Control Theory Homework 2

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2. Calculating Total TF.

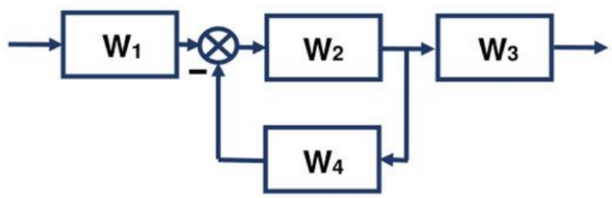


Figure 1: Closed Loop System with 2 inputs.

We can see that W_2 and W_4 are in feedback loop.
Thus we can combine them $W_{24} = \frac{W_2}{1+W_2*W_4}$

Next, notice that $W_1,\ W_{24}\ and\ W_3$ are in series connection.
Thus we can combine them into Total TF $W_0 = W_1 * \frac{W_2}{1+W_2*W_4} * W_3$

After substituting corresponding values of $W_1,\ W_2,\ W_3\ and\ W_4$, we obtain:

$$W_0 = \frac{2}{s+1} * \frac{1}{s} * \frac{1}{1+\frac{1}{s}*\frac{1}{s+3}} * \frac{2}{s+1.5} = \frac{4s+12}{s^4+5.5s^3+10s^2+7s+1.5}$$

Schemas

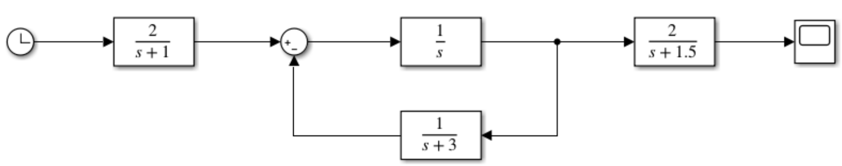


Figure 2: Original Schema.

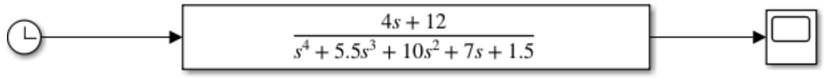


Figure 3: Simplified Schema.

Plots

There are only 2 lines because original and total TFs coincide.

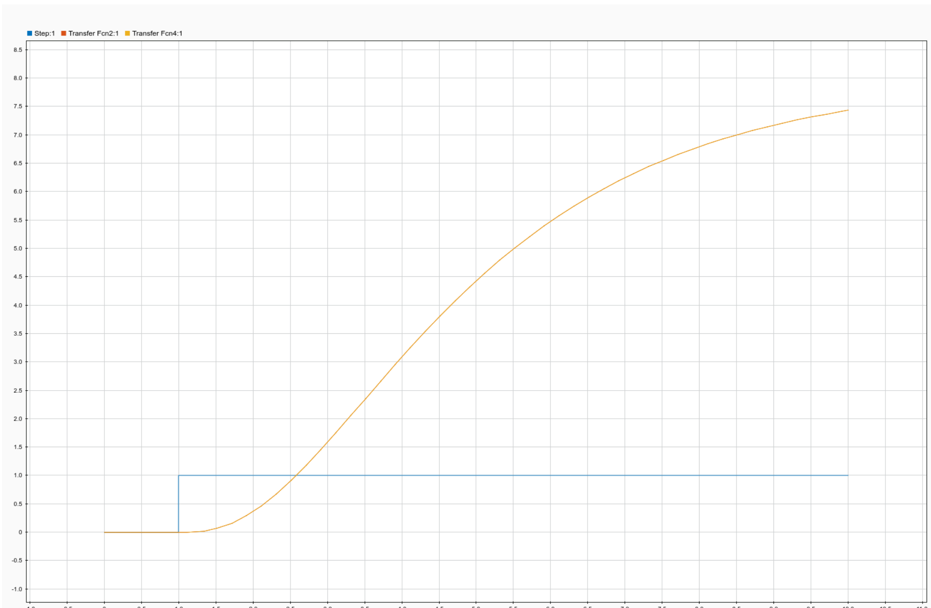


Figure 4: Step Plot.

