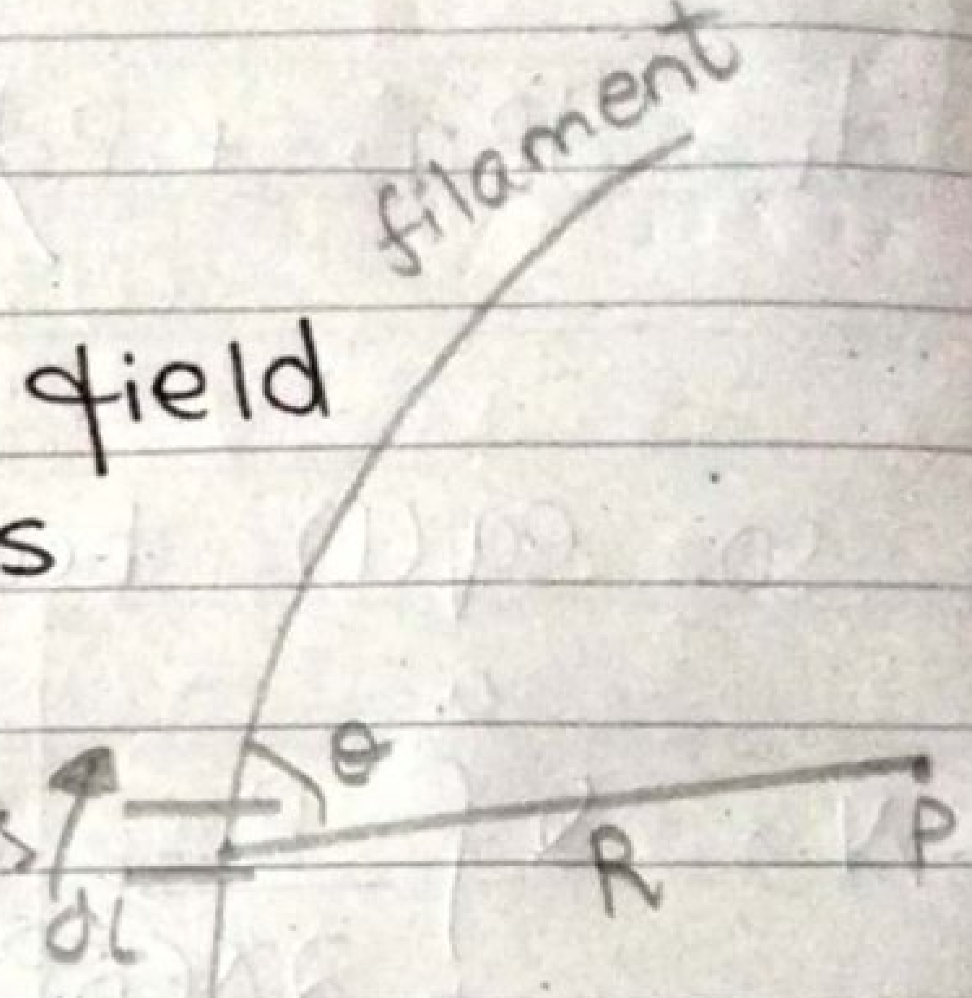
⇒ BIOT SAVART LAW:-

⇒ concerning the magnetic field produced by a differential dc element in free space

Biot savart law states that
" The magnitude of magnetic field intensity at any point 'P' is directly proportional to the product of current 'I' and magnitude of dL and sine of angle between the filament and the line joining the point where the magnetic field is desired and inversely proportional to the square of distance between the filament and point P "



