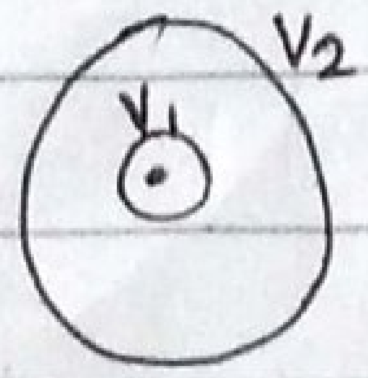
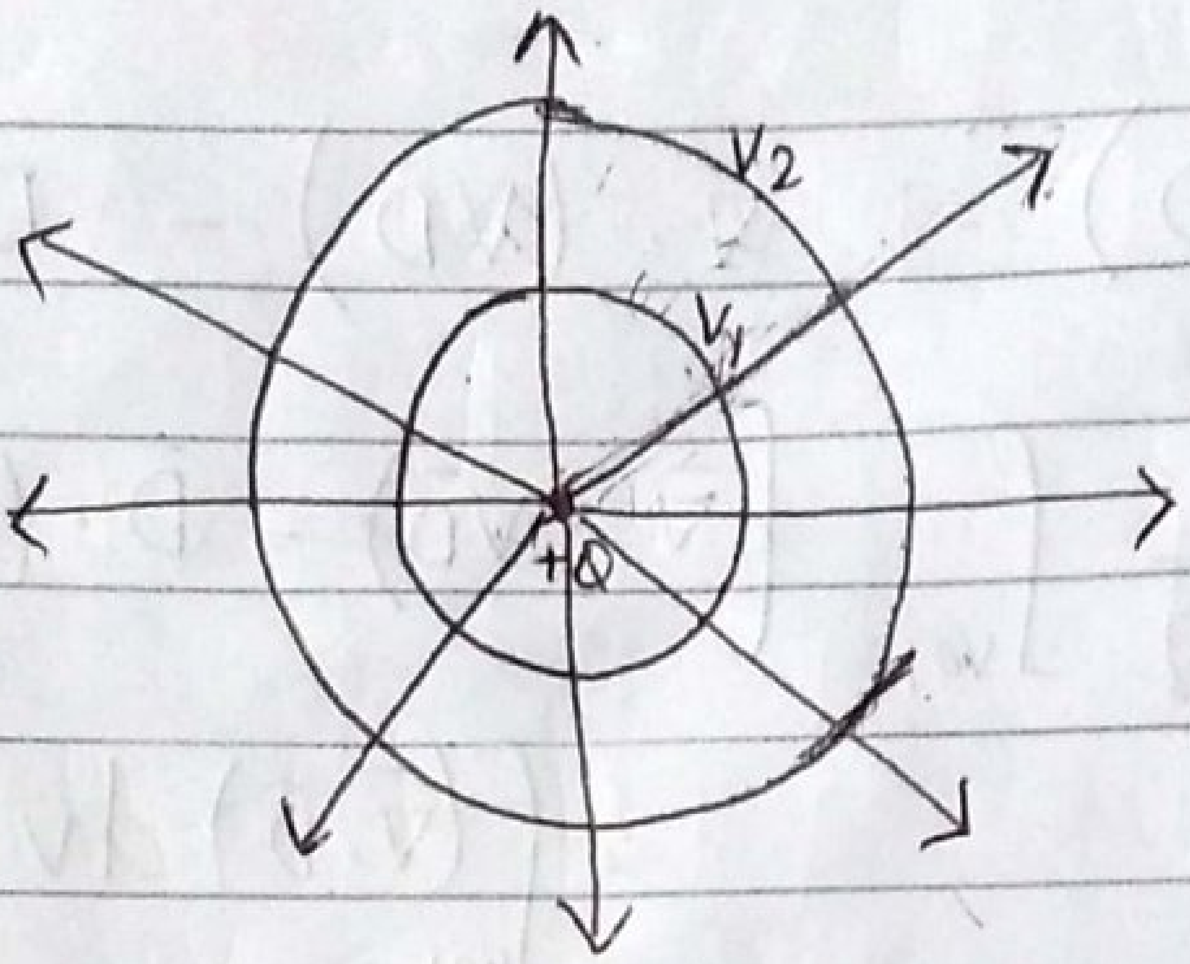
POTENTIAL GRADIANT



$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$V_1 > V_2$



1) Electric field intensity $\perp$ Equipotential surface
2) $\hat{a}_E$ always points in the direction of decreasing values of V.

