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MANUAL FOR TIDAL HEIGHTS ANALYSIS AND PREDICTION

by

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PREFACE

This report is intended to serve as a user's manual to G. Godin's tidal heights analysis and predictions programs, revised along lines suggested by Godin. In addition to describing input and output of these programs, the report gives an outline of the methods used; a full presentation of which can be found in Godin (1972) and Godin and Taylor (1973).

Users who wish to receive updates of these programs and manual should send their names, addresses, and type of computer used, to the author.

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1 USE OF THE TIDAL HEIGHTS ANALYSIS COMPUTER PROGRAM

1.1 General Description

This program analyses the hourly height tide gauge data for a given period of time. Amplitudes and Greenwich phase lags are calculated via a least squares fit method coupled with nodal modulation for only those constituents that can be resolved over the length of the record. Unless specified otherwise, a standard data package of 69 constituents will be considered for inclusion in the analysis. However, up to 77 additional shallow water constituents can be requested. If the record length is such that certain important constituents are not included directly in the analysis, provision is made for the inference of the amplitude and phase of these constituents from others. Gaps within the tidal record are permitted.

1.2 Routines Required

(1) **MAIN** ...... reads in some of data, controls most of the output and calls other routines.

(2) **INPUT** ...... reads in the hourly height data for the desired time period and checks for errors.

(3) **UCON** ...... chooses the constituents to be included in the analysis via the Rayleigh criterion

(4) **SCFIT2** ...... finds the least squares fit to an equally spaced time series using sines and cosines of specified frequencies as fitting functions.

(5) **VUF** ...... reads required information and calculates the nodal corrections for all constituents.

(6) **INFER** ...... reads required information and calculates the amplitude and phase of inferred constituents, as well as adjusting the amplitude and phase of the constituent used for the inference.

(7) **CHLSKY** ...... solves the symmetric positive definite matrix equation resulting from a linear least squares fit.

(8) **GDAY** ...... returns the consecutive day number from a specific origin for any given date and vice versa.

(9) **ASTR** ...... calculates ephermides for the sun and moon.

(10) **OUTPUT** ...... writes predicted hourly heights to the output file.

(11) **SCULP** ...... scales up amplitudes to compensate for moving average filters.

1.3 Data Input

For a computer run of the tidal heights analysis program, two logical units are used for data input. Logical unit number 8 contains the tidal constituent information while logical unit 4 contains the hourly heights and information relating to the type of analysis and output required. A listing of the standard constituent information for logical unit 8 and a sample set of input for logical unit 4 are given in Appendices 7.1 and 7.2 respectively.

Logical unit 8 expects four types of data:

(i) One card each for all possible constituents, `KONTAB`, to be included in the analysis along with their frequencies, `FREQ`, in cycles/h and the constituent with which they should be compared under the Rayleigh criterion, `KMPR`. The format used is `(4X,A5,3X,F13.10,4X,A5)`. Unless `KONTAB` is specifically designated on logical unit 4 for inclusion, a blank data field for `KMPR` results in the constituent not being included in the analysis.

A blank card terminates this data type.

(ii) Two cards specifying values for the astronomical arguments `SO,HO,PO,ENPO,PPO,DS,DH,DP,DNP,DPP` in the format `(5F13.10)`.

`SO` = mean longitude of the moon (cycles) at the reference time origin;
`HO` = mean longitude of the sun (cycles) at the reference time origin;
`PO` = mean longitude of the lunar perigee (cycles) at the reference time origin;
`ENPO` = negative of the mean longitude of the ascending node (cycles) at the reference time origin;
`PPO` = mean longitude of the solar perigee (perihelion) at the reference time origin.

`DS,DH,DP,DNP,DPP` are their respective rates of change over a 365-day period at the reference time origin.

Although these argument values are not used by the program that was revised in October 1992, in order to maintain consistency with earlier programs, they are still required as input. Polynomial approximations are now employed to more accurately evaluate the astronomical arguments and their rates of change.

(iii) At least one card for all the main tidal constituents specifying their Doodson numbers and phase shifts along with as many cards as are necessary for the satellite constituents. The first card for each such constituent is in the format `(6X,A5,1X,6I3,F5.2,I4)` and contains the following information:

`KON` = constituent name;
`II,JJ,KK,LL,MM,NN` = the six Doodson numbers for `KON`;
`SEMI` = the phase correction for `KON`;
`NJ` = the number of satellite constituents.

A blank card terminates this data type.

If `NJ>0`, information on the satellite constituents follows, three satellites per card, in the format `(11X,3(3I3,F4.2,F7.4,1X,I1,1X))`. For each satellite the values read are:

`LDEL,MDEL,NDEL` = the last three Doodson numbers of the main constituent subtracted from the last three Doodson numbers of the satellite constituent;

`PH` = phase correction of the satellite constituent relative to the phase of the main constituent;

`EE` = amplitude ratio of the satellite tidal potential to that of the main constituent;

`IR` = 1 if the amplitude ratio has to be multiplied by the latitude correction factor for diurnal constituents,
= 2 if the amplitude ratio has to be multiplied by the latitude correction factor for semidiurnal constituents,
= otherwise if no correction is required to the amplitude ratio.

(iv) One card specifying each of the shallow water constituents and the main constituents from which they are derived. The format is `(6X,A5,I1,2X,4(F5.2,A5,5X))` and the respective values are:

`KON` = name of the shallow water constituent;
`NJ` = number of main constituents from which it is derived;
`COEF,KONCO` = combination number and name of these main constituents.

