=> FORMULA SHEET

CH:03

=> $\psi = Q$

=> $D_s = \dfrac{\psi}{A} = \dfrac{Q}{4\pi r^2}\hat{a_r}$ → for sphere/point charge

=> $D_{s\rho} = \dfrac{\rho_L}{2\pi\rho}\hat{a_\rho}$ → for line charge

=> $D_s = \dfrac{\rho_s}{2}\hat{a_N}$ → for surface charge

=> $\psi = Q = \oint_{\substack{\text{closed} \\ \text{surface}}} D_s \cdot ds$ → Gauss's law

=> $Q = \int_L \rho_L \, dL$ → for line charge

=> $Q = \int_S \rho_s ds$ → for surface charge

=> $Q = \int_V \rho_v dv$ → volume charge

=> $D = \epsilon_0 E$

=> $Q_{inner} = \rho_{s\,inner} \times a \times 2\pi L$ → coaxial cable

=> $Q_{outer} = \rho_{s\,outer} \times b \times 2\pi L$ → coaxial cable

$\Rightarrow Q_{inner} = -Q_{outer}$

$\Rightarrow \rho_{s\,inner} = \dfrac{-b}{a} \rho_{s\,outer}$

$\Rightarrow D_S = \dfrac{a\,\rho_{s\,inner}}{\rho}$

$\Rightarrow Q = \rho_s \times \Delta S \longrightarrow$ surface charge

$\Rightarrow Q = \rho_L \times L \longrightarrow$ line charge

$\Rightarrow Q = \rho_v \times \Delta V \longrightarrow$ volumetric charge

$$\Rightarrow \rho_v = \bar{\nabla} \cdot D = \frac{Q}{\Delta V}$$

$\Rightarrow \bar{\nabla} \cdot D = \dfrac{\partial}{\partial x} D_x + \dfrac{\partial}{\partial y} D_y + \dfrac{\partial}{\partial z} D_z \rightarrow$ Rectangular co-ordinate system

$\Rightarrow \bar{\nabla} \cdot D = \dfrac{1}{\rho} \dfrac{\partial}{\partial \rho} \rho D_\rho + \dfrac{1}{\rho} \dfrac{\partial}{\partial \phi} D_\phi + \dfrac{\partial}{\partial z} D_z \rightarrow$ cylindrical co-ordinate system

$\Rightarrow \bar{\nabla} \cdot D = \dfrac{1}{r^2} \dfrac{\partial}{\partial r} r^2 (D_r) + \dfrac{1}{r\sin\theta} \dfrac{\partial}{\partial \theta} (\sin\theta D_\theta) + \dfrac{1}{r\sin\theta} \dfrac{\partial}{\partial \phi} D_\phi$

$\hookrightarrow$ spherical co-ordinate system

=> Divergence theorem:

$$\oint_{\text{closed surface}} D_S \cdot ds = \int_v \bar{\nabla} \cdot D \, dv$$

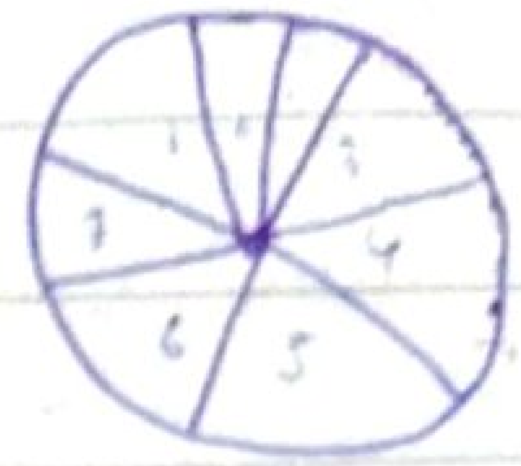
=> surface area of sphere $= \Delta S = 4\pi r^2$

=> volume of sphere $= \frac{4}{3}\pi r^3$

D:3·1

Sol:-

$Q = 60\,\mu C$



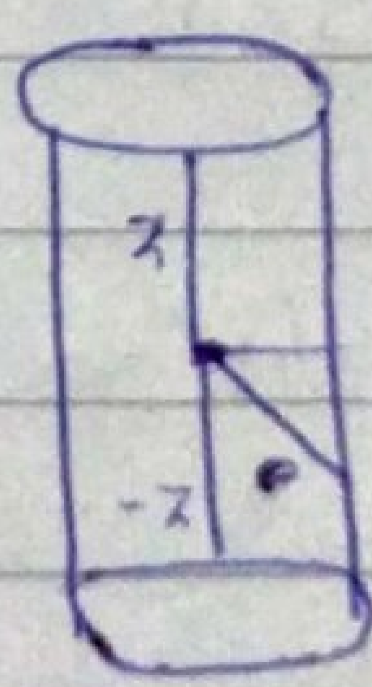
(a)

We know that,

$\psi = Q$

$\psi = \frac{Q}{8} = \frac{60\times10^{-6}}{8} = \boxed{7.5\,\mu C}$

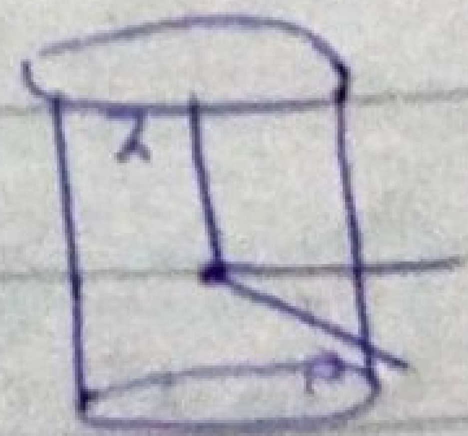
(b)



$\psi = Q$

$\boxed{\psi = 60\,\mu C}$

(c)



$\psi = Q$

$\boxed{\psi = \frac{60}{2} = 30\,\mu C}$

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D : 3.2

P (2, -3, 6)

(a) $Q_A = 55mC$ at Q (-2, 3, -6)

$$\hat{r} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -6 \\ 12 \end{bmatrix}$$

$$|\hat{r}| = \sqrt{4^2 + (-6)^2 + (12)^2}$$

$$= 14$$

$$D = \frac{Q}{4\pi r^2} \hat{a}_r$$

$$= \frac{55 \times 10^{-3}}{4 \times \pi (14)^3} (4\hat{a_x} - 6\hat{a_y} + 12\hat{a_z})$$

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$= 1.595 \times 10^{-6} \left(4\hat{a_x} - 6\hat{a_y} + 12\hat{a_z} \right)$

$= \underbrace{6.38}_{\hat{a_x}} - \underbrace{9.57}_{\hat{a_y}} + \underbrace{19.14}_{\hat{a_z}} \ \mu C/m^2$

$D_1 = \boxed{6.38\,\hat{a_x} - 9.57\,\hat{a_y} + 19.14\,\hat{a_z} \ \mu C/m^2}$

$P_{LB} = 20\,mC/m$ (b)