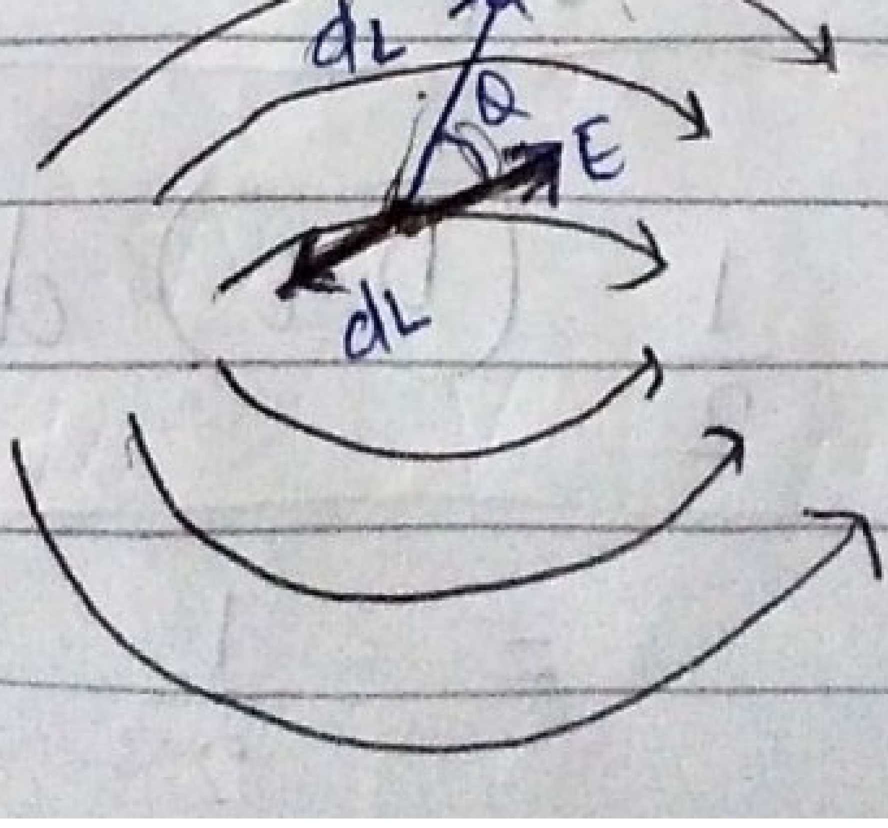
Calculating $\vec{E}$ from V:

$$\underline{V} = -\int \bar{E} \cdot \bar{dL}$$

$$V = -\int |E| \cos Q \, dL$$

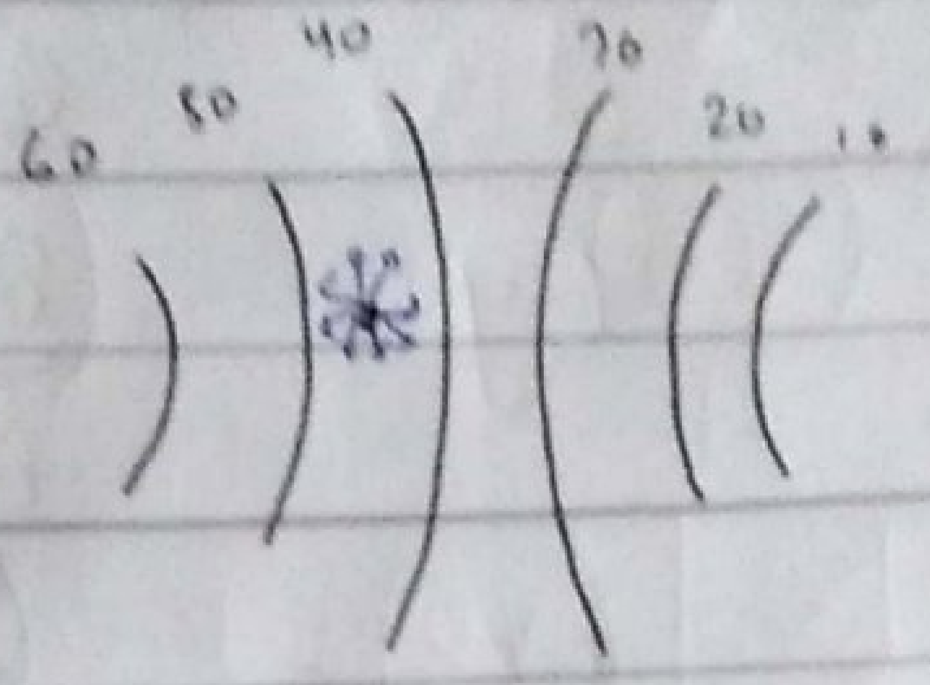
$$dV = -|E| \cos Q \, dL$$

$$\frac{dV}{dL} = -|E| \cos Q$$



If $\theta = \pi = 180^\circ \Rightarrow \cos\theta = -1$

$$\frac{dV}{dL}\bigg|_{max} = |\bar{E}|$$



$$\hat{a}_N = -\hat{a}_E$$
$$\hat{a}_E = -\hat{a}_N$$

$$\bar{E} = |E|\,\hat{a}_E$$
$$\bar{E} = -\frac{dV}{dL}\bigg|_{max}\hat{a}_N$$
$$\boxed{\bar{E} = -\underbrace{\frac{dV}{dN}\,\hat{a}_N}_{\text{grad } V}}$$

$$\boxed{E = -\text{grad } V}$$

$$E = E_x a_x + E_y a_y + E_z a_z$$
$$\boxed{dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz} \quad \text{—(A)}$$
$$d\bar{L} = dx\,a_x + dy\,a_y + dz\,a_z$$

$$dV = -\bar{E}\cdot d\bar{L}$$
$$= -(E_x a_x + E_y a_y + E_z a_z)\cdot(dx\,a_x + dy\,a_y + dz\,a_z)$$

$$\boxed{dV = -E_x\,dx - E_y\,dy - E_z\,dz} \quad \text{(B)}$$

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$\nabla \cdot V$

∇V

Comparing eq (A) and (B)

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$\bar{E} = -\frac{\partial V}{\partial x} a_x - \frac{\partial V}{\partial y} a_y - \frac{\partial V}{\partial z} a_z$$

$$\boxed{\bar{E} = -\left(\frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} a_z\right)}$$

$$\boxed{\text{grad } V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z}$$

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

$$\bar{E} = -\nabla V$$

$\boxed{\bar{E} = -\text{grad } V}$ proved!

Red

Rectangular:

$$\boxed{\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z}$$

Cylindrical:

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z}$$

Spherical:

$$\boxed{\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta} \frac{\partial V}{\partial \phi} a_\phi}$$

— x — x — x — x — x —

$$\boxed{dW = -Q\bar{E} \cdot d\bar{L}} \quad \text{— (i)}$$

$$\boxed{W = -Q \int_{ini}^{final} E \cdot dL} \quad \text{— (ii)}$$

$$\boxed{W = \frac{Q\rho_L}{2\pi\epsilon_0} \ln \frac{b}{a}} \quad \text{— (iii)}$$

$$\boxed{V_{AB} = -\int_B^A \bar{E} \cdot d\bar{L}} \quad \text{— (iv)}$$

$$\boxed{V_{AB} = \frac{\rho_L}{2\pi\epsilon_0} \ln \frac{b}{a}} \quad \text{— (v)}$$

$$\boxed{V_{AB} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)} \quad \text{— (vi)}$$

