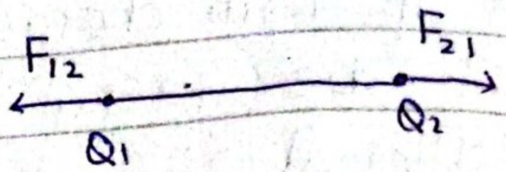
COULOMB'S LAW:-



⇒ According to this law,

$$F \propto q_1 q_2 \longrightarrow (1)$$

$$F \propto \frac{1}{r^2} \longrightarrow (2)$$

combining both equations:

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{F = K \frac{q_1 q_2}{r^2}}$$

$$K = \frac{1}{4\pi\epsilon_0}$$ ⇒ ϵ_0 = permittivity of free space

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \, F/m = 8.854 \times 10^{-12} \frac{F}{m}$$

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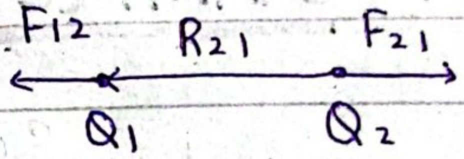
* If both charges are same then the force will be repulsive and if both charges are different then the force will be attractive.

⇒ FORCE ON Q_1 DUE TO Q_2.

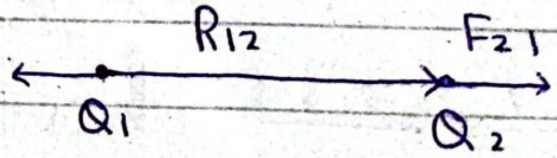


$$F_{12} = \frac{K\,Q_1 Q_2}{R_{21}^2}\,\hat{a}_{21}$$

$$\because \hat{a}_{21} = \frac{R_{21}}{|R_{21}|} = \frac{r_1 - r_2}{|r_1 - r_2|}$$

⇒ FORCE ON Q_2 DUE TO Q_1 :-

$$F_{21} = \frac{K\,Q_1 Q_2}{R_{12}^2}\,\hat{a}_{12}$$



$$\because \hat{a}_{12} = \frac{R_{12}}{|R_{12}|} = \frac{r_2 - r_1}{|r_2 - r_1|}$$

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$$\boxed{F_{12} = -F_{21}}$$

↳ magnitude of both forces are same but the direction is opposite

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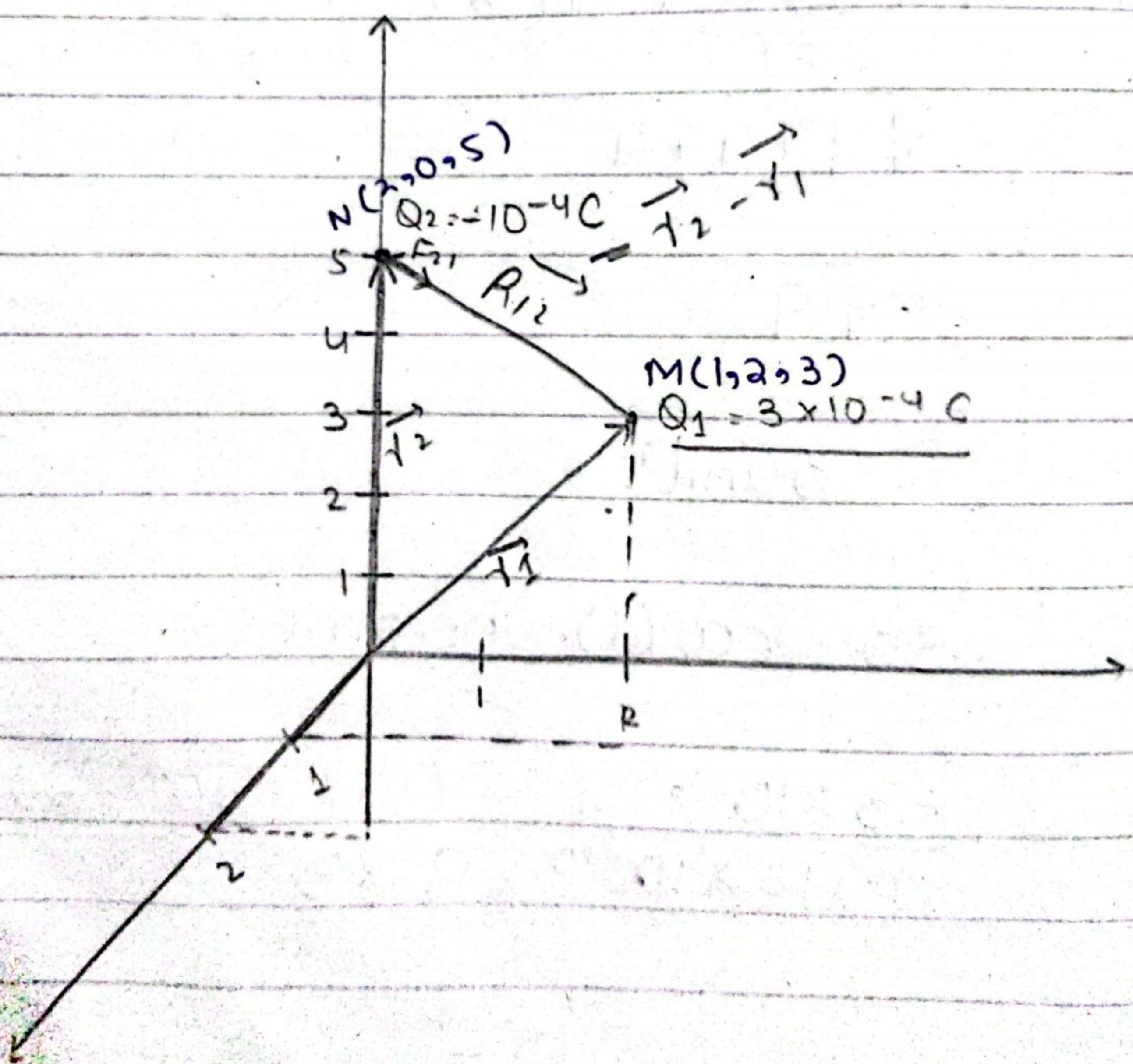
EXAMPLE : 2.1

Sol:-

$Q_1 = 3\times10^{-4}\,C$ at $M(1,2,3)$

$Q_2 = -10^{-4}\,C$ at $N(2,0,5)$

$F_{21} = ??$



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$$\Rightarrow R_{12} = r_2 - r_1$$

$$= -\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$|R_{12}| = \sqrt{1^2 + (-2)^2 + 2^2}$$

$$= \sqrt{9} = 3 \text{ units}$$

$$\hat{a}_{12} = \frac{a_x - 2a_y + 2a_z}{3}$$

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By applying coulomb's law:-

$$F = K\frac{q_1 q_2}{|R_{12}|^2} \hat{a_{12}}$$

$$= \frac{1}{4 \times \pi \times \frac{1}{36\pi} \times 10^{-9}} \frac{(3\times10^{-4}) \times (-10^{-4})}{9} \hat{a_{12}}$$

$$= -30 \left(\frac{\hat{ax} - 2\hat{ay} + 2\hat{az}}{3}\right)$$

$$\underline{F_{21} = -10\hat{ax} + 20ay - 20\hat{az}} \;//$$

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D 2·1 <u>Sol:</u>

$Q_A = -20\,\mu C$ at $A(-6, 4, 7)$

$Q_B = 50\,\mu C$ at $B(5, 8, -2)$

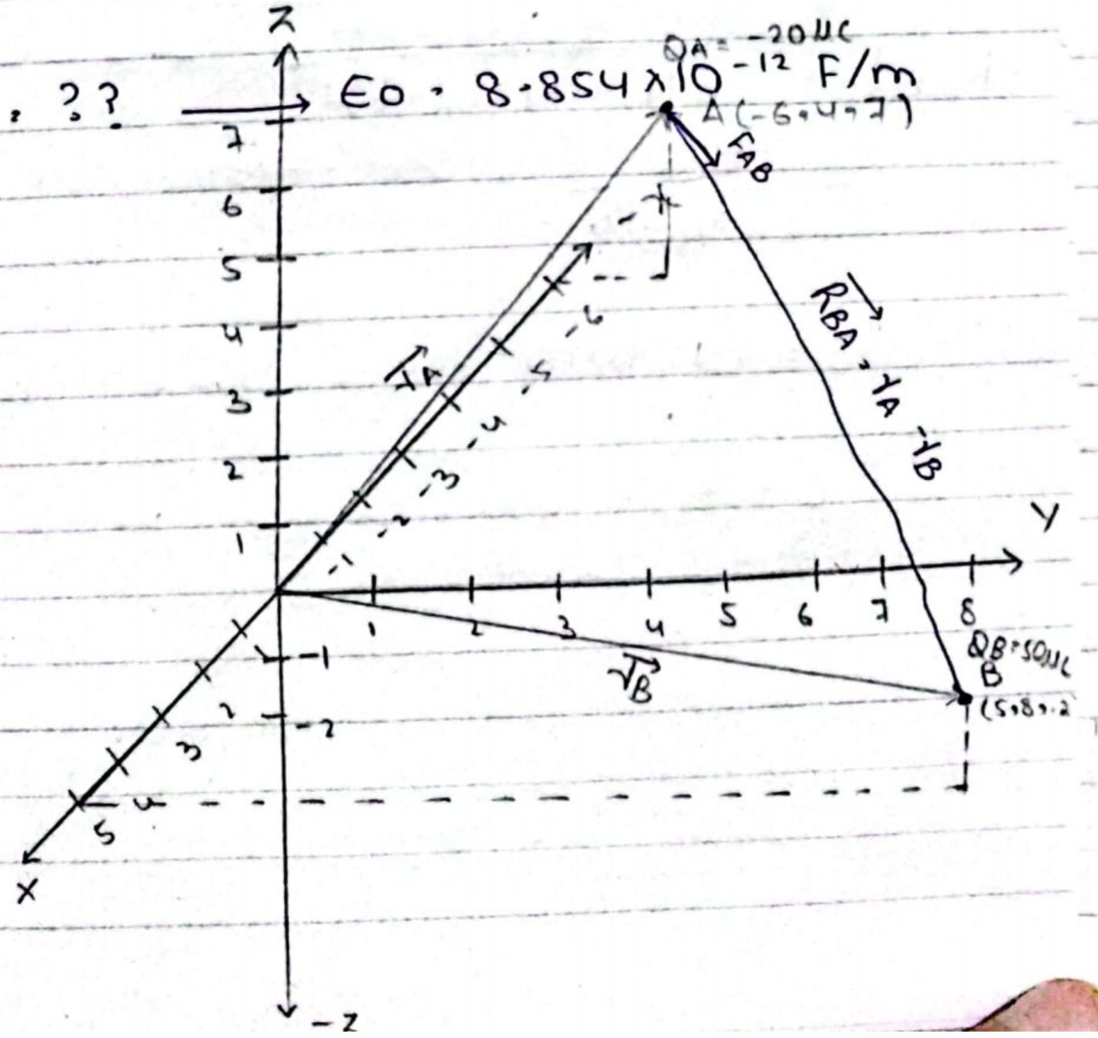
(a)