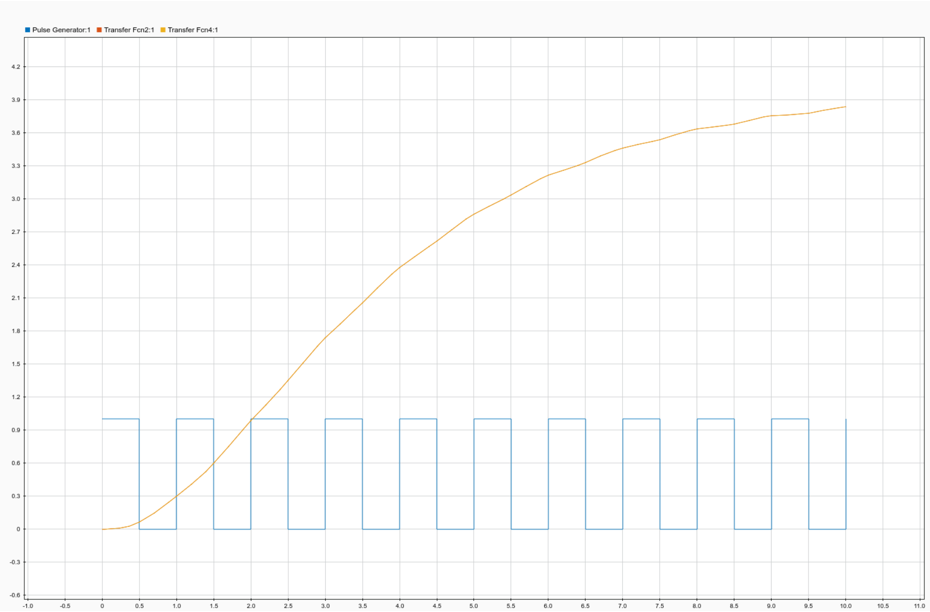


Figure 5: Impulse Plot (Period T = 1, Pulse Width is 50%).

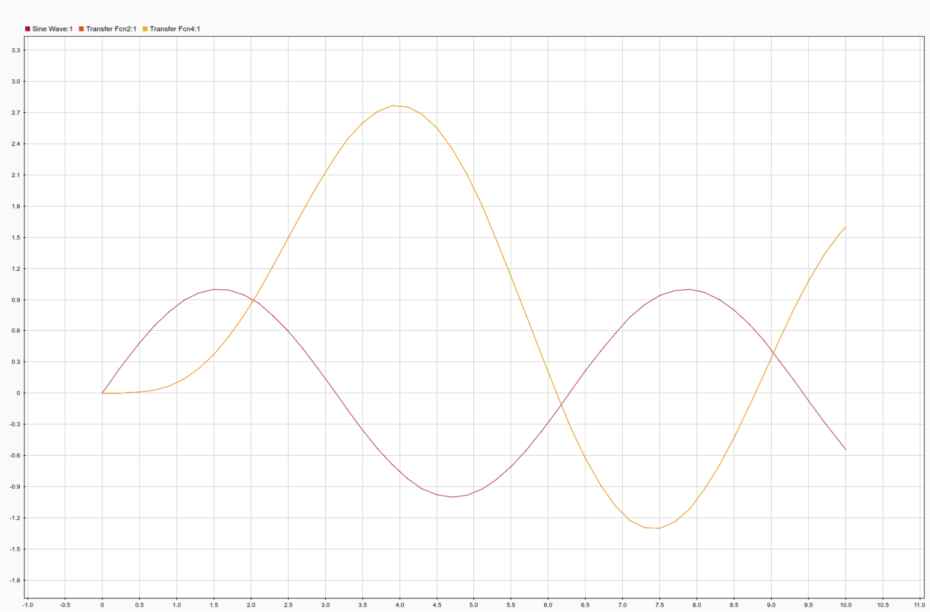


Figure 6: Frequency Plot (Sine is used without any shift or scale).

Bode and Pole-Zero plots.

Sine is used (no scale or shift).