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}} \quad \text{— (vii)}$$

$$\boxed{V_{AB} = V_A - V_B} \quad \text{— (viii)}$$

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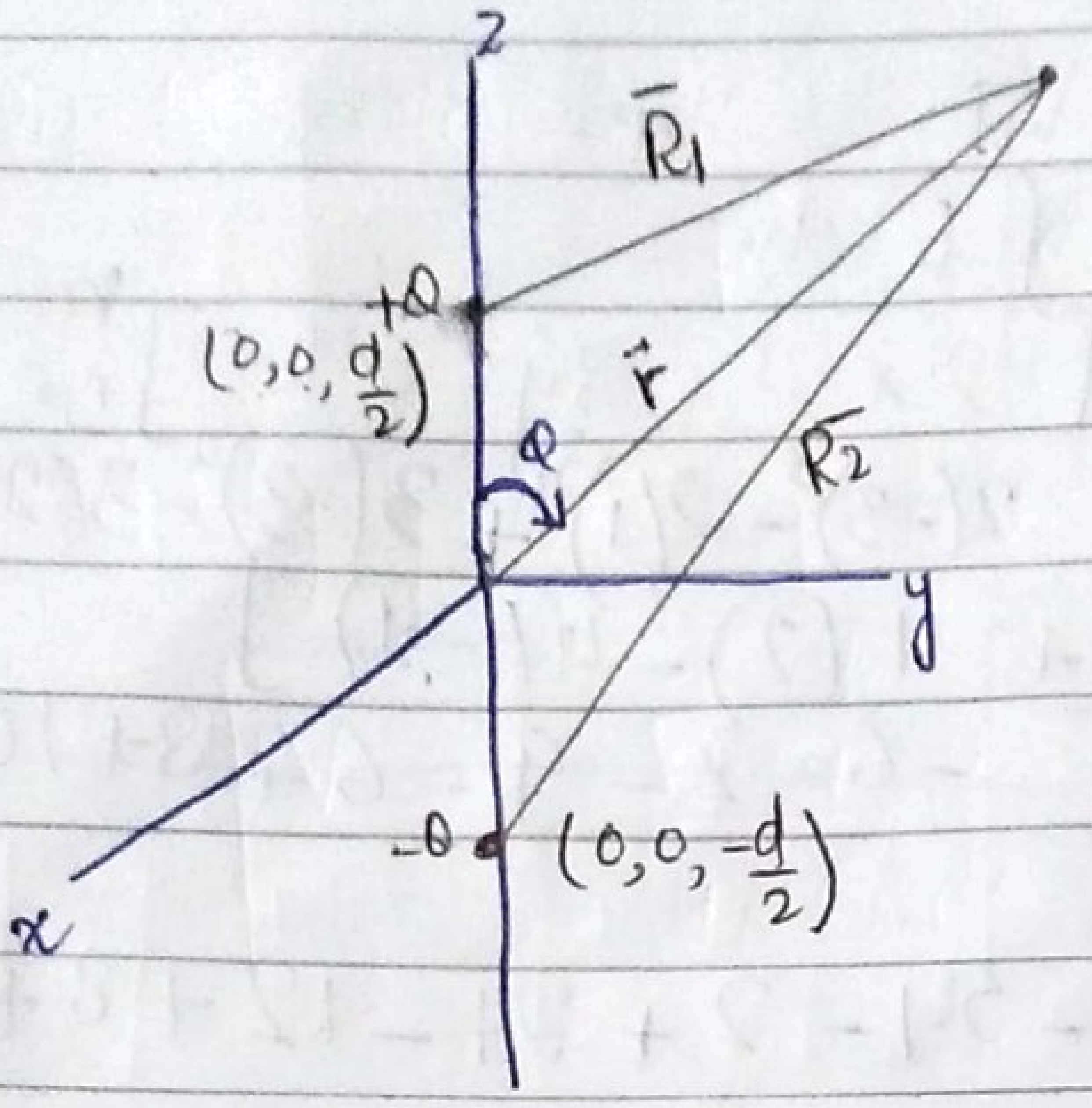
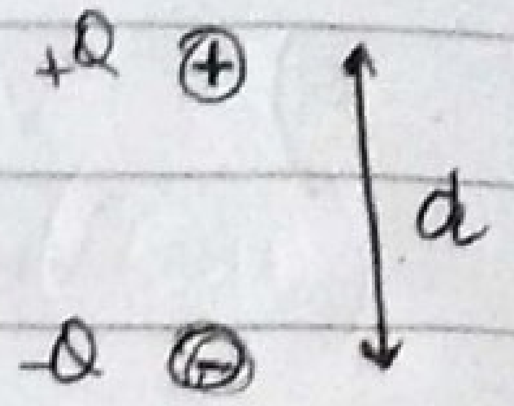
$F = \frac{kq_1q_2}{r^2} \cdot a_r$

