CHAPTER : 01

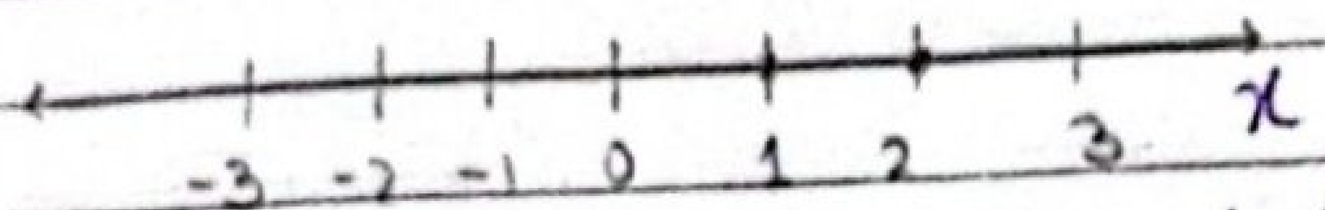
Date:

⇒ VECTOR ALGEBRA:

OBJECT:-
To understand the co-ordinate system.

How a point is formed?

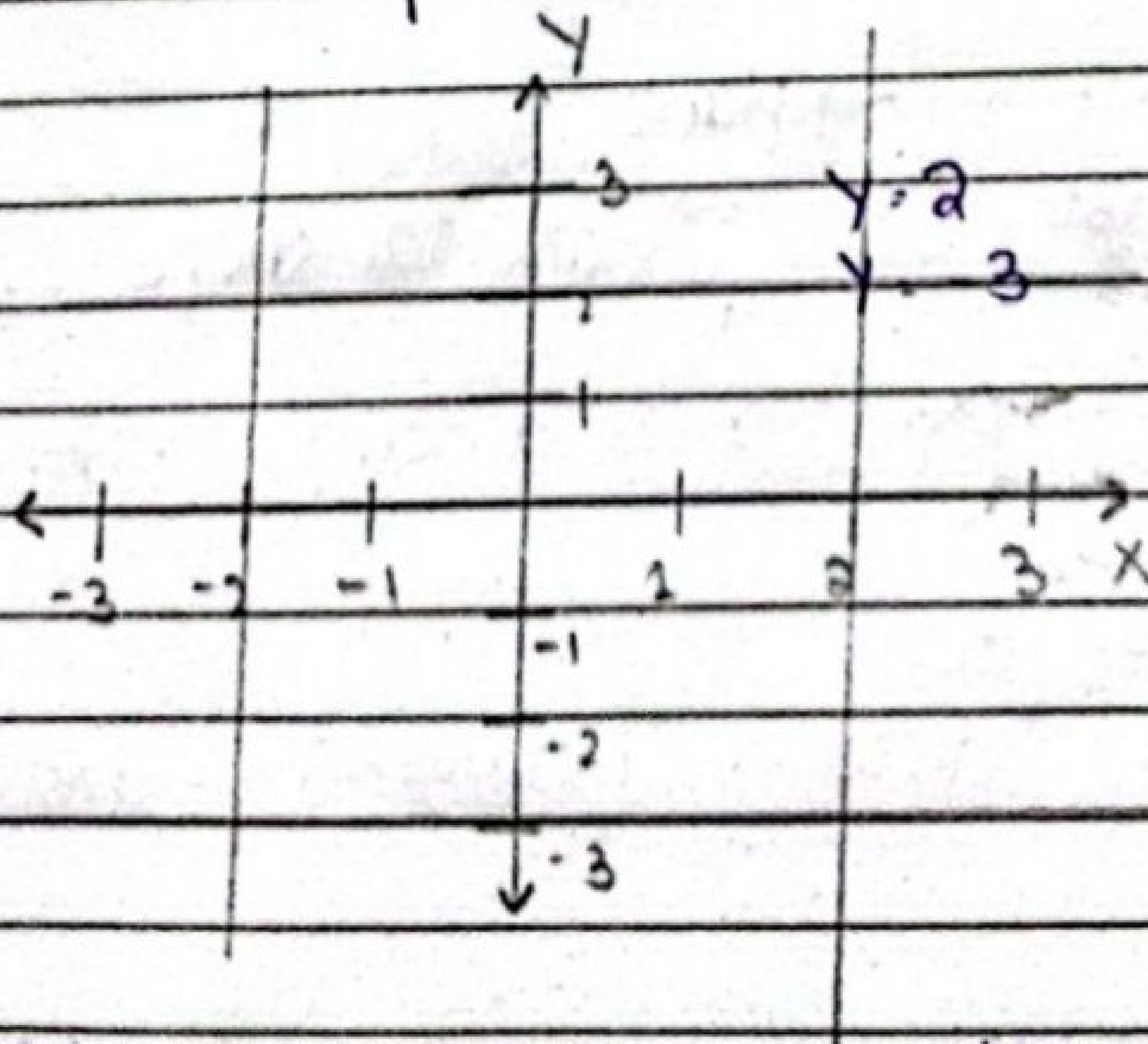
1D:-



$x = 1$
$x = 2$

$x = $ const (point is a fundamental shape)

2D:-



⇒ A straight line having infinite length and parallel to other axis.

" Intersection of two lines is a point ".

$\left.\begin{array}{l} x = \text{const} \\ y = \text{const} \end{array}\right\} \rightarrow$ fundamental shape

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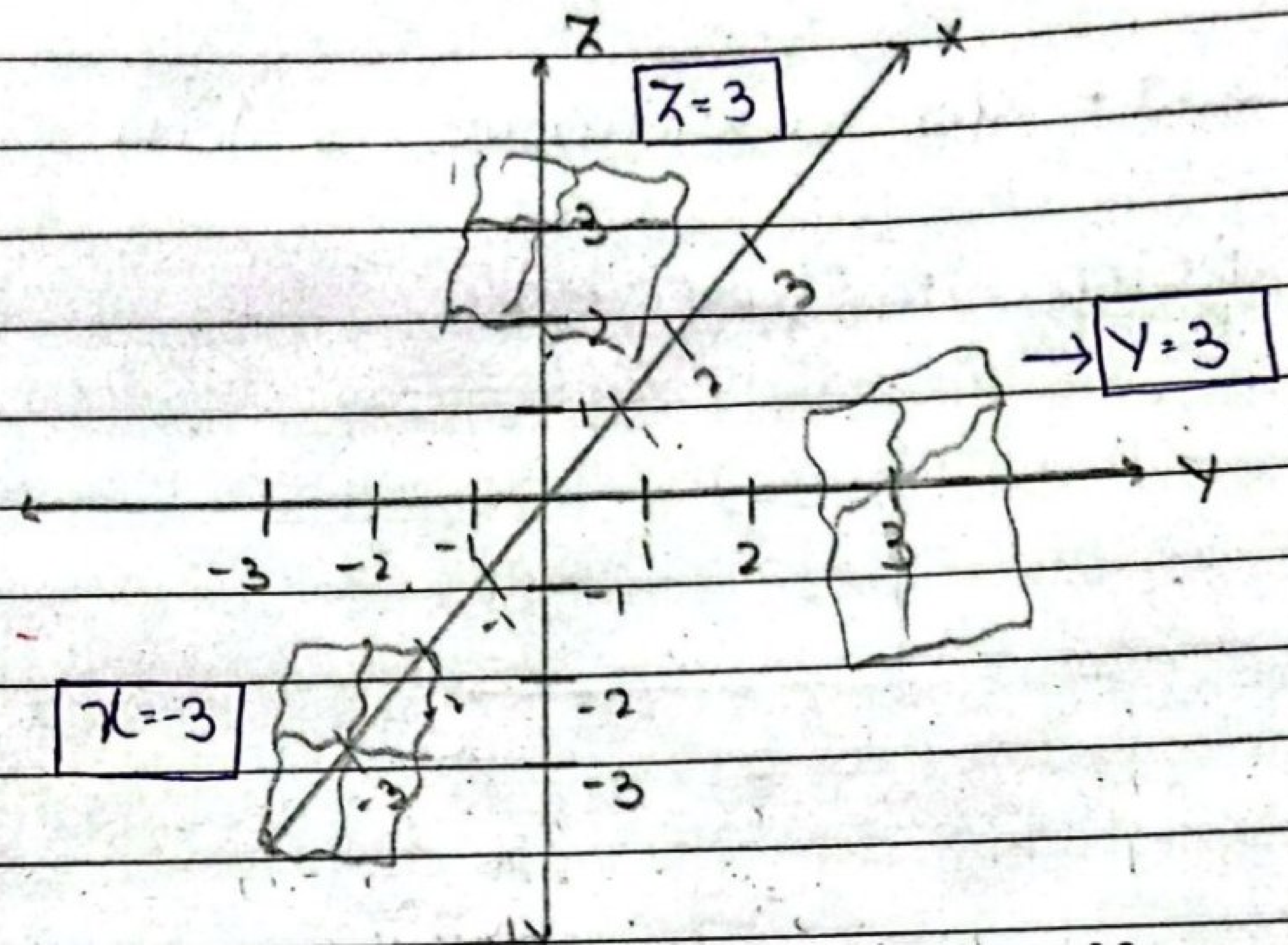
3D:-



" Intersection of two planes "

x = const
y = const
z = const
} → plane → fundamental shape

Q. Why we need co-ordinate system?

* to locate point
* to find distance between two points

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Position Vector:-
A vector directed from origin towards any point is known as position vector at that point.

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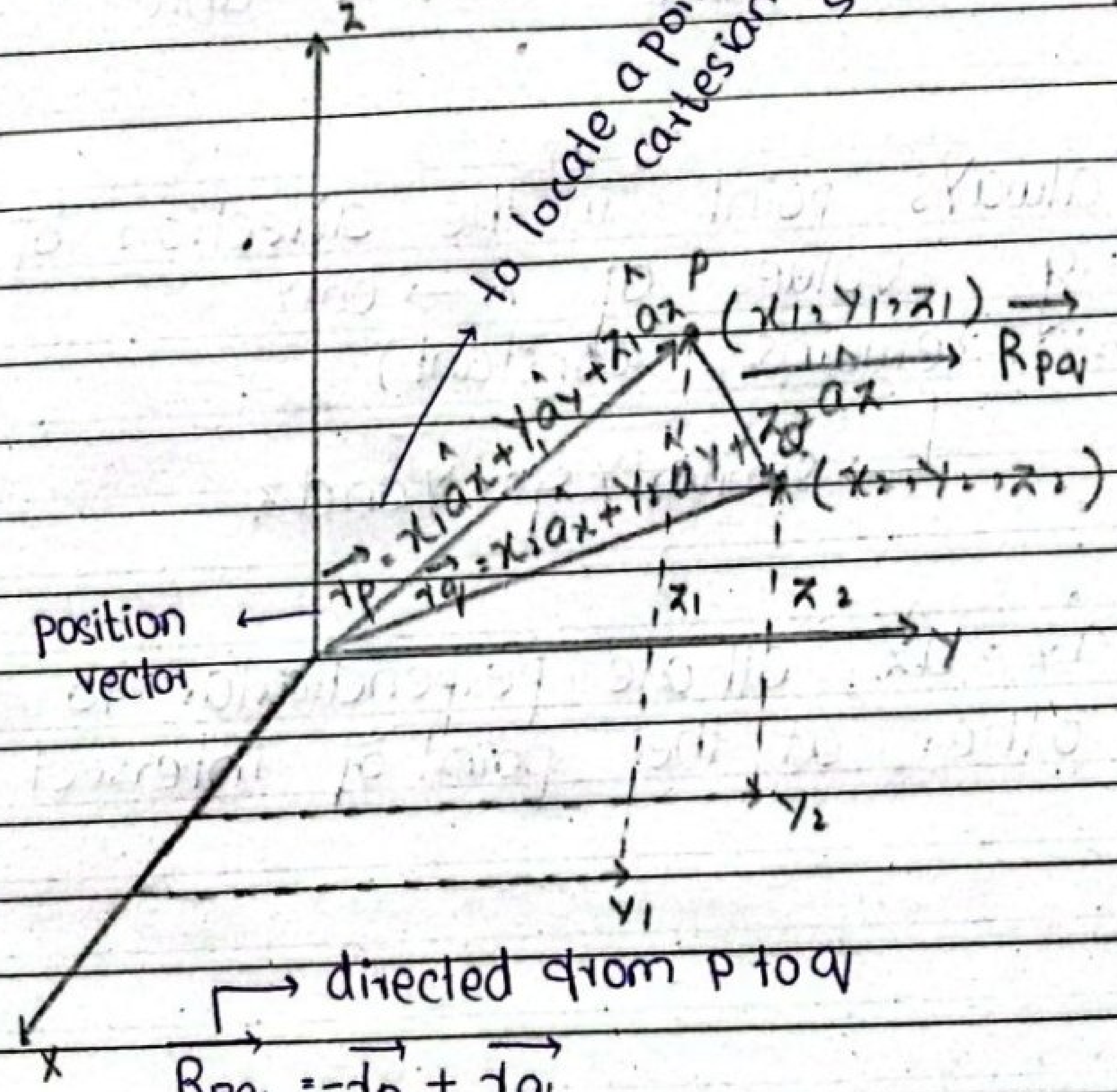
CARTESIAN OR RECTANGULAR CO-ORDINATE SYSTEM:-

$\hat{i}, \hat{j}, \hat{k}$
$\downarrow \quad \downarrow \quad \downarrow$
$\hat{a}_x, \hat{a}_y, \hat{a}_z$



$\overrightarrow{R_{pq}} = -\vec{r_p} + \vec{r_q}$ → directed from p to q

$\overrightarrow{R_{pq}} = \vec{r_q} - \vec{r_p}$

$\overrightarrow{R_{pq}} = (x_2\hat{a}_x + y_2\hat{a}_y + z_2\hat{a}_z) - (x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z)$

$\overrightarrow{R_{pq}} = (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$

$|\overrightarrow{R_{pq}}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

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$\hat{a}_{pa} \cdot \dfrac{\overrightarrow{R_{pa}}}{|\overrightarrow{R_{pa}}|}$

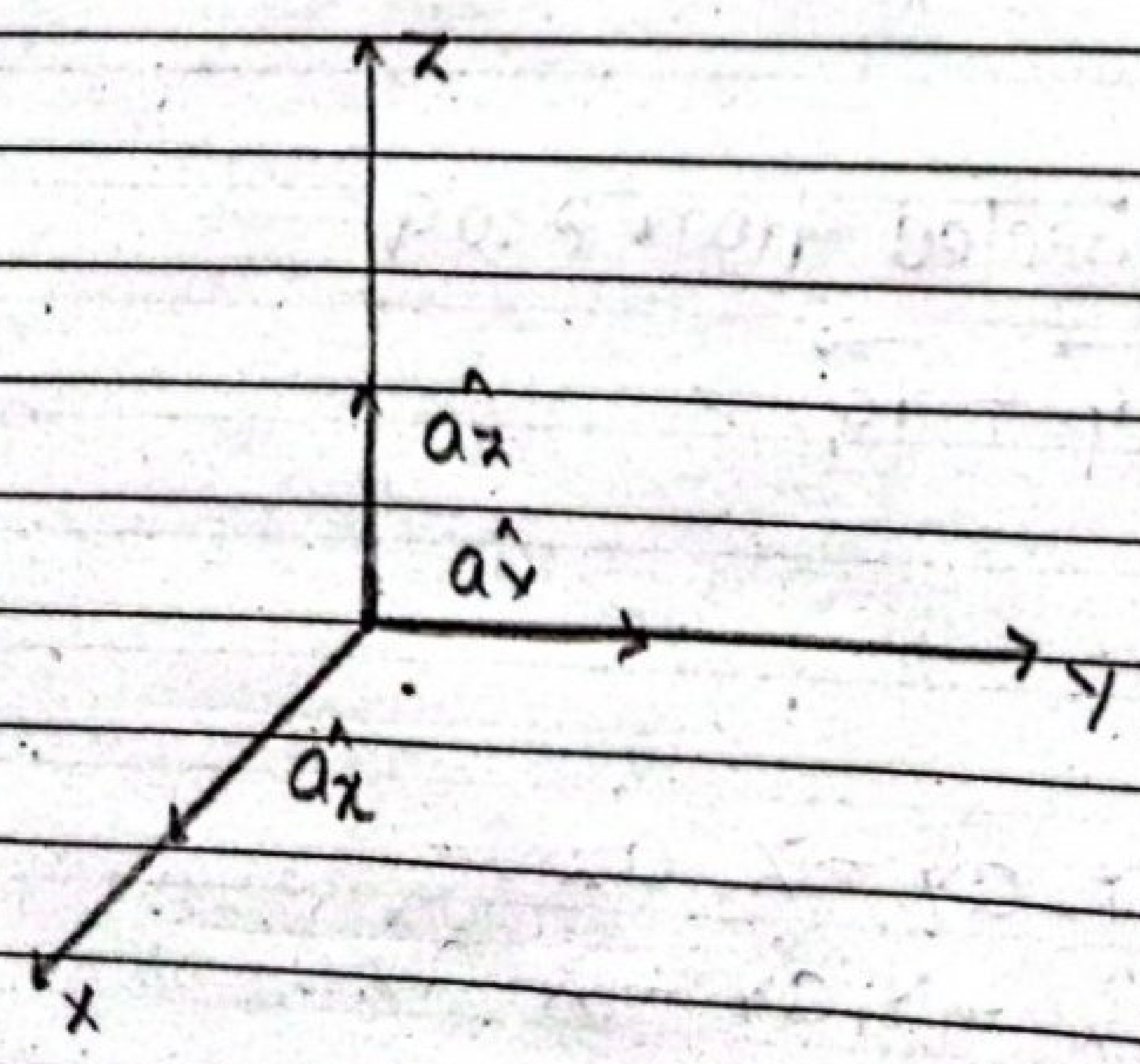
PRIMITIVE UNIT VECTORS:-

"The vector which can not be further simplified are called primitive unit vector"

⇒ $\hat{a}_x$ always point in the direction of increasing values of $x \rightarrow$ only (y, z remains constant)

↳ similarly for y and z

⇒ $\hat{a}_x, \hat{a}_y, \hat{a}_z$ all are perpendicular to each other at the point of intersect



