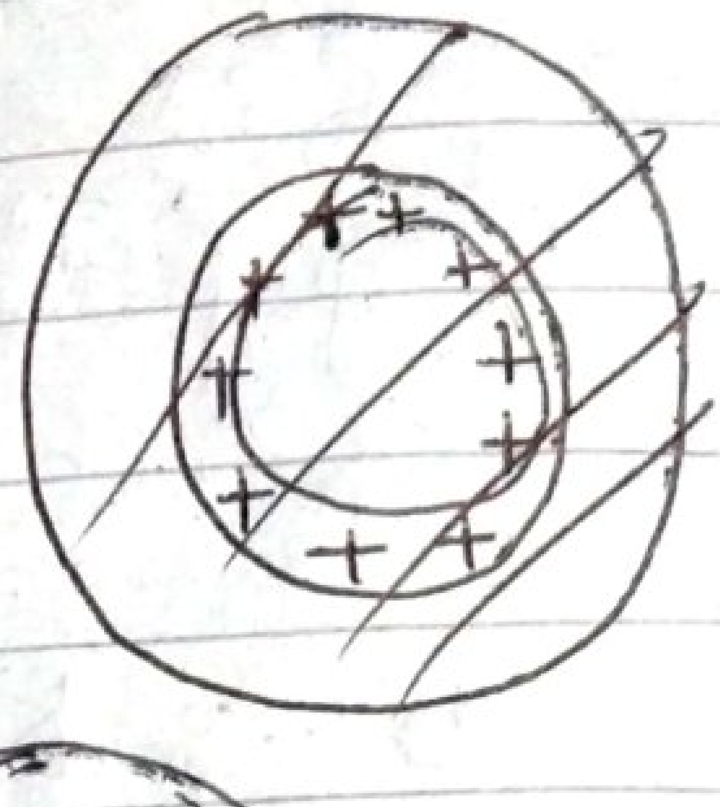
Chapter : 03

Faraday's experiment:

$\cancel{\Psi} = \cancel{\phi}$

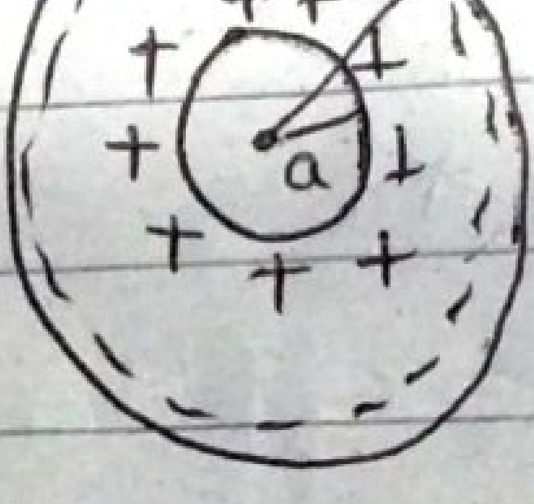


$\Rightarrow \boxed{\Psi = Q}$

$\Rightarrow \Psi \propto Q$

$\Psi = KQ$

$\boxed{\Psi = Q}$



$D_s = \frac{\Psi}{A} = \frac{Q}{4\pi r^2} = \frac{Q}{4\pi a^2}$ inner sphere

$\boxed{D_s = \frac{Q}{4\pi b^2} \quad \text{outer sphere}}$

point charge: (0D) $\rightarrow D_r = \frac{Q}{4\pi r^2} \hat{a_r}$

(1D)

line charge: $D_L = \frac{\rho_L}{2\pi\rho} \hat{a_\rho}$

(2D)

