2.32 A discrete-time LTI system has the impulse response $h[n]$ depicted in Fig. P2.32(a). Use linearity and time invariance to determine the system output $y[n]$ if the input is

(a) $x[n] = 3\delta[n] - 2\delta[n-1]$

(b) $x[n] = u[n+1] - u[n-3]$

(c) $x[n]$ as given in Fig. P2.32(b).

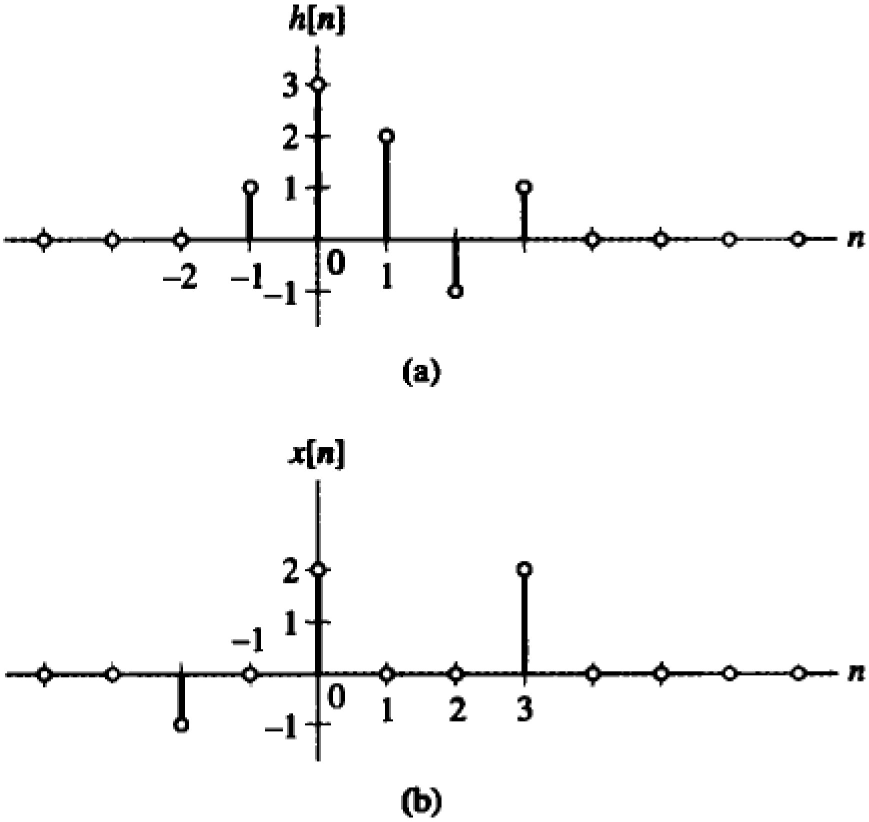

Figure P2.32

2.33 Evaluate the following discrete-time convolution sums:

(a) $y[n] = u[n+3] * u[n-3]$

(b) $y[n] = 3^n u[-n+3] * u[n-2]$

(c) $y[n] = \left(\frac{1}{4}\right)^n u[n] * u[n+2]$

(d) $y[n] = \cos\left(\frac{\pi}{2}n\right)u[n] * u[n-1]$

(e) $y[n] = (-1)^n * 2^n u[-n+2]$

(f) $y[n] = \cos\left(\frac{\pi}{2}n\right) * \left(\frac{1}{2}\right)^n u[n-2]$

(g) $y[n] = \beta^n u[n] * u[n-3], \quad |\beta| < 1$

(h) $y[n] = \beta^n u[n] * \alpha^n u[n-10], \quad |\beta| < 1, |\alpha| < 1$

(i) $y[n] = (u[n+10] - 2u[n] + u[n-4]) * u[n-2]$

(j) $y[n] = (u[n+10] - 2u[n] + u[n-4]) * \beta^n u[n], \quad |\beta| < 1$

(k) $y[n] = (u[n+10] - 2u[n+5] + u[n-6]) * \cos\left(\frac{\pi}{2}n\right)$

(l) $y[n] = u[n] * \sum_{p=0}^{\infty} \delta[n-4p]$

(m) $y[n] = \beta^n u[n] * \sum_{p=0}^{\infty} \delta[n-4p], \quad |\beta| < 1$

(n) $y[n] = \left(\frac{1}{2}\right)^n u[n+2] * \gamma^{|n|}$