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NUMERICAL

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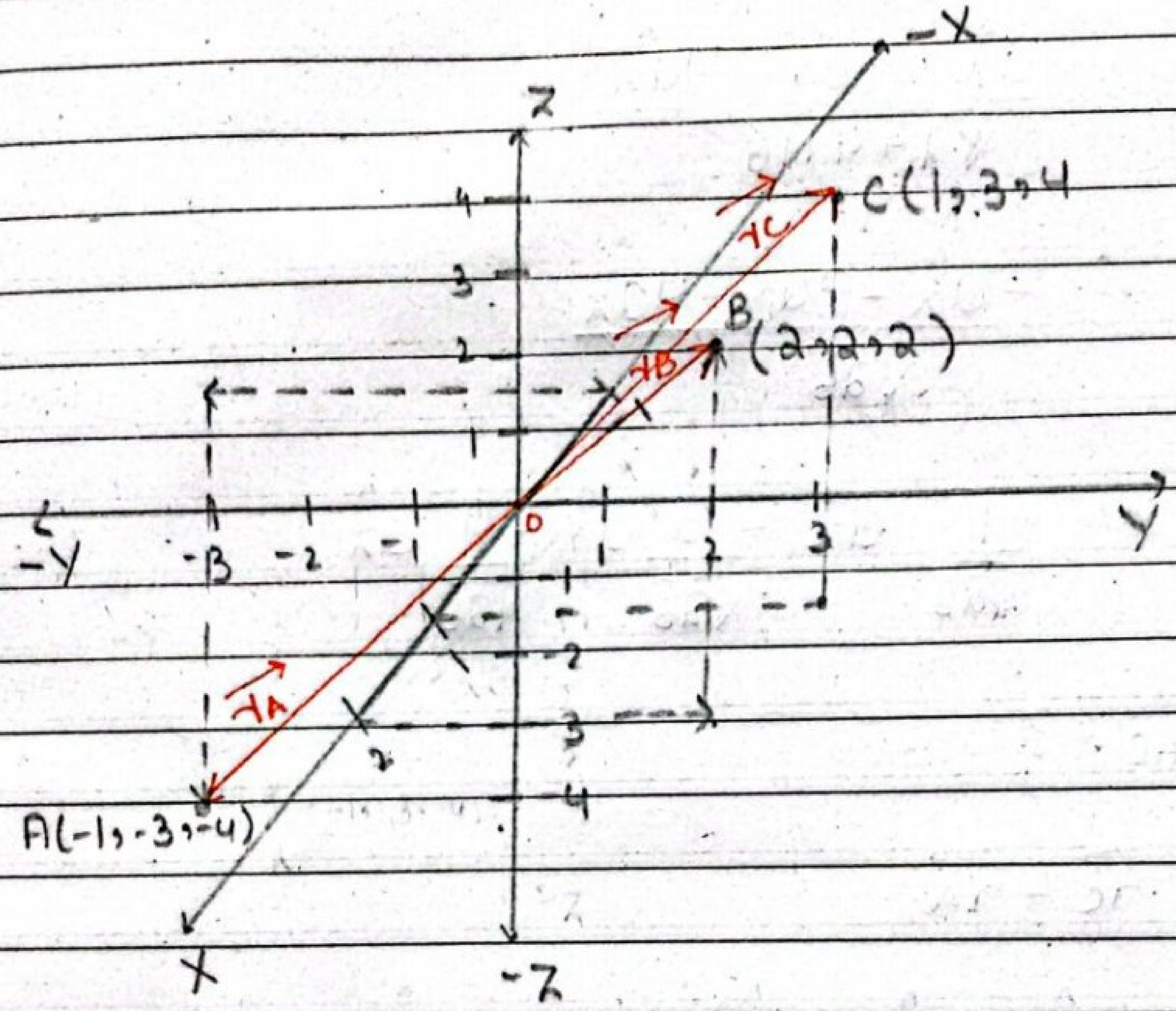
given 2 vector $\vec{r_A} = -\hat{a_x} - 3\hat{a_y} - 4\hat{a_z}$

and $\vec{r_B} = 2\hat{a_x} + 2\hat{a_y} + 2\hat{a_z}$ & point $C(1,3,4)$ (a) $\overrightarrow{R_{AB}}$ (b) $\hat{a}_A$ (c) $\hat{a}_{AC}$



(a.) $\overrightarrow{R_{AB}} = \vec{r_B} - \vec{r_A}$

$= (2\hat{a_x} + 2\hat{a_y} + 2\hat{a_z}) - (-\hat{a_x} - 3\hat{a_y} - 4\hat{a_z})$

$= (2+1)\hat{a_x} + (2+3)\hat{a_y} + (2+4)\hat{a_z}$

$\Rightarrow 3\hat{a_x} + 5\hat{a_y} + 6\hat{a_z}$

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$$\hat{a}_A = \frac{\vec{r}_A}{|\vec{r}_A|}$$

$$= \frac{-\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z}{\sqrt{(-1)^2+(-3)^2+(-4)^2}}$$

$$\Rightarrow \frac{-\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z}{\sqrt{1+9+16}}$$

$$= \frac{-\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z}{\sqrt{26}}$$

$$\hat{a}_A = \frac{-1}{\sqrt{26}}\hat{a}_x - \frac{3}{\sqrt{26}}\hat{a}_y - \frac{4}{\sqrt{26}}\hat{a}_z$$

(c) $\hat{a}_{AC} = ?$

$$\vec{R}_{AC} = \vec{r}_C - \vec{r}_A$$

$$= (\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z) - (-\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z)$$

$$= (1+1)\hat{a}_x + (3+3)\hat{a}_y + (4+4)\hat{a}_z$$

$$= 2\hat{a}_x + 6\hat{a}_y + 8\hat{a}_z$$

$$|\vec{R}_{AC}| = \sqrt{(2)^2+(6)^2+(8)^2}$$

$$= \sqrt{4+36+64}$$

$$= 2\sqrt{26}$$

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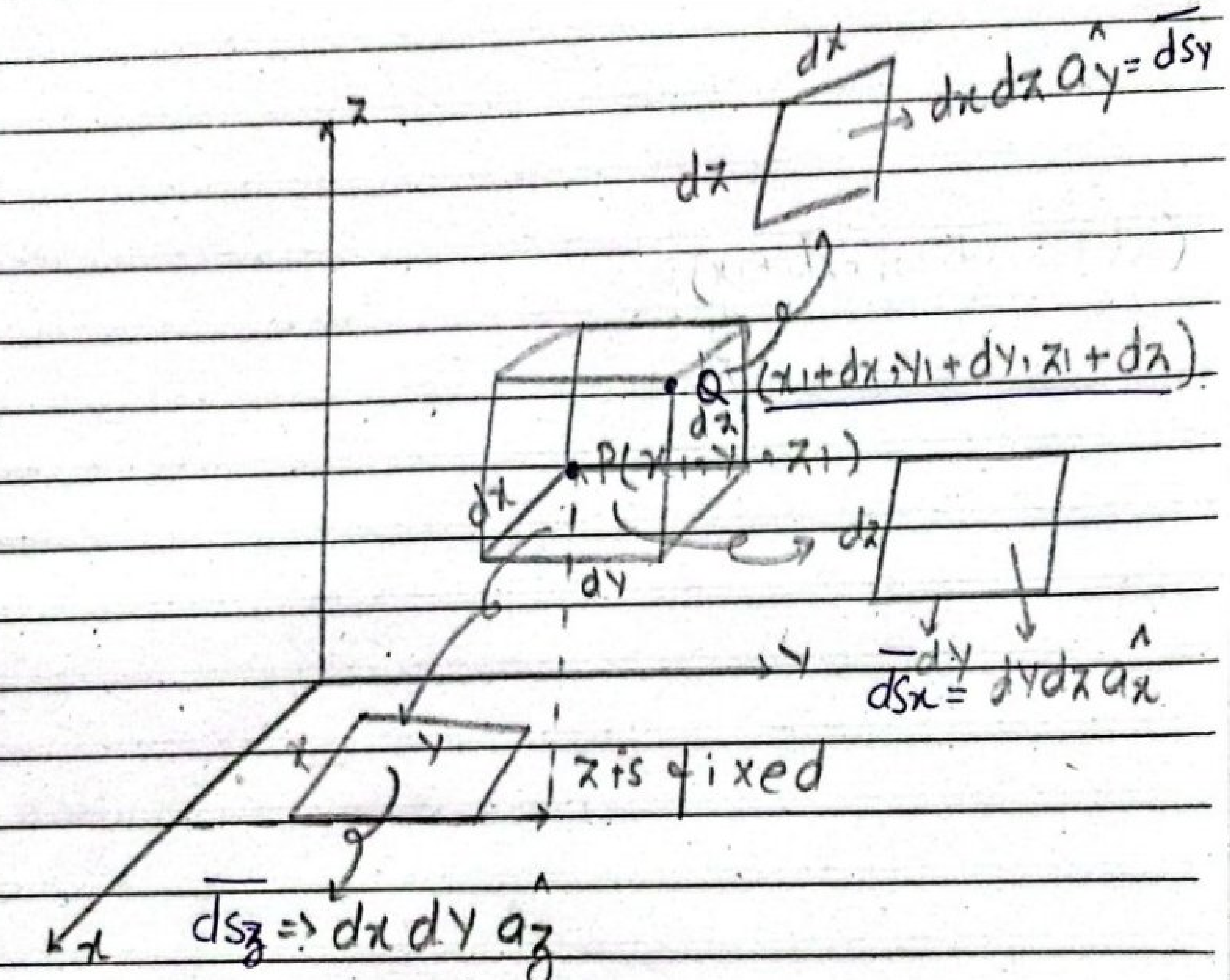
$$\hat{a}_{AC} = \frac{\overrightarrow{R_{AC}}}{|\overrightarrow{R_{AC}}|}$$

$$= \frac{2\hat{a_x} + 6\hat{a_y} + 8\hat{a_z}}{2\sqrt{26}}$$

$$= \frac{1}{\sqrt{26}}\hat{a_x} + \frac{3}{\sqrt{26}}\hat{a_y} + \frac{4}{\sqrt{26}}\hat{a_z}$$



$dl = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$

↳ differential length $= (x_2 - x_1)\hat{a_x} + (y_2 - y_1)\hat{a_y} + (z_2 - z_1)\hat{a_z}$

$\overline{ds_x} = dy\,dz\,\hat{a_x}$

$\overline{ds_y} = dx\,dz\,\hat{a_y}$ → differential surfaces

$\overline{ds_z} = dx\,dy\,\hat{a_z}$

$dV = dx\,dy\,dz$

↳ differential volume

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Cross Product:-

$\vec{A} \times \vec{B} = |A||B| \sin\theta_{AB} \cdot \hat{a}_N$

⇒ direction of unit vector is always perpendicular to the plane containing vectors

$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\left.\begin{aligned} a_x \times a_y &= a_z \\ a_y \times a_z &= a_x \\ a_z \times a_x &= a_y \end{aligned}\right\} \quad \begin{aligned} a_y \times a_x &= -a_z \\ a_z \times a_y &= -a_x \\ a_x \times a_z &= -a_y \end{aligned}$$

$$\boxed{A \times B = -B \times A}$$

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Q. $A = 2\hat{a_x} - 3\hat{a_y} + \hat{a_z}$

$B = -4\hat{a_x} - 2\hat{a_y} + 5\hat{a_z}$

Sol:-

find $A \times B$

$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ 2 & -3 & 1 \\ -4 & -2 & 5 \end{vmatrix}$$

$$= a_x \begin{vmatrix} -3 & 1 \\ -2 & 5 \end{vmatrix} - a_y \begin{vmatrix} 2 & 1 \\ -4 & 5 \end{vmatrix} + a_z \begin{vmatrix} 2 & -3 \\ -4 & -2 \end{vmatrix}$$

$$= a_x(-15+2) - a_y(10+4) + a_z(-4-12)$$

$$\boxed{= -13\hat{a_x} - 14\hat{a_y} - 16\hat{a_z}}$$

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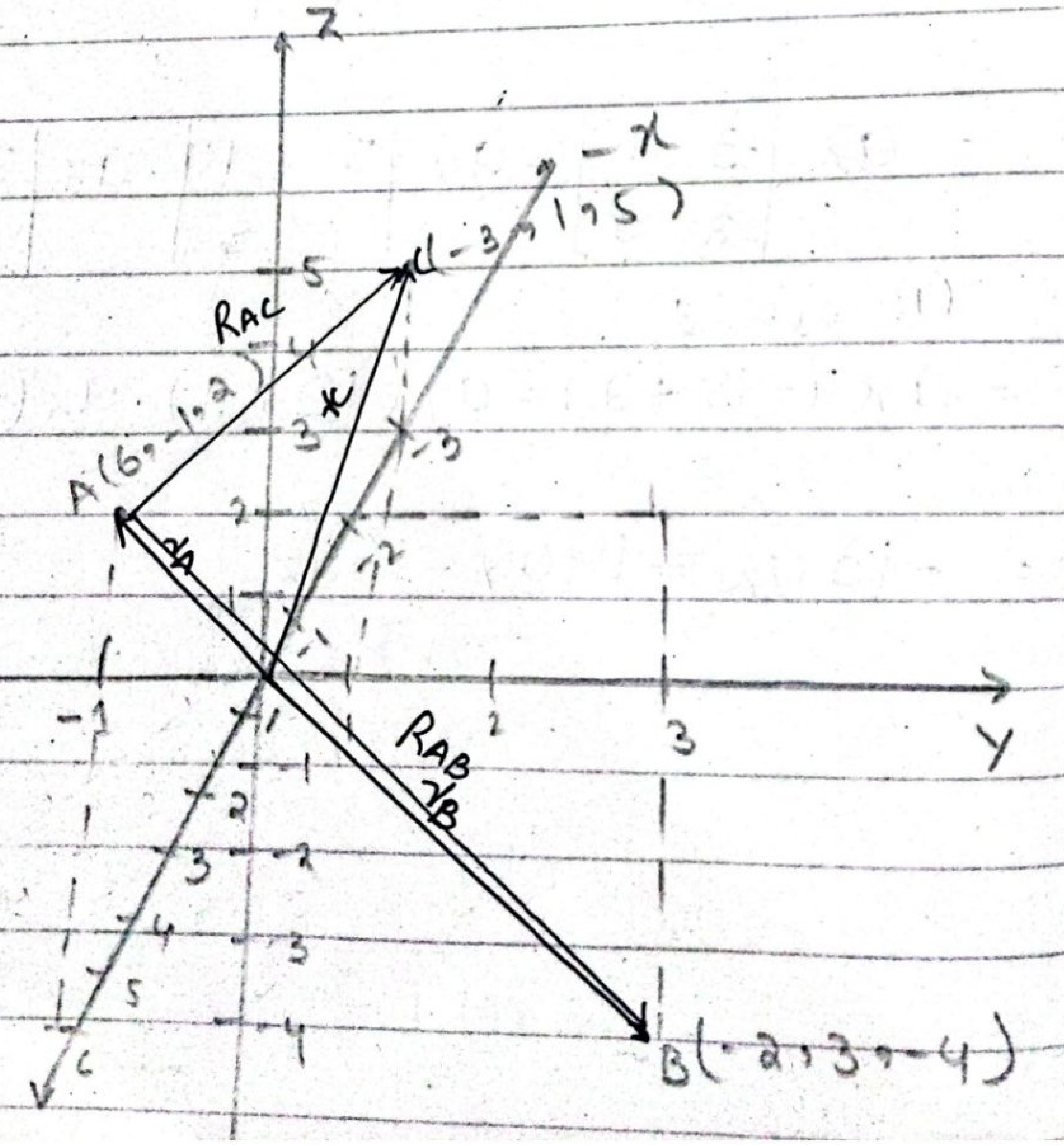
D 1.4

The three vertices of a triangle are located at A(6, −1, 2), B(−2, 3, −4) and C(−3, 1, 5) Find.

(a) $R_{AB} \times R_{AC}$
(b) the area of the triangle
(c) a unit vector perpendicular to the plane in which triangle is located.



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$R_{AB} = \vec{r_B} - \vec{r_A}$

$R_{AB} = (-2\hat{a_x} + 3\hat{a_y} - 4\hat{a_z}) - (6\hat{a_x} - \hat{a_y} + 2\hat{a_z})$

$R_{AB} = -8\hat{a_x} + 4\hat{a_y} - 6\hat{a_z}$

$R_{AC} = \vec{r_C} - \vec{r_A}$

$= (-3a_x + a_y + 5a_z) - (6\hat{a_x} - \hat{a_y} + 2\hat{a_z})$

$= -9\hat{a_x} + 2\hat{a_y} + 3\hat{a_z}$

$$R_{AB} \times R_{AC} = \begin{vmatrix} a_x & a_y & a_z \\ -8 & 4 & -6 \\ -9 & 2 & 3 \end{vmatrix}$$

$$= a_x \begin{vmatrix} 4 & -6 \\ 2 & 3 \end{vmatrix} - a_y \begin{vmatrix} -8 & -6 \\ -9 & 3 \end{vmatrix} + a_z \begin{vmatrix} -8 & 4 \\ -9 & 2 \end{vmatrix}$$

$= a_x(12+12) - a_y(-24-54) + a_z(-16+36)$

$\boxed{= 24\hat{a_x} + 78\hat{a_y} + 20\hat{a_z}}$

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(b)

$\Rightarrow |R_{AB} \times R_{AC}|$

$= \sqrt{(24)^2 + (78)^2 + (20^2)}$

$\boxed{= 84.023}$

Area of triangle $= \frac{1}{2} |R_{AB} \times R_{AC}|$

$= \frac{84.023}{2}$

$= \boxed{42.01 \text{ sq.units}}$

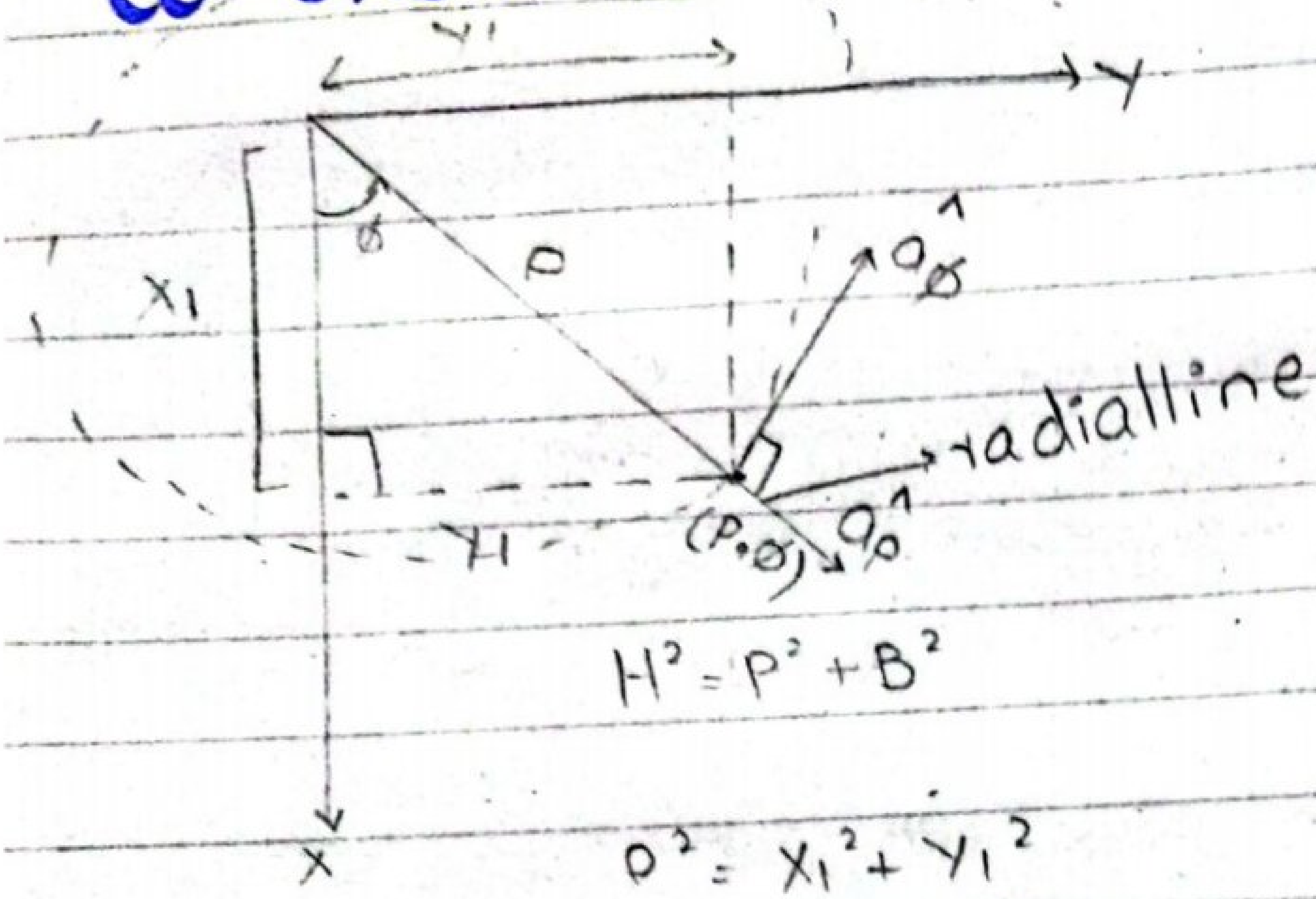
(c)