surface charge: $D_s = \frac{\rho_s}{2} \hat{a_N}$

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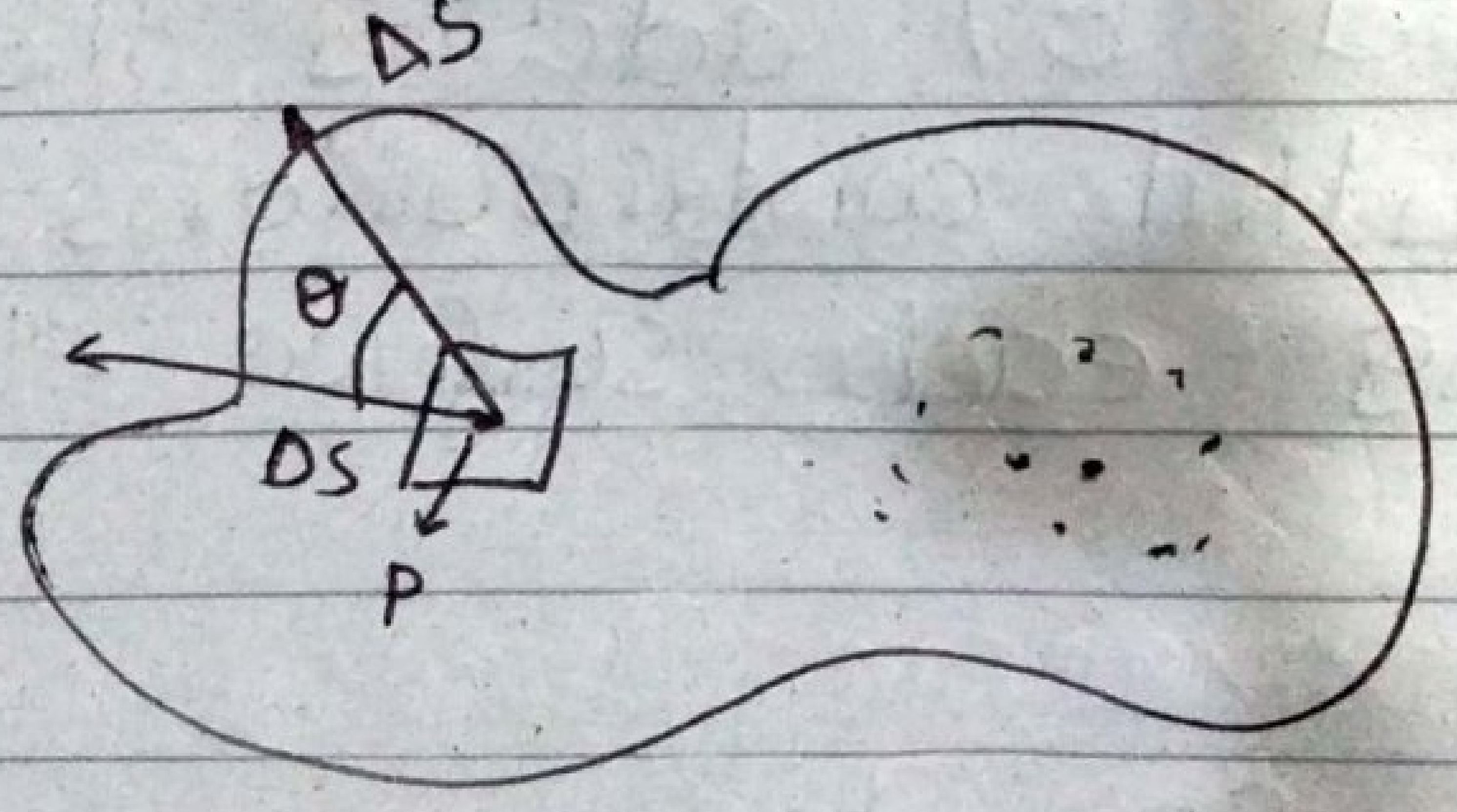
volume charge: $D_v = \frac{\int \rho_v dv}{4\pi r^2} \hat{a}_r$ (3D)

$\boxed{D = \epsilon_0 E}$

⇒ Gauss's Law:

