4.16 Find the FT representations of the following periodic signals:

(a) $x(t) = 2\cos(\pi t) + \sin(2\pi t)$

(b) $x(t) = \sum_{k=0}^{4} \frac{(-1)^k}{k+1} \cos((2k+1)\pi t)$

(c) $x(t)$ as depicted in Fig. P4.16(a).

(d) $x(t)$ as depicted in Fig. P4.16(b).

Sketch the magnitude and phase spectra.

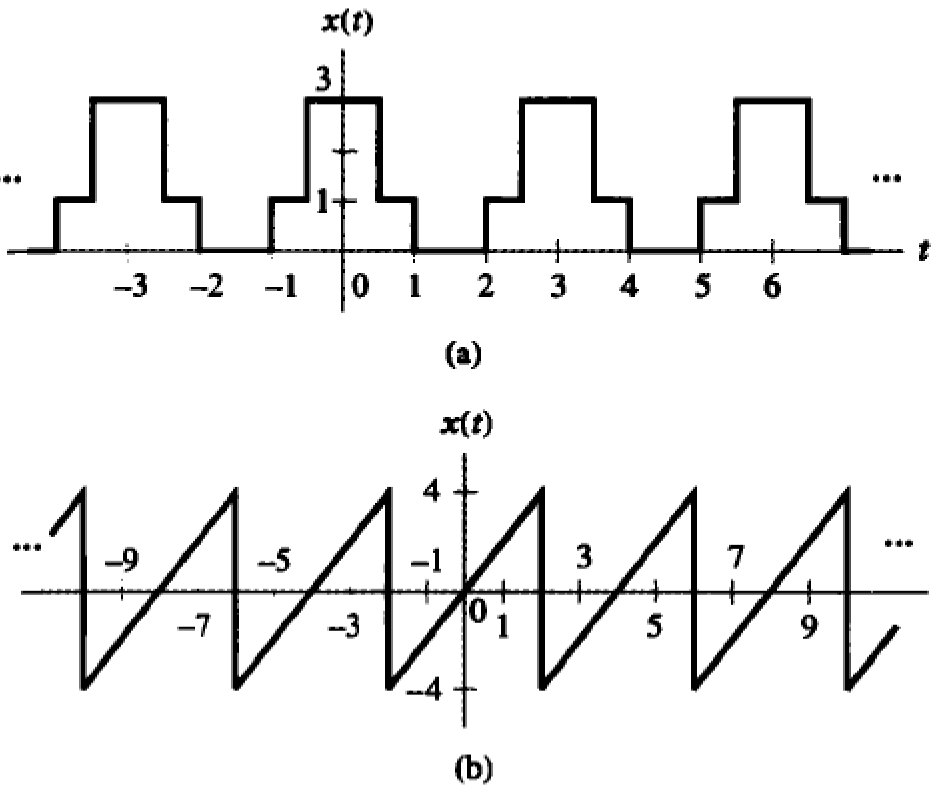

FIGURE P4.16

4.17 Find the DTFT representations of the following periodic signals:

(a) $x[n] = \cos\left(\frac{\pi}{8}n\right) + \sin\left(\frac{\pi}{5}n\right)$

(b) $x[n] = 1 + \sum_{m=-\infty}^{\infty} \cos\left(\frac{\pi}{4}m\right)\delta[n-m]$

(c) $x[n]$ as depicted in Fig. P4.17(a).

(d) $x[n]$ as depicted in Fig. P4.17(b).

(e) $x[n]$ as depicted in Fig. P4.17(c).

Sketch the magnitude and phase spectra.