$R_{AB} = ??$

$|R_{AB}| = ??$

$F_{AB} = ??$ $\longrightarrow \epsilon_0 = \dfrac{10^{-9}}{36\pi}$ F/m

$F_{AB} = ??$ $\longrightarrow \epsilon_0 = 8.854 \times 10^{-12}$ F/m



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(a)

$R_{AB} = \vec{r_B} - \vec{r_A}$

$$= \begin{bmatrix} 5 \\ 8 \\ -2 \end{bmatrix} - \begin{bmatrix} -6 \\ 4 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ 4 \\ -9 \end{bmatrix}$$

(b)

$|R_{AB}| = \sqrt{11^2 + 4^2 + (-9)^2}$

$= \sqrt{218}$

$= 14.76 m$

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(C)

$R_{BA} = r_A - r_B$

$$= \begin{bmatrix} -6 \\ 4 \\ 7 \end{bmatrix} - \begin{bmatrix} 5 \\ 8 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -11 \\ -4 \\ 9 \end{bmatrix}$$

$$|R_{BA}| = \sqrt{(-11)^2 + (-4)^2 + 9^2}$$

$$= \sqrt{218} = 14.76m$$

$$F_{AB} = \frac{K q_1 q_2}{R_{BA}^2} \hat{a}_{BA}$$

$$= \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}} \times \frac{(-20 \times 10^{-6}) \times (50 \times 10^{-6})}{(14.76)^2} \hat{a}_{BA}$$

$$= \frac{-0.0412}{14.76} (-11\hat{a}_x - 4\hat{a}_y + 9\hat{a}_z)$$

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$= -2.796 \times 10^{-3} (-11\hat{a_x} - 4\hat{a_y} + 9\hat{a_z})$

$= 0.0307\,\hat{a_x} + 0.011184\,\hat{a_y} - 0.025\hat{a_z}$

(d)

$$F_{AB} = \frac{K q_1 q_2}{R_{BA}^2}\,\hat{a_{BA}}$$

$$= \frac{1}{4 \times \pi \times 8.854 \times 10^{-12}} \times \frac{(-20 \times 10^{-6})(50 \times 10^{-6})}{218}\,\hat{a_{BA}}$$

$$= \frac{-0.0412}{14.76} (-11\hat{a_x} - 4\hat{a_y} + 9\hat{a_z})$$

$= -2.792 \times 10^{-3} (-11\hat{a_x} - 4\hat{a_y} + 9\hat{a_z})$

$= 0.0307\,\hat{a_x} + 0.011168\,\hat{a_y}$
$- 0.025128\,\hat{a_z}$

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COULOMB'S LAW IN VECTOR FORM:-

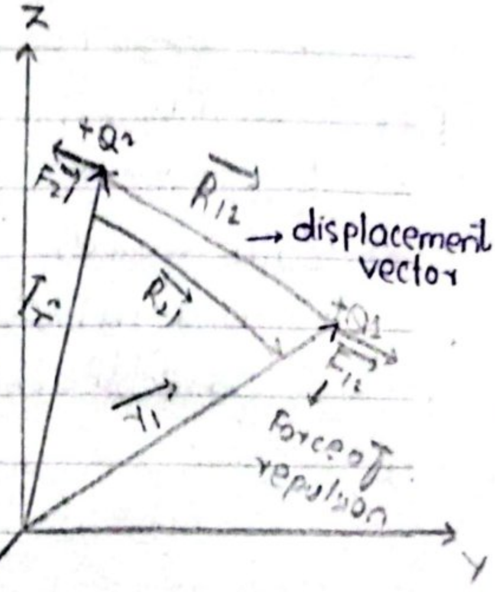
force exerted on q_2 due to q_1

DERIVATION:-

$$\vec{F_{21}} = K\frac{q_1 q_2}{|R_{12}|^2}\,\hat{a_{12}} \longrightarrow (1)$$

$$\vec{F_{12}} = \frac{Kq_1 q_2}{|R_{21}|^2}\,\hat{a_{21}} \longrightarrow (2)$$

Force exerted on q_1 due to q_2



$$\vec{R_{12}} = -\vec{R_{21}} \longrightarrow (3)$$

Put eq (3) in eq (1):

$$\vec{F_{21}} = K\frac{q_1 q_2}{|-R_{21}|^2}\,\hat{a_{12}}$$

$$= \frac{Kq_1 q_2}{|R_{21}|^2}\,\hat{a_{12}} = \frac{Kq_1 q_2}{|R_{21}|^2}\,-\hat{a_{21}}$$

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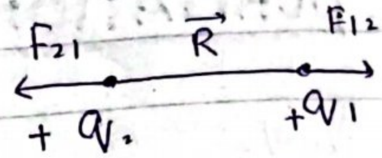
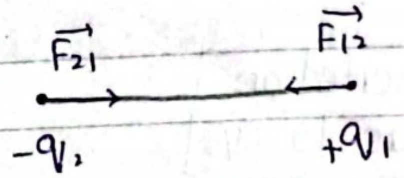
$$\vec{F_{21}} = -\left\{ \frac{K q_1 q_2}{R_{21}^2} \hat{a_{21}} \right\} \rightarrow \text{by eq (2)}$$



$$\boxed{\vec{F_{21}} = -\vec{F_{12}}}$$

=> Both forces will be same in magnitude but opposite in direction.



=> EXTENDED VECTOR FORM:

$$\hat{a_{12}} = \frac{\hat{R_{12}}}{|R_{12}|} \rightarrow (4)$$

$$\hat{r_1} + \hat{R_{12}} - \hat{r_2} = 0$$

$$\boxed{\hat{R_{12}} = \hat{r_2} - \hat{r_1}} \rightarrow (5)$$

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Put eq(5) in eq(4):

$$\hat{a_{12}} = \frac{\hat{r_2} - \hat{r_1}}{|r_2 - r_1|} \longrightarrow (6)$$

Put eq(6) in eq(~~1~~2):

$$\vec{F_{21}} = \frac{K q_1 q_2}{|R_{12}|^2} \frac{(\hat{r_2} - \hat{r_1})}{|r_2 - r_1|}$$

$\cancel{F_{12}}$

$$\vec{F_{21}} = \frac{K q_1 q_2}{|r_2 - r_1|^2} \times \frac{(\hat{r_2} - \hat{r_1})}{|r_2 - r_1|}$$

$$\boxed{\vec{F_{21}} = \frac{K q_1 q_2}{|r_2 - r_1|^3} (\hat{r_2} - \hat{r_1})}$$

=> Force exerted on q_2 due to q_1

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$$\hat{a}_{21} = \frac{\vec{R}_{21}}{|\vec{R}_{21}|} \longrightarrow (7)$$

$$= \vec{r}_2 + \vec{R}_{21} - \vec{r}_1 = 0$$

$$\vec{R}_{21} = \vec{r}_1 - \vec{r}_2 \rightarrow (8)$$

so eq (7) becomes,

$$\hat{a}_{21} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

so eq (2) becomes,

$$\vec{F}_{12} = \frac{K q_1 q_2}{|r_1 - r_2|^2} \times \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

$$\boxed{\vec{F}_{12} = \frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)}$$

