} + AB\bar{C}\bar{D} + ABCD + \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD$

$\Rightarrow ABCD + AB\bar{C}D + ABC\bar{D} + AB\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD$

↳ standard SOP expression

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Date:

b) $AB(\bar{B}\bar{C} + BD)$

Sol:-

=> $AB(\bar{B}\bar{C} + BD)$

=> $AB\bar{B}\bar{C} + ABD$

=> ABD → minimal SOP expression

=> STEP:01

=> DOMAIN OF EXPRESSION:-

There are four variables in the expression. A, B, C, D

=> STEP :02

m_1

A ✓

B ✓

C ✗

D ✓

=> STEP:03

=> $AB \cdot 1 \cdot D$

=> $AB(C + \bar{C})D$

=> $ABCD + AB\bar{C}D$ → standard SOP expression.

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c) $A+B[AC+(B+\bar{C})D]$

Sol:-

$\Rightarrow A+B[AC+BD+\bar{C}D]$

$\Rightarrow A+ABC+BD+B\bar{C}D$

$\Rightarrow A+BD \rightarrow$ minimal SOP expression

STEP : 01

⇒ DOMAIN OF EXPRESSION:-

There are four variables in the expression
A, B, C, D

⇒ STEP : 02

m_1	m_2
A ✓	A X
B X	B ✓
C X	C X
D X	D ✓

⇒ STEP : 03

$\Rightarrow A\cdot1\cdot1\cdot1+1\cdot B\cdot1\cdot D$

$\Rightarrow A(B+\bar{B})(C+\bar{C})(D+\bar{D})+(A+\bar{A})B(C+\bar{C})D$

$\Rightarrow (AB+A\bar{B})(C+\bar{C})(D+\bar{D})+(ABD+\bar{A}BD)(C+\bar{C})$

$\Rightarrow (ABC+A\bar{B}C+AB\bar{C}+A\bar{B}\bar{C})(D+\bar{D})+ABCD+AB\bar{C}D$
$+\bar{A}BCD+\bar{A}B\bar{C}D$

$\Rightarrow ABCD+A\bar{B}CD+AB\bar{C}D+A\bar{B}\bar{C}D+ABC\bar{D}+A\bar{B}C\bar{D}$
$+AB\bar{C}\bar{D}+A\bar{B}\bar{C}\bar{D}+ABCD+AB\bar{C}D+\bar{A}BCD+\bar{A}B\bar{C}D$

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$\Rightarrow A\bar{B}CD + ABCD + AB\bar{C}D + A\bar{B}\bar{C}D + ABC\bar{D}$
$+ A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}D$

↳ standard S·O·P expression.

Q·29

convert each standard SOP expression in problem 25 to standard POS form.

(a) $ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$

Sol:-

$\Rightarrow$ $ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$ → standard SOP expression
111 101 100 011

TRUTH TABLE

A	B	C	Y	Sum term
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	0	$\bar{A}+\bar{B}+C$
1	1	1	1	

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$Y = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$ → standard pos expression

(b) $ABC + A\bar{B}C + \bar{A}\bar{B}C$

Sol:-

$\Rightarrow$ $ABC + A\bar{B}C + \bar{A}\bar{B}C$ → standard sop expression

111 101 001

TRUTH TABLE

A	B	C	Y	Sum term
0	0	0	0	$A+B+C$
0	0	1	1	
0	1	0	0	$A+\bar{B}+C$
0	1	1	0	$A+\bar{B}+\bar{C}$
1	0	0	0	$\bar{A}+B+C$
1	0	1	1	
1	1	0	0	$\bar{A}+\bar{B}+C$
1	1	1	1	

$Y = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)$
$(\bar{A}+\bar{B}+C)$ → standard pos expression

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c) $ABC + A\bar{B}C + AB\bar{C}$

Sol:-

$\Rightarrow ABC + A\bar{B}C + AB\bar{C}$ → standard SOP expression

111 101 110

TRUTH TABLE

A	B	C	Y	Sum term
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	0	$A+\bar{B}+\bar{C}$
1	0	0	0	$\bar{A}+B+C$
1	0	1	1	
1	1	0	1	
1	1	1	1	

$Y = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)$ → standard POS expression.

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Q.30 Date:

convert each standard SOP expression in problem 26 to standard POS form.

(a) $ABCD + AB\bar{C}D + ABC\bar{D} + AB\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD$

Sol:-

=> $ABCD + AB\bar{C}D + ABC\bar{D} + AB\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD$
1111 1101 1110 1100 0111 1011 0011

A	B	C	D	Y	Sumterm
0	0	0	0	0	$A+B+C+D$
0	0	0	1	0	$A+B+C+\bar{D}$
0	0	1	0	0	$A+B+\bar{C}+D$
0	0	1	1	1	
0	1	0	0	0	$A+\bar{B}+C+D$
0	1	0	1	0	$A+\bar{B}+C+\bar{D}$
0	1	1	0	0	$A+\bar{B}+\bar{C}+D$
0	1	1	1	1	
1	0	0	0	0	$\bar{A}+B+C+D$
1	0	0	1	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

$Y = (A+B+C+D)\cdot(A+B+C+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+C+D)$
$(A+\bar{B}+C+\bar{D})\cdot(A+\bar{B}+\bar{C}+D)\cdot(\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})$
$(\bar{A}+B+\bar{C}+D) \rightarrow$ standard POS expression

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(c) $ABCD + A\bar{B}CD + AB\bar{C}D + A\bar{B}\bar{C}D + ABC\bar{D} + A\bar{B}C\bar{D}$
$+ AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}D$

Sol:-

$ABCD + A\bar{B}CD + AB\bar{C}D + A\bar{B}\bar{C}D + ABC\bar{D} + A\bar{B}C\bar{D}$
$+ AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}D \rightarrow$ standard SOP expression.

