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ASSIGNMENT

CHAPTER 1 : VECTOR ANALYSIS

Q 1.1) Given:

$$M = -10a_x + 4a_y - 8a_z$$
$$N = 8a_x + 7a_y - 2a_z$$

Required:

a) a unit vector in direction of $-M + 2N$
b) the magnitude of $5a_x + N - 3M$
c) $|M||2N|(M+N)$

Solution:

a) $$\hat{a}_{-M+2N} = \frac{r_{-M+2N}}{|r_{-M+2N}|}$$

$$r_{-M+2N} = -(-10a_x + 4a_y - 8a_z) + 2(8a_x + 7a_y - 2a_z)$$
$$= 10a_x - 4a_y + 8a_z + 16a_x + 14a_y - 4a_z$$
$$= 26a_x + 10a_y + 4a_z$$

$$|r_{-M+2N}| = \sqrt{(26)^2 + (10)^2 + (4)^2}$$
$$= 6\sqrt{22} = 28.14$$

$$a_{-M+2N} = \frac{26a_x + 10a_y + 4a_z}{6\sqrt{22}} = 0.92a_x + 0.35a_y + 0.14a_z$$

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b) $R_x = 5a_x + 8a_x + 7a_y - 2a_z - 3(-10a_x + 4a_y - 8a_z)$

$= 5a_x + 8a_x + 7a_y - 2a_z + 30a_x - 12a_y + 24a_z$

$= 43a_x - 5a_y + 22a_z$

$|R_x| = \sqrt{(43)^2 + (-5)^2 + (22)^2}$

$|R_x| = 3\sqrt{262} = 48.6$

c) $|M||2N|(M+N) = \left(\sqrt{(-10)^2+(4)^2+(-8)^2}\right)\left(\sqrt{(16)^2+(14)^2+(-4)^2}\right)(-10a_x + 4a_y - 8a_z + 8a_x + 7a_y - 2a_z)$

$= (6\sqrt{5})(6\sqrt{13})(-2a_x + 11a_y - 10a_z)$

$= 36\sqrt{65}(-2a_x + 11a_y - 10a_z)$

$= -580.48a_x + 3193a_y - 2902a_z$

Q 1.2) Given:

$\vec{A} = a_x + 2a_y + 3a_z$

$\vec{B} = 2a_x + 3a_y - 2a_z$

Required:

a) unit vector in direction of $(A - B)$

b) unit vector in direction of line extending from origin to midpoint of line joining ends of A and B

Solution:

a) $\hat{a}_{BA} = \dfrac{\vec{r_{BA}}}{|\vec{r_{BA}}|}$

$r_{BA} = r_A - r_B$

$= a_x + 2a_y + 3a_z - (2a_x + 3a_y - 2a_z)$

$= a_x + 2a_y + 3a_z - 2a_x - 3a_y + 2a_z$

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$\vec{r_{BA}} = -a_x - a_y + 5a_z$

$|\vec{r_{BA}}| = \sqrt{(-1)^2 + (-1)^2 + (5)^2}$
$= 3\sqrt{3} = 5.2$

$\hat{a_{BA}} = \dfrac{-a_x - a_y + 5a_z}{3\sqrt{3}}$
$= -0.192a_x - 0.192\,a_y + 0.96\,a_z$

b) $\hat{a}_{\frac{A+B}{2}} = \dfrac{r_{\frac{A+B}{2}}}{|r_{\frac{A+B}{2}}|}$

$r_{\frac{A+B}{2}} = \dfrac{a_x + 2a_y + 3a_z + 2a_x + 3a_y - 2a_z}{2}$
$= \dfrac{3a_x + 5a_y + a_z}{2}$

$|r_{\frac{A+B}{2}}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$
$= 2.95$

$\hat{a}_{\frac{A+B}{2}} = \dfrac{\frac{3}{2}a_x + \frac{5}{2}a_y + \frac{1}{2}a_z}{2.95}$
$= 0.508\,a_x + 0.847\,a_y + 0.169\,a_z$

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Q1.3) Given:

$$\vec{A} = 6a_x - 2a_y - 4a_z$$

$$\hat{a_B} = \frac{2}{3}a_x - \frac{2}{3}a_y + \frac{1}{3}a_z$$

$$|\vec{B} - \vec{A}| = 10$$

Required:

$$\vec{B} = ?$$

Solution:

$$|\vec{B} - \vec{A}| = 10$$

$$\Rightarrow \left| \frac{2}{3}a_x|B| - \frac{2}{3}a_y|B| + \frac{1}{3}a_z|B| - (6a_x - 2a_y - 4a_z) \right| = 10 \quad \because \hat{a_B} = \frac{\vec{B}}{|B|}, \quad \vec{B} = \hat{a_B} \times |B|$$

$$\Rightarrow \left| \frac{2}{3}a_x|B| - \frac{2}{3}a_y|B| + \frac{1}{3}a_z|B| - 6a_x + 2a_y + 4a_z \right| = 10$$

$$\Rightarrow \left| \left(\frac{2}{3}|B| - 6\right)a_x + \left(\frac{-2}{3}|B| + 2\right)a_y + \left(\frac{1}{3}|B| + 4\right)a_z \right| = 10$$

$$\Rightarrow \sqrt{\left(\frac{2}{3}|B| - 6\right)^2 + \left(\frac{-2}{3}|B| + 2\right)^2 + \left(\frac{1}{3}|B| + 4\right)^2} = 10$$

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$$\Rightarrow \left(\frac{2}{3}|B| - 6\right)^2 + \left(\frac{-2}{3}|B| + 2\right)^2 + \left(\frac{1}{3}|B| + 4\right)^2 = 100$$

$$\Rightarrow \frac{4}{9}|B|^2 - 8|B| + 36 + \frac{4}{9}|B|^2 - \cancel{\frac{8}{3}|B|} + 4 + \frac{1}{9}|B|^2 + \cancel{\frac{8}{3}|B|} + 16 = 100$$

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$\rightarrow |B|^2 - 8|B| - 44 = 0$

After apply quadratic formula:

$|B| = 11.746$, $|B| = -3.74$

we will take +ve value

$|B| = 11.746$

$\vec{B} = \hat{a}_B \cdot |B|$

$= \left(\frac{2}{3} a_x - \frac{2}{3} a_y + \frac{1}{3} a_z\right) \cdot 11.746$

$\vec{B} = 7.83\, a_x - 7.83\, a_y + 3.92\, a_z$

$B(7.83, -7.83, 3.92)$

Q1.5) Given:

$\vec{G} = 24\, xy\, a_x + 12(x^2+2)\, a_y + 18z^2 a_z$

$P(1, 2, -1)$

$Q(-2, 1, 3)$

Required:

a) $\vec{G}$ at P

b) a unit vector in direction of $\vec{G}$ at Q

c) a unit vector directed from Q towards P

d) the equation of surface on which $|\vec{G}| = 60$

Solution:

a) $\vec{r}_{GP} = 24(1)(2)\, a_x + 12(1^2+2)\, a_y + 18(-1)^2\, a_z$

$= 48\, a_x + 36\, a_y + 18\, a_z$

b) $\hat{a}_{GQ} = \frac{\vec{r_{GQ}}}{|r_{GQ}|}$

$\vec{r}_{GQ} = 24(-2)(1)\, a_x + 12((-2)^2+2)\, a_y + 18(3)^2 a_z$

$= -48\, a_x + 72\, a_y + 162\, a_z$

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$$|\vec{r}_{GQ}| = \sqrt{(-48)^2 + (72)^2 + (162)^2}$$
$$= 183.66$$

$$\hat{a}_{GQ} = \frac{-48\,a_x + 72\,a_y + 162\,a_z}{183.66}$$
$$= -0.26\,a_x + 0.39\,a_y + 0.88\,a_z$$

c) $$\hat{a}_{QP} = \frac{\vec{r}_{QP}}{|\vec{r}_{QP}|}$$

$$\vec{r}_{QP} = r_P - r_Q$$
$$= a_x + 2a_y - a_z - (-2a_x + a_y + 3a_z)$$
$$= a_x + 2a_y - a_z + 2a_x - a_y - 3a_z$$
$$= 3a_x + a_y - 4a_z$$

$$|\vec{r}_{QP}| = \sqrt{(3)^2 + (1)^2 + (-4)^2}$$
$$= \sqrt{26} = 5.099$$

$$\hat{a}_{QP} = \frac{3a_x + a_y - 4a_z}{\sqrt{26}}$$
$$= 0.59\,a_x + 0.2\,a_y - 0.78\,a_z$$

d) $$|G| = 60$$

$$\Rightarrow \sqrt{(24xy)^2 + (12x^2 + 24)^2 + (18z^2)^2} = 60$$

S.B.S

$$\Rightarrow (24xy)^2 + (12x^2 + 24)^2 + (18z^2)^2 = 3600$$
$$\Rightarrow 576\,x^2y^2 + 144x^4 + 576x^2 + 576 + 324z^4 = 3600$$
$$\Rightarrow 576\,x^2y^2 + 144\,x^4 + 576x^2 + 324\,z^4 = 3024$$
$$\Rightarrow 16\,x^2y^2 + 4x^4 + 16x^2 + 9z^4 = 84$$

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Q 1.6) Given:

$$\vec{A} = 2a_x + a_y + 3a_z$$
$$\vec{B} = a_x - 3a_y + 2a_z$$

Required:

Find acute angle between $\vec{A}$ and $\vec{B}$ by using definition of:

a) dot product
b) cross product

Solution:

a) $\vec{A} \cdot \vec{B} = (2a_x + a_y + 3a_z) \cdot (a_x - 3a_y + 2a_z)$
$= 2 - 3 + 6$
$= 5$

$|\vec{A}| = \sqrt{(2)^2 + (1)^2 + (3)^2}$
$= \sqrt{14}$

$|\vec{B}| = \sqrt{(1)^2 + (-3)^2 + (2)^2}$
$= \sqrt{14}$

$\because \vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos Q$

$5 = (\sqrt{14})(\sqrt{14}) \cos Q$

$Q = \cos^{-1}\left(\frac{5}{(\sqrt{14})(\sqrt{14})}\right)$

$Q = 69.1°$

b) $A \times B = \begin{vmatrix} a_x & a_y & a_z \\ 2 & 1 & 3 \\ 1 & -3 & 2 \end{vmatrix}$

$= (2+9)a_x - (4-3)a_y + (-6-1)a_z$
$= 11a_x - a_y - 7a_z$

$|A \times B| = \sqrt{(11)^2 + (-1)^2 + (-7)^2}$
$= 3\sqrt{19} = 13.08$

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$|A| = \sqrt{14}$

$|B| = \sqrt{14}$

$\because |A \times B| = |A||B| \sin Q$

$3\sqrt{19} = (\sqrt{14})(\sqrt{14}) \sin Q$

$Q = \sin^{-1}\left(\frac{3\sqrt{19}}{(\sqrt{14})(\sqrt{14})}\right)$

$Q = 69.1°$

Q1.9) Given:

$$\vec{G} = \frac{25}{x^2+y^2}(x a_x + y a_y)$$

Required:

a) a unit vector in direction of $\vec{G}$ at $P(3,4,-2)$

b) the angle b/w G and a_x at P

c) value of following double integral on plane $y = 7$

$$\int_0^4 \int_0^2 \vec{G} \cdot a_y \, dz \, dx$$

Solution:

a) $\hat{a}_{G_P} = \frac{\vec{r}_{G_P}}{|\vec{r}_{G_P}|}$

$\vec{r}_{G_P} = \frac{25}{(3)^2+(4)^2}(3a_x + 4a_y)$

$= 3a_x + 4a_y$

$|\vec{r}_{G_P}| = \sqrt{(3)^2+(4)^2}$

$= 5$

$\hat{a}_{G_P} = \frac{3}{5}a_x + \frac{4}{5}a_y$

$= 0.6\, a_x + 0.8\, a_y$

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b) $\hat{a}_{GP} \cdot \hat{a}_x = (0.6\,a_x + 0.8\,a_y + 0\,a_z) \cdot (1\,a_x + 0\,a_y + 0\,a_z)$

$= 0.6$

$|a_{GP}| = \sqrt{(0.6)^2 + (0.8)^2}$

$= 1$

$|\hat{a}_x| = \sqrt{(1)^2 + (0)^2 + (0)^2}$

$= 1$

$\hat{a}_{GP} \cdot \hat{a}_x = |a_{GP}|\,|a_x| \cos Q$

$0.6 = (1)(1) \cos Q$

$Q = \cos^{-1}(0.6)$

$Q = 53.1°$

c) $\int_0^4 \int_0^2 G \cdot a_y \, dz \, dx$

$\Rightarrow \int_0^4 \int_0^2 \left(\frac{25\,x\,a_x}{x^2+y^2} + \frac{25\,y\,a_y}{x^2+y^2} \right) \cdot a_y \, dz \, dx$

$\Rightarrow \int_0^4 \int_0^2 \frac{25y}{x^2+y^2} \, dz \, dx \qquad \because y = 7$

$\Rightarrow \int_0^4 \int_0^2 \frac{175}{x^2+49} \, dz \, dx$

Integrating w.r.t 'z'

$\frac{175}{x^2+49} \int_0^4 z \Big|_0^2 \, dx$

Applying limit :

$\Rightarrow \int_0^4 \frac{350}{x^2+49} \, dx$

$\Rightarrow 350 \int_0^4 \frac{1}{x^2+49} \, dx$

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$\Rightarrow 350 \left(\frac{1}{7} \tan^{-1} \frac{x}{7} \right) \Big|_0^4$

$\Rightarrow 50 \tan^{-1} \frac{x}{7} \Big|_0^4$

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$\Rightarrow 50 \tan^{-1} \left(\frac{4}{7} \right) - 50 \tan^{-1} \left(\frac{0}{7} \right)$

$\Rightarrow 26$

Q1.11) Given:

$M = 0.1a_x - 0.2a_y - 0.1a_z$
$N = -0.2a_x + 0.1a_y + 0.3a_z$
$P = 0.4a_x + 0a_y + 0.1a_z$

Required:

a) the vector $\vec{R}_{MN}$
b) the dot product $\vec{R}_{MN} \cdot \vec{R}_{MP}$
c) the scalar projection of $\vec{R}_{MN}$ on $\vec{R}_{MP}$
d) the angle b/w $\vec{R}_{MN}$ and $\vec{R}_{MP}$