The end of these shallow water constituents is denoted by a blank card.

Logical unit 4 contains six types of data:

(i) One card for the variables `IOUT1,RAYOPT,ZOFF,ICHK,OBSFAC,INDPR,NSTRP` in the format `(I2,2X,F4.2,2X,F10.0,I2,3X,F10.7,2I5)`.

`IOUT1` = 6 if the only output desired is a line printer listing of results,
= 2 if both analysis output and listing are desired;
`RAYOPT` = Rayleigh criterion constant value if different from 1.0;
`ZOFF` = constant to be subtracted from all the hourly heights;
`ICHK` = 0 if the hourly height input data is to be checked for format errors,
= otherwise if this checking to be waived;
`OBSFAC` = scaling factor, if different from 0.01, which will multiply the hourly observations, in order to produce the desired units for the final constituent amplitudes. (e.g. if the hourly observations are in mm/s and the final units are to be ft/sec, then this variable would be set to 0.0032808.);
`INDPR` = 1 if hourly height predictions based on the analysis results are to be calculated and written onto device number 10. If there is inference, this parameter value will also give the rms residual error after inference adjustments have been made,
= 0 if no such predictions are desired;
`NSTRP` = number of successive moving average filters that have been applied to the original data.

If `NSTRP>0`, then `TIMINT` and `(LSTRP(I),I=1,NSTRP)` will be read on a following card, in the format `(F10.0,10I5)`, and suitable amplitude corrections will be applied to compensate for the smoothing effect of these filters.

`TIMINT` = sampling interval, in minutes, of the original unfiltered record;

`(LSTRP(J),J=1,NSTRP)` = number of consecutive observations used in computing each of the `NSTRP` moving average filters.

(ii) One card for each possible inference pair. The format is `(2(4X,A5,E16.10),2F10.3)` and the respective values read are:

`KONAN & SIGAN` = name and frequency of the analysed constituent to be used for the inference;
`KONIN & SIGIN` = name and frequency of the inferred constituent;
`R` = amplitude ratio of `KONIN` to `KONAN`;
`ZETA` = Greenwich phase lag of the inferred constituent subtracted from the Greenwich phase lag of the analysed constituent.

These are terminated by one blank card.

(iii) One card for each shallow water constituent, other than those in the standard 69 constituent data package, to be considered for inclusion in the analysis. The Rayleigh comparison constituent is also required and the additional shallow water constituent must be found in data type (i) of logical unit 8, but have a blank data field where the Rayleigh comparison constituent is expected. The format is `(6X,A5,4X,A5)` and a blank card is required at the end.

(iv) One card in the format `(I1,1X,10I2)` specifying the following information on the period of the analysis:

`INDY` = 8 indicates an analysis is desired for the upcoming period;
= 0 indicates no further analyses are required;
`IHH1,IDD1,IMM1,IYY1,ICC1` = hour, day, month, year and century of the beginning of the analysis (measured in time `ITZONE` of input data (v));
`IHHL,IDDL,IMML,IYYL,ICCL` = hour, day, month, year and century of the end of the analysis.

If `ICC1` or `ICCL` are zero, their value is reset to 19.

(v) One card in the format `(I1,4X,A5,3A6,A4,A3,1X,2I2,I3,I2,5X,A5)` containing the following information on the tidal station:

`INDIC` = 1 if J card output is desired (no longer used),
= otherwise if not;
`KSTN` = tidal station number;
`(NA(J),J=1,4)` = tidal station name (22 characters maximum length);
`ITZONE` = time zone of the hourly observations;
`LAD,LAM` = station latitude in degrees and minutes;
`LOD,LOM` = station longitude in degrees and minutes;
`IREF` = reference station number.

(vi) The hourly height data cards contain the following information in the format `(I1,1X,I5,7X,3I2,12A4)`.

`KOLI` = 1 or 2 indicates whether this specific card is the first or second

one for that day,
= otherwise indicates a non-data card which is ignored;

`JSTN` = tidal station number;

`ID,IM,IY` = day, month and year of the heights on this card.

`(KARD(J),J=1,12)` = hourly heights in integer form. The final constituent amplitudes unless a are in units 1/100 of those for the hourly height nonzero for `OBSFAC` is read (see (i)). Missing values should be specified as a blank field or 9999.

When `KOLI=1`, the first hourly height on the data card is assumed to be at 0100 h and when `KOLI=2`, it is assumed to be at 1300 h. The time zone of these observations determines the nature of the Greenwich phase lag (see Section 2.3.1).

After the initial analysis of a computer run is completed, control returns to input (iv). Successive cards are read then until either a 0 or 8 value is found for `INDY`.

The hourly height data cards need not begin and end so as to include exactly the analysis period. The program ignores data outside this range. However if more than one analysis is desired from a single job submission and hourly height data cards do extend beyond the first analysis period, care should be taken to ensure that one of these cards does not have `KOLI=0` or blank, otherwise the job will be terminated. This is because all successive cards after the one containing the last hour of the desired analysis period are read in input (iv) format.

1.4 Output

Three logical units are used for the output of results from the tidal heights analysis program. Device number 6 is the line printer, 2 is used for analysis results and 10 contains hourly synthesized values based on the analysis results; 6 is required for all program runs whereas the use of 2 and 10 is controlled by the input variables `IOUT1` and `INDPR` which are read from device 4.

