FORMULA SHEET

CHP: 04

⇒ DIFFERENTIAL WORK:

$$dw = -Q\vec{E}\cdot\vec{dl}$$

$$\vec{dL} = \hat{a}_L \times |\vec{dL}|$$

⇒ LINE INTEGRAL

$$\boxed{W = -Q\int_{init}^{final}\vec{E}\cdot\vec{dl}}$$

⇒ $dL = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$ → Rectangular

⇒ $dL = d\rho\,\hat{a}_\rho + \rho\,d\phi\,\hat{a_\phi} + dz\,\hat{a_z}$ → cylindrical

⇒ $dL = dr\,\hat{a}_r + r\,d\theta\,\hat{a_\theta} + r\sin\theta\,d\phi\,\hat{a}_\phi$
↳ spherical

⇒ POTENTIAL DIFFERENCE:

$$V_{AB} = -\int_B^A E\cdot dL \Rightarrow \boxed{V = \frac{W}{Q}}$$

$$V = -\int_{init}^{final} E\cdot dL$$

$$V_{AB} = V_A - V_B \rightarrow B \rightarrow A$$

=> POTENTIAL FIELD OF A POINT CHARGE:

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

=> ELECTRIC FIELD INTENSITY:

$$E = -\nabla V$$

$$\text{grad } V = \frac{\partial}{\partial x} V \hat{a}_x + \frac{\partial}{\partial y} V \hat{a}_y + \frac{\partial}{\partial z} V \hat{a}_z$$ → Rectangular

$$\text{grad } V = \frac{\partial}{\partial \rho} V \hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} V \hat{a}_\phi + \frac{\partial}{\partial z} V \hat{a}_z$$
↳ cylindrical

$$\text{grad } V = \frac{\partial}{\partial r} V \hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta} V \hat{a}_\phi + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} V \hat{a}_\phi$$
↳ spherical

$$\frac{dV}{dN} = |\overline{\nabla} V|$$

Scanned with CamScanner

$$E = -\frac{dV}{dN}\,\hat{a_N}$$

$$\hat{a_N} = \frac{-E}{dV/dN}$$

Scanned with CamScanner

$F = \frac{Q}{\epsilon}$

$E = \frac{F}{Q}$

$F =$

$F = QE \cdot a_L dl \quad -QE \cdot dl$

4.1

$Q \cdot a_L \times dl$ $\boxed{dW = -QE \cdot dl}$

$Q \cdot dl$

$$E = \frac{1}{z^2}\left(8xyz\,\hat{a}_x + 4x^2z\,\hat{a}_y - 4x^2y\,\hat{a}_z\right) \text{ V/m}$$

$Q = 6\,nC$

$dw = ??$

$|dL| = 2\,\mu m$

$$a_L = \frac{-6}{7}\hat{a}_x + \frac{3}{7}\hat{a}_y + \frac{2}{7}\hat{a}_z$$

(a) $a_L \times |dL| = dL$

$$dL = \frac{-6}{7} \times 2\,\hat{a}_x + \frac{3}{7} \times 2\,\hat{a}_y + \frac{2}{7} \times 2\,\hat{a}_z$$

$$dL = \frac{-12}{7}\hat{a}_x + \frac{6}{7}\hat{a}_y + \frac{4}{7}\hat{a}_z$$

$$dw = -QE \cdot dL$$

$$= -6 \times 10^{-9}\left[\frac{8xy}{z}\hat{a}_x \cdot \frac{-12}{7}\hat{a}_x + \frac{4x^2z}{z^2}\hat{a}_y \cdot \frac{6}{7}\hat{a}_y - \frac{4x^2y}{z^2}\hat{a}_z \cdot \frac{4}{7}\hat{a}_z\right] \times 10^{-6}$$

Scanned with CamScanner

$$dw = -6\times10^{-9}\left[-\frac{96}{7}\frac{xy}{z} + \frac{24}{7}\frac{x^2}{z} - \frac{16}{7}\frac{x^2y}{z^2}\right]$$

$$dw = -6\times10^{-9}\left[-\frac{96}{7}\frac{2x-2}{3} + \frac{24}{7}\frac{4}{3} - \frac{16}{7}\frac{4x-2}{9}\right]$$

$$\boxed{dw = -149.3\,fJ}$$

$dw = -Q$

(b) $\frac{6}{7}\hat{a_x} - \frac{3}{7}\hat{a_y} - \frac{2}{7}\hat{a_z}$

Sol:-

$\vec{dL} = \hat{a_L} \times dL$

$= \left[\frac{6}{7}\hat{a_x} - \frac{3}{7}\hat{a_y} - \frac{2}{7}\hat{a_z}\right] 2\times10^{-6}$

$= \frac{12}{7}\hat{a_x} - \frac{3\times2}{7}\hat{a_y} - \frac{4}{7}\hat{a_z}$ um $\times10^{-6}$

$= \frac{12}{7}\hat{a_x} - \frac{6}{7}\hat{a_y} - \frac{4}{7}\hat{a_z}$ $\times10^{-6}$