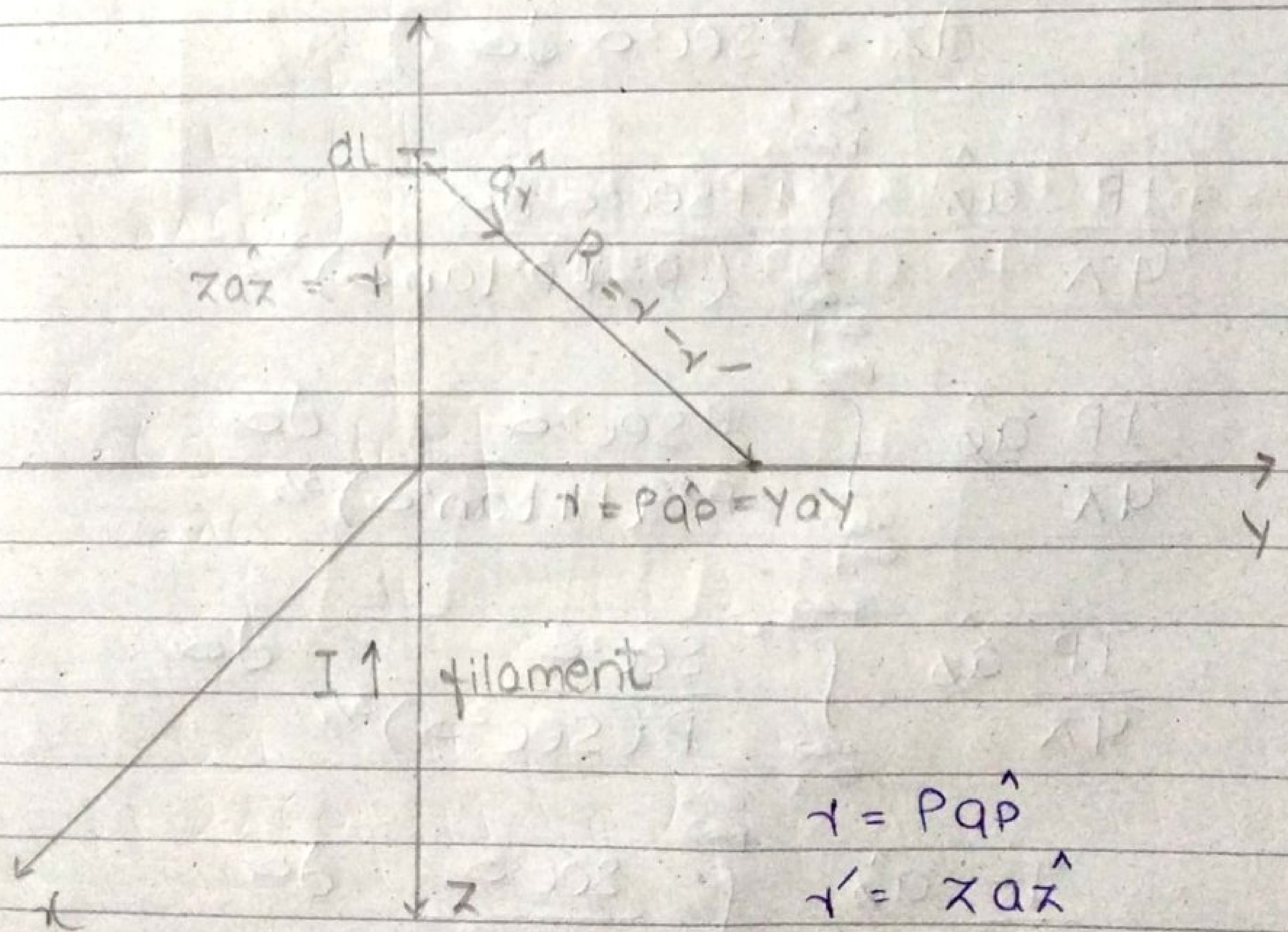
The direction of magnetic field intensity is normal to the plane containing the differential filament and line drawn from filament to point 'P'.

$$dH \propto \frac{I\,dL \sin\theta}{R^2}$$

$$dH = \frac{I\,dL \times \hat{a}_r}{4\pi R^2} \qquad \because \hat{a}_r = \frac{\vec{R}}{|\vec{R}|}$$

$$\boxed{H = \oint \frac{I\,dL \times \hat{a_r}}{4\pi R^2}}$$

An Infinitely Long Straight Filament Carrying Differential DC current I :-



$r = \rho \hat{a_\rho}$

$r' = z \hat{a_z}$

$\hat{R} = r - r' = \rho \hat{a_\rho} - z \hat{a_z}$

$dL = dz\,a_z$

$|\hat{R}| = \sqrt{\rho^2 + z^2}$

Scanned with CamScanner

$$H = \oint \frac{I dL}{4\pi R^2} \times \hat{a_r}$$

$$= \oint_{-\infty}^{+\infty} \frac{I\, dz \hat{a_z} \times (\rho \hat{a_\rho} - \cancel{z \hat{a_z}}^{\,0})}{4\pi (\rho^2 + z^2)^{3/2}}$$

