CHAPTER: 05

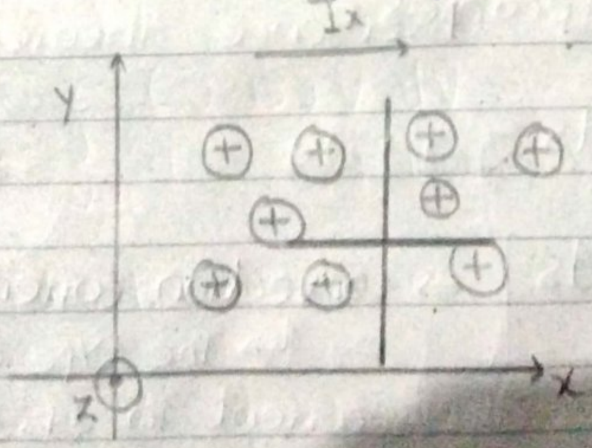
=> CURRENTS AND CONDUCTORS:

CURRENT:-

=> Electric charges in motion constitute a current. The unit of current is amphere. It is defined as the Rate of movement of charges passing a given reference point (or crossing a given reference plane) of one coloumb per second.

$$I = \frac{dq}{dt}$$ => Rate of change of charges constitute a current.

=> current is a scalar quantity.



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Current Density:

⇒ The increment of current ΔI crossing an incremental surface ΔS normal to the current density is:

$$J_N = \frac{\Delta I}{\Delta S} \Rightarrow \Delta I = J_N \Delta S$$

⇒ current density is normal to the surface.

⇒ When current density is not perpendicular to ΔS

$$\Delta I = J \cdot \Delta S$$

$$\lim_{\Delta S \to 0} \Delta I = \lim_{\Delta S \to 0} J \cdot \Delta S \longrightarrow (1)$$

$$\Delta I = dI$$
$$\Delta S = ds$$

so eq(1) becomes,

$$dI = J \cdot ds$$

$$\boxed{I = \int_S J \cdot ds}$$ ⇒ convection current.

↳ The type of current in which charges are moving

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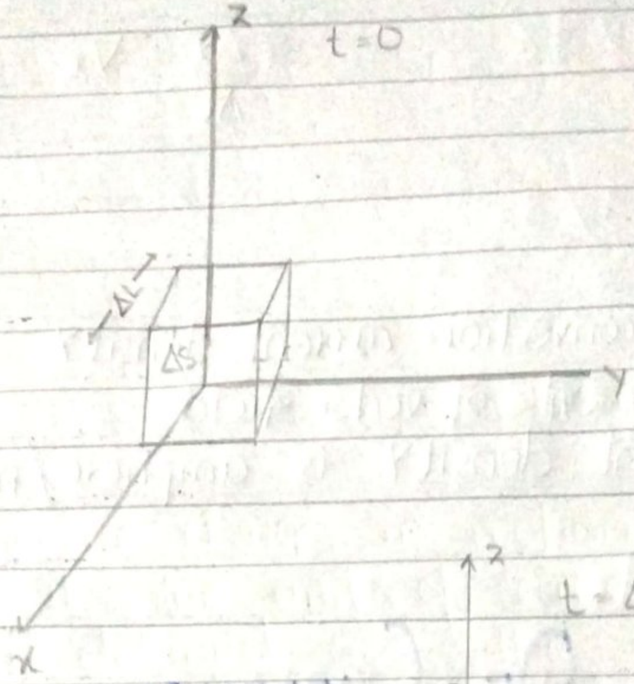

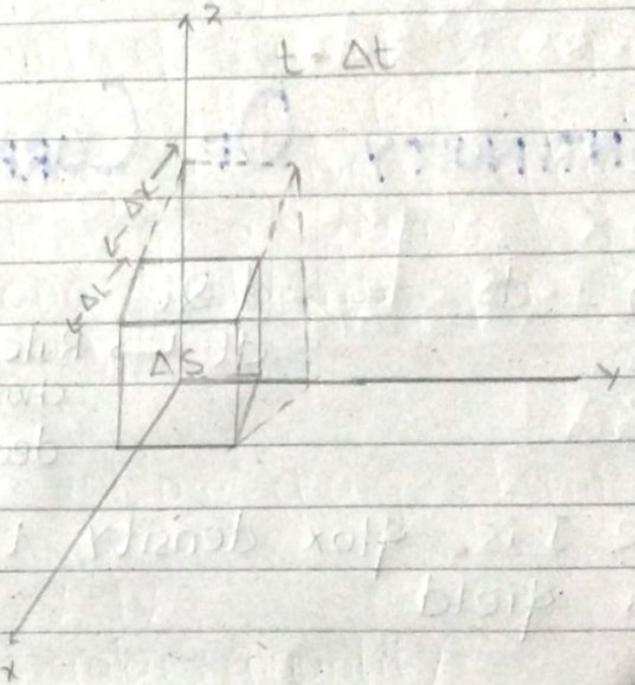

let ΔQ_i be the charge inside the cube of volume ΔV

$$\Delta Q_i = \rho_v \Delta V = \rho_v \Delta S \Delta L$$

$$\Delta Q = \rho_v \Delta S \Delta x$$

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta S \Delta x}{\Delta t}$$

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$\Delta I = \rho_v \Delta s V_x$ $\quad \because \frac{\Delta x}{\Delta t} = V_x$

$\frac{\Delta I}{\Delta S} = \rho_v V$

$\boxed{J = \rho_v V}$ $\Rightarrow$ convection current density

↳ is flux density type of vector field.

=> unit of current density is amphere/m^2

=> CONTINUITY OF CURRENT:-

$I = \oint_S J \cdot ds = -\frac{dQ_i}{dt}$ ↳ Rate with which charges are decreasing.

=> since J is flux density type of vector field

=> So applying divergence theorem,

$\rho_v(t)$

Volume

$+Q_i$

$\oint_S J \cdot ds = \int_{vol} (\bar{\nabla} \cdot J)\, dv$

=> $\int_{vol} \bar{\nabla} \cdot J\, dv = -\frac{d}{dt} \int_{vol} \rho_v dv$

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$$\int_{VOL} \bar{\nabla}\cdot J \, dv = \int_{VOL} -\frac{\partial}{\partial t} \rho_v \, dv$$

$$\boxed{\bar{\nabla}\cdot J = -\frac{\partial}{\partial t} \rho_v}$$ ⇒ continuity equation.

D: 5.1

$J = 10\rho^2 z \, \hat{a}_\rho - 4\rho\cos^2\phi \, \hat{a}_\phi$ mA/m²

$J_P = ??$ $\quad P(\rho = 3, \phi = 30°, z = 2)$ (a)

(b) $I = ??$ $\quad \rho = 3, \; 0 < \phi < 2\pi,$
$2 < z < 2.8$

Sol:-

(a)

$$J_P = 10(9)(2)\,\hat{a}_\rho - 4\times(3)\times[\cos 30°]^2\,\hat{a}_\phi$$

$$J_P = 180\,\hat{a}_\rho - 9\,\hat{a}_\phi \text{ mA/m}^2$$

(b)

$$I = \oint_S J\cdot ds$$

$$I = \int_S (10\rho^2 z\,\hat{a}_\rho - 4\rho\cos^2\phi\,\hat{a}_\phi)\cdot \rho\, d\phi\, dz\, \hat{a}_\rho$$