⇒ Force exerted on q_1 due to q_2

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EXAMPLE: 2.2

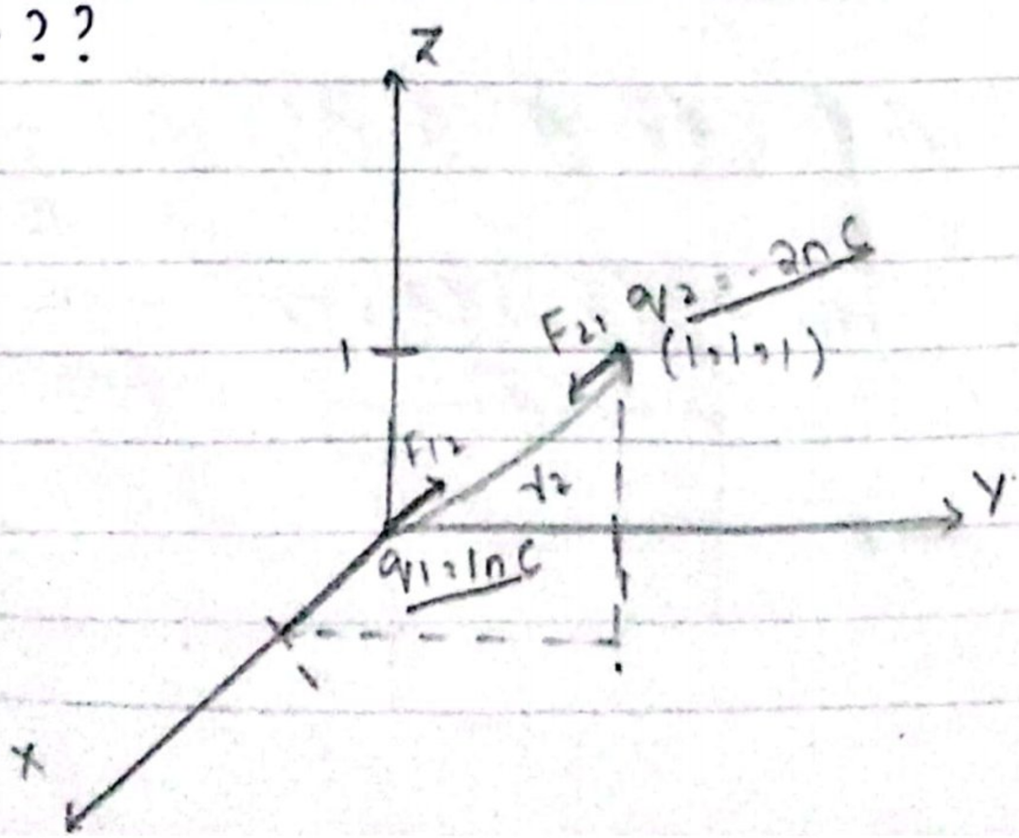
Sol:

$q_1 = 1nC$ at $(0,0,0)$

$q_2 = -2nC$ at $(1,1,1)$

$F_{12} = ??$

$F_{21} = ??$



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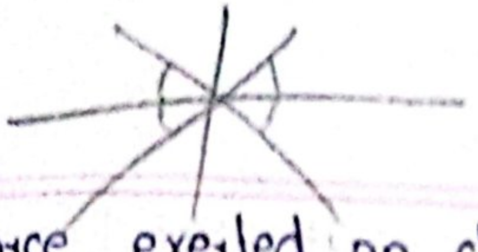

Force exerted on charge q_1 due to q_2 is:

$$\boxed{F_{12} = \frac{K q_1 q_2}{R_{21}^2}\, \hat{a}_{21} \longrightarrow (1)}$$

$$\vec{R}_{21} = \vec{r}_1 - \vec{r}_2$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$|\vec{R}_{21}| = \sqrt{3}$$

so eq(1) becomes:

$$= \frac{1}{4 \times \pi \times \epsilon_0} \quad \frac{1 \times -2}{3} \quad \times \frac{\vec{R}_{21}}{R_{21}}$$

$$= -3.46\,(-\hat{a}_x - \hat{a}_y - \hat{a}_z)$$

$$= 3.46\,\hat{a}_x + 3.46\,\hat{a}_y + 3.46\,\hat{a}_z \quad //$$

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$\Rightarrow$ Force exerted on charge q_2 due to q_1 is:

$$F_{21} = \frac{K q_1 q_2}{R_{12}^2} \hat{a_{12}} \longrightarrow (1a)$$

$$R_{12} = \vec{r_2} - \vec{r_1}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$|R_{12}| = \sqrt{3}$$

so equation 1(a) becomes:

$$F_{21} = \frac{1}{4\pi\epsilon_0} \times \frac{(1)(-2)}{(\sqrt{3})^2} \times \hat{a_{12}}$$

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$$\Rightarrow -3.46(\hat{a}_x + \hat{a}_y + \hat{a}_z)$$

$$\Rightarrow -3.46\,\hat{a}_x - 3.46\,\hat{a}_y - 3.46\,\hat{a}_z$$

$$\boxed{F_{12} = -F_{21}}$$

EXAMPLE 2.3

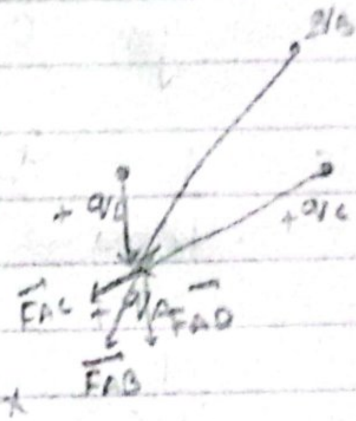
Sol:

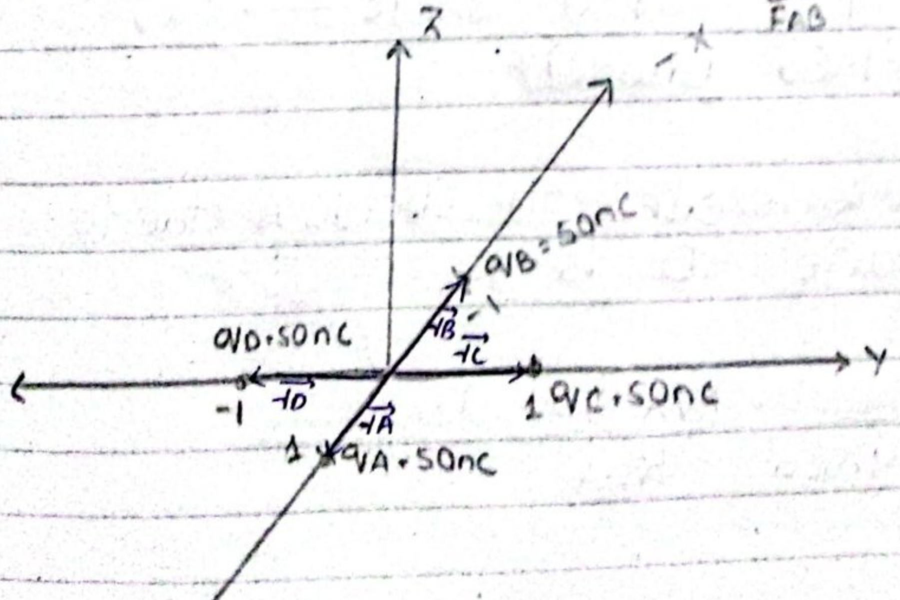
$q_A = 50nC$ at $(1,0,0)$

$q_B = 50nC$ at $(-1,0,0)$

$q_C = 50nC$ at $(0,1,0)$

$q_D = 50nC$ at $(0,-1,0)$





Force exerted on A due to B

$$F_{AB} = \frac{1}{4\pi\epsilon_0} \frac{q_A q_B}{R_{BA}^2} \hat{a}_{BA}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{50 \times 50}{(r_A - r_B)^3} (r_A - r_B) \longrightarrow (1)$$

$\Rightarrow$ Force exerted on charge A due to charge C is:

$$F_{AC} = \frac{1}{4\pi\epsilon_0} \frac{q_A q_C}{R_{CA}^2} \hat{a}_{CA}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{50 \times 50}{(r_A - r_C)^3} r_A - r_C \longrightarrow (2)$$

$\Rightarrow$ Force exerted on Charge A due to charge D is:

$$F_{AD} = \frac{1}{4\pi\epsilon_0} \frac{q_A q_D}{R_{DA}} \hat{a}_{DA}$$

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$$= \frac{1}{4\pi\epsilon_0} \frac{50\times 50}{(r_A - r_D)^3} (r_A - r_D) \rightarrow (3)$$

$$r_A - r_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \Bigg| = |r_A - r_B| = 2$$

$$r_A - r_C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \Bigg| = |r_A - r_C| = \sqrt{1^2 + (-1)^2}$$

$$= \sqrt{2}$$

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$$r_A - r_D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad |r_A - r_D| = \sqrt{2}$$

$\Rightarrow$ Total force exerted on point charge A is :

$$\vec{F} = \vec{F_{AB}} + \vec{F_{AC}} + \vec{F_{AD}}$$

$$= \frac{50 \times 50}{4\pi\epsilon_0}\left[\frac{r_A - r_B}{(r_A - r_B)^3} + \frac{r_A - r_C}{(r_A - r_C)^3} + \frac{r_A - r_D}{(r_A - r_D)^3}\right]$$

$$= 2.246 \times 10^{13}\left[\frac{2a_x}{2^3} + \frac{(a_x - a_y)}{(\sqrt{2})^3} + \frac{a_x + a_y}{(\sqrt{2})^3}\right]$$

$$= \cancel{2.246 \times 10^{13}\left[\frac{2a_x + a_x - a_y + a_x + a_y}{64}\right]}$$

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$$= \frac{2\cdot 246\times 10^{13}}{64} \times 4\hat{a_x}$$

$$= 1{:}4\times 10^{3}\hat{a_x}$$

$$= 2\cdot 246\times 10^{13}\left[\frac{2a_x}{2^3} + \frac{a_x - a_y + a_x + a_y}{(\sqrt{2})^3}\right]$$

$$= 2\cdot 246\times 10^{13}\left[\frac{4a_x}{16\sqrt{2}}\right]$$

$$= \boxed{21\cdot 6\,\hat{a_x}\ \mu N}$$

$\downarrow$

=> is the total force exerted on A

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⇒ FIELD OF AN INFINITE UNIFORM SHEET OF CHARGES:-

⇒ Symmetry check:

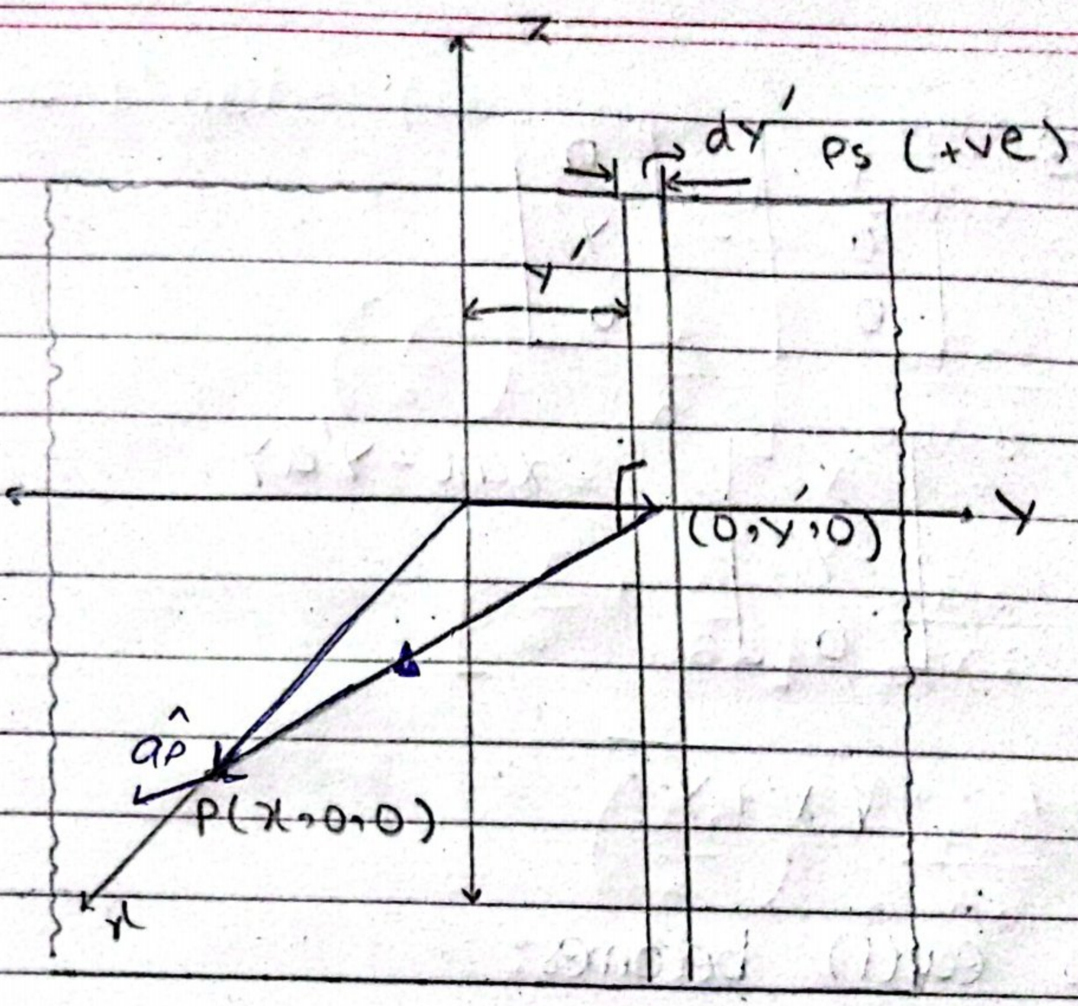
⇒ the co-ordinate for which the magnitude of field is expected to change.

1) $\overset{\checkmark}{x}, \overset{\times}{y}, \overset{\times}{z}$

2) $\overset{\checkmark}{\rho}, \overset{\checkmark}{\phi}, \overset{\times}{z}$ ×

3) $\overset{\checkmark}{r}, \overset{\checkmark}{\theta}, \overset{\checkmark}{\phi}$ ×

⇒ we choose cartesian co-ordinate system to derive expression for electric field of an infinite uniform sheet of charges.



$\Rightarrow$ Electric field intensity due to a line charge is,

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\hat{a}_\rho$$

$\because \rho_L = \rho_s \, dy'$

$$\overline{dE} = \frac{\rho_s \, dy'}{2\pi\epsilon_0 \rho}\hat{a}_\rho \longrightarrow (1)$$

$$\vec{R} = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ y' \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ -y' \\ 0 \end{bmatrix} = x a_x - y' a_y$$

$$|R| = \sqrt{x^2 + y'^2}$$

so eq(1) becomes:

$$\overline{dE} = \frac{\rho_s \, dy' \, (x \hat{a}_x - y \hat{a}_y)}{2\pi\epsilon_0 (x^2 + y'^2)} \rightarrow (3)$$

Symmetry check clause: 02
Check which of the component exist

$$\overline{E} = E_x \hat{a}_x + \cancel{E_y}^{0} \hat{a}_y + \cancel{E_z}^{0} \hat{a}_z$$

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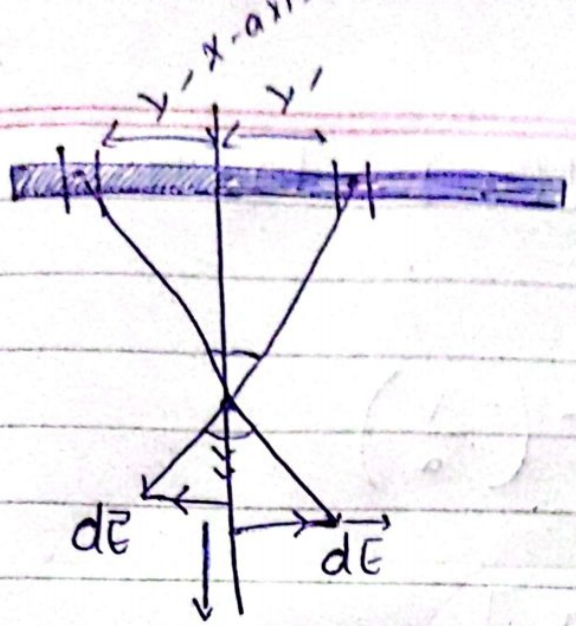

Y-component will cancel out

so eq (3) becomes:

$$\overline{dE_x} = \frac{\rho_s\, dy' \,(x \hat{a}_x)}{2\pi\epsilon_0 (x^2+y'^2)}$$

$$= \frac{\rho_s \hat{a}_x\, x}{2\pi\epsilon_0} \int_{-\infty}^{+\infty} \frac{dy'}{(x^2+y'^2)}$$

$$= \frac{\rho_s \hat{a}_x \cancel{x}}{2\pi\epsilon_0} \left[\frac{1}{\cancel{x}} \tan^{-1}\left(\frac{y'}{x}\right) \right]_{-\infty}^{+\infty}$$

$$= \frac{\rho_s \hat{a}_x}{2\pi\epsilon_0} \left[\tan^{-1}\left(\frac{y'}{x}\right) \right]_{-\infty}^{+\infty}$$

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$$= \frac{\rho_s \hat{a_x}}{2\pi\epsilon_0}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right]$$

$$= \frac{\rho_s \hat{a_x}}{2\pi\epsilon_0}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{\rho_s \hat{a_x}}{2\pi\epsilon_0}\left[\frac{\pi}{2} + \frac{\pi}{2}\right]$$

$$= \frac{\rho_s \hat{a_x}}{2\pi\epsilon_0}\left(\frac{2\pi}{2}\right)$$

$$= \frac{\rho_s \hat{a_x}}{2\epsilon_0}$$

$$= \frac{\rho_s}{2\epsilon_0}\hat{a_x}$$

$$\vec{E} = \frac{\rho_s}{2\epsilon_0}\hat{a_N}$$

↳ perpendicular

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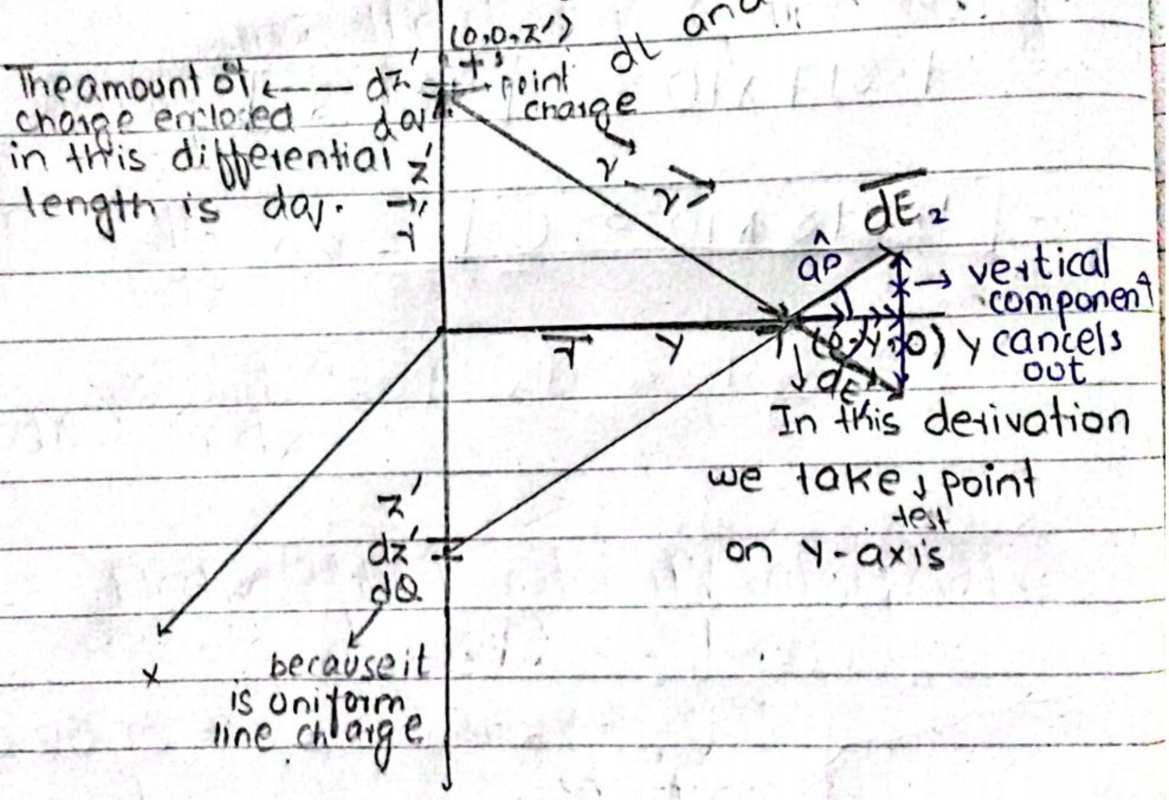
ELECTRIC FIELD DUE TO A LINE CHARGE:-

infinite $\boxed{\text{uniform}}$ → line charges k kisi bhi portion ko cut krenga to some length & charges hogi which is dl and dq.