A	B	C	D	Y	Sumterm
0	0	0	0	0	$A+B+C+D$
0	0	0	1	0	$A+B+C+\bar{D}$
0	0	1	0	0	$A+B+\bar{C}+D$
0	0	1	1	0	$A+B+\bar{C}+\bar{D}$
0	1	0	0	0	$A+\bar{B}+C+D$
0	1	0	1	1	
0	1	1	0	0	$A+\bar{B}+\bar{C}+D$
0	1	1	1	1	
1	0	0	0	1	
1	0	0	1	1	
1	0	1	0	1	
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

$Y = (A+B+C+D)\cdot(A+B+C+\bar{D})\cdot(A+B+\bar{C}+D)\cdot$
$(A+B+\bar{C}+\bar{D})\cdot(A+\bar{B}+C+D)\cdot(A+\bar{B}+\bar{C}+D)$

↳ standard POS expression.

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b) $ABCD + AB\bar{C}D$

Sol:-

$ABCD + AB\bar{C}D$ → standard SOP expression
1111 1101

A	B	C	D	Y	Sumterm
0	0	0	0	0	$A+B+C+D$
0	0	0	1	0	$A+B+C+\bar{D}$
0	0	1	0	0	$A+B+\bar{C}+D$
0	0	1	1	0	$A+B+\bar{C}+\bar{D}$
0	1	0	0	0	$A+\bar{B}+C+D$
0	1	0	1	0	$A+\bar{B}+C+\bar{D}$
0	1	1	0	0	$A+\bar{B}+\bar{C}+D$
0	1	1	1	0	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	0	$\bar{A}+B+C+D$
1	0	0	1	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	0	$\bar{A}+B+\bar{C}+\bar{D}$
1	1	0	0	0	$\bar{A}+\bar{B}+C+D$
1	1	0	1	1	
1	1	1	0	0	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	1	

$Y = (A+B+C+D)\cdot(A+B+C+\bar{D})\cdot(A+B+\bar{C}+D)\cdot(A+B+\bar{C}+\bar{D})$
$(A+\bar{B}+C+D)\cdot(A+\bar{B}+C+\bar{D})\cdot(A+\bar{B}+\bar{C}+D)\cdot(A+\bar{B}+\bar{C}+\bar{D})$
$(\bar{A}+B+C+D)\cdot(\bar{A}+B+C+\bar{D})\cdot(\bar{A}+B+\bar{C}+D)\cdot(\bar{A}+B+\bar{C}+\bar{D})$
$(\bar{A}+\bar{B}+\bar{C}+D)$

↳ standard POS expression.

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SECTION 4-7

Date:

BOOLEAN EXPRESSIONS AND TRUTH TABLES

Q-31

Develop a truth table for each of the following standard SOP expression

(a) $\bar{A}BC + A\bar{B}\bar{C} + ABC$

Sol:

$\bar{A}BC + A\bar{B}\bar{C} + ABC$

1 0 1 0 1 0 1 1 1

TRUTH TABLE

A	B	C	Y	Product term
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	1	$\bar{A}BC$
1	1	0	0	
1	1	1	1	ABC

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D) $\bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z} + X\bar{Y}Z + \bar{X}YZ$

Sol:-

$\bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z} + X\bar{Y}Z + \bar{X}YZ$

000 001 110 101 011

TRUTH TABLE:

X	Y	Z	O	Product term
0	0	0	1	$\bar{X}\bar{Y}\bar{Z}$
0	0	1	1	$\bar{X}\bar{Y}Z$
0	1	0	0	
0	1	1	1	$\bar{X}YZ$
1	0	0	0	
1	0	1	1	$X\bar{Y}Z$
1	1	0	1	$XY\bar{Z}$
1	1	1	0	

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Q·32 Date:

Develop a truth table for each of the following standard SOP expression:

a) $\bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D}$

Sol:-

$\bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D}$

0101 0110 1001 0000

TRUTH TABLE

A	B	C	D	Y	Product term
0	0	0	0	1	$\bar{A}B\bar{C}\bar{D}$
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\bar{A}B\bar{C}D$
0	1	1	0	1	$\bar{A}BC\bar{D}$
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	$A\bar{B}\bar{C}D$
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	

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b) $WXYZ + WXY\bar{Z} + \bar{W}XYZ + W\bar{X}YZ + WX\bar{Y}Z$

Sol:-

$\Rightarrow WXYZ + WXY\bar{Z} + \bar{W}XYZ + W\bar{X}YZ + WX\bar{Y}Z$

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TRUTH TABLE

W	X	Y	Z	O	Product term
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	1	$\bar{W}XYZ$
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	1	$W\bar{X}YZ$
1	1	0	0	0	
1	1	0	1	1	$WX\bar{Y}Z$
1	1	1	0	1	$WXY\bar{Z}$
1	1	1	1	1	$WXYZ$

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Q-33 Date:

Q. Develop a truth table for each of the SOP expression:

a) $\bar{A}B + AB\bar{C} + \bar{A}\bar{C} + A\bar{B}C$

Sol:-

$\Rightarrow \bar{A}B + AB\bar{C} + \bar{A}\bar{C} + A\bar{B}C$

↳ minimal SOP expression

converting minimal SOP into standard SOP

STEP:01

=> There are three variables in the SOP expression; A, B, C

STEP:02

m_1	m_2	m_3	m_4
A ✓	A ✓	A ✓	A ✓
B ✓	B ✓	B ✗	B ✓
C ✗	C ✓	C ✓	C ✓

STEP:03

$\Rightarrow \bar{A}B \cdot 1 + AB\bar{C} + \bar{A} \cdot 1 \cdot \bar{C} + A\bar{B}C$

$\Rightarrow \bar{A}B(C+\bar{C}) + AB\bar{C} + \bar{A}(B+\bar{B})\bar{C} + A\bar{B}C$

$= \bar{A}BC + \bar{A}B\bar{C} + AB\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

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$\Rightarrow \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

↳ standard SOP expression

TRUTH TABLE

A	B	C	Y	Product term
0	0	0	1	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	0	

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(b) $\bar{X} + Y\bar{Z} + WZ + X\bar{Y}Z$

Sol:

$\Rightarrow \bar{X} + Y\bar{Z} + WZ + X\bar{Y}Z$

↳ minimal SOP expression

$\Rightarrow$ convert minimal SOP into standard SOP expression

STEP : 01

DOMAIN OF THE EXPRESSION

There are four variables in the expression. W, X, Y, Z

STEP : 02

m_1	m_2	m_3	m_4
W x	W x	W ✓	W x
x ✓	x x	x x	x ✓
y x	y ✓	y x	y ✓
z x	z ✓	z ✓	z ✓

STEP : 03

$\Rightarrow 1 \cdot \bar{X} \cdot 1 \cdot 1 + 1 \cdot 1 \cdot Y\bar{Z} + W \cdot 1 \cdot 1 \cdot Z + 1 \cdot X\bar{Y}Z$

$\Rightarrow (W+\bar{W})\bar{X}(Y+\bar{Y})(Z+\bar{Z}) + (W+\bar{W})(X+\bar{X})Y\bar{Z}$
$+ W(X+\bar{X})(Y+\bar{Y})Z + (W+\bar{W})X\bar{Y}Z$

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Date:

$= (W\bar{X} + \bar{W}\bar{X})(Y+\bar{Y})(Z+\bar{Z}) + (W+\bar{W})(XY\bar{Z} + \bar{X}Y\bar{Z})$
$+ (WX + \bar{W}\bar{X})(Y+\bar{Y})Z + \bar{W}X\bar{Y}Z + \bar{W}X\bar{Y}Z$

$\Rightarrow (W\bar{X}Y + W\bar{X}\bar{Y} + \bar{W}\bar{X}Y + \bar{W}\bar{X}\bar{Y})(Z+\bar{Z})$

$+ WXY\bar{Z} + W\bar{X}Y\bar{Z} + \bar{W}XY\bar{Z} + \bar{W}\bar{X}Y\bar{Z} +$
$WXZY + WX\bar{Y}Z + W\bar{X}YZ + W\bar{X}\bar{Y}Z + WX\bar{Y}Z +$
$\bar{W}X\bar{Y}Z$

$= W\bar{X}YZ + W\bar{X}\bar{Y}Z + \bar{W}\bar{X}YZ + \bar{W}\bar{X}\bar{Y}Z +$
$W\bar{X}Y\bar{Z} + W\bar{X}\bar{Y}\bar{Z} + \bar{W}\bar{X}Y\bar{Z} + \bar{W}\bar{X}\bar{Y}\bar{Z} + WXY\bar{Z} +$
$W\bar{X}Y\bar{Z} + \bar{W}XY\bar{Z} + \bar{W}XY\bar{Z} + WXYZ + WX\bar{Y}Z$
$+ W\bar{X}\bar{Y}Z + W\bar{X}\bar{Y}Z + WX\bar{Y}Z + \bar{W}X\bar{Y}Z$

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$= W\bar{X}YZ + W\bar{X}\bar{Y}Z + \bar{W}\bar{X}YZ + \bar{W}\bar{X}\bar{Y}Z + W\bar{X}Y\bar{Z}$
$+ W\bar{X}\bar{Y}\bar{Z} + \bar{W}\bar{X}Y\bar{Z} + \bar{W}\bar{X}\bar{Y}\bar{Z} + WXY\bar{Z} + \bar{W}XY\bar{Z}$
$+ WX\bar{Y}Z + WXYZ + \bar{W}X\bar{Y}Z$

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TRUTH TABLE

Date:

W	X	Y	Z	O	Product term
0	0	0	0	1	$\bar{w}\bar{x}\bar{y}\bar{z}$
0	0	0	1	0 1	$\bar{w}\bar{x}\bar{y}z$
0	0	1	0	1	$\bar{w}\bar{x}y\bar{z}$
0	0	1	1	1	$\bar{w}\bar{x}yz$
0	1	0	0	0	
0	1	0	1	0 1	$\bar{w}x\bar{y}z$
0	1	1	0	1	$\bar{w}xy\bar{z}$
0	1	1	1	0	
1	0	0	0	1	$w\bar{x}\bar{y}\bar{z}$
1	0	0	1	1	$w\bar{x}\bar{y}z$
1	0	1	0	1	$w\bar{x}y\bar{z}$
1	0	1	1	1	$w\bar{x}yz$
1	1	0	0	0	
1	1	0	1	1	$wx\bar{y}z$
1	1	1	0	1	$wxy\bar{z}$
1	1	1	1	1	$wxyz$