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$$I = \int_{2}^{2.8}\int_{0}^{2\pi} 10\rho^3 z \, d\phi \, dz$$

$$I = \int_{2}^{2.8}\int_{0}^{2\pi} 10(3)^3 z \, d\phi \, dz$$

$$I = 270 \int_{2}^{2.8} \phi \Big|_{0}^{2\pi} z \, dz$$

$$I = 270 \times 2\pi \int_{2}^{2.8} z \, dz$$

$$I = 540\pi \left. \frac{z^2}{2} \right|_{2}^{2.8}$$

$$\boxed{I = 3.26A}$$

D 5.2

$J = -10^6 z^{1.5} a_z$ A/m²

(a) $I = ??$ $z = 0.1m$

(b) $v = 2 \times 10^6$ m/s at $z = 0.1m$
$\rho_v = ??$

(c) $\rho_v = -2000$ C/m³
$z = 0.15m$
$v = ??$

$$I = \int_S J \cdot ds$$

$$= \int_S (-10^6 z^{1.5} a_z) \cdot ds_z$$

$$= \int_0^{2\pi} \int_0^{20\times10^{-6}} (-10^6 z^{1.5} \hat{a_z}) \cdot \rho \, d\rho \, d\phi \, \hat{a_z}$$

$$= \int_0^{2\pi} \int_0^{20\times10^{-6}} -10^6 (0.1)^{1.5} \rho \, d\rho \, d\phi$$

$$= -31622.7766 \int_0^{2\pi} \frac{\rho^2}{2} \Big|_0^{20\times10^{-6}} d\phi$$

$$= -15811.3883 \times 4\times10^{-10} \times 2\pi$$

$$\boxed{I = -39.7 \ \mu A}$$

b) $P_v = ??$

$J = P_v \times V$

$$P_v = \frac{J}{V} = \frac{-10^6 (0.1)^{1.5}}{2 \times 10^6}$$

$$\boxed{P_v = -15.8 \text{ m } C/m^3}$$

(C) $J = P_v \times V$

$$V = \frac{J}{P_v} = \frac{-10^6 (0.15)^{1.5}}{-2000}$$

$$\boxed{V = 29.0 \text{ m/s}}$$

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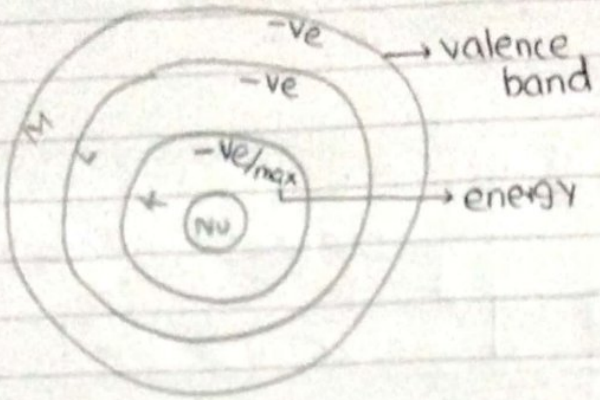
METALLIC CONDUCTORS

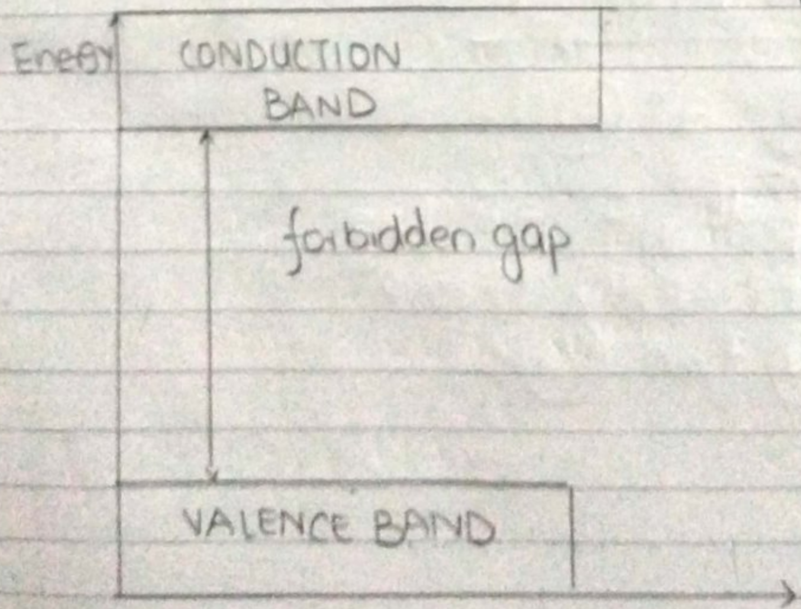


$E = P \cdot E + K \cdot E$

→ At $T = 0K$

⇒ number of electrons surrounding the nucleus is equal to number of protons.

⇒ The energy absorbed by electrons to jump from lower state to higher state must be in discrete form.
→ quanta.



Insulators.

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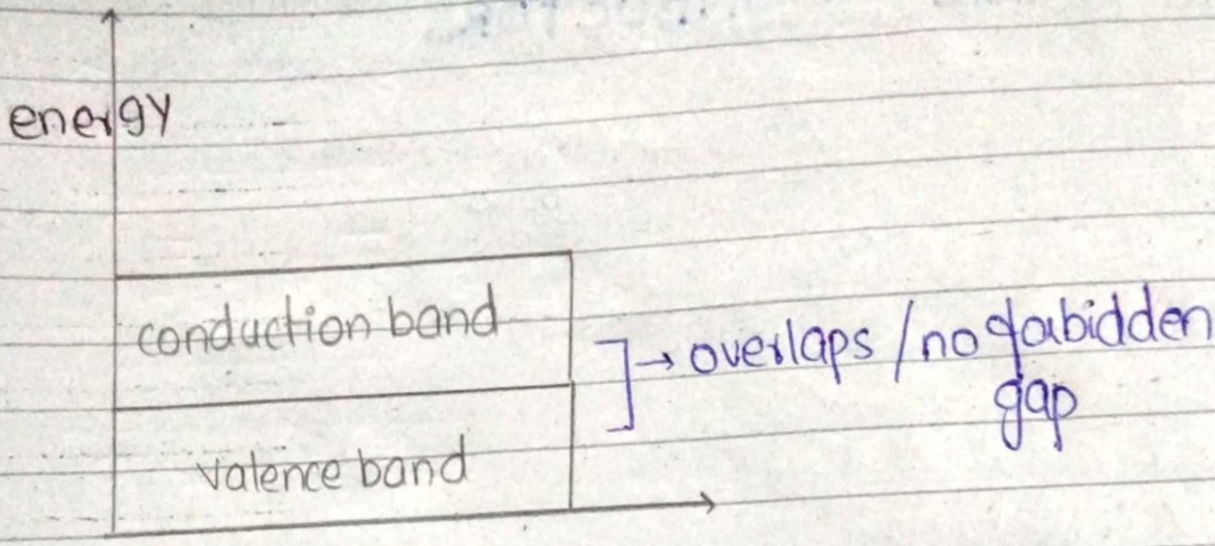
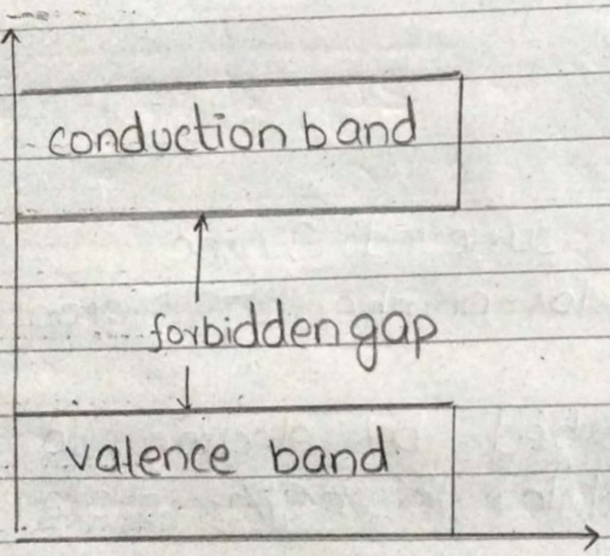
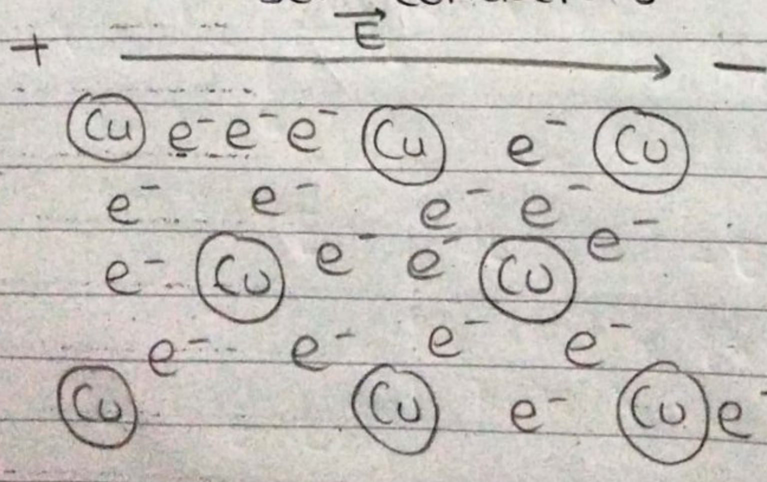


conductors



semi conductors



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⇒ The arrangement of conductor is such that the electric field intensity inside the conductor is zero.