$D = ??$

(on the x axis) = (2, 0, 0)

$$\hat{P} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 6 \end{bmatrix}$$

$|P| = \sqrt{(-3)^2 + 6^2} = 3\sqrt{5}$

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$$D = \frac{\rho_L}{2 \times \pi \times \rho} \hat{a_\rho}$$

$$= \frac{20 \times 10^{-3}}{2 \times \pi \times (3\sqrt{5})^2} (-3\hat{a_y} + 6\hat{a_z})$$

$$\boxed{D_P = -212 \hat{a_y} + 424 \hat{a_z} \ \mu C/m^2}$$

(c)

$\rho_{sc} = 120 \ \mu C/m^2$

$z = -5 \longrightarrow (2, -3, -5)$

$$N = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 11 \end{bmatrix} = 11\hat{a_z}$$

$$D_s = \frac{\rho_s}{2} \hat{a_N} = \frac{120 \times 10^{-6}}{2} \times \frac{11}{(\sqrt{11^2})} \hat{a_z}$$

$$\boxed{D_s = \underset{\hat{a_z}}{60} \mu C/m^2}$$

Example: 3.1

Proof:

$$\psi = \oint_{\substack{closed \\ surface}} D_s \cdot ds$$

$$= \oint_{\substack{closed \\ surface}} D_{sr} \cdot ds_r$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{Q}{4\pi a^2} \hat{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{Q}{4\pi a^2} \times r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{Q \cancel{a^2}}{4\pi \cancel{a^2}} \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{Q}{4\pi} \int_0^{2\pi} -\cos\theta \Big|_0^{\pi} \, d\phi$$

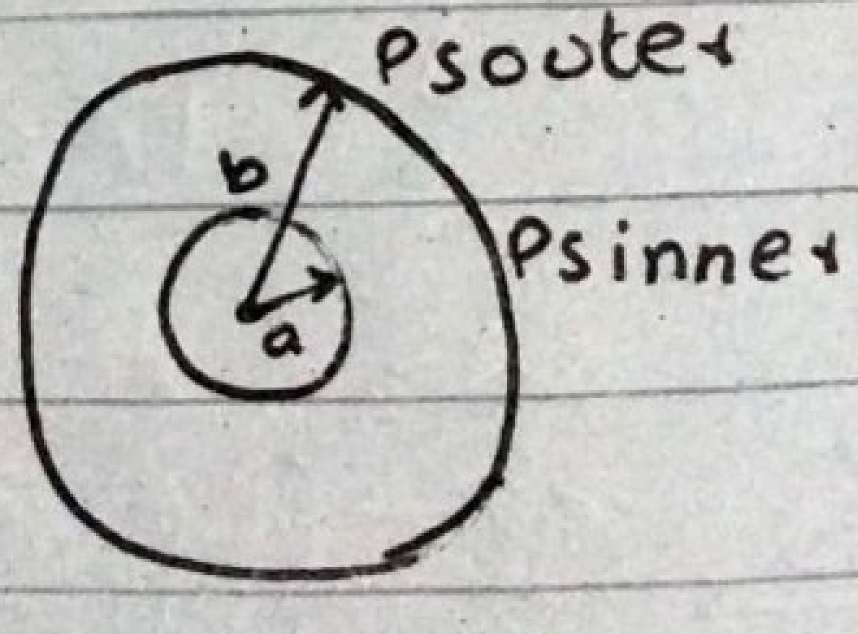
$$= \frac{Q}{4\pi}\left[-\cos\pi + \cos 0^\circ\right]\int_0^{2\pi} d\phi$$

$$= \frac{Q}{4\pi}\left[1 + 1\right]\int_0^{2\pi} d\phi$$

$$= \frac{Q}{{}_{2}\cancel{4}\pi} \times \cancel{2}1 \quad 2\cancel{\pi}$$

$$\boxed{= Q \;//}$$

Example: 3.2



$L = 50cm$

$a = 1mm$

$b = 4mm$

$Q_{inner} = 30nC$

$\rho_{s\,inner} = ??$

$\rho_{s\,outer} = ??$

$E = ??$

$D = ??$

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$$Q_{inner} = \rho_{s\,inner} \times a \times 2\pi l$$

$$\rho_{s\,inner} = \frac{Q_{inner}}{a \times 2\pi l}$$

$$\rho_{s\,inner} = \frac{30 \times 10^{-9}}{1 \times 10^{-3} \times 2 \times \pi \times \frac{50}{100}}$$

$$\boxed{\rho_{s\,inner} = 9.54\ \mu C/m^2}$$

$$\rho_{s\,inner} = \frac{-b}{a}\,\rho_{s\,outer}$$

$$-\frac{9.54 \times 10^{-6} \times 1 \times 10^{-3}}{4 \times 10^{-3}} = \rho_{s\,outer}$$

$$\rho_{s\,outer} = -2.385 \times 10^{-6}\ C/m^2$$

$$D_s = \frac{a\,\rho_{s\,inner}}{\rho} = \frac{1 \times 10^{-3} \times 9.54 \times 10^{-6}}{\rho}$$

$$= \frac{9.54 \times 10^{-9}}{\rho}$$

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$D = \epsilon_0 E$

$E = \dfrac{D_s}{\epsilon_0}$

$= \dfrac{9.54 \times 10^{-9}}{\rho} \div \epsilon_0$

$= \dfrac{9.54 \times 10^{-9}}{8.85 \times 10^{-12} \rho}$

$E = \dfrac{1077.96}{\rho}$ N/C or V/m

D:3.3

Sol:

$D = 0.3r^2\, a_r\ nC/m^2$

$E = ??\quad P(r=2, \theta=25°, \phi=90°)$

$D = \epsilon_0 E$

$E = \dfrac{D}{\epsilon_0} = \dfrac{0.3r^2 \times 10^{-9}}{8.85 \times 10^{-12}}$

$E = \dfrac{0.3 \times (2)^2 \times 10^{-9}}{8.85 \times 10^{-12}}$

$\boxed{E = 135.5\ V/m}$

$\boxed{\text{at } r=3}$

(b)

BY ussing gauss's law

$Q = \oint_{\text{closed surface}} D_s \cdot ds$

$$= \int_0^{2\pi}\int_0^{\pi} 0.3r^2 \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= \int_0^{2\pi}\int_0^{\pi} 0.3r^2 \, r^2 \sin\theta \, d\theta \, d\phi$$

$$= 0.3r^4 \int_0^{2\pi} -\cos\theta \Big|_0^{\pi} \, d\phi$$

$$= 0.3\,(3)^4 \int_0^{2\pi} \left[-\cos\pi + \cos 0^\circ\right] d\phi$$

$$= 0.3 \times (3)^4 \times 2 \int_0^{2\pi} d\phi$$

$$= 0.3 \times (3)^4 \times 2 \times 2\pi$$

$$\boxed{Q = 305 nC}$$

(c)

$\boxed{\text{at } r = 4}$

$$\frac{\psi}{4\pi r^2} = D_r$$

$$\psi = D_r \times 4\pi r^2$$

$$\psi = 0.3 r^2 \times 4 \times \pi \times r^2$$

$$\boxed{\psi = 965 nC}$$

Drill
~~Example~~ 3.4

(a)