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Q Develop a truth table for each of the standard POS expressions:

a) $(\bar{A}+\bar{B}+\bar{C})(A+B+C)(A+\bar{B}+C)$

Sol:-

$\Rightarrow (\bar{A}+\bar{B}+\bar{C})(A+B+C)(A+\bar{B}+C)$

1 1 1 0 0 0 0 1 0

TRUTH TABLE

A	B	C	O	Sum term
0	0	0	0	$A+B+C$
0	0	1	1	
0	1	0	0	$A+\bar{B}+C$
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	0	$\bar{A}+\bar{B}+\bar{C}$

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(b)

Date:

$(\bar{A}+B+\bar{C}+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$
$(\bar{A}+B+C+\bar{D})$

Sol:-

$\Rightarrow (\bar{A}+B+\bar{C}+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$
$(\bar{A}+B+C+\bar{D})$

TRUTH TABLE

A	B	C	D	O	Sumterm
0	0	0	0	1	
0	0	0	1	1	
0	0	1	0	1	
0	0	1	1	1	
0	1	0	0	1	
0	1	0	1	0	$A+\bar{B}+C+\bar{D}$
0	1	1	0	0	$A+\bar{B}+\bar{C}+D$
0	1	1	1	1	
1	0	0	0	1	
1	0	0	1	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

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Q.35

Date:

Develop a truth table for each of the standard POS expressions:-

a) $(A+B)(A+C)(A+B+C)$

Sol:-

$\Rightarrow (A+B)(A+C)(A+B+C)$

↳ minimal POS expression

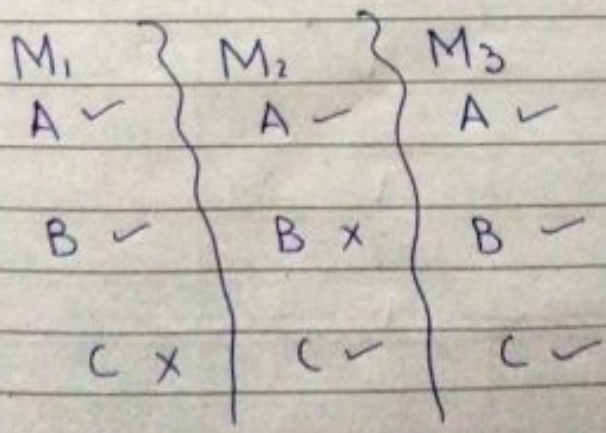
⇒ convert minimal POS to standard POS expression

STEP: 01

DOMAIN OF THE EXPRESSION:-

⇒ There are three variables in the expression A, B, C

STEP: 02



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STEP : 03

$\Rightarrow (A+B+0)(A+0+C)(A+B+C)$

$\Rightarrow (A+B+C\bar{C})(A+B\bar{B}+C)(A+B+C)$

$\Rightarrow (A+B+C)(A+B+\bar{C})(A+B+C)(A+\bar{B}+C)$
$(A+B+C)$

$\Rightarrow (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$
0 0 0 0 0 1 0 → standard POS expression

A	B	C	Y	Sum term
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	1	

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Date:

b) $(A+\bar{B})(A+\bar{B}+\bar{C})(B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

Sol:-

$\Rightarrow (A+\bar{B})(A+\bar{B}+\bar{C})(B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

↳ minimal POS expression

⇒ convert this minimal POS to standard POS expression

STEP: 01

⇒ DOMAIN OF THE EXPRESSION:-

There are four variables in the expression. A, B, C, D

STEP: 02

M_1	M_2	M_3	M_4
A ✓	A ✓	A ✗	A ✓
B ✓	B ✓	B ✓	B ✓
C ✗	C ✓	C ✓	C ✓
D ✗	D ✗	D ✓	D ✓

STEP: 03

$\Rightarrow (A+\bar{B}+0+0)(A+\bar{B}+\bar{C}+0)(0+B+C+\bar{D})$
$(\bar{A}+B+\bar{C}+D)$

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$\Rightarrow (A+\bar{B}+C\bar{C}+D\bar{D})(A+B+\bar{C}+D\bar{D})(A\bar{A}+B+C+\bar{D})$
$(\bar{A}+B+\bar{C}+D)$

$\Rightarrow [(A+\bar{B}+C)(A+B+\bar{C})+D\bar{D}](A+\bar{B}+\bar{C}+D)$
$(A+\bar{B}+\bar{C}+\bar{D})(A+B+C+\bar{D})(\bar{A}+B+C+\bar{D})$
$(\bar{A}+B+\bar{C}+D)$

$\Rightarrow (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$
$(A+\bar{B}+\bar{C}+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})$
$(A+B+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

$\Rightarrow (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$
$(A+\bar{B}+\bar{C}+\bar{D})(A+B+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

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TRUTH TABLE

Date:

A	B	C	D	Y	Sum term
0	0	0	0	1	$A+\bar{A}$
0	0	0	1	0	$A+B+C+\bar{D}$
0	0	1	0	1	
0	0	1	1	1	
0	1	0	0	0	$A+\bar{B}+C+D$
0	1	0	1	0	$A+\bar{B}+C+\bar{D}$
0	1	1	0	0	$A+\bar{B}+\bar{C}+D$
0	1	1	1	0	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	1	
1	0	0	1	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	1	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

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Q·36 Date:

From each truth table derive a standard SOP and standard POS expression.