$\hat{a}_{R_{AB} \times R_{AC}} = \frac{R_{AB} \times R_{AC}}{|R_{AB} \times R_{AC}|}$

$= \frac{24\hat{a}_x + 78\hat{a}_y + 20\hat{a}_z}{84.023}$

CIRCULAR CYLINDRICAL → (ρ, ϕ, z)

CO-ORDINATE SYSTEM



$H^2 = P^2 + B^2$

$\rho^2 = x_1^2 + y_1^2$

$\rho = \sqrt{x_1^2 + y_1^2}$

$\tan\phi = \frac{y_1}{x_1}$ $\phi = \tan^{-1}\left(\frac{y_1}{x_1}\right)$

P = radius of circle

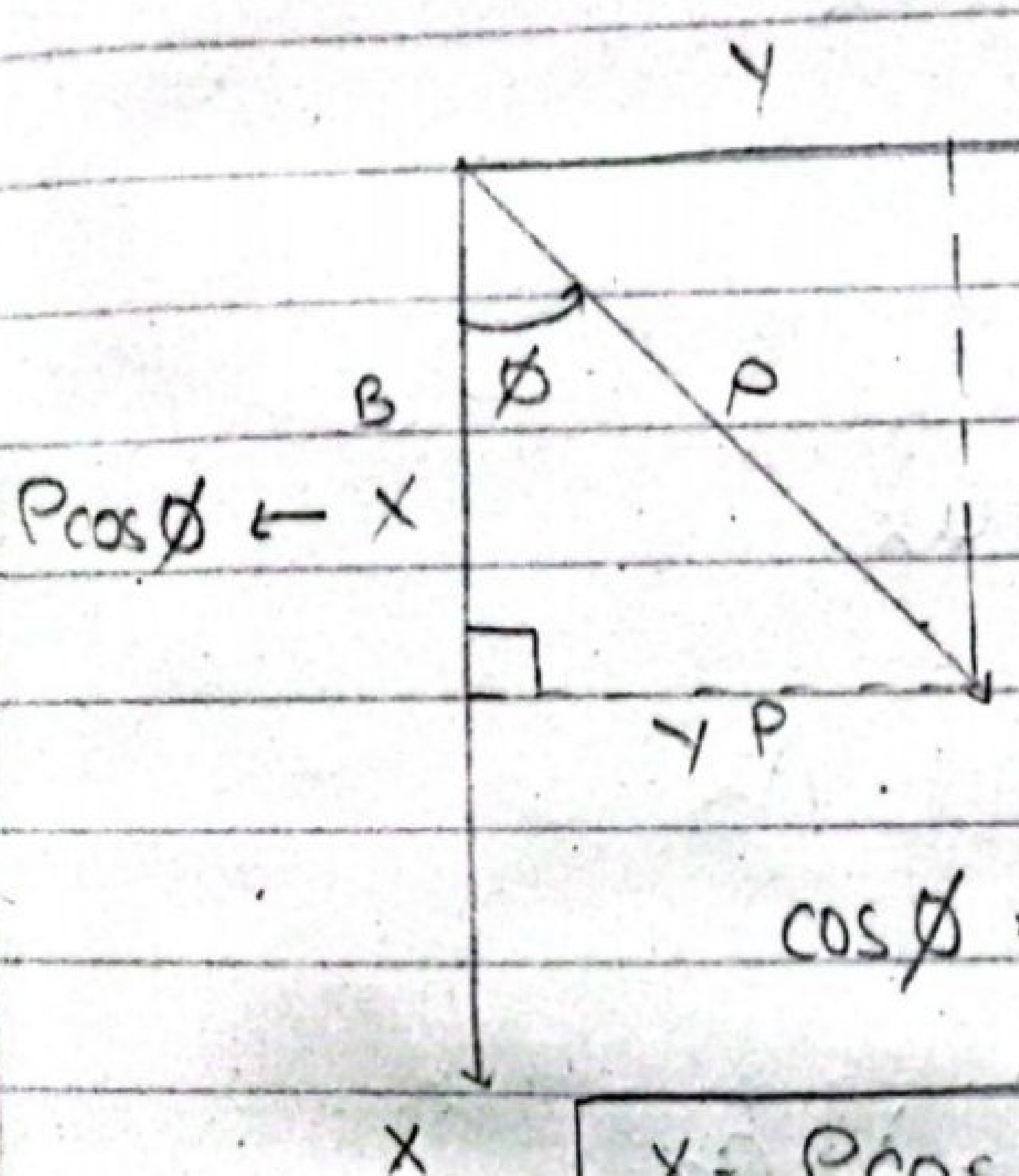
Ø = Angle between the radial line and reference axis

⇒ origin will be the center of the circle

$\boxed{\vec{P}\cdot\hat{a}_x = |P|\cos\phi}$

⇒ $\hat{a}_\rho$ must be pointing in that direction where value of ρ is increasing

⇒ $\hat{a}_\phi$ must be perpendicular to $\hat{a}_\rho$



→ $\cos(90-\theta) = \sin\theta$

⇒ $\rho\cos(90-\phi)$
$= \rho\sin\phi$

$\cos\phi = \dfrac{X}{\rho}$ $\quad$ $\sin\phi = \dfrac{Y}{\rho}$

$\boxed{X = \rho\cos\phi}$ $\quad$ $\boxed{Y = \rho\sin\phi}$

$\boxed{Z = Z}$

⇒ $\hat{a}_\phi$ must be pointing in that direction where value of ϕ is increasing.

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RECTANGULAR TO CYLINDRICAL CO-ORDINATE CONVERSION:-

$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

to

$\vec{A} = \overset{?}{A_\rho} \hat{a}_\rho + \overset{?}{A_\phi} \hat{a}_\phi + \overset{?}{A_z} \hat{a}_z$

To find A_ρ, A_ϕ and A_z

$$\begin{aligned} A_\rho &= A \cdot \hat{a}_\rho \\ &= (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot \hat{a}_\rho \\ &= A_x (\hat{a}_x \cdot \hat{a}_\rho) + A_y (\hat{a}_y \cdot \hat{a}_\rho) + A_z (\hat{a}_z \cdot \hat{a}_\rho) \end{aligned}$$

$$\begin{aligned} A_\phi &= A \cdot \hat{a}_\phi \\ &= (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot \hat{a}_\phi \\ &\Rightarrow A_x ((\hat{a}_x \cdot \hat{a}_\phi)) + A_y ((\hat{a}_y \cdot \hat{a}_\phi)) \\ &\quad + A_z (\hat{a}_z \cdot \hat{a}_\phi) \end{aligned}$$

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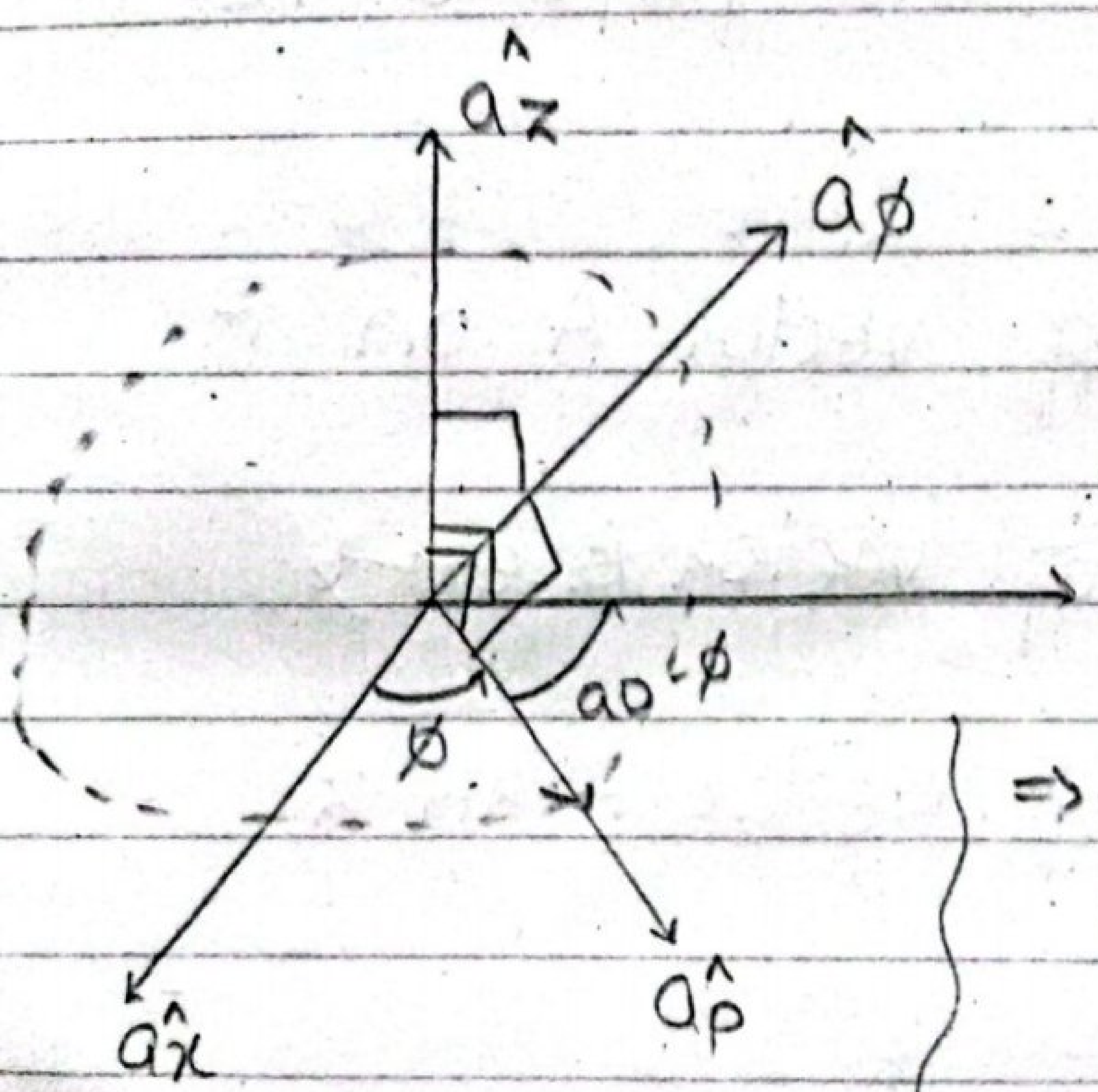
$A_z = A \cdot \hat{a}_z$

$= (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot \hat{a}_z$

$\Rightarrow A_x ((\hat{a}_x \cdot \hat{a}_z)) + A_y ((\hat{a}_y \cdot \hat{a}_z))$
$+ A_z ((\hat{a}_z \cdot \hat{a}_z))$



$90° - (90° - \phi)$

$\Rightarrow \hat{a}_x \cdot \hat{a}_\rho = \cos\phi$

$\hat{a}_y \cdot \hat{a}_\rho = \cos(90 - \phi)$
$= \sin\phi$

$\Rightarrow \hat{a}_x \cdot \hat{a}_\phi = \cos(90 + \phi)$
$= -\sin\phi$

$\cdot$	$\hat{a}_\rho$	$\hat{a}_\phi$	$\hat{a}_z$
$\hat{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\hat{a}_y$	$\sin\phi$	$\cos\phi$	0
$\hat{a}_z$	0	0	1

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$\hat{a_y} \cdot \hat{a_\phi} = \cos(90^\circ - 90^\circ + \phi)$

$\boxed{\hat{a_y} \cdot \hat{a_\phi} = \cos\phi}$

$\Rightarrow$ $\hat{a_z}$ is perpendicular to all unit vector.

$\Rightarrow$ $\hat{a_\rho}$, $\hat{a_\theta}$, $\hat{a_z}$ are perpendicular to each other

$A_\rho \rightarrow$ projection of vector A on ρ

$A_\phi \rightarrow$ projection of vector A on ϕ

$A_z \rightarrow$ projection of vector A on z

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EXAMPLE : 1.3

Q. Transform the vector $B = y a_x - x a_y + z a_z$ into cylindrical co-ordinates.

Sol:-

$$B = \overset{?}{B_\rho} \hat{a}_\rho + \overset{?}{B_\phi} \hat{a}_\phi + \overset{?}{B_z} \hat{a}_z \rightarrow (1)$$

$$B_\rho = \vec{B} \cdot \hat{a}_\rho$$
$$= (y\hat{a}_x - x\hat{a}_y + z\hat{a}_z) \cdot \hat{a}_\rho$$
$$= y(\hat{a}_x \cdot \hat{a}_\rho) - x(\hat{a}_y \cdot \hat{a}_\rho) + z(\hat{a}_z \cdot \hat{a}_\rho)$$

$$\Rightarrow y\cos\phi - x\sin\phi$$

$$B_\phi = \vec{B} \cdot \hat{a}_\phi$$
$$= (y a_x - x a_y + z a_z) \cdot a_\phi$$
$$= y(\hat{a}_x \cdot \hat{a}_\phi) - x(\hat{a}_y \cdot \hat{a}_\phi) + z(\hat{a}_z \cdot \hat{a}_\phi)$$
$$= -y\sin\phi - x\cos\phi$$

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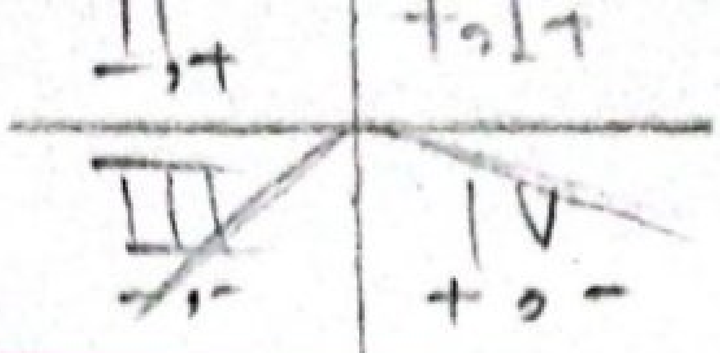

D1.5

(a) Give the rectangular co-ordinates of the point $C(\rho = 4.4, \phi = -115^\circ, z = 2)$

(b) Give the cylindrical co-ordinates of the point $D(x = -3.1, y = 2.6, z = -3)$

(c) specify the distance from C to D.

Sol:- (a)

$x = \rho \cos\phi$

$x = 4.4 \times \cos(-115^\circ)$

$x = -1.859 \Rightarrow \boxed{(-1.859, -3.98, 2)}$

$y = \rho \sin\phi$

$y = 4.4 \times \sin(-115^\circ)$

$y = -3.98$

$z = 2$

(b)

$\phi = \tan^{-1}\left(\frac{2.6}{-3.1}\right)$

$\phi = 140^\circ$

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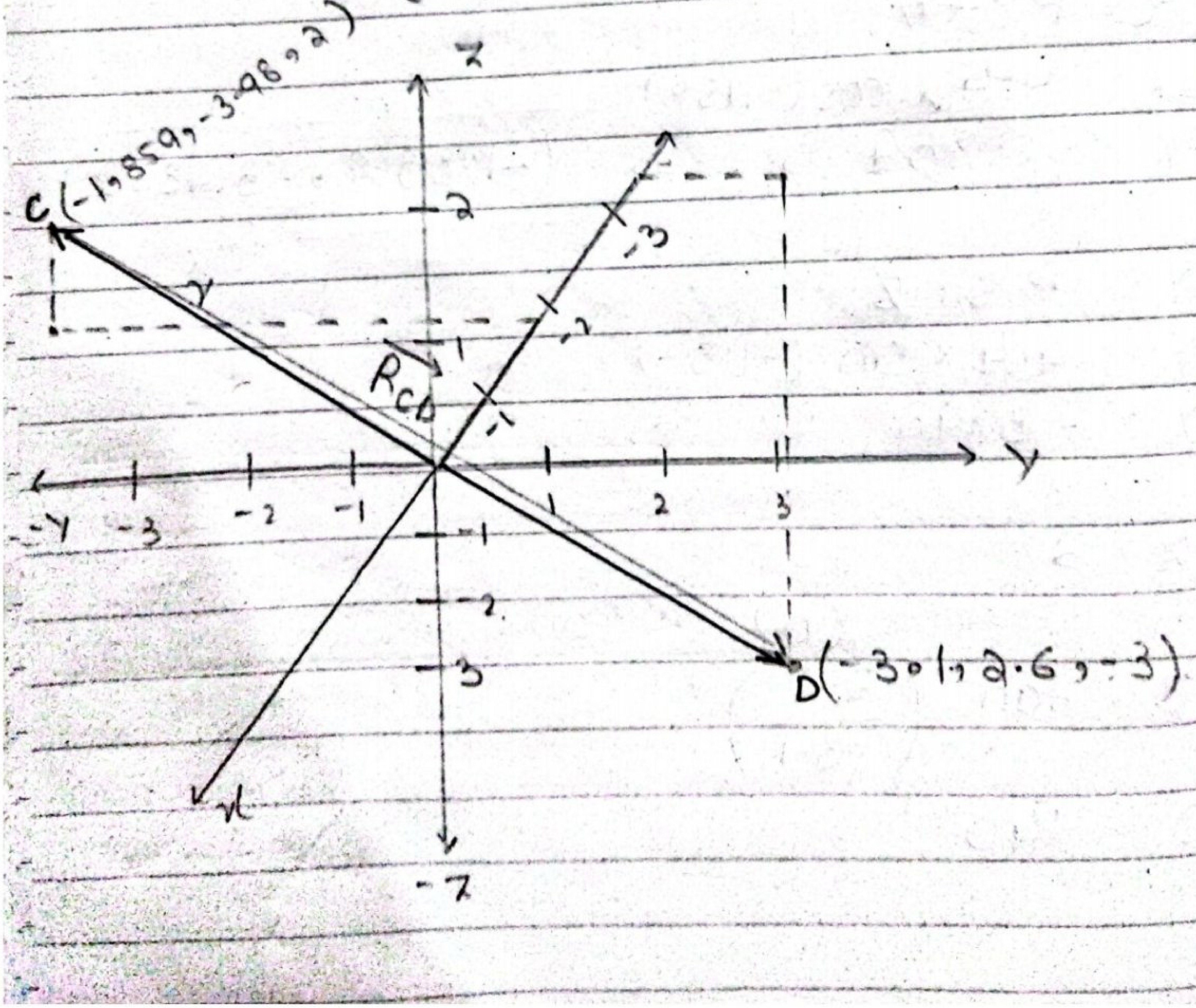
$$\rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{(2.6)^2 + (-3.1)^2}$$

$$\boxed{\rho = 4.04}$$

$$= (4.04, 140°, -3)$$

(C)



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$R_{CD} = \vec{r_D} - \vec{r_C}$