$Q_1 = 0.1 \mu C \quad (1, -2, 3)$
$Q_2 = \frac{1}{7} \mu C \quad (-1, 2, -2)$

$$\psi = Q_1 + Q_2$$
$$\psi = 0.1 \times 10^{-6} + \frac{1}{7} \times 10^{-6}$$

$$\boxed{\psi = 0.243 \mu C}$$

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(b) $P_L = \pi \mu C/m$

$x = -2, \ y = 3$

$Q = P_L \times L$

$= \pi \times 10^{-6} \times 10$

$= 31.4 \mu C$

$\psi = Q$

$\boxed{\psi = 31.4 \mu C}$

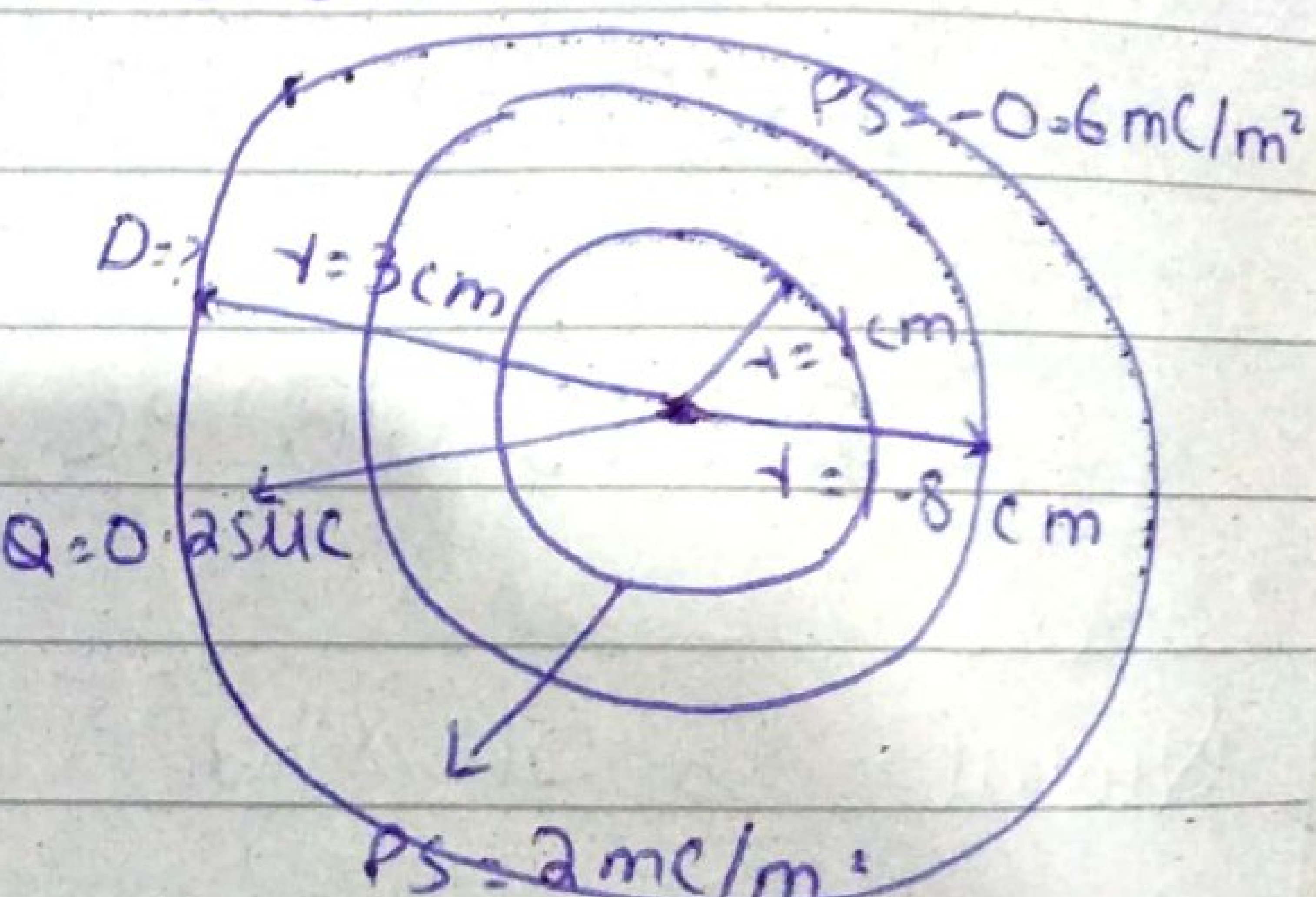
D 3.5

$Q = 0.25\ \mu C$ at $r=0$

$\rho_s = 2\ mC/m^2$ at $r = 1cm$

$\rho_s = -0.6\ mC/m^2$ at $r = 1.8cm$

(i) $D = ??$ at $r = 0.5cm$



=> at $0 < r < 1$

=> $Q = 0.25\ \mu C$

$$D_r = \frac{Q}{4\pi r^2} = \frac{0.25 \times 10^{-6}}{4 \times \pi \times \left(\frac{0.5}{100}\right)^2}\ \hat{a}_r$$

$$\boxed{D_r = 795.7\,\hat{a}_r\ \mu C/m^2}$$

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(b) $D = ??$ at $r = 1.5cm$

at $1 < r < 1.8$

$Q = \rho_s \times \Delta S$

$= 2 \times 10^{-3} \times 4 \times \pi \times r^2$

$= 2 \times 10^{-3} \times 4 \times \pi \times \left(\frac{1}{100}\right)^2$

$= 2.513 \times 10^{-6} C$

$Q_{total} = 2.513 \times 10^{-6} + 0.25 \times 10^{-6}$

$Q_{total} = 2.763 \times 10^{-6} C$

$$D_r = \frac{2.763 \times 10^{-6}}{4 \times \pi \times \left(\frac{1.5}{100}\right)^2} \hat{a}_r$$

$$\boxed{D_r = 972 \, \mu C/m^2 \ \hat{a}_r}$$

(c) at $r = 2.5cm$ $D_r = ??$

$r > 1.8cm$

$Q = \rho_s \times \Delta s$

$Q = -0.6 \times 10^{-3} \times 4 \times \pi \times \left(\frac{1.8}{100}\right)^2$

$Q = -2.442 \times 10^{-6} C$

$Q_{total} = -2.442 \times 10^{-6} + 2.763 \times 10^{-6}$

$Q_{total} = 3.21 \times 10^{-7}$

$$D_r = \frac{3.21 \times 10^{-7}}{4 \times \pi \times \left(\frac{2.5}{100}\right)^2} \hat{a}_r$$

$$\boxed{D_r = 40.8 \hat{a}_r \ \mu C/m^2}$$

(d) $\rho_s = \frac{Q}{\Delta s} = \frac{-3.21 \times 10^{-7}}{4 \times \pi \times r^2}$

$$= \frac{-3.21 \times 10^{-7}}{4 \times \pi \times \left(\frac{3}{100}\right)^2}$$

$$\boxed{\rho_s = -28.3 \mu C/m^2}$$

Example: 3·3, 3·4 → same

$Q = ?$ $(x, y, z) = (0, 0, 0)$

$\Delta V = 10^{-9} m^3$

$D = e^{-x} \sin y\, a_x - e^{-x} \cos y\, a_y + 2z\, a_z \quad C/m^2$

$\Rightarrow \bar{\nabla} \cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