$$\bar{F} = q\bar{E}$$

⇒ The force experienced by one electron after applying the electric field intensity is

$$\boxed{\bar{F} = -e\bar{E}}$$

⇒ The direction of force is opposite to the field.

$$\bar{V_d} \propto -\bar{E}$$

$$\bar{V_d} = -\mu_e \bar{E} \longrightarrow (1)$$

↓

measure of ease with which an electron can move through crystal structure.

convection current density is:

$J = \rho_v \times V$ → drift velocity

By using eq(1)

$$\boxed{J = -\rho_e \mu_e \bar{E}}$$

$$\rho \times \mu \rightarrow \frac{m^2}{Vs} \times \frac{C}{m^3} = \frac{A}{Vm} = \frac{S}{m}$$

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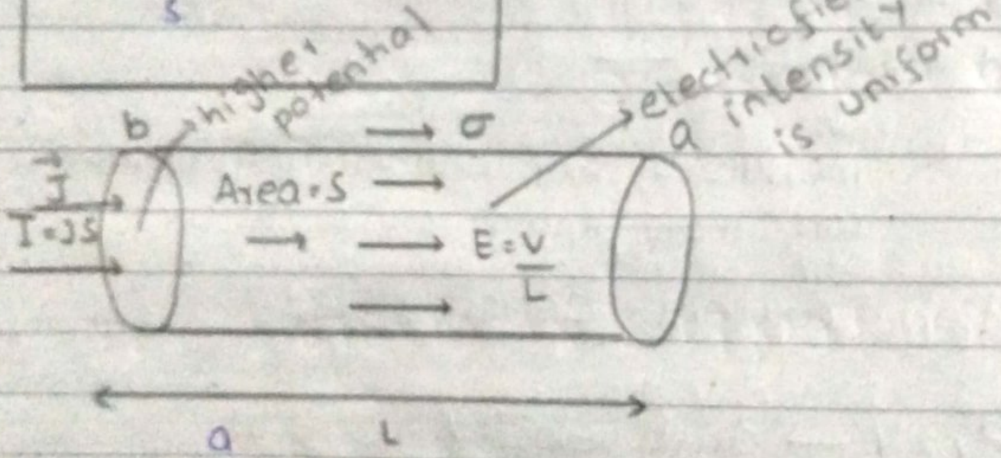
The product of charge density and mobility of charge gives "conductivity".

$$\boxed{\bar{J} = \sigma \bar{E}}$$

$$\boxed{\sigma = -\rho_e \mu_e}$$

conductivity

$$\boxed{I = \int_S J \cdot ds = JS}$$ ⇒ current flowing in conductor



$$V_{ab} = -\int_b^a E \cdot dL$$

$$= -E \cdot \int_b^a dL = -E \cdot L_{ba}$$

$$= E \cdot L_{ba}$$

$$\boxed{V = E \times L}$$

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$$J = \frac{I}{S}$$

$$= \sigma E = \frac{I}{S}$$

$$= \frac{V}{L}\sigma = \frac{I}{S}$$

$$\boxed{V = \frac{IL}{\sigma S}} \Rightarrow \boxed{V = IR}$$

$$R = \frac{V}{I} = \frac{IL}{\sigma S} \div I$$

$\sigma S \rightarrow \rho$ = resistivity

$$\boxed{R = \frac{L}{\sigma S}}$$

$$\boxed{R = \frac{V}{I} = \frac{-\int_b^a E \cdot dl}{\int_s J \cdot ds}}$$

$\Rightarrow$ If current is perependicular to electric field intensity.

$\rightarrow$ at higher potential where current is entering the surface

D: 5.3

$|J| = ??$
$\sigma = 6.17 \times 10^7$ S/m
$\mu_e = 0.0056 m^2/V$

(a) $V_d = 1.5$ μm/s

$|J| = \sigma E \longrightarrow (1)$

$E \mu_e = V_d$

$V_d = 0.0056 \times E$

$$E = \frac{1.5 \times 10^{-6}}{0.0056} = 2.678 \times 10^{-4} \text{ V/m}$$

$|J| = 2.678 \times 10^{-4} \times 6.17 \times 10^7$

$\boxed{|J| = 16.5 \text{ K A/m}^2}$

(b)
$E = 1$ mV/m

$|J| = 6.17 \times 10^7 \times 1 \times 10^{-3}$
$\boxed{|J| = 61.7 \text{ K A/m}^2}$

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(C) $L = 2.5mm$
$V = 0.4mV$

$V = E \times L$

$E = \frac{V}{L}$

$E = \left(\frac{2.5 \times 10^{-3}}{0.4 \times 10^{-3}} \right)^{-1}$

$E = 0.16 \; V/m$

$|J| = \sigma \times E$

$|J| = 6.17 \times 10^{7} \times 0.16$

$\boxed{|J| = 9.9 M \; A/m^2}$

(d)

$L = 2.5mm$
$I = 0.5A$

$I = J \times S$

$J = \frac{I}{S}$

$J = \frac{0.5}{(2.5 \times 10^{-3})^2} = \boxed{80 \; K \; A/m^2}$

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DRILL 5·4

Sol:

$d = 0.6$ inch $\qquad \sigma = 5.80 \times 10^7$ S/m

$L = 1200$ ft

$I = 50A$

$R = ?$ (a)

$J = ?$ (b)

$V = ?$ (c)

$P = ?$ (d)

$$d = 0.6 \times \frac{2.54}{100} = 0.01524 \text{ m}$$

$$L = 1200 \text{ ft} = 365.76 \text{ m}$$

$$r = \frac{d}{2} = 7.62 \times 10^{-3} \text{ m}$$

$S = \pi r^2$ = surface area of circle

$$S = \pi (7.62 \times 10^{-3})^2$$

$$S = 1.824 \times 10^{-4} \text{ m}^2$$

$$R = \frac{L}{\sigma S} = \frac{365.76}{(5.80 \times 10^7)(1.824 \times 10^{-4})}$$

$$\boxed{R = 0.034\ \Omega}$$

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(b) $I = J \cdot S$

$$J = \frac{I}{S} = \frac{50}{1.824 \times 10^{-4}} = 2.74 \times 10^{5}\ A/m^2$$

(c) $V = IR$

$V = 50 \times 0.034$

$\boxed{V = 1.7\ \text{Volts}}$

(d) $P = VI$

$P = 1.7 \times 50$

$\boxed{P = 85W}$

EXAMPLE : 5.1

$L = 1\text{mile} = 1609\,m$ $\quad \sigma = 5.80 \times 10^{7}\ S/m$

$d = 0.0508\ in = 1.290 \times 10^{-3}$

$R = ?$

$$r = \frac{d}{2} = \frac{1.290 \times 10^{-3}}{2} = 6.45 \times 10^{-4}\ m$$

$S = \pi r^2 \rightarrow$ surface area of circle

$S = \pi \times (6.45 \times 10^{-4})^2$

$S = 1.306 \times 10^{-6}\ m^2$

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$$R = \frac{L}{\sigma S} = \frac{1609}{(5.80\times10^{7})(1.306\times10^{-6})}$$

$$\boxed{R = 21.2\,\Omega}$$

⇒ POINT FORM OF OHM'S LAW:-

$$\bar{J} = \sigma \bar{E}$$

$$\frac{A}{m^2} = \frac{S}{m}\cdot\frac{V}{m}$$

$$A = SV$$

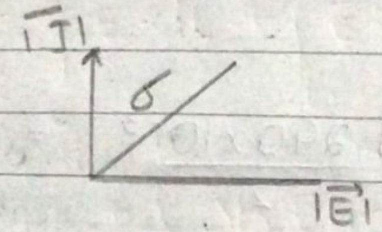
$$V = \frac{A}{S}$$

$$V = \Omega A$$

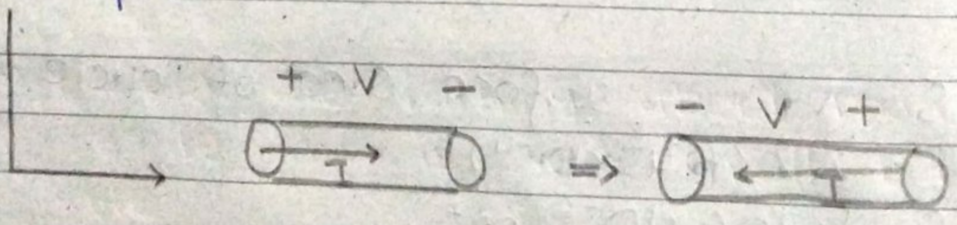
$$\boxed{V = IR}$$

Ohm's law is:

i) Linear →

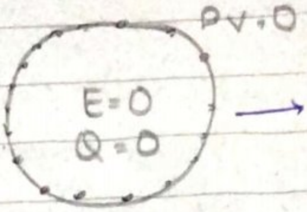


ii) Isotropic



⇒ The direction of the input → voltage does not change the property of material which is resistance.