$R_{CD} = (-3.1\hat{a_x} + 2.6\hat{a_y} - 3\hat{a_z}) -$

$(-1.859\hat{a_x} - 3.98\hat{a_y} + 2\hat{a_z})$

$R_{CD} = -1.241\hat{a_x} + 6.58\hat{a_y} - 5\hat{a_z}$

$|R_{CD}| = \sqrt{(-1.241)^2 + (6.58)^2 + (-5)^2}$

$$\boxed{|R_{CD}| = \cancel{6.77} \quad 8.35}$$

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D1.6

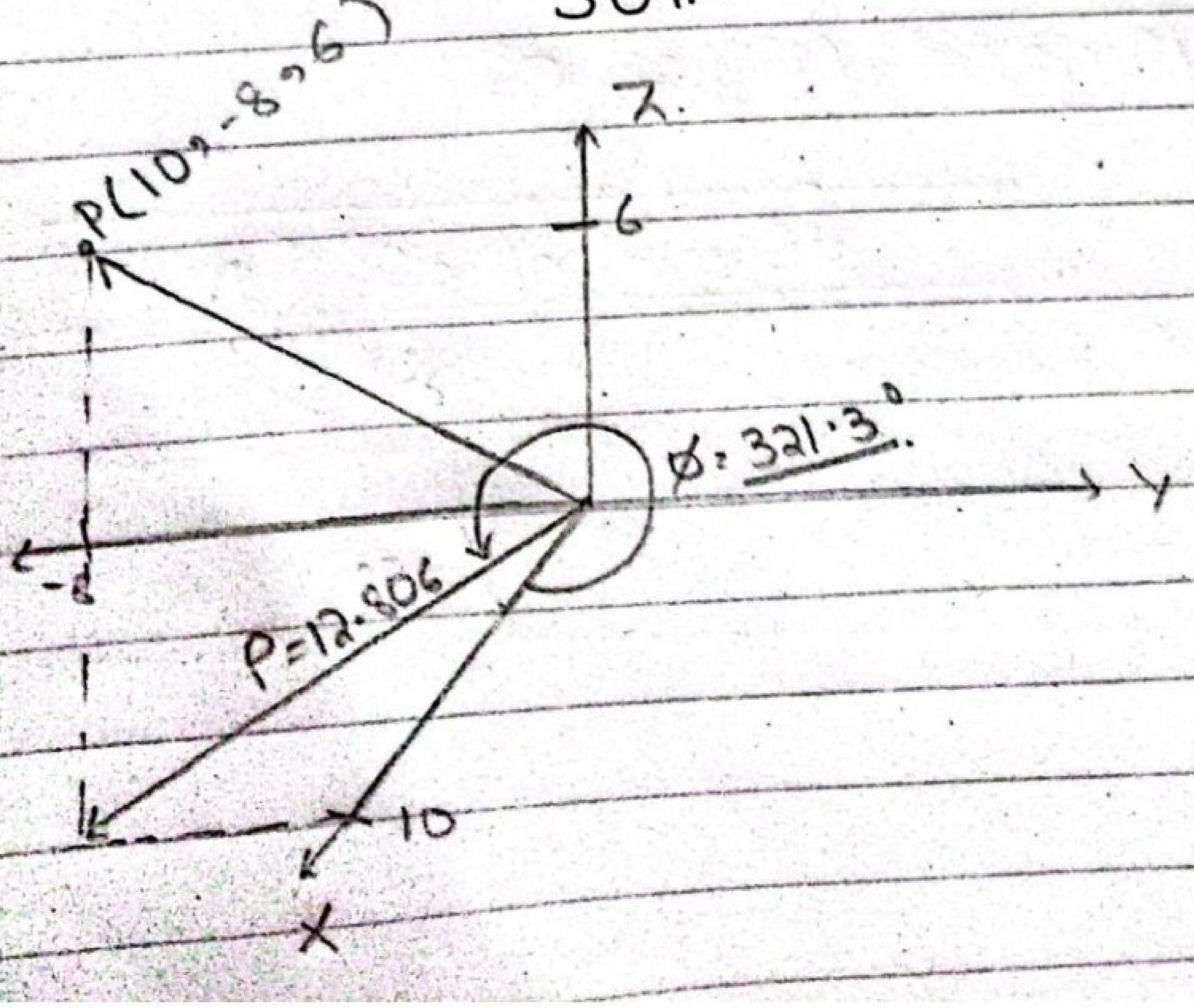
Transform to cylindrical co-ordinates

(a) $F = 10a_x - 8a_y + 6a_z$ at point $P(10, -8, 6)$;

(b) $G = (2x+y)a_x - (y-4x)a_y$ at point $Q(\rho, \phi, z)$

(c) Give the rectangular components of the vector $H = 20a_\rho - 10a_\phi + 3a_z$ at $P(x=5, y=2, z=-1)$

Sol:-



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$$\phi = \tan^{-1}\left(\frac{-8}{10}\right)$$

$$\boxed{\phi = 321.3^\circ}$$

$$P = \sqrt{x^2+y^2}$$

$$P = \sqrt{10^2+(-8)^2}$$

$$\boxed{P = 12.806}$$

$$F = \overset{?}{F_P}\,\hat{a_P} + \overset{?}{F_\phi}\,\hat{a_\phi} + \overset{?}{F_z}\,\hat{a_z} \longrightarrow (1)$$

$$F_P = \vec{F}\cdot\hat{a_P}$$

$$= 10\,(\hat{a_x}\cdot\hat{a_P}) - 8\,(\hat{a_y}\cdot\hat{a_P}) + 6\,(\hat{a_z}\cdot\hat{a_P})$$

$$= 10\cos\phi - 8\sin\phi + 0$$

$$= 10\cos(321.3^\circ) - 8\sin(321.3^\circ)$$

$$= 12.806$$

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$F_\phi = \vec{F} \cdot \hat{a}_\phi$

$= (10\,\hat{a}_x \cdot \hat{a}_\phi) - 8\,(\hat{a}_y \cdot \hat{a}_\phi) + 6\,(\hat{a}_z \cdot \hat{a}_\phi)$

$= -10 \sin\phi - 8\cos\phi + 0$

$= -10 \sin(321.3^\circ) - 8\cos(321.3^\circ)$

$= 0$

$F_z = \vec{F} \cdot \hat{a}_z$

$= 6(\hat{a}_z \cdot \hat{a}_z)$

$= 6$

So eq (1) becomes,

$\boxed{= 12.806\,\hat{a}_\rho + 6\,\hat{a}_z}$ //

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(b)

$$G = \overset{?}{G_\rho}\,\hat{a_\rho} + \overset{?}{G_\phi}\,\hat{a_\phi} + \overset{?}{G_z}\,\hat{a_z} \rightarrow (1)$$

$$G_P = \vec{G} \cdot \hat{a_P}$$

$$= (2x+y)\,\hat{a_x} \cdot \hat{a_P} - (y-4x)\,\hat{a_y} \cdot \hat{a_P}$$

$$\Rightarrow (2x+y)\cos\phi - (y-4x)\sin\phi$$

$$\Rightarrow (2P\cos\phi + \sin\phi P)\cos\phi -$$

$$(P\sin\phi - 4P\cos\phi)\sin\phi$$

$$= 2P\cos^2\phi + \sin\phi\cos\phi P) -$$

$$(P\sin^2\phi - 4P\cos\phi\sin\phi)$$

$$\Rightarrow 2P\cos^2\phi - P\sin^2\phi + P\sin\phi\cos\phi + 4P\cos\phi\sin\phi$$

$$= 2P\cos^2\phi - P\sin^2\phi + 5p\sin\phi\cos\phi$$

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$G_\phi = \vec{G} \cdot \hat{a}_\phi$

$= (2x+y)\, \hat{a}_x \cdot \hat{a}_\phi - (y-4x)\, \hat{a}_y \cdot \hat{a}_\phi$

$= (2\rho\cos\phi + \rho\sin\phi) - \sin\phi$

$- (\rho\sin\phi - 4\rho\cos\phi)\cos\phi$

$\Rightarrow -2\rho\cos\phi\sin\phi - \rho\sin^2\phi$
$- \rho\sin\phi\cos\phi + 4\rho\cos^2\phi$

$\Rightarrow -3\rho\cos\phi\sin\phi \cancel{+3\rho\sin^2\phi}$
$- \rho\sin^2\phi + 4\rho\cos^2\phi$

so eq (1) becomes:

$G = [2\rho\cos^2\phi - \rho\sin^2\phi + 5\rho\sin\phi\cos\phi]\,\hat{a}_\rho$

$+ [4\rho\cos^2\phi - \rho\sin^2\phi - 3\rho\sin\phi\cos\phi]$
$\hat{a}_\phi$

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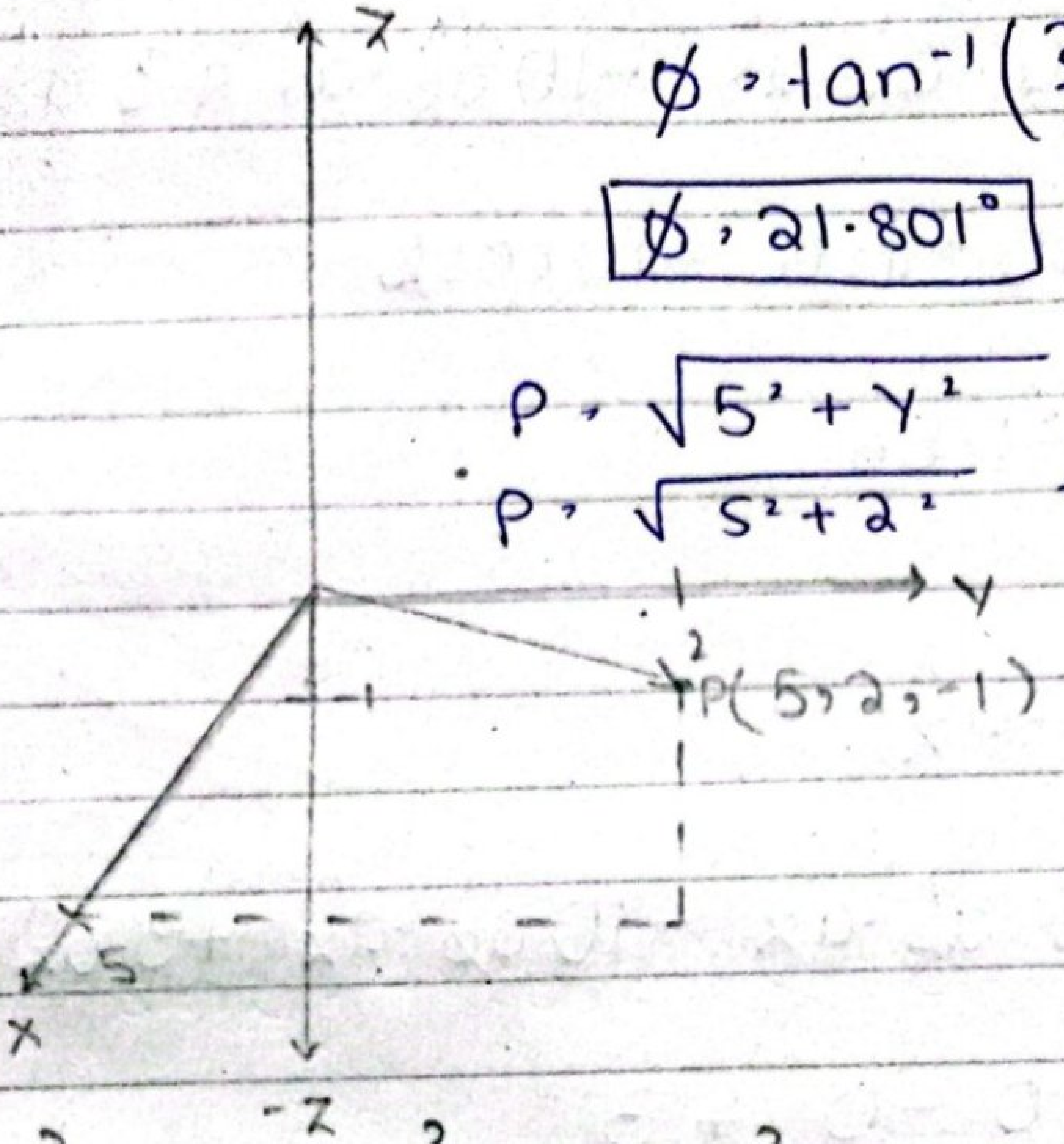
(C)

$P(x=5, y=2, z=-1)$



$\phi = \tan^{-1}\left(\frac{2}{5}\right)$

$\boxed{\phi = 21.801^\circ}$

$\rho = \sqrt{5^2 + y^2}$

$\rho = \sqrt{5^2 + 2^2} = \boxed{\sqrt{29}}$

$P = P_x \hat{a_x} + P_y \hat{a_y} + P_z \hat{a_z} \longrightarrow (1)$

$P_x = \vec{P} \cdot \hat{a_x}$

$= 20\, \hat{a_\rho} \cdot \hat{a_x} - 10 (a_\phi \cdot \hat{a_x})$

$+ 3\, \hat{a_z} \cdot \hat{a_x}$

$= 20 \cos\phi - 10(-\sin\phi)$

$\boxed{P_x = 22.3}$

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$P_y = \vec{P} \cdot \hat{a_y}$

$= 20\, \hat{a_\rho} \cdot \hat{a_y} - 10\, \hat{a_\phi} \cdot \hat{a_y} + 3\, \hat{a_z} \cdot \hat{a_y}$

$= 20 \sin\phi - 10 \cos\phi$

$= -1.86$.

$P_z = \vec{P} \cdot \hat{a_z}$

$= 20\, \hat{a_\rho} \cdot \hat{a_z} - 10\, \hat{a_\phi} \cdot \hat{a_z} + 3\, \hat{a_z} \cdot \hat{a_z}$

$= 0 - 0 + 3$

$= 3$.

So eq(1) becomes,

$$\boxed{P = 22.3\, \hat{a_x} - 1.86\, \hat{a_y} + 3\hat{a_z}}$$

//

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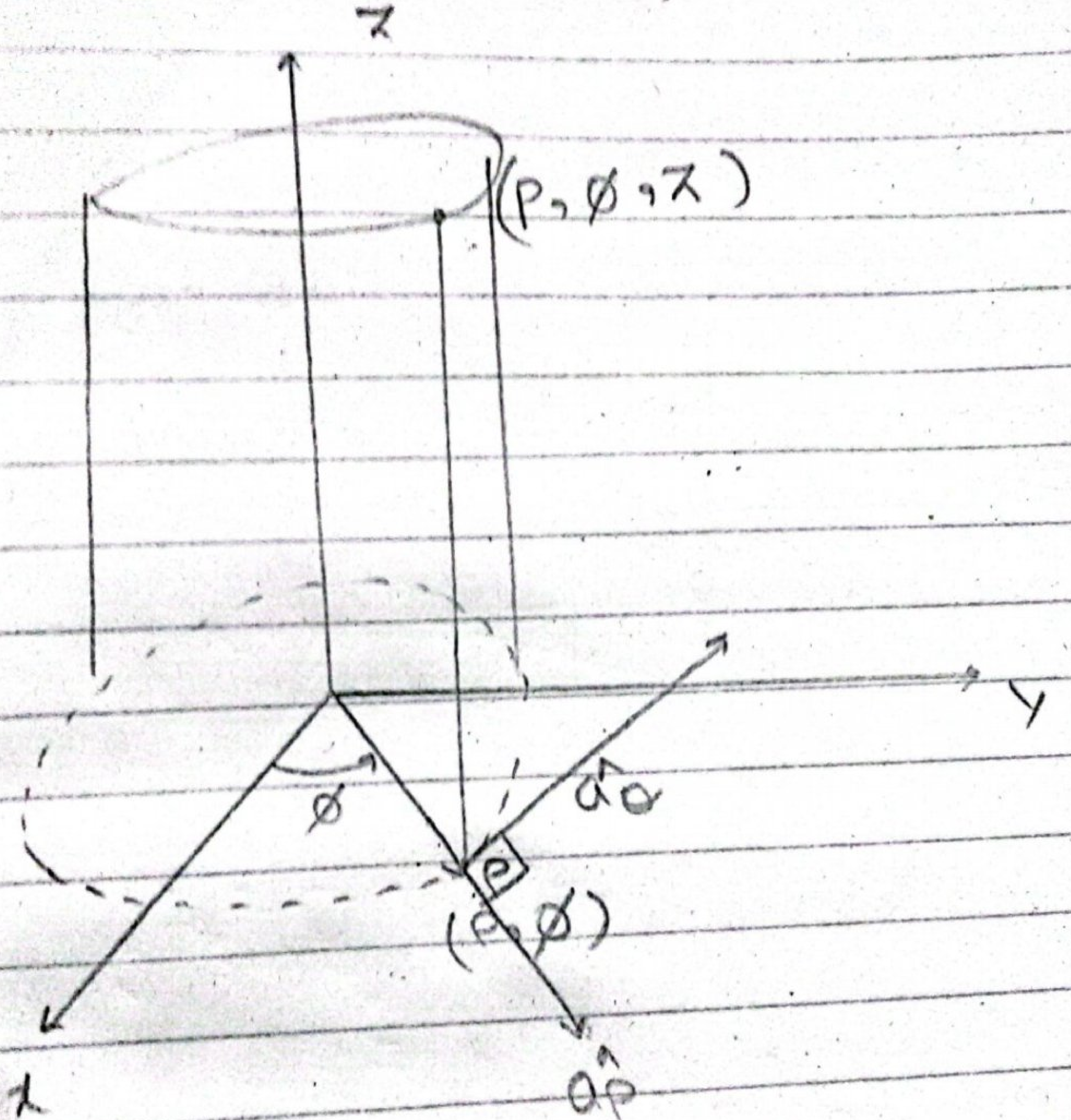

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DIFFERENTIAL VOLUME & SURFACES IN RECTANGULAR CO-ORDINATE SYSTEM:

$dv = dx\,dy\,dz$

$ds_{x_1} = dy\,dz\,\hat{a_x}$

$ds_y = dx\,dz\,\hat{a_y}$

$ds_z = dx\,dy\,\hat{a_z}$

$d_{-sx} = dy\,dz\,(-\hat{a_x})$

$d_{-sy} = dx\,dz\,(-\hat{a_y})$

$d_{-sz} = dx\,dy\,(-\hat{a_z})$

$dl = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$

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Method : 01

Q. suppose the problem is of rectangular co-ordinate system with following limits

$2 < x < 5$, $3 < y < 7$, $9 < z < 11$

find area & volume.