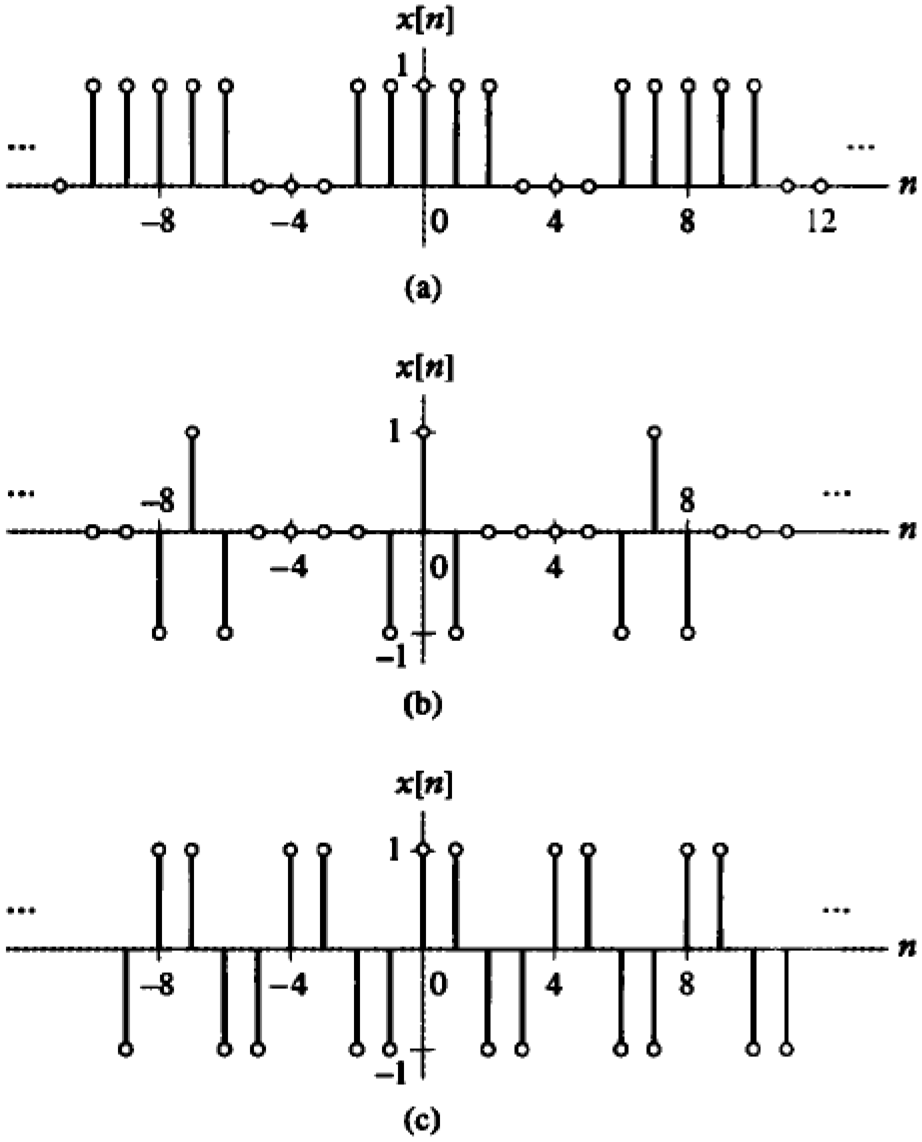

FIGURE P4.17

4.18 An LTI system has the impulse response

$$h(t) = 2\frac{\sin(2\pi t)}{\pi t}\cos(7\pi t).$$

Use the FT to determine the system output if the input is

(a) $x(t) = \cos(2\pi t) + \sin(6\pi t)$

(b) $x(t) = \sum_{m=-\infty}^{\infty}(-1)^m\delta(t - m)$

(c) $x(t)$ as depicted in Fig. P4.18(a).

(d) $x(t)$ as depicted in Fig. P4.18(b).

(e) $x(t)$ as depicted in Fig. P4.18(c).