⇒ BOUNDARY CONDITIONS:-

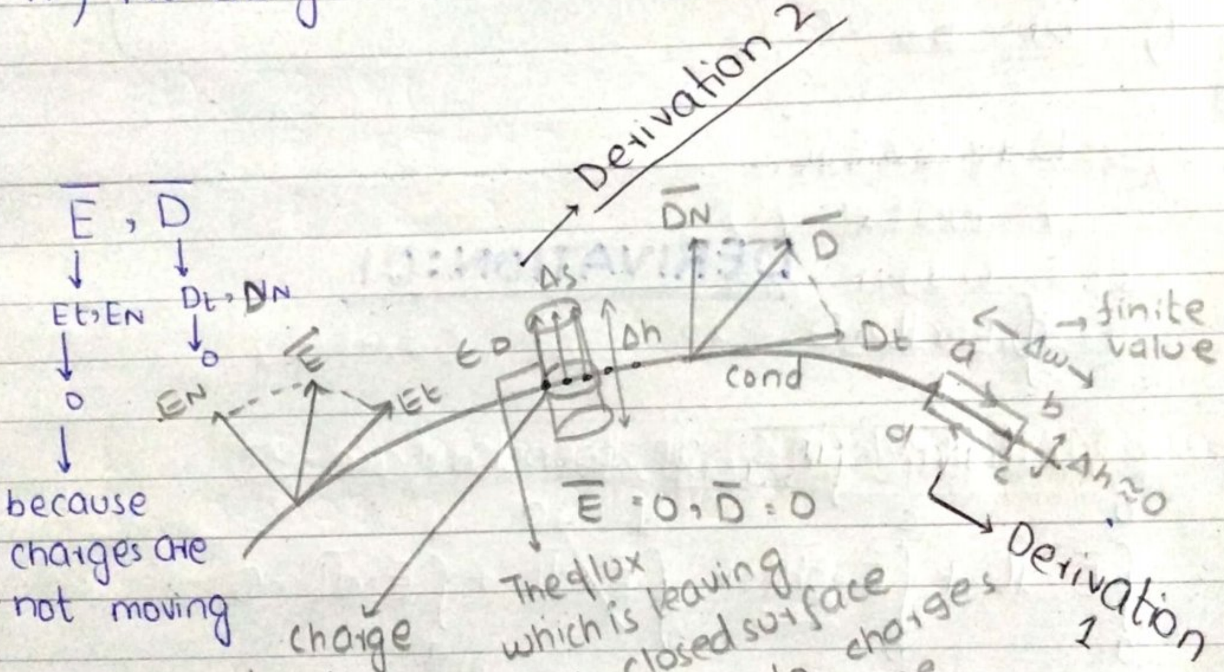


→ charges are at boundary

i) $\overline{E} = 0$

ii) No charge

→ within the conductor



charge density is equal to flux density

The flux which is leaving this closed surface is equal to charges at that surface

$$Q = \oint_s D \cdot ds = \psi$$

$$\boxed{|D_N| = \rho_s = \epsilon_0 |E|}$$

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=> WITH IN THE CONDUCTOR:-

1) $\bar{D} = \epsilon_0 E = 0$

Electric field intensity is zero with in the conductor

=> ON THE SURFACE /OUTSIDE THE CONDUCTOR:

1) $D_t = E_t = 0$

2) $D_N = \epsilon_0 E_N = \rho_s$

DERIVATION:01

$\oint E \cdot dl = 0$

apply this on boundary

$$\int_a^b \bar{E} \cdot \bar{dl} + \int_b^c \bar{E} \cdot \bar{dl} + \int_c^d \cancel{\bar{E} \cdot \bar{dl}}^{\,0} + \int_d^a \bar{E} \cdot \bar{dl} = 0$$

$$E_t \Delta w - \frac{1}{2} \Delta h \cancel{E_{N at b}}^{\,0} + \frac{1}{2} \Delta h \cancel{E_{N at a}}^{\,0} = 0$$

=> since we are talking about boundary $\Delta h \to 0$ while Δw is very small but finite

$E_t \Delta w = 0$

$\boxed{E_t = 0}$ $\therefore$ $\boxed{D_t = 0}$

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DERIVATION : 02

Applying Gauss's law:

$$\oint_S \bar{D} \cdot \bar{ds} = Q$$

$$\int_{top} D \cdot ds + \int_{bot} \cancel{D \cdot \vec{ds}}^{0} + \int_{cyl} \cancel{\vec{D} \cdot ds}^{0} = Q$$

$$\Rightarrow D_N \Delta \cancel{s} = Q = \rho_s \Delta \cancel{s}$$

$$\boxed{D_N = \rho_s}$$

$$\boxed{\epsilon_0 E_N = \rho_s}$$

$\Rightarrow$ $E_t = 0$ & Electric field has only the normal component E_N,

$\therefore$ conductor surface is equipotential surface

$\downarrow$

$\Rightarrow$ voltages at all points are equal

SUMMARY:

⇒ The static electric field intensity inside a conductor is zero.

⇒ The static electric field intensity at the surface of a conductor is everywhere directed normal to that surface.

⇒ The conductor surface is an equipotential surface.

EXAMPLE 5.2

$V = 100(x^2 - y^2)$

$P(2, -1, 3)$

$V, E, D, \rho_s = ??$ at $(2, -1, 3)$

equation of conductor surface = ??

Sol:

first finding V at P

$V = 100(x^2 - y^2)$

$V = 100(4 - 1)$

$V = 100(3)$

$\boxed{V = 300\,V}$

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⇒ Since, the conductor surface is equipotential surface the voltages at all point remains 300V

$$300 = 100\,(x^2 - y^2)$$

$$\frac{300}{100} = (x^2 - y^2)$$

$$\boxed{3 = (x^2 - y^2)}$$

$x^2 - y^2 = 3$

↳ equation of surface

⇒ Now finding Electric field intensity at P

$E = -\text{grad}V$

$$E = -\left(\frac{\partial}{\partial x} V \hat{a_x} + \frac{\partial}{\partial y} V \hat{a_y} + \frac{\partial}{\partial z} V \hat{a_z}\right)$$

$$E = -\frac{\partial}{\partial x} 100\,(x^2 - y^2) - \frac{\partial}{\partial y} 100\,(x^2 - y^2)$$

$$E = -100 \times 2x\,\hat{a_x} - 100 \times -2y\,\hat{a_y}$$

$$E = -200x\,\hat{a_x} + 200y\,\hat{a_y}$$

$$E = -200\,(2)\,\hat{a_x} + 200\,(-1)\,\hat{a_y}$$

$$\boxed{E = -400\,\hat{a_x} - 200\,\hat{a_y} \quad V/m}$$

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Now finding D at P :

$D = \epsilon_0 E$

$D = 8.85 \times 10^{-12} [-400\hat{a_x} - 200\hat{a_y}]$

$\boxed{D = -3.54\hat{a_x} - 1.77 \; nC/m^2}$

$\rho_s = D_N$

$|D| = \sqrt{(-3.54)^2 + (-1.77)^2}$

$|D| = 3.96 \; nC/m^2$

$\boxed{\rho_s = 3.96 \; nC/m^2}$

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D 5.5

SOL:-

$V = 100 \sinh 5x \sin 5y$ V
$P\ (0.1, 0.2, 0.3)$

find (a) V (b) E (c) |E|

(d) $|\rho_s|$ point P lies on a conductor surface

first finding V at P

$V = 100 \sinh 5(0.1) \sin 5(0.2)$
$\boxed{V = 43.8 \text{ V}}$

Now finding E at P:

$E = -\text{grad}\, V$

$$E = -\frac{\partial}{\partial x} 100 \sinh 5x \sin 5y\, \hat{a_x} - \frac{\partial}{\partial y} 100 \sinh 5x \sin 5y\, \hat{a_y}$$

$$E = -100\, 5\cosh 5x \sin 5y\, \hat{a_x} - 100 \sinh 5x\, 5\cos 5y\, \hat{a_y}$$

$$E = -500 \cosh 5(0.1) \sin 5(0.2)\, \hat{a_x} - 500 \sinh 5(0.1) \cos 5(0.2)\, \hat{a_y}$$

$$\boxed{E = -474.4\, \hat{a_x} - 140.7\, \hat{a_y}}$$

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$$|E| = \sqrt{(-474.4)^2 + (-140.7)^2}$$

$$\boxed{|E| = 494.825 \text{ V/m}}$$

$\rho_s = \epsilon_0 E = D_N$

$\rho_s = 494.825 \times 8.825 \times 10^{-12}$

$$\boxed{\rho_s = 4.36 \text{ nC/m}^2}$$

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⇒ ENERGY EXPENDED IN MOVING A POINT CHARGE IN AN ELECTRIC FIELD:-

⇒ Force experienced is given by

$F_E = \vec{F} \cdot \hat{a}_L$

⇒ suppose we wish to move a charge Q a distance dL in an electric field

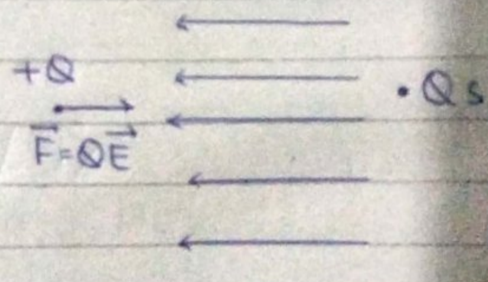
$F_E = \vec{F} \cdot \hat{a}_L$

$F_E = Q\vec{E} \cdot \hat{a}_L$ → since it is a point charge

$F_{APP} = -Q\vec{E} \cdot \hat{a}_L$ → The force that we must apply is equal and opposite to the force associated with the field.

⇒ differential work to move a point charge is

$dw = -Q\vec{E} \cdot \hat{a}_L \times |dL|$



$dw = -Q\vec{E} \cdot \vec{dL}$

$$\boxed{W = -Q\int_b^a \vec{E} \cdot \vec{dL}}$$

$\hat{a}_L$ = unit vector in the direction of dL

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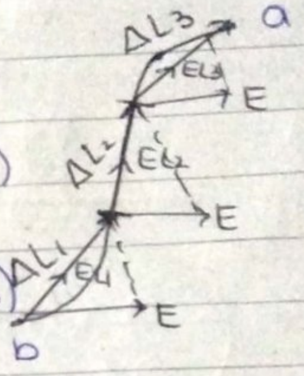
$$W = -Q \int_b^a \vec{E} \cdot \hat{a}_L \, dl$$

$$W = -Q \int_b^a E_L \, dL \Rightarrow \text{Line integral}$$



$$W = -Q \left(E_{L1} \Delta L_1 + E_{L2} \Delta L_2 + E_{L3} \Delta L_3 \right)$$

$$W = -Q \left[E_1 \cdot \Delta L_1 + E_2 \cdot \Delta L_2 + E_3 \cdot \Delta L_3 \right]$$

and because we have assumed a uniform field

$$E = E_1 = E_2 = E_3$$

$$W = -Q E \cdot \left(\Delta L_1 + \Delta L_2 + \Delta L_3 \right)$$

$$W = -Q E \cdot L_{ba}$$

$$\boxed{W = -Q \int_b^a E \cdot dL}$$

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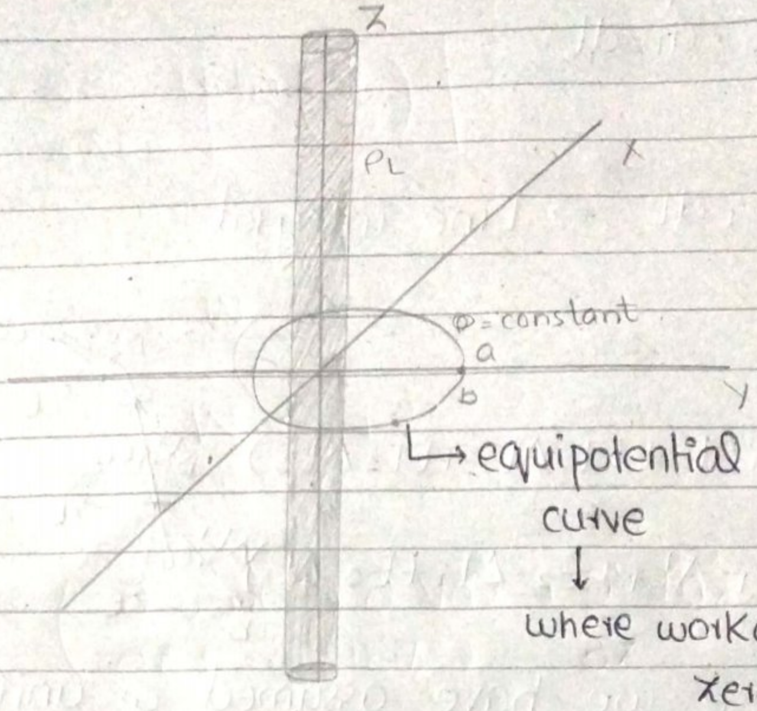
CASE : 01



↳ equipotential curve
↓
where workdone is zero

$$W = -Q\int_b^a E \cdot dL \longrightarrow (1)$$

$$E = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a_\rho}$$

$$dL = \hat{a_\rho}\cancel{d\rho}^{\,0} + \rho\, d\phi\, \hat{a_\phi} + \cancel{dz}^{\,0}\hat{a_z}$$

so eq(1) becomes,

$$W = -Q\int_b^a \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a_\rho} \cdot \rho\, d\phi\, \hat{a_\phi}$$

$$W = -Q\frac{\rho_L \cancel{\rho}}{2\pi\epsilon_0\cancel{\rho}}\int_b^a \hat{a_\rho}\cdot\hat{a_\phi}\, d\phi$$

$$\boxed{W = 0}$$

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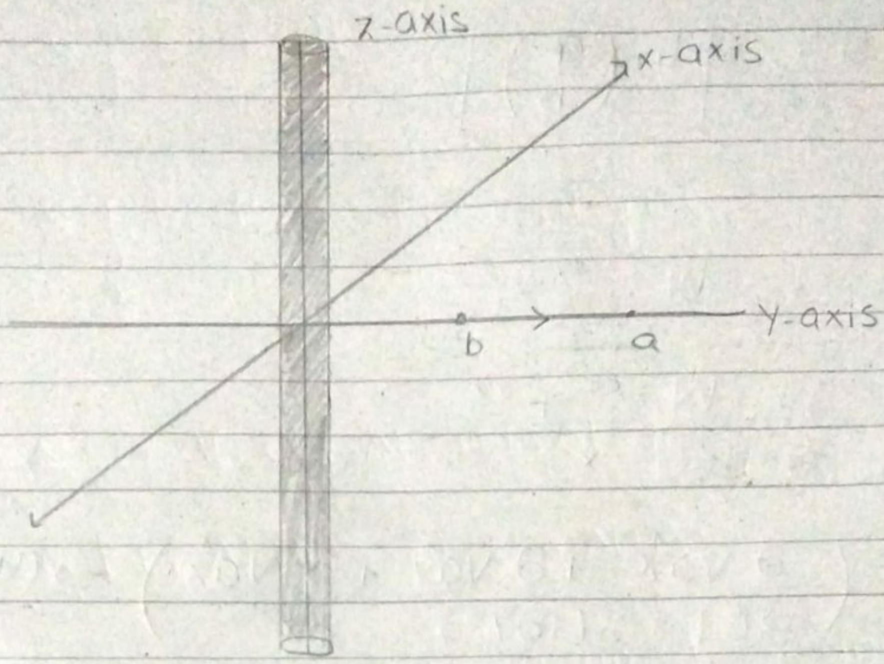
CASE: 02



$$W = -Q \int_b^a E \cdot dl \longrightarrow (1)$$

$$E = \frac{\rho_L}{2\pi\epsilon_0} \hat{a_\rho}$$

$$d_L = \partial\rho \hat{a_\rho} + \cancel{\rho \partial\phi \hat{a_\phi}}^{0} + \cancel{\partial z \hat{a_z}}^{0}$$

So eq(1) becomes,

$$W = -Q \int_b^a \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a_\rho} \cdot \partial\rho \hat{a_\rho}$$

$$W = -\frac{Q\rho_L}{2\pi\epsilon_0} \int_b^a \frac{1}{\rho} \partial\rho \Rightarrow \frac{-Q}{2\pi\epsilon_0} \rho_L \ln \rho \Big|_b^a$$

$$W = \frac{-Q\rho_L}{2\pi\epsilon_0} [\ln(a) - \ln(b)]$$

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$$\boxed{W = \frac{-QPL}{2\pi\epsilon_0}\left[\ln\left(\frac{a}{b}\right)\right]}$$

