

# Control Theory Homework 1

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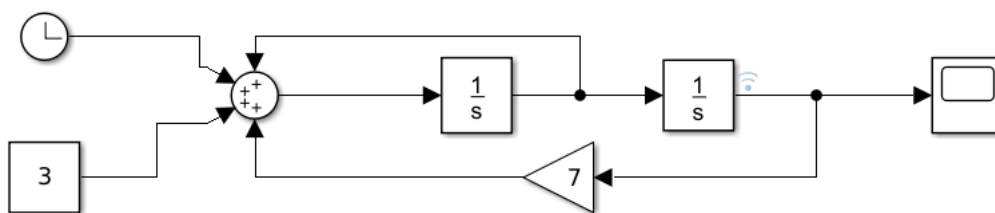
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2.A

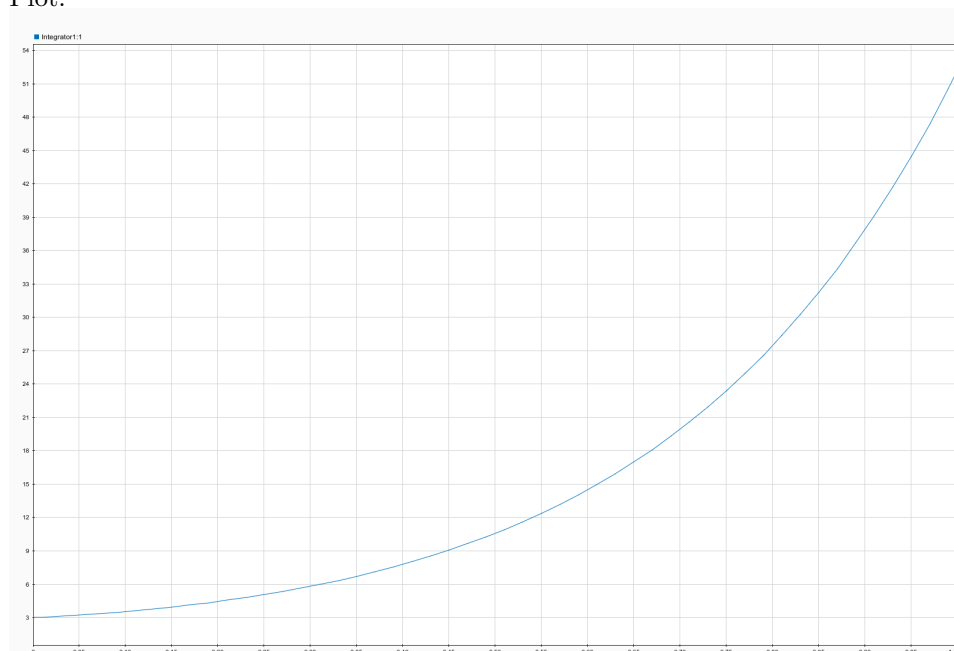
$$x'' - 5x = x' + t + 2x + 3, x'(0) = 4, x(0) = 3$$

$$x'' = x' + t + 7x + 3$$

Simulink schema:



Plot:



## 2.B Calculations of TF:

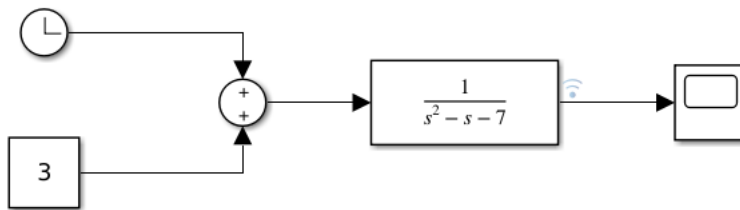
$$x'' = x' + t + 7x + 3$$

$$x'' - x' - 7x = t + 3$$

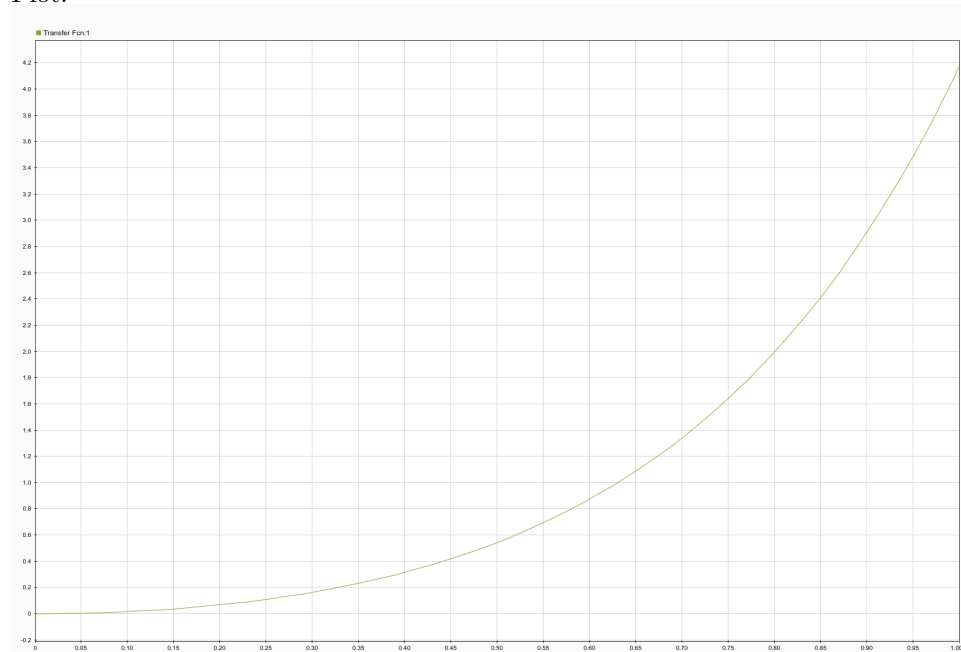
$$(p^2 - p - 7)x = t + 3$$

$$\frac{x}{t + 3} = \frac{1}{p^2 - p - 7}$$

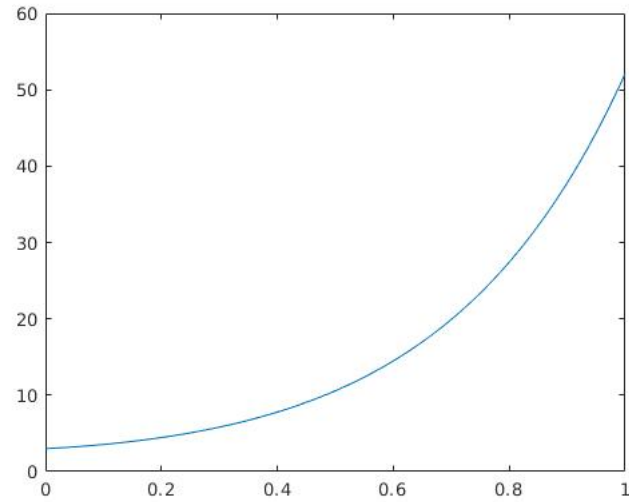
Simulink schema:



Plot:



**2.C** Plot:



Listing 1: MATLAB code

```
1  syms x(t);
2
3  egn = diff(x, t, 2) - 5*x == diff(x, t, 1) + t + 2*x
4      + 3;
5  Dx = diff(x,t, 1);
6  cond = [Dx(0) == 4, x(0) == 3];
7
8  xSolt(t) = dsolve(egn, cond);
9
10 t = linspace(0, 1);
11
12 figure
13 plot(t, xSolt(t));
```

## 2.D Code:

Listing 2: MATLAB code

```

1 syms p t X;
2 f = t + 3;
3 F = laplace(f, t, p);
4 X1 = p * X - 3;
5 X2 = p * X1 - 4;
6 S = solve(X2 - X1 - 7*X == F, X);
7 s = ilaplace(S, p, t);

```

3

$$\begin{aligned}
 3x'' + 3x' - 3 &= 2t - 2, y = 3x' \\
 x'' + x' - 1 &= \frac{2t - 2}{3} \\
 x'' &= \frac{2t}{3} - x' + \frac{1}{3}
 \end{aligned}$$

Let  $X' = \begin{bmatrix} x' \\ x'' \end{bmatrix}$ , then  $X = \begin{bmatrix} x \\ x' \end{bmatrix}$  and our state space model is:

$$\begin{aligned}
 \begin{bmatrix} x' \\ x'' \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix} \cdot t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
 y &= [0 \quad 3] \cdot X = [0 \quad 3] \cdot \begin{bmatrix} x \\ x' \end{bmatrix}
 \end{aligned}$$

4

$$3x'''' + 2x''' - 3x'' + 2x' - 3 = u_1 + 5u_2, y = x' + u_2$$

$$\begin{aligned}
 x'''' + \frac{2}{3}x''' - x'' + \frac{2}{3}x' - 1 &= \frac{u_1 + 5u_2}{3} \\
 x'''' &= \frac{u_1 + 5u_2}{3} - \frac{2}{3}x''' + x'' - \frac{2}{3}x' + 1
 \end{aligned}$$

Let  $X' = \begin{bmatrix} x' \\ x'' \\ x''' \\ x'''' \end{bmatrix}$ , then  $X = \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix}$  and our state space model is:

$$\begin{aligned}
 \begin{bmatrix} x' \\ x'' \\ x''' \\ x'''' \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2}{3} & 1 & -\frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{3} & \frac{5}{3} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
 y &= [0 \quad 1 \quad 0 \quad 0] \cdot X = [0 \quad 1 \quad 0 \quad 0] \cdot \begin{bmatrix} x \\ x' \\ x'' \\ x''' \end{bmatrix} + [0 \quad 1] \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
 \end{aligned}$$