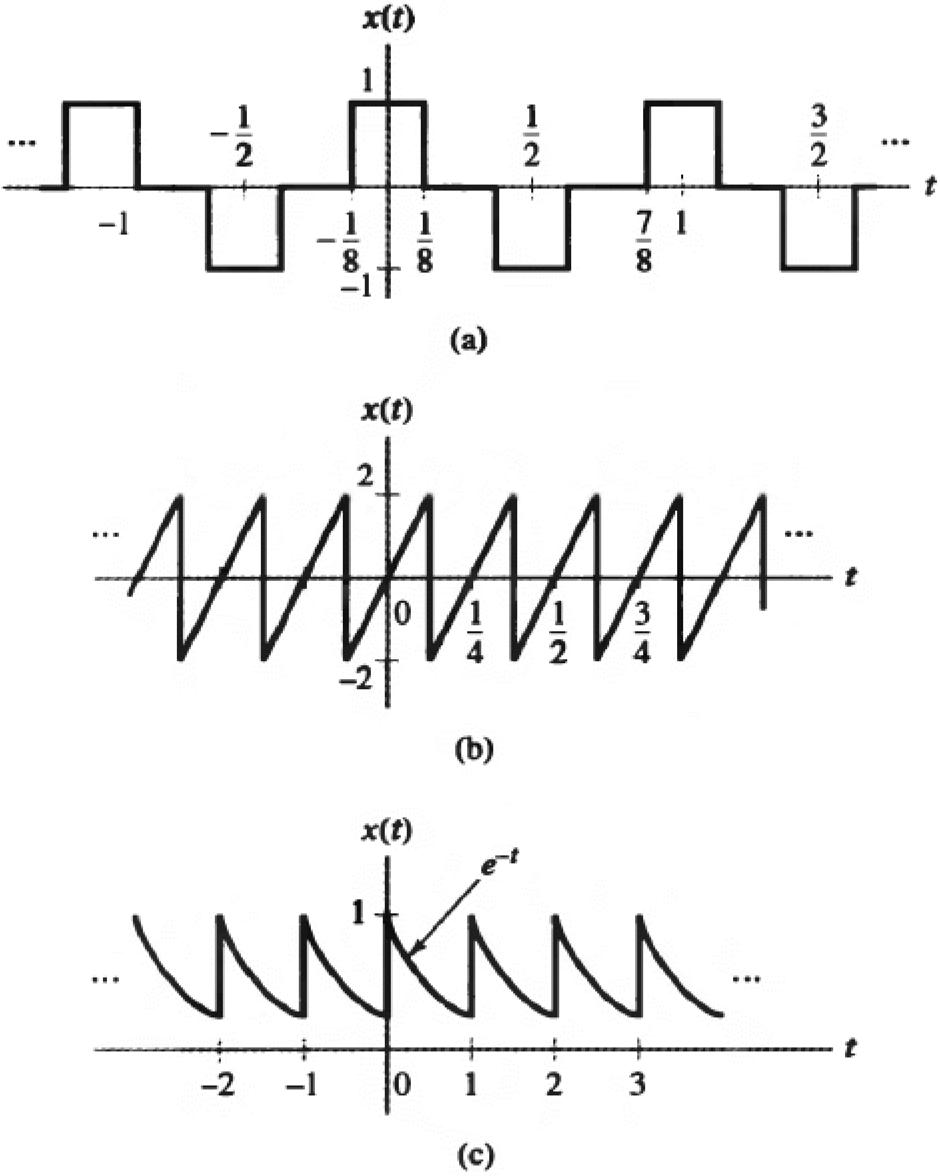

FIGURE P4.18

4.24 Determine and sketch the FT representation, $X_\delta(j\omega)$, for the following discrete-time signals with the sampling interval T_s as given:

(a) $x[n] = \dfrac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}$, $T_s = 2$

(b) $x[n] = \dfrac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}$, $T_s = \frac{1}{4}$

(c) $x[n] = \cos\left(\frac{\pi}{2}n\right)\dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$, $T_s = 2$

(d) $x[n]$ depicted in Fig. P4.17(a) with $T_s = 4$.

(e) $x[n] = \sum_{p=-\infty}^{\infty}\delta[n - 4p]$, $T_s = \frac{1}{8}$

4.26 The continuous-time signal $x(t)$ with FT as depicted in Fig. P4.26 is sampled.

(a) Sketch the FT of the sampled signal for the following sampling intervals:

(i) $T_s = \frac{1}{14}$

(ii) $T_s = \frac{1}{7}$

(iii) $T_s = \frac{1}{5}$

In each case, identify whether aliasing occurs.

(b) Let $x[n] = x(nT_s)$. Sketch the DTFT of $x[n]$, $X(e^{j\Omega})$, for each of the sampling intervals given in (a).

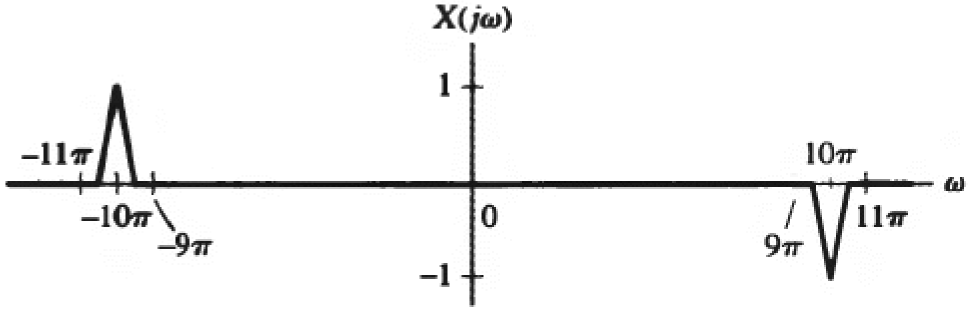

FIGURE P4.26

4.29 For each of the following signals, sampled with sampling interval T_s, determine the bounds on T_s, which guarantee that there will be no aliasing:

(a) $x(t) = \frac{1}{t}\sin 3\pi t + \cos(2\pi t)$

(b) $x(t) = \cos(12\pi t)\dfrac{\sin(\pi t)}{2t}$

(c) $x(t) = e^{-6t}u(t) * \dfrac{\sin(Wt)}{\pi t}$

(d) $x(t) = w(t)z(t)$, where the FT's $W(j\omega)$ and $Z(j\omega)$ are depicted in Fig. P4.29.

