when using `fft` to compute DTFS coefficients. We have adopted the DTFS in this text because the terminology involved is more descriptive and less likely to lead to confusion with the DTFT. The reader should be aware that he or she will likely encounter DFT terminology in other texts and references.

3. A general treatment of Fourier analysis is presented in
   - Kammler, D. W., *A First Course in Fourier Analysis* (Prentice-Hall, 2000)
   - Bracewell, R. N., *The Fourier Transform and Its Applications*, 2nd ed. (McGraw-Hill, 1978)
   - Papoulis, A. *The Fourier Integral and Its Applications* (McGraw-Hill, 1962)

   The text by Kammler provides a mathematical treatment of the FT, FS, DTFT, and DTFS. The texts by Bracewell and Papoulis are application oriented and focus on the FT.

4. The role of the FS and FT in solving partial-differential equations such as the heat equation, wave equation, and potential equation is described in
   - Powers, D. L., *Boundary Value Problems* 2nd ed.(Academic Press, 1979)

5. The uncertainty principle, Eq. (3.65), is proved in Bracewell, *op. cit.*

ADDITIONAL PROBLEMS

3.48 Use the defining equation for the DTFS coefficients to evaluate the DTFS representation of the following signals:

(a) $x[n] = \cos\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right)$

(b) $x[n] = 2\sin\left(\frac{14\pi}{19}n\right) + \cos\left(\frac{10\pi}{19}n\right) + 1$

(c) $x[n] = \sum_{m=-\infty}^{\infty}(-1)^m(\delta[n - 2m] + \delta[n + 3m])$

(d) $x[n]$ as depicted in Figure P3.48(a).

(e) $x[n]$ as depicted in Figure P3.48(b).

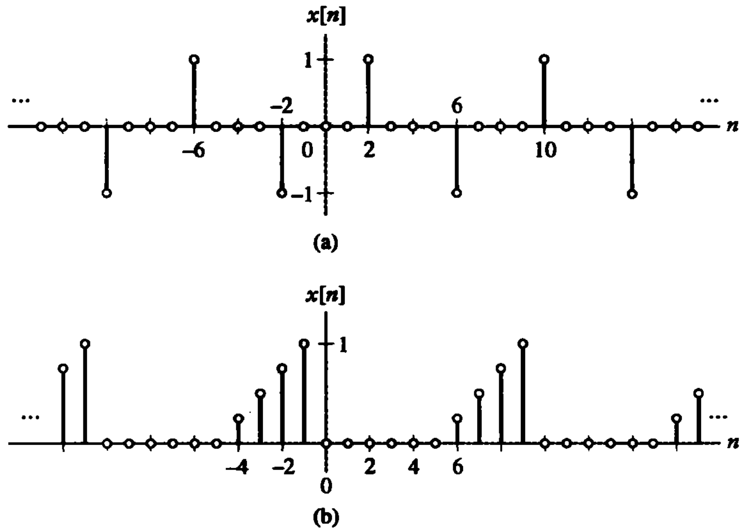

FIGURE P3.48

3.49 Use the definition of the DTFS to determine the time-domain signals represented by the following DTFS coefficients:

(a) $X[k] = \cos\left(\frac{8\pi}{21}k\right)$

(b) $X[k] = \cos\left(\frac{10\pi}{19}k\right) + j2\sin\left(\frac{4\pi}{19}k\right)$

(c) $X[k] = \sum_{m=-\infty}^{\infty}(-1)^m(\delta[k-2m] - 2\delta[k+3m])$

(d) $X[k]$ as depicted in Figure P3.49(a).

(e) $X[k]$ as depicted in Figure P3.49(b).

(f) $X[k]$ as depicted in Figure P3.49(c).

