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CHAPTER # 2 : Coulomb's law & Electric Field Intensity

Q2.2). Given:

$Q_1 = 1nC$ at $M(0,0,0)$

$Q_2 = -2nC$ at $N(1,1,1)$

Required:

a) $F_{12} = ?$

b) $F_{21} = ?$

Solution:

a) $\because F_{12} = \dfrac{kQ_1Q_2}{|r_{12}|^2}\,\hat{a}_{12}$

$F_{12} = \dfrac{kQ_1Q_2}{|r_{12}|^3} \cdot \vec{r}_{12}$

$r_{12} = r_2 - r_1$

$= ax + ay + az - (+0ax + 0ay + 0az)$

$= ax + ay + az$

$|r_{12}| = \sqrt{(1)^2+(1)^2+(1)^2} = \sqrt{3}$

$F_{12} = \dfrac{1}{4\pi\epsilon_0} \dfrac{(1\times10^{-9})(2\times10^{-9})}{(\sqrt{3})^3} \cdot ax+ay+az$

$= \dfrac{1}{4\pi(8.85\times10^{-12})} \dfrac{(1\times10^{-9})(2\times10^{-9})}{(\sqrt{3})^3} \cdot ax+ay+az$

$= (3.46\,ax + 3.46\,ay + 3.46\,az)\ nC$

$\because F_{21} = \dfrac{kQ_1Q_2}{|r_{21}|^2}\,\hat{a}_{21}$

$= \dfrac{1}{4\pi\epsilon_0} \dfrac{Q_1Q_2}{|\vec{r}_{21}|^3}\,\vec{r}_{21}$

$r_{21} = r_1 - r_2$

$= 0ax + 0ay + 0az - (ax + ay + az)$

$r_{21} = -ax - ay - az$

$r_{21} = \sqrt{(-1)^2+(-1)^2+(-1)^2} = \sqrt{3}$

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$$F_{21} = \frac{1}{4\pi(8.85\times10^{-12})}\frac{(1\times10^{-9})(2\times10^{-9})}{(\sqrt{3})^3}\left[-a_x - a_y - a_z\right]$$

$$= (-3.46 a_x - 3.46 a_y - 3.46 a_z)\ nC$$

$$= -(3.46 a_x + 3.46 a_y + 3.46 a_z)\ nC$$

Hence

$$F_{21} = -F_{12}$$

Q23) Given:

$Q_1 = 50nC$ at $A(1,0,0)$

$Q_2 = 50nC$ at $B(-1,0,0)$

$Q_3 = 50nC$ at $C(0,1,0)$

$Q_4 = 50nC$ at $D(0,-1,0)$

Required:

$F_A = ?$

Solution:

$$\because F_A = F_{AB} + F_{AC} + F_{AD} \quad \text{—(i)}$$

$$F_{AB} = \frac{kQ_1Q_2}{|\vec{r}_{BA}|^2}\hat{a}_{BA}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{|r_{BA}|^3}\cdot\vec{r}_{BA}$$

$$r_{BA} = r_A - r_B$$

$$= a_x + 0a_y + 0a_z - (-a_x + 0a_y + 0a_z)$$

$$= a_x + a_x$$

$$= 2a_x$$

$$|r_{BA}| = \sqrt{(2)^2} = \sqrt{4} = 2$$

$$F_{AB} = \frac{1}{4\pi(8.85\times10^{-12})}\times\frac{(50\times10^{-9})(50\times10^{-9})}{(\sqrt{2})^3}\cdot 2a_x$$

$$F_{AB} = \frac{7.95\times10^{-5}\ \cancel{a_x}}{\cancel{6.26\times10^{-16}}} \quad 5.62\times10^{-6}\ a_x$$

Date:

$$F_{AC} = \frac{kQ_1Q_2}{|\vec{R_{CA}}|^2} \cdot \hat{a_{CA}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{|\vec{R_{CA}}|^3} \cdot \{\vec{R_{CA}}\}$$

$$\vec{r_{CA}} = \vec{r_A} - \vec{r_C}$$
$$= ax - (ay)$$
$$= ax - ay$$
$$r_{\vec{CA}} = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$$

$$F_{AC} = \frac{1}{4\pi(8.85\times10^{-12})} \frac{(50\times10^{-9})(50\times10^{-9})}{(\sqrt{2})^3} \cdot ax - ay$$
$$= 7.94\times10^{-6} \, (ax - ay)$$
$$= (7.94ax - 7.94ay) \ \mu N$$