$a > b$; $W =$ " $-$ "

$b > a$; $W =$ " $+$ "

X ——— X ——— X ——— X

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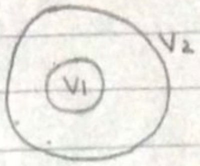
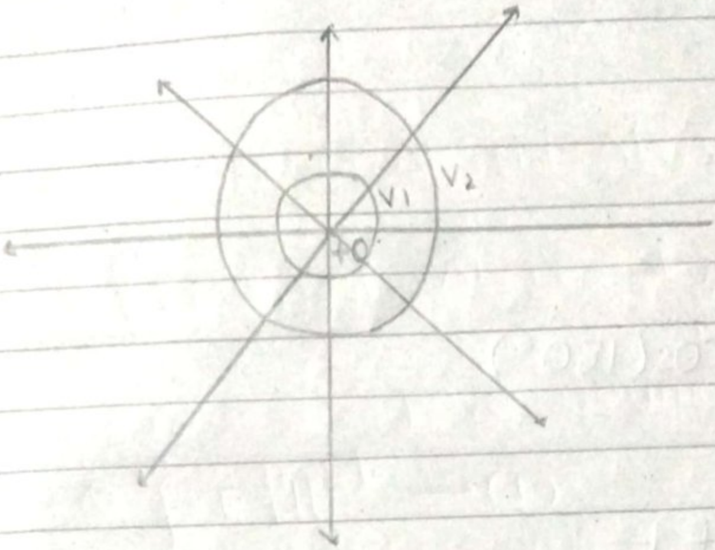
POTENTIAL GRADIENT:-





$V_1 > V_2$

* Electric field intensity is perpendicular to equipotential surface

* $\hat{a}_E$ always point in the direction of decreasing values of V.

calculating E from V:

$V = -\int E \cdot dL$

$V = -\int |E| \, dL \cos\theta$

$dV = -|E| \cos\theta \, dL$

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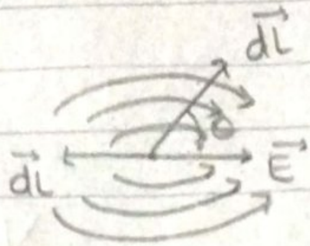
$$\frac{dV}{dl} = -|E|\cos\theta$$



If $\theta = 180°$

$$\frac{dV}{dl} = -|E| \cos(180°)$$

$$\boxed{\left.\frac{dV}{dL}\right|_{max} = |E|}$$

we know that,

$$\hat{a_E} = -\hat{a_N}$$

$$\vec{E} = |\vec{E}|\, \hat{a_E}$$

$$\vec{E} = -\frac{dV}{dL}\, \hat{a_N}$$

$$\boxed{\vec{E} = \underbrace{-\frac{dV}{dN}\, \hat{a_N}}_{\text{grad } V}}$$

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$\vec{E} = E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z$

$\vec{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$

$dV = -\vec{E}\cdot\vec{dl}$

$= -(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z)\cdot(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$

$dV = -E_x dx - E_y dy - E_z dz \longrightarrow (3)$

$dV = \frac{\partial}{\partial x} V dx + \frac{\partial}{\partial y} V dy + \frac{\partial}{\partial z} V Dz \longrightarrow (4)$

comparing eq(3) and eq(4):

$-E_x \cancel{dx} = \frac{\partial}{\partial x} V \cancel{dx}$

$E_x = -\frac{\partial}{\partial x} V$

$E_y = -\frac{\partial}{\partial y} V$

$E_z = -\frac{\partial}{\partial z} V$

$E = -\frac{\partial}{\partial x} V \hat{a}_x - \frac{\partial}{\partial y} V \hat{a}_y - \frac{\partial}{\partial z} V \hat{a}_z$

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$$E = -\left(\frac{\partial}{\partial x} V \hat{a_x} + \frac{\partial}{\partial y} V \hat{a_y} + \frac{\partial}{\partial z} V \hat{a_z}\right) \rightarrow (4)$$

we Know that,

$$\bar{\nabla} = \frac{\partial}{\partial x} \hat{a_x} + \frac{\partial}{\partial y} \hat{a_y} + \frac{\partial}{\partial z} \hat{a_z}$$

$$V\bar{\nabla} = \frac{\partial}{\partial x} V \hat{a_x} + \frac{\partial}{\partial y} V \hat{a_y} + \frac{\partial}{\partial z} V \hat{a_z}$$

so,

$$\boxed{E = -V\bar{\nabla}}$$

$E = -$ Gradient of V

$$V\bar{\nabla} = \frac{\partial}{\partial x} V \hat{a_x} + \frac{\partial}{\partial y} V \hat{a_y} + \frac{\partial}{\partial z} V \hat{a_z} \rightarrow \text{Rectangular}$$

$$V\bar{\nabla} = \frac{\partial}{\partial \rho} V \hat{a_\rho} + \frac{1}{\rho} \frac{\partial}{\partial \phi} V \hat{a_\phi} + \frac{\partial}{\partial z} V \hat{a_z} \rightarrow \text{8p cylindrical}$$

$$V\bar{\nabla} = \frac{\partial}{\partial r} V \hat{a_r} + \frac{1}{r} \frac{\partial}{\partial \theta} V \hat{a_\theta} + \frac{1}{r\sin\theta} \frac{\partial}{\partial \phi} V \hat{a_\phi} \rightarrow \text{spherical}$$

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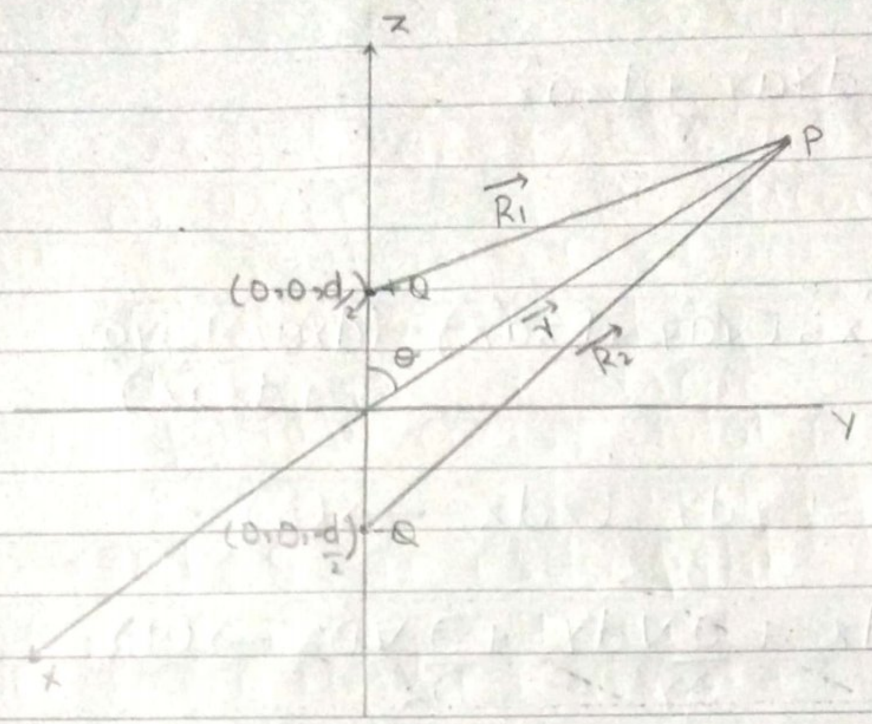
THE ELECTRIC DIPOLE :-



$$V^+ = \frac{Q}{4\pi\epsilon_0 |\vec{R_1}|} = \frac{Q}{4\pi\epsilon_0 R_1}$$

$$V^- = \frac{-Q}{4\pi\epsilon_0 R_2}$$

$$V = V^+ + V^-$$

$$V = \frac{Q}{4\pi\epsilon_0 R_1} + \frac{Q(-1)}{4\pi\epsilon_0 R_2}$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R_1} + \frac{1 \times -1}{R_2}\right]$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