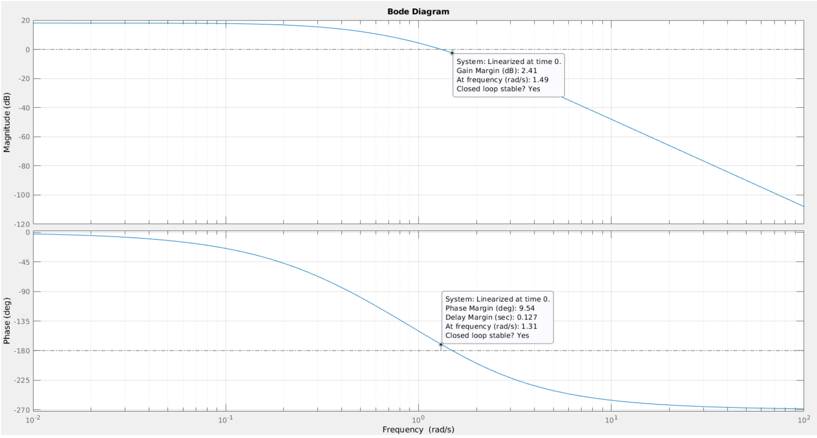


Figure 7: Bode Plot.

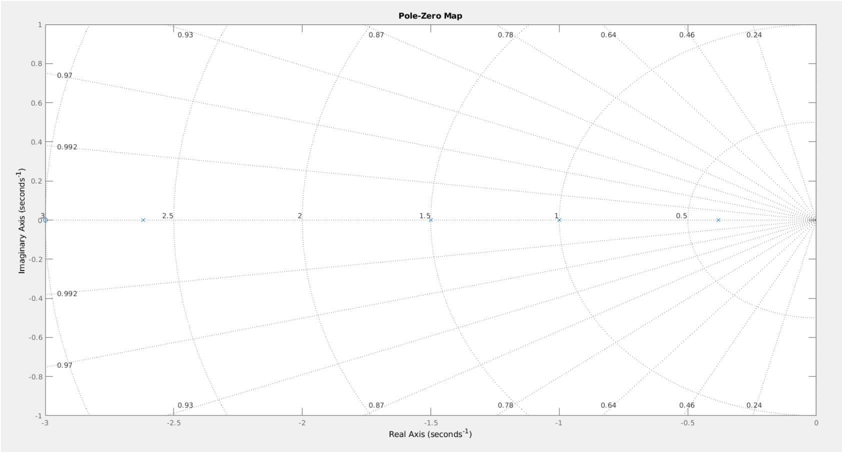


Figure 8: Pole-Zero Plot.

As you can see from pictures, the system is stable (all points in Pole-Zero are in negative real axis).

Bode plot analysis.

Let's rewrite our TF:

$$W_0 = \frac{4s+12}{s^4+5.5s^3+10s^2+7s+1.5} = \frac{4(s+3)}{(s+0.381966)(s+1)(s+1.5)(s+2.61803)}$$

Zero: -3

Poles: -0.381966, -1, -1.5, -2.61803

As you can see on Figure 7, Margin plot intersects zero at 1.49 and Phase plot intersects -180 at 1.31.

3. We have the following schema:

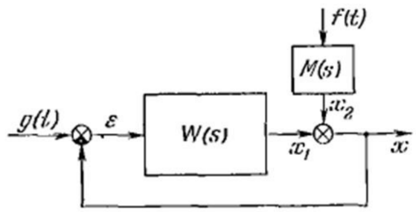


Figure 9: Closed Loop System with 2 inputs.

In system with 2 inputs, to calculate total TF W_0, we need to calculate TFs for each input individually (omitting other input).

Let's find TF with respect to $g(t)$ first.
After "hiding" $f(t)$, we are left with negative feedback loop. Hence,

$$\frac{X}{G} = \frac{W(s)}{1 + W(s)}$$

Now, let's perform the similar actions with respect to $g(t)$.
After "hiding" $g(t)$, we are left with the following schema:

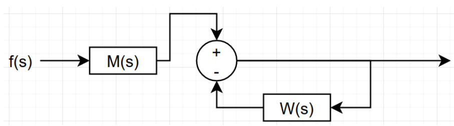


Figure 10: Schema when omitting g(t).