Recommendations for the use of moving average filters on the elevation data prior to submission for analysis, and the scaling compensation method used in the improved analysis program are found in Foreman (1978) or Godin (1972).

When `IOUT1` is 6, `INDPR` is other than 1, and there are no inferred constituents, the only output is two pages on the line printer. The first of these lists the constituents included in the least squares fit, their frequencies in cycles/h (although eight decimal places are given, depending on computer accuracy, less than this number may be significant), the C and S coefficient values (see Section 2.2.1) measured in units `OBSFAC` times those for the hourly heights, and their respective standard deviation estimates. It also specifies the number of hourly height observations (excluding gaps) within the analysis period, the average and standard deviation of the original observations, the root mean square residual error, and the matrix condition number. In the columns titled `AL`, `GL`, `A`, and `G`, the second page respectively lists the amplitudes and phases (degrees) obtained for each constituent from the C and S coefficient values, and the same amplitudes and phases after nodal modulation and astronomical argument adjustments. The initial and final hour of the analysis are also specified along with the Rayleigh criterion constant ('separation'), the midpoint of the analysis period, the total number of possible hourly observations in the analysis period, and the total number of possible observations used in the analysis. This last value includes gaps in the record and is the largest odd number less than or equal to the total number of possible hourly observations (if the total number of possible hourly observations is an even number, the last hour is ignored). If there is at least one inferred constituent, page 2 results are repeated with the inclusion of inferred constituents and appropriate adjustments to the constituents from which the inferences were made. Appendix 7.3 lists the final page of results obtained from the input value of Appendix 7.2.

The only effect of changing the value of IOUT1 to 2 (regardless of INDPR's value) is to store on file 2, the same information as the second (and third) page(s) of the line printer. The list of constituent names, amplitudes and Greenwich phase lags begins on line 5 of this file and is in the correct format for input to the tidal heights prediction program, namely (5X,A5,28X,F8.4,F7.2).

When INDPR equals 4, device 10 will contain hourly predictions calculated from the analysis results. Values are specified only for the analysis period, including those intervals where there were gaps in the original record, and are in the same measurement units and scaling as the original data. The format used is the same as for input type (vi) of logical unit 4.

1.5 Program Conversion, Modifications, Storage and Dimension Guidelines

The source program and constituent data package described in this manual have been tested on various mainframe, PC and workstation computers at the Institute of Ocean Sciences, Patricia Bay. Although as much of the program as possible was written in basic FORTRAN, some changes may be required before the program and data package can be used on other installations. Please write or call the author if any problems are encountered.

The program in its present form requires approximately 68,000 bytes for the storage of its instructions and arrays.

Changing the number or type of constituents in the standard data package may require some alterations to the analysis program. If constituents are added to the standard data package, the dimensions of several arrays may have to be altered. Restrictions on the minimum dimension of such arrays are now given.

Let

- MTOT be the total number of possible constituents contained in the data package (presently 146),
- M be the number of constituents considered for inclusion in the analysis (presently 69 plus the number of shallow water constituents specifically designated for inclusion),
- MCON be the number of main constituents in the standard data package (presently 45),
- MSAT be the sum of the total number of satellites for these main constituents and the number of main constituents with no satellites (presently 162 plus 8 for the version of the constituent data package, listed in Appendix 7.1, that contains no third-order satellites for both N_2 and L_2),
- MSHAL be the sum for all shallow water constituents, of the number of main constituents from which each is derived (presently 251).

Then in the main program, arrays KONTAB,FREQ and KMPR should have minimum dimension MTOT+1; arrays KON,C,S,SIG,ERC,ERS,A,EPS,KO,AA and GD should have minimum dimension M; array NKON should have dimension at least as large as the number of extra shallow water constituents specifically designated for analysis inclusion (its present maximum is 15); and arrays Z and XP should be large enough to contain the hourly heights (and gaps) in the analysis period (its present maximum is 375 days).

In subroutine **INPUT** array Z should be dimensioned the same as in the main program, while KARD and IHT should be dimensioned 12.

In the other subroutine **OUTPUT**, Z is in a common block and should be dimensioned as in the main program, XP is in the argument list and need only have dimension 2, and arrays MONTH and IHT should have dimension 12 and 24 respectively.

In subroutine **VUF**, arrays VU and F should have minimum dimension MTOT; arrays KON and NJ should have minimum dimension MTOT+1; arrays II,JJ,KK,LL,MM,NN and SEMI should have minimum dimension MCON+1; arrays EE,LDEL,MDEL,NDEL,IR and PH should have minimum dimension MSAT; and KONCO, COEF should have minimum dimension MSHAL+4.

In subroutine **INFER**, arrays KONAN,KONIN,SIGAN,SIGIN,R and ZETA can presently accommodate a maximum of nine inferred constituents.

In subroutine **SCFIT2**, arrays X,XP,C,S,ERC,ERS and F should have the same dimension as Z,XP,C,S,ERC,ERS and SIG in the main program and arrays RHS and A should have minimum dimension 2M-1 and M(2M-1) respectively. AC and AS should have the size of A and care should be taken that through their equivalence relationships, neither AC and AS, nor RHSC and RHSS overlap.