2.34 Consider the discrete-time signals depicted in Fig. P2.34. Evaluate the following convolution sums:

(a) $m[n] = x[n] * z[n]$

(b) $m[n] = x[n] * y[n]$

(c) $m[n] = x[n] * f[n]$

(d) $m[n] = x[n] * g[n]$

(e) $m[n] = y[n] * z[n]$

(f) $m[n] = y[n] * g[n]$

(g) $m[n] = y[n] * w[n]$

(h) $m[n] = y[n] * f[n]$

(i) $m[n] = z[n] * g[n]$

(j) $m[n] = w[n] * g[n]$

(k) $m[n] = f[n] * g[n]$

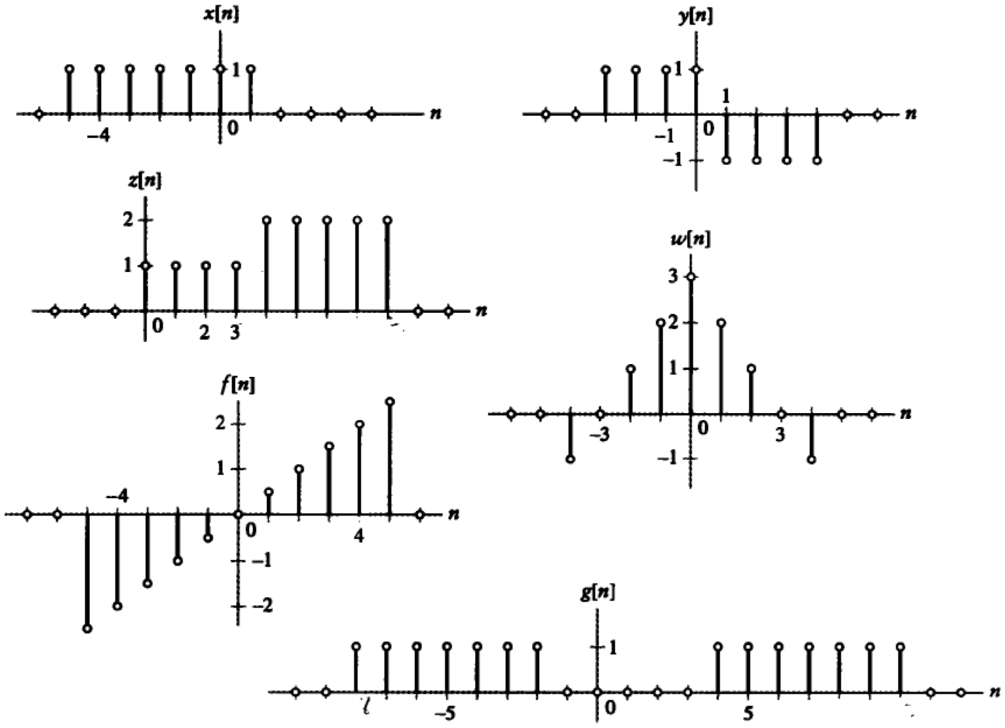

FIGURE P2.34

2.37 The convolution sum evaluation procedure actually corresponds to a formal statement of the well-known procedure for multiplying polynomials. To see this, we interpret polynomials as signals by setting the value of a signal at time n equal to the polynomial coefficient associated with monomial z^n. For example, the polynomial $x(z) = 2 + 3z^2 - z^3$ corresponds to the signal $x[n] = 2\delta[n] + 3\delta[n-2] - \delta[n-3]$. The procedure for multiplying polynomials involves forming the product of all polynomial coefficients that result in an nth-order monomial and then summing them to obtain the polynomial coefficient of the nth-order monomial in the product. This corresponds to determining $w_n[k]$ and summing over k to obtain $y[n]$.