$$F_{AD} = \frac{kQ_1Q_2}{|r_{DA}|^2} \hat{a_{DA}}$$
$$= \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{|\vec{r_{DA}}|^3} \cdot \vec{r_{DA}}$$

$$\vec{r_{DA}} = \vec{r_A} - \vec{r_D}$$
$$= ax - (-ay)$$
$$= ax + ay$$
$$|\vec{r_{DA}}| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$$

$$F_{AC} = \frac{1}{4\pi(8.85\times10^{-12})} \frac{(50\times10^{-9})(50\times10^{-9})}{(\sqrt{2})^3} \cdot ax + ay$$
$$= (7.94\times10^{-6}) \, (ax + ay)$$
2ax
$$= (7.94ax + 7.94ay) \ \mu N$$

Date:

Now eq (i) becomes:

$$F_A = \cancel{7.95\times10^{-5}\,a_x}\ 5.62\times10^{-6} + 7.94\times10^{-6}\,a_x - \cancel{7.94\times10^{-6}\,a_y}$$
$$+ 7.94\times10^{-6}\,a_x + \cancel{7.94\times10^{-6}\,a_y}$$

$$F_A = 21.4 a_x\ \mu N$$

Q2.5) Given:

$Q_1 = 25nC$ at $P_1(4,-2,7)$
$Q_2 = 60nC$ at $P_2(-3,4,-2)$

Required:

a) find $\vec{E}$ at $P_3(1,2,3)$
b) At what point on y-axis is $E_x = 0$?

Solution:

a) $\because \vec{E} = \dfrac{kQ}{|r|^3}\cdot\vec{r}$

$$\vec{E_3} = \vec{E_{31}} + \vec{E_{32}} \quad \text{—— (i)}$$

$$\vec{E_{31}} = \frac{1}{4\pi\epsilon_0}\frac{Q_1\cdot\vec{r_{13}}}{|\vec{r_{13}}|^3}$$

$$\vec{r_{13}} = \vec{r_3} - \vec{r}$$
$$= a_x + 2a_y + 3a_z - (4a_x - 2a_y + 7a_z)$$
$$= a_x + 2a_y + 3a_z - 4a_x + 2a_y - 7a_z$$
$$= -3a_x + 4a_y - 4a_z$$

$$|\vec{r_{13}}| = \sqrt{(-3)^2+(4)^2+(-4)^2}$$
$$= \sqrt{41}$$

$$\vec{E_{31}} = \frac{25\times10^{-9}\cdot(-3a_x + 4a_y - 4a_z)}{4\pi(8.85\times10^{-12})(\sqrt{41})^3}$$
$$= 0.856(-3a_x + 4a_y - 4a_z)$$
$$= -2.57a_x + 3.4a_y - 3.4a_z$$

Date:

$$\vec{E}_{32} = \frac{Q_2 \cdot \vec{r}_{23}}{4\pi\epsilon_0 |\vec{r}_{23}|^2}$$

$$\vec{r}_{23} = \vec{r}_3 - \vec{r}_2$$
$$= a_x + 2a_y + 3a_z - (-3a_x + 4a_y - 2a_z)$$
$$= a_x + 2a_y + 3a_z + 3a_x - 4a_y + 2a_z$$
$$= 4a_x - 2a_y + 5a_z$$

$$|\vec{r}_{23}| = \sqrt{(4)^2 + (-2)^2 + (5)^2}$$
$$= 3\sqrt{5}$$

$$\vec{E}_{32} = \frac{60 \times 10^{-9} \cdot (4a_x - 2a_y + 5a_z)}{4\pi(8.85 \times 10^{-12})(3\sqrt{5})^3}$$
$$= 1.787\,(4a_x - 2a_y + 5a_z)$$
$$= 7.149\,a_x - 3.57\,a_y + 8.9\,a_z$$