"The total electric flux passing through any closed surface is equal to the total charge enclosed by that surface"

⇒ 

⇒ At any point 'p' consider an incremental element of surface Δs and let D_s make an angle 'θ' with Δs

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⇒ The flux crossing ΔS is then the product of the normal component of $D_{S\,normal}$ and ΔS.

$$\Delta\psi = D_{S\,normal}\,\Delta S$$

$$\Delta\psi = D_S \cdot \Delta S$$

$$d\psi = D_S \cdot dS = D_S \cos\theta \, dS$$

⇒ The total electric flux through any closed surface is obtained by adding the differential contributions crossing each surface element ΔS.

$$\int d\psi = \int D_S \cdot dS$$

$$\boxed{\psi = \oint_{\text{closed surface}} D_S \cdot dS} = Q$$

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$$Q = \int_L \rho_L \, dL \quad \text{(Line charge)}$$

$$Q = \int_S \rho_S \, dS \quad \text{(surface charge)}$$

$$Q = \int_V \rho_V \, dV \quad \text{(volume charge)}$$

$$\boxed{\oint_{\substack{\text{closed} \\ \text{surface}}} D_S \cdot dS = \int_V \rho_V \, dV}$$

=> <u>APPLICATIONS OF GAUSS'S LAW:</u>

=> <u>SYMMETRIC CHARGE DISTRIBUTION:</u>

=> <u>CONDITIONS:</u>

=> D_S is everywhere either normal or tangential to the closed surface. so that $D_s \cdot ds$ is either $D_s ds$ or zero

⇒ on that portion of the closed surface for which $D_s \cdot ds$ is not zero ; D_s = constant

⇒ point charge

⇒ condition 2

$$Q = \oint_S D_s \cdot ds = \oint_S D_s ds$$

$$= D_s \oint_S ds$$

$$= D_s \iint ds_r$$

$$= D_s \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= D_s r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= D_s r^2 \int_0^{2\pi} -\cos\theta \Big|_0^{\pi} d\phi \, \hat{a}_r$$

$$= D_s r^2 \left[-\cos(\pi) + \cos 0^\circ\right] \int_0^{2\pi} d\phi \, \hat{a}_r$$