$$= \oint_{-\infty}^{+\infty} \frac{I\, dz\, \hat{a_z} \times \hat{a_\rho}\, \rho}{4\pi (\rho^2 + z^2)^{3/2}} \qquad \because \hat{a_z} \times \hat{a_\rho} = \hat{a_\phi}$$

$$K \times i = J$$

$$= \frac{I}{4\pi} \oint_{-\infty}^{+\infty} \frac{dz\, a_\phi\, \rho}{(\rho^2 + z^2)^{3/2}}$$

$$= \frac{I \hat{a_\phi}}{4\pi} \oint_{-\infty}^{+\infty} \frac{dz\, \rho}{(\rho^2 + z^2)^{3/2}}$$

let $z = \rho \cdot \tan\alpha$

$dz = \rho \cdot \sec^2\alpha \, d\alpha$

$dz = \rho \sec^2\alpha \, d\alpha$

$$= \frac{I \hat{a_\phi}}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\alpha \, d\alpha}{(\rho^2 + \rho^2 \tan^2\alpha)^{3/2}}$$

$$= \frac{I \hat{a_\phi}}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\alpha \, d\alpha}{\rho^{\cancel{2} \times 3/\cancel{2}} (1 + \tan^2\alpha)^{3/2}}$$

$$= \frac{I}{4\pi\rho} \hat{a_\phi} \int_{\alpha_1}^{\alpha_2} \frac{\sec^2\alpha \, d\alpha}{(\sec^2\alpha)^{3/2}}$$

$$= \frac{I}{4\pi\rho} \hat{a_\phi} \int_{\alpha_1}^{\alpha_2} \frac{\sec^2\alpha}{\sec^3\alpha} \, d\alpha$$

$$= \frac{I}{4\pi\rho} \hat{a_\phi} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha$$

$$= \frac{I}{4\pi\rho} \hat{a_\phi} \left| \sin\alpha \right|_{\alpha_1}^{\alpha_2}$$