Finally, in subroutine **CHLSKY**, arrays A and F should have minimum dimensions M(2M-1) and 2M-1 respectively.

2 TIDAL HEIGHTS ANALYSIS PROGRAM DETAILS

2.1 Constituent Data Package

2.1.1 Astronomical variables

The astronomical variables required by the tidal analysis program were used by Doodson (1921) in his development of the tidal potential. From them one can calculate the position of the sun or moon, and hence the tide generating forces, at any time. These varaibles are:

$$\begin{aligned} S(t) &= \text{ mean longitude of the moon;} \\ H(t) &= \text{ mean longitude of the sun;} \\ P(t) &= \text{ mean longitude of the lunar perigee;} \\ N'(t) &= \text{ negative of the longitude of the mean ascending node;} \\ P'(t) &= \text{ mean longitude of the solar perigee (perihelion).} \end{aligned}$$

For H, N' and P' these longitudes are measured along the ecliptic eastward from the mean vernal equinox position at time t; while for S and P they are measured in the ecliptic eastward from the mean vernal equinox position at time t to the mean ascending mode of the lunar orbit, and then along this orbit. Together with the rates of change of these variables, τ the local mean lunar time, and the Doodson numbers for each tidal constituent, one can calculate the constituent frequencies, their astronomical argument phase angles, V, and their nodal modulation phase, u, and amplitude, f, corrections.

The values of the astronomical variables and constituent frequencies in the program are calculated using the power series expansion formulae given on pages 98 and 107 of the *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac* (1961). These formulae were derived from Newcomb's *Tables of the Sun* and a revision of Brown's lunar theory (used in the development of his *Tables of Motion of the Moon*) so that it is in accord with Newcomb's.

(For those interested, even higher ordered approximations can be found in *Astronomical Formulae for Calculators* by Jean Meeus.) In particular, the astronomical variables and frequencies are calculated at the central hour of the analysis period and in order to gain precision t_0, the reference time origin, is taken to be 0000 ET.[1] This latter date, it was felt, would be closer to the analysis period of most records than the previous choice of 0000 ET January 1, 1901, and hence would yield more accurate results via the linear approximation.

In keeping with the choice of reference time origin and astronomical variable specifications, t should be measured in Ephemeris time. However, the correction from Universal time is irregular and in most cases small, so it has been assumed for computational purposes that all observations are recorded in ET.

2.1.2 Choice of constituents and Rayleigh comparison pairs

There is a maximum of 146 possible tidal constituents that can be included in the tidal analysis, 45 of these are astronomical in origin (main constituents) while the remaining 101 are

[1] Ephemeris Time (ET) is the uniform measure of time defined by the laws of dynamics and determined in principle from the orbital motion of the Earth as represented by Newcomb's *Tables of the Sun*. Universal or Greenwich Mean Time is defined by the rotational motion of the Earth and is not rigorously uniform.

shallow water constituents.[2] Because computation time (and cost) of the computer program increases approximately as the square of the number of constituents included in the analysis, and because for many tidal stations, most of the shallow water constituents are insignificant, a smaller standard package was seen as adequate for general use. Based on the suggestions of G. Godin, it was decided that this package contain all the main constituents and 24 of the shallow water. However, provision was made so that other shallow water constituents among the 77 remaining could be included if desired.

The Rayleigh comparison constituent is used for the purpose of deciding whether or not a specific constituent should be included in the analysis. If F_0 is the frequency of such a constituent, F_1 is the frequency of its Rayleigh comparison constituent and T is the time span of the proposed record to be analysed, then the constituent will be included in the analysis only if $|F_0 - F_1|T \geq RAY$. RAY is commonly given the value 1 although it can be specified differently in the program.

In order to determine the set of Rayleigh comparison pairs, it is important to consider, within a given constituent group (e.g. diurnal or semidiurnal), the order of constituent inclusion in the analysis as T (the time span of the record to be analysed) increases. Assuming this point of view, the specific objectives used when constructing the set listed in Appendix 7.1 were:

(i) within each constituent group, when possible, have the order of constituent selection correspond with decreasing magnitude of tidal potential amplitude (as calculated by Cartwright and Edden (1973)),

(ii) when possible, compare a candidate constituent with whichever of the neighbouring, already selected constituents, that is nearest in frequency,

(iii) when there are two neighbouring constituents of relatively equal tidal potential amplitude, rather than waiting until the record length is sufficient to permit the selection of both at the same time (i.e. by comparing them to each other), choose a representative of the pair whose inclusion will be as early as possible. This will give information sooner about that frequency range, and via inference, still enable some information to be obtained on both constituents.

The Rayleigh comparison pairs chosen for the low frequency, diurnal, semidiurnal and terdiurnal constituent groups are given in Tables 1, 2, 3 and 4 respectively. Figures given for the length of record required for constituent inclusion assume a Rayleigh criterion constant value (input variable `RAYOPT`) of 1.0.

$2Q_1$ and SIG_1 provide an example of objective (iii). Because $2Q_1$ has a greater frequency separation for Q_1 and hence would appear in an analysis of shorter record length than SIG_1, it was chosen as the representative.