=> SYMMETRY CHECK:-

1) the co-ordinate for which the magnitude of field is expected to change.

2) which component of the field would

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exist.

1) $x, y, z \mid \hat{a_x}, \hat{a_y}, \hat{a_z} \rightarrow$ Rectangular co-ordinate system.

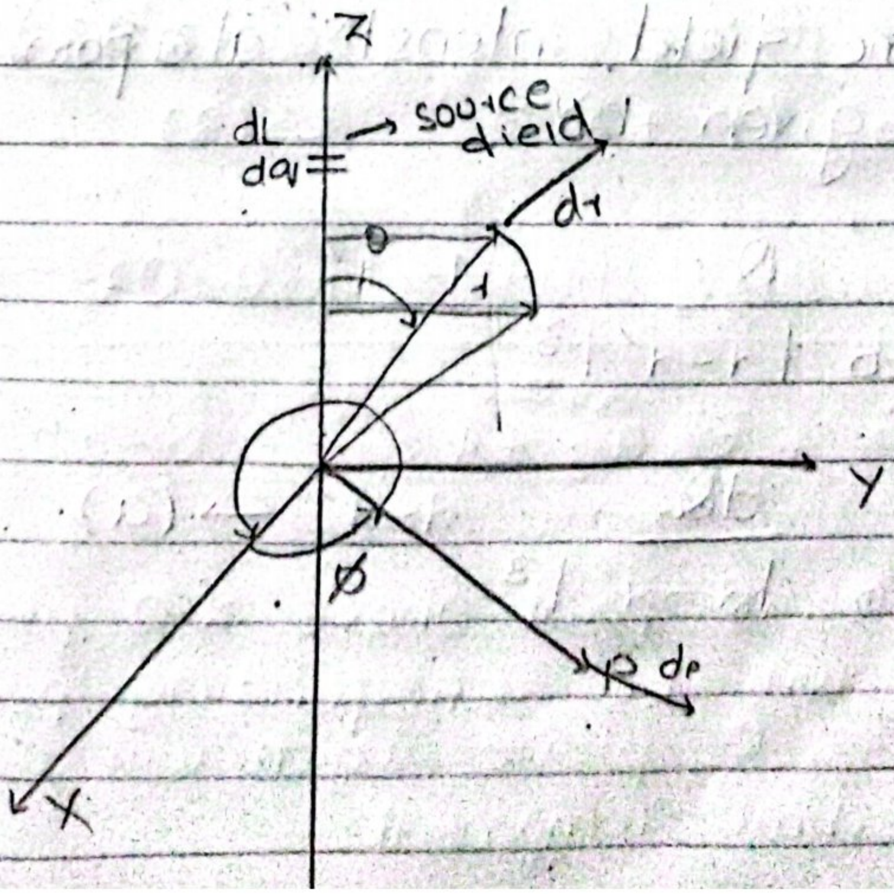
$\vec{E} = E_x\hat{a_x} + E_y\hat{a_y} + E_z\hat{a_z}$

2) $\rho, \phi, z \mid \hat{a_\rho}, \hat{a_\phi}, \hat{a_z} \rightarrow$ cylindrical co-ordinate system

$\vec{E} = E_\rho\hat{a_\rho} + E_\phi\hat{a_\phi} + E_z\hat{a_z}$

3) $r, \theta, \phi \mid \hat{a_r}, \hat{a_\theta}, \hat{a_\phi} \rightarrow$ spherical co-ordinate system

$\vec{E} = E_r\hat{a_r} + E_\theta\hat{a_\theta} + E_\phi\hat{a_\phi}$



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⇒ we will use cylindrical co-ordinate system to get one variable in final formula.

⇒ By using the formula of line charge density.

$$\rho_L = \frac{dQ}{dz}$$

$$dQ = \rho_L dz$$

→ At first we will treat as a point charge

⇒ Electric field intensity at a point is given by:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|r-r'|^3} r-r'$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dQ}{|r-r'|^3} r-r' \rightarrow (a)$$

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$r' = z' a_z$

$r = y \hat{a}_y = \rho \hat{a}_\rho$

$r - r' = \rho \hat{a}_\rho - z' \hat{a}_z$

$|r - r'| = \sqrt{\rho^2 + (-z')^2}$

$= \sqrt{\rho^2 + z'^2}$

so eq(a) becomes:

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\rho_L dz'}{(\rho^2 + z'^2)^{3/2}} (\rho \hat{a}_\rho - z' \hat{a}_z)$$

$\hookrightarrow (2)$

Symmetry check clause: 02

$E = E_\rho \hat{a}_\rho + \cancel{E_\phi a_\phi}^{0} + \cancel{E_z \hat{a}_z}^{0}$

$\Rightarrow$ for every point above $z=0$ plane there will be corresponding point below $z=0$ plane which will cancel out vertical component.

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so eq (2) becomes:

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\rho_L \, dz'}{(\rho^2 + z'^2)^{3/2}} \rho \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \hat{a}_\rho \int_{-\infty}^{+\infty} \frac{dz'}{(\rho^2+z'^2)^{3/3}}$$

substitute $z' = \rho \tan \alpha$

$$E = \boxed{\frac{\rho_L \cancel{\rho}}{\cancel{4}\, 2\pi\epsilon_0 \rho} \hat{a}_\rho} \quad V/m$$

ρ_L = line charge density

ρ = perpendicular distance b/w the test point and line charge

PROBLEM:01

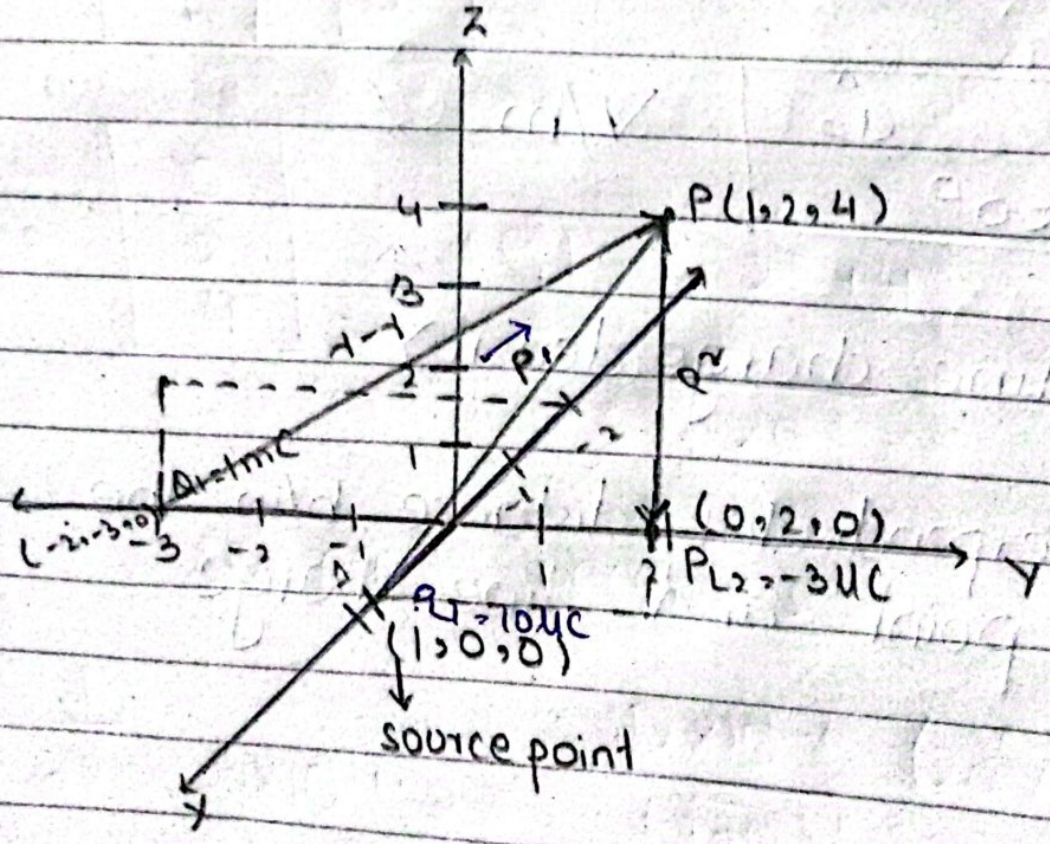
$E = ??$

$P(1,2,4)$ in free space

i) $Q_1 = 1mC$ at $(-2,-3,0)$

ii) $P_{L1} = 10\mu C$ on x-axis

iii) $P_{L2} = -3\mu C$ on Y-axis.