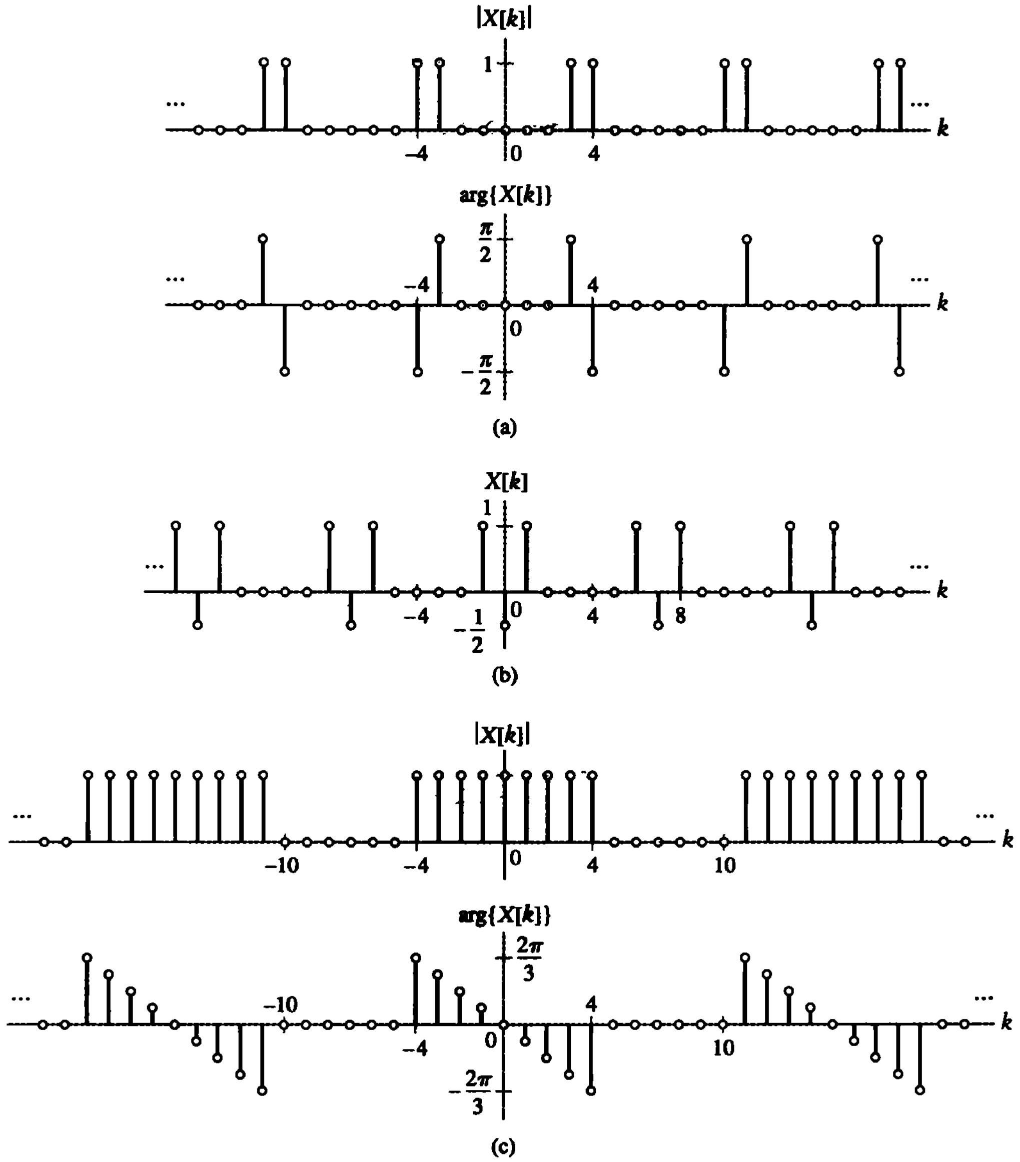

FIGURE P3.49

3.50 Use the defining equation for the FS coefficients to evaluate the FS representation of the following signals:

(a) $x(t) = \sin(3\pi t) + \cos(4\pi t)$

(b) $x(t) = \sum_{m=-\infty}^{\infty}\delta(t - m/3) + \delta(t - 2m/3)$

(c) $x(t) = \sum_{m=-\infty}^{\infty} e^{j\frac{2\pi}{7}m}\delta(t - 2m)$

(d) $x(t)$ as depicted in Figure P3.50(a).

(e) $x(t)$ as depicted in Figure P3.50(b).

(f) $x(t)$ as depicted in Figure P3.50(c).