$W = -Q\, E \cdot dL$

$= -6\times10^{-9}\left[\frac{8xy}{z}\hat{a_x} \cdot \frac{12}{7}\hat{a_x} - \frac{6}{7}\hat{a_y} \cdot \frac{4x^2}{z}\hat{a_y} - \frac{4}{7}\hat{a_z} \cdot -\frac{4x^2y}{z^2}\hat{a_z}\right]$

$$= -6\times10^{-9}\left[\frac{96}{7}\frac{xy}{z} - \frac{24}{7}\frac{x^2}{z} + \frac{16}{7}\frac{x^2y}{z^2}\right]$$

$$= -6\times10^{-9}\left[\frac{96}{7}\frac{2x-2}{3} - \frac{24}{7}\frac{4}{3} + \frac{16}{7}\frac{4x-2}{9}\right]$$

$$= -6\times10^{-9}\left[\frac{-384}{21} - \frac{96}{21} - \frac{128}{63}\right]$$

$$\boxed{w = 149.3\,fJ}$$

(c) $\frac{3}{7}\hat{ax} + \frac{6}{7}\hat{ay}$

<u>Sol:-</u>

$$dL = \hat{a_L} \times |\vec{dL}|$$

$$= \frac{3}{7}\hat{ax} + \frac{6}{7}ay \times 2\times10^{-6}$$

Scanned with CamScanner

$$= \frac{6}{7}\hat{a_x} + \frac{12}{7}\hat{a_y} \text{ Um}$$

$$dW = -Q\, E \cdot dL$$

$$= -6\times10^{-9}\left[\frac{8xy}{z}\hat{a_x}\cdot\frac{6}{7}\hat{a_x} + \frac{4x^2}{z}\hat{a_y}\cdot\frac{12}{7}\hat{a_y}\right]$$

$$= -6\times10^{-9}\left[\frac{48}{7}\frac{xy}{z} + \frac{48}{7}\frac{x^2}{z}\right]$$

$$= -6\times10^{-9}\left[\frac{48\times2\times-2}{7\times3} + \frac{48}{7}\frac{4}{3}\right]$$

$$\boxed{dw = 0\,J}$$

Scanned with CamScanner

Example: 4.1

$E = y a_x + x a_y + 2 a_z$

$Q = 2C$

$B\ (1, 0, 1)$ to $A\ (0.8, 0.6, 1)$

$\Rightarrow \quad W = -Q \int_B^A E \cdot dL$

$W = -2 \int_B^A (y a_x + x a_y + 2 a_z) \cdot (dx\, \hat{a}_x + dy\, \hat{a}_y + dz\, \hat{a}_z)$

$= -2 \int_B^A y\, dx + \int_B^A x\, dy + \int_B^A 2 dz$

$= -2 \int_1^{0.8} y\, dx + \int_0^{0.6} x\, dy + 2 \int_1^1 dz$

$= \because \quad x^2 + y^2 = 1, \quad z = 1$

$y = \sqrt{1 - x^2}$

$\because \int \sqrt{1-x^2} = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}a^2 \sin^{-1}\left(\frac{x}{a}\right) + c$

$\because \int \sqrt{a^2-x^2} = \frac{1}{2} x \sqrt{a^2-x^2} + \frac{1}{2} a^2 \sin^{-1}\left(\frac{x}{a}\right) + c$

$$= -\frac{2}{2} \int_1^{0.8} \sqrt{1-x^2}\, dx + \int_0^{0.6} \sqrt{1-y^2}\, dy + 0$$

$$= -\frac{2}{2} \left[x\sqrt{1-x^2} + \sin^{-1} x\right]\Big|_1^{0.8} + y\sqrt{1-y^2} + \sin^{-1} y \Big|_0^{0.6}$$

$$= -\frac{2}{2} \left[0.8\sqrt{1-0.8^2} + \sin^{-1}(0.8)\right] - \left[1\sqrt{1-1^2} + \sin^{-1}(1)\right]$$

$$+ \left[0.6\sqrt{1-0.6^2} + \sin^{-1}(0.6) - 0 - \sin^{-1}(0)\right]$$

$$= -\frac{2}{2} [0.48 + 0.927 - 1.570 + 0.48 + 0.643]$$

$$= -0.96\, J$$

Scanned with CamScanner

Example 4.2

$W = ?$
$Q = 2C$

$B(1, 0, 1)$ to $A(0.8, 0.6, 1)$
init ——— final

$$y - y_1 = m(x - x_1)$$
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{0.6 - 0}{0.8 - 1}(x - 1)$$

$$\Rightarrow y = -3(x - 1) \quad \Rightarrow -\frac{y}{3} + 1 = x$$

$$W = -Q \int_{init}^{final} E \cdot dL$$

$$W = -2 \int_{init}^{final} (y a_x + x a_y + 2 a_z) \cdot (dx\, a_x + dy\, a_y + dz\, a_z)$$