Sol:

shows direction

$ds_1 = dx\,dy\,\hat{a_z}$; $ds_4 = dx\,dy(-\hat{a_z})$

$ds_2 = dy\,dz\,\hat{a_x}$; $ds_5 = dy\,dz\,(-\hat{a_x})$

$ds_3 = dx\,dz\,\hat{a_y}$; $ds_6 = dx\,dz\,(-\hat{a_y})$

combining ds_1 & ds_4

$= 2\,dx\,dy$

combining ds_2 & ds_5

$= 2\,dy\,dz$

combining ds_3 and ds_6

$= 2\,dx\,dz$

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$$8.\ 2\int_{3}^{7}\int_{2}^{5} dx\,dy + 2\int_{9}^{11}\int_{3}^{7} dy\,dz + 2\int_{9}^{11}\int_{2}^{5} dx\,dz$$

$$\Rightarrow 2\int_{3}^{7} x \Big|_{2}^{5} dy + 2\int_{9}^{11} y \Big|_{3}^{7} dz$$

$$+ 2\int_{9}^{11} x \Big|_{2}^{5} dz$$

$$= 2\int_{3}^{7} 3\,dy + 2\int_{9}^{11} 4\,dz + 2\int_{9}^{11} 3\,dz$$

$$= 6\,y \Big|_{3}^{7} + 8\,z \Big|_{9}^{11} + 2\times 3\,z \Big|_{9}^{11}$$

$$= 6\times 4 + 8(2) + 6(2)$$

$$= 24 + 16 + 12$$

$$= \boxed{52 \text{ sq. units}}$$

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$$dV = \int_9^{11}\int_3^{7}\int_2^{5} dx\,dy\,dz$$

$$= \int_9^{11}\int_3^{7} x \Big|_2^5 \,dy\,dz$$

$$= \int_9^{11}\int_3^{7} 3 \,dy\,dz$$

$$= 3\int_9^{11} y \Big|_3^7 \,dz$$

$$= 3\int_9^{11} 4 \,dz$$

$$= 12\int_9^{11} dz$$

$$= 12\, z \Big|_9^{11}$$

$$= 12(2)$$

$$= \boxed{24 \text{ units}^3}$$

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$= 4$

$dy = 4$

Method :02

Q. suppose the problem is of rectangular coordinate system. with following limits.

$2 < x < 5$; $3 < y < 7$; $9 < z < 11$

find volume & surface area.

Sol:

$dx = x_2 - x_1$

$dx = 5 - 2$

$\boxed{dx = 3}$

$dy = y_2 - y_1$

$dy = 7 - 3$

$\boxed{dy = 4}$

$dz = z_2 - z_1$

$dz = 11 - 9$

$\boxed{dz = 2}$

Volume $= dx\,dy\,dz$

$= 3 \times 4 \times 2$

$\boxed{V = 24 \text{ units}^3}$

$dS_x = dy\,dz$

$= 4 \times 2$

$= 8$

$= 8 \times 2$

$\boxed{dS_x = 16}$

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$dsy = dx\,dz$

$= 3 \times 2$

$= 6$

$= 6 \times 2$

$\boxed{dsy = 12}$

$dsz = dx\,dy$

$= 3 \times 4$

$= 12$

$dsz = 12 \times 2$

$\boxed{dsz = 24}$

$s = dsx + dsy + dsz$

$= 16 + 12 + 24$

$= 24 + 28$

$\boxed{s = 52 \text{ units}^2}$

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DIFFERENTIAL VOLUME & SURFACES IN CIRCULAR CYLINDRICAL CO-ORDINATE SYSTEM:

$$\boxed{dV = \rho\, \partial\rho\, \partial\phi\, \partial z}$$

$$dS_\rho = \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$dS_\phi = d\rho\, dz\, \hat{a}_\phi$$

$$dS_z = \rho\, \partial\phi\, \partial\rho\, \hat{a}_z$$

$$d\text{-}S_\rho = \rho\, \partial\phi\, \partial z (-\hat{a}_\rho)$$

$$d\text{-}S_\phi = \partial\rho\, dz\, (-\hat{a}_\phi)$$

$$d\text{-}S_z = \rho\, d\phi\, \partial\rho\, (-\hat{a}_z)$$

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METHOD : 01

Q. Suppose the problem is of cylindrical system with following limits

$3 < \rho < 5$; $0.1\pi < \phi < 0.3\pi$;

$2 < z < 5$

find total surface area & volume?

Sol:-

$dS_1 = \rho\, \partial\phi\, dz\, \hat{a}_\rho$; $dS_4 = \rho\, \partial\phi\, dz(-\hat{a}_\rho)$

$dS_2 = \rho\, d\rho\, d\phi\, \hat{a}_z$; $dS_5 = \rho\, d\rho\, \partial\phi(\hat{a}_z)$

$dS_3 = d\rho\, dz\, \hat{a}_\phi$; $dS_6 = d\rho\, dz\,(-\hat{a}_\phi)$

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combing dS_1 & dS_4

$= 3\, d\phi\, dz + 5\, d\phi\, dz$

$= 8\, d\phi . dz$

combinig dS_2 and dS_5

$= 2\rho\, d\rho\, d\phi$

combining dS_3 and dS_6

$= 2\, d\rho\, dz$

$$S = 8\int_2^5\int_{0.1\pi}^{0.3\pi} d\phi\, dz + 2\int_{0.1\pi}^{0.3\pi}\int_3^5 \rho\, d\rho\, d\phi$$

$$+ 2\int_2^5\int_3^5 d\rho . dz$$

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$$S = 8\int_2^5 \phi \Big|_{0.1\pi}^{0.3\pi} dz + 2\int_{0.1\pi}^{0.3\pi} \frac{\rho^2}{2}\Big|_3^5 d\phi$$

$$+ 2\int_2^5 \rho \Big|_3^5 dz$$

$$S = 8\int_2^5 0.628\, dz + \frac{2}{2}\int_{0.1\pi}^{0.3\pi} 16\, d\phi$$

$$+ 2\int_2^5 2\, dz$$

$$= 5.024\, z\Big|_2^5 + 16\,\phi\Big|_{0.1\pi}^{0.3\pi}$$

$$+ 4\, z\Big|_2^5$$

$$= 5.024 \times 3 + 16 \times 0.628$$
$$+ 4 \times 3$$

$$= 15.072 + 10.048 + 12$$

$$\boxed{= 37.12 \text{ units}^2}$$

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$$V = \int_{2}^{5}\int_{0.1\pi}^{0.3\pi}\int_{3}^{5} \rho \, \partial\rho \, d\phi \, dz$$

$$= \int_{2}^{5}\int_{0.1\pi}^{0.3\pi} \frac{\rho^2}{2}\Big|_{3}^{5} \, d\phi \, dz$$

$$= \frac{1}{2}\int_{2}^{5}\int_{0.1\pi}^{0.3\pi} 16 \, d\phi \, dz$$

$$= 8\int_{2}^{5} \phi \Big|_{0.1\pi}^{0.3\pi} dz$$

$$= 8\int_{2}^{5} 0.628 \, dz$$

$$= 5.026\int_{2}^{5} dz$$

$$= 5.026 \, z \Big|_{2}^{5}$$

$$\boxed{= 15.079 \text{ units}^3}$$

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METHOD: 02

Q. suppose the problem is of cylindrical system with following limits:

$3 < \rho < 5$; $0.1\pi < \phi < 0.3\pi$;

$2 < z < 5$

surface area = ? volume = ?

Sol:-

$dS_\rho = \rho \, d\phi \, dz \, \hat{a}_\rho$

$dS_\phi = d\rho \, dz \, \hat{a}_\phi$

$dS_z = \rho \, d\rho \, d\phi \, \hat{a}_z$

$d\rho = \rho_2 - \rho_1$

$d\rho = 5 - 3$

$\boxed{d\rho = 2}$

$d\phi = \phi_2 - \phi_1$

$d\phi = 0.3\pi - 0.1\pi$

$\boxed{d\phi = 0.628}$

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$dz = z_2 - z_1$

$dz = 5 - 2$

$\boxed{dz = 3}$

$V = \rho \, d\rho \, d\phi \, dz$

$= \left(\frac{5+3}{2}\right) \times 2 \times 0.628 \times 3$

$\boxed{V = 15.072 \text{ units}^3}$

$dS_\rho = \rho \, d\phi \, dz$

$= \left(\frac{5+3}{2}\right) \times 0.628 \times 3$

$= 7.536 \times 2$

$\boxed{dS_\rho = 15.072}$

$dS_\phi = d\rho \, dz$

$= 3 \times 2$

$= 6$

$= 6 \times 2$

$\boxed{dS_\phi = 12}$

$dS_z = \rho \, d\rho \, d\phi$

$= \left(\frac{5+3}{2}\right) \times 2 \times 0.628$

$dS_z = 5.024 \times 2$

$\boxed{dS_z = 10.048}$

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$$\boxed{dL = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z}$$

$S = 15.072 + 10.048 + 12$

$$\boxed{S = 37.12 \text{ units}^2}$$

⇒ DIFFERENTIAL VOLUME & SURFACES IN SPHERICAL CO-ORDINATE SYSTEM:-

$V = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$dS_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$dS_\theta = r \sin\theta \, dr \, d\phi \, \hat{a}_\theta$

$dS_\phi = r \, d\theta \, d\phi \, \hat{a}_\phi$

$dS_{-r} = r^2 \sin\theta \, d\phi \, d\theta \, (-\hat{a}_r)$

$dS_{-\theta} = r \sin\theta \, dr \, d\phi \, (-\hat{a}_\theta)$

$dS_{-\phi} = r \, d\theta \, d\phi \, (-\hat{a}_\phi)$

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$dl = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$

Q.

Q. Find the total surface area and volume

$3 < r < 5$; $0.1\pi < \theta < 0.3\pi$;

$1.2\pi < \phi < 1.6\pi$

Sol:-

$d_1 = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$

$d_2 = r\sin\theta\, dr\, d\phi\, \hat{a}_\theta$

$d_3 = r\, dr\, d\theta\, \hat{a}_\phi$

$d_4 = r^2 \sin\theta\, d\theta\, d\phi\, (-\hat{a}_r)$

$d_5 = r\sin\theta\, dr\, d\phi\, (-\hat{a}_\theta)$

$d_6 = r\, dr\, d\theta (-\hat{a}_\phi)$

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combining d_1 & d_4

$= (r^1 + r^2) \sin\theta \, d\theta \, d\phi$

$= 9 + 25 \sin\theta \, d\theta \, d\phi$

$= 34 \sin\theta \, d\theta \, d\phi$

combining d_2 & d_5

$= (\sin\theta + \sin\theta) r \, dr \, d\phi$

$= [\sin(0.1\pi) + \sin(0.3\pi)] r \, dr \, d\phi$

$= 1.118 \, r \, dr \, d\phi$

combining d_3 & d_6

$= 2r \, dr \, d\theta$

$$\Rightarrow 34 \int_{1.2\pi}^{1.6\pi}\int_{0.1\pi}^{0.3\pi} \sin\theta \, d\theta \, d\phi \quad +$$

$$1.118 \int_{1.2\pi}^{1.6\pi}\int_{3}^{5} r \, dr \, d\phi$$

$$2 + 2 \int_{0.1\pi}^{0.3\pi}\int_{3}^{5} r \, dr \, d\theta$$

$$, \; 34 \int_{1.2\pi}^{1.6\pi} -\cos\theta \Big|_{0.1\pi}^{0.3\pi} d\phi$$

$$+ \frac{1.118}{2} \int_{1.2\pi}^{1.6\pi} r^2 \Big|_3^5 \, d\phi \quad +$$

$$\frac{2}{2} \int_{0.1\pi}^{0.3\pi} r^2 \Big|_3^5 \, d\theta$$

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$$= +12.35 \int_{1.2\pi}^{1.6\pi} d\phi$$

$$+ 8.944 \int_{1.2\pi}^{1.6\pi} d\phi$$

$$+ 16 \int_{0.1\pi}^{0.3\pi} d\theta$$

$$= +12.35 \times \phi \Big|_{1.2\pi}^{1.6\pi} +$$

$$8.944 \times \phi \Big|_{1.2\pi}^{1.6\pi} +$$

$$16 \times \theta \Big|_{0.1\pi}^{0.3\pi}$$

$$= +15.519 + 11.239 + 10.05$$

$$\boxed{= 36.8 \text{ units}^2}$$

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$$dv = \int_{1.2\pi}^{1.6\pi}\int_{0.1\pi}^{0.3\pi}\int_{3}^{5} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_{1.2\pi}^{1.6\pi}\int_{0.1\pi}^{0.3\pi} \frac{r^3}{3} \Big|_{3}^{5} \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{3}\int_{1.2\pi}^{1.6\pi}\int_{0.1\pi}^{0.3\pi} 98 \sin\theta \, d\theta \, d\phi$$

$$= \frac{98}{3}\int_{1.2\pi}^{1.6\pi} -\cos\theta \Big|_{0.1\pi}^{0.3\pi} d\phi$$

$$= \frac{98}{3} \times \cancel{1.\ldots}\ 0.363 \int_{1.2\pi}^{1.6\pi} d\phi$$

$$= 11.858 \ \phi \Big|_{1.2\pi}^{1.6\pi}$$

$$\Rightarrow 11.858\,(1.6\pi - 1.2\pi)$$

$$\Rightarrow \boxed{15 \text{ units}^3}$$

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Method :02

$3 < r < 5$; $0.1\pi < \theta < 0.3\pi$;

$1.2\pi < \phi < 1.6\pi$

$dr = r_2 - r_1$
$dr = 5 - 3$
$\boxed{dr = 2}$

$d\theta = \theta_2 - \theta_1$
$d\theta = 0.3\pi - 0.1\pi$
$\boxed{d\theta = 0.2\pi}$

$d\phi = \phi_2 - \phi_1$
$d\phi = 1.6\pi - 1.2\pi$
$\boxed{d\phi = 0.4\pi}$

$dS_1 = r^2 \sin\theta \, d\theta d\phi$

$= \left(\frac{5+3}{2}\right)^2 \sin\left(\frac{0.1\pi + 0.3\pi}{2}\right) d\theta d\phi$

$= (4)^2 \sin(0.2\pi) \times 0.2\pi \times 0.4\pi$

$\boxed{= 7.425} \times 2$

$\boxed{= 14.85}$

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$dS_2 = r\sin\theta \, dr \, d\phi$

$$= \left(\frac{5+3}{2}\right) \sin\left(\frac{0.1\pi + 0.3\pi}{2}\right)$$

$$\times 2 \times 0.4\pi$$

$$= 5.909 \times 2$$

$$= 11.818$$

$dS_3 = r \, dr \, d\theta$

$$= 4 \times 2 \times 0.2\pi$$

$$= 5.026$$

$$= 5.026 \times 2$$

$$= 10.053$$

$S = 14.85 + 11.818 + 10.053$

$\boxed{S = 36.721 \text{ units}^2}$

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$V = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= (4)^2 \sin(0.2\pi) \times 2 \times 0.2\pi \times 0.4\pi$

$= \boxed{14.85 \text{ units}^3}$

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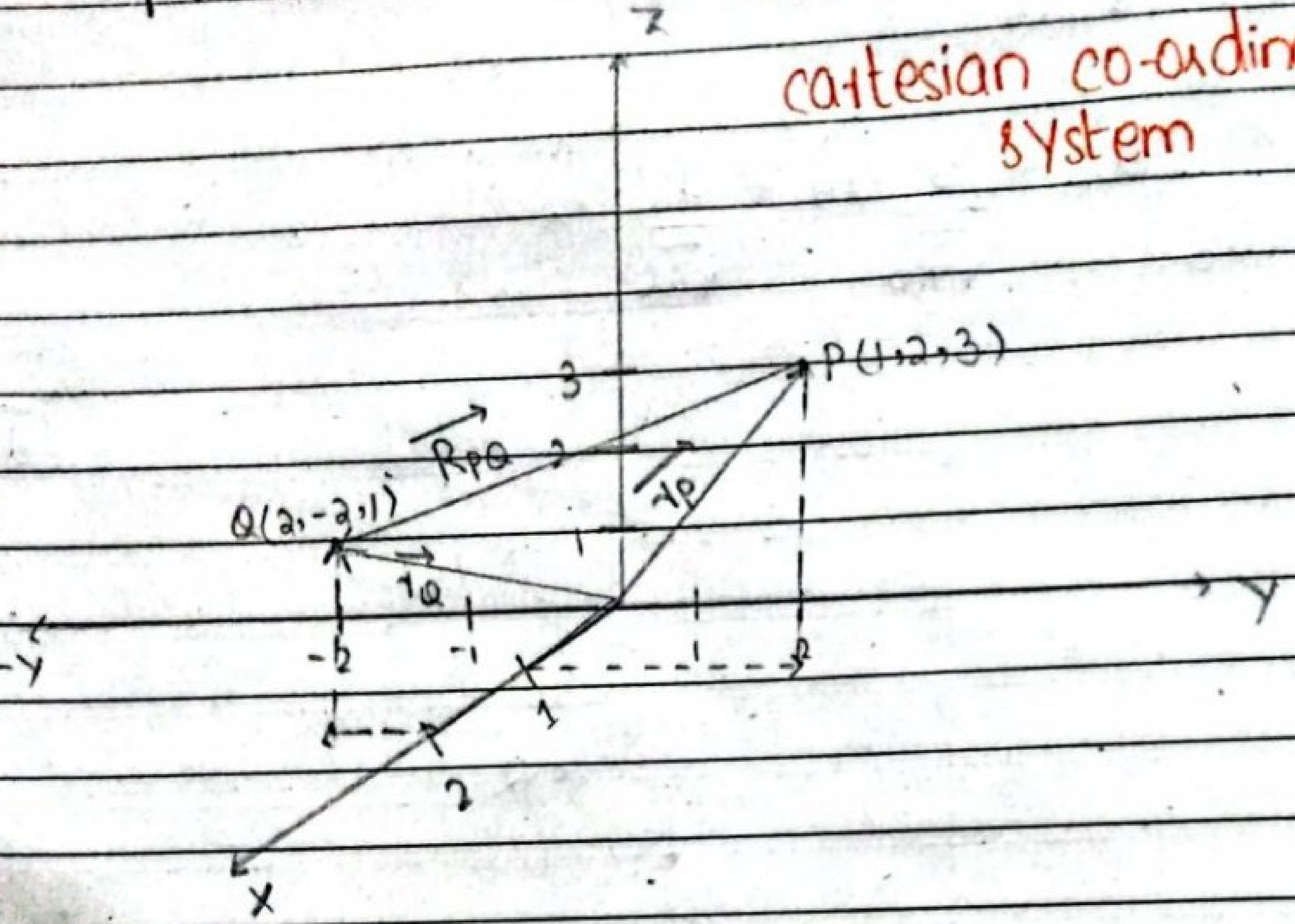
Date:

Q.

