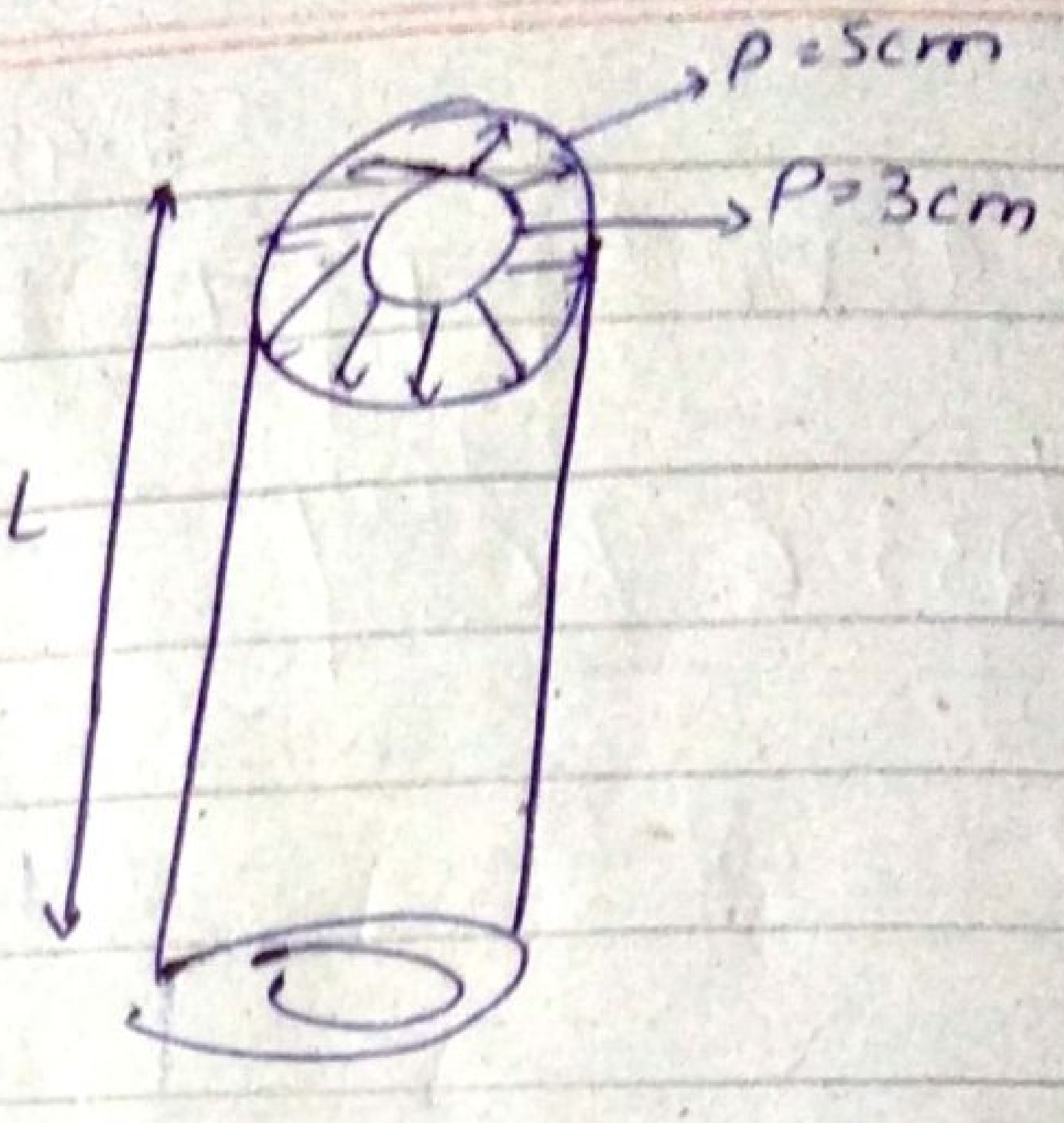
5.11

$\rho = 3cm$
$\& \rho = 5cm$
$I = 3A$
$V = ?$
$R = ?$
$E = ?$
$\sigma = 0.05 \, S/m$