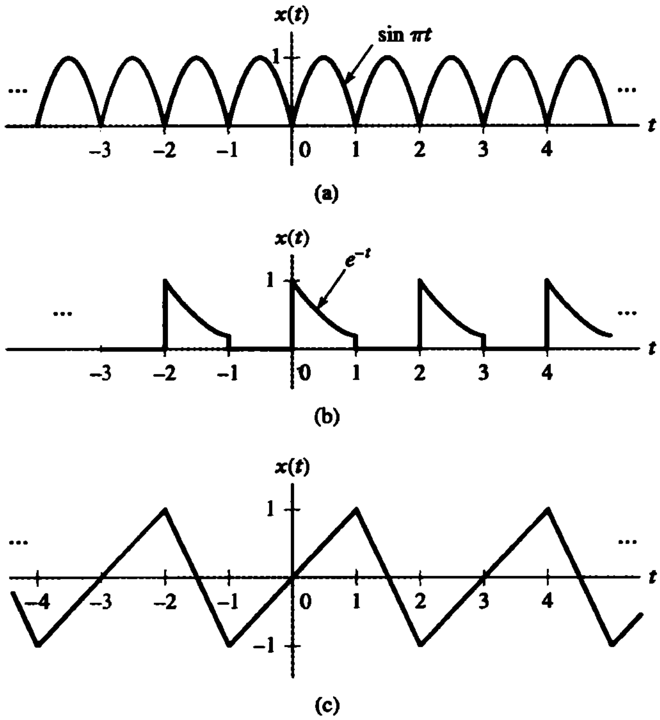

FIGURE P3.50

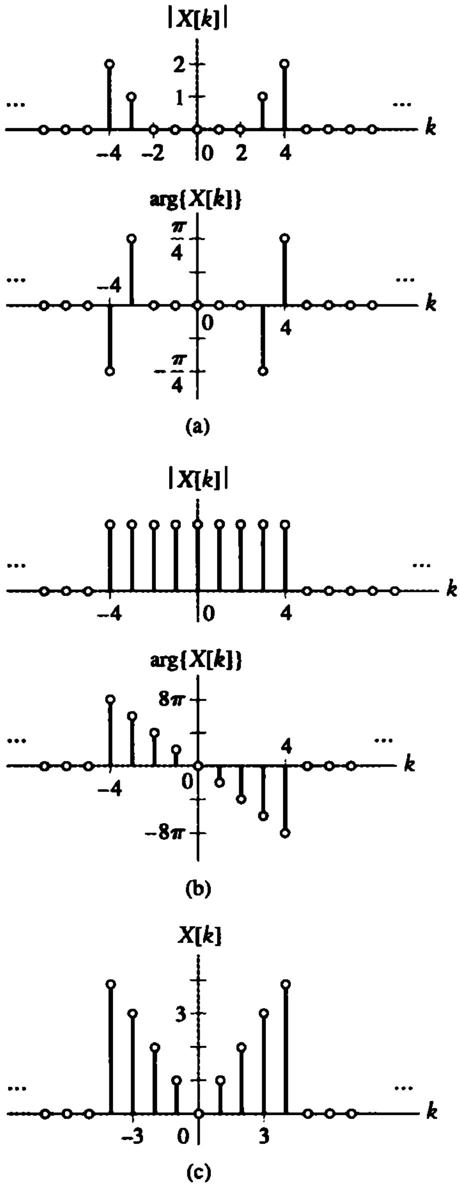

FIGURE P3.51

3.51 Use the definition of the FS to determine the time-domain signals represented by the following FS coefficients:

(a) $X[k] = j\delta[k - 1] - j\delta[k + 1] + \delta[k - 3] + \delta[k + 3], \quad \omega_o = 2\pi$

(b) $X[k] = j\delta[k - 1] - j\delta[k + 1] + \delta[k - 3] + \delta[k + 3], \quad \omega_o = 4\pi$

(c) $X[k] = \left(\frac{-1}{3}\right)^{|k|}, \quad \omega_o = 1$

(d) $X[k]$ as depicted in Figure P.3.51(a), $\omega_o = \pi$.

(e) $X[k]$ as depicted in Figure P.3.51(b), $\omega_o = 2\pi$.

(f) $X[k]$ as depicted in Figure P.3.51(c), $\omega_o = \pi$.

3.52 Use the defining equation for the DTFT to evaluate the frequency-domain representations of the following signals:

(a) $x[n] = \left(\frac{3}{4}\right)^n u[n - 4]$

(b) $x[n] = a^{|n|} \quad |a| < 1$

(c) $x[n] = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{N}n\right), & |n| \le N \\ 0, & \text{otherwise} \end{cases}$

(d) $x[n] = 2\delta[4 - 2n]$

(e) $x[n]$ as depicted in Figure P3.52(a).

(f) $x[n]$ as depicted in Figure P3.52(b).

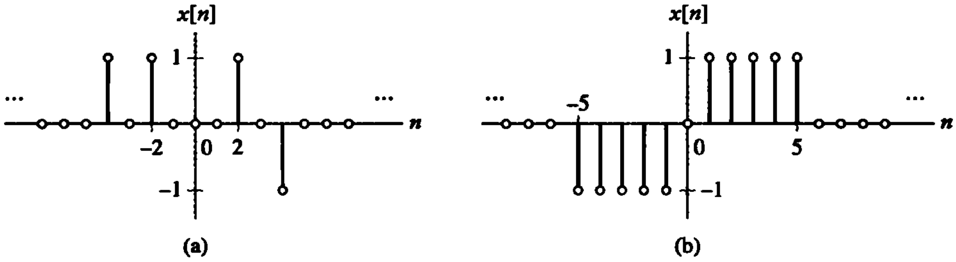

FIGURE P3.52

3.53 Use the equation describing the DTFT representation to determine the time-domain signals corresponding to the following DTFTs:

(a) $X(e^{j\Omega}) = \cos(2\Omega) + j\sin(2\Omega)$

(b) $X(e^{j\Omega}) = \sin(\Omega) + \cos\left(\frac{\Omega}{2}\right)$.

(c) $|X(e^{j\Omega})| = \begin{cases} 1, & \pi/4 < |\Omega| < 3\pi/4, \\ 0, & \text{otherwise} \end{cases}$

$\arg\{X(e^{j\Omega})\} = -4\Omega$

(d) $X(e^{j\Omega})$ as depicted in Figure P3.53(a).

(e) $X(e^{j\Omega})$ as depicted in Figure P3.53(b).

(f) $X(e^{j\Omega})$ as depicted in Figure P3.53(c).

3.54 Use the defining equation for the FT to evaluate the frequency-domain representations of the following signals:

(a) $x(t) = e^{-2t}u(t-3)$

(b) $x(t) = e^{-4|t|}$

(c) $x(t) = te^{-t}u(t)$

(d) $x(t) = \sum_{m=0}^{\infty} a^m \delta(t-m)$, $|a| < 1$

(e) $x(t)$ as depicted in Figure P3.54(a).

(f) $x(t)$ as depicted in Figure P3.54(b).

3.55 Use the equation describing the FT representation to determine the time-domain signals corresponding to the following FTs:

(a) $X(j\omega) = \begin{cases} \cos(2\omega), & |\omega| < \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases}$

(b) $X(j\omega) = e^{-2\omega}u(\omega)$

(c) $X(j\omega) = e^{-2|\omega|}$

(d) $X(j\omega)$ as depicted in Figure P3.55(a).

(e) $X(j\omega)$ as depicted in Figure P3.55(b).

(f) $X(j\omega)$ as depicted in Figure P3.55(c).