The Electric Dipole





d is extremely small

$$V_+ = \frac{+Q}{4\pi\epsilon_0 R_1}$$

$$V_- = \frac{-Q}{4\pi\epsilon_0 R_2}$$

$$V = V_+ + V_-$$

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

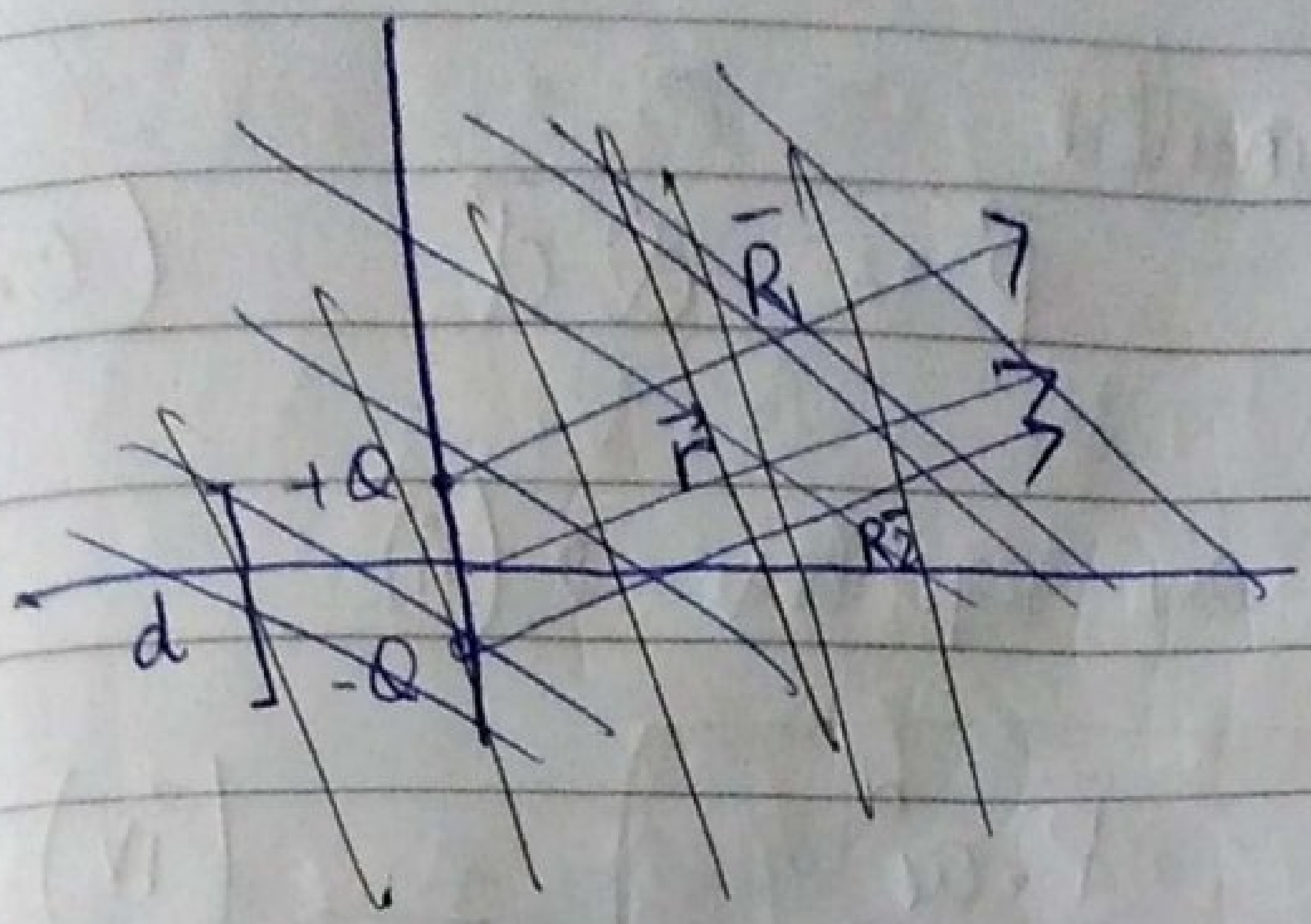
$$\boxed{V = \frac{Q}{4\pi\epsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)} \quad \text{—(i)}$$