A	B	C	X	Product term/ sum term
0	0	0	0	$A+B+C$
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	$A+\bar{B}+C$
0	1	1	0	$A+\bar{B}+\bar{C}$
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	1	$A\bar{B}C$
1	1	0	0	$\bar{A}+\bar{B}+C$
1	1	1	1	ABC

STANDARD SOP EXPRESSION:-

$X = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$

STANDARD POS EXPRESSION:

$X = (A+B+C)\cdot(A+\bar{B}+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+\bar{B}+C)$

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(b)

Date:

A	B	C	X	Product term / Sum term
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	0	$A+\bar{B}+\bar{C}$
1	0	0	0	$\bar{A}+B+C$
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

=> STANDARD SOP EXPRESSION:-

$X = A\bar{B}C + AB\bar{C} + ABC$

=> STANDARD POS EXPRESSION:-

$X = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+C)$

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(C) Date:

A	B	C	D	O	sum term/ product term
0	0	0	0	1	$\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$
0	0	0	1	1	$\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot D$
0	0	1	0	0	$(A+B+\bar{C}+D$
0	0	1	1	1	$\bar{A}\cdot\bar{B}\cdot C\cdot D$
0	1	0	0	0	$A+\bar{B}+C+D$
0	1	0	1	1	$\bar{A}\cdot B\cdot\bar{C}\cdot D$
0	1	1	0	1	$\bar{A}\cdot B\cdot C\cdot\bar{D}$
0	1	1	1	0	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	0	$\bar{A}+B+C+D$
1	0	0	1	1	$A\cdot\bar{B}\cdot\bar{C}\cdot D$
1	0	1	0	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	0	$\bar{A}+B+\bar{C}+\bar{D}$
1	1	0	0	1	$A\cdot B\cdot\bar{C}\cdot\bar{D}$
1	1	0	1	0	$(\bar{A}+\bar{B}+C+\bar{D})$
1	1	1	0	0	$(\bar{A}+\bar{B}+\bar{C}+D)$
1	1	1	1	0	$(\bar{A}+\bar{B}+\bar{C}+\bar{D})$

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Date:

⇒ STANDARD SOP EXPRESSION:-

$$O = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D} + \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot D + \bar{A}\cdot\bar{B}\cdot C\cdot D + \bar{A}\cdot B\cdot\bar{C}\cdot D + \bar{A}\cdot B\cdot C\cdot\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D}$$

⇒ STANDARD POS EXPRESSION:-

$$O = (A+B+\bar{C}+D)\cdot(A+\bar{B}+C+D)\cdot(A+\bar{B}+\bar{C}+\bar{D})\cdot(\bar{A}+B+C+D)\cdot(\bar{A}+B+\bar{C}+D)\cdot(\bar{A}+B+\bar{C}+\bar{D})\cdot(\bar{A}+\bar{B}+C+\bar{D})\cdot(\bar{A}+\bar{B}+\bar{C}+D)\cdot(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

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(d)

A	B	C	D	X	Sum term/ Product term
0	0	0	0	0	$A+B+C+D$
0	0	0	1	0	$(A+B+C+\bar{D})$
0	0	1	0	1	$\bar{A}\bar{B}C\bar{D}$
0	0	1	1	0	$A+B+\bar{C}+\bar{D}$
0	1	0	0	1	$\bar{A}B\bar{C}\bar{D}$
0	1	0	1	1	$\bar{A}B\bar{C}D$
0	1	1	0	0	$A+\bar{B}+\bar{C}+D$
0	1	1	1	1	$\bar{A}BCD$
1	0	0	0	0	$\bar{A}+B+C+D$
1	0	0	1	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	1	$A\bar{B}CD$
1	1	0	0	1	$AB\bar{C}\bar{D}$
1	1	0	1	0	$\bar{A}+\bar{B}+C+\bar{D}$
1	1	1	0	0	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	1	$ABCD$

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Date:

SOP EXPRESSION:-

$X = \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}CD + AB\bar{C}\bar{D} + ABCD$ //

POS EXPRESSION:-

$X = (A+B+C+D) \cdot (A+B+C+\bar{D}) \cdot (A+B+\bar{C}+\bar{D}) \cdot (A+\bar{B}+\bar{C}+D) \cdot (\bar{A}+B+C+D) \cdot (\bar{A}+B+C+\bar{D}) \cdot (\bar{A}+B+\bar{C}+D) \cdot (\bar{A}+\bar{B}+C+\bar{D}) \cdot (\bar{A}+\bar{B}+\bar{C}+D)$ //

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Date:

Q. Find the standard/conanical SOP form also minimal SOP form using truth table

	A	B	C	F	Product term
m_0	0	0	0	0	
m_1	0	0	1	0	
m_2	0	1	0	1	$\bar{A}B\bar{C}$
m_3	0	1	1	0	
m_4	1	0	0	1	$A\bar{B}\bar{C}$
m_5	1	0	1	1	$A\bar{B}C$
m_6	1	1	0	1	$AB\bar{C}$
m_7	1	1	1	1	ABC