However, it can be seen in several cases, that it was not possible or feasible to adhere to all the objectives just outlined. Choosing a Rayleigh comparison constituent from the list of those constituents already included in the analysis proved to be difficult near the frequency edges of constituent groups. Upward arrows indicate failure to uphold this objective. OO_1 is such a case. For it, the potential comparison pairs were SO_1, K_1 and J_1. The first of these would result in both SO_1 and OO_1 appearing at the same later time than had J_1 or K_1 been

[2] The criterion for selecting these main constituents was to include all the diurnal and semidiurnal constituents with Cartwright and Edden (1973) tidal potential amplitudes greater than 0.00250, along with M_3 and the most important low frequency constituents. Section 2.1.3 gives the analogous shallow water constituent criterion.

Table 1 Order of Slower-than-Diurnal Constituent Selection in Accordance with the Rayleigh Criterion. Tidal Potential Amplitude for Main Constituents Shown within Brackets. Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Length of Record (h) Required for Constituent Inclusion	Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents												
		0.11407		0.11408		1.08162		0.20237		1.30978		0.22816	
	ZO		SA		SSA		MSM		MM		MSF		MF
13	ZO												
355											(1369) MSF		
764									(8254) MM				
4383					(7281) SSA								(15647) MF
4942							(1579) MSM						
8766			(1156) SA										

Table 2 Order of Constituent Selection in Accordance with the Rayleigh Criterion. Tidal Potential Amplitude for Main Constituents is Shown within Brackets. Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents

1.30978, 0.20237, 1.30978, 0.20237, 1.30978, 0.22816, 1.08162, 0.22816, 0.20237, 0.96758, 0.11407, 0.11408, 0.11407, 0.11407, 0.11408, 1.08162, 0.20237, 1.30978, 0.22816, 151215

Length of Record (h) Required for Constituent Inclusion	ALP_1	$2Q_1$	SIG_1	Q_1	RHO_1	O_1	TAU_1	BET_1	NO_1	CHI_1	PI_1	P_1	S_1	K_1	PSI_1	PHI_1	THE_1	J_1	SO_1	OO_1	UPS_1
24														(53011) K_1							
328						(37694) O_1															
651																				(1624) OO_1	
662		(955) $2Q_1$		(7217) Q_1					(2964) NO_1									(2964) J_1			(311) UPS_1
764	(278) ALP_1																				
4383							(493) TAU_1	(278) BET_1				(17543) P_1				(755) PHI_1			SO_1		
4942			(1152) SIG_1		(1371) RHO_1					(567) CHI_1							(567) THE_1				
8767											(1028) PI_1		(416) S_1		(422) PSI_1						

Table 3 Order of Semidiurnal Constituent Selection in Accordance with the Rayleigh Criterion. Tidal Potential Amplitude for Main Constituents is Shown within Brackets. Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents

OQ_2 0.20237 EPS_2 1.30978 $2N_2$ 0.20237 MU_2 1.30978 N_2 0.20237 NU_2 1.10741 GAM_2 0.08830 H_1 0.11408 M_2 0.11408 H_2 0.11408 MKS_2 1.08162 LDA_2 0.20237 L_2 1.19570 T_2 0.11407 S_2 0.11407 R_2 0.11408 K_2 1.28399 MSN_2 0.22816 ETA_2

Length of Record (h) Required for Constituent Inclusion	OQ_2	EPS_2	$2N_2$	MU_2	N_2	NU_2	GAM_2	H_1	M_2	H_2	MKS_2	LDA_2	L_2	T_2	S_2	R_2	K_2	MSN_2	ETA_2
13									(90809) M_2										
355															(42248) S_2				
662					(17386) N_2														(643) ETA_2
764		(671) EPS_2		(2776) MU_2									(2597) L_2						
4383											MKS_2						(11498) K_2	MSN_2	
4942	(259) OQ_2		(2301) $2N_2$			(3302) NU_2						(670) LDA_2							
8767								(313) H_1		(277) H_2				(2476) T_2		(355) R_2			
11326							(273) GAM_2												

Table 4 Order of Terdiurnal Constituent Selection in Accordance with the Rayleigh Criterion. Tidal Potential Amplitude for Main Constituents is Shown within Brackets. Lines with Arrows Denote Links with Rayleigh Comparison Pairs.

Length of Record (h) Required for Constituent Inclusion	Frequency Differences (cycles/h) $\times 10^3$ between Neighbouring Constituents				
	1.52505	1.29689	0.22816	2.82193	
	MO_3	M_3	SO_3	MK_3	SK_3
25		(1188) M_3			
355					SK_3
656	MO_3			MK_3	
4383			SO_3		

chosen. Hence, information about OO_1 would be unnecessarily delayed. Although, due to the tidal potential amplitude of J_1, objective (i) is violated with both the second and third choices, it was felt that the third was a better compromise. With it, OO_1 only appears 11 h sooner than J_1.

K_2 is an example of an unavoidable violation of objective (i). Because it is so close in frequency to S_2, its importance as a major semidiurnal constituent does not insure it an early inclusion in the analysis package.

Because shallow water constituents do not have a tidal potential amplitude, objective (i) does not apply to them. However, based on his experience, Godin was able to suggest a hierarchy of their relative importance. A further criteria used when selecting comparison pairs for them was that no shallow water constituent should appear in an analysis before all the main constituents, from which it is derived, have also been selected. Table 5 shows that this has

Table 5 Shallow Water Constituents in the Standard Data Package.