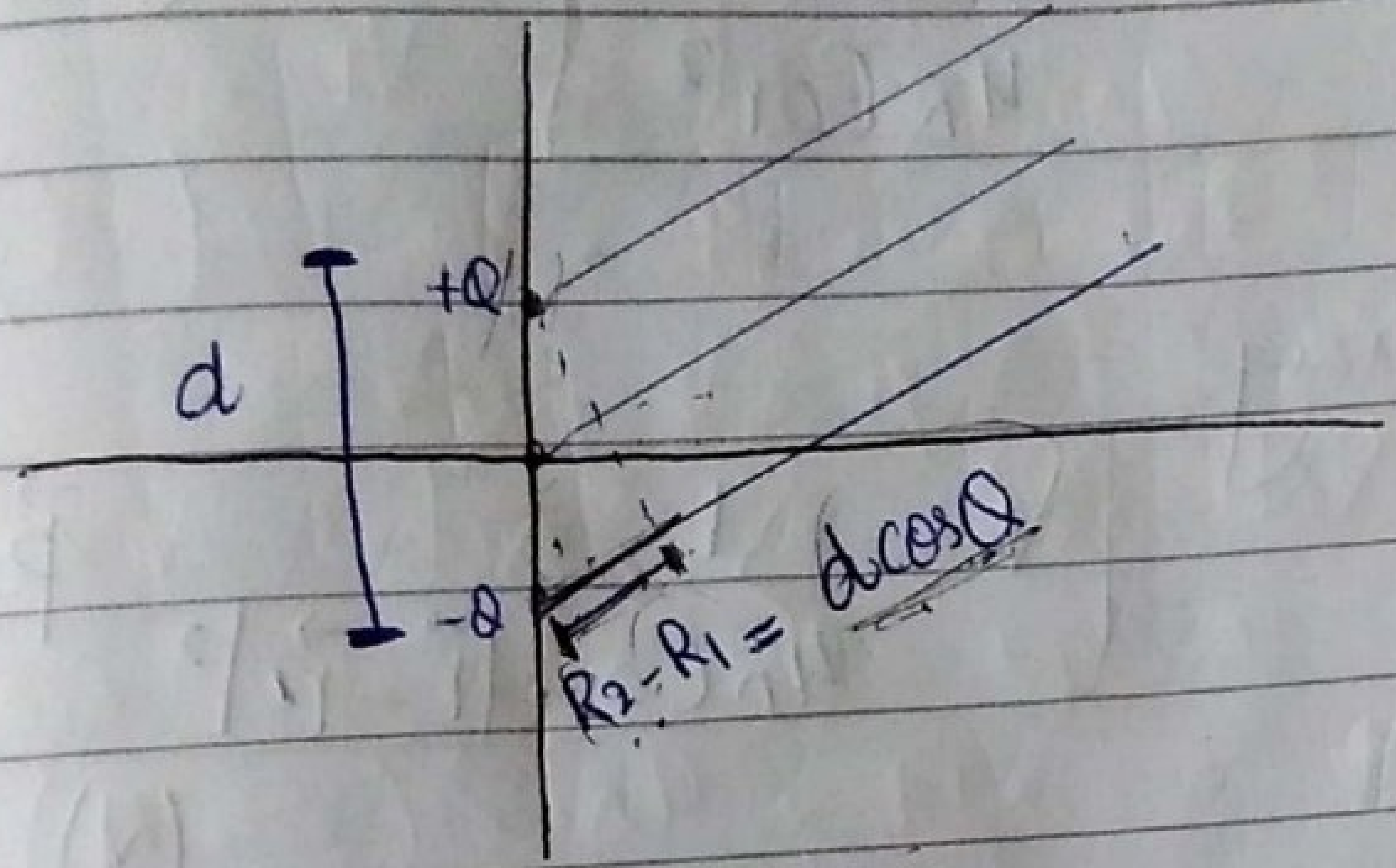
Since $\underbrace{|\vec{r}|}_{10^{-3}m} >>>>>>>>>> \underbrace{d}_{10^{-9}m}$

So,

$$R_1 = R_2 = r$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) \Rightarrow \frac{\partial}{\partial r}\left(r^{-2}\right) \Rightarrow -2\, r^{-3} \Rightarrow \frac{-2}{r^3}$$





$$\boxed{V = \frac{Q\, d \cos\theta}{4\pi \epsilon_0 r^2}} \quad \text{— (ii)}$$

$$E = -\nabla V$$

$$= -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right)$$

$$= -\left[\frac{\partial}{\partial r}\left(\frac{Q d \cos\theta}{4\pi\epsilon_0 r^2}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{Q d\cos\theta}{4\pi\epsilon_0 r^2}\right)\hat{a}_\theta\right]$$

$$= -\left[\frac{-2 Q d \cos\theta}{2\pi\epsilon_0 r^3}\hat{a}_r - \frac{Q d}{4\pi\epsilon_0 r^3}\sin\theta\, \hat{a}_\theta\right]$$

$$\boxed{E = \frac{Q d \cancel{\cos}}{4\pi\epsilon_0 r^3}\left[2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta\right]} \quad \text{(iii)}$$

b)