Scanned with CamScanner

$$W = -2 \int_{init}^{final} y\,dx + \int_{init}^{final} x\,dy + \int_{init}^{final} 2\,dz$$

$$W = -2 \left[\int_{1}^{0.8} -3(x-1)\,dx + \int_{0}^{0.6} 1 - \frac{y}{3}\,dy + \int_{1}^{1} 2\,dz \right]$$

$$W = -2 \left[-\frac{3}{2}x^2 + 3x \Big|_{1}^{0.8} + y - \frac{y^2}{6} \Big|_{0}^{0.6} \right]$$

$$W = -2 \left[-\frac{3}{2}(0.8)^2 + 3(0.8) + \frac{3}{2}(1)^2 - 3(1) \right]$$

$$+ \left[0.6 - \frac{0.6^2}{6} - 0 + 0 \right]$$

$$W = -2 \left[0.48 \right] \Rightarrow \boxed{W = -0.96\,J}$$

D: 4.2

$Q = 4C$

$B(1,0,0)$ to $A(0,2,0)$

$y = 2 - 2x \,,\, z = 0$

(a) $E = 5a_x$ V/m

$$\vec{E} \cdot \vec{dl} = 5\hat{a_x} \cdot (dx\hat{a_x} + dy\hat{a_y} + dz\hat{a_z})$$

$$= 5dx$$

$$W = -Q\int_{init}^{final} E \cdot dl$$

$$= -Q\int_{1}^{0} 5dx$$

$$= -4\int_{1}^{0} 5dx$$

$$= -4 \times 5x \Big|_{1}^{0}$$

$$= -4 \;\; 5[0-1]$$

Scanned with CamScanner

$W = 20 J$ //

(b) $5x \hat{a_x}$

$$\vec{E} \cdot \vec{dl} = 5x \hat{a_x} \cdot (dx \hat{a_x} + dy \hat{a_y} + dz \hat{a_z})$$

$$= 5x\,dx$$

$$W = -Q \int_{init}^{final} E \cdot dl$$

$$= -4 \int_{1}^{0} 5x\,dx$$

$$= -\overset{2}{\cancel{4}}\ \frac{5}{\underset{1}{\cancel{2}}}\ x^2 \Big|_1^0$$

$$= -10\ [0^2 - 1^2]$$

$$= -10\ [-1]$$

$$= \cancel{2}\ 10 J$$

Scanned with CamScanner

(c) $5x\,a\hat{x} + 5y\,a\hat{y}$ V/m

$\vec{E}\cdot\vec{dl} = (5x\,a\hat{x} + 5y\,a\hat{y}) \cdot (dx\,a\hat{x} + dy\,a\hat{y} + dz\,a\hat{z})$

$= 5x\,dx + 5y\,dy$

$$W = -Q\int_{init}^{final} E\cdot dl$$

$$= -4\left[\int_1^0 5x\,dx + \int_0^2 5y\,dy\right]$$

$$= -4\,\frac{5}{2}\,x^2\Big|_1^0 + \frac{5}{2}\,y^2\Big|_0^2 \times -4$$

$$= -10\,[-1] + \frac{5}{2}\,[4]^2 \times -4$$

$= \cancel{20} + \cancel{10}\;\; 10 - 40$

$= \cancel{30J}\;\; -30J$

Scanned with CamScanner

D: 4.3

$E = y a_x$ V/m
$Q = 3C$

(a)

$\vec{E} \cdot \vec{dl} = (y \hat{a_x} \cdot dx \hat{a_x} + dy \hat{a_y} + dz \hat{a_z})$

$= y\,dx$

$$W = -Q \int_{init}^{final} \vec{E} \cdot \vec{dl}$$ (1,3,5) to (2,3,5)

$$= -3 \int_1^2 y\,dx$$

$$= -3 \int_1^2 3\,dx$$

$$= -3 \times 3 \int_1^2 dx$$

$$= -9 \times \Big|_1^2 = -9J$$

Scanned with CamScanner

$(1,0,3)$ to $(2,0,3)$

(b) $W = -Q\int_{init}^{final} E \cdot dL$

$= -3\int_1^2 y\,dx$

$= -3\int_1^2 0\,dx$

$= 0J$.