Shallow Water Constituent	Record Length (h) Required for Constituent Inclusion	Component Main Constituents and Record Lengths (h) Required for Their Inclusion in the Analysis					
SO_1	4383	S_2	355	O_1	328		
MKS_2	4383	M_2	13	K_2	4383	S_2	356
MSN_2	4383	M_2	13	S_2	355	N_2	662
MO_3	656	M_2	13	O_1	328		
SO_3	4383	S_2	355	O_1	328		
MK_3	656	M_2	13	K_1	24		
SK_3	355	S_2	355	K_1	24		
MN_4	662	M_2	13	N_2	662		
M_4	25	M_2	13				
SN_4	764	S_2	355	N_2	662		
MS_4	355	M_2	13	S_2	355		
MK_4	4383	M_2	13	K_2	4383		
S_4	355	S_2	355				
SK_4	4383	S_2	355	K_2	4383		
$2MK_5$	24	M_2	13	K_1	24		
$2SK_5$	178	S_2	355	K_1	24		
$2MN_6$	662	M_2	13	N_2	662		
M_6	26	M_2	13				
$2MS_6$	355	M_2	13	S_2	355		
$2MK_6$	4383	M_2	13	K_2	4383		
$2SM_6$	355	S_2	355	M_2	13		
MSK_6	4383	M_2	13	S_2	355	K_2	4383
$3MK_7$	24	M_2	13	K_1	24		
M_8	26	M_2	13				

been upheld for all shallow water constituents in the standard 69 constituent data package.

We recommend that the objectives outlined here be employed when choosing the Rayleigh comparison constituent for any additions to the list of possible constituents to be included in the analysis.

2.1.3 Satellite constituents and nodal modulation

Doodson's (1921) development of the tidal potential contains a very large number of constituents. Due to the great length of record required for their separation, several of these can be considered, for all intents and purposes, unanalysable. The standard approach to this problem is to form clusters consisting of all constituents with the same first three Doodson numbers. The major contributor in terms of tidal potential amplitude lends its name to the cluster and the lesser constituents are called satellites.

The method of analysis uses this main and satellite constituent approach in the following manner. The Rayleigh criteria is applied to the main constituent frequencies to determine whether or not they are to be included in the analysis. For each of those so chosen, we analyse at its frequency and obtain an apparent amplitude and phase. However, because these results are actually due to the cumulative effect of all the constituents in that cluster, an adjustment is made so that only the contribution due to the main constituent is found. This adjustment is called the nodal modulation.

In order to make the nodal modulation correction to the amplitude and phase of a main constituent, it is necessary to know the relative amplitudes and phases of the satellites. As is commonly done, it is assumed in this program that the same relationship as is found with the equilibrium tide (tidal potential), holds with the actual tide. That is, the tidal potential amplitude ratio of a satellite to its main constituent is assumed to be equal to the corresponding tidal heights amplitude ratio, and the difference in tidal potential phase equals the difference in tidal height phase.

The source of the tidal potential amplitude ratio, as found in the constituent data package of Appendix 7.1, is Cartwright and Tayler (1971) and Cartwright and Edden (1973). Using new computation methods and the latest values for the astronomical constants, they obtained more accurate results than those from the previously used Doodson computations. It should be noted that in several cases (whenever the satellite arises via the third-order term), this version of the constituent data package requires that the amplitude ratio be multiplied by a latitude correction factor.

Phase differences between satellites and main constituents arise when the tidal potential development yields different trigonometric terms for these constituents. The common convention is to express all terms in cosine form and so an extra $-\frac{1}{4}$ cycle phase shift is introduced if the term was originally a sine. Satellites requiring such a shift are called third order. A further $\frac{1}{2}$ cycle change is also introduced when all negative amplitudes are made positive.

Because several test analyses indicate less consistent results when third-order satellites are included in the N_2 and L_2 nodal modulation, Godin has decided to delete these from the present standard constituent data package. Instead he suggests that the results of analyses with this package should be compared with those of previous analyses in order to find the most suitable adjustment for these constituents.

The only other main constituents that do not have all their satellites included for nodal modulation are the slow frequency constituents. For them, no satellites are specified. Because low frequency noise may be as much as an order of magnitude greater than the satellite contributions, and M_m, M_{sf} and M_f when they are detectable are often of shallow water origin, the effect of making corrections for the expected satellites would be to obscure further, rather than clarify the actual low frequency periodic signal.

Section 2.3.2 gives further details on the nodal modulation correction.

2.1.4 Shallow water constituents

Shallow water tidal constituents arise from the distortion of main constituent tidal oscillations in shallow water. Because the speed of propagation of a progressive wave is approximately proportional to the square root of the depth of water in which it is travelling, shallow water has the effect of retarding the trough of a wave more than the crest. This distorts the original sinusoidal wave shape and introduces harmonic signals that are not predicted in tidal potential development. The frequencies of these derived harmonics can be found by calculating the effect of non-linear terms in the hydrodynamic equations of motion on a signal due to one or more main constituents (see Godin (1972), pp. 154–164 for further details).

The shallow water constituents chosen for inclusion in the standard 69 constituent data package were suggested by G. Godin. They are listed in Table 5 and are derived only from the largest main constituents, namely M_2, S_2, N_2, K_2, K_1 and O_1, using the lowest types of possible interaction. The 77 additional shallow water constituents that can be included in the analysis if so desired are derived from lesser main constituents and higher types of interaction. In the constituent data package listing of Appendix 7.1, they can be spotted by their lack of a Rayleigh comparison constituent.