Evaluate the convolutions $y[n] = x[n] * h[n]$, both using the convolution sum evaluation procedure and taking a product of polynomials.

(a) $x[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$,
$h[n] = u[n] - u[n-3]$

(b) $x[n] = u[n-1] - u[n-5]$,
$h[n] = u[n-1] - u[n-5]$

2.39 Evaluate the following continuous-time convolution integrals:

(a) $y(t) = (u(t) - u(t-2)) * u(t)$

(b) $y(t) = e^{-3t}u(t) * u(t+3)$

(c) $y(t) = \cos(\pi t)(u(t+1) - u(t-1)) * u(t)$

(d) $y(t) = (u(t+3) - u(t-1)) * u(-t+4)$

(e) $y(t) = (tu(t) + (10-2t)u(t-5) - (10-t)u(t-10)) * u(t)$

(f) $y(t) = 2t^2(u(t+1) - u(t-1)) * 2u(t+2)$

(g) $y(t) = \cos(\pi t)(u(t+1) - u(t-1)) * (u(t+1) - u(t-1))$

(h) $y(t) = \cos(2\pi t)(u(t+1) - u(t-1)) * e^{-t}u(t)$

(i) $y(t) = (2\delta(t+1) + \delta(t-5)) * u(t-1)$

(j) $y(t) = (\delta(t+2) + \delta(t-2)) * (tu(t) + (10-2t)u(t-5) - (10-t)u(t-10))$

(k) $y(t) = e^{-\gamma t}u(t) * (u(t+2) - u(t))$

(l) $y(t) = e^{-\gamma t}u(t) * \sum_{p=0}^{\infty}\left(\frac{1}{4}\right)^p\delta(t-2p)$

(m) $y(t) = (2\delta(t) + \delta(t-2)) * \sum_{p=0}^{\infty}\left(\frac{1}{2}\right)^p\delta(t-p)$

(n) $y(t) = e^{-\gamma t}u(t) * e^{\beta t}u(-t)$

(o) $y(t) = u(t) * h(t)$, where $h(t) = \begin{cases} e^{2t} & t < 0 \\ e^{-3t} & t \geq 0 \end{cases}$