$= \frac{\partial}{\partial x} e^{-x} \sin y + \frac{\partial}{\partial y} -e^{-x} \cos y$

$+ \frac{\partial}{\partial z} 2z\, \hat{a_z} \cdot \hat{a_z}$

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$= -e^{-x} \cancel{\sin y} + e^{-x} \cancel{\sin y} + 2$

$= 2$

$\bar{\nabla} \cdot D = \dfrac{Q}{\Delta V}$

$Q = 2 \times 10^{-9}$

$\boxed{Q = 2nC}$

D : 3.6

$D = 8xyz^4 a_x + 4x^2 z^4 a_y + 16x^2 y z^3 a_z \ pC/m^2$

(a) $\phi = ??$

$z = 2, \ 0 < x < 2, \ 1 < y < 3$

(b) E at $(2, -1, 3)$

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(c) $Q = ??$ $(2, -1, 3)$

$\Delta V = 10^{-12} m^3$

Sol:

By using gauss's law,

$\psi = \oint D_s \cdot ds$

$= \oint D_s \cdot ds_z$

$= \int_1^3 \int_0^2 (8xyz^4 \, a_x + 4x^2z^4 \, a_y + 16x^2yz^3 a_z) \cdot dx\,dy\,\hat{a_z}$

$= \int_1^3 \int_0^2 16x^2yz^3 \, dx\,dy$

$= \int_1^3 \int_0^2 16x^2y \times 8 \, dx\,dy$

$= \int_1^3 \int_0^2 128x^2y \, dx\,dy$

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$$= \int_1^3 \int_0^2 128\, x^2 y \, dx\, dy$$

$$= 128y \int_1^3 \frac{x^3}{3} \Big|_0^2 \, dy$$

$$= \frac{128y}{3} \int_1^3 8\, dy$$

$$= \frac{128}{3} \times 8 \int_1^3 y\, dy$$

$$= \frac{1024}{3} \frac{y^2}{2} \Big|_1^3$$

$$= \frac{1024}{6} \left[3^2 - 1^2 \right]$$

$$= \frac{1024}{6} \times 8$$

$$\boxed{\psi = 1365\ PC}$$

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(b) E at P (2, -1, 3)

$D = \epsilon_0 E$

$E = \frac{D}{\epsilon_0}$

$$E = \frac{8xyz^4 \hat{ax} + 4x^2z^4 \hat{ay} + 16x^2yz^3 \hat{az}}{8.85 \times 10^{-12}}$$

$$E = \frac{8 \times 2 \times -1 \times (3)^4 \hat{ax}}{8.85 \times 10^{-12}}$$

$$+ \frac{4 \times (2)^2 \times (3)^4 \hat{ay}}{8.85 \times 10^{-12}} + \frac{16 (2)^2 \times (-1)(3)^3 \hat{az}}{8.85 \times 10^{-12}}$$

$$= -1.464 \times 10^{14} \hat{ax} + 1.464 \times 10^{14} \hat{ay}$$

$$- 1.952 \times 10^{14} \hat{az}$$

$$= -1.464 \times 10^{14} \times 10^{-12} \hat{ax}$$

$$+ 1.464 \times 10^{14} \times 10^{-12} \hat{ay}$$

$$- 1.952 \times 10^{14} \times 10^{-12} \hat{az}$$

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$$\boxed{E = -146.4\,\hat{a_x} + 146.4\,\hat{a_y} - 195.2\,\hat{a_z}\ \text{V/m}}$$

(c)

$$\bar{\nabla}\cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$= \frac{\partial}{\partial x} 8xyz^4 + \frac{\partial}{\partial y} 4x^2z^4 + \frac{\partial}{\partial z} 16x^2yz^3$$

$$= 8yz^4 + 0 + 48x^2yz^2$$

$$= 8(-1)(3)^4 + 48(4)(-1)(3)^2$$

$$= -648 - 1728$$

$$= -2376\ \rho$$

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$$\bar{\nabla} \cdot D = \frac{Q}{\Delta V}$$

$Q = -2376 \times 10^{-12} \times 10^{-12}$

$$\boxed{Q = -2.38 \times 10^{-21} C}$$

<u>D: 3.7</u>

(a) $D = (2xyz - y^2)\hat{a_x} + (x^2z - 2xy)\hat{a_y} + x^2y\,\hat{a_z}$ C/m²

$$\bar{\nabla} \cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$= \frac{\partial}{\partial x}(2xyz - y^2) + \frac{\partial}{\partial y}(x^2z - 2xy) + \frac{\partial}{\partial z}(x^2y)$$

$= 2yz + (-2x) + 0$

$= 2yz - 2x$

at $P(2, 3, -1)$

$= 2(3)(-1) - 2(2)$

$= -6 - 4$

$\boxed{= -10}$

(b) $D = 2\rho z^2 \sin^2\phi \, \hat{a}_\rho + \rho z^2 \sin 2\phi \, \hat{a}_\phi + 2\rho^2 z \sin^2\phi \, a_z$

C/m^2 at $(2, 110°, -1)$

$$\bar{\nabla} \cdot D = \frac{1}{\rho}\frac{\partial \rho D_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho} 2\rho^2 z^2 \sin^2\phi + \frac{1}{\rho}\frac{\partial}{\partial \phi} \rho z^2 \sin 2\phi$$

$$+ \frac{\partial}{\partial z} 2\rho^2 z \sin^2\phi$$

$$= \frac{1}{\cancel{\rho}} 4\cancel{\rho} z^2 \sin^2\phi + \frac{1}{\cancel{\rho}} 2\cos 2\phi \cancel{\rho} z^2$$

$$+ 2\rho^2 \sin^2\phi$$

$$= 4z^2 \sin^2\phi + 2\cos 2\phi \, z^2$$
$$+ 2\rho^2 \sin^2\phi$$

$$= 4 \times 1 \times \sin^2(110°) + 2\cos(220°)$$
$$+ 2 \times 4 \times \sin^2(110°)$$

$$\boxed{= 9.06}$$

(c) $D = 2r\sin\theta\cos\phi \, \hat{a_r} + r\cos\theta\cos\phi \, \hat{a_\theta}$
$- r\sin\phi \, \hat{a_\phi}$

$$\Rightarrow \bar{\nabla} \cdot D = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} \sin\theta D_\theta$$
$$+ \frac{1}{r\sin\theta} \frac{\partial D_\phi}{\partial\phi}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} 2r^3 \sin\theta\cos\phi + \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} r\cos\theta\sin\theta\cos\phi +$$