When shallow water effects are noticeable, main constituents, if they are close in frequency, may coexist or be masked by constituents of non-linear origin. The resultant nodal modulation will be due to the pair and thus will not coincide to the calculated modulation of the main constituent. In suspected cases, the effectiveness of nodal corrections in a series of successive analyses will indicate the presence of pairs or emphasize the predominance of one constituent over the other. Table 6 (taken from unpublished notes of Godin) lists compound constituents which may coexist with or mask constituents of direct astronomical origin. In all cases except SO_1 and MO_3, the main rather than the compound constituent is included in the standard constituent data package.

2.2 The Least Squares Method of Analysis

2.2.1 Formulation of the problem

The first stage in the actual analysis of tidal records is the least squares fit for constituent amplitude and phase. If the tidal record is of minimum length 13 h, the present program and data package insure that the constant constituents Z_0 and M_2 are always included in the analysis. If σ_j for $j = 1, M$ are the frequencies (cycles/h) of the other tidal constituents chosen for inclusion in the analysis by the Rayleigh criterion, then the problem is to find the amplitudes, A_j, and phases, ϕ_j, of the function $C_0 + \sum_{j=1}^{M} A_j \cos[2\pi(\sigma_j t_i - \phi_j)]$ that best fit the series of observations $y(t_i)$, $i = 1, N$.[3] Assuming $N > 2M + 1$ we see that it is impossible to

[3] In order to minimize the loss of accuracy due to round off, the average of the hourly heights observations is subtracted from all original values. The $y(t_i)$ values mentioned in all computations henceforth are actually the resultant deviations. At the end of all calculations, C_0 is adjusted by this mean value.

Table 6 Shallow Water Constituents that May Mask Main Constituents.

Main Constituent	Component Constituent which May Coexist at or Near its Frequency
Q_1	NK_1
O_1	NK_1**
TAU_1	MP_1**
NO_1*	NO_1**
P_1	SK_1**
K_1	MO_1
J_1	MQ_1
SO_1	SO_1
OQ_2	OQ_2**
EPS_2	MNS_2
$2N_2$	O_2**
MU_2	$2MS_2$
N_2	KQ_2**
GAM_2	OP_2**
M_2	KO_2**
L_2	$2MN_2$**
S_2	KP_2
K_2	K_2
MO_3	MO_3**
M_3	NK_3**

* With M_1 as a satellite.

** The modulation or frequency of the compound constituent is sufficiently different that the pair could be separated if a long enough record of high precision were available.

solve the system $y(t_i) = C_0 + \sum_{j=1}^{M} A_j \cos[2\pi(\sigma_j t_i - \phi_j)]$ exactly because it is overdetermined. Hence, it is necessary to adopt a criterion which will enable unique optimum values for the parameters A_j and ϕ_j to be found. The most common optimization criterion used, and the one chosen here, is the least squares technique.

Re-expressing $\sum_{j=1}^{M} A_j \cos\left[2\pi(\sigma_j t_i - \phi_j)\right]$ as

$$\sum_{j=1}^{M} \left[C_j \cos(2\pi\sigma_j t_i) + S_j \sin(2\pi\sigma_j t_i)\right],$$

where $A_j = (C_j^2 + S_j^2)^{1/2}$ and $2\pi\phi j = \arctan S_j/C_j$, so that the fitting function is linear in the parameters S_j and C_j and hence more easily solved, and rewriting $y(t_i)$ as y_i, the objective of the least squares technique is to minimize

$$T = \sum_{i=1}^{M} \left[y_i - C_0 - \sum_{j=1}^{M} (C_j \cos 2\pi\sigma_j t_i + S_j \sin 2\pi\sigma_j t_i)\right]^2,$$

$$C_k = \sum_{i=1}^{N} \cos 2\pi\sigma_k t_i$$

$$S_k = \sum_{i=1}^{N} \sin 2\pi\sigma_k t_i$$

$$CC_{kj} = \sum_{i=1}^{N} (\cos 2\pi\sigma_k t_i)(\cos 2\pi\sigma_j t_i) = CC_{jk}$$

$$SS_{kj} = \sum_{i=1}^{N} (\sin 2\pi\sigma_k t_i)(\sin 2\pi\sigma_j t_i) = SS_{jk}$$

$$CS_{kj} = \sum_{i=1}^{N} (\cos 2\pi\sigma_k t_i)(\sin 2\pi\sigma_j t_i) = SC_{jk}$$

$$\begin{pmatrix} N & C_1 & C_2 & \dots & C_M & S_1 & S_2 & \dots & S_M \\ C_1 & CC_{11} & C_{12} & \dots & CC_{1_M} & CS_{11} & CS_{12} & \dots & CS_{1_M} \\ C_2 & CC_{21} & CC_{22} & \dots & CC_{2_M} & CS_{21} & CS_{22} & \dots & CS_{2_M} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \\ C_M & CC_{M_1} & CC_{M_2} & \dots & CC_{MM} & CS_{M_1} & CS_{M_2} & \dots & CS_{MM} \\ S_1 & SC_{11} & SC_{12} & \dots & SC_{1_M} & SS_{11} & SS_{12} & \dots & SS_{1_M} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \\ S_M & SC_{M_1} & SC_{M_2} & \dots & SC_{MM} & SS_{M_1} & SS_{M_2} & \dots & SS_{MM} \end{pmatrix} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ \vdots \\ C_M \\ S_1 \\ \vdots \\ S_M \end{pmatrix} \begin{pmatrix} \sum_{i=1}^N y_i \\ \sum_{i=1}^N y_i \cos 2\pi\sigma_1 t_i \\ \sum_{i=1}^N y_i \cos 2\pi\sigma_2 t_i \\ \vdots \\ \sum_{i=1}^N y_i \cos 2\pi\sigma_M t_i \\ \sum_{i=1}^N y_i \sin 2\pi\sigma_1 t_i \\ \vdots \\ \sum_{i=1}^N y_i \sin 2\pi\sigma_M t_i \end{pmatrix}$$