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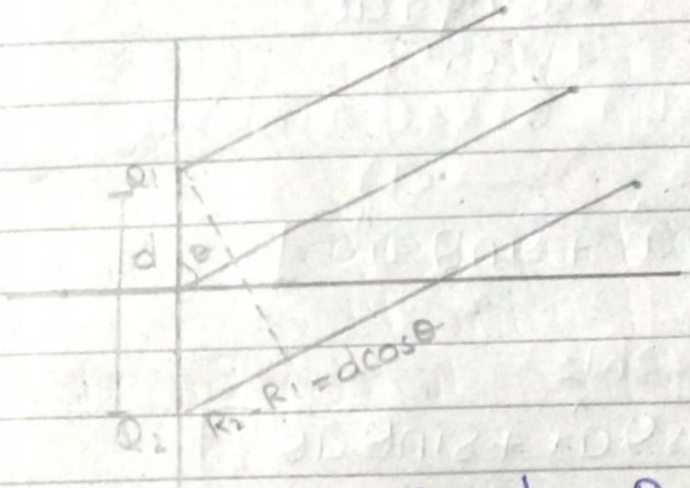
$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{R_2 - R_1}{R_1 R_2}\right] \longrightarrow (1)$$

since, $|\vec{r}| >>>>>>> d$

d is extremely small

so, $R_1 = R_2 = r$



$$R_2 - R_1 = d\cos\theta$$

so eq(1) becomes,

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{d\cos\theta}{r^2}\right]$$

$$\boxed{V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2}} \longrightarrow (2)$$

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$E = -\bar{\nabla} V$

$$= -\frac{\partial}{\partial r}\frac{Qd\cos\theta}{4\pi\epsilon_0 r^2}\hat{a_r} - \frac{1}{r}\frac{\partial}{\partial\theta}\frac{Qd\cos\theta}{4\pi\epsilon_0 r^2}\hat{a_\theta}$$

$$= -\frac{Qd\cos\theta}{4\pi\epsilon_0}\frac{-2}{r^3}\hat{a_r} - \frac{1}{r}\frac{-Qd\sin\theta}{4\pi\epsilon_0 r^2}\hat{a_\theta}$$

$$= \frac{1\cancel{2}Qd\cos\theta}{{}_2\cancel{4}\pi\epsilon_0 r^3}\hat{a_r} + \frac{Qd\sin\theta \cdot 1}{4\pi\epsilon_0 r^2 r}\hat{a_\theta}$$

$$= \frac{Qd}{2\pi\epsilon_0 r^3}\left[\cos\theta\,\hat{a_r} + \frac{\sin\theta}{2}\hat{a_\theta}\right]$$

$$\boxed{E = \frac{Qd}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{a_r} + \sin\theta\,\hat{a_\theta}\right]}$$

modifying our eq(2): $\boxed{\because P = Qd}$

$$V = \frac{Q \cdot d}{4\pi\epsilon_0 r^2}$$

$$\boxed{V = \frac{\vec{P}\cdot\hat{a_r}}{4\pi\epsilon_0 r^2}}$$

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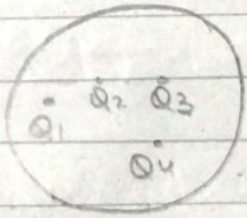
In generalized terms,

$$V = \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^3}$$

$$\boxed{V = \frac{\vec{p} \cdot (\vec{r} - \vec{r}')}{4 \times \pi \times \epsilon_0 \, |r - r'|^3}}$$

=> Energy Density In Electrostatics Field:-



=> Work to position $Q_1 = Q_1 V$

=> since we are starting our work by visualizing an empty universe. so bringing a charge Q_1 from infinity to any position requires no work.

work to position $Q_1 = 0$

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work to position $Q_2 = Q_2 V_{21}$

work to position $Q_3 = Q_3 V_{31} + Q_3 V_{32}$

work to position $Q_4 = Q_4 V_{41} + Q_4 V_{42} + Q_4 V_{43}$

Total potential energy = total positioning work of whole field.

$$W_E = Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32} + Q_4 V_{41} + Q_4 V_{42} + Q_4 V_{43} \longrightarrow (1)$$

$$Q_3 V_{31} = Q_3 \frac{Q_1}{4\pi\epsilon_0 R_{13}}$$

$$Q_3 V_{31} = Q_1 \frac{Q_3}{4\pi\epsilon_0 R_{13}}$$

$$Q_3 V_{31} = Q_1 V_{13}$$

so eq (1) becomes,

$$W_E = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23} + Q_1 V_{14} + Q_2 V_{24} + Q_3 V_{34} \longrightarrow (2)$$

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Adding eq(1) & eq(2)

$$2W_E = Q_1[\underbrace{V_{12} + V_{13} + V_{14} + \cdots}_{V_1}]$$
$$+ Q_2[\underbrace{V_{21} + V_{23} + V_{24} + \cdots}_{V_2}]$$
$$+ Q_3[\underbrace{V_{31} + V_{32} + V_{34} + \cdots}_{V_3}]$$

$$2W_E = Q_1V_1 + Q_2V_2 + Q_3V_3$$

$$2W_E = \sum_{m=1}^{n} Q_m V_m$$

$$W_E = \frac{1}{2} \sum_{m=1}^{n} Q_m V_m$$

for continous volume charge distribution.

$$W_E = \frac{1}{2} \int_{vol} \rho_v V \, dv$$

$$W_E = \frac{1}{2} \int_{vol} (\bar{\nabla} \cdot D) V \, dv \quad \because \rho_v = \bar{\nabla} \cdot D$$

$\hookrightarrow (2)$

using identity

$$\bar{\nabla} \cdot (VD) = V(\bar{\nabla} \cdot D) + D \cdot (\bar{\nabla} V)$$

$$V(\bar{\nabla} \cdot D) = \bar{\nabla} \cdot (VD) - D \cdot (V \bar{\nabla})$$

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so eq(2) becomes,

$$W_E = \frac{1}{2}\int_{vol} [\bar{\nabla}\cdot(VD) - D\cdot(V\bar{\nabla})]\, dv$$

$$= \frac{1}{2}\int_{vol} \bar{\nabla}\cdot(VD)\, dv - \frac{1}{2}\int_{vol} D\cdot(V\bar{\nabla})\, dv$$

$$\because \int_{vol} \bar{\nabla}\cdot D\, dv = \oint_{S} \bar{D}\cdot ds$$

$$= \frac{1}{2}\oint_{S} \cancelto{0}{\bar{D}\cdot ds} + \frac{1}{2}\int_{vol} D\cdot E\, dv$$

$$\boxed{W_E = \frac{1}{2}\int_{vol} D\cdot E\, dv}$$

$$\boxed{W_E = \frac{1}{2}\int_{vol} \epsilon_0 E^2\, dv}$$

$$\boxed{dW_E = \frac{1}{2} D\cdot E\, dv}$$

$$\boxed{\frac{dW_E}{dv} = \frac{1}{2} D\cdot E}$$

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CHAPTER: 05

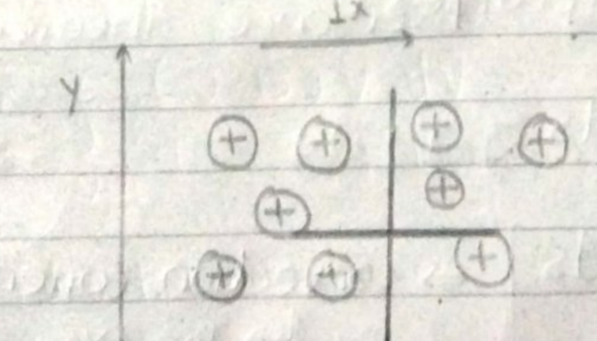
=> CURRENTS AND CONDUCTORS:

CURRENT:-

=> Electric charges in motion constitute a current. The unit of current is amphere. It is defined as the Rate of movement of charges passing a given reference point (or crossing a given reference plane) of one coloumb per second.

$$I = \frac{dq}{dt}$$ => Rate of change of charges constitute a current.

=> current is a scalar quantity.



Current Density:

⇒ The increment of current ΔI crossing an incremental surface ΔS normal to the current density is:

$$J_N = \frac{\Delta I}{\Delta S} \Rightarrow \Delta I = J_N \Delta S$$

⇒ current density is normal to the surface.

⇒ When current density is not perpendicular to ΔS

$$\Delta I = J \cdot \Delta S$$

$$\lim_{\Delta S \to 0} \Delta I = \lim_{\Delta S \to 0} J \cdot \Delta S \longrightarrow (1)$$

$$\Delta I = dI$$
$$\Delta S = dS$$

so eq (1) becomes,

$$dI = J \cdot dS$$

$$\boxed{I = \int_S J \cdot dS}$$ ⇒ convection current.