Which is a serial connection of $M(s)$ and a feedback loop. Thus we can reduce it to

$$\frac{X}{F} = \frac{M(s)}{1 + W(s)}$$

After substituting corresponding values of $M(s)$ *and* $W(s)$ we obtain:

$$\frac{X}{G} = \frac{s-1}{s^2-s+1} * \frac{1}{1+\frac{s-1}{s^2-s+1}} = \frac{s-1}{s^2}$$

$$\frac{X}{F} = \frac{s+1}{s+3} * \frac{1}{1+\frac{s-1}{s^2-s+1}} = \frac{s^3+1}{s^3+3s^2}$$

$$X = \frac{X}{G} \cdot G + \frac{X}{F} \cdot F = \frac{s-1}{s^2} \cdot G + \frac{s^3+1}{s^3+3s^2} \cdot F$$

4. At first, let's derive a formula allowing us to find TF of SS.

$$\begin{cases} x' = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$\begin{cases} sX = AX + BU \\ Y = CX + DU \end{cases}$$

$$(sI - A)X = BU$$

$$X = (sI - A)^{-1}BU$$

And now let's substitute X into Y's equation.

$$Y = C(sI - A)^{-1}BU + DU$$

Hence, our TF W_0 is:

$$W_0 = C(sI - A)^{-1}B + D$$

Now, let's substitute corresponding values from variant e into the equation.

$$W_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} (s \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix})^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s+1 & -2 \\ 0 & s-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{s-1} \end{bmatrix} + \begin{bmatrix} 3 \end{bmatrix}$$

$$W_0 = \frac{1}{s-1} + 3$$

$$W_0 = \frac{3s-2}{s-1}$$

5. Use the formula for TF derived in previous exercise:

$$W_0 = C(sI - A)^{-1}B + D$$

Substitute corresponding values from variant e into the equation.

$$W_0 = \begin{bmatrix} 3 & 0 \end{bmatrix} (s \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix})^{-1} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 3 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} s-1 & 2 \\ -1 & s-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 3 \end{bmatrix}$$

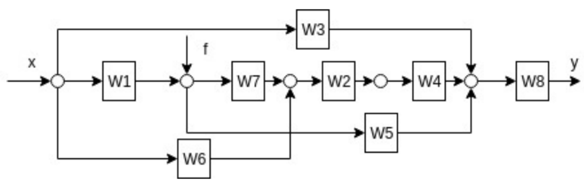
$$W_0 = \frac{1}{s^2 - 2s + 3} \begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} s-1 & -2 \\ 1 & s-1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 3 \end{bmatrix}$$

$$W_0 = \frac{3}{s^2 - 2s + 3} \begin{bmatrix} s-5 & 2s-4 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} \frac{3s-15}{s^2-2s+3} & \frac{3s^2-3}{s^2-2s+3} \end{bmatrix}$$

We got a matrix. Hence, the system has 2 inputs.

6. Notice that we have to inputs f and x.



(e)

Figure 11: Given Schema.

First, let's set x to 0 and calculate TF for f:

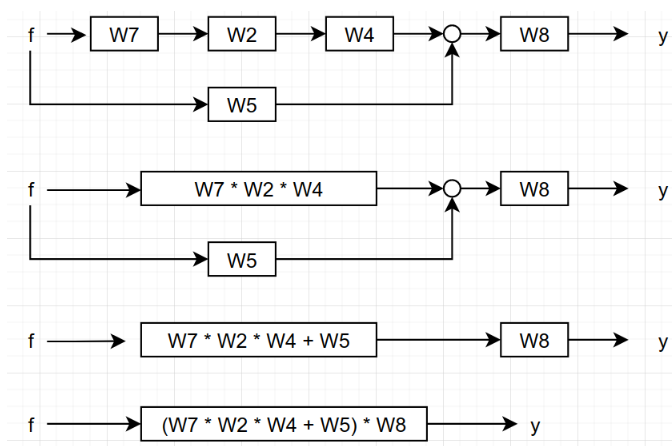


Figure 12: Finding TF for f.

Hence, $\frac{Y}{F} = (W_7 \cdot W_2 \cdot W_4 + W_5) \cdot W_8$

Now, set f to 0 and calculate TF for x.
(Latex doesn't want to put picture here, so, please, scroll down to see steps for finding TF for x).