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$\Rightarrow$ Electric field intensity due to Q_1 is,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{|r-r'|^3} \, r-r'$$

$$= \frac{1\times10^{-3}}{4\times\pi\times8.85\times10^{-12}} \frac{r-r'}{|r-r'|^3} \rightarrow (1)$$

$$r-r' = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} - \begin{bmatrix} -2 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$$

$$\boxed{|r-r'| = 5\sqrt{2}}$$

So eq(1) becomes,

$$= \frac{1\times10^{-3}}{4\times\pi\times8.85\times10^{-12}} \frac{3\hat{a_x}+5\hat{a_y}+4\hat{a_z}}{(5\sqrt{2})^3}$$

$$= 25432.664\,(3\hat{a_x}+5\hat{a_y}+4\hat{a_z})$$

$$\boxed{= 76.2\,\hat{a_x} + 127\,\hat{a_y} + 101.7\,\hat{a_z}} \text{ KV/m}$$

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⇒ Electric field intensity due to line charge on x-axis is:

$$E_{P_1} = \frac{1}{2\pi\epsilon_0} \frac{\rho_L}{\rho_1} \hat{a}_{P_1} \longrightarrow (1)$$

$$P_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} \qquad |P_1| = \sqrt{2^2 + 4^2}$$

$$= \sqrt{20}$$

⇒ <u>So eq (1) becomes:</u>

$$\frac{1}{2 \times 8.85 \times 10^{-12} \times \pi} \cdot \frac{10 \times 10^{-6}}{(\sqrt{20})^2} (2\hat{a}_y + 4\hat{a}_z)$$

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$\Rightarrow 8991.804 \, (2\hat{a_y} + 4\hat{a_z})$

$\Rightarrow 17.9\hat{a_y} + 35.9\hat{a_z}$ KV/m

$\Rightarrow$ Electric field intensity due to line charge on y-axis at point (1,2,4) is:-

$$E P_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ -4 \end{bmatrix} = |P_2| = \sqrt{(-1)^2 + (-4)^2}$$

$$= \sqrt{17}$$

$$E_{P_2} = \frac{1}{2\pi\epsilon_0} \frac{\rho_L}{P_2} \hat{a}_{P_2}$$

$$= \frac{1}{2 \times \pi \times 8.85 \times 10^{-12}} \frac{3 \times 10^{-6}}{17} (-a_x - 4a_z)$$

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$= 3173.578 \, (-a_x - 4a_z)$

$= \boxed{-3.1 \, \hat{a}_x - 12.6 \, \hat{a}_z}$ kV/m

DRILL 2.5

(a)

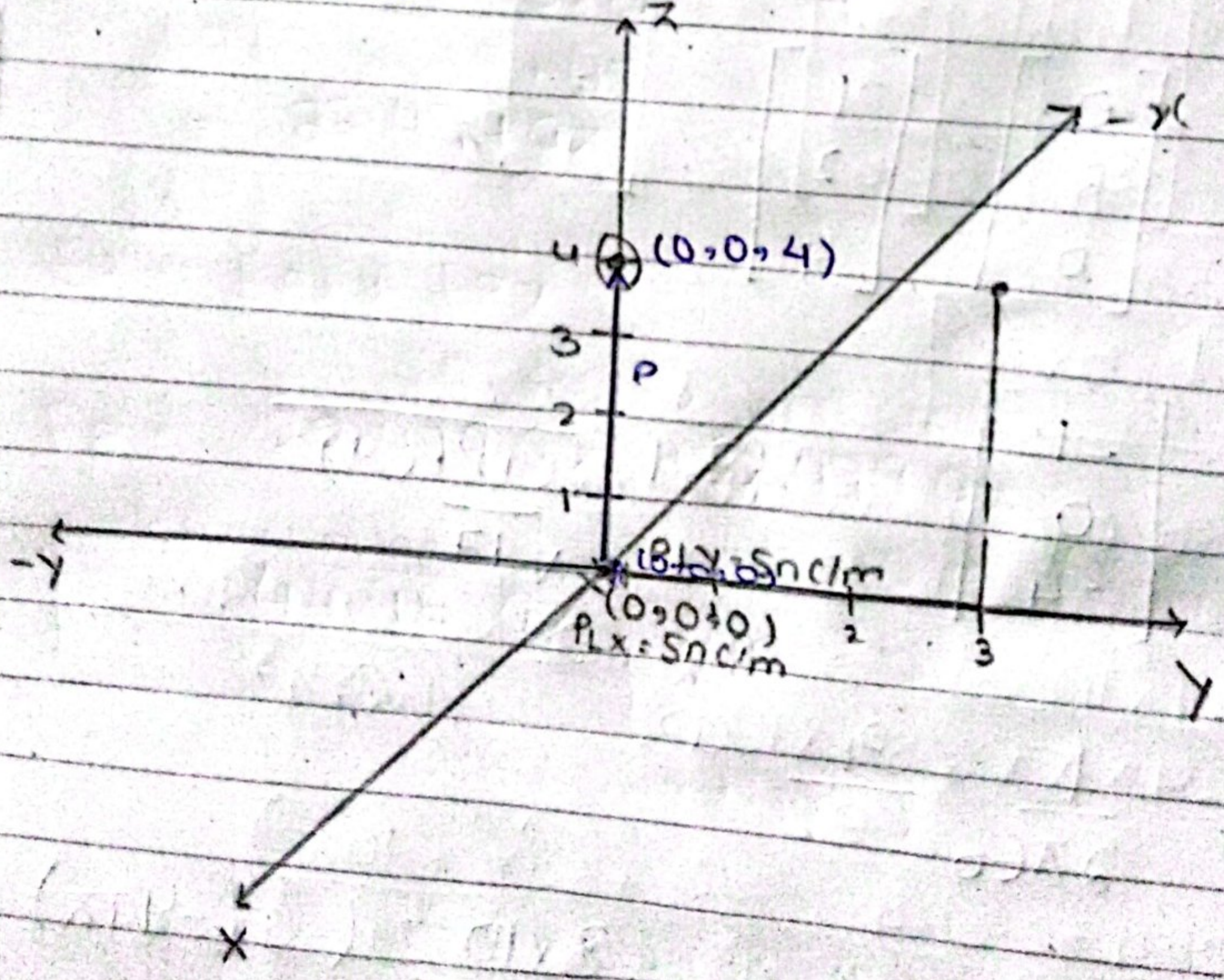
$P_A (0, 0, 4)$

$\rho_{Lx} = 5$ nC/m

$\rho_{Ly} = 5$ nC/m

(b)

$P_B (0, 3, 4)$



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⇒ Electric field intensity due to line charge on x-axis at point (0,0,4) is,

$$E_{Px} = \frac{1}{2\pi\epsilon_0}\frac{P_{Lx}}{P}\hat{a}_P \rightarrow (1)$$

$$P = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = 4 = |P|$$

so eq (1) becomes:

$$= \frac{1}{2 \times \pi \times \epsilon_0}\frac{5 \times 10^{-9}}{4^2} 4\hat{a}_z$$

$$= 22.479\,\hat{a}_z$$

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⇒ Electric field intensity due to line charge on y-axis at point (0,0,4) is,

$$E_{Py} = \frac{1}{2\pi\epsilon_0} \frac{\rho_{LY}}{\rho} \hat{a}_\rho$$

$$\rho = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rho = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \quad \boxed{|\rho| = 4}$$

$$= \frac{1}{2 \times \pi \times 8.85 \times 10^{-12}} \frac{5 \times 10^{-9}}{4^2} 4\hat{a}_z$$

$$= 22.479 \hat{a}_z$$

$$E_P = E_{Px} + E_{Py}$$

$$E_P = 22.479 \hat{a}_z + 22.479 \hat{a}_z$$

$$\boxed{E_P = 44.96 \hat{a}_z} \text{ V/m}$$

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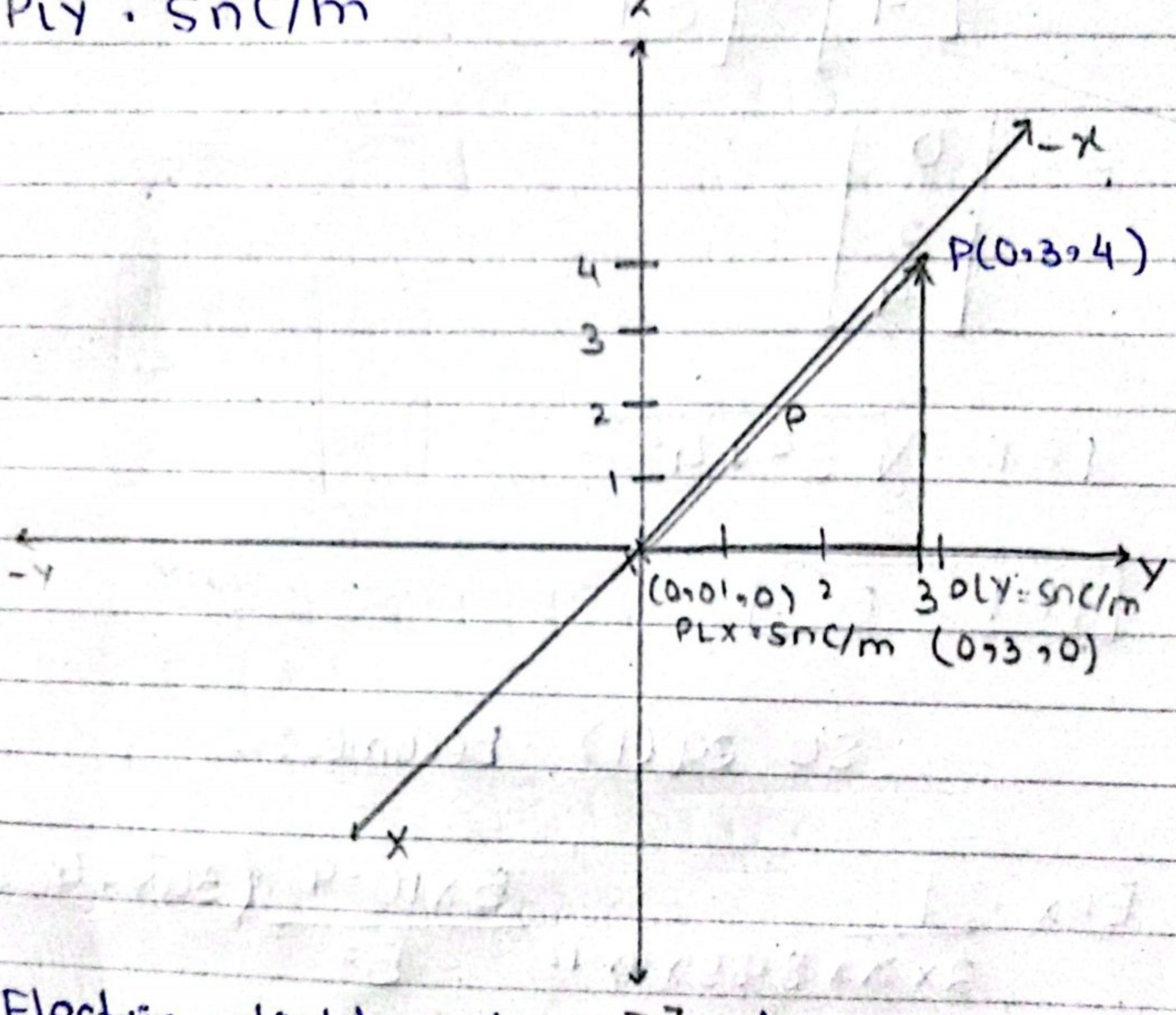
(b)