$$= 2D_s r^2 \int_0^{2\pi} d\phi \, \hat{a_r}$$

$$= 2D_s r^2 \, 2\pi \, \hat{a_r}$$

$$Q = 2 \times 2 \times D_s \times \pi \times r^2 \hat{a_r}$$

$$\boxed{D_{s_r} = \frac{Q}{4\pi r^2} \hat{a_r}}$$

$\Rightarrow$ <u>line charge</u>

$$Q = \oint_{\substack{\text{closed}\\\text{surface}}} D_s \cdot ds$$

$$= D_s \int_0^L \int_0^{2\pi} P \, d\phi \, dz \, \hat{a_P}$$

$$= D_s P \int_0^L \phi \Big|_0^{2\pi} dz \, \hat{a_P}$$

$$= D_s P \, 2\pi \int_0^L dz \, \hat{a_P}$$

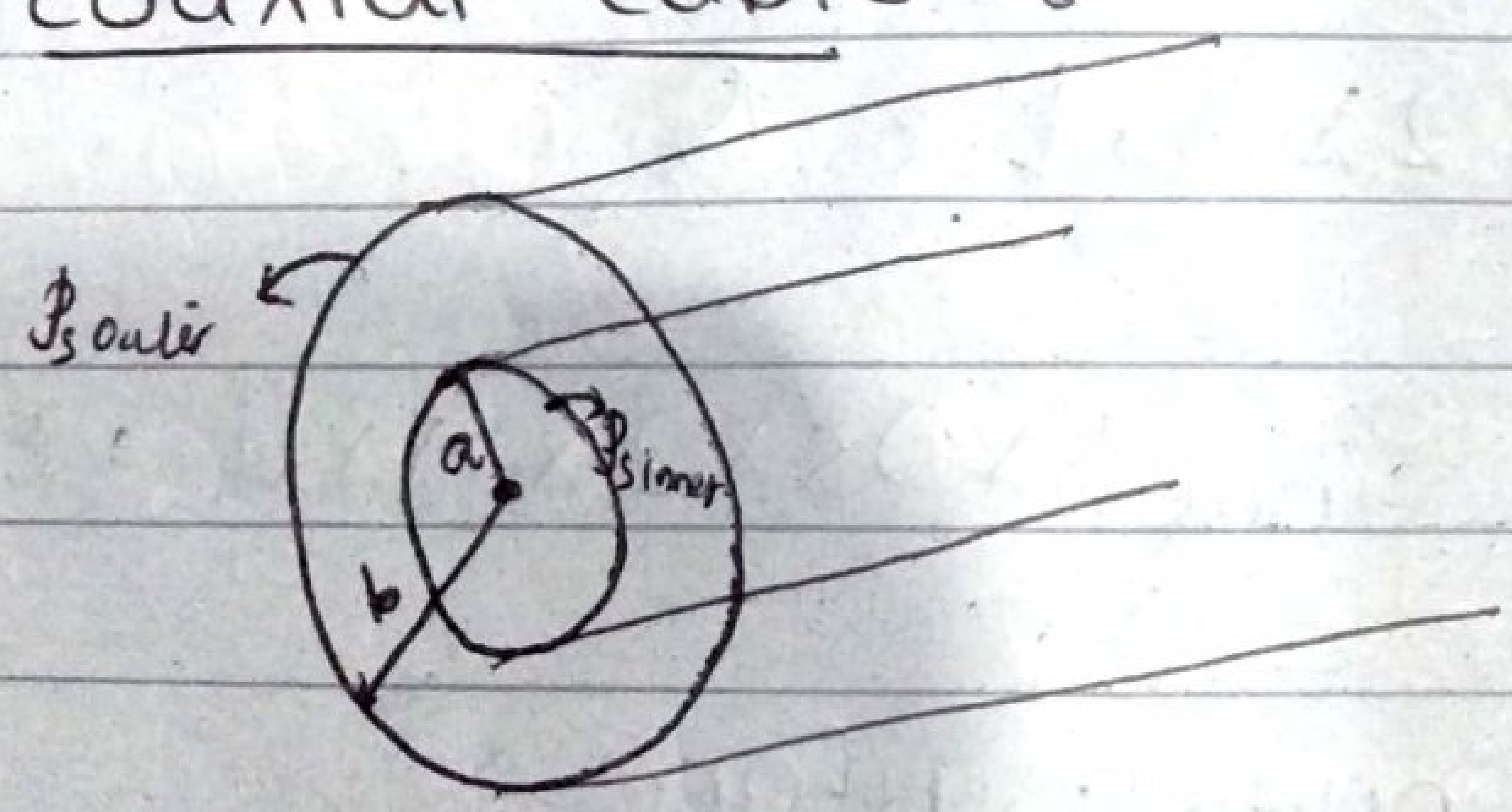
$$Q = D_s P \, 2 \times \pi \times L \, \hat{a_P}$$

$$D_{s\rho} = \frac{Q}{2\pi\rho_L}\hat{a}_\rho$$

$$D_{s\rho} = \frac{\rho_L \cancel{L}}{2\pi\rho \cancel{L}} a_\rho \qquad \boxed{\because Q = \rho_L \times L}$$

$$\boxed{D_s = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho}$$

<u>coaxial cable :</u>



$\Rightarrow$ BY applying Gauss's law:

$$Q = \oint_{\text{closed surface}} D_s \cdot ds \qquad D_s \parallel ds$$

$$= D_s \int_0^{2\pi} \int_0^{\pi} \cancel{r^2} \sin\theta \, d\cancel{\phi} \, d\phi \, \cancel{a_r}$$