$3 < \rho < 5cm$

$P = ?$

$$V = -\int E \cdot dL \longrightarrow (1)$$

$$E\sigma = J \longrightarrow (2)$$

$I = JS$

$J = \frac{I}{S} \longrightarrow$ surface area of cylinder $= 2\pi\rho L$

$J = \frac{3}{2\pi\rho L} \hat{a_\rho} \longrightarrow$ radially outward

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so eq(2) becomes,

$$E = \frac{J}{\sigma}$$

$$= \frac{3}{2\pi \rho L \times 0.05} \hat{a_\rho}$$

$$= \frac{3}{0.1\pi \rho L} \hat{a_\rho}$$

so eq(1) becomes

$$V = -\int_{0.05}^{0.03} \frac{0.3}{0.1\pi \rho L} \hat{a_\rho} \cdot \rho \, d\rho \, \hat{a_\rho}$$

$$= -\frac{3}{0.1\pi L} \int_{0.05}^{0.03} \frac{1}{\rho} d\rho$$

$$= \frac{-03}{0.1\pi L} \ln \rho \Big|_{0.05}^{0.03}$$

$$= \frac{-03}{0.1\pi L} \left[\ln\left(\frac{0.03}{0.05}\right) \right]$$

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$$\boxed{V = \frac{4.88}{L}\ V}$$

$$R = \frac{V}{I} = \frac{4.88}{L \times 3} = \frac{1.62}{L}\ \Omega$$

$$\boxed{\frac{dP}{dV} = E \cdot J}$$

$$P = \int_{VOL} E \cdot J \, dv$$

$$P = \int_{VOL} E \cdot J \, dv \longrightarrow (1)$$

$$E \cdot J = \left(\frac{3}{2\pi \rho L} \hat{a_\rho} \right) \cdot \left(\frac{3}{0.1\pi \rho L} \right) \hat{a_\rho}$$

$$= \frac{9}{0.2\pi^2 \rho^2 L^2}$$

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so eq(1) becomes

$$= \int_0^L \int_0^{2\pi} \int_{0.03}^{0.05} \frac{9}{0.2\pi^2 \rho^2 L^2} \rho \, d\rho \, d\phi \, dz$$

$$= \frac{9}{0.2\pi^2 L^2} \int_0^L \int_0^{2\pi} \int_{0.03}^{0.05} \frac{1}{\rho} \, d\rho \, d\phi \, dz$$

$$= \frac{9}{0.2\pi^2 L^2} \int_0^L \int_0^{2\pi} \ln\left(\frac{0.05}{0.03}\right) d\phi \, dz$$

$$= \frac{4.597}{0.2\pi^2 L^2} \int_0^L \int_0^{2\pi} d\phi \, dz$$

$$= \frac{4.597 \times 2\pi \times L}{0.2\pi^2 L^2}$$

$$\boxed{P = \frac{14.63}{L}} \, W$$

5.19

$V = 20x^2yz - 10z^2\ V$

(a) $V = 0$ → equipotential voltage
$V = 60$
equation of surface = ??

(b) $\rho_s = ??$
$V = 60V$
$x = 2, z = 1$
$0 \leq V \leq 60\ V$

Sol:-

Equation of surfaces:

at 0V :

$0 = 20x^2yz - 10z^2$

at 60V

$60 = 20x^2yz - 10z^2$

(b)

$60 = 20(2)^2(y)(1) - 10$

$60 = 80y - 10$

$70 = 80y$

$y = \frac{70}{80}$

$\boxed{y = \frac{7}{8}}$

$\boxed{\rho_s = D_s = \epsilon_0 E_N} \rightarrow (1)$

$E = -\text{grad} V$

$E = -\left(\frac{\partial}{\partial x} (20x^2yz - 10z^2)\, \hat{a_x} \right.$

$+ \frac{\partial}{\partial y} (20x^2yz - 10z^2)\, \hat{a_y} +$

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$$\frac{\partial}{\partial z}\left(20x^2yz - 10z^2\right)\hat{az}\Big)$$

$$E_N = -40xyz\,\hat{ax} - 20x^2z\,\hat{ay}$$

$$-\left(20x^2y - 20z\right)\hat{az}$$

$\rho_s = \epsilon_0 E_N$

$$= -40\times 8.85\times 10^{-12}\,xyz\,\hat{ax}$$
$$-20x^2z\times 8.85\times 10^{-12}\,\hat{ay}$$

$$-\ 8.85\times 10^{-12}\left(20x^2y - 20z\right)\hat{az}$$

$$= -40\times 8.85\times 10^{-12}\times(2)(1)\times(7/8)\ \hat{ax}$$

$$-\ 20(4)(1)\times 8.85\times 10^{-12}\ \hat{ay}$$

$$-\ 8.85\times 10^{-12}\left(20\times 4\times \frac{7}{8} - 20\right)\hat{az}$$

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$$= -6.195\times10^{-10}\,\hat{a}_x - 7.08\times10^{-10}\,\hat{a}_y$$

$$- 4.425\times10^{-10}\,\hat{a}_z$$

$$\boxed{|P| = 1.04\,nC/m^2}$$

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