2.40 Consider the continuous-time signals depicted in Fig. P2.40. Evaluate the following convolution integrals:

(a) $m(t) = x(t) * y(t)$

(b) $m(t) = x(t) * z(t)$

(c) $m(t) = x(t) * f(t)$

(d) $m(t) = x(t) * a(t)$

(e) $m(t) = y(t) * z(t)$

(f) $m(t) = y(t) * w(t)$

(g) $m(t) = y(t) * g(t)$

(h) $m(t) = y(t) * c(t)$

(i) $m(t) = z(t) * f(t)$

(j) $m(t) = z(t) * g(t)$

(k) $m(t) = z(t) * b(t)$

(l) $m(t) = w(t) * g(t)$

(m) $m(t) = w(t) * a(t)$

(n) $m(t) = f(t) * g(t)$

(o) $m(t) = f(t) * d(t)$

(p) $m(t) = z(t) * d(t)$

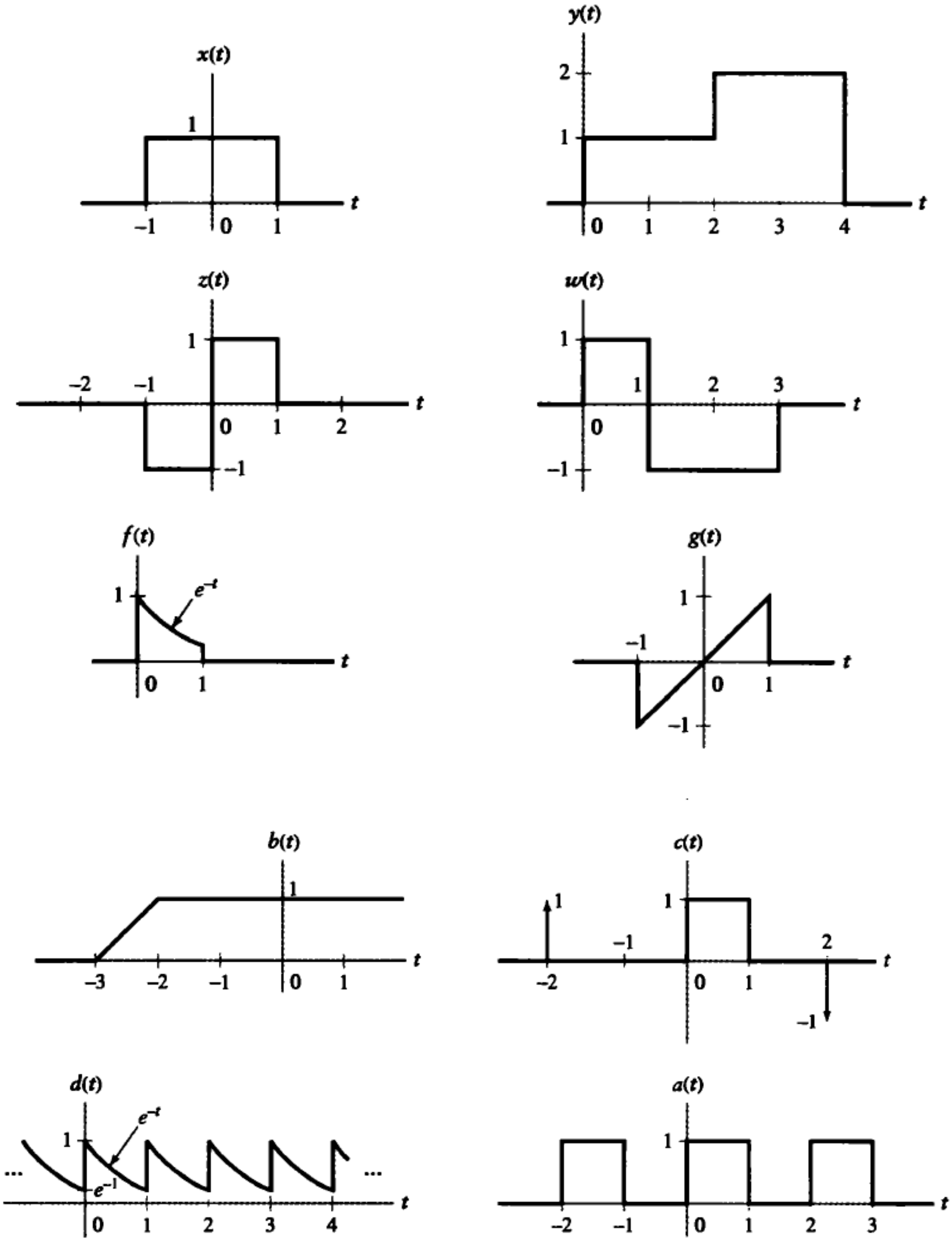

Figure P2.40

2.42 Use the definition of the convolution sum to derive the following properties:

(a) Distributive: $x[n] * (h[n] + g[n]) = x[n] * h[n] + x[n] * g[n]$

(b) Associative:
$x[n] * (h[n] * g[n]) = (x[n] * h[n]) * g[n]$

(c) Commutative: $x[n] * h[n] = h[n] * x[n]$

2.44 Show that if $y(t) = x(t) * h(t)$ is the output of an LTI system with input $x(t)$ and impulse response $h(t)$, then

$$\frac{d}{dt}y(t) = x(t) * \left(\frac{d}{dt}h(t)\right)$$

and

$$\frac{d}{dt}y(t) = \left(\frac{d}{dt}x(t)\right) * h(t).$$

2.48 For the interconnection of LTI systems depicted in Fig. P2.46(c), the impulse responses are $h_1(t) = \delta(t-1)$, $h_2(t) = e^{-2t}u(t)$, $h_3(t) = \delta(t-1)$ and $h_4(t) = e^{-3(t+2)}u(t+2)$. Evaluate $h(t)$, the impulse response of the overall system from $x(t)$ to $y(t)$.

