Control Theory Homework 1

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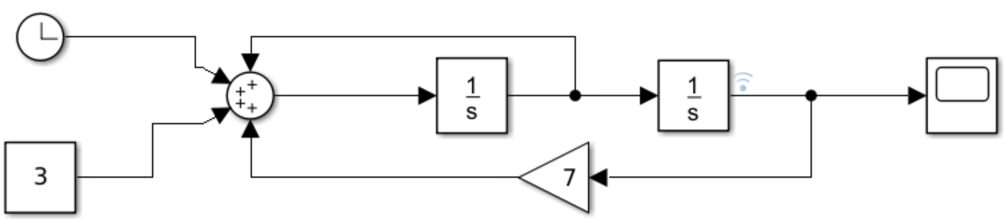
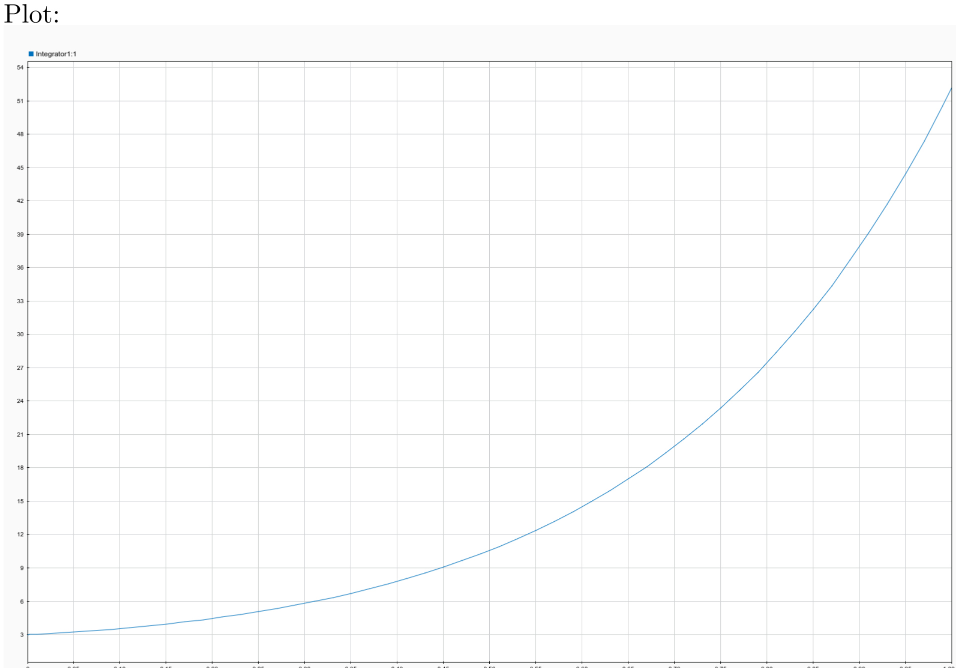
2.A

$$x'' - 5x = x' + t + 2x + 3, x'(0) = 4, x(0) = 3$$
$$x'' = x' + t + 7x + 3$$

Simulink schema:



Plot:



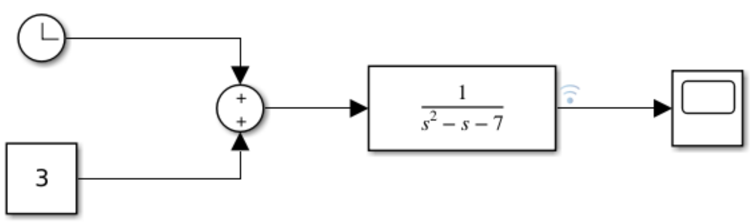
2.B Calculations of TF:

$$x'' = x' + t + 7x + 3$$
$$x'' - x' - 7x = t + 3$$

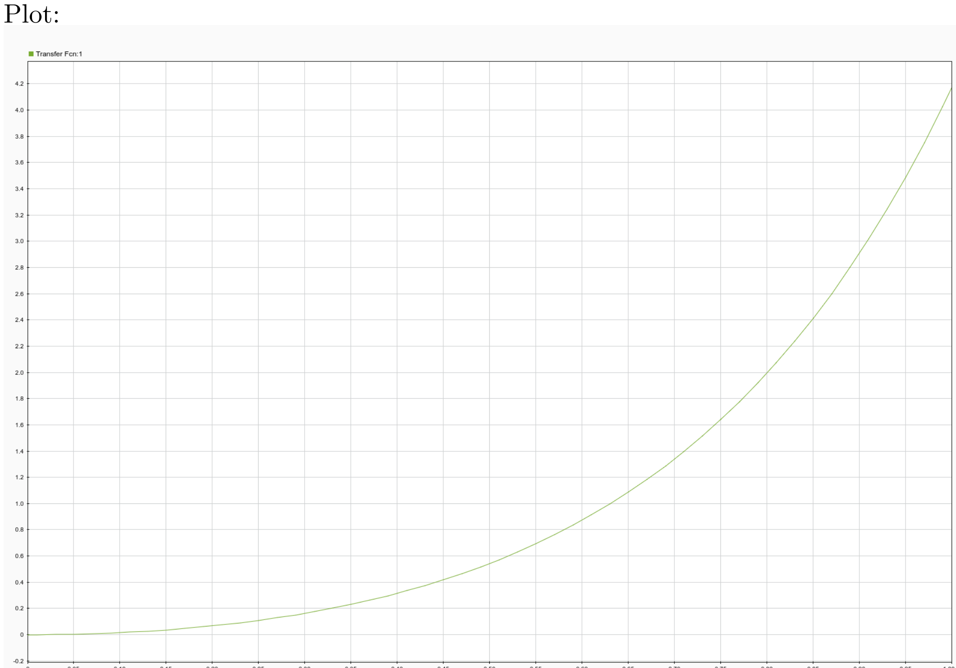
Substitute $u(t) = t + 3$ and apply Laplace transform to both sides.

$$LT(x'' - x' - 7x) = LT(u(t))$$
$$X(p)(p^2 - p - 7) = U(p)$$
$$\frac{X(p)}{U(p)} = \frac{1}{p^2 - p - 7}$$

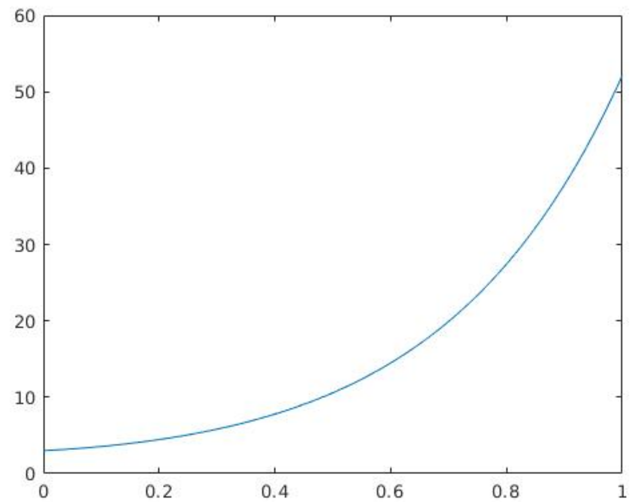
Simulink schema:



Plot:



2.C Plot:



Listing 1: MATLAB code

```
syms x(t);

egn = diff(x, t, 2) - 5*x == diff(x, t, 1) + t + 2*x
    + 3;
Dx = diff(x,t, 1);
cond = [Dx(0) == 4, x(0) == 3];

xSolt(t) = dsolve(egn, cond);

t = linspace(0, 1);

figure
plot(t, xSolt(t));
```

2.D Code:

Listing 2: MATLAB code

```
syms p t X;
f = t + 3;
F = laplace(f, t, p);
X1 = p * X - 3;
X2 = p * X1 - 4;
S = solve(X2 - X1 - 7*X == F, X);
s = ilaplace(S, p, t);
```

3

$$3x'' + 3x' - 3 = 2t - 2, y = 3x'$$
$$x'' + x' - 1 = \frac{2t-2}{3}$$
$$x'' = \frac{2t}{3} - x' + \frac{1}{3}$$

Let $X' = \left[\begin{smallmatrix} x' \\ x'' \end{smallmatrix}\right]$, then $X = \left[\begin{smallmatrix} x \\ x' \end{smallmatrix}\right]$ and our state space model is:

$$\begin{bmatrix} x' \\ x'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix} \cdot t + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$y = \begin{bmatrix} 0 & 3 \end{bmatrix} \cdot X = \begin{bmatrix} 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

4

$$3x'''' + 2x''' - 3x'' + 2x' - 3 = u_1 + 5u_2, y = x' + u_2$$

$$x'''' + \frac{2}{3}x''' - x'' + \frac{2}{3}x' - 1 = \frac{u_1 + 5u_2}{3}$$

$$x'''' = \frac{u_1 + 5u_2}{3} - \frac{2}{3}x''' + x'' - \frac{2}{3}x' + 1$$

Let $X' = \left[\begin{smallmatrix} x' \\ x'' \\ x''' \\ x'''' \end{smallmatrix}\right]$, then $X = \left[\begin{smallmatrix} x \\ x' \\ x'' \\ x''' \end{smallmatrix}\right]$ and our state space model is:

$$\begin{bmatrix} x' \\ x'' \\ x''' \\ x'''' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2}{3} & 1 & -\frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{3} & \frac{5}{3} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \cdot X = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$