$$Q \Rightarrow D_s \int_0^L \int_0^{2\pi} \rho \, d\phi \, dz$$

$$Q = D_s \rho \int_0^L \phi \Big|_0^{2\pi} dz$$

$$Q = D_s \rho \, 2\pi \int_0^L dz$$

$$Q = D_s \rho \, 2\pi \times L$$

$$D_s \times \rho \times 2 \times \pi \times L = Q$$

$$\boxed{Q_{inner} = \rho_{inner} \times D_s \times 2 \times \pi \times L} \rightarrow (1)$$

<u>For inner surface :</u>

$$Q_{inner} = \int_S \rho_s \, ds$$

$$= \rho_s \int_0^L \int_0^{2\pi} \rho \, d\phi \, dz$$

$$= \rho_s \, a \, 2\pi L$$

$$Q_{inner} = \rho_{s_{inner}} \, a \, 2\pi L \longrightarrow (2)$$

For outer surface:

$$Q_{outer} = \int_S \rho_s \, ds$$

$$= \rho_s \int_0^L \int_0^{2\pi} \rho \, d\phi \, dz \qquad \because \rho = b$$

$$= \rho_s b \int_0^L \int_0^{2\pi} d\phi \, dz$$

$$= \rho_s b \, 2\pi L$$

$$\boxed{Q_{outer} = \rho_s b \times 2 \times \pi \times L} \rightarrow (3)$$

comparing eq (1) & (2):

$$\rho_{inner} \times D_s \times \cancel{2 \times \pi \times L} = \rho_s \times a \times \cancel{2 \pi \times L}$$

$$\boxed{D_s = \frac{a \, \rho_{s_{inner}}}{\rho_{inner}}}$$

$$Q_{inner} = -Q_{outer}$$

$$\rho_{s_{inner}} a \, 2\pi L = -\rho_{s_{outer}} b \, 2\pi L$$

$$\boxed{\rho_{s_{inner}} = -\frac{b}{a} \rho_{s_{outer}}}$$

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Divergence theorem

$$\boxed{\oint_S D_s \cdot ds = \int_V \bar{\nabla} \cdot D \, dv}$$

rectangular co-ordinates

$$\text{Div } D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$\bar{\nabla} \cdot D = \frac{1}{P} \frac{\partial}{\partial P} P D_P + \frac{1}{P} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

cylindrical co-ordinates

$$\boxed{P_V = \bar{\nabla} \cdot D = \frac{\Delta Q}{\Delta V}} = \frac{\oint_S D_s \cdot ds}{\Delta V}$$