D. 4.4

$E = 6x^2\hat{a_x} + 6y\hat{a_y} + 4\hat{a_z}$ V/m

(a) $V_{MN} = ?$ $M(2,6,-1)$
$N(-3,-3,2)$

Scanned with CamScanner

$$\boxed{V = \frac{W}{Q}}$$

$$V_{MN} = -\int_N^M E \cdot dL \longrightarrow (1)$$

$$= E \cdot dl$$

$$= (6x^2 \hat{a_x} + 6y \hat{a_y} + 4 \hat{a_z}) \cdot (dx \hat{a_x} + dy \hat{a_y} + dz \hat{a_z})$$

$$= 6x^2 dx + 6y dy + 4 dz$$

$$= -\int_N^M 6x^2 dx + 6y \cdot dy + 4 dz$$

$$= -\left[\int_{-3}^{2} 6x^2 dx + \int_{-3}^{6} 6y dy + \int_{2}^{-1} 4 dz \right]$$

$$= -\left[\frac{6}{3} x^3 \Big|_{-3}^{2} + \frac{6}{2} y^2 \Big|_{-3}^{6} + 4z \Big|_{2}^{-1} \right]$$

$$= -\left[\frac{6}{3}\left[2^3-(-3)^3\right]\right]+3\left[6^2-(-3)^2\right]$$

$$+4[-1-2]$$

$$= -\left[\frac{6}{3}\left[8+27\right]\right]+3\left[36-9\right]$$

$$+4[-3]$$

$$= -70-81+12$$

$$= \boxed{-139V}$$

(b) $V_M = ??$

$V_Q = 0$

$$V_{MQ} = -\int_Q^M E\cdot dl$$

$$= - \int_{4}^{2} 6x^2\,dx - \int_{-2}^{6} 6y\,dy - \int_{-35}^{-1} 4\,dz$$

$$= - \frac{\cancel{6}^{2}}{\cancel{3}_{1}} x^3 \Big|_4^2 - \frac{6}{2} y^2 \Big|_{-2}^{6} - 4z \Big|_{-35}^{-1}$$

$$= -2\,[8-64] - 3\,[36-4] - 4\,[-1+35]$$

$$= -2\,[-56] - 3\,[32] - 4\,[34]$$

$$= 112 - 96 - 136$$

$$\boxed{= -120\,V} = V_{MQ}$$

$$V_{MQ} = V_M - V_Q$$

$$V_{MQ} = V_M - 0$$

$$\boxed{V_M = -120V}$$

Scanned with CamScanner

(C) $V_N = ??$

$V_P = 2$

$$V_{NP} = -\int_P^N E \cdot dL$$

$$= -\int_1^{-3} 6x^2 dx - \int_2^{-3} 6y\,dy - \int_{-4}^{2} 4\,dz$$

$$= -\frac{6}{3} x^3 \Big|_1^{-3} - \frac{6}{2} y^2 \Big|_2^{-3}$$

$$-4z \Big|_{-4}^{2}$$

$$= -2\left[(-3)^3 - (1)^3\right] - 3\left[(-3)^2 - 2^2\right]$$

$$-4\left[2 - (-4)\right]$$

$$= -2[-27-1] - 3(9-4)$$

Scanned with CamScanner

$- 4 [2+4]$

$= -2[-28] - 3(5) - 4(6)$

$= 56 - 15 - 24$

$= 17 = V_{NP}$

$V_{NP} = V_N - V_P$

$V_{NP} + V_P = V_N$

$17 + 2 = V_N$

$\boxed{V_N = 19V}$

Scanned with CamScanner

Q: 4.5

Sol:-

$Q = 15nC$

$V_P = ??$ $(-2, 3, -1)$

$V = 0$ $(6, 5, 4)$

$V_Q = 0$

$$V_{PQ} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_p} - \frac{1}{r_Q}\right]$$

$$= \frac{15\times10^{-9}}{4\times\pi\times8.85\times10^{-12}}\left[\frac{1}{\sqrt{14}} - \frac{1}{\sqrt{77}}\right]$$

$$\boxed{V_{PQ} = 20.676 \text{ volts}}$$

$V_{PQ} = V_P - V_Q$

$V_{PQ} = V_P$

$$\boxed{V_P = 20.676 \text{ Volts}}$$

Scanned with CamScanner

(b)

$$V_{P\infty} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_p} - \frac{1}{r_\infty}\right]$$

$$= \frac{15\times10^{-9}}{4\times\pi\times8.85\times10^{-12}}\left[\frac{1}{\sqrt{14}} - \frac{1}{\infty}\right]$$

$$= \frac{15\times10^{-9}}{4\times\pi\times8.85\times10^{-12}}\left[\frac{1}{\sqrt{14}}\right]$$

$$= 36.0V$$

(C) $V = 5V$ at $(2, 0, 4)$

$V_M = 5V$

$$V_{PM} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_p} - \frac{1}{r_M}\right]$$

$$= \frac{15\times10^{-9}}{4\times\pi\times8.85\times10^{-12}}\left[\frac{1}{\sqrt{14}} - \frac{1}{2\sqrt{5}}\right]$$