Solution:

a) $\vec{R}_{MN} = \vec{R}_N - \vec{R}_M$
$= -0.2a_x + 0.1a_y + 0.3a_z$
$- (0.1a_x - 0.2a_y - 0.1a_z)$
$= -0.3a_x + 0.3a_y + 0.4a_z$

b) $\vec{R}_{MN} = -0.3a_x + 0.3a_y + 0.4c_z$
$\vec{R}_{MP} = \vec{R}_P - \vec{R}_M$
$= (0.4a_x + 0a_y + 0.1a_z) - (0.1a_x - 0.2a_y$
$- 0.1c_z)$
$= 0.4a_x + 0a_y + 0.1c_z - 0.1a_x + 0.2a_y + 0.1a_z$
$= 0.3a_x + 0.2a_y + 0.2a_z$

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$$\vec{R_{MN}} \cdot \vec{R_{MP}} = (-0.3a_x + 0.3a_y + 0.4a_z) \cdot (0.3a_x + 0.2a_y + 0.2a_z)$$
$$= -0.09 + 0.06 + 0.08$$
$$= 0.05$$

c) $$\frac{\vec{R_{MN}} \cdot \vec{R_{MP}}}{|\vec{R_{MP}}|} = \frac{0.05}{\sqrt{(0.3)^2 + (0.2)^2 + (0.2)^2}}$$
$$= 0.1212$$

d) $\because$ $$\vec{R_{MN}} \cdot \vec{R_{MP}} = |\vec{R_{MN}}|\,|\vec{R_{MP}}| \cos Q$$
$$0.05 = \left(\sqrt{(-0.3)^2 + (0.3)^2 + (0.4)^2}\right)\left(\sqrt{(0.3)^2 + (0.2)^2 + (0.2)^2}\right)\cos Q$$
$$Q = 78^\circ$$

Q1.13) Required:

a) the vector component of $\vec{F} = 10a_x - 6a_y + 5a_z$ that is parallel to $\vec{G} = 0.1a_x + 0.2a_y + 0.3a_z$

b) the vector component of $\vec{F}$ that is perpendicular to $\vec{G}$

c) the vector component of $\vec{G}$ that is perpendicular to $\vec{F}$.

Solution:

a) $$\vec{F}_{\parallel G} = \left(\frac{\vec{F} \cdot \vec{G}}{|\vec{G}|}\right) \hat{a}_G$$
$$= \frac{\vec{F} \cdot \vec{G}\ (\vec{G})}{|\vec{G}|^2}$$
$$= \frac{\left[(10a_x - 6a_y + 5a_z) \cdot (0.1a_x + 0.2a_y + 0.3a_z)\right] \; 0.1a_x + 0.2a_y + 0.3a_z}{\left(\sqrt{(0.1)^2 + (0.2)^2 + (0.3)^2}\right)^2}$$

$$= \frac{1.3\ (0.1a_x + 0.2a_y + 0.3a_z)}{0.14}$$
$$= 9.2857\ (0.1a_x + 0.2a_y + 0.3a_z)$$

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$= 0.93 a_x + 1.86 a_y + 2.79 a_z$

b) $F_{\perp G} = \vec{F} - \overline{F_{\parallel G}}$

$= 10a_x - 6a_y + 5a_z - (0.93 a_x + 1.86 a_y + 2.79 a_z)$

$= 10a_x - 6a_y + 5a_z - 0.93 a_x - 1.86 a_y - 2.79 a_z$

$= 9.07 a_x - 7.86 a_y + 2.21 a_z$

c) Step 1: First we will find the vector component of $\vec{G}$ that is ~~perpendicular~~ parallel to $\vec{F}$

$$G_{\parallel F} = \frac{\vec{G}\vec{F}}{|\vec{F}|}(\hat{a}_F)$$

$$= \frac{\vec{G}\cdot\vec{F}}{|F|^2}(\vec{F})$$

$$= \frac{[(0.1a_x + 0.2a_y + 0.3a_z)\cdot(10a_x - 6a_y + 5a_z)]\,[10a_x - 6a_y + 5a_z]}{(\sqrt{(10)^2 + (-6)^2 + (5)^2})^2}$$

$$= \frac{1.3(10a_x - 6a_y + 5a_z)}{161}$$

$$= 8.07\times10^{-3}(10a_x - 6a_y + 5a_z)$$

$$= 0.08 a_x - 0.048 a_y + 0.04 a_z$$

Step 2: Now we will find the vector component of $\vec{G}$ that is perpendicular to $\vec{F}$

$$G_{\perp F} = \vec{G} - \vec{G_{\parallel F}}$$

$$= 0.1 a_x + 0.2 a_y + 0.3 c_z - (0.08 a_x - 0.048 a_y + 0.04 a_z)$$

$$= 0.1 a_x + 0.2 a_y + 0.3 c_z - 0.08 a_x + 0.048 a_y - 0.04 a_z$$

$$= 0.02 a_x + 0.248 a_y + 0.258$$

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Q 1.15) Given:

$\vec{r_1} = 7a_x + 3a_y - 2a_z$
$\vec{r_2} = -2a_x + 7a_y - 3a_z$
$\vec{r_3} = 0a_x + 2a_y + 3a_z$

Required: c) area of Δ defined by $\vec{r_1}$ and $\vec{r_2}$
a) unit vector perpendicular to both $\vec{r_1}$ and $\vec{r_2}$
b) unit vector perpendicular to vector $\vec{r_1} - \vec{r_2}$ and $\vec{r_2} - \vec{r_3}$
d) area of triangle defined by the heads of r_1, r_2 and r_3

Solution:

a) $\hat{a}_{\vec{r_1} \times \vec{r_2}} = \dfrac{\vec{r_1} \times \vec{r_2}}{|\vec{r_1} \times \vec{r_2}|}$

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} a_x & a_y & a_z \\ 7 & 3 & -2 \\ -2 & 7 & -3 \end{vmatrix}$$

$$= (-9 + 14)a_x - (-21 - 4)a_y + (\cancel{6 - 49} \; 49+6)a_z$$

$$= 5a_x + 25a_y + 55a_z$$

$$|\vec{r_1} \times \vec{r_2}| = \sqrt{(5)^2 + (25)^2 + (55)^2}$$

$$= 35\sqrt{3}$$

$$\hat{a}_{r_1 \times r_2} = \frac{5}{35\sqrt{3}} a_x + \frac{25}{35\sqrt{3}} a_y + \frac{55}{35\sqrt{3}} a_z$$

$$= 0.08\, a_x + 0.41\, a_y + 0.9\, a_z$$

b) $\hat{a}_{\vec{r_1} - \vec{r_2} \times \vec{r_2} - \vec{r_3}} = \dfrac{\vec{r_1} - \vec{r_2} \times \vec{r_2} - \vec{r_3}}{|\vec{r_1} - \vec{r_2} \times \vec{r_2} - \vec{r_3}|}$

$$\vec{r_1} - \vec{r_2} = 7a_x + 3a_y - 2a_z - (-2a_x + 7a_y - 3a_z)$$

$$= 7a_x + 3a_y - 2a_z + 2a_x - 7a_y + 3a_z$$

$$= 9a_x - 4a_y + a_z$$

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$$\vec{r_2} - \vec{r_3} = -2a_x + 7a_y - 3a_z - (0 + 2a_y + 3a_z)$$
$$= -2a_x + 7a_y - 3a_z - 2a_y - 3a_z$$
$$= -2a_x + 5a_y - 6a_z$$