Now eq (i) becomes,

$$E_3 = -2.57a_x + 3.4a_y - 3.4a_z + 7.149a_x - 3.57a_y + 8.9a_z$$
$$= 4.58a_x - 0.17a_y + 5.5\,a_z$$

b) $\vec{E}_y = \vec{E}_{y1} + \vec{E}_{y2}$ —(ii)

$$\vec{E}_{y1} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{|\vec{r}_{1y}|^3} \cdot \vec{r}_{1y}$$

$$\vec{r}_{1y} = \vec{r}_y - \vec{r}_1$$
$$= 0a_x + y\,a_y + 0z - (4a_x - 2a_y + 7a_z)$$
$$= y\,a_y - 4a_x + 2a_y - 7a_z$$
$$= -4a_x + (y+2)\,a_y - 7a_z$$

$$|\vec{r}_{1y}| = \sqrt{(-4)^2 + (y+2)^2 + (-7)^2}$$
$$= \sqrt{16 + y^2 + 4y + 4 + 49}$$

$$|\vec{r}_{1y}| = \sqrt{y^2 + 4y + 69}$$

Date:

$$E_{Y_1} = \frac{25\times10^{-9}}{4\pi\epsilon_0} \cdot \frac{(-4a_x + (y+2)a_y - 7a_z)}{[\sqrt{y^2+4y+69})^{3/2}]}$$

$$E_{Y_1} = \frac{224.8}{(y^2+4y+69)^{3/2}} (-4a_x + (y+2)a_y - 7a_z)$$

$$E_{Y_2} = \frac{Q_2 \, \vec{r_{2Y}}}{4\pi\epsilon_0 |\vec{r_{2Y}}|^3}$$

$$\vec{r_{2Y}} = r_Y - r_2$$
$$= y a_y - (-3a_x + 4a_y - 2a_z)$$
$$= y a_y + 3a_x - 4a_y + 2a_z$$
$$= 3a_x + (y-4)a_y + 2a_z$$

$$|\vec{r_{2Y}}| = \sqrt{(3)^2 + (y-4)^2 + (2)^2}$$
$$= \sqrt{9 + y^2 - 8y + 16 + 4}$$
$$= \sqrt{y^2 - 8y + 29}$$

$$E_{Y_2} = \frac{60\times10^{-9} \cdot [3a_x + (y-4)a_y + 2a_z]}{4\pi\epsilon_0 (y^2 - 8y + 29)^{3/2}}$$

Now eq (i) becomes

$$\vec{E_Y} = \frac{224.8}{(y^2+4y+69)^{3/2}} (-4a_x + (y+2)a_y - 7a_z) + \frac{539.5 \, [3a_x + (y-4)a_y + 2a_z]}{(y^2-8y+29)^{3/2}}$$

Now taking only x-component of E_Y

$$\vec{E_x} = \frac{224.8}{(y^2+4y+69)^{3/2}}(-4) + \frac{539.5}{(y^2-8y+29)^{3/2}}(3)$$

$$0 = \frac{-899.2}{(y^2+4y+69)^{3/2}} + \frac{1618.5}{(y^2-8y+29)^{3/2}}$$

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$$\frac{899.2}{(y^2+4y+69)^{3/2}} = \frac{1618.5}{(y^2-8y+29)^{3/2}}$$

$$\left[899.2\,(y^2-8y+29)^{3/2}\right]^{2/3} = \left[(y^2+4y+69)^{3/2} \cdot 1618.5\right]^{2/3}$$

$$[899.2]^{2/3}\,(y^2-8y+29)^{3/2\times 2/3} = (y^2+4y+69)^{3/2\cdot 2/3}\,(1618.5)^{2/3}$$

$$93.16\,(y^2-8y+29) = (y^2+4y+69)\,(137.8)$$

$$93.16y^2 - 745.28 + 2701.6 = 137.8y^2 + 551.2y + 9508$$

$$44.64y^2 + 1296.48y + 6806.4 = 0$$

$$y = -6.87 \quad , \quad y = -22.16$$

Q2.7) Given:

$$Q = 2\mu C \quad \text{at} \quad A(4,3,5)$$

Required:

a) E_ρ at $P(8,12,2)$

b) E_ϕ at $P(8,12,2)$

c) E_z at $P(8,12,2)$

Solution:

→ First we will find electric field intensity in rectangular coordinates:

$$\vec{E} = \frac{kQ}{|r|^3}\vec{r}$$

$$\vec{E_{PA}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{|\vec{r_{AP}}|}\vec{r_{AP}}$$

$$\vec{r_{AP}} = \vec{r_P} - \vec{r_A}$$

$$= 8a_x + 12a_y + 2a_z - (4a_x + 3a_y + 5a_z)$$

Date:

$$\vec{r}_{AP} = 8a_x + 12a_y + 2a_z - 4a_x - 3a_y - 5a_z$$
$$= 4a_x + 9a_y - 3a_z$$
$$|\vec{r}_{AP}| = \sqrt{(4)^2 + (9)^2 + (-3)^2}$$
$$= \sqrt{106}$$

$$E_{PA} = \frac{2\times10^{-6}}{4\pi(8.85\times10^{-12})(\sqrt{106})^3} \cdot (4a_x + 9a_y - 3a_z)$$
$$= 16.48\,(4a_x + 9a_y - 3a_z)$$
$$= 65.9a_x + 148.3a_y - 49.4a_z$$

$$\vec{E} = \vec{E}_\rho \hat{a}_\rho + \vec{E}_\phi \hat{a}_\phi + \vec{E}_z \hat{a}_z$$

$$\vec{E}_\rho = \vec{E} \cdot a_\rho$$
$$= (65.9a_x + 148.3a_y - 49.4a_z)\,a_\rho$$
$$= 65.9\,a_x a_\rho + 148.3\,a_y a_\rho - 49.4\,a_z a_\rho$$
$$= 65.9\cos\phi + 148.3\sin\phi$$

	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$\sin\phi$	$\cos\phi$	0
a_z	0	0	1

$$\vec{E}_\phi = \vec{E} \cdot \hat{a}_\rho$$
$$= (65.9a_x + 148.3a_y - 49.4a_z)\,a_\phi$$
$$= 65.9\,a_x a_\phi + 148.3\,a_y a_\phi - 49.4\,a_z a_\phi$$
$$= -65.9\sin\phi + 148.3\cos\phi$$

$$\vec{E}_z = \vec{E} \cdot a_z$$
$$= (65.9a_x + 148.3a_y - 49.4a_z)\,a_z$$
$$= 65.9\,a_x a_z + 148.3\,a_y a_z - 49.4\,a_z a_z$$
$$\vec{E}_z = -49.4$$

Converting rectangular coordinate to cylindrical coordinate

$$\rho = \sqrt{x^2 + y^2}$$
$$\rho = \sqrt{(8)^2 + (12)^2}$$
$$\rho = 4\sqrt{13}$$

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$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{12}{8}\right)$

$\phi = 56.3^\circ$

$z = 2$

$E_\rho = 65.9 \cos(56.3) + 148.3 \sin(56.3)$

$\boxed{E_\rho = 159.9 \text{ V/m}}$

$E_\phi = -65.9 \sin(56.3) + 148.3 \cos(56.3)$

$\boxed{E_\phi = 27.4 \text{ V/m}}$

$\boxed{E_z = -49.4 \text{ V/m}}$

Q 2.17) Given:

$\rho_L = 16 \text{ nC/m}$

$y = -2$

$z = 5$

Required:

a) Find $\vec{E}$ at $P(1,2,3)$

b) Find $\vec{E}$ at that point in $z=0$ plane where direction of $\vec{E}$ is given by $\frac{1}{3}a_y - \frac{2}{3}a_z$.

Solution:

a) $E = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho$

$E = \frac{\rho_L}{2\pi\epsilon_0} \frac{\vec{R}_\rho}{|R_\rho|^2}$

$\vec{R}_\rho = (1a_x + 2a_y + 3a_z) - (1a_x - 2a_y + 5a_z)$

$= a_x + 2a_y + 3a_z - a_x + 2a_y - 5a_z$

$= 4a_y - 2a_z$

$|\vec{R}_\rho| = \sqrt{(4)^2 + (-2)^2} = 2\sqrt{5}$

Date:

$$\vec{E} = \frac{16\times10^{-9}}{2\pi(8.85\times10^{-12})} \frac{(4a_y - 2a_z)}{(2\sqrt{5})^2}$$

$$= 14.39\,(4a_y - 2a_z)$$

$$= 57.54\,a_y - 28.77\,a_z$$

b) $$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\,\rho}\,\hat{a}_\rho$$

$$= \frac{\rho_L}{2\pi\epsilon_0} \frac{\vec{RQ}}{|\vec{RQ}|^2}$$

$\cancel{\vec{RQ}}$ $$\vec{R_Q} = x a_x + y a_y + 0 a_z - (x a_x - 2a_y + 5a_z)$$

$$= x a_x + y a_y - x a_x + 2a_y - 5a_z$$

$$= (y+2)\,a_y - 5\,a_z$$

$$|\vec{R_Q}| = \sqrt{(y+2)^2 + (-5)^2}$$

$$= \sqrt{(y+2)^2 + 25}$$

$$\vec{E} = \frac{16\times10^{-9}}{2\pi(8.85\times10^{-12})} \frac{[(y+2)a_y - 5a_z]}{[\sqrt{(y+2)^2+25}]^{\cancel{2}}}$$

$$= \frac{16\times10^{-9}}{2\pi(8.85\times10^{-12})} \frac{[(y+2)a_y - 5a_z]}{((y+2)^2+25)} \quad \text{— (i)}$$