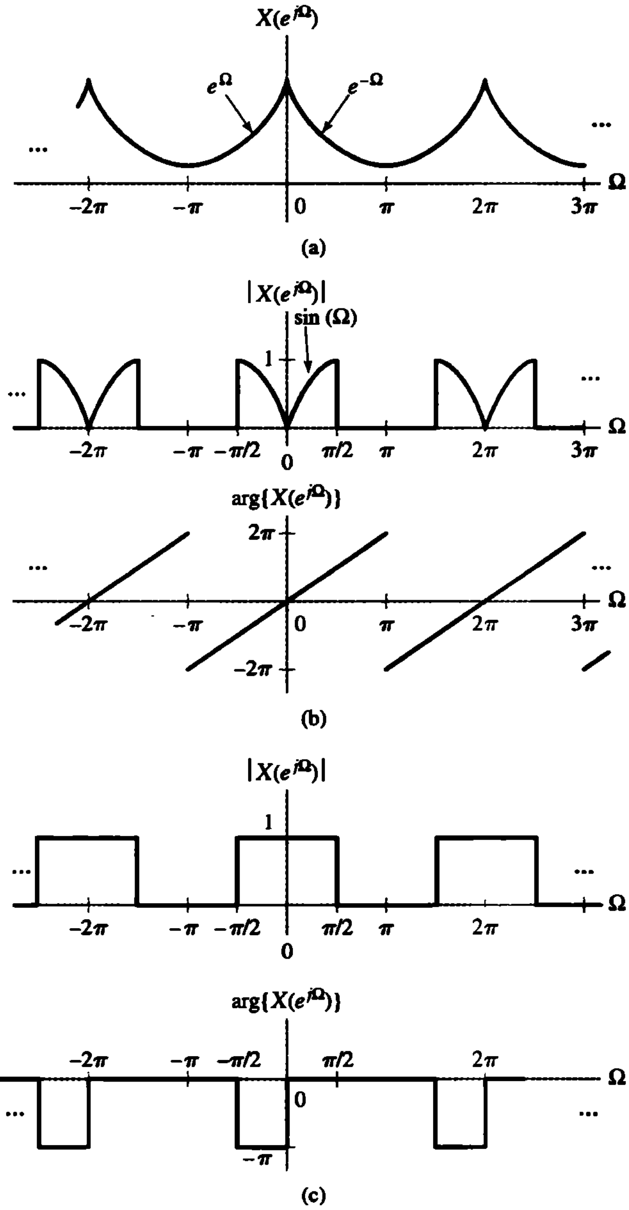

FIGURE P3.53

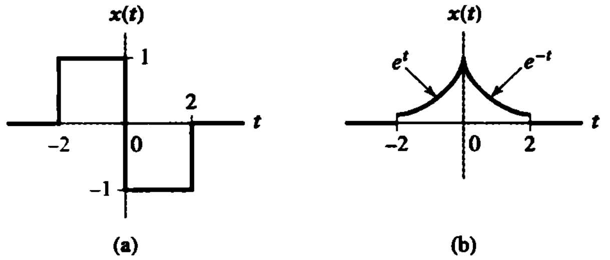

FIGURE P3.54

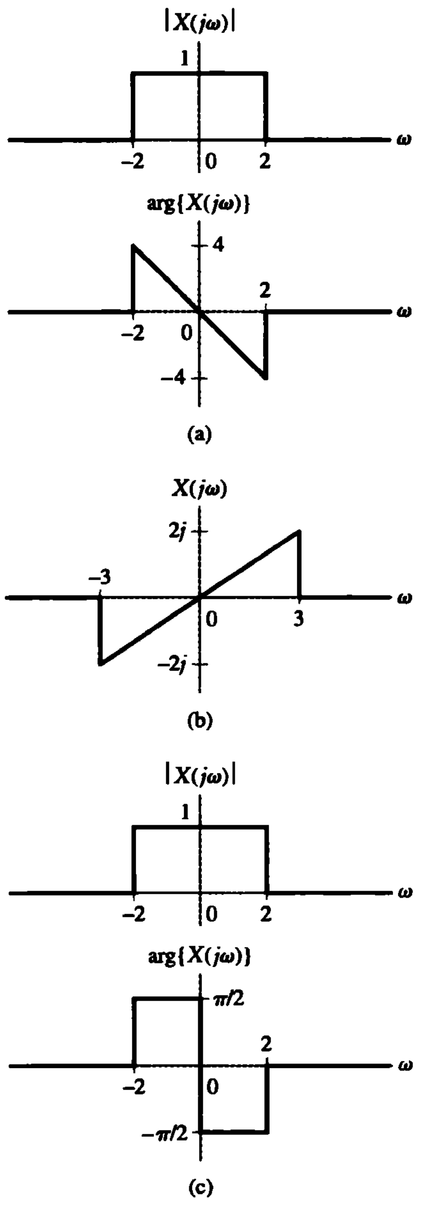

FIGURE P3.55

3.56 Determine the appropriate Fourier representations of the following time-domain signals, using the defining equations:

(a) $x(t) = e^{-t}\cos(2\pi t)u(t)$

(b) $x[n] = \begin{cases} \cos\left(\frac{\pi}{10}n\right) + j\sin\left(\frac{\pi}{10}n\right), & |n| < 10 \\ 0, & \text{otherwise} \end{cases}$

(c) $x[n]$ as depicted in Figure P3.56(a).

(d) $x(t) = e^{1+t}u(-t + 2)$

(e) $x(t) = |\sin(2\pi t)|$

(f) $x[n]$ as depicted in Figure P3.56(b).

(g) $x(t)$ as depicted in Figure P3.56(c).