Dipole moment:

$$\vec{p} = Q\vec{d} \quad (C.m)$$

Modifying our equation (ii)

$$\boxed{V = \frac{\bar{P} \cdot \hat{a_r}}{4\pi \epsilon_0 r^2}} \quad \text{— (iv)}$$

Generalizing,

$$\boxed{V = \frac{1}{4\pi \epsilon_0 \cdot |\vec{r} - \vec{r'}|^2} \; \vec{p} \cdot \frac{\vec{r} - \vec{r'}}{|\vec{r} - \vec{r'}|}} \quad \text{(v)}$$

Drill 4.9)

a) $V = \frac{\bar{p} \cdot \hat{a_r}}{4\pi \epsilon_0 r^2}$

$= \frac{3a_x - 2a_y + a_z}{4\pi \epsilon_0 r^2} \cdot \hat{a_r}$

$r = 2a_x + 3a_y + 4a_z$

$\vec{r} = \sqrt{29}$

$V = \frac{3a_x - 2a_y + a_z}{4\pi \epsilon_0 (\sqrt{29})^3} \cdot 2a_x + 3a_y + 4a_z$

$= 57.5\,M\,[6 - 6 + 4] \times 10^{-9}$

$= 0.23\ V$

D

b) $r = 2.5$, $\theta = 30°$, $\phi = 40°$

$x = r \sin\theta \cos\phi$

$x = 0.957$

$y = r \sin\theta \sin\phi$

$y = 0.803$

$z = r\cos\theta$

$z = 2.165$

$\vec{r} = 0.957 a_x + 0.803 a_y + 2.165 a_z$

$|\vec{r}| = 2.5$

$$V = \frac{(3a_x - 2a_y + a_z)\,n \cdot (0.957 a_x + 0.803 a_y + 2.165 a_z)}{4\pi\epsilon_0 (2.5)^3}$$

$= 575.2\,M\,[2.871 - 1.606 + 2.165] \times 10^{-9}$

$= 1.97\ V$

Drill 4.10)

a) $x = 1.368$

$y = 0$

$z = 3.758$

$\vec{r} = 1.368 a_x + 3.758 a_z$

$|\vec{r}| = 3.83$

$$V = \frac{6 a_z (n) \cdot (1.368 a_x + 3.758 a_z)}{4\pi\epsilon_0 (3.83)^3}$$

$$V = 160.048M \times 22.548n$$

$$V = 3.4\ V$$

b) $\cancel{E = -\nabla V}$

$\cancel{= -(}$

$$E = \frac{Qd}{4\pi\epsilon_0 r^3}\left[2\cos\theta\, a_r + \sin\theta\, a_r\right]$$

Since we dont know the value of (Qd) so will calculate it using this formula:

$$V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2} \Rightarrow Qd = \frac{V \times 4\pi\epsilon_0 r^2}{\cos\theta}$$

$$Qd = \frac{3.17 \times 4\pi\epsilon_0 (4)^2}{\cos(20)}$$

$$Qd = 6.0054\ n\ Cm$$

$$E = \frac{6.0054n}{4\pi\epsilon_0 (4)^3}\left[2\cos 20\, a_r + \sin 20\, a_\theta\right]$$

$$= 0.84\left[2\cos 20\, a_r + \sin 20\, a_\theta\right]$$

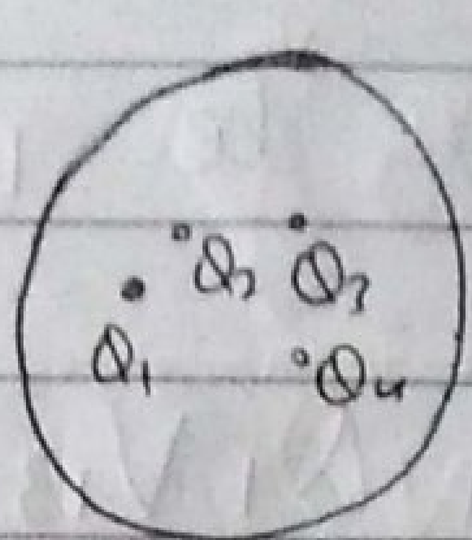
$$= 1.585\, a_r + 0.288\, a_\theta$$

$$-\nabla V$$

$$= -\frac{\partial}{\partial x}\hat{a}_x - \frac{\partial}{\partial y}\hat{a}_y - \frac{\partial}{\partial z}\hat{a}_z$$

z

ENERGY DENSITY IN ELECTROSTATIC FIELD



$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Work to position $Q_1 = Q_1 V$