Hence, $\frac{Y}{X} = (W_1 \cdot W_5 + W_3 + W_2 \cdot W_4 \cdot (W_1 \cdot W_7 + W_6)) \cdot W_8$
As a result, we have

$$\frac{Y}{F} \cdot F + \frac{Y}{X} \cdot X$$

where

$$\frac{Y}{X} = (W_1 \cdot W_5 + W_3 + W_2 \cdot W_4 \cdot (W_1 \cdot W_7 + W_6)) \cdot W_8$$

and

$$\frac{Y}{F} = (W_7 \cdot W_2 \cdot W_4 + W_5) \cdot W_8$$

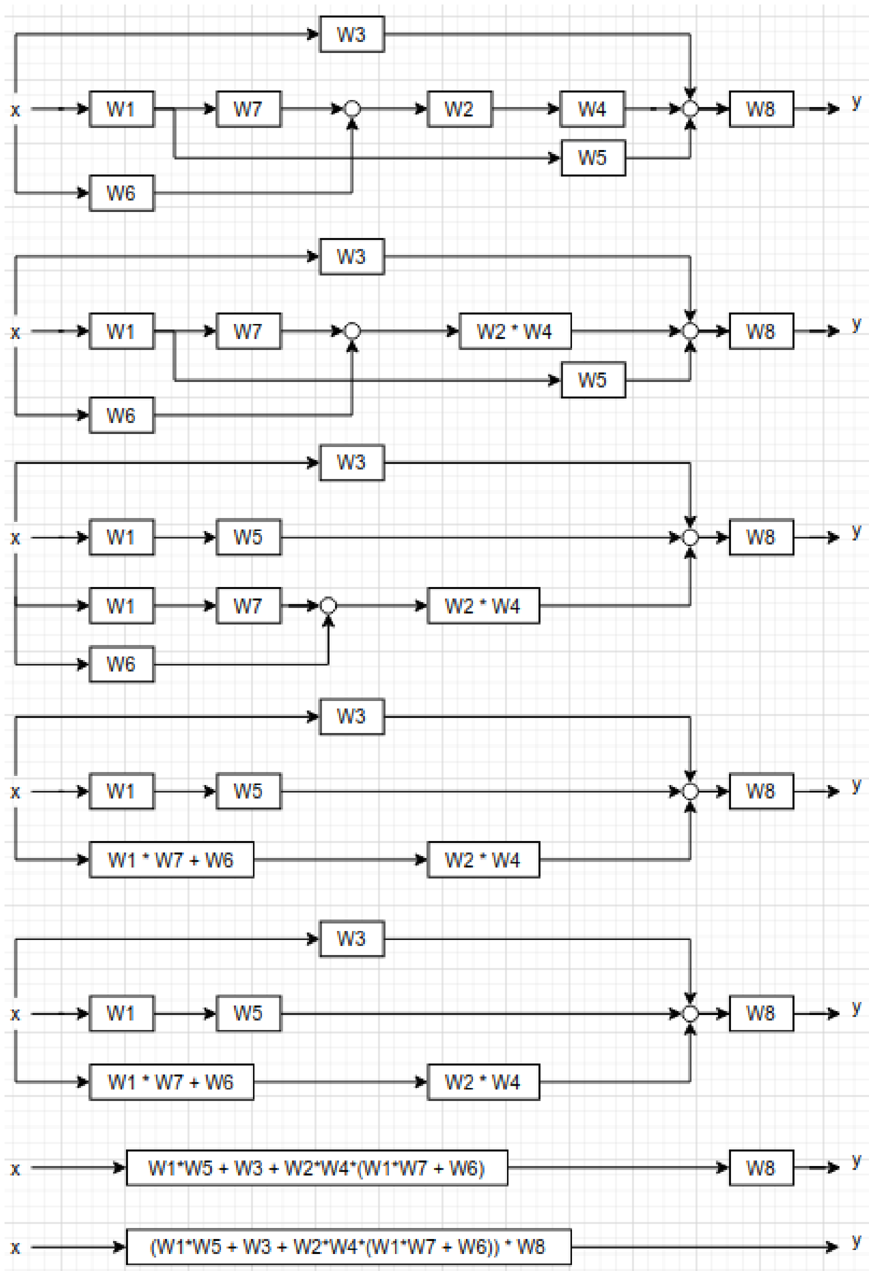


Figure 13: Finding TF for x.