3.57 Determine the time-domain signal corresponding to each of the following frequency-domain representations:

(a) $X[k] = \begin{cases} e^{-jk\pi/2}, & |k| < 10 \\ 0, & \text{otherwise} \end{cases}$

Fundamental period of time domain signal is $T = 1$.

(b) $X[k]$ as depicted in Figure P3.57(a).

(c) $X(j\omega) = \begin{cases} \cos\left(\frac{\omega}{4}\right) + j\sin\left(\frac{\omega}{4}\right), & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$

(d) $X(j\omega)$ as depicted in Figure P3.57(b).

(e) $X(e^{j\Omega})$ as depicted in Figure P3.57(c).

(f) $X[k]$ as depicted in Figure P3.57(d).

(g) $X(e^{j\Omega}) = |\sin(\Omega)|$

3.58 Use the tables of transforms and properties to find the FT's of the following signals:

(a) $x(t) = \sin(2\pi t)e^{-t}u(t)$

(b) $x(t) = te^{-3|t-1|}$

(c) $x(t) = \left[\frac{2\sin(3\pi t)}{\pi t}\right]\left[\frac{\sin(2\pi t)}{\pi t}\right]$

(d) $x(t) = \frac{d}{dt}(te^{-2t}\sin(t)u(t))$

(e) $x(t) = \int_{-\infty}^{t} \frac{\sin(2\pi\tau)}{\pi\tau}d\tau$

(f) $x(t) = e^{-t+2}u(t - 2)$

(g) $x(t) = \left(\frac{\sin(t)}{\pi t}\right) * \frac{d}{dt}\left[\left(\frac{\sin(2t)}{\pi t}\right)\right]$