$$\vec{r_1} - \vec{r_2} \times \vec{r_2} - \vec{r_3} = \begin{vmatrix} a_x & a_y & a_z \\ 9 & -4 & 1 \\ -2 & 5 & -6 \end{vmatrix}$$
$$= (24-5)a_x - (-54+2) + (45-8)a_z$$
$$= 19a_x + 52a_y + 37a_z$$
$$|\vec{r_1} - \vec{r_2} \times \vec{r_2} - \vec{r_3}| = \sqrt{(19)^2 + (52)^2 + (37)^2}$$
$$= 66.58$$

$$\hat{a}_{\vec{r_1} - \vec{r_2} \times \vec{r_2} - \vec{r_3}} = \frac{19}{66.58} a_x + \frac{52}{66.58} a_y + \frac{37}{66.58} a_z$$
$$= 0.28 a_x + 0.78 a_y + 0.55 a_z$$

c) $\because$ Area of triangle $= \frac{1}{2} |\vec{r_1} \times \vec{r_2}|$

$|\vec{r_1} \times \vec{r_2}| = 35\sqrt{3}$ → We have already found out value of $|\vec{r_1} \times \vec{r_2}|$ in part a

$$\text{Area of triangle} = \frac{1}{2}(35\sqrt{3})$$
$$= 30.3$$

d) $\because$ Area of triangle $= \frac{1}{2} |\vec{r_2} - \vec{r_1} \times \vec{r_2} - \vec{r_3}|$

$$\vec{r_2} - \vec{r_1} = -2a_x + 7a_y - 3a_z - (7a_x + 3a_y - 2a_z)$$
$$= -2a_x + 7a_y - 3a_z - 7a_x - 3a_y + 2a_z$$
$$= -9a_x + 4a_y - a_z$$

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$$\vec{r_2} - \vec{r_3} = -2a_x + 5a_y - 6a_z$$

$$\vec{r_2} - \vec{r_1} \times \vec{r_2} - \vec{r_3} = \begin{vmatrix} a_x & a_y & a_z \\ -9 & 4 & -1 \\ -2 & 5 & -6 \end{vmatrix}$$

$$= (-24+5)a_x - (54-2)a_y + (-45+8)a_z$$

$$= -19a_x - 52a_y - 37a_z$$

$$|\vec{r_2} - \vec{r_1} \times \vec{r_2} - \vec{r_3}| = \sqrt{(-19)^2 + (-52)^2 + (-37)^2}$$

$$= 66.59$$

$$\text{Area of triangle} = \frac{1}{2}(66.59)$$

$$= 33.2$$

Q 1.17) Given:

$$\vec{A} = -4a_x + 2a_y + 5a_z$$

$$\vec{R_{AM}} = 20a_x + 18a_y - 10a_z$$

$$\vec{R_{AN}} = -10a_x + 8a_y + 15a_z$$

Required:

a) a unit vector perpendicular to the triangle

b) a unit vector in plane of triangle and perpendicular to $\vec{R_{AN}}$

c) a unit vector in plane of triangle that bisects interior angle at $\vec{A}$

Solution:

a) $$\hat{a}_{AM \times AN} = \frac{\vec{r_{AM}} \times \vec{r_{AN}}}{|\vec{r_{AM}} \times \vec{r_{AN}}|}$$

$$\vec{r_{AM}} \times \vec{r_{AN}} = \begin{vmatrix} a_x & a_y & a_z \\ 20 & 18 & -10 \\ -10 & 8 & 15 \end{vmatrix}$$

$$= (270+80)a_x - (300-100)a_y + (160+180)a_z$$

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$$= 350\,a_x - 200\,a_y + 340\,a_z$$

$$|\vec{r}_{AM} \times \vec{r}_{AN}| = \sqrt{(350)^2 + (-200)^2 + (340)^2}$$

$$= 30\sqrt{309}$$

$$\hat{a}_{AM \times AN} = \frac{350}{30\sqrt{309}}\,a_x - \frac{200}{30\sqrt{309}}\,a_y + \frac{340}{30\sqrt{309}}\,a_z$$

$$= 0.66\,a_x - 0.38\,a_y + 0.65\,a_z$$

b) $$\hat{a}_{AM \times AN} \times \hat{a}_{AN} = \hat{a}_{AM \times AH} \times \frac{\vec{r}_{AN}}{|\vec{r}_{AN}|}$$

$$= \hat{a}_{AM \times AN} \times \frac{(-10a_x + 8a_y + 15a_y)}{\sqrt{(-10)^2 + (8)^2 + (15)^2}}$$

$$= \hat{a}_{AM \times AN} \times \frac{-10a_x + 8a_y + 15a_y}{\sqrt{389}}$$

$$= 0.66a_x - 0.38a_y + 0.65a_z \times (-0.507a_x + 0.405a_y + 0.76\,a_z)$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ 0.66 & -0.38 & 0.65 \\ -0.507 & 0.405 & 0.76 \end{vmatrix}$$

$$= (-0.2888 - 0.26325)\,a_x - (0.5016 + 0.33)\,a_y + (0.2673 - 0.19266)\,a_z$$

$$= -0.55\,a_x - 0.83\,a_y + 0.07\,a_z$$

c) $$\frac{1}{2}(a_{An} + a_{AM}) = \frac{1}{2}\left(\frac{20a_x + 18a_y - 10a_z}{\sqrt{(20)^2 + (18)^2 + (-10)^2}} + \frac{\cancel{(0.55a_x - 0.83a_y + 0.07a_z)}}{\cancel{32}}\right)$$

$-0.507a_x + 0.405a_y + 0.76a_z$

$$= \frac{1}{2}\left(\frac{20a_x + 18a_y - 10a_z}{2\sqrt{206}} - 0.507a_x + 0.405a_y + 0.76a_z\right)$$

$$= \frac{1}{2}\left(0.696\,a_x + 0.627\,a_y - 0.35a_z \cancel{- 0.55a_x - 0.83a_y + 0.07a_z} - 0.507a_x + 0.405a_y + 0.761a_z\right)$$

$$= \frac{1}{2}\left(0.189\,a_x + 1.032\,a_y + 0.411\,a_z\right)$$

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$= 0.095\, a_x + 0.516\, a_y + 0.2055\, a_z$

$$a_{bis} = \frac{0.0.95\, a_x + 0.516\, a_y + 0.2055\, a_z}{\sqrt{(0.95)^2 + (0.516)^2 + (0.2055)^2}}$$

$$= \frac{0.095\, a_x + 0.516\, a_y + 0.2055\, a_z}{0.56}$$

$= 0.16\, a_x + 0.92\, a_y + 0.366\, a_z$

Q1.19) Given:

$$\vec{D} = \frac{x a_x + y a_y}{x^2 + y^2}$$

Required.

a) Express field $\vec{D}$ in ~~vector~~ cylindrical components and cylindrical variables

b) Evaluate $\vec{D}$ at pt where $\rho = 2$, $\phi = 0.2\pi$, and $z = 5$, expressing result in cylindrical and rectangular components

Solution:

a) $\vec{D} = D_\rho a_\rho + D_\phi a_\phi + D_z a_z$ —(i)