$$\boxed{Q = P_s \, ds}$$

$$\boxed{Q = \int_S P_s \, ds}$$

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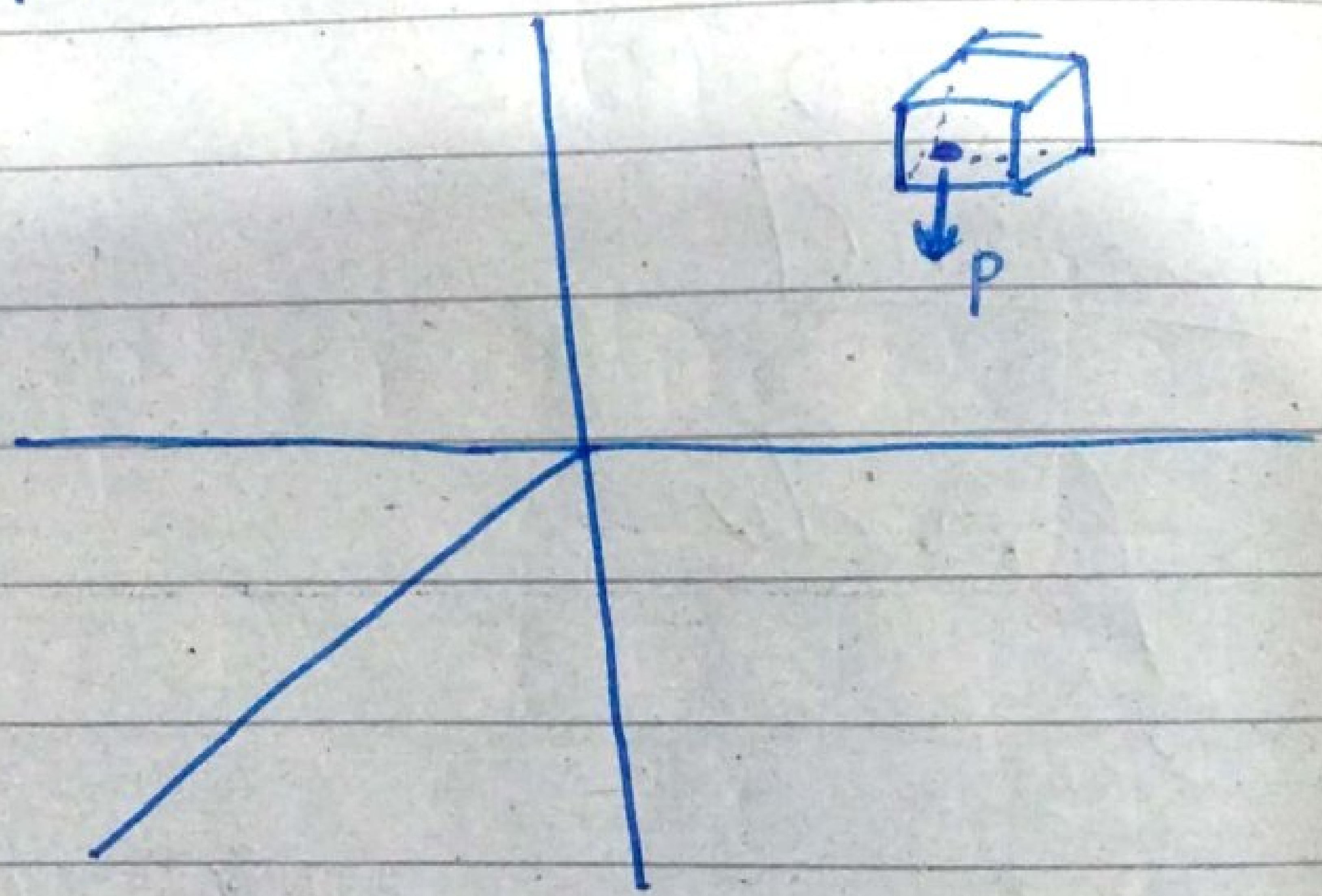
Guass's law on asymmetric:

$$\psi = Q = \oint D \cdot ds$$

$$D = D_x \hat{a_x} + D_y \hat{a_y} + D_z \hat{a_z}$$

where D_x, D_y & D_z are function of (x, y, z)

At pt P:



$$D_0 = D_{0x} \hat{a_x} + D_{0y} \hat{a_y} + D_{0z} \hat{a_z}$$

$$\psi = Q = \oint \vec{D_0} \cdot \vec{ds}$$

$$\oint \vec{D} \cdot \vec{ds} = \int_{front} \vec{D} \cdot \vec{ds} + \int_{back} \vec{D} \cdot \vec{ds}$$

$$+ \int_{top} \vec{D} \cdot \vec{ds} + \int_{bottom} \vec{D} \cdot \vec{ds} +$$

$$\int_{left} \vec{D} \cdot \vec{ds} + \int \vec{D} \cdot \vec{ds}$$

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First we are working for:

$$\Psi_{front} = \int_{front} \vec{D} \cdot \vec{ds}$$

$$\Psi_{front} = \int_{front} D \cdot ds = \int \vec{D}_{front} \cdot \vec{\Delta S}$$