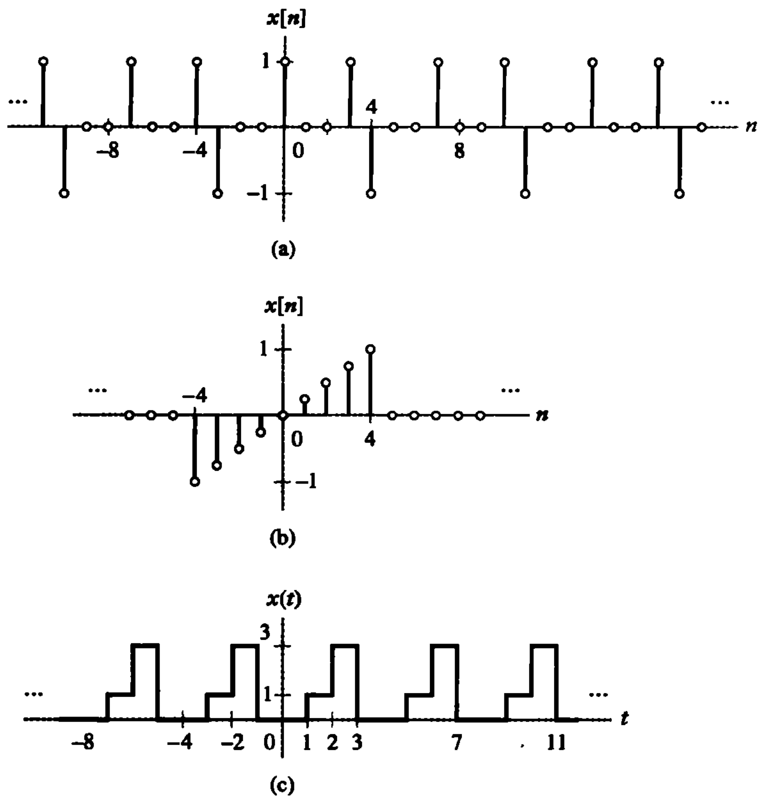

FIGURE P3.56

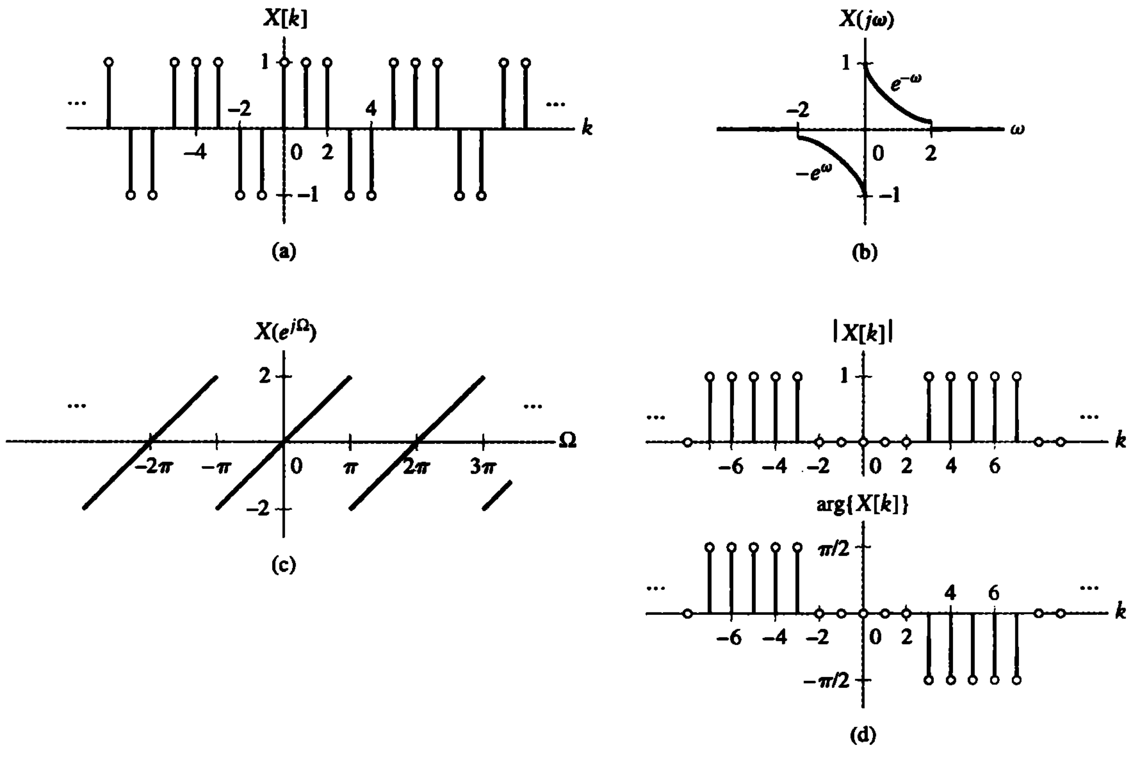

FIGURE P3.57

3.59 Use the tables of transforms and properties to find the inverse FTs of the following signals:

(a) $X(j\omega) = \dfrac{j\omega}{(1 + j\omega)^2}$

(b) $X(j\omega) = \dfrac{4\sin(2\omega - 4)}{2\omega - 4} - \dfrac{4\sin(2\omega + 4)}{2\omega + 4}$

(c) $X(j\omega) = \dfrac{1}{j\omega(j\omega + 2)} - \pi\delta(\omega)$

(d) $X(j\omega) = \dfrac{d}{d\omega}\left[4\sin(4\omega)\dfrac{\sin(2\omega)}{\omega}\right]$

(e) $X(j\omega) = \dfrac{2\sin(\omega)}{\omega(j\omega + 2)}$

(f) $X(j\omega) = \dfrac{4\sin^2(\omega)}{\omega^2}$

3.60 Use the tables of transforms and properties to find the DTFTs of the following signals:

(a) $x[n] = \left(\frac{1}{3}\right)^n u[n + 2]$

(b) $x[n] = (n - 2)(u[n + 4] - u[n - 5])$

(c) $x[n] = \cos\left(\frac{\pi}{4}n\right)\left(\frac{1}{2}\right)^n u[n - 2]$

(d) $x[n] = \left[\dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\right] * \left[\dfrac{\sin\left(\frac{\pi}{4}(n - 8)\right)}{\pi(n - 8)}\right]$

(e) $x[n] = \left[\dfrac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}\right]^2 * \dfrac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$

3.61 Use the tables of transforms and properties to find the inverse DTFTs of the following signals:

(a) $X(e^{j\Omega}) = j\sin(4\Omega) - 2$

(b) $X(e^{j\Omega}) = \left[e^{-j2\Omega}\dfrac{\sin\left(\frac{15}{2}\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}\right] \circledast \left[\dfrac{\sin\left(\frac{7}{2}\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}\right]$

(c) $X(e^{j\Omega}) = \cos(4\Omega)\left[\dfrac{\sin\left(\frac{3}{2}\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}\right]$

(d) $X(e^{j\Omega}) = \begin{cases} e^{-j4\Omega} & \frac{\pi}{4} < |\Omega| < \frac{3\pi}{4} \\ 0 & \text{otherwise} \end{cases}, \text{ for } |\Omega| < \pi$

(e) $X(e^{j\Omega}) = e^{-j\left(4\Omega + \frac{\pi}{2}\right)}\dfrac{d}{d\Omega}\left[\dfrac{2}{1 + \frac{1}{4}e^{-j\left(\Omega - \frac{\pi}{4}\right)}} + \dfrac{2}{1 + \frac{1}{4}e^{-j\left(\Omega + \frac{\pi}{4}\right)}}\right]$

3.62 Use the FT pair

$$x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & \text{otherwise} \end{cases} \xleftrightarrow{FT} X(j\omega) = \frac{2\sin(\omega)}{\omega}$$

and the FT properties to evaluate the frequency-domain representations of the signals depicted in Figures P3.62(a)–(g).