~~$\hat{a}_Q = 0, \frac{1}{3} a_y, \frac{2}{}$~~

$$\hat{a}_Q = \left(0, \frac{1}{3}, \frac{-2}{3}\right)$$

$$= \left(0, \frac{1}{3}, \frac{-2}{3}\right)$$

Hence,

$$|E_z| = -2\,|E_y|$$

Date:

so, $-5 = -2(y+2)$

$5 = 2y+4$

$2y = 1$

$y = 0.5$

Putting value of y in eq (i)

$$\vec{E} = \frac{16\times10^{-9}\left[(0.5+2)a_y - 5a_z\right]}{2\pi(8.85\times10^{-12})\left((0.5+2)^2+25\right)}$$

$$= 9.2\,(2.5a_y - 5a_z)$$

$$\vec{E} = 23\,a_y - 46\,a_z$$

Q 2.19) Given:

$\rho_L = 2\mu C/m$ on z-axis

Required:

Find $\vec{E}$ in rectangular coordinates at $P(1,2,3)$ if charge exist from

a) $-\infty < z < \infty$

b) $-4 \le z \le 4$

Solution:

a) In field of a line charge we assume that straight line charge extending along the z-axis from $-\infty$ to ∞ and its formula is:

$$E = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho$$

$$E = \frac{\rho_L}{2\pi\epsilon_0}\frac{\vec{R_p}}{|\vec{R_p}|^2}$$

$$\vec{R_p} = a_x + 2a_y + 3a_z - (0a_x + 0a_y + 3a_z)$$

$$= a_x + 2a_y + 3a_z - 3a_z$$

Date:

$\vec{R_P} = a_x + 2a_y$

$|\vec{R_P}| = \sqrt{(1)^2 + (2)^2}$

$|\vec{R_P}| = \sqrt{5}$

$$\vec{E} = \frac{2\times10^{-6}}{2\pi(8.85\times10^{-12})}\frac{(a_x + 2a_y)}{(\sqrt{5})^2}$$

$$= (7193.4)\ a_x + 2a_y$$

$$= 7193.4a_x + 14386.88\ a_y$$

b) $\vec{E} = \frac{kQ\vec{r}}{\cancel{4\pi\epsilon_0}|r|^3}$

$\vec{E} = \frac{Q\vec{r}_Q}{4\pi\epsilon_0|\vec{r}|^3}$ $\qquad \rho_L = \frac{Q}{\text{Length}}$

$\vec{E} = \frac{\rho_L dz\ \vec{r}_Q}{4\pi\epsilon_0|\vec{r}_Q|^3}$ $\qquad Q = dz\ \rho_L$

$\vec{r}_Q = a_x + 2a_y + 3a_z - (0a_x + 0a_y + za_z)$

$= a_x + 2a_y + (3-z)a_z$

$|\vec{r}_Q| = \sqrt{(1)^2 + (2)^2 + (3-z)^2}$

$= \sqrt{5 + (3-z)^2}$

$$\vec{E} = \frac{2\times10^{-6}}{4\pi(8.85\times10^{-12})}\int_{-4}^{4}\frac{a_x + 2a_y + (3-z)a_z\ dz}{[5 + (3-z)^2]^{3/2}}$$

After apply integral

$$= 3597\left[\frac{(a_x + 2a_y)(z-3) + 5a_z}{z^2 - 6z + 14}\right]_{-4}^{4}$$

$$= 4.9\ a_x + 9.8\ a_y + 4.9\ a_z\ \ kV/m$$

Date:

Q2.1) Given:

$Q_1 = 5nC$ at $y = 5cm$
$Q_2 = -10nC$ at $y = -5cm$
$Q_3 = 15nC$ at $x = -5cm$
$Q_4 = 20nC$

Required:

Find coordinates of 2 Q_4 charge

Solution:

$$\vec{E_i} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1}{|\vec{r_1}|^2}\hat{a}_{r_1} + \frac{Q_2}{|\vec{r_2}|^2}\hat{a}_{r_2} + \frac{Q_3}{|\vec{r_3}|^2}\hat{a}_{r_3}\right]$$

$\vec{r_1} = 0a_x + 0a_y + 0a_z - (0a_x + 5a_y + 0a_z)$
$\vec{r_1} = -5a_y$
$|\vec{r_1}| = \sqrt{(-5)^2} = 5$

$\vec{r_2} = 0a_x + 0a_y + 0a_z - (0a_x - 5a_y + 0a_z)$
$= 5a_y$
$|\vec{r_2}| = 5$

$\vec{r_3} = 0a_x + 0a_y + 0a_z - (-5a_x + 0a_y + 0a_z)$
$\vec{r_3} = 5a_x$
$|\vec{r_3}| = 5$

$$\cancel{\vec{E_i} = \frac{1}{4\pi\epsilon_0}\left[\frac{5\times10^{-9}}{(5)^3}\cdot(-5a_y) + \frac{(-10\times10^{-9})}{(5)^3}\cdot 5a_y + \frac{15\times10^{-9}}{5}\right.}$$

$$\vec{E_i} = \frac{1}{4\pi\epsilon_0}\left[\frac{-5}{(5)^2}a_y - \frac{10}{(5)^2}a_y + \frac{15}{(5)^2}a_z\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{3}{5}\right)[a_x - a_y] \ nC/m$$

Date:

$$\vec{E_F} = \frac{-20}{4\pi \epsilon_0 |r_f|^2} \frac{1}{\sqrt{2}} [a_x - a_y]$$

$$\vec{E_1} = \vec{E_F}$$

$$\frac{-1}{4\pi\epsilon_0}\left(\frac{3}{5}\right)[a_x - a_y] = \frac{-20}{4\pi\epsilon_0 p^2}\left(\frac{1}{\sqrt{2}}\right)[a_x - a_y]$$

$$p = \sqrt{\frac{100}{3\sqrt{2}}} = 4.85$$

$$x \text{ and } y\text{- coordinate} = \frac{4.85}{\sqrt{2}} = 3.43$$

Hence coordinates of $Q_4 = 20\,nC$ = $(3.43, -3.43)$

Date:

Q 2.21) Given:

$$\rho_{L+} = 75 \, nC/m$$
$$\rho_{L-} = 75 \, nC/m$$
$$x = 0$$
$$y = \pm 0.4m$$

Required:

What force per unit length does each line charge exert on other?

Solution:

$$\because \vec{E} = \frac{\rho_L}{2\pi\epsilon_0} \frac{\vec{r_p}}{|\vec{r_p}|^2}$$

For +ve charge:

$$\vec{r_p} = 0ax + 0.4ay + 0az - (0ax - 0.4ay + 0az)$$
$$= 0.4ay + 0.4ay$$
$$= 0.8ay$$
$$|\vec{r_p}| = \sqrt{(0.8)^2}$$
$$= 0.8$$

$$\cancel{\vec{E} = \frac{75 \times 10^{-9}}{2\pi(8.85 \times 10^{-12})} \cdot \frac{0.8ay}{(0.8)^2}}$$

$$d\vec{F} = dQ \cdot \vec{E}$$

$$\vec{F} = \int Q \cdot E \qquad \because \rho_L = \frac{Q}{dz}$$
$$Q = \rho_L \, dz$$

$$\vec{F} = \int_0^1 \rho_L \, dz \, \frac{\rho_L}{2\pi\epsilon_0} \frac{\vec{r_p}}{|\vec{r_p}|^2}$$

$$\frac{\vec{F}}{dz} = \int_0^1 \frac{\rho_L^2}{2\pi\epsilon_0} \frac{\vec{r_p}}{|r_p|^2}$$