$+\frac{1}{r\sin\theta}\frac{\partial\, r\sin\phi}{\partial\phi}$

$= \frac{6r^2}{r^2}\sin\theta\cos\phi + \frac{1}{r\sin\theta} r\cos\phi\,[-\sin\theta\sin\theta + \cos\theta\cos\theta]$

$+\frac{1}{r\sin\theta} - r\cos\phi$

$= 6\sin\theta\cos\phi - \frac{\cos\phi}{\sin\theta} + \frac{\cos\phi}{\sin\theta}[\cos^2\theta - \sin^2\theta]$

$\Rightarrow \cancel{6\sin\phi\cos\phi} \cancel{- \frac{2\cos\phi}{\sin\phi}}$

$= 6\sin\theta\cos\phi - \frac{\cos\phi}{\sin\theta} + \frac{\cos\phi}{\sin\theta}[\cos^2\theta - \sin^2\theta]$

$\Rightarrow \boxed{1.28}$

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D = 3-8

sol:-

$$D = \frac{4xy}{z} a_x + \frac{2x^2}{z} a_y - \frac{2x^2 y}{z^2} a_z \quad (a)$$

$P_v = ??$

$P_v = \bar{\nabla} \cdot D$

$$= \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$= \frac{\partial}{\partial x} \frac{4xy}{z} + \frac{\partial}{\partial y} \frac{2x^2}{z} + \frac{\partial}{\partial z}\left(\frac{-2x^2 y}{z^2}\right)$$

$$= \frac{4y}{z} + 0 + 4\frac{x^2 y}{z^3}$$

$$= \frac{4y}{z} + \frac{4x^2 y}{z^3} \,//$$

(b) $D = z \sin\phi \, \hat{a_\rho} + z\cos\phi \, \hat{a_\phi}$
$+ \rho \sin\phi \, a_z$

$$\nabla \cdot D = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho D_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}D_\phi + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho z\sin\phi + \frac{1}{\rho} z\cos\phi + \frac{\partial}{\partial z}\rho\sin\phi$$

$$= \frac{z\sin\phi}{\rho} - \frac{z\sin\phi}{\rho} + 0$$

$$= 0$$

(C) $D = \sin\theta\sin\phi\,\hat{a_r} + \cos\theta\sin\phi\,\hat{a_\theta} + \cos\phi\,\hat{a_\phi}$

$$\bar{\nabla}\cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}r^2 D_r + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\sin\theta D_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}D_\phi$$

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$\cos\theta$

$$= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \sin\theta \sin\phi + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta} \sin\theta\cos\theta\sin\phi$$

$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\cos\phi$$

$$= \frac{1}{r^2} 2r\sin\theta\sin\phi + \frac{1}{r\sin\theta}\sin\phi\,[\sin\theta - \sin\theta$$

$$+ \cos\theta\cos\theta] + \frac{1}{r\sin\theta} - \sin\phi$$

$$= \frac{2}{r}\sin\theta\sin\phi + \frac{1}{r\sin\theta}\sin\phi\,[\cos^2\theta - \sin^2\theta]$$

$$- \frac{1}{r\sin\theta}\sin\phi$$

$$= \frac{2}{r}\sin\theta\sin\phi + \frac{1}{r\sin\theta}\sin\phi\,[\cos^2\theta - \sin^2\theta - 1]$$

$$= \frac{2}{r}\sin\theta\sin\phi + \frac{1}{r\sin\theta}\sin\phi\,[\cos^2\theta - (1-\cos^2\theta) - 1]$$

$$= \frac{2\sin\theta\sin\phi}{r} + \frac{\sin\phi}{r\sin\theta}\,[\cos^2\theta - 1 + \cos^2\theta - 1]$$

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continued drill 3.8

$$= \frac{2}{r} \sin\theta \sin\phi + \frac{\sin\phi}{r \sin\theta} \left[2\cos^2\theta - 2 \right]$$

$$= \frac{2}{r} \sin\theta \sin\phi + \frac{\sin\phi}{r \sin\theta} 2 \left[\cos^2\theta - 1 \right]$$

$$= \frac{2}{r} \sin\theta \sin\phi + \frac{\sin\phi \, 2 - \sin^2\theta}{r \sin\theta}$$

$$= \frac{2}{r} \sin\theta \sin\phi - \frac{2}{r} \sin\theta \sin\phi$$

$$= 0 \;//$$

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Example: 3.5

Sol:

$D = 2xy\hat{a}_x + x^2\hat{a}_y \ C/m^2$

$x = 0$ and 1
$y = 0$ and 2
$z = 0$ and 3

Divergence theorem:

$$\oint_{\text{closed surface}} D_s \cdot ds = \int_v \bar{\nabla} \cdot D \, dv$$

$\Rightarrow$ first solving L.H.S.

$$\Rightarrow \oint D_s \cdot ds$$

$$\Rightarrow \iint D_{sx} \cdot ds_x + \iint D_{s-x} \cdot ds_{-x}$$

$$+ \iint D_{sy} \cdot ds_y + \iint D_{s-y} \cdot ds_{-y}$$

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$+\iint D_{s_z} \cdot ds_z \quad + \iint D_{s\text{-}z} \cdot ds_{-z}$

$\Rightarrow \iint 2xy\hat{a}_x \cdot dydz\,\hat{a}_x$

$+ \iint 2xy\hat{a}_x \cdot dydz\,-\hat{a}_x$

$+ \iint x^2\hat{a}_y \cdot dxdz\,\hat{a}_y$

$+ \iint x^2\hat{a}_y \cdot dxdz\,-\hat{a}_y$

$\Rightarrow \iint 2xy\,dydz + \iint -2xy\,dydz$

$+ \iint \cancel{x^2 dxdz} + \iint \cancel{-x^2 dxdz}$

$= \iint 2xy\,dydz + \iint -2(0)y\,dydz$

$= \int_0^3\int_0^2 2y\,dydz$

$= \frac{2y^2}{2}\Big|_0^2 \int_0^3 dz$

$= 4\; z\Big|_0^3$

$= 4 \times 3$

$= 12$ //

Now solving R.H.S

$$\Rightarrow \int_V \bar{\nabla}\cdot D \, dv$$

$$\bar{\nabla}\cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$= \frac{\partial}{\partial x} 2xy + \frac{\partial}{\partial z} 0 + \frac{\partial}{\partial y} x^2$$

$$= 2y + 0 + 0$$

$$= 2y$$

$$= \int_0^1 \int_0^2 \int_0^3 2y \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^2 2y \, z \Big|_0^3 \, dy \, dx$$

$$= 2y \int_0^1 \int_0^2 3 \, dy \, dx$$

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$$= 6 \int_0^1 \int_0^2 y \, dy \, dx$$

$$= 6 \int_0^1 \frac{y^2}{2} \Big|_0^2 dx$$

$$= 3 \int_0^1 4 \, dx$$

$$= 12 \int_0^1 dx$$

$$= 12 \times 1$$

$$= 12 \;//$$

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D:3.9

$$D = 6\rho \sin \frac{1}{2}\phi \, \hat{a}_\rho + 1.5\rho \cos \tfrac{1}{2}\phi \, a_\phi \; C/m^2$$