Scanned with CamScanner

$V_{PM} = 5.887\,V$

$V_{PM} = V_P - V_M$

$V_P = V_{PM} + V_M$

$V_P = 5.887 + 5$

$\boxed{V_P = 10.887\,V}$

$\Rightarrow$ Example 4.4

$V = 2x^2 y - 5z$

$V_P = ??$ $(-4, 3, 6)$
$E = ??$
$a_N = ??$
$D = ??$
$\rho_V = ??$

Scanned with CamScanner

$$V \text{ at } p = 2x^2 y - 5z$$

$$= 2(-4)^2(3) - 5(6)$$

$$= 2(16)(3) - 30$$

$$= 96 - 30$$

$$= 66\,V$$

$$E = -\nabla V$$

$$= -\left(\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z} \right)$$

$$= -\left(\frac{\partial}{\partial x}(2x^2 y - 5z)\hat{a_x} + \frac{\partial}{\partial y}(2x^2 y - 5z)\hat{a_y} \right.$$

$$\left. + \frac{\partial}{\partial z}(2x^2 y - 5z)\hat{a_z} \right)$$

$$= -\left(4xy\,\hat{a_x} + 2x^2\,\hat{a_y} - 5\,\hat{a_z} \right)$$

Scanned with CamScanner

$E =$

$E = -4(-4)(3)\,\hat{a_x} - 2(-4)^2\hat{a_y} + 5\hat{a_z}$

$$\boxed{E = 48\hat{a_x} - 32\hat{a_y} + 5\hat{a_z}\ V/m}$$

$D = \epsilon_0 E$

$D = -4 \times 8.85 \times 10^{-12}\, xy\, \hat{a_x}$
$-2 \times 8.85 \times 10^{-12} x^2 \hat{a_y} + 5 \times 8.85 \times 10^{-12} \hat{a_z}$

$$\boxed{D = -35.4\, xy\, \hat{a_x} - 17.7x^2\hat{a_y} + 44.25\,\hat{a_z}\ pC/m^2}$$

$\rho_v = \bar{\nabla} \cdot D$

$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial}{\partial x} -35.4xy + \frac{\partial}{\partial y} -17.7x^2 + \frac{\partial}{\partial z} 44.25$$

$$= -35.4y + 0 + 0$$

$$= -35.4 \times (3)$$

$$\boxed{\rho_v = -106.2 \, pC/m^3}$$

D: 4.8

$V = \frac{100}{z^2+1} \rho \cos\phi \ V$

V at P = ?? $(\rho = 3m, \phi = 60^\circ, z = 2m)$

E = ??

|E| = ??

$\frac{dV}{dN}$ = ??

a_N = ??

ρ_V = ??

$V = \frac{100}{4+1} 3 \cos 60^\circ \ V$

$\boxed{V = 30V}$

Scanned with CamScanner

(z^2+1)
$-1(z^2+1)^{-2}$

$$E = -\nabla \overline{V}$$

$$= -\left[\frac{\partial V}{\partial \rho}\hat{a_\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a_\phi} + \frac{1\partial V}{\partial z}\hat{a_z} \right]$$

$$= -\left[\frac{\partial}{\partial \rho}\frac{100}{z^2+1}\rho\cos\phi\,\hat{a_\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}\frac{100}{z^2+1}\rho\cos\phi\,\hat{a_\phi} \right.$$

$$\left. + \frac{\partial}{\partial z}\frac{100}{z^2+1}\rho\cos\phi\,\hat{a_z} \right]$$

$$= -\left[\frac{100}{z^2+1}\cos\phi\,\hat{a_\rho} + \frac{1}{\cancel{\rho}}\frac{100\cancel{\rho}}{z^2+1} - \sin\phi\,\hat{a_\phi} \right.$$

$$\left. + 100\rho\cos\phi\frac{\partial}{\partial z}\frac{1}{z^2+1}\hat{a_z} \right]$$

$$= -\left[\frac{100}{4+1}\times 3\cos 60^\circ\,\hat{a_\rho} + \frac{100}{4+1} - \sin 60^\circ\,\hat{a_\phi} \right.$$

$$\left. + 100\times 3\times\cos 60^\circ\frac{-1}{(z^2+1)^2}2z\,\hat{a_z} \right]$$

$$\boxed{E = -10\hat{a_\rho} + 10\sqrt{3}\,\hat{a_\phi} + 24\hat{a_z}\ V/m}$$

$|E| = 31.24\ V/m$

$$\frac{dV}{dN} = |\nabla \bar{V}|$$

$$\frac{dV}{\partial N} = 31.24\ V/m$$

$$\vec{E} = -\frac{\partial V}{dN} \times \hat{a_N}$$

$$E = -31.24\ \hat{a_N}$$

$$\hat{a_N} = \frac{E}{-31.24}$$

$$\hat{a_N} = \frac{-10}{-31.24}\hat{a_\rho} + \frac{10\sqrt{3}}{-31.24}\hat{a_\phi} + \frac{24}{-31.24}\hat{a_z}$$

$$\boxed{\hat{a_N} = 0.320\,\hat{a_\rho} - 0.554\,\hat{a_\phi} - 0.768\,\hat{a_z}}$$

$$D = \epsilon_0 E$$

$$= \left[\frac{-100}{z^2+1}\cos\phi \times 8.85\times10^{-12}\,\hat{a_\rho} + \frac{100}{z^2+1}\sin\phi\,\hat{a_\phi} \times 8.85\times10^{-12} + 100\rho\cos\phi\,\frac{2z}{(z^2+1)^2}\times 8.85\times10^{-12}\,\hat{a_z}\right]$$

$$= -8.85\times10^{-10}\,\frac{\cos\phi}{z^2+1}\,\hat{a_\rho} + 8.85\times10^{-10}\,\frac{\sin\phi}{z^2+1}\,\hat{a_\phi} + 1.77\times10^{-9}\,\frac{\rho\cos\phi\, z}{(z^2+1)^2}\,\hat{a_z}$$