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$$\frac{\vec{F}}{dz} = \int_0^1 \frac{(75\times10^{-9})^2}{2\pi(8.85\times10^{-12})} \cdot \frac{0.8\,a_y}{(0.8)^2}$$

$$\frac{\vec{F}}{d\vec{z}} = 1.26\times10^{-4}\,a_y \text{ N/m}$$

For negative charge:

$$\vec{r_p} = 0a_x - 0.4a_y + 0a_z - (0a_x + 0.4a_y + 0a_z)$$
$$= -0.4a_y - 0.4a_y$$
$$= -0.8\,a_y$$
$$|\vec{r_p}| = 0.8$$

$$\frac{\vec{F}}{dz} = \int \frac{\rho_L^2}{2\pi\epsilon_0} \frac{\vec{r_p}}{\vec{r_p}}$$
$$= \int \frac{(75\times10^{-9})^2}{2\pi(8.85\times10^{-12})} \frac{-0.8\,a_y}{(0.8)^2}$$
$$\frac{\vec{F}}{dz} = -1.26\times10^{-4}\,a_y \text{ N/m}$$

Q 2.22) Given:

$\rho_a = \rho_s = 100\ nC/m^2$

$z = \pm 2.0$ cm

Required:

What force per unit area does each sheet exert on the other?

Solution:

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} a_z$$

$$d\vec{F} = dQ \cdot \vec{E} \qquad \because \rho_a = \frac{Q}{da}$$

Date:

$\vec{F} = \rho a \, da . \vec{E}$ $\quad ; dQ = \rho a \, da$

$\frac{\vec{F}}{da} = \rho a \vec{E}$

$= \rho a . \frac{\rho a}{2\epsilon_0} a_z$

$= \frac{\rho_s^2}{2\epsilon_0} a_z$

$= \frac{(100 \times 10^{-9})^2}{2(8.85 \times 10^{-12})}$

$\frac{\vec{F}}{da} = 5.6 \times 10^{-4} \; N/m^2$

So, 5.6×10^{-4} force per unit area will each sheet exert on the other.

Q2.25) Given:

$\oint Q = 12nC$ at $P(2,0,6)$

$\rho_L = 3nC/m$ at $x = -2, y = 3$

$\rho_s = 0.2 \; nC/m^2$ at $x = 2$

Required.

Find $\vec{E}$ at origin

Solution:

Total electric field intensity = Electric field intensity due to point charge + Electric field intensity due to line charge + Electric field intensity due to surface charge —— (i)

Date:

Electric field intensity due to point charge:

$$\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{\vec{r_p}}{|\vec{r_p}|^3}$$

$$\vec{r_p} = 0a_x + 0a_y + 0a_z - (2a_x + 0a_y + 6a_z)$$
$$= -2a_x - 6a_z$$
$$|\vec{r_p}| = \sqrt{(-2)^2 + (-6)^2}$$
$$= 2\sqrt{10}$$

$$\vec{E} = \frac{12\times10^{-9}\,(-2a_x - 6a_z)}{4\pi\,(8.85\times10^{-12})\,[2\sqrt{10}]^3}$$
$$= 0.426\,(-2a_x - 6a_z)$$
$$= -0.853\,a_x - 2.559\,a_z$$

Electric field intensity due to line charge

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\,\rho}\,\hat{a_\rho}$$

$$\rho = 0a_x + 0a_y + 0a_z - (-2a_x + 3a_y + 0)$$
$$= 2a_x - 3a_y$$
$$|\vec{\rho}| = \sqrt{(2)^2 + (-3)^2}$$
$$= \sqrt{13}$$

$$\vec{E} = \frac{3\times10^{-9}\,(2a_x - 3a_y)}{2\pi\,(8.85\times10^{-12})\,(\sqrt{13})^2}$$
$$= 4.15\,(2a_x - 3a_y)$$

$$\vec{E} = 8.3\,a_x - 12.45\,a_y$$

Date:

Electric field intensity due to surface charge:

$$\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a_x}$$

$$\vec{x} = 0a_x + 0a_y + 0a_z - (+2a_x + 0a_y + 0a_z)$$
$$\vec{x} = -2a_x$$
$$|\vec{x}| = \sqrt{(-2)^2} = 2$$
$$\hat{a_x} = \frac{-2a_x}{2} = -a_x$$

$$\vec{E} = \frac{0.2 \times 10^{-9}}{2(8.85 \times 10^{-12})} - a_x$$
$$\vec{E} = -11.3\, a_x$$

Total electric field intensity $= -0.853\, a_x - 2.559\, a_z + 8.3 a_x - 12.45\, a_y - 11.3 a_x$

$$\vec{E} = -3.583\, a_x - 12.45 a_y - 2.56 a_z$$

Q2.29) Given:

$$\vec{E} = 20\, e^{-5y} (\cos 5x\, a_x - \sin 5x\, a_y)$$

Required:

a) $|\vec{E}|$ at $P(\pi/6, 0.1, 2)$

b) a unit vector in the direction of $\vec{E}$ at P

c) the equation of the direction line passing throught P.