$P_B\ (0,3,4)$

$\rho_{Lx} = 5nC/m$
$\rho_{Ly} = 5nC/m$



Electric field intensity due to line charge on x-axis at point (0,3,4) is

$$E_{\rho x} = \frac{1}{2\pi\epsilon_0} \frac{\rho_{Lx}}{\rho} \hat{a}_\rho \longrightarrow (1)$$

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$$P = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

$$|P| = \sqrt{3^2 + 4^2}$$

$$\boxed{|P| = 5}$$

so eq(1) becomes:-

$$E_{Px} = \frac{1}{2 \times \pi \times 8.85 \times 10^{-12}} \frac{5 \times 10^{-9}}{5^2} (3\hat{a_y} + 4\hat{a_z})$$

$$= 3.596 (3\hat{a_y} + 4\hat{a_z})$$

$$= 10.79 \hat{a_y} + 14.384 \hat{a_z}$$

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⇒ Electric field intensity due to line charge on y-axis at P (0, 3, 4)

$$P = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = |P| = 4$$

$$E_{Py} = \frac{1}{2\pi\epsilon_0} \frac{5 \times 10^{-9}}{4^2} (4\hat{a_z})$$

$$= 5.619 \times 4 \, \hat{a_z}$$

$$= 22.479 \, \hat{a_z}$$

$$E = E_{Px} + E_{Py}$$

$$E = 10.79 \, \hat{a_y} + 14.384 \, \hat{a_z} + 22.479 \, \hat{a_z}$$

$$\boxed{E = 10.79 \, \hat{a_y} + 36.86 \, \hat{a_z} \ \ V/m}$$

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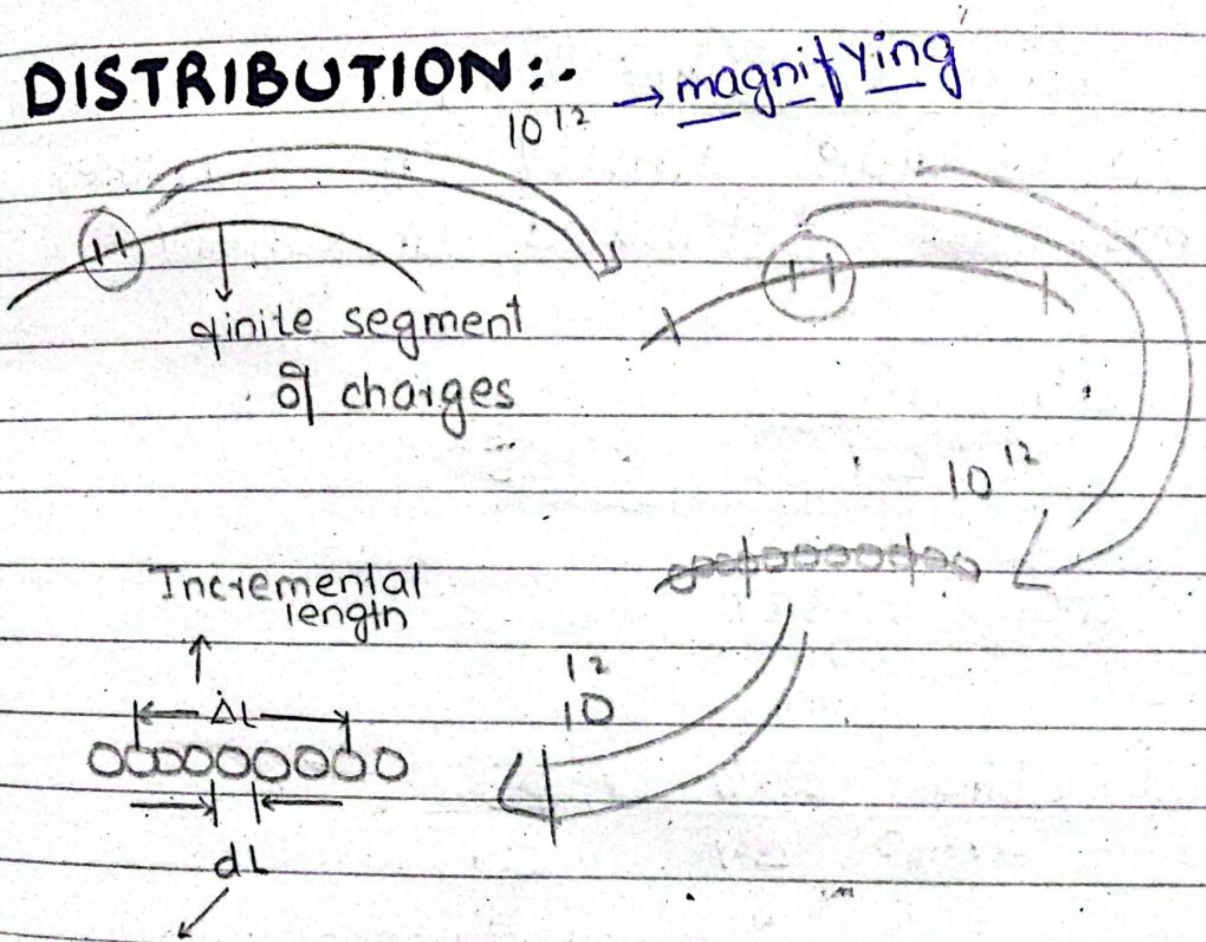
⇒ FIELD ARISING FROM A CONTINOUS VOLUME CHARGE DISTRIBUTION :-



⇒ only one charge or one point is differential length

⇒ differential entities are always equal to point size

⇒ Incremential entities can never be treated as a point.

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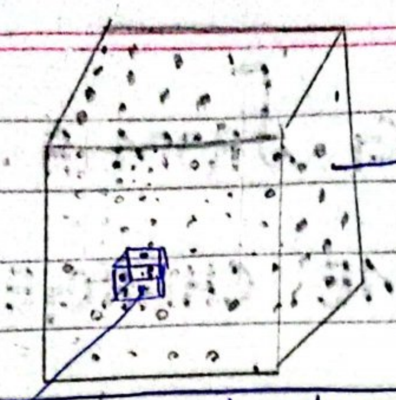
→ three dimensional

⇒ VOLUME CHARGE DENSITY:



$\boxed{\text{incremental volume} = \Delta V}$

⇒ To calculate density the molecules must be distributed uniformly.

$$\rho_v = \frac{\Delta Q}{\Delta V}$$ → only one charge in volume??

↓ volume charge density

$$\rho_v = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dv} = \rho_v$$

ΔV ↓ ⇒ very small volume in which always one charge is present

$$= \rho_v = \frac{dQ}{dv}$$

$$\boxed{dQ = \rho_v \, dv}$$

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* we use the concept of charge density to calculate total charges in a specific area.

Integrating both sides:

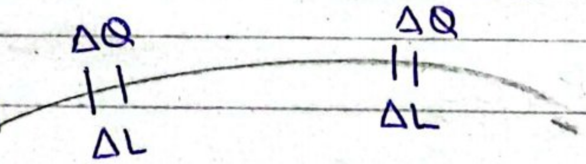
$$\int dQ = \int \rho_v \, dv$$

$$\boxed{Q = \int_{vol} \rho_v \, dv}$$ → integration with respect to three variables

↳ To calculate total charges enclosed in a volume. $= \boxed{C/m^3}$

=> LINE CHARGE DENSITY:-

one dimensional



$$\rho_L = \frac{\Delta Q}{\Delta L}$$

$$\rho_L = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L} = \frac{dQ}{dL}$$

$$\rho_L = \frac{dQ}{dL} \Rightarrow dQ = \rho_L \, dL$$

Integrate both sides

$$\boxed{Q = \int \rho_L \, dL}$$ → integration w.r.t one variable.