C/m^2

Scanned with CamScanner

$\rho_v = \bar{\nabla} \cdot D$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{-8.85 \times 10^{-10} \cos\phi}{z^2+1}$$

$$+ \frac{1}{\rho} \frac{\partial}{\partial \phi} \frac{8.85 \times 10^{-10} \sin\phi}{z^2+1}$$

$$+ \frac{\partial}{\partial z} \frac{1.77 \times 10^{-9} \rho \cos\phi \, z}{(z^2+1)^2}$$

$$= \frac{1}{\rho} \frac{-8.85 \times 10^{-10} \cos\phi}{z^2+1} + \frac{1}{\rho(z^2+1)} 8.85 \times 10^{-10} \cos\phi + 1.77 \times 10^{-9} \rho \cos\phi - \left[\frac{3z^2 - 1}{(z^2+1)^3}\right]$$

$$= \frac{-8.85 \times 10^{-10}}{3 \times 5} \cos 60^\circ + \frac{1}{3(5)} 8.85 \times 10^{-10} \cos 60^\circ$$

$$- 1.77 \times 10^{-9} \, 3 \cos 60^\circ \left[\frac{3 \times 4 - 1}{(5)^3}\right]$$

Scanned with CamScanner

$\boxed{Q = -233 pC}$

4.5

$G = 2y a_x$

(a) A(1, -1, 2) to B(1, 1, 2) to P(2, 1, 2)

$$\Rightarrow \int_A^P G \cdot dl$$

$$\Rightarrow \int_A^P 2y a_x \cdot (dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z})$$

$$\Rightarrow \int_A^P 2y\, dx$$

(1, 1, 2) → (2, 1, 2)

$$= \int_1^2 2y\, dx$$

$$= \int_1^2 2\, dx$$

$$= 2x \Big|_1^2$$

$$= 2(2-1)$$

$$= 2$$

(b) $$\int_A^P G \cdot dl$$

$$= \int_A^P 2y a_x \cdot (dx a_x + dy a_y + dz a_z)$$

$$= \int_1^2 2y \, dx \qquad (1,-1,2) \rightarrow (2,-1,2)$$

$$= 2(-1) \int_1^2 dx$$

$$= -2 \; x \Big|_1^2$$

$$= -2(1)$$

$$= -2$$

4.7

$G = 3xy^2 a_x + 2z a_y$ $\quad P(2,1,1)$
$\quad Q(4,3,1)$

(a) $y = x - 1$
$z = 1$

$$\Rightarrow \int_{init}^{final} G \cdot dl \longrightarrow (1)$$

$\Rightarrow G \cdot dl$

$$\Rightarrow (3xy^2 a_x + 2z a_y) \cdot (dx\, a_x + dy\, a_y + dz\, a_z)$$

$$= 3xy^2 dx + 2z\, dy$$

$$= \int_2^4 3xy^2 dx + 2z \int_1^3 dy$$

$$= \int_2^4 3x(x-1)^2 dx + 2(1) \int_1^3 dy$$

$$\Rightarrow \int_2^4 3x(x^2+1-2x)\,dx + 2\int_1^3 dy$$

$$= \int_2^4 3x^3+3x-6x^2\,dx + 2\int_1^3 dy$$

$$= \frac{3}{4}x^4 + \frac{3}{2}x^2 - \frac{6}{3}x^3 \Big|_2^4 + 2y\Big|_1^3$$

$$= \left[\frac{3}{4}(4)^4 + \frac{3}{2}(4)^2 - 2(4)^3 - \frac{3}{4}(2)^4 - \frac{3}{2}(2)^2 + 2(2)^3\right] + 2[3-1]$$

$$= \left[86+4\right] = 90$$

Scanned with CamScanner

(b) $6y = x^2 + 2$, $z = 1$

$\Rightarrow G \cdot dl$

$\Rightarrow (3xy^2 a_x + 2z a_y) \cdot (dx a_x + dy a_y + dz a_z)$

$\Rightarrow 3xy^2 dx + 2z dy + 0$

$\Rightarrow 3xy^2 dx + 2z dy$

$\Rightarrow \int_2^4 3xy^2 dx + 2z \int_1^3 dy$

$\Rightarrow \int_2^4 3x \left[\frac{x^2+2}{6}\right]^2 dx + 2z \int_1^3 dy$

$\Rightarrow \int_2^4 \cancel{3}x \left[\frac{x^4 + 4 + 4x^2}{\cancel{36}\ 12}\right] dx + 2\int_1^3 dy$