↳ The type of current in which charges are moving

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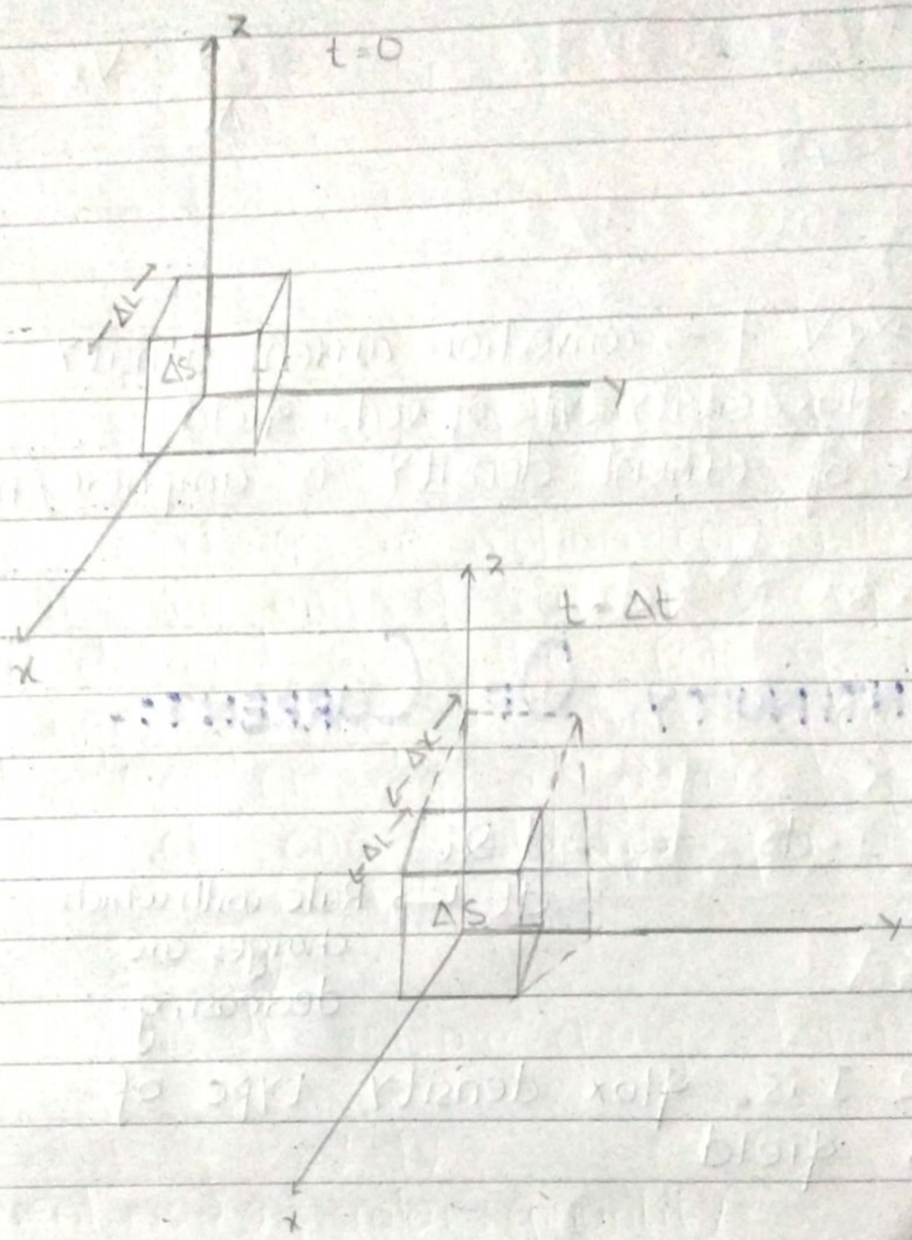

let ΔQ_i be the charge inside the cube of volume ΔV

$$\Delta Q_i = \rho_v \Delta V = \rho_v \Delta S \Delta L$$

$$\Delta Q = \rho_v \Delta S \Delta x$$

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta S \Delta x}{\Delta t}$$

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$$\Delta I = \rho_v \Delta S V_x \qquad \because \frac{\Delta x}{\Delta t} = V_x$$

$$\frac{\Delta I}{\Delta S} = \rho_v V$$

$\boxed{J = \rho_v V}$ ⇒ convection current density

↳ is flux density type of vector field.

⇒ unit of current density is amphere/m^2

⇒ CONTINUITY OF CURRENT:-

$$I = \oint_S J \cdot ds = -\frac{dQ_i}{dt}$$

↳ Rate with which charges are decreasing.

⇒ since J is flux density type of vector field

⇒ so applying divergence theorem,

$$\oint_S J \cdot ds = \int_{vol} (\bar{\nabla} \cdot J)\, dv$$

Volume

$+Q_i$

$\rho_v(t)$

$$\Rightarrow \int_{vol} \bar{\nabla} \cdot J \, dv = -\frac{d}{dt} \int_{vol} \rho_v \, dv$$

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$$\int_{VOL} \bar{\nabla}\cdot J \, dv = \int_{VOL} -\frac{\partial}{\partial t} \rho_v \, dv$$

$$\boxed{\bar{\nabla}\cdot J = -\frac{\partial}{\partial t}\rho_v}$$ ⇒ continuity equation.

D: 5.1

$J = 10\rho^2 z \, \hat{a_\rho} - 4\rho\cos^2\phi \, \hat{a_\phi} \ mA/m^2$

$J_P = ??$ $P(\rho = 3, \phi = 30°, z = 2)$ (a)

(b) $I = ??$ $\rho = 3, 0 < \phi < 2\pi,$
$2 < z < 2.8$

Sol:-

(a)

$J_P = 10(9)(2)\hat{a_\rho} - 4\times(3)\times[\cos 30°]^2 \hat{a_\phi}$

$J_P = 180\hat{a_\rho} - 9\hat{a_\phi} \ mA/m^2$

(b)

$I = \oint_S J\cdot ds$

$I = \int_S (10\rho^2 z \hat{a_\rho} - 4\rho\cos^2\phi \hat{a_\phi})\cdot \rho \, d\phi \, dz \, \hat{a_\rho}$

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$$I = \int_{2}^{2\cdot8}\int_{0}^{2\pi} 10\rho^3 z \, d\phi \, dz$$

$$I = \int_{2}^{2\cdot8}\int_{0}^{2\pi} 10(3)^3 z \, d\phi \, dz$$

$$I = 270 \int_{2}^{2\cdot8} \phi \Big|_{0}^{2\pi} z \, dz$$

$$I = 270 \times 2\pi \int_{2}^{2\cdot8} z \, dz$$

$$I = 540\pi \, \frac{z^2}{2} \Big|_{2}^{2\cdot8}$$

$$\boxed{I = 3\cdot26A}$$

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D 5.2

$J = -10^6 z^{1.5} a_z \ A/m^2$

(a) $I = ??$ $\quad z = 0.1m$

(b) $v = 2\times10^6 \ m/s$ at $z = 0.1m$
$\rho_v = ??$

(c) $\rho_v = -2000 \ C/m^3$
$z = 0.15m$
$v = ??$

$$I = \int_S J \cdot ds$$

$$= \int_S (-10^6 z^{1.5} a_z) \cdot ds_z$$

$$= \int_0^{2\pi}\int_0^{20\times10^{-6}} (-10^6 z^{1.5} \hat{a}_z) \cdot \rho \, d\rho \, d\phi \, \hat{a}_z$$

$$= \int_0^{2\pi}\int_0^{20\times10^{-6}} -10^6 (0.1)^{1.5} \rho \, d\rho \, d\phi$$

$$= -31622.7766 \int_0^{2\pi} \frac{\rho^2}{2} \Big|_0^{20\times10^{-6}} d\phi$$

$$= -15811.3883 \times 4\times10^{-10} \times 2\pi$$

$$\boxed{I = -39.7 \ \mu A}$$

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b) $P_v = ??$

$J = P_v \times V$

$$P_v = \frac{J}{V} = \frac{-10^6 (0.1)^{1.5}}{2 \times 10^6}$$

$\boxed{P_v = -15.8 \text{ m } C/m^3}$

(C) $J = P_v \times V$

$$V = \frac{J}{P_v} = \frac{-10^6 (0.15)^{1.5}}{-2000}$$

$\boxed{V = 29.0 \text{ m/s}}$

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