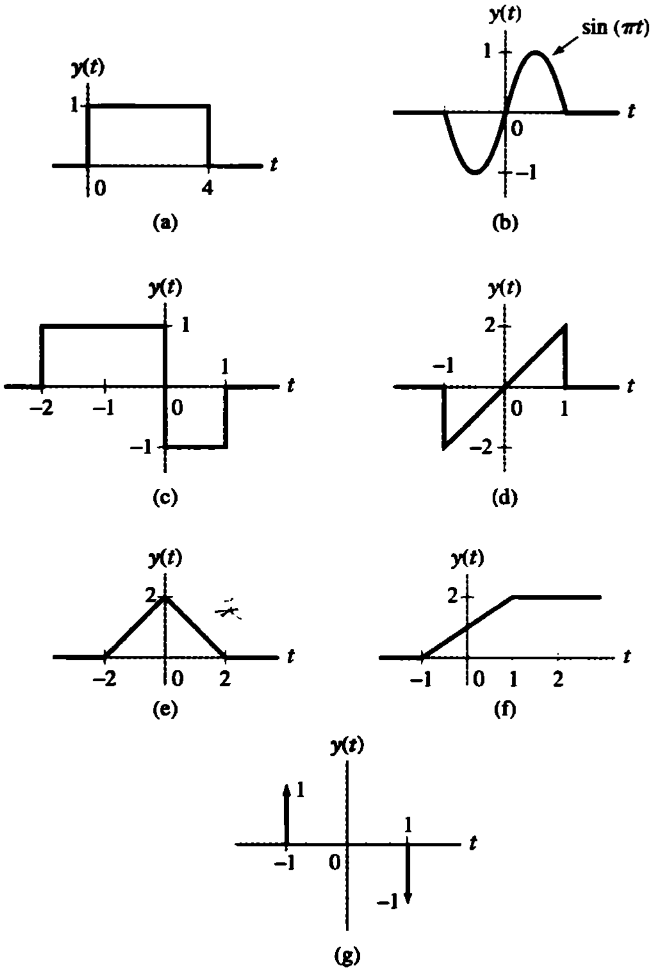

FIGURE P3.62

3.63 You are given $x[n] = n\left(\frac{3}{4}\right)^{|n|} \xleftrightarrow{DTFT} X(e^{j\Omega})$. Without evaluating $X(e^{j\Omega})$, find $y[n]$ if

(a) $Y(e^{j\Omega}) = e^{-j4\Omega}X(e^{j\Omega})$

(b) $Y(e^{j\Omega}) = \text{Re}\{X(e^{j\Omega})\}$

(c) $Y(e^{j\Omega}) = \dfrac{d}{d\Omega}X(e^{j\Omega})$

(d) $Y(e^{j\Omega}) = X(e^{j\Omega}) \circledast X(e^{j(\Omega - \pi/2)})$

(e) $Y(e^{j\Omega}) = \dfrac{d}{d\Omega}X(e^{j2\Omega})$

(f) $Y(e^{j\Omega}) = X(e^{j\Omega}) + X(e^{-j\Omega})$

(g) $Y(e^{j\Omega}) = \dfrac{d}{d\Omega}\left\{e^{-j4\Omega}\left[X\left(e^{j\left(\Omega+\frac{\pi}{4}\right)}\right) + X\left(e^{j\left(\Omega-\frac{\pi}{4}\right)}\right)\right]\right\}$

3.64 A periodic signal has the FS representation $x(t) \xleftrightarrow{FS;\,\pi} X[k] = -k2^{-|k|}$. Without determining $x(t)$, find the FS representation ($Y[k]$ and ω_o) if

(a) $y(t) = x(3t)$ (b) $y(t) = \dfrac{d}{dt}x(t)$

(c) $x(t) = x(t-1)$ (d) $y(t) = \text{Re}\{x(t)\}$

(e) $y(t) = \cos(4\pi t)x(t)$

(f) $y(t) = x(t) \circledast x(t-1)$

3.65 Given

$$x[n] = \frac{\sin\left(\frac{11\pi}{20}n\right)}{\sin\left(\frac{\pi}{20}n\right)} \xleftrightarrow{DTFS;\,\frac{\pi}{10}} X[k],$$

evaluate the time signal $y[n]$ with the following DTFS coefficients, using only DTFS properties:

(a) $Y[k] = X[k-5] + X[k+5]$

(b) $Y[k] = \cos\left(k\frac{\pi}{5}\right)X[k]$

(c) $Y[k] = X[k] \circledast X[k]$

(d) $Y[k] = \text{Re}\{X[k]\}$

3.66 Sketch the frequency response of the systems described by the following impulse responses:

(a) $h(t) = \delta(t) - 2e^{-2t}u(t)$

(b) $h(t) = 4e^{-2t}\cos(50t)u(t)$

(c) $h[n] = \frac{1}{8}\left(\frac{7}{8}\right)^n u[n]$

(d) $h[n] = \begin{cases} (-1)^n & |n| \le 10 \\ 0 & \text{otherwise} \end{cases}$

Characterize each system as low pass, band pass, or high pass.

3.67 Find the frequency response and the impulse response of the systems having the output $y(t)$ for the input $x(t)$:

(a) $x(t) = e^{-t}u(t)$, $y(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(b) $x(t) = e^{-3t}u(t)$, $y(t) = e^{-3(t-2)}u(t-2)$

(c) $x(t) = e^{-2t}u(t)$, $y(t) = 2te^{-2t}u(t)$

(d) $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $y[n] = \frac{1}{4}\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n]$

(e) $x[n] = \left(\frac{1}{4}\right)^n u[n]$,
$y[n] = \left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{4}\right)^{n-1} u[n-1]$

3.68 Determine the frequency response and the impulse response for the systems described by the following differential and difference equations:

(a) $\dfrac{d}{dt}y(t) + 3y(t) = x(t)$

(b) $\dfrac{d^2}{dt^2}y(t) + 5\dfrac{d}{dt}y(t) + 6y(t) = -\dfrac{d}{dt}x(t)$

(c) $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n] - \frac{3}{4}x[n-1]$

(d) $y[n] + \frac{1}{2}y[n-1] = x[n] - 2x[n-1]$

3.69 Determine the differential- or difference-equation descriptions for the systems with the following impulse responses:

(a) $h[t] = \dfrac{1}{a}e^{-\frac{t}{a}}u(t)$

(b) $h(t) = 2e^{-2t}u(t) - 2te^{-2t}u(t)$

(c) $h[n] = \alpha^n u[n]$, $|\alpha| < 1$

(d) $h[n] = \delta[n] + 2\left(\frac{1}{2}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n]$