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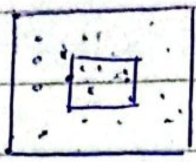
Two dimensional

=> SURFACE CHARGE DENSITY:-



$$\rho_s = \frac{\Delta Q}{\Delta S}$$

$$\rho_s = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

$$dQ = \rho_s \, dS$$

=> <u>Integrating both sides:</u>

$$\int dQ = \int \rho_s \, dS$$

$\boxed{Q = \int \rho_s \, ds}$ => integration w.r.t two variable.

=> C/m^2

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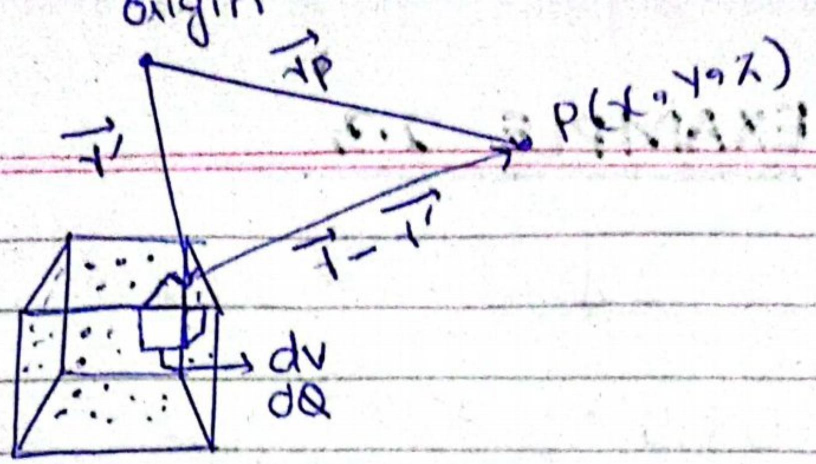

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r} - \vec{r}'|^2} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\vec{dE} = \frac{1}{4\pi\epsilon_0} \frac{dQ}{|\vec{r} - \vec{r}'|^2} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\vec{dE} = \frac{1}{4\pi\epsilon_0} \frac{\rho_v \, dv}{|\vec{r} - \vec{r}'|^2} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \int_{vol} \frac{\rho_v \, dv}{|\vec{r} - \vec{r}'|^2} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}}$$

⇒ To find total charges enclosed in small differential volumes enclosed in whole region.

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EXAMPLE : 2.3

$$\rho_v = -5\times10^{-6} e^{-10^5 \rho z} \ C/m^3$$

$$Q = \int_{vol} \rho_v \, dv$$

$$= \int_0^{0.01} \int_0^{2\pi} \int_{0.02}^{0.04} \rho_v \cdot \rho \, dz \, d\phi \, d\rho$$

$$= \int_0^{0.01} \int_0^{2\pi} \int_{0.02}^{0.04} -5\times10^{-6} e^{-10^5 \rho z} \cdot \rho \, dz \, d\phi \, d\rho$$

$$= \int_0^{0.01} \int_0^{2\pi} \frac{-5\times10^{-6} e^{-10^5 \rho z} \cdot \cancel{\rho}}{-10^5 \cancel{\rho}} \Bigg|_{0.02}^{0.04} d\phi \, d\rho$$

$$= 5\times10^{-11} \int_0^{0.01} \int_0^{2\pi} e^{-10^5 \rho (0.04)} - e^{-10^5 \rho (0.02)} \, d\phi \, d\rho$$

$$= 5\times10^{-11} \int_0^{0.01} \int_0^{2\pi} e^{-4000\rho} - e^{-2000\rho} \, d\phi \, d\rho$$

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$$= 5\times10^{-11}\int_0^{0.01}\left(e^{-4000P} - e^{-2000P}\right)\phi\Big|_0^{2\pi} dP$$

$$= 5\times10^{-11}\left(e^{-4000P} - e^{-2000P}\right)\int_0^{0.01}(2\pi - 0)\,dP$$

$$= 3.1415\times10^{-10}\int_0^{0.01}\left(e^{-4000P} - e^{-2000P}\right)dP$$

$$= 3.1415\times10^{-10}\left[\frac{e^{-4000P}}{-4000} - \frac{e^{-2000P}}{-2000}\right]\Bigg|_0^{0.01}$$

$$= 3.1415\times10^{-10}\left[\frac{-e^{-4000P}}{4000} + \frac{e^{-2000P}}{2000}\right]\Bigg|_0^{0.01}$$

$$= 3.1415\times10^{-10}\left[\frac{-e^{-4000\times0.01}}{4000} + \frac{e^{-2000\times0.01}}{2000}\right] -$$

$$\left[\frac{-1}{4000} + \frac{1}{2000}\right]$$

$= 3.1415 \times 10^{10} \left[1.0305 \times 10^{-12} - 2.5 \times 10^{-4} \right]$

$\boxed{= 0.0785 \ PC}$

D 2.4

(b)

$0 \leq \rho \leq 0.1 \ ; \ 0 \leq \phi \leq \pi \ ;$

$2 \leq z \leq 4 \ ; \ \rho_v = \rho^2 z^2 \sin 0.6 \phi$

$$Q = \int_{VOL} \rho_v \, dv$$

$$= \int_2^4 \int_0^{\pi} \int_0^{0.1} \rho^2 z^2 \sin 0.6\phi \, \rho \, d\rho \, d\phi \, dz$$

$$= z^2 \sin 0.6\phi \int_2^4 \int_0^{\pi} \frac{\rho^4}{4} \Big|_0^{0.1} d\phi \, dz$$

$$= \frac{z^2 \sin 0.6\phi}{4} \int_2^4 \int_0^{\pi} (0.1)^4 - (0)^4 \, d\phi \, dz$$

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$$= \frac{1 \times 10^{-4} z^2}{4} \int_2^4 \int_0^{\pi} \sin 0.6\phi \, d\phi \, dz$$

$$= -\frac{1 \times 10^{-4} z^2}{4} \int_2^4 \frac{\cos 0.6\phi}{0.6} \Big|_0^{\pi} dz$$

$$\Rightarrow -4.166 \times 10^{-5} z^2 \int_2^4 \cos 0.6 \times \pi - \cos 0 \; dz$$

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$$= -4.166 \times 10^{-5} \times -1.309 \int_2^4 z^2 \, dz$$

$$= \frac{5.453 \times 10^{-5}}{3} \; z^3 \Big|_2^4$$

$$= 1.817 \times 10^{-5} \left[4^3 - 2^3 \right]$$

$$= \boxed{1.018 \times 10^{-3} \; C}$$

D 2.4

(a)

$0.1 \leq x, y, z \leq 0.2$;

$\rho_v = \dfrac{1}{x^3 y^3 z^3}$

$Q = ??$

$$Q = \int_{vol} \rho_v \, dv$$

$$= \int_{0.1}^{0.2} \int_{0.1}^{0.2} \int_{0.1}^{0.2} \frac{1}{x^3 y^3 z^3} \, dx\,dy\,dz$$

$$= \int_{0.1}^{0.2} \int_{0.1}^{0.2} \int_{0.1}^{0.2} x^{-3} y^{-3} z^{-3} \, dx\,dy\,dz$$

$$= \int_{0.1}^{0.2} \int_{0.1}^{0.2} \frac{x^{-2}}{-2} y^{-3} z^{-3} \Big|_{0.1}^{0.2} \, dy\,dz$$

$$= \frac{-y^{-3} z^{-3}}{2} \int_{0.1}^{0.2} \int_{0.1}^{0.2} \left[(0.2)^{-2} - (0.1)^{-2}\right] dy\,dz$$

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$$= 37.5 \int_{0.1}^{0.2} \int_{0.1}^{0.2} y^{-3} z^{-3} \, dy \, dz$$

$$= \frac{37.5}{z^3} \int_{0.1}^{0.2} \frac{y^{-2}}{-2} \Big|_{0.1}^{0.2} dz$$

$$= \frac{-18.75}{z^3} \int_{0.1}^{0.2} [0.2]^{-2} - (0.1)^{-2} \, dz$$

$$= 1406.25 \int_{0.1}^{0.2} z^{-3} \, dz$$

$$= 1406.25 \frac{z^{-2}}{-2} \Big|_{0.1}^{0.2}$$

$$= -703.125 \left[(0.2)^{-2} - (0.1)^{-2} \right]$$

$$= 52734.375 \longrightarrow (1)$$

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$$-2 \Big|$$

$$\Rightarrow \int_{-0.2}^{-0.1} \int_{-0.2}^{-0.1} \int_{-0.2}^{-0.1} \frac{1}{x^3 y^3 z^3} \, dx\, dy\, dz$$

$$\Rightarrow \int_{-0.2}^{-0.1} \int_{-0.2}^{-0.1} \frac{x^{-2}}{-2} \Bigg|_{-0.2}^{-0.1} \frac{1}{y^3 z^3} \, dy\, dz$$

$$= -\frac{1}{2y^3z^3} \int_{-0.2}^{-0.1} \int_{-0.2}^{-0.1} (-0.1)^{-2} - (-0.2)^{-2} \, dy\, d$$

$$= \frac{-75}{2} \int_{-0.2}^{-0.1} \int_{-0.2}^{-0.1} \frac{1}{y^3 z^3} \, dy\, dz$$

$$= \frac{-37.5}{z^3} \int_{-0.2}^{-0.1} \frac{y^{-2}}{-2} \Bigg|_{-0.2}^{-0.1} dz$$

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$$= 18.75 \times 75 \int_{-0.2}^{-0.1} \cancel{y^{-0}}\, z^{-3}\, dz$$

$$= 1406.25 \left. \frac{z^{-2}}{-2} \right|_{-0.2}^{-0.1}$$

$$= -703.125 \times 75$$

$$= -52734.375 \longrightarrow (2)$$

= Adding eq (1) & (2):

$$= 52734.375 - 52734.375$$

$$= 0\,C$$

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