$$\int \vec{D}_{front} = \vec{D}_{front\,x}\,\hat{a_x} + \vec{D}_{front\,y}\,\hat{a_y} + \vec{D}_{front\,z}\,\hat{a_z}$$

$$\Psi_{front} = \int D_{front\,x}\, a_x \cdot \Delta y \Delta z\, a_x$$

$$\Psi_{front} = \int D_{front\,x}\, \Delta y \Delta z$$

$$D_{front\,x} = D_{x0} + \underbrace{\frac{\partial D_x}{\partial x}\,\frac{\Delta x}{2}}_{\Delta D_x}$$

$$\boxed{\Psi_{front} = \left[D_{x0} + \frac{\partial D_x}{\partial x} \cdot \frac{\Delta x}{2} \right] \Delta y \Delta z} \quad (i)$$

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$$\boxed{\psi_{back} = -\left[D_{x0} - \frac{\partial}{\partial x} D_x \frac{\Delta x}{2}\right] \Delta y \Delta z} \quad (ii)$$

Adding eq (i) & (ii)

$$\psi_{front} + \psi_{back} = \left[D_{x0} + \frac{\partial}{\partial x} D_x \frac{\Delta x}{2}\right] \Delta y \Delta z - \left[D_{x0} - \frac{\partial}{\partial x} D_x \frac{\Delta x}{2}\right] \Delta y \Delta z$$

$$= \left[D_{x0} + \frac{\partial}{\partial x} D_x \frac{\Delta x}{2}\right] \Delta y \Delta z$$

$$\left[- D_{x0} + \frac{\partial}{\partial x} D_x \frac{\Delta x}{2}\right] \Delta y \Delta z$$

$$= \left[\cancel{D_{x0}} + \frac{\partial}{\partial x} D_x \frac{\Delta x}{2} - \cancel{D_{x0}} + \frac{\partial}{\partial x} D_x \frac{\Delta x}{2}\right.$$

$$\Delta y \Delta z$$

$$= \left[\cancel{2} \frac{\partial}{\partial x} D_x \frac{\Delta x}{\cancel{2}}\right] \Delta y \Delta z$$

$$= \left[\frac{\partial}{\partial x} D_x \Delta x\right] \Delta y \Delta z$$

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$$= \frac{\partial}{\partial x} D_x \quad \Delta x \Delta y \Delta z$$

$$\frac{\partial}{\partial x} D_x \ \Delta V \quad \text{—— (iii)}$$

$$\psi_{front} + \psi_{back} = \frac{\partial}{\partial x}$$

Similarly :

$$\psi_{top} + \psi_{bottom} = \frac{\partial}{\partial z} D_z \ \Delta V \quad \text{— (iv)}$$

$$\psi_{left} + \psi_{Right} = \frac{\partial}{\partial y} D_y \ \Delta V \quad \text{— (v)}$$

Add eq (iii), (iv) and (v)

$$\oint D \cdot ds = Q = \psi = \left(\frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} \right) \Delta V$$

$$\boxed{\oint \frac{D \cdot ds}{\Delta V} = \frac{Q}{\Delta V} = \frac{\psi}{\Delta V} = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} = \rho_v}$$

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$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

$$\bar{D} = D_x a_x + D_y a_y + D_z a_z$$

$$\nabla \cdot \bar{D} = \left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z\right) \cdot \left(D_x a_x + D_y a_y + D_z a_z\right)$$

$$\nabla \cdot \bar{D} = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$\nabla \cdot \bar{D} = \text{div } \vec{D}$$

$$\boxed{\text{div } \bar{D} = \nabla \cdot \bar{D} = \rho_v}$$

↓

This is the first of four Maxwell's equation

$$\boxed{\oint \vec{D} \cdot \vec{ds} = \int_{vol} \left(\vec{\nabla} \cdot \vec{D}\right) dv}$$

↓

This is divergence theorem

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