$\rho = 2, \; \phi = 0, \; \phi = \pi$

$z = 0 \; \& \; z = 5$

Divergence theorem:

$$\oint_{\text{closed surface}} D_s \cdot ds = \int_v \bar{\nabla} \cdot D \, dv$$

L.H.S

$\Rightarrow \oint D_s \cdot ds$

$\Rightarrow \iint D_{s\rho} \cdot ds_\rho + \iint D_{s-\rho} \cdot ds_{-\rho}$

$+ \iint D_{s\phi} \cdot ds_\phi + \iint D_{s-\phi} \cdot ds_{-\phi}$

$+ \iint D_{sz} \cdot ds_z + \iint D_{s-z} \cdot ds_{-z}$

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$$= \iint 6\rho \sin\tfrac{1}{2}\phi\, \hat{a}_\rho \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$+ \iint 6\rho \sin\tfrac{1}{2}\phi\, \hat{a}_\rho \cdot \rho\, d\phi\, dz\, (-\hat{a}_\rho)$$

$$+ \iint 1.5\rho \cos\tfrac{1}{2}\underset{\hat{a}_\phi}{\phi} \cdot \partial\rho\, dz\, \hat{a}_\phi$$

$$+ \iint 1.5\rho \cos\tfrac{\phi}{2}\, \hat{a}_\phi \cdot \partial\rho\, \partial z\, (-\hat{a}_\phi)$$

$$= \iint 6\rho^2 \sin\tfrac{\phi}{2}\, d\phi\, dz - \iint 6\rho^2 \sin\tfrac{\phi}{2}\, d\phi\, dz$$

$$+ \iint 1.5\rho \cos\tfrac{\phi}{2}\, \partial\rho\, dz - \iint 1.5\rho \cos\tfrac{\phi}{2}\, \partial\rho\, dz$$

$$= \iint 24 \sin\tfrac{\phi}{2}\, d\phi\, dz - 0$$

$$1.5 \iint \cos 0\, \partial\rho\, dz + \iint 1.5 \cos 90^\circ\, \partial\rho\, dz$$

$$= 24 \int_0^5\int_0^\pi \sin\tfrac{\phi}{2}\, \partial\phi\, dz \quad 1.5 \int_0^5\int_0^2 \partial\rho\, dz$$

$$= 24 \int_0^5 \frac{-\cos\frac{\phi}{2}}{\frac{1}{2}}\Big|_0^\pi dz \quad 1.5\, \rho\Big|_0^2 \int_0^5 dz$$

$$= 48\left[-\cos\frac{\pi}{2} + \cos 0^\circ\right]\int_0^5 dz$$

$$1.5 \times 2 \int_0^5 dz$$

$$= 48 \times 5 - 3 \times 5$$

$$= 240 - 15$$

$$= 225$$

R.H.S

$$\Rightarrow \int_v \bar{\nabla} \cdot D \, dv$$

$$\Rightarrow \bar{\nabla} \cdot D$$

$$= \frac{1}{\rho}\frac{\partial \rho D_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{P}\frac{\partial}{\partial P} 6P^2 \sin\frac{\phi}{2} + \frac{1}{P}\frac{\partial}{\partial \phi} 1.5P\cos\frac{\phi}{2}$$

$$= \frac{12\cancel{P}\sin\frac{\phi}{2}}{\cancel{P}} + \frac{1.5\cancel{P} - \sin\frac{\phi}{2}\cdot\frac{1}{2}}{\cancel{P}}$$

$$= 12\sin\frac{\phi}{2} - \frac{1}{2}\times 1.5\sin\frac{\phi}{2}$$

$$= 12\sin\frac{\phi}{2} - 0.75\sin\phi$$

$$= 11.25\sin\frac{\phi}{2}$$

$$= \int_0^5\int_0^\pi\int_0^2 \sin\frac{\phi}{2}\,\partial P\, d\phi\, dz \times 11.25$$

$$= \int_0^5\int_0^\pi \sin\frac{\phi}{2}\; P\Big|_0^2\, d\phi\, dz \times 11.25$$

$$= 2\int_0^5\int_0^\pi \sin\frac{\phi}{2}\, d\phi\, dz \times 11.25$$

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$$= \int_0^5 dz$$

$$=$$

$$\Rightarrow 2 \int_0^5 \frac{-\cos\frac{\phi}{2}}{1/2} \Big|_0^{\pi} dz \times 11.25$$

$$= +4 \int_0^5 \left[-\cos\frac{\pi}{2} + \cos 0\right] dz \times 11.25$$

$$= +4 \int_0^5 dz$$

$$= +4 \times z \Big|_0^5 \Rightarrow 4 \times 5 \times 11.25$$

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$=225$

3.3 Assignment questions

Sol:-

$P = 8cm$
$Ps = 5e^{-20|z|} \, nC/m^2$
$Q = ??$ $\rightarrow$ (a)

$$Q = \int_S Ps \, ds$$

$$= 2\int_0^\infty \int_0^{2\pi} 5e^{-20z} \times 10^{-9} \, P \, d\phi \, dz$$

$$= 2\int_0^\infty \int_0^{2\pi} 5e^{-20z} \times 10^{-9} \frac{8}{100} \, d\phi \, dz$$

$$= \frac{2 \times 8}{100} \times 5 \times 10^{-9} \int_0^\infty \phi \Big|_0^{2\pi} e^{-20z} \, dz$$

$$= 8 \times 10^{-10} \times 2\pi \int_0^\infty e^{-20z} \, dz$$

$$= 5.026 \times 10^{-9} \frac{e^{-20z}}{-20} \Big|_0^\infty$$

$= -\frac{5.026 \times}{20}$

$= -2.51$

$= 2.513$

$\boxed{= 0.25}$

(b) ϕ

$= \phi$

$= \phi \int_{0.0}^{0.0}$

$= 4 \times 10$

$= 4 \times 10$

$= \frac{4.188}{-20}$

$$= -\frac{5.026\times10^{-9}}{20}\left[e^{-\infty} - e^{0}\right]$$

$$= -2.513\times10^{-10}\left[0-1\right]$$

$$= 2.513\times10^{-10}\ C$$

$$\boxed{= 0.25\,nC}$$

(b) $\phi = Q = \int_S \rho_s\, ds$

$$= \phi = \int_{0.01}^{0.05}\int_{30^\circ}^{90^\circ} 5e^{-20z}\times10^{-9}\rho\, d\phi\, dz$$

$$= \phi \int_{0.01}^{0.05}\int_{30^\circ}^{90^\circ} 5\times10^{-9}\times\frac{8}{100}\, e^{-20z}\, d\phi\, dz$$

$$= 4\times10^{-10}\int_{0.01}^{0.05} e^{-20z}\ \phi\Big|_{30^\circ}^{90^\circ} dz$$

$$= 4\times10^{-10}\int_{0.01}^{0.05} e^{-20z}\, dz\ \left[\pi/2 - \pi/6\right]$$