Solution:

a) $\vec{E} = 20 e^{-5(0.1)} [\cos 5(\pi/6)\, a_x - \sin 5(\pi/6)\, a_y]$

$$\vec{E} = -10.5\, a_x - 6.06\, a_y$$
$$|\vec{E}| = \sqrt{(-10.5)^2 + (-6.06)^2}$$
$$|\vec{E}| = 12.12$$

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b) $\hat{a}_E = \frac{\vec{E}}{|\vec{E}|}$

$$= \frac{-10.5a_x - 6.06a_y}{12.12}$$

$$= -0.866\, a - 0.5\, a_y$$

c) $\frac{E_y}{E_x} = \frac{dy}{dx} = \frac{-\sin 5x}{\cos 5x}$

$$\frac{dy}{dx} = -\tan 5x$$

$$\int dy = \int -\tan 5x\, dx$$

$$y = \frac{\ln \cos 5x}{5} + c$$

For c:

$$0.1 = \frac{\ln \cos\left(5 \times \frac{\pi}{6}\right)}{5} + c$$

$$y = \frac{-\ln \sin 5x}{5} + c$$

For c:

$$0.1 = \frac{-\ln \sin \frac{5\pi}{6}}{5} + c$$

Evaluating at P, we find c = 0.13

$$y = \frac{\ln \cos 5x}{5} + 0.13$$

Q 2.30) Given:

$$\frac{E_\rho}{E_\phi} = \frac{d\rho}{\rho\, d\phi}$$

Date:

$\vec{E} = \rho \cos 2\phi \, a_\rho - \rho \sin 2\phi \, a_\phi$

$P(2, 30°, 0)$

Required:

Find equation of line.

Solution:

$$\frac{E_\rho}{E_\phi} = \frac{d\rho}{\rho d\phi} = \frac{\rho \cos 2\phi}{-\rho \sin 2\phi}$$

$$\frac{d\rho}{\rho} = -\cot 2\phi \, d\phi$$

Taking integral on both sides

$$\int \frac{d\rho}{\rho} = -\int \cot 2\phi \, d\phi$$

$$\ln \rho = -\frac{\ln \sin 2\phi}{2} + c$$

$$2 \ln \rho = -\ln \sin 2\phi + c$$

$$\ln(\rho^2) = \ln\left(\frac{c}{\sin 2\phi}\right)$$

$$\rho^2 = \frac{c}{\sin 2\phi} \quad \text{—(i)}$$

At point $(2, 30°, 0)$

$$2^2 = \frac{c}{\sin(2 \times 30°)}$$

$$c = 2\sqrt{3}$$

So eq (i) becomes:

$$\rho^2 = \frac{2\sqrt{3}}{\sin 2\phi}$$

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Date:

Q 2.27) Given:

$$\vec{E} = (4x - 2y)a_x - (2x + 4y)a_y$$

Required:

a) Find eq of streamline that passes through point $P(2, 3, -4)$

b) a unit vector specifying direction of $\vec{E}$ at $Q(3, -2, 5)$

Solution:

a)
$$\frac{E_y}{E_x} = \frac{dy}{dx} = \frac{-(2x + 4y)}{4x - 2y}$$

$$2(x\,dy + y\,dx) = y\,dy - x\,dx$$

$$C_1 + 2xy = \frac{1}{2}y^2 - \frac{1}{2}x^2$$

$$y^2 - x^2 = 4xy + C_2$$

At point $P(2, 3, -4)$

$$9 - 4 = 24 + C_2$$

$$C_2 = -19$$

$$y^2 - x^2 = 4xy - 19$$

b)
$$\vec{E} = [4(3) - 2(-2)]a_x - [2(3) + 4(-2)]a_y$$

$$\vec{E} = 16a_x + 2a_y$$

$$|\vec{E}| = \sqrt{(16)^2 + (2)^2}$$

$$= 2\sqrt{65}$$

$$\hat{a}_E = \frac{16}{2\sqrt{65}}a_x + \frac{2}{2\sqrt{65}}a_y$$

$$= 0.99x + 0.124\,a_y$$