$D_\rho = \vec{D} \cdot a_\rho$

$$= \frac{x a_x + y a_y}{x^2 + y^2} \cdot a_\rho$$

R.W

	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$\sin\phi$	$\cos\phi$	0
a_z	0	0	1

$$= \frac{x a_x \cdot a_\rho}{x^2 + y^2} + \frac{y a_y \cdot a_\rho}{x^2 + y^2}$$

$$= \frac{\rho\cos\phi \cos\phi}{(\rho\cos\phi)^2 + (\rho\sin\phi)^2} + \frac{\rho\sin\phi \cdot \sin\phi}{(\rho\cos\phi)^2 + (\rho\sin\phi)^2}$$ $\because x = \rho\cos\phi$, $\because y = \rho\sin\phi$

$$= \frac{\rho\cos^2\phi + \rho\sin^2\phi}{\rho^2\cos^2\phi + \rho^2\sin^2\phi}$$

$$= \frac{\rho(\cos^2\phi + \sin^2\phi)}{\rho^2(\cos^2\phi + \sin^2\phi)}$$ $\because \cos^2\phi + \sin^2\phi = 1$

Date:

$D_\rho = \frac{1}{\rho}$

$D_\phi = \vec{D} \cdot a_\phi$

$= \frac{x a_x + y a_y}{x^2+y^2} \cdot a_\phi$

$= \frac{x\, a_x \cdot a_\phi}{x^2+y^2} + \frac{y\, a_y\, a_\phi}{x^2+y^2}$

$= \frac{\rho\cos\phi(-\sin\phi) + \rho\sin\phi\cos\phi}{\rho^2\cos^2\phi + \rho^2\sin^2\phi}$

$= 0$

$D_z = \vec{D} \cdot a_z$

$= \frac{x a_x + y \cancel{a} a_y}{x^2+y^2} \cdot a_z$

$= \frac{x\, a_x\, a_z}{x^2+y^2} + \frac{y\, a_y \cdot a_z}{x^2+y^2}$

$= 0$

Now eq (1) becomes

$\vec{D} = \frac{1}{\rho} a_\rho$

b) At $\rho = 2$, $\phi = 0.2\pi$ and $z = 5$

$\vec{D} = \frac{1}{2} a_\rho$

$\vec{D} = 0.5\, a_\rho$

$\because x = \rho\cos\phi$

$x = 1.618$

$\because y = \rho\sin\phi$

$y = 1.17557$

$z = 2$

$z = 5$

At $x = 1.618$, $y = 1.17557$ and $z = 5$

$D = \frac{x a_x + y a_y}{x^2+y^2}$

$= \frac{1.618\; a_x}{(1.618)^2 + (1.17557)^2} + \frac{1.17557\; a_y}{(1.618)^2 + (1.17557)^2}$

Date:

$\vec{D} = 0.4\, a_x + 0.29\, a_y$

Q (1.21) Required:

Express in cylindrical components

a) the vector from $\vec{C}(3, 2, -7)$ to $\vec{D}(-1, -4, 2)$

b) a unit vector at $\vec{D}$ directed toward $\vec{C}$

c) a unit vector at $\vec{D}$ directed toward origin

Solution:

a) $\vec{R_{CD}} = \vec{R_D} - \vec{R_C}$

$= -a_x - 4a_y + 2a_z - (3a_x + 2a_y - 7a_z)$

$= -a_x - 4a_y + 2a_z - 3a_x - 2a_y + 7a_z$

$= -4a_x - 6a_y + 9a_z$

$\vec{R_{CD}} = R_{CD\rho}\, a_\rho + R_{CD\phi}\, a_\phi + R_{CDz}\, a_z$ —(i)

$R_{CD\rho} = R_{CD}\, a_\rho$

$= (-4a_x - 6a_y + 9a_z)\, a_\rho$

$= -4\, a_x \cdot a_\rho - 6\, a_y \cdot a_\rho + 9\, a_z a_\rho$

$= -4\cos\phi - 6\sin\phi + 9(0)$

$= -4\cos\phi - 6\sin\phi$

R.W

	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$\sin\phi$	$\cos\phi$	0
a_z	0	0	1

At $\rho = \sqrt{13}$, $\phi = 33.7°$ and $z = -7$

$= -4\cos(33.7) - 6\sin(33.7)$

$= -6.66$

$x = 3, y = 2, z = -7$

$\rho = \sqrt{x^2 + y^2}$

$\rho = \sqrt{3^2 + 2^2}$

$= \sqrt{13}$

$\phi = \tan^{-1}(y/x)$

$\phi = 33.7°$

$z = -7$

$R_{CD\phi} = R_{CD} \cdot a_\phi$

$= (-4a_x - 6a_y + 9a_z)\, a_\phi$

$= -4\, a_x a_\phi - 6\, a_y \cdot a_\phi + 9 a_z a_\phi$

$= 4\sin\phi - 6\cos\phi$

At $\phi = 33.7$

$= 4\sin(33.7) - 6\cos(33.7)$

$= -2.77$

Date:

$$R_{CD z} = R_{CD} \cdot a_z$$
$$= (-4a_x - 6a_y + 9a_z)\, a_z$$
$$= -4a_x a_z - 6a_y \cdot a_z + 9 a_z a_z$$
$$= 9$$

Now eq (i) becomes:

$$\bar{R_{CD}} = -6.66\, a_\rho - 2.77\, a_\phi + 9\, a_z$$

b) $\vec{R_{DC}} = 4a_x + 6a_y - 9a_z$

$$\vec{R_{DC}} = R_{DC\rho}\, a_\rho + R_{DC\phi}\, a_\phi + R_{DC z}\, a_z \quad \text{— (ii)}$$

$$R_{DC\rho} = R_{DC} \cdot a_\rho$$
$$= 4a_x \cdot a_\rho + 6a_y a_\rho - 9a_z a_\rho$$
$$= 4\cos\phi + 6\sin\phi$$

At $\phi = 256°$

$$= 4\cos(256°) + 6\sin(256)$$
$$= -6.8$$

$x = -1,\ y = -4,\ z = 2$

$\rho = \sqrt{x^2 + y^2}$

$\rho = \sqrt{(-1)^2 + (-4)^2}$

$\rho = 3$

$\phi = \tan^{-1}(y/x)$

$\phi = \tan^{-1}(-4/-1)$

$\phi = 256°$

$$R_{DC\phi} = R_{DC} \cdot a_\phi$$
$$= 4a_x a_\phi + 6a_y a_\phi - 9a_z a_\phi$$
$$= -4\sin\phi + 6\cos\phi$$

At $\phi = 256°$

$$= -4\sin(256) + 6\cos(256)$$
$$= 2.43$$

$$R_{DC z} = R_{DC} \cdot a_z$$
$$= 4a_x a_z + 6a_y a_z - 9a_z a_z$$
$$= -9$$

Now eq (ii) becomes

$$\vec{R_{DC}} = -6.8\, a_\rho + 2.43\, a_\phi - 9a_z$$

Date:

c) $\vec{R_{D0}} = +a_x + 4a_y - 2a_z$

$\vec{R_D} = R_{D\rho}\, a_\rho + R_{D\phi}\, a_\phi + R_{Dz}\, a_z$

$R_{D\rho} = R_D \cdot a_\rho$

$= (+a_x + 4a_y - 2a_z)\, a_\rho$

$= +a_x \cdot a_\rho + 4\, a_y\, a_\rho - 2\, a_z\, a_\rho$

$= +\cos\phi + 4\sin\phi$

At $\phi = 256^\circ$

$= +\cos(256) + 4\sin(256)$

$= 4.123$

b) $\hat{a}_{DC} = \dfrac{\vec{r_{DC}}}{|\vec{r_{DC}}|}$

$\vec{r_{DC}} = r_C - r_D$

$= 3a_x + 2a_y - 7a_y - (-a_x - 4a_y + 2a_z)$

$= 3a_x + 2a_y - 7a_y + a_x + 4a_y - 2a_z$

$= 4a_x + 6a_y - 9a_z$

$|\vec{r_{DC}}| = \sqrt{(4)^2 + (6)^2 + (-9)^2}$

$= \sqrt{133}$

$\hat{a}_{DC} = \dfrac{4}{\sqrt{133}} a_x + \dfrac{6\, a_y}{\sqrt{133}} - \dfrac{9}{\sqrt{133}} a_z$

