The optimality of (s, S) policies in the dynamic inventory problem

1. Summary

This paper considers the dynamic inventory problem with an ordering cost composed of a unit cost plus a reorder cost. It is shown that if the holding and shortage costs are linear then the optimal policy in each period is always of the (s, S) type. More general conditions on the holding and shortage costs are given which imply the same result. A similar result is also given in the case of a time lag in delivery.

2.Introduction

An elaborate discussion of the history and general features of problem may be found in [2]. We shall content ourselves here with a brief description of the type of model introduction in [1] and discussed by a number of subsequent authors [2,3,4].

A sequence of purchasing decision is made at the beginning of a number of regularly spaced intervals. These purchases contribute to a buildup of inventories which are then depleted by demands during the various intervals. We shall assume the demands to be independent observations from a common distribution function, though varying distributions may be treated by the same technique.

Various costs are charged during the successive periods and the objective is to select the purchasing decisions so as to minimize the expectation of the discounted value of all costs. There are, general speaking, three types of costs: a purchasing cost c (z), where z is the amount purchased; a holding cost h (.), which is a function of excess of supply over demand at the end of the period, and a shortage cost p (.) which is a function of excess demand over supply at the end of the period. Holding or shortage costs are charged at the end of the period and purchasing costs are charged when a purchase is made. We shall assume initially that purchases are made only at the beginning of the period and that delivery is instantaneous. In section 4 the case of a time lag in delivery will be discussed.

If the stock level, immediately after purchases are delivered, is y, then the expected holding and shortage costs to be charged during that period are given by:

A function here!

Let us assume that the inventory problem has a finite horizon of n periods and that the problem is begun with an initial inventory of x units, where x >=0. Let’s Cn(x) represent the expected value of the discounted costs during this n period program if the provisioning is done optimally (the discount factor will be denoted by alpha, and will between 0 and 1). Then it is easy to see that Cn(x) satisfies the following functional equation:

A function here!

and that if yn(x) is the minimizing value of y in (2), then yn(x)-x represents the optimal initial purchase. The purpose of this paper will be to show that under surprisingly weak conditions the optimal policy will be of a very simple type.

Let us begin by review some of the work that has been done on the one-period problem (n=1, and C0=0). The single-period problem is essentially a problem in the calculous and a considerable amount is known about it, in distinction to the sequential problem [2]. The simplest case is when the ordering cost is linear, i.e., c(z)=c\*z. In this case, the optimal policy for the single period model is frequently defined by a single critical number x^, as follows: if x<x^, buy x^-x, and if x>x^, do not buy. Analogous results frequently hold in the sequential problem, the optimal policy being defined by a sequence of critical numbers x1^, x2^, … [3].

A sufficient condition for these results to hold is that L(y) be convex, a condition which obtains when the holding and shortage costs are each convex increasing functions which vanish at the origin.