P (1, 2, 3) and Q (2, -2, 1) are two points find $\vec{R}_{PQ}$, $|\vec{R}_{PQ}|$, $\hat{a}_{PQ}$

cartesian co-ordinate system



$$\vec{R}_{PQ} = \vec{r}_Q - \vec{r}_P$$

$$= (2\hat{a}_x - 2\hat{a}_y + \hat{a}_z) - (\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)$$

$$= (2-1)\hat{a}_x + (-2-2)\hat{a}_y + (1-3)\hat{a}_z$$

$$= \hat{a}_x - 4\hat{a}_y - 2\hat{a}_z$$

$$|\vec{R}_{PQ}| = \sqrt{1^2 + (-4)^2 + (-2)^2}$$

$$= \sqrt{1+16+4}$$

$$= \sqrt{21}$$

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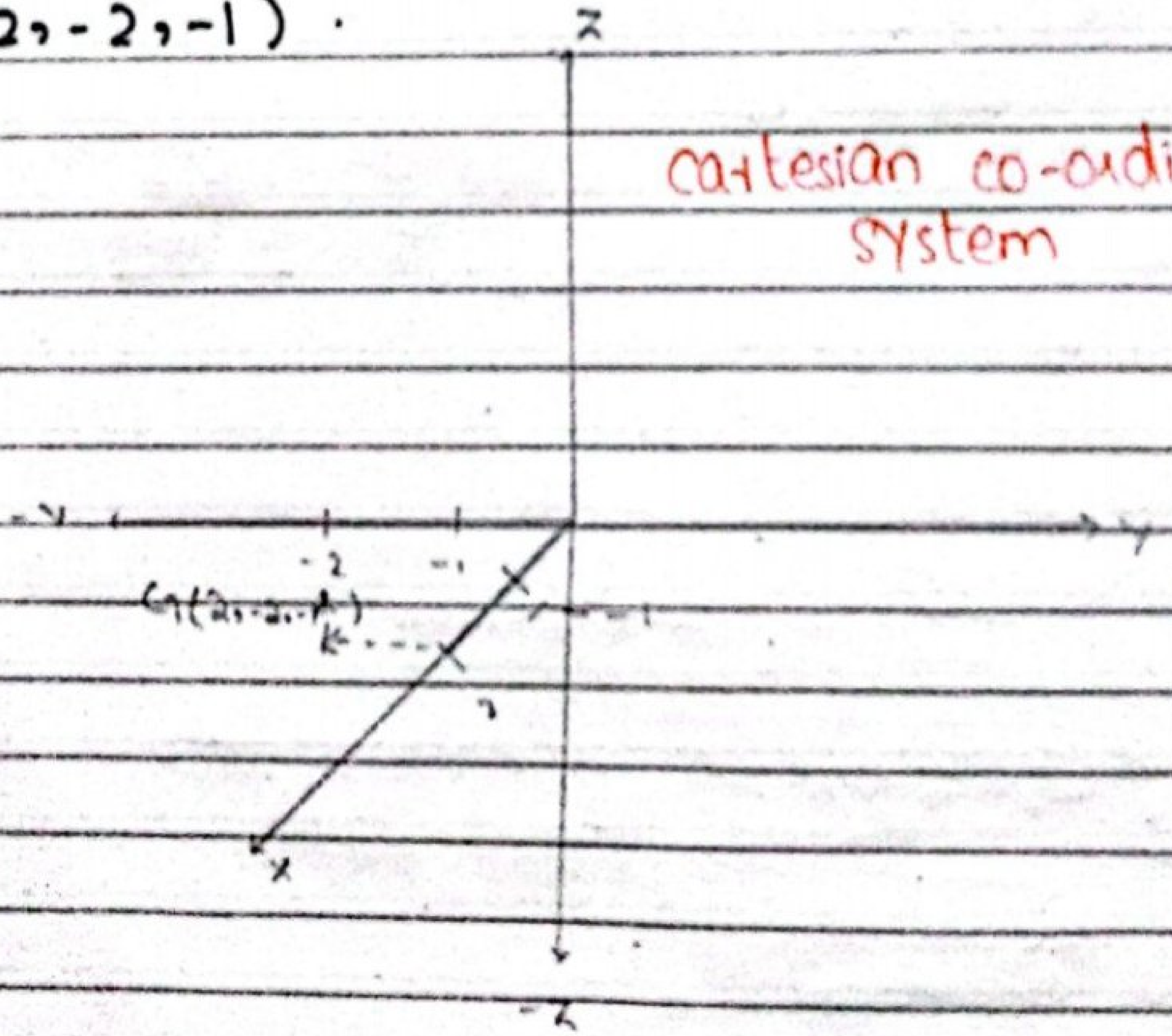
$$\hat{a}_{pq} = \frac{\vec{R}_{PQ}}{|\vec{R}_{PQ}|}$$

$$= \frac{\hat{a}_x - 4\hat{a}_y - 2\hat{a}_z}{\sqrt{21}}$$

$$= \frac{1}{\sqrt{21}}\hat{a}_x - \frac{4}{\sqrt{21}}\hat{a}_y - \frac{2}{\sqrt{21}}\hat{a}_z$$

EXAMPLE 1·1

Specify the unit vector extending from the origin toward the point $G(2,-2,-1)$.



$\vec{G} = 2a_x - 2a_y - a_z$

$|\vec{G}| = \sqrt{(2)^2+(-2)^2+(-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

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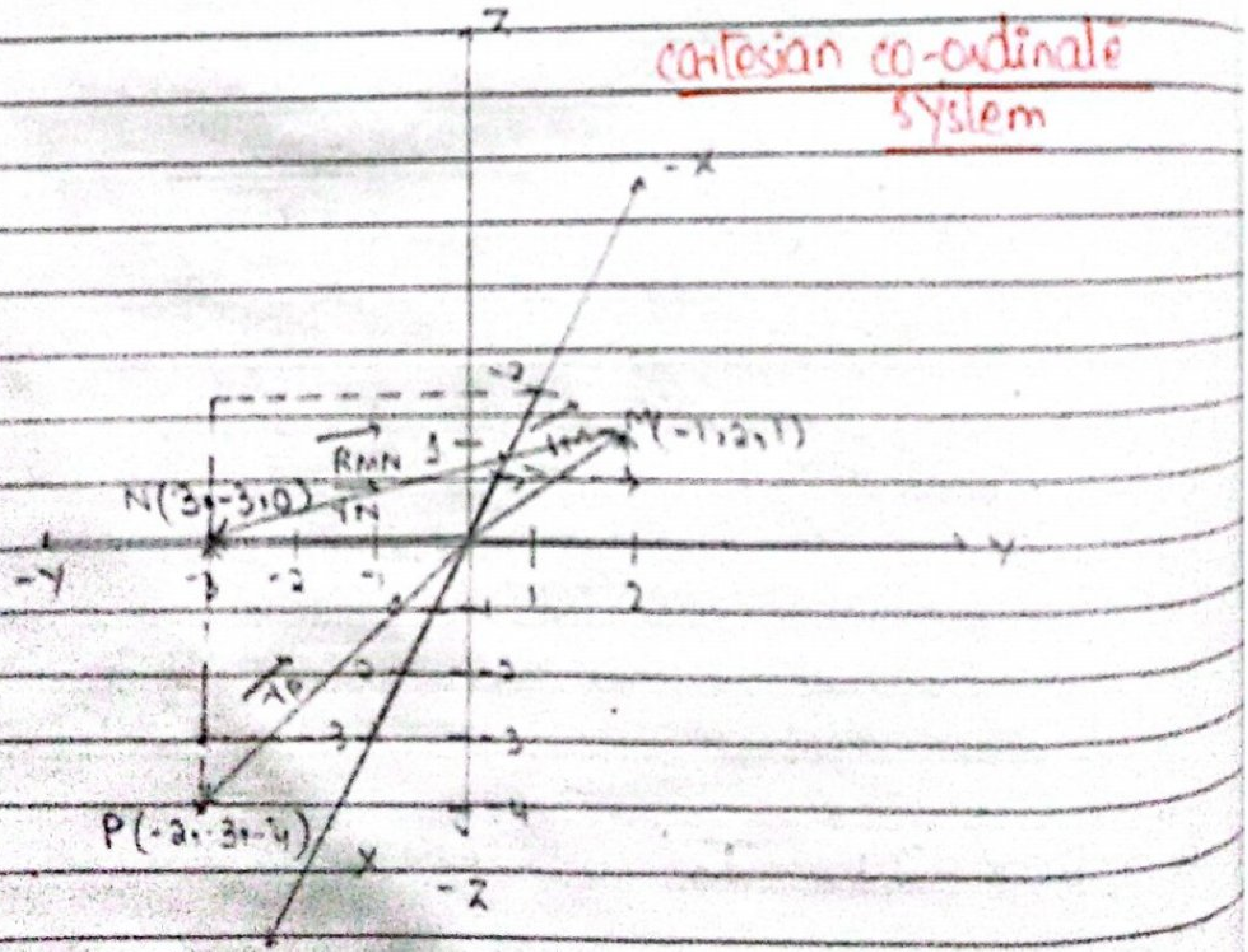
$$\hat{a}_G = \frac{\vec{G}}{|\vec{G}|}$$

$$= \frac{2}{3}\hat{a}_x - \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_x$$

D1.1. Given points M(-1, 2, 1), N(3, -3, 0) and P(-2, -3, -4) find

(a) R_{MN} ; (b) $R_{MN} + R_{MP}$ (c) $|R_M|$

(d) a_{MP} (e) $|2r_P - 3r_N|$



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Date:

(a)

$\vec{R}_{MN} = \vec{r}_N - \vec{r}_M$

$= (3\hat{a}_x - 3\hat{a}_y + 0\hat{a}_z) - (-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$

$= (3+1)\hat{a}_x + (-3-2)\hat{a}_y + (0-1)\hat{a}_z$

$= 4\hat{a}_x - 5\hat{a}_y - \hat{a}_z$

(b)

$\vec{R}_{MP} = \vec{r}_P - \vec{r}_M$

$= (-2\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z) - (3\hat{a}_x - 3\hat{a}_y + 0\hat{a}_z)$

$= (-2-3)\hat{a}_x + (-3+3)\hat{a}_y + (-4-0)\hat{a}_z$

$= -5\hat{a}_x - 4\hat{a}_z$

$\vec{R}_{MN} + \vec{R}_{MP} = (4\hat{a}_x - 5\hat{a}_y - \hat{a}_z) + (-5\hat{a}_x - 4\hat{a}_z)$

$= -\hat{a}_x - 5\hat{a}_y - 5\hat{a}_z$

(b)

$\vec{R}_{MP} = \vec{r}_P - \vec{r}_M$

$= (-2\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z) - (-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$

$= (-2+1)\hat{a}_x + (-3-2)\hat{a}_y + (-4-1)\hat{a}_z$

$= -\hat{a}_x - 5\hat{a}_y - 5\hat{a}_z$

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$R_{MN} + R_{MP} = (4\hat{a_x} - 5\hat{a_y} - \hat{a_z}) +$

$(-\hat{a_x} - 5\hat{a_y} - 5\hat{a_z})$

$= 3\hat{a_x} - 10\hat{a_y} - 6\hat{a_z}$

(c)

$|R_M| = ?$

$|R_M| = \sqrt{(-1)^2 + (2)^2 + (1)^2}$

$= \sqrt{1 + 4 + 1}$

$= \sqrt{6}$

(d)

$\hat{a}_{MP} = \frac{\vec{R}_{MP}}{|\vec{R}_{MP}|}$

$|\vec{R}_{MP}| = \sqrt{(-1)^2 + (-5)^2 + (-5)^2}$

$= \sqrt{1 + 25 + 25}$

$= \sqrt{51}$

$\hat{a}_{MP} = \frac{-1}{\sqrt{51}} \hat{a_x} - \frac{5}{\sqrt{51}} \hat{a_y} - \frac{5}{\sqrt{51}} \hat{a_z}$

(e) $|2r_P - 3r_N|$

Sol:-

$$2r_P - 3r_N = 2(-2\hat{a}_x - 3\hat{a}_y - 4\hat{a}_z) - 3(3\hat{a}_x - 3\hat{a}_y)$$

$$= (-4\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z) - (9\hat{a}_x - 9\hat{a}_y)$$

$$= -13\hat{a}_x + 3\hat{a}_y - 8\hat{a}_z$$

$$|2r_P - 3r_N| = \sqrt{(-13)^2 + (3)^2 + (-8)^2}$$

$$= 15.5$$

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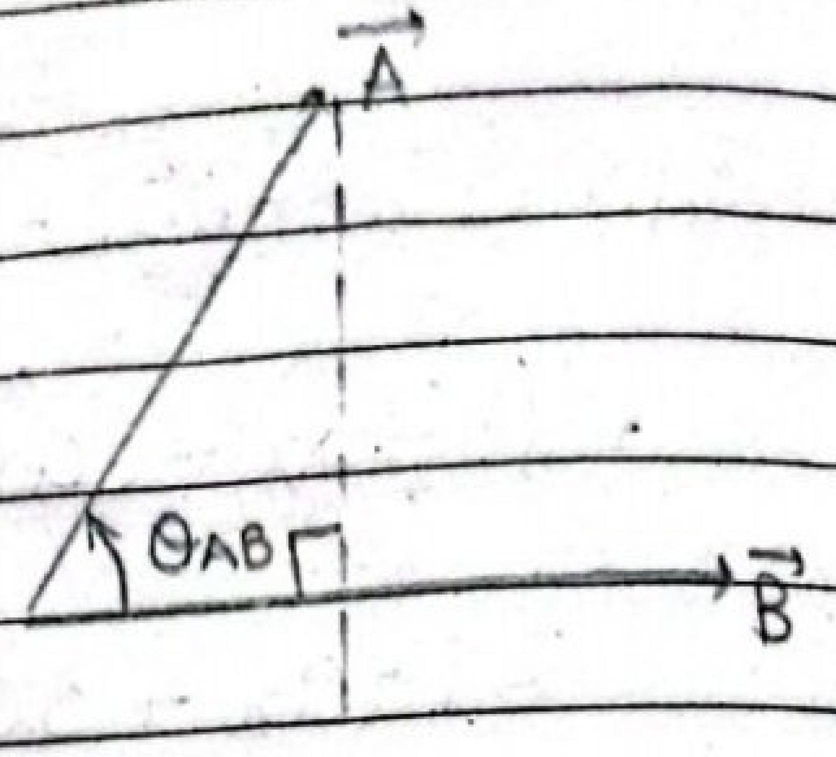
THE DOT PRODUCT:

⇒ It is also known as scalar product.

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$



$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

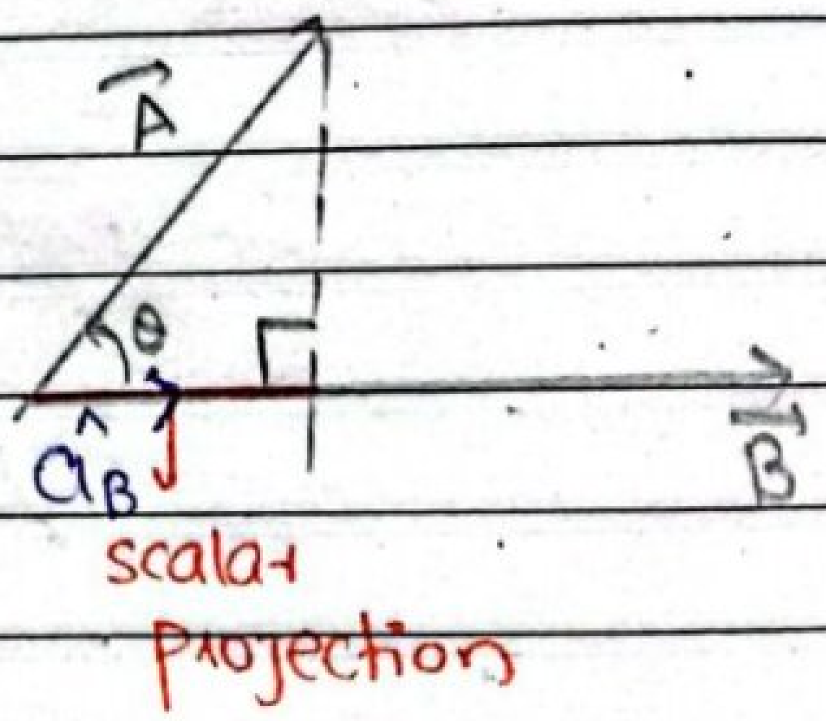
$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

scalar projection of $\vec{A}$ on $\vec{B}$:

$\Rightarrow \vec{A}\cdot\hat{a}_B$

$\Rightarrow |\vec{A}|\cos\theta_{AB}$



Vector projection of $\vec{A}$ on $\vec{B}$:

↳ vector projection doesn't mean $A \times B$

$\Rightarrow (\vec{A}\cdot\hat{a}_B)\hat{a}_B$

$a_x\cdot a_y = a_y\cdot a_x = a_y\cdot a_z = a_z\cdot a_y$
$= a_z\cdot a_x = a_x\cdot a_z = 0$

↳ $90^\circ \Rightarrow \cos 90^\circ = 0$

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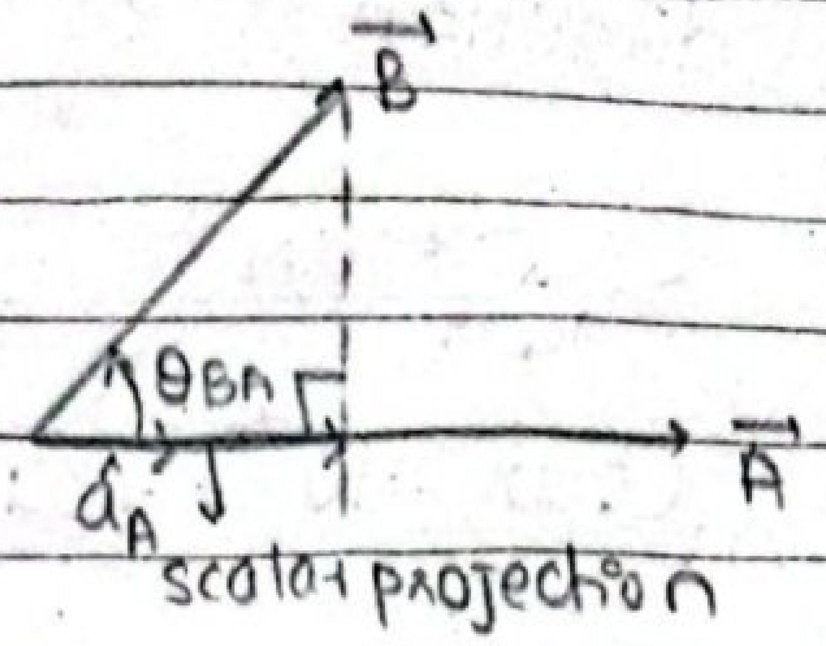
=> Scalar projection of $\vec{B}$ on $\vec{A}$:

=> $\vec{B} \cdot \hat{a}_A$

=> $|\vec{B}| \cos\theta_{BA}$



=> Vector projection of $\vec{B}$ on $\vec{A}$

=> $(\vec{B} \cdot \hat{a}_A)\,\hat{a}_A$

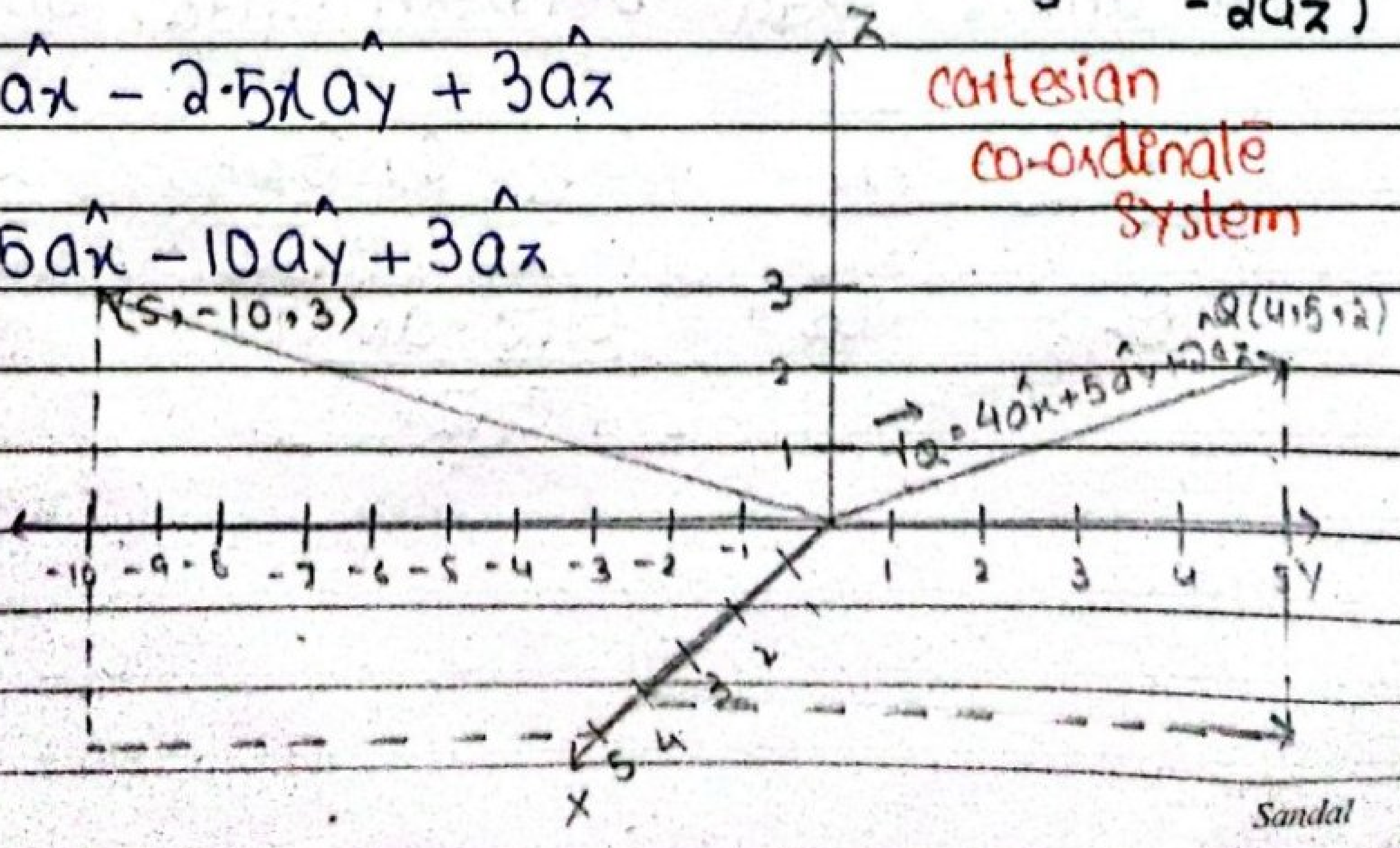
EXAMPLE 1-2

consider the vector field $G = y\hat{a}_x - 2.5x\hat{a}_y + 3\hat{a}_z$ and the point $Q(4,5,2)$. we wish to find G at Q; the scalar component of G at Q in the direction of a_N; and finally, the angle θ_{GA} between $G(r_Q)$ and a_N also find vector component of G at Q in the direction of $a_N = \frac{1}{3}(2\hat{a}_x + \hat{a}_y - 2\hat{a}_z)$

Sol:-

$G = y\hat{a}_x - 2.5x\hat{a}_y + 3\hat{a}_z$

$G_{\vec{r}_Q} = 5\hat{a}_x - 10\hat{a}_y + 3\hat{a}_z$



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scalar component of G at Q in the direction of $\hat{a}_N$

$$\Rightarrow G_{\vec{r}Q} \cdot \hat{a}_N$$

$$\Rightarrow (5\hat{a_x} - 10\hat{a_y} + 3\hat{a_z}) \cdot \left(\frac{2}{3}\hat{a_x} + \frac{1}{3}\hat{a_y} - \frac{2}{3}\hat{a_z}\right)$$

$$\Rightarrow \frac{10}{3} - \frac{10}{3} - \frac{6}{3}$$

$$\Rightarrow -2$$

vector component of G at Q in the direction of $\hat{a}_N$

$$\Rightarrow (G_{\vec{r}Q} \cdot \hat{a}_N)\hat{a}_N$$

$$= -2\left(\frac{2}{3}\hat{a_x} + \frac{1}{3}\hat{a_y} - \frac{2}{3}\hat{a_z}\right)$$

$$= \frac{-4}{3}\hat{a_x} - \frac{2}{3}\hat{a_y} + \frac{4}{3}\hat{a_z}$$

The angle θ_{Ga} between $G(\vec{r}Q)$ and $\hat{a}_N$

$$\vec{G}_{\vec{r}Q} \cdot \hat{a}_N = |G_{\vec{r}Q}| \cos\theta_{G\hat{a}}$$

$$\theta_{G\hat{a}} = \cos^{-1}\left[\frac{G_{\vec{r}Q} \cdot \hat{a}_N}{|G_{\vec{r}Q}|}\right]$$

$|\vec{G}_{Q}| = \sqrt{(5)^2 + (-10)^2 + (3)^2}$

$= \sqrt{134}$

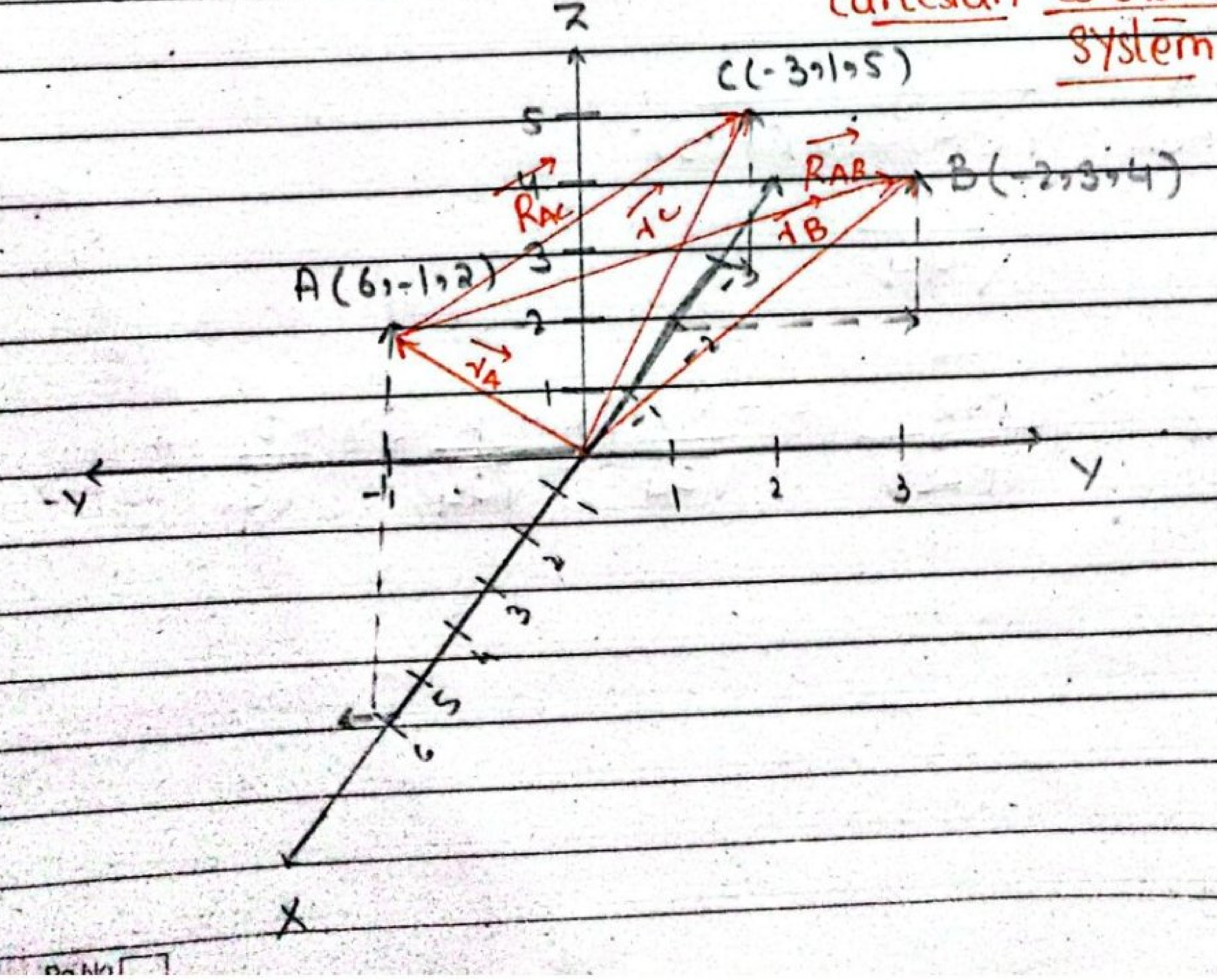
$\theta_{G\hat{a}} = \cos^{-1}\left[\dfrac{-2}{\sqrt{134}}\right]$

$\boxed{\theta_{G\hat{a}} = 99.9^{\circ}}$

D 1.3.

The three vertices of a triangle are located at $A(6,-1,2)$, $B(-2,3,4)$, and $C(-3,1,5)$ Find (a) R_{AB} (b) R_{AC} (c) the angle θ_{BAC} at vertex A; (d) the vector projection of R_{AB} on R_{AC}.

cartesian co-ordinate system



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(a) $\vec{R}_{AB} = \vec{r}_B - \vec{r}_A$

$= (-2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z) - (6\hat{a}_x - \hat{a}_y + 2\hat{a}_z)$

$= -8\hat{a}_x + 4\hat{a}_y + \overset{2}{\cancel{6}}\hat{a}_z$

(b) $\vec{R}_{AC} = \vec{r}_C - \vec{r}_A$

$= (-3\hat{a}_x + \hat{a}_y + 5\hat{a}_z) - (6\hat{a}_x - \hat{a}_y + 2\hat{a}_z)$

$= -9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

(d)

the vector projection of $\vec{R}_{AB}$ on $\vec{R}_{AC}$

$\Rightarrow (\vec{R}_{AB} \cdot \hat{a}_{\vec{R}_{AC}})\, \hat{a}_{\vec{R}_{AC}} \longrightarrow (1)$

$|\vec{R}_{AC}| = \sqrt{(-9)^2 + (2)^2 + (3)^2}$

$= \sqrt{94}$

$\hat{a}_{\vec{R}_{AC}} = \dfrac{\vec{R}_{AC}}{|\vec{R}_{AC}|} = \dfrac{-9}{\sqrt{94}}\hat{a}_x + \dfrac{2}{\sqrt{94}}\hat{a}_y + \dfrac{3}{\sqrt{94}}\hat{a}_z$

$\Rightarrow$ eq(1) becomes

$$\Rightarrow (-8\hat{a_x} + 4\hat{a_y} + 2\hat{a_z}) \cdot \left(\frac{-9}{\sqrt{94}}\hat{a_x} + \frac{2}{\sqrt{94}}\hat{a_y} + \frac{3}{\sqrt{94}}\hat{a_z}\right) \cdot$$

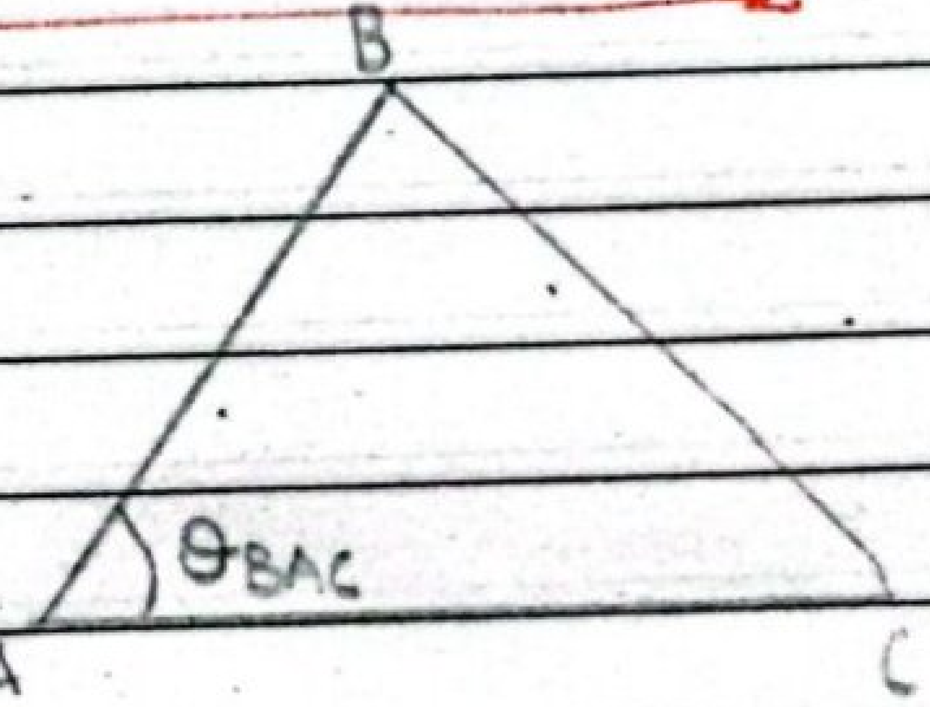
$$\hat{a}_{\overrightarrow{RAC}}$$

$$\Rightarrow (7.426 + 0.825 + 0.618)\,\hat{a}_{\overrightarrow{RAC}}$$

$$= 8.869\left(\frac{-9}{\sqrt{94}}\hat{a_x} + \frac{2}{\sqrt{94}}\hat{a_y} + \frac{3}{\sqrt{94}}\hat{a_z}\right)$$

vector Projection $= \boxed{-8.232\,\hat{a_x} + 1.829\,\hat{a_y} + 2.74\,\hat{a_z}}$

(c)



$$\overrightarrow{R_{AB}} \cdot \overrightarrow{R_{AC}} = |\overrightarrow{R_{AB}}||\overrightarrow{R_{AC}}|\cos\theta \quad \text{(1)}$$

$$|\overrightarrow{R_{AB}}| = \sqrt{(-8)^2 + (4)^2 + (2)^2}$$

$$= 2\sqrt{21}$$

$$|\overrightarrow{R_{AC}}| = \sqrt{94}$$

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Date:

$$\overrightarrow{R_{AB}} \cdot \overrightarrow{R_{AC}} = (-8\hat{a}_x + 4\hat{a}_y + 2\hat{a}_z) \cdot (-9\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)$$

$$= 72 + 8 + 6$$

$$= 86$$

so eq(1) becomes

$$86 = 2\sqrt{21} \times \sqrt{94} \cos\theta_{BAC}$$

$$\theta_{BAC} = \cos^{-1}\left[\frac{86}{88.85}\right]$$

$$\boxed{\theta_{BAC} = 14.55^{\circ}}$$

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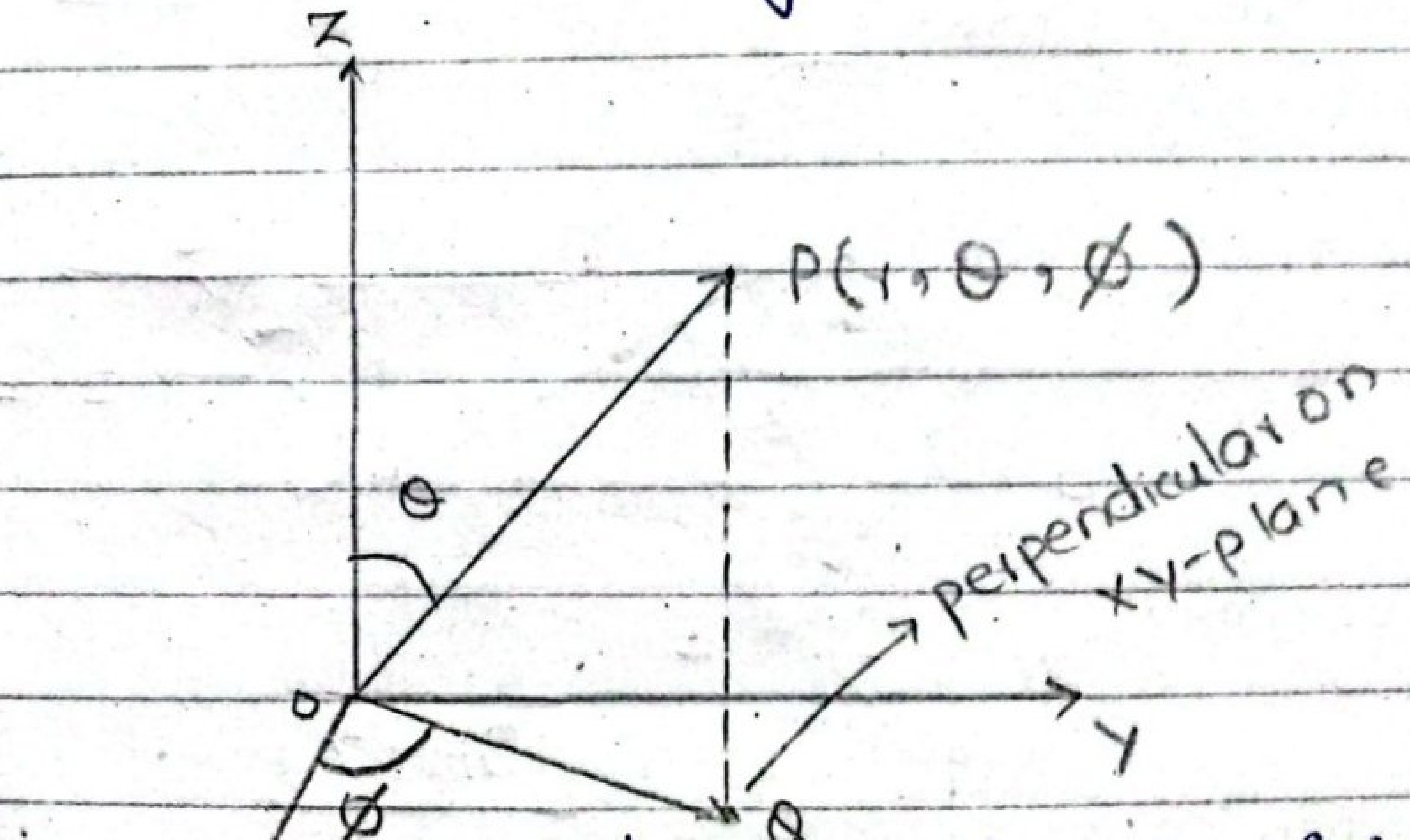
⇒ SPHERICAL POLAR CO-ORDINATE SYSTEM:

↳ (r, θ, ϕ)

r = Radial distance

θ : zenith angle

ϕ : azimuthal angle



r = distance between origin & observation point

θ : line OP makes an angle with z-axis

ϕ = line OQ makes an angle with x-axis.