STANDARD /CONANICAL SOP FORM:-

$$F = \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

MINIMAL SOP FORM:-

$F = \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

$F = \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}C + ABC + A\bar{B}\bar{C}$

$F = B\bar{C}(\bar{A}+A) + AC(B+\bar{B}) + A\bar{B}\bar{C}$

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$F = B\bar{C}(1) + A\bar{C}(1) + AB\bar{C}$

$F = B\bar{C} + A\bar{C} + AB\bar{C}$

$F = B\bar{C} + A[C + \bar{B}\bar{C}]$

$F = B\bar{C} + A[C + \bar{B}]$

$F = B\bar{C} + A\bar{C} + A\bar{B}$ (redundant term)

$$F = A\bar{B} + B\bar{C}$$

MIN TERMS:-

$F(A, B, C) = m_2 + m_4 + m_5 + m_6 + m_7$

$$F(A, B, C) = \sum m(2, 4, 5, 6, 7)$$

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Q.

Q. For the given truth table, minimize the SOP expression.

	A	B	F	Product terms
m_0	0	0	0	
m_1	0	1	1	$\bar{A}B$
m_2	1	0	0	
m_3	1	1	1	AB

STANDARD / CONANICAL SOP EXPRESSION:-

$F = \bar{A}B + AB$

Now finding the minimal SOP form:

$F = B(A+\bar{A})$

$F = B$

$F(A,B) = m_1 + m_3$ → minterms

$F(A,B) = \Sigma m(1,3)$

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Q. Simplify the expression for:

$Y(A,B) = \sum m(0,2,3)$

find the standard / conanical SOP form and also minimal SOP form.

SOL:-

A	B	Y.	Product term
0	0	1	$\bar{A}\bar{B}$
0	1	0	
1	0	1	$A\bar{B}$
1	1	1	AB

STANDARD / CONANICAL SOP FORM:-

$Y = \bar{A}\bar{B} + A\bar{B} + AB$

Now finding the minimal SOP form:

$Y = \bar{B}(A+\bar{A}) + AB$

$Y = \bar{B} + AB$

$Y = \bar{B} + A$

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Date:

Q.

Find standard / conanical POS form and also minimal POS form using truth table.

	A	B	C	Y	Sumterm
M_0	0	0	0	0	$A+B+C$
M_1	0	0	1	0	$A+B+\bar{C}$
M_2	0	1	0	0 1	
M_3	0	1	1	0	$A+\bar{B}+\bar{C}$
	1	0	0	1	
	1	0	1	1	
M_6	1	1	0	1	
M_7	1	1	1	1	

STANDARD / CONANICAL POS FORM:

$Y = (A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$

Now finding the minimal POS form:

$Y = (A+B+C\cdot\bar{C})(A+\bar{B}+\bar{C})$

$Y = (A+B)(A+\bar{B}+\bar{C})$

$Y = A + B\cdot(\bar{B}+\bar{C})$

$Y = A + B\bar{B} + B\bar{C}$

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$Y = A + B\bar{C}$

$Y = (A+B)(A+\bar{C})$

↳ minimal POS form

MAX TERMS :-

$Y(A,B,C) = M_0 \cdot M_1 \cdot M_3$

$Y(A,B,C) = \pi M(0,1,3)$

Q.

For the given truth table, minimize the P.O.S expression

	A	B	Y	Sum term
M_0	0	0	0	$A+B$
M_1	0	1	1	
M_2	1	0	0	$\bar{A}+B$
M_3	1	1	1	

STANDARD / CONANICAL POS FORM :-

$Y = (A+B)(\bar{A}+B)$

Now finding the minimal POS form:

$Y = B + A\bar{A}$

$Y = B$

↓

minimal POS form

$Y(A,B) = M_0 \cdot M_2$ → maxterm

$Y(A,B) = \pi M(0,2)$ → maxterm

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Q. For the given truth table minimize the POS expression and also write min terms and maxterms.

	A	B	Y	Sumterm/product
M_0	0	0	1	
M_1	0	1	0	$A+\bar{B}$
M_2	1	0	1	
M_3	1	1	0	$\bar{A}+\bar{B}$

STANDARD/ CONANICAL POS FORM:-

$Y = (A+\bar{B})(\bar{A}+\bar{B})$

Now finding the minimal POS form

$Y = \bar{B} + A\cdot\bar{A}$

$Y = \bar{B}$ → minimal POS form

MAX TERMS:-	MIN TERMS
$Y = M_1 \cdot M_3$	$Y = m_0 + m_2$
$Y = \Pi M(1,3)$	$Y = \Sigma m(0,2)$

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$Y(A,B,C) = \sum m(0,2,3,6,7)$ minimize the SOP expression.

A	B	C	O	Product term
0	0	0	1	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	1	$\bar{A}BC$
1	0	0	0	
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

STANDARD / CONANICAL SOP FORM:-

$O = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$ → standard/conanical SOP form

$O = \bar{A}\bar{C}(B+\bar{B}) + \bar{A}BC + AB(C+\bar{C})$

$O = A\bar{C} + \bar{A}BC + AB$

$O = \bar{A}\bar{C} + B(A+\bar{A}C)$

$O = A\bar{C} + AB + B\bar{C}$ → redundant term

$O = AB + A\bar{C}$ → minimal SOP form

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Q. $Y(A,B,C) = \Pi M(1,4,5)$ minimize the POS expression.