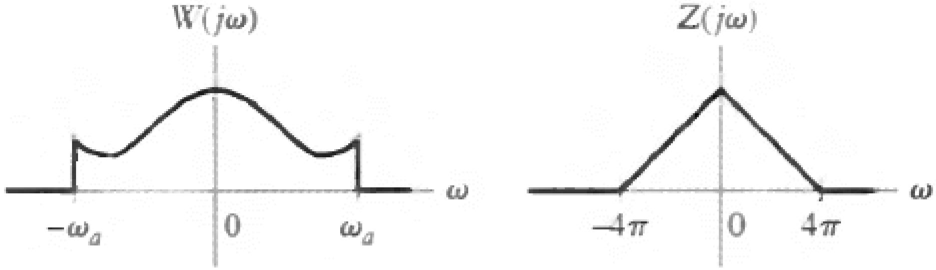

FIGURE P4.29

4.31 Let $|X(j\omega)| = 0$ for $|\omega| > \omega_m$. Form the signal $y(t) = x(t)[\cos(3\pi t) + \sin(10\pi t)]$. Determine the maximum value of ω_m for which $x(t)$ can be reconstructed from $y(t)$, and specify a system that will perform the reconstruction.

4.56 The rectangular window is defined as

$$w_r[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}.$$

We may truncate the duration of a signal to the interval $0 \le n \le M$ by multiplying the signal with $w[n]$. In the frequency domain, we convolve the DTFT of the signal with

$$W_r(e^{j\Omega}) = e^{-j\frac{M}{2}\Omega}\frac{\sin\left(\frac{\Omega(M+1)}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}.$$

The effect of this convolution is to smear detail and introduce ripples in the vicinity of discontinuities. The smearing is proportional to the mainlobe width, while the ripple is proportional to the size of the sidelobes. A variety of alternative windows are used in practice to reduce sidelobe height in return for increased mainlobe width. In this problem, we evaluate the effect of windowing time-domain signals on their DTFT. The role of windowing in filter design is explored in Chapter 8.

The Hanning window is defined as

$$w_h[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

(a) Assume that $M = 50$ and use the MATLAI command `fft` to evaluate the magnitude spec trum of the rectangular window in dB at inter vals of $\frac{\pi}{50}$, $\frac{\pi}{100}$, and $\frac{\pi}{200}$.

(b) Assume that $M = 50$ and use the MATLAI command `fft` to evaluate the magnitude spec trum of the Hanning window in dB at interval of $\frac{\pi}{50}$, $\frac{\pi}{100}$, and $\frac{\pi}{200}$.

(c) Use the results from (a) and (b) to evaluate th mainlobe width and peak sidelobe height in dl for each window.

(d) Let $y_r[n] = x[n]w_r[n]$ and $y_h[n] = x[n]w_h[n]$ where $x[n] = \cos\left(\frac{26\pi}{100}n\right) + \cos\left(\frac{29\pi}{100}n\right)$ an $M = 50$. Use the the MATLAB command `ff` to evaluate $|Y_r(e^{j\Omega})|$ in dB and $|Y_h(e^{j\Omega})|$ in dB a intervals of $\frac{\pi}{200}$. Does the choice of window af fect whether you can identify the presence o two sinusoids? Why?

(e) Let $y_r[n] = x[n]w_r[n]$ and $y_h[n] = x[n]w_h[n]$ where $x[n] = \cos\left(\frac{26\pi}{100}n\right) + 0.02\cos\left(\frac{51\pi}{100}n\right)$ an $M = 50$. Use the the MATLAB command `ff`

to evaluate $|Y_r(e^{j\Omega})|$ in dB and $|Y_h(e^{j\Omega})|$ in dB at intervals of $\frac{\pi}{200}$. Does the choice of window affect whether you can identify the presence of two sinusoids? Why?