$$\boxed{H = \frac{I}{4\pi\rho} (\sin\alpha_2 - \sin\alpha_1) \hat{a_\phi}}$$

Scanned with CamScanner

Ampere's Circuital Law:-

$$\boxed{\oint H \cdot dl = I}$$

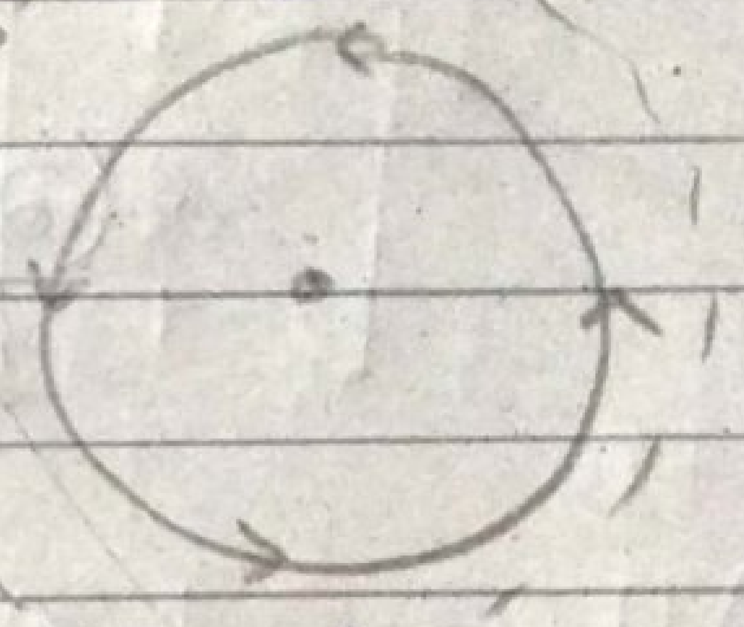
conditions for choosing surface

i) $dl \perp H$

$\vec{H} \cdot \vec{dl} = 0$ $\quad\because \cos(90^\circ) = 0$

• $dl \parallel H$ $\quad\because \cos(0^\circ) = 1$

$\bar{H} \cdot \bar{dl} = |H|\,|dl|$



ii) whenever $\bar{dl} \parallel H$, $|H|$ must be constant

$$\oint H \cdot dl = \int_0^{2\pi} H \cdot P\, d\phi\, \hat{a_\phi}$$

$$= HP \int_0^{2\pi} d\phi$$

$$I = HP\, 2\pi$$

$$\boxed{H_\phi = \frac{I}{2\pi P}\, \hat{a_\phi}}$$

Scanned with CamScanner

7.1

<u>Sol:</u>

(a) $P_1(0,0,2)$, $P_2(4,2,0)$

$I_1 \Delta L_1 = 2\pi a_z \mu$ Am

$\Delta H_2 = ??$

<u>Sol:-</u>

$$\Delta H_2 = \frac{I_1 \Delta L_1 \times \hat{a_r}\ \mu}{4\pi R^2}$$

$$= \frac{(2\pi a_z) \times \hat{a_r}\ \mu}{4\pi R^2}$$

$$= \frac{(2\pi a_z) \times \vec{R}\ \mu}{4\pi R^3} \longrightarrow (1)$$

$$\vec{R} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$$

$$= \frac{(2\pi \hat{a_z}) \times (4\hat{a_x} + 2\hat{a_y} - 2\hat{a_z})\ \mu}{4\pi (2\sqrt{6})^3}$$

$$= \frac{4 \times 2\pi \times 10^{-6}\ \hat{a_y} - 2\pi \times 2\hat{a_x} \times 10^{-6}}{1477.497}$$

$a_x \times a_y = a_z$
$a_y \times a_z = a_x$
$a_z \times a_x = a_y$

$= 17.01\,\hat{a_y} - 8.51\,\hat{a_x}$ n A/m

(b) $P_1(0,2,0)$, $P_2(4,2,3)$, $2\pi\, a_z$ UA·m

Sol:-

$$R = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$$

$$\Delta H_2 = \frac{I_1 \Delta L_1 \times \hat{a_r}}{4\pi R^2}$$

$$= \frac{(2\pi \times 10^{-6})\overset{\hat{a_z}}{1} \times (4\hat{a_x} + 3\hat{a_z})}{4\pi R^3}$$

$$= \frac{2 \times 4 \times \pi \times 10^{-6}\,\hat{a_y}}{4 \times \pi\,(5)^3}$$

$= 16\,\hat{a_y}$ n A/m

Scanned with CamScanner

(C)

P_1 (1, 2, 3)

P_2 (−3, −1, 2)

$I_1 \Delta L_1 = 2\pi(-a_x + a_y + 2a_z)\ \mu Am$

Sol:-

$$R = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ -3 \\ -1 \end{bmatrix}$$

$$\Delta H_2 = \frac{I_1 \Delta L_1 \times \hat{a}_r}{4\pi R^2}$$

$$= \frac{2\pi(-a_x + a_y + 2a_z) \times 10^{-6} \times (-4\hat{a}_x - 3\hat{a}_y - \hat{a}_z)}{4\pi R^3}$$

$$= \frac{(-2\pi a_x + 2\pi a_y + 4\pi a_z) \times (-4a_x - 3\hat{a}_y - a_z)}{4\pi R^3} \to (1)$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ -2\pi & 2\pi & 4\pi \\ -4 & -3 & -1 \end{vmatrix}$$

= a_x

= 4

=

=>

=

$= \hat{a_x}(-2\pi + 12\pi) - \hat{a_y}(2\pi + 16\pi)$
$+ a_z(6\pi + 8\pi)$

$= \hat{a_x}(10\pi) - 18\pi\hat{a_y} + 14\pi\hat{a_z}$

$\therefore$ so eq(1) becomes,

$\hat{a_z})$

$$\Delta H_2 = \frac{10\pi}{4\pi\, 26\sqrt{26}}\hat{a_x} - \frac{18\pi}{4\pi 26\sqrt{26}}\hat{a_y} + \frac{14\pi}{4\cdot\pi 26\sqrt{26}}\hat{a_z}\, \mu$$

$a_z)\ \mu$

$$\Delta H_2 = 18\cdot 9\hat{a_x} - 33\cdot 9\hat{a_y} + 26\cdot 4\, a_z\ nA/m$$

Scanned with CamScanner

Drill 7-2

$H = \frac{I}{2\pi\rho}\hat{a}_\rho$

$I = 15A$

$H_x = ??$
$H_y = ??$
$H_z = ??$

$P_A(\sqrt{20}, 0, 4)$

$P_s(0, 0, 4)$

$$\rho = \begin{bmatrix} \sqrt{20} \\ 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} \sqrt{20} \\ 0 \\ 0 \end{bmatrix}$$