Since we are starting our work by visualizing an empty universe. Since bringing a charge Q_1 from infinity to any position requires no work.

work to position $Q_1 = 0$

work to position $Q_2 = Q_2 V_{21}$

work to position $Q_3 = Q_3 V_{31} + Q_3 V_{32}$

work to position $Q_4 = Q_4 V_{41} + Q_4 V_{42} + Q_4 V_{43}$

Total potential energy = Total positioning work of whole field

$$\boxed{W_E = Q_2 V_{21} + \underline{Q_3 V_{31}} + Q_3 V_{32} + Q_4 V_{41} + Q_4 V_{42} + Q_4 V_{43}} \quad \text{—(i)}$$

$$Q_3 V_{31} = Q_3 \frac{Q_1}{4\pi\epsilon_0 R_{13}}$$

$$= Q_1 \frac{Q_3}{4\pi\epsilon_0 R_{13}}$$

$$Q_3 V_{31} = Q_1 V_{13}$$

So now eq (i) can be rewritten as:

$$\boxed{W_e = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23} + Q_1 V_{14} + Q_2 V_{24} + Q_3 V_{34}} \quad \text{---(ii)}$$

Adding eq (i) and (ii)

$$\Rightarrow 2W_e = Q_1 (\overbrace{V_{12} + V_{13} + V_{14} + \cdots}^{V_1}) + Q_2 (V_{21} + V_{23} + V_{24} + \cdots) + Q_3 (V_{31} + V_{32} + V_{34} + \cdots) + \cdots$$

$$\Rightarrow 2W_e = Q_1 V_1 + Q_2 V_2 + Q_3 V_3 + \cdots$$

$$\Rightarrow 2W_e = \sum_{m=1}^{N} Q_m V_m$$

$$\Rightarrow W_e = \frac{1}{2} \sum_{m=1}^{N} Q_m V_m$$

For continous volume charge distribution:

$$W_e = \frac{1}{2} \int_{vol} \rho_v V \, dv$$

$$W_e = \frac{1}{2} \int_{vol} (\bar{\nabla} \cdot \bar{D}) V \, dv \quad \because \rho_v = \bar{\nabla} \cdot \bar{D}$$

using Identity:

$$\bar{\nabla}\cdot(V\bar{D}) = V(\bar{\nabla}\cdot\bar{D}) + \bar{D}\cdot(\bar{\nabla}V)$$

$$V(\bar{\nabla}\cdot\bar{D}) = \bar{\nabla}\cdot(V\bar{D}) - \bar{D}\cdot(\bar{\nabla}V)$$

$$We = \frac{1}{2}\int_{vol}\left[\bar{\nabla}\cdot(V\bar{D}) - \bar{D}\cdot(\bar{\nabla}V)\right] dv$$

$$\because \int_{vol}(\bar{\nabla}\cdot\bar{D})\,dv = \oint_s \bar{D}\cdot\bar{ds}$$

$$We = \frac{1}{2}\int_{vol}(\nabla\cdot V\bar{D})\,dv - \frac{1}{2}\int_{vol}[\bar{D}\cdot(\bar{\nabla}V)]\,dv$$

$$\because \bar{E} = -\nabla V$$

$$We = \cancel{\frac{1}{2}\oint_s (V\bar{D})\cdot\bar{ds}}^{\,0} + \frac{1}{2}\int_{vol}(\bar{D}\cdot\bar{E})\,dv$$

$$\boxed{We = \frac{1}{2}\int_{vol}(\bar{D}\cdot\bar{E})\,dv}$$

$$\boxed{We = \frac{1}{2}\int_{vol}\epsilon_0 E^2\,dv}$$

$$dWe = \frac{1}{2}(\bar{D}\cdot\bar{E})\,dv$$

$$\boxed{\frac{d\,We}{dv} = \frac{1}{2}\bar{D}\cdot\bar{E}}$$

This is energy density equation