3.70 Determine the differential- or difference-equation descriptions for the systems with the following frequency responses:

(a) $H(j\omega) = \dfrac{2 + 3j\omega - 3(j\omega)^2}{1 + 2j\omega}$

(b) $H(j\omega) = \dfrac{1 - j\omega}{-\omega^2 - 4}$

(c) $H(j\omega) = \dfrac{1 + j\omega}{(j\omega + 2)(j\omega + 1)}$

(d) $H(e^{j\Omega}) = \dfrac{1 + e^{-j\Omega}}{e^{-j2\Omega} + 3}$

(e) $H(e^{j\Omega}) = 1 + \dfrac{e^{-j\Omega}}{\left(1 - \frac{1}{2}e^{-j\Omega}\right)\left(1 + \frac{1}{4}e^{-j\Omega}\right)}$

3.71 Consider the RL circuit depicted in Fig. P3.71.

(a) Let the output be the voltage across the inductor, $y_L(t)$. Write a differential-equation description for this system and find the frequency response. Characterize the system as a filter.

(b) Determine and plot the voltage across the inductor, using circuit analysis techniques, if the input is the square wave depicted in Fig. 3.21 with $T = 1$ and $T_o = 1/4$.

(c) Let the output be the voltage across the resistor, $y_R(t)$. Write a differential-equation description for this system and find the frequency response. Characterize the system as a filter.

(d) Determine and plot the voltage across the resistor, using circuit analysis techniques, if the input is the square wave depicted in Fig. 3.21 with $T = 1$ and $T_o = 1/4$.

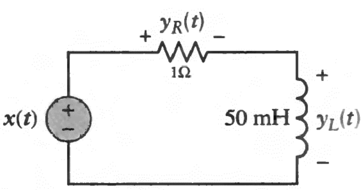

FIGURE P3.71

signal $x(t)$ as one period of the T-periodic signal $\tilde{x}(t)$; that is,

$$x(t) = \begin{cases} \tilde{x}(t), & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases}$$

(a) Graph an example of $x(t)$ and $\tilde{x}(t)$ to demonstrate that, as T increases, the periodic replicates of $x(t)$ in $\tilde{x}(t)$ are moved farther and farther away from the origin. Eventually, as T approaches infinity, these replicates are removed to infinity. Thus, we write

$$x(t) = \lim_{T\to\infty} \tilde{x}(t).$$

(b) The FS representation for the periodic signal $\tilde{x}(t)$ is

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_o t},$$

where

$$X[k] = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \tilde{x}(t)e^{-jk\omega_o t}\,dt.$$

Show that $X[k] = \frac{1}{T}X(jk\omega_o)$, where

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{j\omega t}\,dt.$$

(c) Substitute the preceding definition of $X[k]$ into the expression for $\tilde{x}(t)$ in (b), and show that

$$\tilde{x}(t) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(jk\omega_o)e^{jk\omega_o t}\omega_o.$$

(d) Use the limiting expression for $x(t)$ in (a), and define $\omega \approx k\omega_o$ to express the limiting form of the sum in (c) as the integral

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega.$$

COMPUTER EXPERIMENTS

3.102 Use MATLAB to repeat Example 3.7 for $N = 50$ and (a) $M = 12$, (b) $M = 5$, and (c) $M = 20$.

3.103 Use MATLAB's `fft` command to repeat Problem 3.48.

3.104 Use MATLAB's `ifft` command to repeat Problem 3.49.

3.105 Use MATLAB's `fft` command to repeat Example 3.8.

3.106 Use MATLAB to repeat Example 3.14. Evaluate the peak overshoot for $J = 29, 59$, and 99.

3.107 Let $x(t)$ be the triangular wave depicted in Fig. P3.107.

(a) Find the FS coefficients $X[k]$.

(b) Show that the FS representation for $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=0}^{\infty} B[k]\cos(k\omega_o t).$$

(c) Define the J-term partial-sum approximation to $x(t)$ as

$$\hat{x}_J(t) = \sum_{k=0}^{J} B[k]\cos(k\omega_o t).$$

Use MATLAB to evaluate and plot one period of the Jth term in this sum and $\hat{x}_J(t)$ for $J = 1, 3, 7, 29$, and 99.

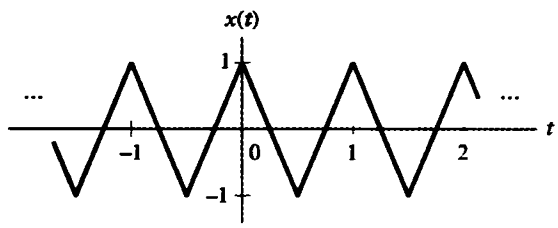

FIGURE P3.107

3.108 Repeat Problem 3.107 for the impulse train given by

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n).$$

3.109 Use MATLAB to repeat Example 3.15, with the following values for the time constant:

(a) $RC = 0.01$ s.

(b) $RC = 0.1$ s.

(c) $RC = 1$ s.

3.110 This experiment builds on Problem 3.71.

(a) Graph the magnitude response of the circuit depicted in Fig. P3.71, assuming that the voltage across the inductor is the output. Use logarithmically spaced frequencies from 0.1 rad/s to 1000 rad/s. You can generate N logarithmically spaced values between 10^{d1} and 10^{d2} by using the MATLAB command `logspace(d1,d2,N)`.