Scanned with CamScanner

$$\Rightarrow \int_2^4 \frac{x^4}{12}\cdot x + \frac{1}{3}x + \frac{4}{12}x^2\cdot x\,dx + 2\int_1^3 dy$$

$$\Rightarrow \frac{1}{12}\frac{x^5}{5} + \frac{1}{3}x + \frac{1}{3}\frac{x^3}{3}\Bigg|_2^4 + 2y\Bigg|_1^3$$

$$= \frac{1}{60}x^5 + \frac{1}{3}x + \frac{1}{9}x^3\Bigg|_2^4 + 2(3-1)$$

$$= \frac{1}{60}(4)^5 + \frac{1}{3}(4) + \frac{1}{9}(4)^3 - \frac{1}{60}(2)^5 - \frac{1}{3}(2) - \frac{1}{9}(2)^3 + 2(2)$$

$$= \int_2^4 \frac{x^5}{12} + \frac{1}{3}x + \frac{4}{12}x^3\,dx + 2\int_1^3 dy$$

$$= \frac{x^6}{12\times 6} + \frac{1}{3}\frac{x^2}{2} + \frac{\cancel{4}}{12\times\cancel{4}}x^4\Bigg|_2^4 + 2y\Bigg|_1^3$$

$$\Rightarrow \frac{x^6}{72} + \frac{x^2}{6} + \frac{x^4}{12} \Big|_2^4 + 2(3-1)$$

$$= \left[\frac{(4)^6}{72} + \frac{(4)^2}{6} + \frac{(4)^4}{12} - \frac{(2)^6}{72} - \frac{(2)^2}{6} - \frac{(2)^4}{12} \right] + 2(2)$$

$$= 78 + 4$$

$$= 82$$

Scanned with CamScanner

$\oint D_s / \frac{ds_r}{\partial s_p}$ / $Q = \int \rho_s ds$

4.9

$\rho_s = 20 nC/m^2$

$r = 0.6 cm$

$V = ??$ at $P(r = 1cm, \theta = 25°, \phi = 50°)$.

$$\rho_s = \frac{Q}{\Delta S}$$

$$\rho_s = \frac{Q}{4\pi r^2}$$

$$20 \times 10^{-9} \times 4 \times \pi \times \left(\frac{0.6}{100}\right)^2 = Q$$

$$\boxed{Q = 9.047 pC}$$

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{9.047 \times 10^{-12}}{4 \times \pi \times 8.85 \times 10^{-12} \times \frac{1}{100}}$$

$$= 8.134 V$$

(b) $V_{AB} = ??$

$A(r = 2cm, \theta = 30°, \phi = 60°)$ &

$B(r = 3cm, \theta = 45°, \phi = 90°)$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$= \frac{9.047 \times 10^{-12}}{4 \times \pi \times 8.85 \times 10^{-12}}\left[\frac{1}{2/100} - \frac{1}{3/100}\right]$$

$$\boxed{V_{AB} = 1.36V}$$

Scanned with CamScanner

4.21

Sol:

$V = 2xy^2z^3 + 3\ln(x^2+2y^2+3z^2)$

at $P(3,2,-1)$

(a) V

(b) $|V|$

(c) E

(d) $|E|$

(e) a_N

(f) D

$V = 2xy^2z^3 + 3\ln(x^2+2y^2+3z^2)$

$V = 2(3)(4)(-1) + 3\ln(9+2(4)+3(1))$

$V = -24 + 3\ln(9+8+3)$

$V = -24 + 3\ln(20)$

$\boxed{V = -15.01 V}$ $(3, 2, -1)$

$\boxed{|V| = 15.01 V}$

$E = -\bar{\nabla} V$

$= -\left(\frac{\partial}{\partial x} V \hat{a_x} + \frac{\partial}{\partial y} V \hat{a_y} + \frac{\partial}{\partial z} V \hat{a_z} \right)$

$= -\left(\frac{\partial}{\partial x} \left[2xy^2z^3 + 3\ln(x^2 + 2y^2 + 3z^2) \right] \hat{a_x} \right.$

$+ \frac{\partial}{\partial y} \left[2xy^2z^3 + 3\ln(x^2 + 2y^2 + 3z^2) \right] \hat{a_y}$

$+ \frac{\partial}{\partial z} \left[2xy^2z^3 + 3\ln(x^2 + 2y^2 + 3z^2) \right] \hat{a_z}$