$$= \frac{4.188\times10^{-10}}{-20}\ e^{-20z}\Big|_{0.01}^{0.05}$$

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$$\Rightarrow \frac{4.188 \times 10^{-10}}{-20}\left[e^{-20\times0.05} - e^{-20\times0.01}\right]$$

$$= -2.094 \times 10^{-11}\left[-0.4508\right]$$

$$= \boxed{9.4\ PC}$$

Example: 3.4

$\Rightarrow$ Sol:-

$$E = \left(\frac{5z^3}{\epsilon_0}\right)\hat{a}_z \ V/m$$

$Q = ??$ $(r = 3m \text{ radius})$

$$D = E\epsilon_0$$

$$= \left(\frac{5z^3}{\epsilon_0}\right) \times \epsilon_0$$

$$= 5z^3 \hat{a}_z \ C/m^2$$

$$Q = \oint_{\substack{\text{closed} \\ \text{surface}}} D_s \cdot ds$$

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$Q ? \frac{\underline{DL \times DC}}{\underline{Ds}}$ $= 5z^3 \cdot 4 \times \pi \times r^2$

$$Q = \int_0^{2\pi}\int_0^{\pi} 5z^3 \, \hat{a_z} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$$

$$Q = \int_0^{2\pi}\int_0^{\pi} 5z^3 r^2 \sin\theta \cos\theta \, d\theta \, d\phi$$

$$= 5\int_0^{2\pi}\int_0^{\pi} \cancel{5} (r\cos\theta)^3 r^2 \sin\theta \cos\theta \, d\theta \, d\phi$$

$$= 5\int_0^{2\pi}\int_0^{\pi} \cancel{5} \, r^3 \cos^3\theta \, r^2 \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= 5\int_0^{2\pi}\int_0^{\pi} r^5 \cos^4\theta \sin\theta \, d\theta \, d\phi$$

$$= -5\int_0^{2\pi} r^5 \frac{\cos^5\theta}{5} \Big|_0^{\pi} d\phi$$

$$= -\frac{\cancel{5}(3)^5}{\cancel{5}} [\cos^5\pi - \cos^5 0] \int_0^{2\pi} d\phi$$

$$= -(3)^5 [-1-1] \, 2\pi$$

$$\boxed{Q = 972\pi \, C}$$

Example: 3.5

Sol:-

$D = 4xy\hat{a}_x + 2(x^2+z^2)a_y +$
$4yz\hat{a}_z \; nC/m^2$

$0 < x < 2, \quad 0 < y < 3, \quad 0 < z < 5$

$\psi = \oint_{\substack{\text{closed} \\ \text{surface}}} D_s \cdot ds$

$= \iint D_{sx} \cdot ds_x + \iint D_{s-x} \cdot ds_{-x}$

$+ \iint D_{sy} \cdot ds_y + \iint D_{s-y} \cdot ds_{-y}$

$+ \iint D_{sz} \cdot ds_z + \iint D_{s-z} \cdot ds_{-z}$

$= \iint 4xy\hat{a}_x \cdot dy\,dz\,\hat{a}_x + \iint 4xy\hat{a}_x \cdot dy\,dz\,(-\hat{a}_x)$

$+ \iint 2(x^2+z^2)\hat{a}_y \cdot dx\,dz\,\hat{a}_y$

$+ \iint 2(x^2+z^2)\hat{a}_y \cdot dx\,dz\,(-\hat{a}_y)$

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$$+\iint 4yz\,\hat{a_z}\cdot dx\,dy\,\hat{a_z} + \iint 4yz\,\hat{a_z}\cdot dx\,dy\,(-\hat{a_z})$$

$$\Rightarrow \iint 4xy\,dy\,dz - \iint 4\overset{0}{\vec{x}}y\,dy\,dz$$

$$+\iint 2(x^2+\cancel{z^2})\,dx\,dz - \iint 2(x^2+\cancel{z^2})\,dx\,dz$$

$$+\iint 4yz\,dx\,dy \quad \iint 4y\overset{0}{z}\,dx\,dy$$

$$\Rightarrow \int_0^5\int_0^3 4(2)(y)\,dy\,dz + \int_0^3\int_0^2 4y(5)\,dx\,dy$$

$$= \int_0^5\int_0^3 8y\,dy\,dz + \int_0^3\int_0^2 20y\,dx\,dy$$

$$= \frac{\cancel{8}^{4}}{\cancel{2}_{1}}\,y^2\Big|_0^3 \int_0^5 dz + \frac{20}{2}\,y^2\Big|_0^3 \int_0^2 dx$$

$$= 4\,[9-0]\int_0^5 dz + 10\times 9\int_0^2 dx$$

$$= 36\,z\Big|_0^5 + 90\,x\Big|_0^2$$

$$= 36\times 5 + 90\times 2$$

$$= 180+180$$

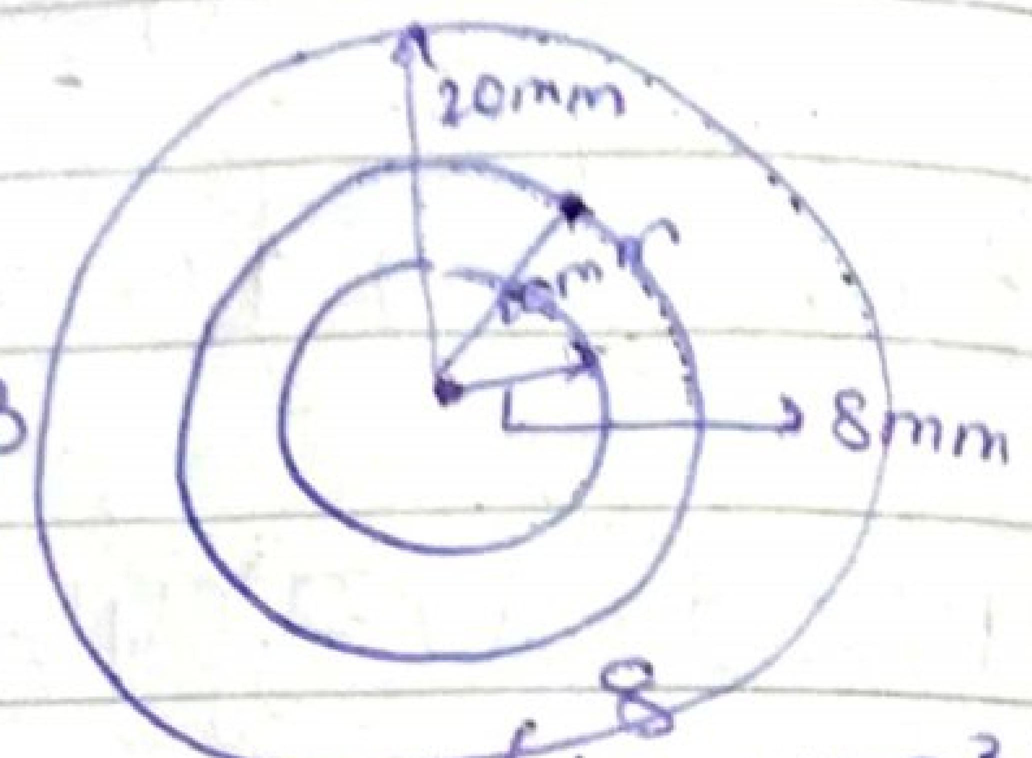
$$\boxed{= 360C}$$

Example: 3.9

$P_v = 80\,\mu C/m^3$

$\hookrightarrow 8mm < r < 10mm$

$Q = ??$ $\longrightarrow r = 10\,mm$ (a)