$= \sqrt{20}\,\hat{a}_x$ $\quad |\rho| = 4.472$

$H = \frac{I}{2\pi(4.472)}\hat{a}_\phi$

$= \frac{15}{2\pi(4.472)}\hat{a}_\phi$

$= 0.534\,\hat{a}_\phi$

$H_x = 0.534\,\hat{a}_\phi \cdot \hat{a}_x$

$H_x = 0.534 - \sin\phi$
$H_x = 0$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \tan^{-1}\left(\frac{0}{\sqrt{20}}\right)$

$\phi = 0$

$H_y = 0.534\, \hat{a_\phi} \cdot \hat{a_y}$

$= 0.534 \cos\phi$

$= 0.534$

$H_z = 0.534\, \hat{a_\phi} \cdot \hat{a_z}$

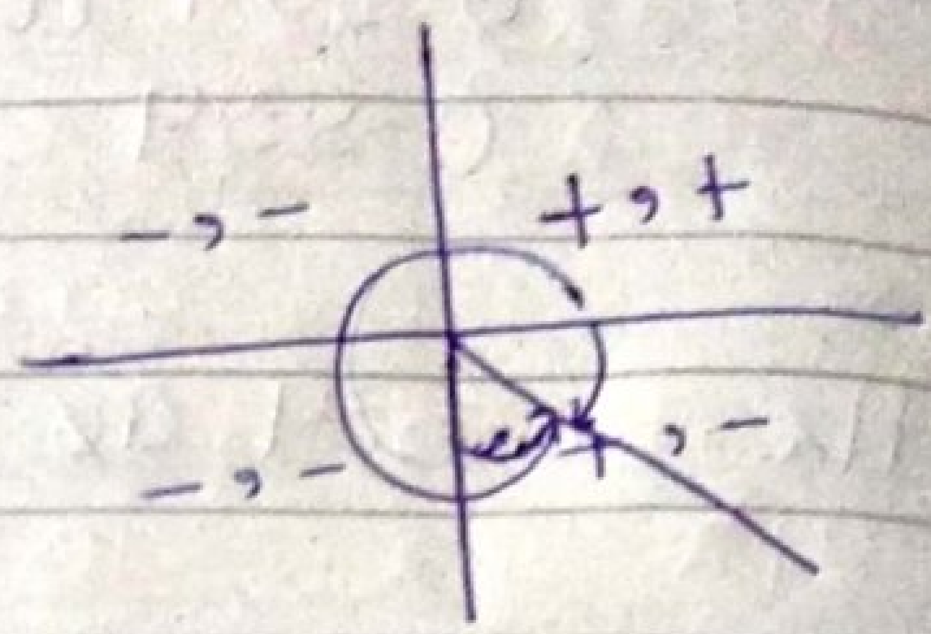
$= 0$

$\boxed{H = 0.534\, \hat{a_{\cancel{\phi}y}}} = 0.534\, \hat{a_y}$ A/m

Scanned with CamScanner

(b) $P_B\ (2,-4,4)$

Sol:-



$P_S = (0,0,4)$

$P_B = (2,-4,4)$

$$P = \begin{bmatrix} 2 \\ -4 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 0 \end{bmatrix}$$

$|P| = 4.472$

$$H = \frac{I}{2\pi\rho}\hat{a_\phi}$$

$$H = \frac{15}{2\pi \times 4.472}\hat{a_\phi}$$

$= 0.534\,\hat{a_\phi}$

$H_x = 0.534\,\hat{a_\phi}\cdot\hat{a_x}$

$= -0.534\sin\phi$

$= -0.534 \times \sin(296.6)$

$= 0.4774$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \tan^{-1}\left(\frac{-4}{2}\right)$

$\phi = -63.4 + 360^\circ$

$\phi = 296.6^\circ$

Scanned with CamScanner

$H_y = 0.534\,\hat{a_\phi} \cdot \hat{a_y}$

$= 0.534 \cos\phi$

$= 0.534 \cos(296.6^\circ)$

$= 0.239$

$H = 0.4774\,\hat{a_x} + 0.239\,\hat{a_y}$ A/m