TRUTH TABLE

A	B	C	Y	sum term
0	0	0	1	
0	0	1	0	$(A+B+\bar{C})$
0	1	0	1	
0	1	1	1	
1	0	0	0	$\bar{A}+B+C$
1	0	1	0	$\bar{A}+B+\bar{C}$
1	1	0	1	
1	1	1	1	

STANDARD/ CONANICAL POS EXPRESSION:-

$Y = (A+B+\bar{C})\cdot(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

Now finding the minimal POS expression

$Y = (A+B+\bar{C})(\bar{A}+B+C\cdot\bar{C})$

$Y = (A+B+\bar{C})(\bar{A}+B)$

$Y = B + \bar{A}\cdot(A+\bar{C})$

$Y = B + A\bar{A} + \bar{A}\bar{C}$

$Y = B + \bar{A}\bar{C}$

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$Y = (B+\bar{A})(B+\bar{C})$

MINIMAL TO CANONICAL FORM CONVERSION

S·O·P FORM

$Y = A + B'C$

convert this minimal SOP form to canonical SOP form

SOL:-

STEP:01

⇒ There are three variables in the expression

A, B, C

STEP:02

m_1	m_2
A ✓	A ✗
B ✗	B ✓
C ✗	C ✓

STEP:03

$Y = A \cdot 1 \cdot 1 + 1 \cdot B' \cdot C$

$Y = A \cdot (B+B')(C+C') + (A+A')B'C$

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Date:

$= A \cdot [BC + BC' + B'C + B'C'] + A'B'C + AB'C$

$= ABC + ABC' + AB'C + AB'C' + A'B'C + AB'C$

$\Rightarrow$ $ABC + ABC' + AB'C + AB'C' + A'B'C$

↳ standard/ canonical SOP form

POS FORM:

$F = (A+B+C')(A'+C)$

convert this minimal POS form to canonical POS form.

SOL:-

STEP:01

=> There are three variables in the expression.

STEP:02

M_1	M_2
A ✓	A ✓
B ✓	B ✗
C ✓	C ✓

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Date:

STEP: 03

$F = (A+B+C')(A'+0+C)$

$F = (A+B+C')(A' + B \cdot B' + C)$

$F = (A+B+C')[(A'+B)(A'+B')]+C$

$F = (A+B+C')(A'+C+B)(A'+C+B')$

$F = (A+B+C')(A'+B+C)(A'+B'+C)$

↳ standard/canonical SOP form.

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* S·O·P is written when the function is high

⇒ Sum of Product (S·O·P)

S·O·P

Q. Find standard/conanical form and also minimal S·O·P form using truth table.

	A	B	C	F	
m_0	0	0	0	0	0
m_1	0	0	1	0	1
m_2	0	1	0	1	2
m_3	0	1	1	0	3
m_4	1	0	0	1	4
m_5	1	0	1	1	5
m_6	1	1	0	1	6
m_7	1	1	1	1	7

Standard/ conanical S·O·P form

↳ directly written from truth table

Sol:-

First Writing standard/ conanical S·O·P form

$$F = \bar{A}\cdot B\cdot \bar{C} + A\cdot \bar{B}\cdot \bar{C} + A\cdot \bar{B}\cdot C + A\cdot B\cdot \bar{C} + A\cdot B\cdot C$$

Now finding minimal S·O·P form:

$$F = \bar{A}\cdot B\cdot \bar{C} + A\cdot \bar{B}\,(C+\bar{C}) + AB\,(\bar{C}+C)$$

$$F = \bar{A}\cdot B\cdot \bar{C} + A\cdot \bar{B}\,(1) + AB\,(1)$$

$$F = \bar{A}\cdot B\cdot \bar{C} + A\bar{B} + AB$$

$$F = \bar{A}\cdot B\cdot \bar{C} + A\,(B+\bar{B})$$

$$F = \bar{A}\cdot B\cdot \bar{C} + A\,(1)$$

$F = \bar{A} \cdot B \cdot \bar{C} + A$

$F = (A + \bar{A})(A + B\bar{C})$

$\boxed{F = A + B\bar{C}}$ → minimal S.O.P form

Also write min terms for above Function:

$F(A, B, C) = m_2 + m_4 + m_5 + m_6 + m_7$

$\boxed{F(A, B, C) = \sum m(2, 4, 5, 6, 7)}$

⇒ CANONICAL / STANDARD SOP FORM:-

Each minterm is having all the variables in normal or complimented form

i.e: $F = \bar{A}B + A\bar{B} + \bar{A}\bar{B}$

⇒ MINIMAL SOP FORM:-

Each minterm does not have all the variables in normal or complimented form

i.e: $G = A + \bar{B}C$

For the given truth table, minimize the S·O·P expression.