$\hat{a}_{DC} = 0.3468\, a_x + 0.52\, a_y - 0.78\, a_z$

Now converting in cylindrical components

$a_{DC} = a_{DC\rho}\, a_\rho + a_{DC\phi}\, a_\phi + a_{DCz}\, a_z$ — (i)

$a_{DC\rho} = a_{DC}\, a_\rho$

$= 0.3468\, a_x a_\rho + 0.52\, a_y a_\rho - 0.78\, a_z a_\rho$

$= 0.3468\cos\phi + 0.52\sin\phi - 0$

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$= 0.3468\cos(256) + 0.52\sin(256)$ $\quad x = -1,\ y = -4,\ z = 2$

$= -0.59$ $\quad \rho = \sqrt{x^2+y^2} = 3$

$\phi = \tan^{-1}(y/x) = 256°$

$z = 2$

$a_{x\phi} = a_{x}\, a_\phi$

$= 0.3468\, a_x a_\phi + 0.52\, a_y a_\phi - 0.78\, a_z a_\phi$

$= -0.3468 \sin\phi + 0.52\cos\phi$

At $\phi = 256$

$= -0.3468\sin(256) + 0.52\cos(256)$

$= 0.21$

$a_{xz} = a_{xz}\, a_z$

$= 0.3468\, a_x a_z + 0.52\, a_y a_z - 0.78\, a_z a_z$

$= -0.78$

Now eq (i) becomes:

$a_x = -0.59\, a_\rho + 0.21\, a_\phi - 0.78\, a_z$

c) $a_{DO} = \dfrac{\vec{r}_{DO}}{|\vec{r}_{DO}|}$

$\vec{r}_{DO} = \vec{P}_0 - \vec{P}_D$

$= 0a_x + 0a_y + 0a_z - (-a_x - 4a_y + 2a_z)$

$= a_x + 4a_y - 2a_z$

$|\vec{r}_{DO}| = \sqrt{(1)^2 + (4)^2 + (-2)^2}$

$= \sqrt{21}$

$\vec{a}_{DO} = \dfrac{1}{\sqrt{21}} a_x + \dfrac{4}{\sqrt{21}} a_y - \dfrac{2}{\sqrt{21}} a_z$

$\vec{a}_{DO} = 0.21\, a_x + 0.87\, a_y - 0.44\, a_z$

Now converting in cylindrical components

$\vec{a}_{DO} = a_{DO\rho}\, a_\rho + a_{DO\phi}\, a_\phi + a_{DOz}\, a_z$ — (i)

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$a_{DO\rho} = a_{DO} \cdot a_\rho$

$= 0.29\, a_x a_\rho + 0.87\, a_y a_\rho - 0.44\, a_z a_\rho$

$= 0.29 \cos\phi + 0.87 \sin\phi$

At $\phi = 256^\circ$

$= 0.29 \cos(256) + 0.87 \sin(256)$

$= -0.9$

$a_{DO\phi} = a_{DO} \cdot a_\phi$

$= 0.29\, a_x a_\phi + 0.87\, a_y a_\phi - 0.44\, a_z a_\phi$

$= -0.291 \sin\phi + 0.87 \cos\phi$

At $\phi = 256^\circ$

$= -0.291 \sin(256) + 0.87 \cos(256)$

$= -6.7 \times 10^{-3} \simeq 0$

$a_{DOz} = a_{DO} \cdot a_z$

$= 0.29\, a_x a_z + 0.87\, a_y a_z - 0.44\, a_z a_z$

$= -0.44$

Now eq (i) becomes:

$$\hat{a}_{DO} = -0.9\, a_\rho + 0\, a_\phi - 0.44\, a_z$$

Q1.23) Given:

$3 \le \rho \le 5$, $100^\circ \le \phi \le 130^\circ$, $3 \le z \le 4.5$

$\frac{5\pi}{9} \le \phi \le \frac{13\pi}{18}$

Required:

a) the enclosed volume
b) the total area of enclosing surface
c) total length of 12 edges of surfaces
d) length of the longest straight line that lies entirely within the volume

Solution:

Date: ____________

a) $\cancel{d}V = \iiint \rho \; d\rho \, d\phi \, dz$

$\cancel{d}V = \int_{3}^{4.5} \int_{\frac{5\pi}{9}}^{\frac{13\pi}{18}} \int_{3}^{5} \rho \; d\rho \, d\phi \, dz$

Integrating w.r.t 'ρ'

$\cancel{d}V = \int_{3}^{4.5} \int_{\frac{5\pi}{9}}^{\frac{13\pi}{18}} \frac{\rho^2}{2} \Big|_{3}^{5} \; d\phi \, dz$

$\cancel{d}V = \int_{3}^{4.5} \int_{\frac{5\pi}{9}}^{\frac{13\pi}{18}} \frac{25}{2} - \frac{9}{2} \; d\phi \, dz$

$\cancel{d}V = 8 \int_{3}^{4.5} \int_{\frac{5\pi}{9}}^{\frac{13\pi}{18}} d\phi \, dz$

Integrating w.r.t 'ϕ'

$V = 8 \int_{3}^{4.5} \phi \Big|_{\frac{5\pi}{9}}^{\frac{13\pi}{18}} dz$

$V = 8 \int_{3}^{4.5} \frac{13\pi}{18} - \frac{5\pi}{9} \; dz$

$V = \cancel{\frac{77\pi}{9}} \frac{4\pi}{3} \int_{3}^{4.5} dz$

Integrating w.r.t 'z'

$V = \cancel{\frac{77\pi}{9}} \frac{4\pi}{3} \; z \Big|_{3}^{4.5}$

$V = \cancel{\frac{77\pi}{9}} \frac{4\pi}{3} (4.5 - 3)$

$V = 2\pi$

$V = 6.28$

b) $\cancel{ds_T} = ds_1 = \rho \, d\phi \, dz \; a_\rho$ —(i)

$ds_2 = \rho \, d\phi \, dz (-a_\rho)$ —(ii)

$ds_3 = d\rho \, dz \; a_\phi$ —(iii)

$ds_4 = d\rho \, dz \, (-a_\phi)$ —(iv)

$ds_5 = \rho \, d\rho \, d\phi \; (a_z)$ —(v)