$Q = P_v \times \Delta V$

$Q = P_v \times \frac{4}{3} \times \pi \times r^3$

$Q = 80 \times 10^{-6} \times \frac{4}{3} \times \pi \times (\overset{8}{\cancel{10}} \times 10^{-3})^3$

$\boxed{Q = 171\,pC}$

$D_r = \frac{Q}{4\pi r^2}\,\hat{a}_r$

$= \frac{171 \times 10^{-12}}{4 \times \pi (10)^2}$

$\boxed{D_r = 0.36\,\mu C/m^2\,\hat{a}_r}$

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(c)

$r > 10mm$

$Q = 0$

$Q_{total} = 171 pC$

$$D_r = \frac{171 \times 10^{-12}}{4 \times \pi \times (20 \times 10^{-3})^2} \hat{a_r}$$

$$\boxed{D_r = 0.034 \mu C/m^2 \ \hat{a_r}}$$

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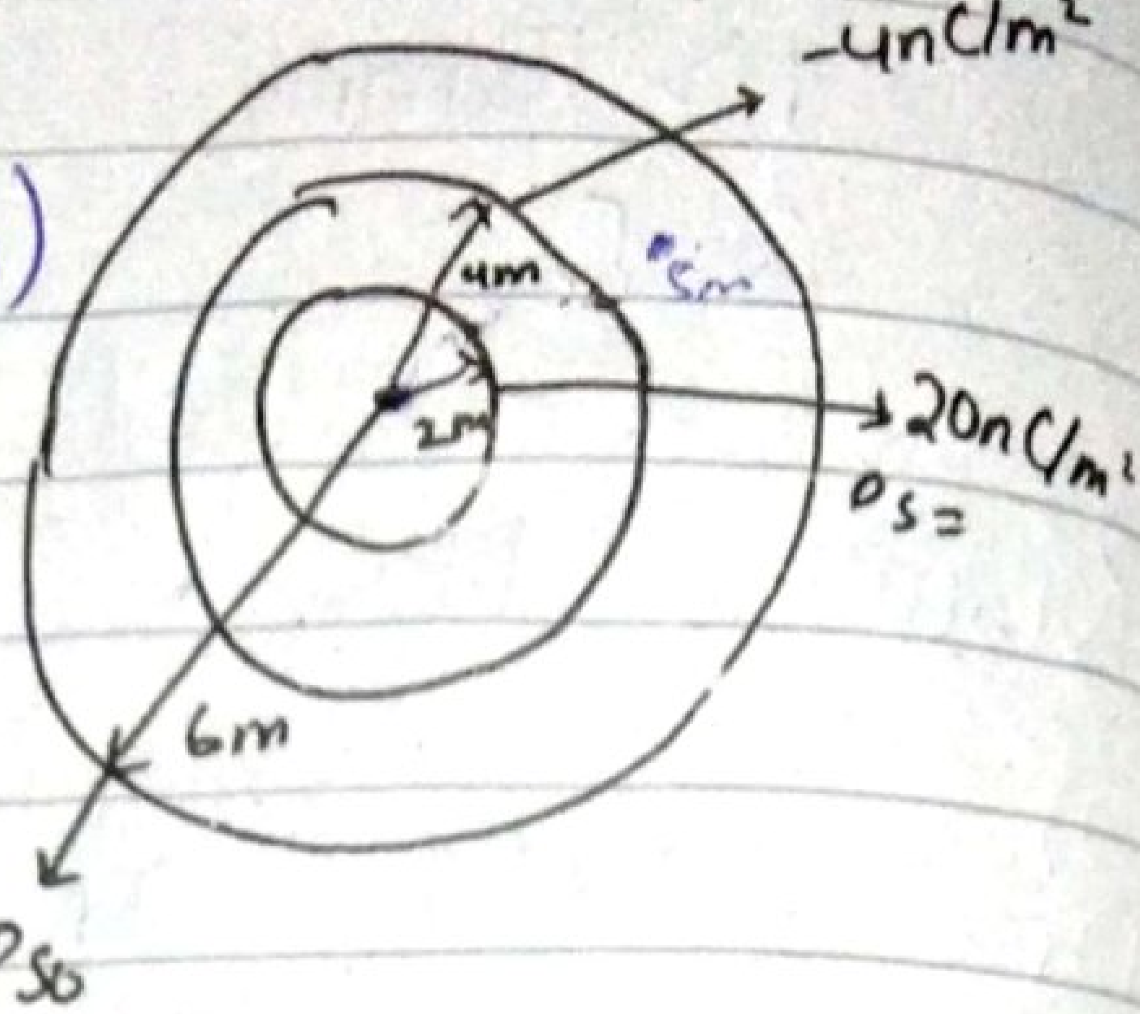
3.13

Sol:-

$D = ?$ (at $r = 1m$)



$0 < r < 2$

$Q = 0$

$D_r = 0$

$D_r = ??$ at $r = 3m$

at $2 < r < 4$

$Q = \rho_s \Delta s$

$= 20 \times 10^{-9} \times 4 \times \pi \times (2)^2$

$= 1.005 \times 10^{-6} C$

$D_r = \dfrac{Q}{4\pi r^2} \hat{a_r}$ at $(r = 3m)$

$= \dfrac{1.005 \times 10^{-6}}{4 \times \pi \times (3)^2} \hat{a_r}$

$= 8.8 \, nC/m^2$
$\hat{a_r}$

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$D_r = ??$ at $r = 5m$

~~at r~~

at $4 < r < 6$

$Q = \rho_s \times \Delta s$

$= -4 \times 10^{-9} \times 4 \times \pi \times (4)^2$

$= -8.042 \times 10^{-7} C$

$Q_{total} = 1.005 \times 10^{-6} - 8.042 \times 10^{-7}$

$Q_{total} = 2.007 \times 10^{-7} C$

$D_r = \frac{Q}{4\pi r^2} \hat{a_r}$

$= \frac{2.007 \times 10^{-7}}{4 \times \pi \times 25}$

$= 0.63 \, nC/m^2$

$\hat{a_r}$

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$r > 6$

$Q = \rho_s \times \Delta s$

$Q = \rho_s \times 4\pi r^2$

$Q = \rho_{s0} \times 4 \times \pi \times 36$

$Q = 144\pi \rho_{s0}$

$Q_{total} = 2.007 \times 10^{-7} + 144\pi \rho_{s0}$

$D = \dfrac{Q}{4\pi r^2} \hat{a_r}$

$0 = Q$

$-2.007 \times 10^{-7} = 144\pi \rho_{s0}$

$\boxed{\rho_{s0} = -0.44 \, nC/m^2}$

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