A	B	F
0	0	0
0	1	1
1	0	0
1	1	1

Sol:

→ directly written from truth table

⇒ first finding the standard / canonical form

$F = A'B + AB$

Now finding the minimal S.O.P form

$F = (A' + A)B$

$F = (1) \cdot B$

$F = B$

Q. Simplify the expression for :-

$Y(A,B) = \sum m(0,2,3)$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Sol:-

=> first finding the standard / canonical S·O·P form

=> $Y = m_0 + m_2 + m_3$

=> $Y = A'B' + AB' + AB$

Now finding minimal S·O·P form

=> $Y = A'B' + AB' + AB$

$Y = A'B' + A(B' + B)$

$Y = A'B' + A(1)$

$Y = A'B' + A$

$Y = A + B'$

=> Product of Sum (P·O·S)

* product of sum is written when the output is low

* standard / conanical form is written directly from truth table

Q. Find standard / conanical P·O·S form and also minimal P·O·S form using truth table.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

=> First finding the standard/ conanical P.O.S form.

$$Y = (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+\bar{C})$$

=> Now finding the minimal P.O.S form

$$Y = [(A+B)+(C \cdot \bar{C})] \cdot (A+\bar{B}+\bar{C})$$

$$Y = [(A+B)+0] \cdot (A+\bar{B}+\bar{C})$$

$$Y = (A+B) \cdot (A+\bar{B}+\bar{C})$$

$$Y = A+(B \cdot (\bar{B}+\bar{C}))$$

$$Y = A+(B \cdot \bar{B} + B\bar{C})$$

$$Y = A + B\bar{C}$$

$$Y = (A+B) \cdot (A+\bar{C})$$

Q.

For the given truth table, minimize the P.O.S expression.

A	B	Y	
0	0	0	m_0
0	1	1	m_1
1	0	0	m_2
1	1	1	m_3

⇒ first finding the standard/ conanical P.O.S form

$$\boxed{Y = (A+B) \cdot (\bar{A}+B)}$$

Now finding the minimal P·O·S form

$Y = (A+B)(\bar{A}+B)$

$Y = B + (A \cdot \bar{A})$

$Y = B + (0)$

$\boxed{Y = B}$ → minimal Pos form

Also finding the maxterms

$Y = m_0 \cdot m_2$ $\quad$ $Y = M_0 \cdot M_2$

$Y = \Pi(m_0, m_2)$ $\quad$ $Y = \Pi(M_0, M_2)$

$Y = \Pi$ $\quad$ $\boxed{Y = \Pi M(0, 2)}$

* In S·O·P form there are more "AND" gates

* In P·O·S form there are more "OR" gates.

Q. For the given truth table minimize the POS expression. And also write min terms and max terms.

A	B	Y	
0	0	1	M_0
0	1	0	M_1
1	0	1	M_2
1	1	0	M_3

standard/conanical POS form:

$$\boxed{Y = (A + \bar{B}) \cdot (\bar{A} + \bar{B})}$$

↳ directly written from truth table

minimal POS form:

$$Y = (A + \bar{B}) \cdot (\bar{A} + \bar{B})$$

$$Y = \bar{B} + (A \cdot \bar{A})$$

$$Y = \bar{B} + 0$$

$$\boxed{Y = \bar{B}}$$

Max terms:

$$Y(A, B) = M_1 \cdot M_3$$

$$Y(A, B) = \pi(M_1, M_3)$$

$$\boxed{Y(A, B) = \pi M(1, 3)}$$

Min terms:

$Y(A,B) = m_0 + m_2$

$$\boxed{Y(A,B) = \sum m(0,2)}$$

PRACTICE PROBLEMS

Q. $Y(A,B,C) = \Sigma m(0,2,3,6,7)$
minimize the S.O.P expression

Sol:-

Truth table

	A	B	C	Z
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

⇒ first writing the standard/ conanical S·O·P form

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C} + A\cdot B\cdot\bar{C} + ABC$$

⇒ Now finding the minimal S·O·P form

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B(C+\bar{C}) + A\cdot B(C+\bar{C})$$

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B(1) + A\cdot B(1)$$

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B + A\cdot B$$

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + B(A+\bar{A})$$

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + B$$

$Y = (B+\bar{B}) \cdot (B+A\bar{C})$

$Y = B + \bar{A}\bar{C}$

Q. $Y(A,B,C) = \Pi M(1,4,5)$ minimize the P.O.S expression

Truth table

A	B	C	Y	
0	0	0	1	0
0	0	1	0	1
0	1	0	1	2
0	1	1	1	3
1	0	0	0	4
1	0	1	0	5
1	1	0	1	6
1	1	1	1	7

⇒ first writing the standard / conanical P·O·S form

$$Y = (A+B+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+\bar{C})$$

⇒ Now writing the minimal P·O·S form

$$Y = (B+A+\bar{C})\cdot(B+\bar{A}+C)\cdot(\bar{A}+B+\bar{C})$$

$$Y = [B+(A+\bar{C})\cdot(\bar{A}+C)]\cdot(\bar{A}+B+\bar{C})$$

$$Y = [B+A\bar{A}+AC+\bar{A}\bar{C}+C\bar{C}](\bar{A}+B+\bar{C})$$

$$Y = (B+0+AC+\bar{A}\bar{C}+0)(\bar{A}+B+\bar{C})$$

$$Y = (B+AC+\bar{A}\bar{C})(\bar{A}+B+\bar{C})$$

$$Y = B+[(AC+\bar{A}\bar{C})(\bar{A}+\bar{C})]$$

$$Y = B + [\bar{A}AC + \bar{A}\bar{A}\bar{C} + AC\bar{C} + \bar{A}\bar{C}\bar{C}]$$

$$Y = B + [0 + \bar{A}\bar{C} + 0 + \bar{A}\bar{C}]$$

$$Y = B + \bar{A}\bar{C}$$

$$\boxed{Y = (\bar{A} + B) \cdot (B + \bar{C})}$$

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