Date:

$$ds_6 = \rho\, d\rho\, d\phi\, (-a_z) \quad — \text{(vi)}$$

Add eq (i) and (ii)

$$= (\rho + \rho)\ d\phi\, dz$$
$$= (3+5)\ d\phi\, dz$$
$$= 8\ d\phi\, dz$$

Add eq (iii) and (iv)

$$= 2\ d\rho\, dz$$

Add eq (v) and (vi)

$$= 2\rho\ d\rho\ d\phi$$

$$S = 8\int_3^{4.5}\int_{5\pi/9}^{13\pi/18} d\phi\, dz + 2\int_3^{4.5}\int_3^{5} d\rho\, dz + 2\int_{5\pi/9}^{13\pi/18}\int_3^{5} \rho\, d\rho\, d\phi$$

$$S = 8\int_3^{4.5} \phi\Big|_{5\pi/9}^{13\pi/18} dz + 2\int_3^{4.5} \rho\Big|_3^{5} dz + 2\int_{5\pi/9}^{13\pi/18} \frac{\rho^2}{2}\Big|_3^{5} d\phi$$

$$S = \frac{4\pi}{3}\int_3^{4.5} dz + 4\int_3^{4.5} dz + 16\int_{5\pi/9}^{13\pi/18} d\phi$$

$$S = \frac{4\pi}{3}\, z\Big|_3^{4.5} + 4\, z\Big|_3^{4.5} + 16\, \phi\Big|_{5\pi/9}^{13\pi/18}$$

$$S = \frac{4\pi}{3}(4.5-3) + 4(4.5-3) + 16\left(\frac{13\pi}{8} - \frac{5\pi}{9}\right)$$

$$S = 20.7$$

Date:

c) length $= 4 \times 1.5 + 4 \times 2 + 2\left[\frac{30^\circ}{360} \times 2\pi \times 3 + \frac{30^\circ}{360^\circ} \times 2\pi \times 5\right]$

$= 22.4$

d) $A(\rho = 3, \phi = 100^\circ, z = 3)$
$B(\rho = 5, \phi = 130^\circ, z = 4.5)$

→ Now we will convert cylindrical coordinate to rectangular coordinate

$A(-0.52, 2.95, 3)$
$B(-3.21, 3.8, 4.5)$

R.W

For pt A,
$x = \rho\cos\phi$
$x = 3\cos(100)$
$x = -0.52$
$y = \rho\sin\phi$
$y = 3\sin(100)$
$y = 2.95$
$z = 3$
$A(-0.52, 2.95, 3)$

For pt B.
$x = 5\cos(130^\circ)$
$x = -3.21$
$y = 5\sin(130)$
$y = 3.8$
$z = 4.5$
$B(-3.21, 3.8, 4.5)$

~~length~~ $R_{AB} = R_B - R_A$
$= -3.2a_x + 3.8a_y + 4.5a_z - (-0.52a_x + 2.95a_y + 3a_z)$
$= -3.2a_x + 3.8a_y + 4.5a_z + 0.52a_x - 2.95a_y - 3a_z$
$= -2.68a_x + 0.85a_y + 1.5a_z$

$\because$ length $= |R_{AB}|$

length $= |R_{AB}| = \sqrt{(-2.68)^2 + (0.85)^2 + (1.5)^2}$
$\doteq 3.2$

Q 1.25) Given:

$P(r = 0.8, \phi = 30^\circ, \phi = 45^\circ)$

$E = \frac{1}{r^2}\left(\cos\phi \, a_r + \frac{\sin\phi}{\sin\theta} a_\theta\right)$

Required:

a) $\vec{E}$ at P

b) $|\vec{E}|$ at P

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c) a unit vector in direction of $\vec{E}$ at P

Solution:

a) $\vec{E}_p = \frac{1}{(0.8)^2}\left[\cos 45\, a_r + \frac{\sin 45}{\sin 30^\circ}\, a_\phi\right]$

$= 1.5625\left[0.707\, a_r + \sqrt{2}\, a_\phi\right]$

$= 1.104\, a_r + 2.21\, a_\phi$

b) $|E_p| = \sqrt{(1.104)^2 + (2.21)^2}$

$= 2.47$

c) $\hat{a}_{EP} = \frac{\vec{r}_{EP}}{|\vec{r}_{EP}|}$

$= \frac{1.104\, a_r + 2.21\, a_\phi}{2.47}$

$= 0.45\, a_r + 0.89\, a_\phi$

Q 1.26) Given:

$\vec{F} = 5a_x$

Required:

Express uniform vector field $\vec{F}$ in

a) cylindrical components

b) spherical components

Solution:

a) $\vec{F} = F_\rho a_\rho + F_\phi a_\phi + F_z a_z$ — (i)

$F_\rho = \vec{F} \cdot a_\rho$

$= 5a_x \cdot a_\rho$

$= 5\cos\phi$

R W

	a_ρ	a_ϕ	a_z
a_x	$\cos\phi$	$-\sin\phi$	0
a_y	$\sin\phi$	$\cos\phi$	0
a_z	0	0	1

$F_\phi = \vec{F} \cdot a_\phi$

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$F_\phi = 5a_x \cdot a_\phi$
$= -5\sin\phi$

$F_z = 5a_x \cdot a_z$
$F_z = 0$

Now eq (i) becomes
$\vec{F} = 5\cos\phi \, a_\rho - 5\sin\phi \, a_\phi$

b) $\vec{F} = F_r a_r + F_\theta a_\theta + F_\phi a_\phi$ — (ii)

$F_r = \vec{F} \cdot a_r$
$= 5a_x \cdot a_r$
$= 5\sin\theta\cos\phi$

R.W

	a_r	a_θ	a_ϕ
a_x	sc	cc	$-\sin\phi$
a_y	ss	cs	$\cos\phi$
a_z	$\cos\theta$	$\sin\theta$	0

$F_\theta = \vec{F} \cdot a_\theta$
$= 5a_x \cdot a_\theta$
$= 5\cos\theta\cos\phi$

$F_\phi = \vec{F} \cdot a_\phi$
$= 5a_x \cdot a_\phi$
$= -5\sin\phi$

Now eq (ii) becomes

$\vec{F} = 5\sin\theta\cos\phi \, a_r + 5\cos\theta\cos\phi \, a_\theta$
$- 5\sin\phi \, a_\phi$

Date:

Q 1.27) Given:

$2 \le r \le 4$, $30^\circ \le Q \le 50^\circ$, $20^\circ \le \phi \le 60^\circ$

$2 \le r \le 4$, $\frac{\pi}{6} \le Q \le \frac{5\pi}{18}$, $\frac{\pi}{9} \le \phi \le \frac{\pi}{3}$

Required:

a) the enclosed volume

b) total area of closing surface

c) total length of 12 edges of surface

d) length of the longest straight line that lies entirely within the surface.