$= -\left[2y^2z^3 + \frac{3}{x^2 + 2y^2 + 3z^2} \cdot 2x \; \hat{a_x} \right.$

Scanned with CamScanner

$$+ 4xyz^3 + \frac{3}{x^2+2y^2+3z^2} 4y \, \hat{a_y}$$

$$+ 6xy^2z^2 + \frac{3}{x^2+2y^2+3z^2} 6z \, \hat{a_z} \Big]$$

$$= -\Big[2 \times 4 \times -1 + \frac{3}{9+2(4)+3(1)} \times 2(3) \quad \hat{a_x}$$

$$+ 4(3)(2)(-1) + \frac{3 \times 4 \times (2)}{9+8+3} \quad \hat{a_y}$$

$$+ 6(3)(4)(1) + \frac{3}{9+8+3} \times 6 \times -1 \; \hat{a_z} \Big]$$

$$= -\left[-8 + \frac{18}{20} \right] a_x - \left[-24 + \frac{24}{20} \right] \hat{a_y}$$

$$- \left[72 - \frac{18}{20} \right] \hat{a_z}$$

$$= 7.1 \, \hat{a_x} + 22.8 \, \hat{a_y} - 71.1 \, \hat{a_z} \; V/m$$

Scanned with CamScanner

$$|\vec{E}| = \sqrt{(7.1)^2 + (22.8)^2 + (-71.1)^2}$$

$$|\vec{E}| = 75.00 \text{ V/m}$$

$$a_N = \frac{-\vec{E}}{dV/dN} \longrightarrow (1)$$

$$\frac{dV}{dN} = |\nabla \bar{V}|$$

$$= 75.00$$

$$= -\frac{7.1}{75.00}\hat{a}_x - \frac{22.8}{75.00}\hat{a}_y + \frac{71.1}{75.00}\hat{a}_z$$

Scanned with CamScanner

$= -0.094\,\hat{a}_x \quad 0.304\,\hat{a}_y + 0.948\,\hat{a}_z$

$D = \epsilon_0 E$

$= 7.1 \times 8.85 \times 10^{-12}\,\hat{a}_x$
$\quad + 22.8 \times 8.85 \times 10^{-12}\,\hat{a}_y \quad 71.1 \times 8.85 \times 10^{-12}\,\hat{a}_z$

$= 62.8\,\hat{a}_x + 201.7\,\hat{a}_y - 629.2\,\hat{a}_z$ pC/m^2

Scanned with CamScanner

4.23

$V = 80\rho^{0.6}\ V$

$E = ??$

$\rho_v = ??\ \cancel{0.5m} \rightarrow \boxed{\rho = 0.5m}$

$Q = ??$ $\qquad \rho = 0.6,\ 0 < z < 1$

$$E = -\bar{\nabla} V$$

$$= -\left(\frac{\partial}{\partial \rho} V \hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} V \hat{a}_\phi + \frac{\partial}{\partial z} V \hat{a}_z \right)$$

$$= -\frac{\partial}{\partial \rho} 80\rho^{0.6}\ \hat{a}_\rho$$

$$= -48\rho^{-0.4}\ \hat{a}_\rho\ \ V/m$$

Scanned with CamScanner

$D = \epsilon_0 E$

$D = 8.85 \times 10^{-12} \times -48 \times \rho^{-0.4}\ \hat{a}_\rho$

$D = -4.248 \times 10^{-10}\ \rho^{-0.4}\ \hat{a}_\rho\ C/m^2$

$\rho_v = \bar{\nabla} \cdot D$

$= \frac{1}{\rho} \frac{\partial}{\partial \rho}\ \rho(-4.248 \times 10^{-10} \rho^{-0.4})$

$+ \frac{1}{\rho} \frac{\partial}{\partial \phi}\ 0 + \frac{\partial}{\partial z}\ 0$

$= \frac{1}{\rho} -4.248 \times 10^{-10}\ \frac{\partial}{\partial \rho}\ \rho^{0.6}$

$= \frac{1}{\rho} -4.248 \times 10^{-10}\ 0.6\ \rho^{-0.4}$

$= -2.548 \times 10^{-10}\ \frac{\rho^{-0.4}}{\rho}$

$$= -2.548 \times 10^{-10} \frac{(0.5)^{-0.4}}{0.5}$$

$$\boxed{\rho_v = -672\ pC/m^3}$$

$$Q = \int_v \rho_v \, dv$$

$$= \int_0^1 \int_0^{2\pi} \int_0^{0.6} -672 \times 10^{-12}\, \rho \, d\rho \, d\phi \, dz$$

$$= -672 \times 10^{-12} \int_0^1 \int_0^{2\pi} \int_0^{0.6} \rho \, d\rho \, d\phi \, dz$$

$$= -672 \times 10^{-12} \int_0^1 \int_0^{2\pi} \frac{\rho^2}{2} \Big|_0^{0.6} d\phi \, dz$$

$$= -3.36 \times 10^{-10} \int_0^1 \int_0^{2\pi} [0.6^2 - 0] \, d\phi \, dz$$

Scanned with CamScanner

$$D = \frac{\psi}{\Delta S}$$

$$\psi = D \cdot \Delta S$$

$$Q = \left(-4.248 \times 10^{-10} \times 0.6^{-0.4}\right) \times 2\pi r h$$

$$Q = \left(-4.248 \times 10^{-10} \times 0.6^{-0.4}\right) \times 2 \times \pi \times 0.6 \times 1$$

$$\boxed{Q = -1.96\,nC}$$