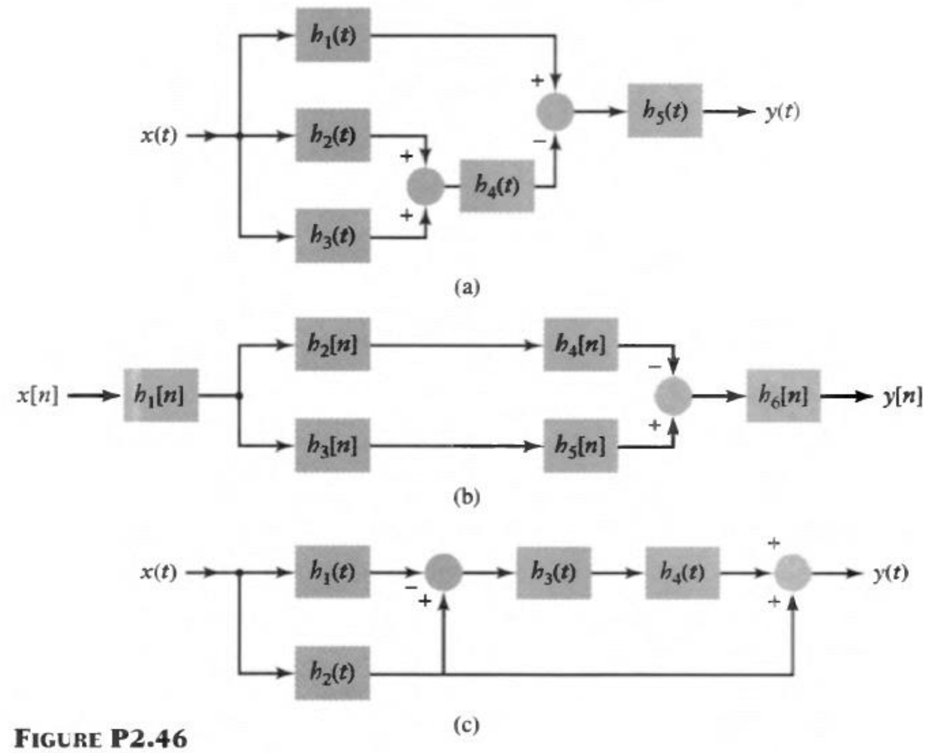

FIGURE P2.46

2.49 For each of the following impulse responses, determine whether the corresponding system is (i) memoryless, (ii) causal, and (iii) stable.

(a) $h(t) = \cos(\pi t)$

(b) $h(t) = e^{-2t}u(t - 1)$

(c) $h(t) = u(t + 1)$

(d) $h(t) = 3\delta(t)$

(e) $h(t) = \cos(\pi t)u(t)$

(f) $h[n] = (-1)^n u[-n]$

(g) $h[n] = (1/2)^{|n|}$

(h) $h[n] = \cos\left(\frac{\pi}{8}n\right)\{u[n] - u[n - 10]\}$

(i) $h[n] = 2u[n] - 2u[n - 5]$

(j) $h[n] = \sin\left(\frac{\pi}{2}n\right)$

(k) $h[n] = \sum_{p=-1}^{\infty} \delta[n - 2p]$

2.50 Evaluate the step response for the LTI systems represented by the following impulse responses:

(a) $h[n] = (-1/2)^n u[n]$

(b) $h[n] = \delta[n] - \delta[n - 2]$

(c) $h[n] = (-1)^n\{u[n + 2] - u[n - 3]\}$

(d) $h[n] = nu[n]$

(e) $h(t) = e^{-|t|}$

(f) $h(t) = \delta^{(2)}(t)$

(g) $h(t) = (1/4)(u(t) - u(t - 4))$

(h) $h(t) = u(t)$

2.86 Two systems have impulse responses

$$h_1[n] = \begin{cases} \frac{1}{4}, & 0 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h_2[n] = \begin{cases} \frac{1}{4}, & n = 0, 2 \\ -\frac{1}{4}, & n = 1, 3 \\ 0, & \text{otherwise} \end{cases}.$$

Use the MATLAB command `conv` to plot the first 20 values of the step response.