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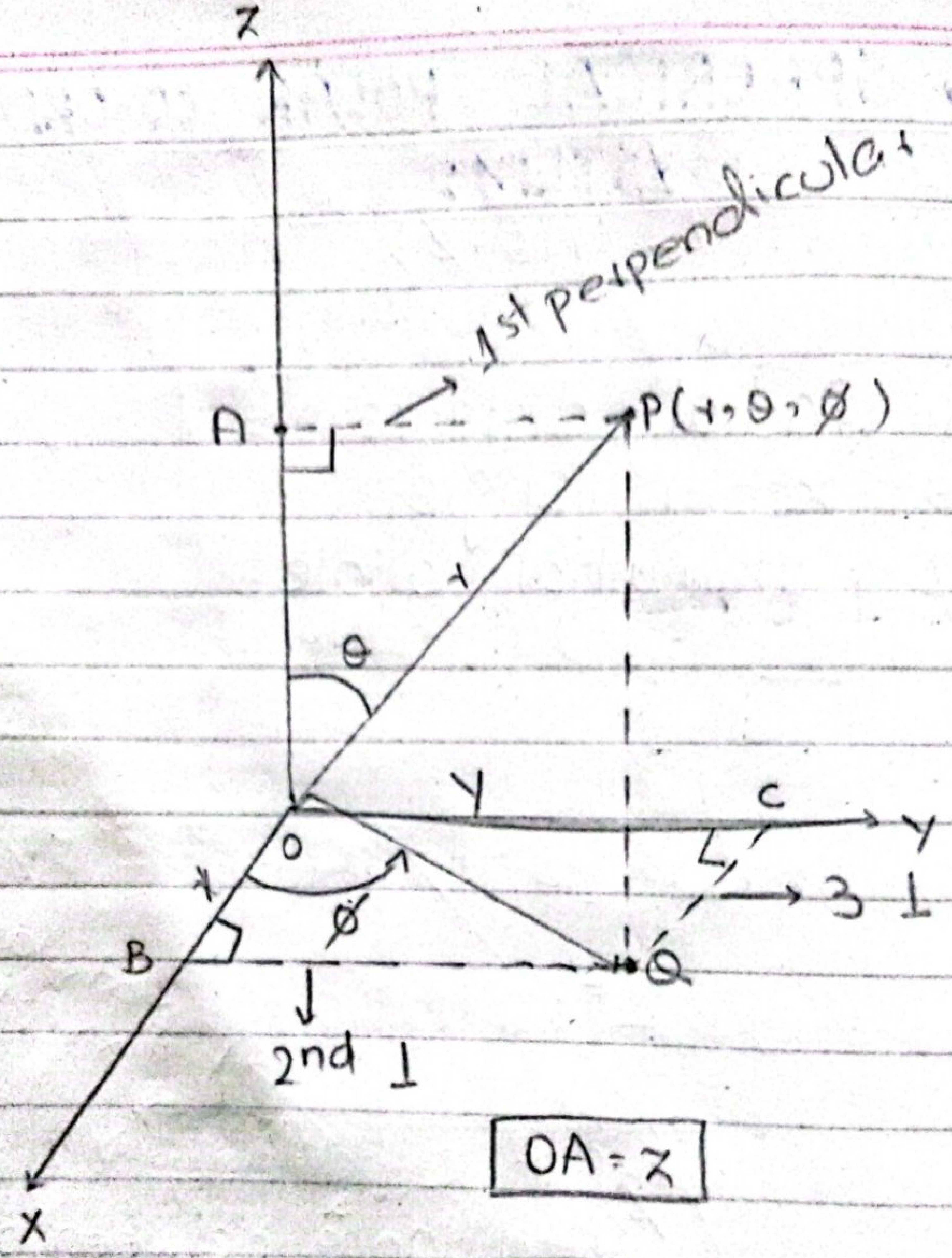


$\boxed{OA = z}$

$\boxed{OB = x}$

$\boxed{OC = y}$

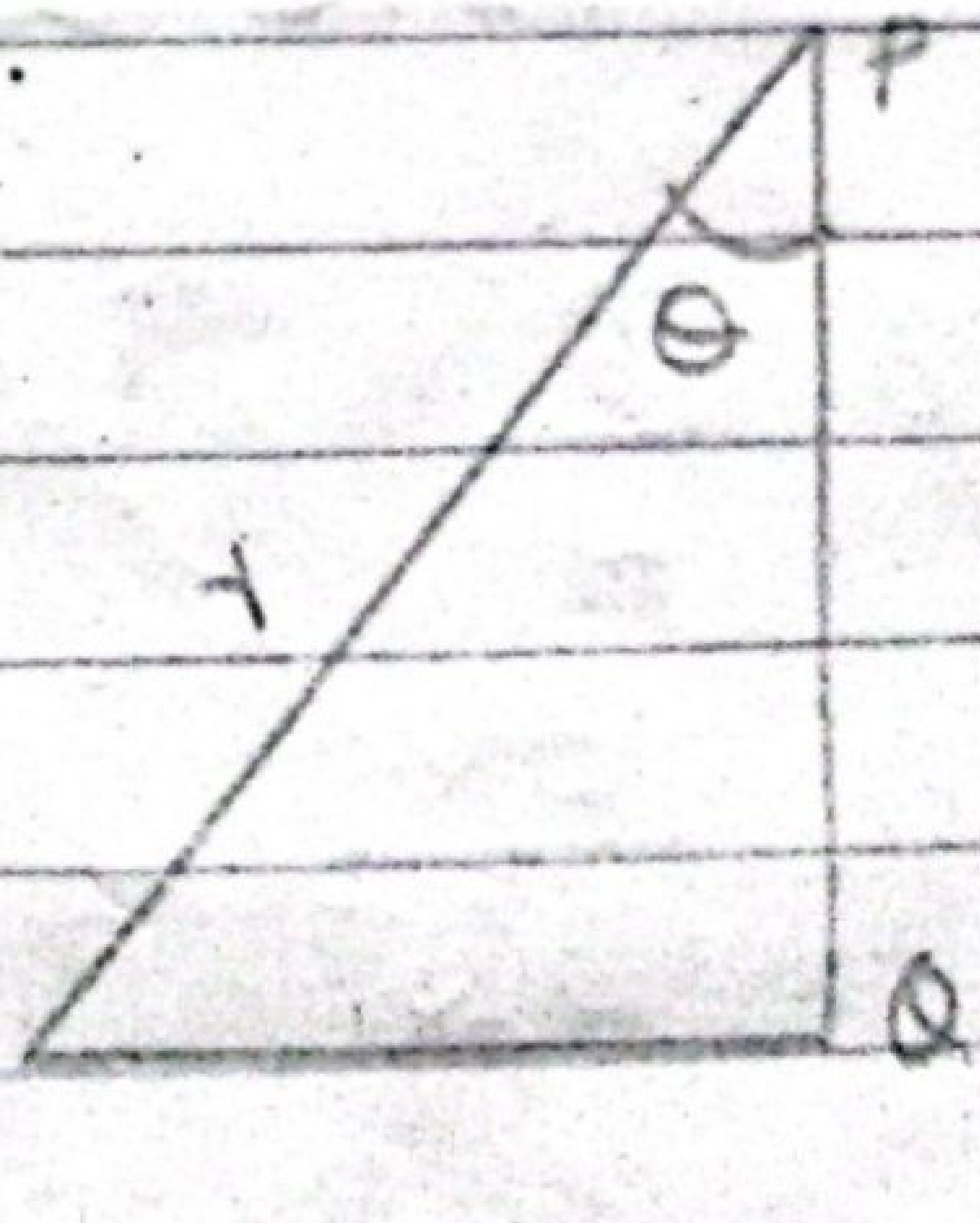
$\Delta\ OPA$

$$\cos\theta = \frac{B}{H} = \cos\theta = \frac{OA}{OP}$$

$$\cos\theta = \frac{x}{r}$$

$$\boxed{x = r\cos\theta}$$

ΔOPQ



$$\sin\theta = \frac{P}{H}$$

$$= \frac{OQ}{OP}$$

$$= \frac{OQ}{r}$$

$$\boxed{OQ = r\sin\theta}$$

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ΔOBQ

$$\cos\phi = \frac{B}{H}$$

$$\cos\phi = \frac{x}{OQ}$$

$$\cos\phi = \frac{x}{r\sin\theta}$$

$$\boxed{x = r\sin\theta\cos\phi}$$

$$\sin\phi = \frac{P}{H}$$

$$\sin\phi = \frac{BQ}{OQ}$$

$$\sin\phi = \frac{Y}{r\sin\theta}$$

$$\boxed{Y = r\sin\theta\sin\phi}$$

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$$\boxed{x = r\sin\theta\cos\phi} \longrightarrow (1)$$

$$\boxed{y = r\sin\theta\sin\phi} \longrightarrow (2)$$

$$\boxed{z = r\cos\theta} \longrightarrow (3)$$

squaring & adding eq(1) & (2) & (3)

$$x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$$

$$= r^2\sin^2\theta(\cos^2\phi+\sin^2\phi) + r^2\cos^2\phi$$

$$x^2+y^2+z^2 = r^2\sin^2\theta + r^2\cos^2\theta$$

$$x^2+y^2+z^2 = r^2(\sin^2\theta+\cos^2\theta)$$

$$x^2+y^2+z^2 = r^2(1)$$

$$x^2+y^2+z^2 = r^2$$

$$\boxed{r = \sqrt{x^2+y^2+z^2}}$$

$$z = r\cos\theta$$

$$\cos\theta = \frac{z}{r}$$

$$\cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right]$$

Divide eq(1) & (2)

$$\frac{x}{y} = \frac{r\cancel{\sin\theta}\cos\phi}{r\cancel{\sin\theta}\sin\phi}$$

$$\frac{x}{y} = \frac{\cos\phi}{\sin\phi}$$

$$\frac{y}{x} = \frac{\sin\phi}{\cos\phi}$$

$$\frac{y}{x} = \tan\phi$$

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$$\boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right]$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

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=> Example 1.4

$G = \left(\frac{xz}{y}\right) \hat{a_x}$ into spherical components & variables

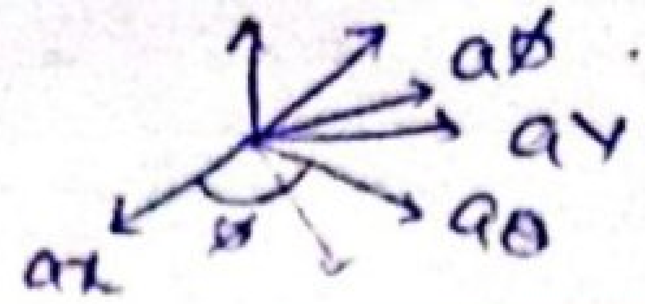
Sol:-

$G_r = \vec{G} \cdot \hat{a_r}$

$= \frac{xz}{y} \hat{a_x} \cdot \hat{a_r}$

$= \frac{\cancel{r \sin\theta} \cos\phi \; r\cos\theta}{\cancel{r \sin\theta} \sin\phi} \sin\theta \cos\phi$

$= \frac{r \cos\theta \sin\theta \cos^2\phi}{\sin\phi}$



$G_\theta \,.\, \vec{G} \cdot \hat{a}_\theta$

$= \dfrac{xz}{y} \quad \hat{a}_x \cdot \hat{a}_\theta$

$= \dfrac{r\sin\theta\cos\phi \cdot r\cos\theta}{r\sin\theta\sin\phi} \quad \cos\theta\cos\phi$

$= \dfrac{r\cos^2\theta\,\cos^2\phi}{\sin\phi}$

$G_\phi - \vec{G} \cdot \hat{a}_\phi$

$= \dfrac{xz}{y} \quad \hat{a}_x \cdot \hat{a}_\phi$

$= \dfrac{xz}{y} \quad -\sin\phi$

$= \dfrac{r\sin\theta\cos\phi \cdot r\cos\theta \; -\sin\phi}{r\sin\theta\sin\phi}$

$= -r\cos\theta\cos\phi$

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$$= \frac{r\cos\theta\sin\theta\cos^2\phi}{\sin\phi}\,\hat{a}_r$$

$$+ \frac{r\cos^2\theta\cos^2\phi}{\sin\phi}\,\hat{a}_\theta - r\cos\theta\cos\phi\,\hat{a}_\phi$$

D 1.7

Given two points $C(-3, 2, 1)$ and $D(r=5, \theta=20°, \phi=-70°)$

(a) find the spherical co-ordinates of C;

(b) the rectangular co-ordinates of D

(c) the distance from C to D

Sol:

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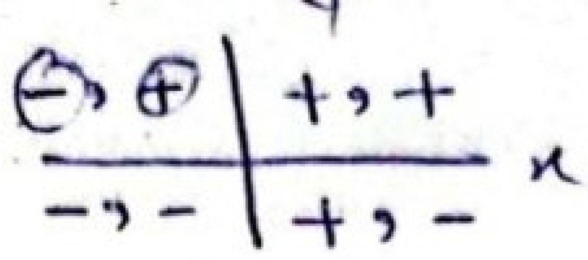
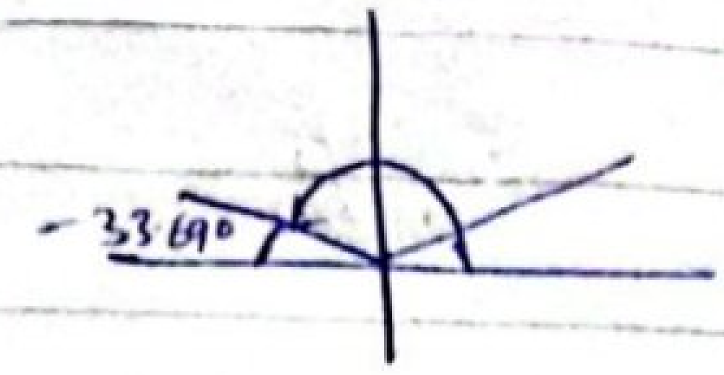


(a)

$C(-3, 2, 1)$



$r = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{(-3)^2+(2)^2+(1)^2}$

$r = \sqrt{14}$ units

$\phi = \tan^{-1}\left(\frac{2}{-3}\right)$

$\phi = -33.69^\circ + 180^\circ$

$= 146.3^\circ$

$\theta = \cos^{-1}\left[\frac{z}{r}\right]$

$= \cos^{-1}\left[\frac{1}{\sqrt{14}}\right] = 74.49^\circ$

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$(\sqrt{14}, 74.49°, 146.3°)$

(b)

$D(r = 5, \theta = 20°, \phi = -70°)$

$x = r \sin\theta \cos\phi$

$x = 5 \sin 20° \cos(-70°)$

$\boxed{x = 0.584}$

$y = r \sin\theta \sin\phi$

$y = 5 \sin 20° \sin(-70°)$

$\boxed{y = -1.606}$

$z = r \cos\theta$

$z = 5 \cos 20°$

$\boxed{z = 4.698}$

$D(0.584, -1.606, 4.698)$

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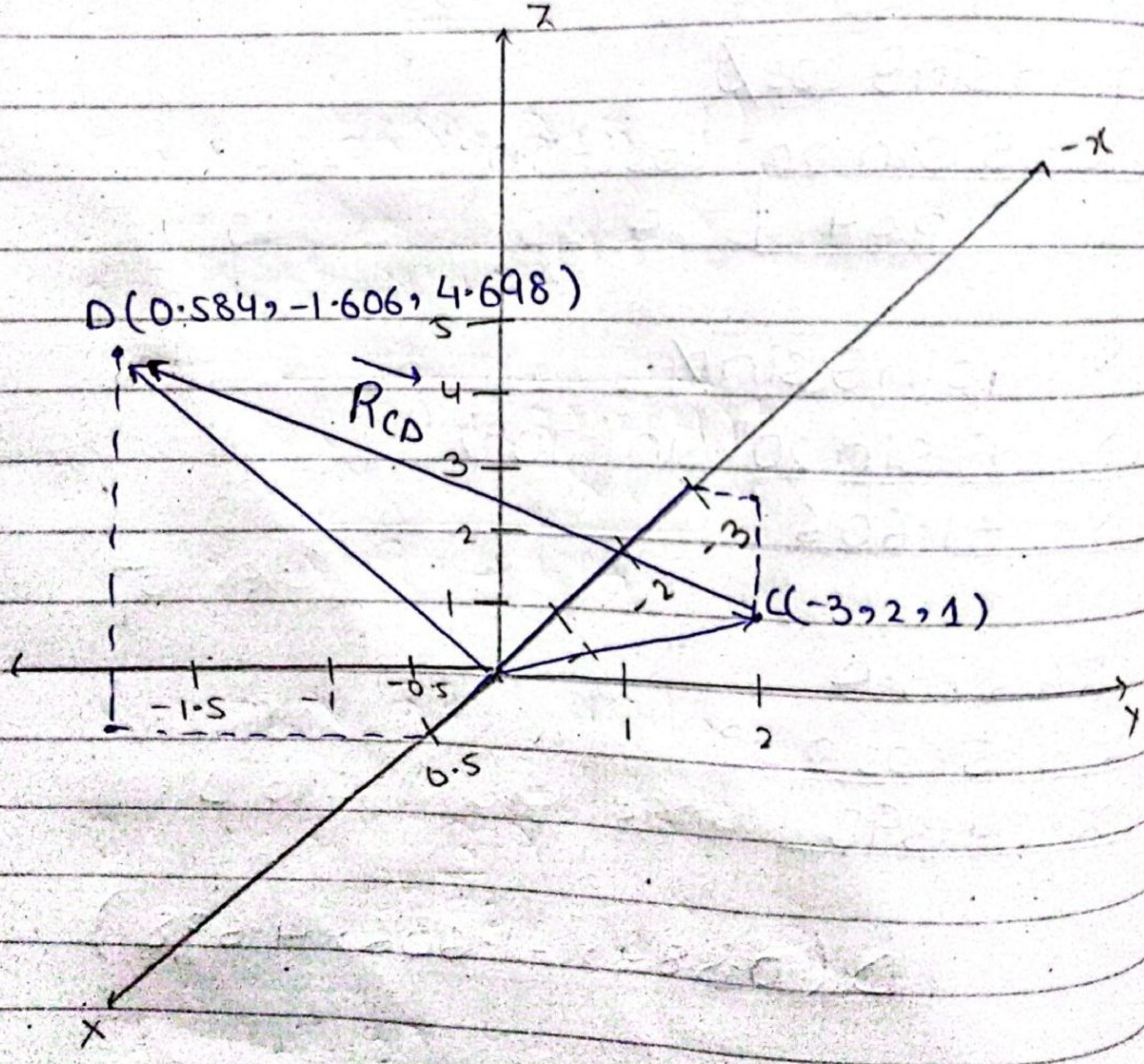
(C)

The distance from C to D

C , $(-3, 2, \underline{1})$

$D = (0.584, -1.606, 4.698)$



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$R_{CD} = \vec{r_D} - \vec{r_C}$

$= (0.584, -1.606, 4.698) -$

$(-3, 2, 1)$

$= (3.584, -3.606, 3.698)$

$|R_{CD}| = \sqrt{(3.584)^2 + (-3.606)^2 + (3.698)^2}$

$\boxed{|R_{CD}| = 6.286 \text{ units}}$

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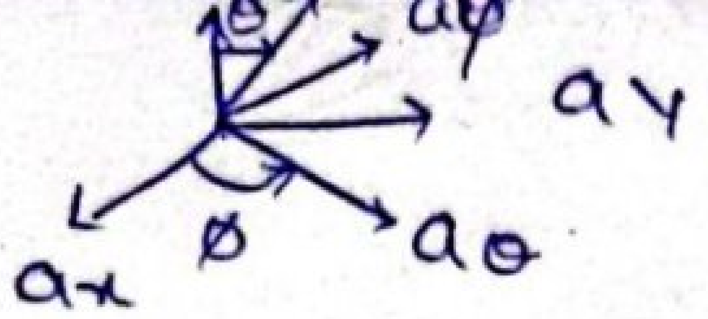

(C) $10\,\hat{a}_z$ at $M(r=4, \theta=110^\circ, \phi=120^\circ)$

Sol:-

$$A_r = \vec{A}\cdot\hat{a}_r$$
$$= 10\,\hat{a}_z\cdot\hat{a}_r$$
$$= 10\cos\theta$$
$$= 10\cos 110^\circ$$
$$= -3.42$$

$$A_\theta = \vec{A}\cdot\hat{a}_\theta$$
$$= 10\,\hat{a}_z\cdot\hat{a}_\theta$$
$$= -10\sin\theta$$
$$= -10\sin(110^\circ)$$
$$= -9.396$$

$$A_\phi = \vec{A}\cdot\hat{a}_\phi$$
$$= 10\,\hat{a}_z\cdot\hat{a}_\phi$$
$$= 0$$

$$= -3.42\,\hat{a}_r - 9.396\,\hat{a}_\theta$$

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