Solution:

a) $V = \iiint r^2 \sin Q \, dr \, dQ \, d\phi$

$$V = \int_{\frac{\pi}{9}}^{\pi/3} \int_{\pi/6}^{\frac{5\pi}{18}} \int_{2}^{4} r^2 \sin Q \, dr \, dQ \, d\phi$$

Integrating w.r.t. 'r'

$$V = \sin Q \int_{\pi/9}^{\pi/3} \int_{\pi/6}^{\frac{5\pi}{18}} \frac{r^3}{3} \Big|_2^4 \, dQ \, d\phi$$

$$= \frac{56}{3} \int_{\pi/9}^{\pi/3} \int_{\pi/6}^{\frac{5\pi}{18}} \sin Q \, dQ \, d\phi$$

$$= -\frac{56}{3} \int_{\pi/9}^{\pi/3} \cos Q \Big|_{\pi/6}^{\frac{5\pi}{18}} \, d\phi$$

$$= 4.167 \int_{\pi/9}^{\pi/3} d\phi$$

$$= 4.167 \ \phi \Big|_{\pi/9}^{\pi/3}$$

$$= 4.167 \left(\frac{\pi}{3} - \frac{\pi}{9} \right)$$

$$= 2.91$$

Date:

b) $ds_1 = r\, dr\, d\theta\ a_\phi$ — (i)
$ds_2 = r\, dr\, d\theta\ (-a_\phi)$ — (ii)
$ds_3 = r^2 \sin\theta\, d\theta\, d\phi\ a_r$ — (iii)
$ds_4 = r^2 \sin\theta\, d\theta\, d\phi\ (-a_r)$ — (iv)
$ds_5 = r\sin\theta\, dr\, d\phi\ a_\theta$ — (v)
$ds_6 = r\sin\theta\, dr\, d\phi\ (-a_\theta)$ — (vi)

Add eq (i) and (ii)

$$= 2r\, dr\, d\theta$$

Add eq (iii) and (iv)

$$= (r^2 + r^2) \sin\theta\, d\theta\, d\phi$$
$$= (2^2 + 4^2) \sin\theta\, d\theta\, d\phi$$
$$= 20 \sin\theta\, d\theta\, d\phi$$

Add eq (v) and (vi)

$$= (\sin\theta + \sin\theta)\, r\, dr\, d\phi$$
$$= \left(\sin\frac{\pi}{6} + \sin\frac{5\pi}{18}\right) r\, dr\, d\phi$$
$$= 1.266\, r\, dr\, d\phi$$

$$S = 2\int_{\pi/6}^{5\pi/18}\int_2^4 r\, dr\, d\theta + 20\int_{\pi/9}^{\pi/3}\int_{\pi/6}^{5\pi/18} \sin\theta\, d\theta\, d\phi$$
$$+ 1.266 \int_{\pi/9}^{\pi/3}\int_2^4 r\, dr\, d\phi$$

$$S = 2\int_{\pi/6}^{5\pi/18} \left.\frac{r^2}{2}\right|_2^4 d\theta + 20\int_{\pi/9}^{\pi/3} \left.-\cos\theta\right|_{\pi/6}^{5\pi/18} d\phi$$
$$+ 1.266 \int_{\pi/9}^{\pi/3} \left.\frac{r^2}{2}\right|_2^4 d\phi$$

Date:

$$S = 12\int_{\pi/6}^{5\pi/18} dQ + \cancel{0.223}\ 4.464\int_{\pi/9}^{\pi/3} d\phi$$
$$+ 7.596\int_{\pi/9}^{\pi/3} d\phi$$

$$S = 12\left(\frac{5\pi}{18} - \frac{\pi}{6}\right) + \cancel{0.223}\ 4.464\left(\frac{\pi}{3} - \frac{\pi}{9}\right) + 7.596\left(\frac{\pi}{3} - \frac{\pi}{9}\right)$$

$$S = 12.61$$

c) $\text{length} = 4\int_2^4 dr + 2\int_{\pi/6}^{5\pi/18}(4+2)\,dQ + \int_{\pi/9}^{\pi/3}\left[4\sin\left(\frac{5\pi}{18}\right) + 4\sin\left(\frac{\pi}{6}\right) + 2\sin\left(\frac{5\pi}{18}\right) + 2\sin\left(\frac{\pi}{6}\right)\right] d\phi$

$= 17.49$

d) $A(r = 2,\ Q = 30°,\ \phi = 20°)$
$B(r = 4,\ Q = 30°,\ \phi = 60°)$

→ Now we will convert spherical coordinate to rectangular coordinate

$A(1.44, 0.52, 1.29)$
$B(1, 1.73, 3.46)$

$R_{AB} = R_B - R_A$
$= a_x + 1.73 a_y + 3.46 a_z - (1.44 a_x + 0.52 a_y + 1.29 a_z)$
$= -0.44 a_x + 1.21 a_y + 2.18 a_z$

$\text{Length} = |R_{AB}| = \sqrt{(-0.44)^2 + (1.21)^2 + (2.18)^2}$
$= 2.53$

R.W

For A:
$x = r\sin Q\cos\phi$
$x = 2\sin(30)\cos(20)$
$x = 1.44$
$y = r\sin Q\sin\phi$
$y = 0.52$
$z = r\cos Q$
$z = 1.28$

For pt B
$x = 4\sin(30)\cos(60)$
$x = 1$
$y = 4\sin(30)\sin(60)$
$y = 1.73$
$z = 4\cos 30 = 3.46$

Q 1.29) Given:
$F = a_x$

Required:
Express $\vec{F}$ in spherical components at pt

Date:

a) $r = 2$, $Q = 1$, $\phi = 0.8$

b) $x = 3$, $y = 2$, $z = -1$

c) $\rho = 2.5$, $\phi = 0.7$, $z = 1.5$

Solution:

a) $\vec{F} = F_r a_r + F_Q a_Q + F_\phi a_\phi$ — (i)

$F_r = \vec{F} \cdot a_r$

$= a_x \cdot a_r$

$= \sin Q \cos \phi$

$= \sin(1)\cos(0.8) = 0.586$

$F_Q = \vec{F} \cdot a_Q = a_x \cdot a_Q = \cos Q \cos\phi$

$F_Q = \cos(1)\cos(0.8) = 0.376$

$F_\phi = \vec{F} \cdot a_\phi = a_x \cdot a_\phi = -\sin\phi$

$F_\phi = -\sin(0.8) = -0.717$

Now eq(i) becomes: $\vec{F} = 0.59\, a_r + 0.38\, a_Q - 0.717\, a_\phi$

~~b) $\vec{F} = F_x a_x + F_y a_y + F_z a_z$ — (ii)~~

~~$F_x = \vec{F} \cdot a_x$~~ ~~$F_y = \vec{F} \cdot a_y$~~

~~$F_x = a_x \cdot a_x = 1$~~ ~~$F_y = a_x \cdot a_y = 0$~~

b) $F_r = \sin Q \cos \phi = \sin(105.5)\cos(33.7)$

$F_r = 0.80$

$F_Q = \cos Q \cos \phi = \cos(105.5)\cos(33.7)$

$F_Q = -0.22$

$F_\phi = -\sin\phi = -\sin 33.7$

$F_\phi = -0.55$

Now eq(i) becomes $\vec{F} = 0.8\, a_r - 0.22\, a_Q - 0.55\, a_\phi$

R.W

b) $r = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

$Q = \cos^{-1}\left(\frac{-1}{\sqrt{14}}\right) = 105.5$

$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$

c) $F_r = \sin Q \cos \phi = \sin(1.02)\cos(0.7)$

$= 0.65$

$F_Q = \cos Q \cos \phi = \cos(1.02)\cos(0.7)$

$= 0.4$

$F_\phi = -\sin\phi = -\sin(0.7)$

$= -0.64$

Now eq (i) becomes

$\vec{F} = 0.65\, a_r + 0.4\, a_Q - 0.64\, a_\phi$

R.W

$x = \rho\cos\phi$

$= 2.5\cos 0.7 = 1.9$

$y = \rho \sin\phi$

$= 2.5 \sin 0.7 = 1.6$

$z = 1.5$

$r = \sqrt{x^2 + y^2 + z^2} = 2.9$

$Q = \cos^{-1}\left(\frac{z}{r}\right) = 1.02$

$\phi = \tan^{-1}(y/x)$

$\phi = 0.7$