Figure 1 The matrix equation $B\mathbf{x} = \mathbf{y}$ resulting from the least squares fit for constituent amplitudes and phases.

for C_0 and all $C_j, S_j \quad j = 1, M$. This is done by solving the following $2M + 1$ simultaneous equations for $j = 1, M$:

$$0 = \frac{\partial T}{\partial C_0} = 2 \sum_{i=1}^{N} \left(y_1 - C_0 - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i \right)(-1);$$

$$0 = \frac{\partial T}{\partial C_0} = 2 \sum_{i=1}^{N} \left(y_1 - C_0 - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i \right)(-\cos 2\pi\sigma_j t_i);$$

$$0 = \frac{\partial T}{\partial C_0} = 2 \sum_{i=1}^{N} \left(y_1 - C_0 - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i \right)(-\sin 2\pi\sigma_j t_i).$$

This results in the matrix equation $B\mathbf{x} = \mathbf{y}$ of Figure 1.

Gaps in the data record (i.e. missing hourly observations) are easily handled by the least squares method because it is not necessary that the observation times, t_i, for $i = 1, N$ be evenly spaced. For example, if the analysis covers the total time period of 100 h but hours 50 to 74 inclusive are missing, then t_{50} will correspond to the seventy-fifth hour. However, because the following identities which simplify the summations require that the observation times be evenly spaced, it is necessary that each of the matrix terms be calculated as the sum of contributions over the data periods that contain no gaps. Assuming that $[n_0, n_1]$ is the hour range of a section of record containing no gaps, we can substitute $t_k = k$ in the matrix coefficients expressions since the times are at successive hours.

Using the relationships

$$\begin{aligned}
\cos a \cos b &= \tfrac{1}{2}[\cos(a+b) + \cos(a-b)] \\
\sin a \sin b &= \tfrac{1}{2}[\cos(a-b) - \cos(a+b)] \\
\sin a \cos b &= \tfrac{1}{2}[\sin(a+b) + \sin(a-b)],
\end{aligned}$$

the formula for the sum of a geometric series, namely

$$\frac{a + ar+, \ldots, +ar^n = a(r^{n+1} - 1)}{(r-1)},$$

and expressing $\cos x$ and $\sin x$ as the real and imaginary parts of e^{ix}, we obtain the identities:

$$\sum_{k=n_0}^{n_1} \cos kx = \frac{\sin\{[(n_1 - n_0 + 1)x]/2\} \cos\{[(n_1 + n_0)x]/2\}}{\sin(x/2)},$$

and

$$\sum_{k=n_0}^{n_1} \sin kx = \frac{\sin\{[(n_1 - n_0 + 1)x]/2\} \sin\{[(n_1 + n_0)x]/2\}}{\sin(x/2)}.$$

Hence the summation expressions in the least squares matrix can be simplified (with regard to computer execution time) as follows.

$$\begin{aligned}
\sum_{k=n_0}^{n_1} \cos(2\pi\sigma_1 k)\cos(2\pi\sigma_2 k) &= \frac{1}{2}\sum_{k=n_0}^{n_1} \{\cos[2\pi k(\sigma_1+\sigma_2)] + \cos[2\pi k(\sigma_1-\sigma_2)]\} \\
&= \frac{1}{2}\Bigg(\frac{\sin[(n_1-n_0+1)\pi(\sigma_1+\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1+\sigma_2)]}{\sin\pi(\sigma_1+\sigma_2)} \\
&\qquad + \frac{\sin[(n_1-n_0+1)\pi(\sigma_1-\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1-\sigma_2)]}{\sin\pi(\sigma_1-\sigma_2)}\Bigg),
\end{aligned}$$

$$\begin{aligned}
\sum_{k=n_0}^{n_1} \sin(2\pi\sigma_1 k)\sin(2\pi\sigma_2 k) &= \frac{1}{2}\sum_{k=n_0}^{n_1} \{\cos[2\pi k(\sigma_1-\sigma_2)] - \cos[2\pi k(\sigma_1+\sigma_2)]\} \\
&= \frac{1}{2}\Bigg(\frac{\sin[(n_1-n_0+1)\pi(\sigma_1-\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1-\sigma_2)]}{\sin\pi(\sigma_1-\sigma_2)} \\
&\qquad - \frac{\sin[(n_1-n_0+1)\pi(\sigma_1+\sigma_2)]\cos[(n_1+n_0)\pi(\sigma_1+\sigma_2)]}{\sin\pi(\sigma_1+\sigma_2)}\Bigg),
\end{aligned}$$

$$\sum_{k=n_0}^{n_1} \sin(2\pi\sigma_1 k)\cos(2\pi\sigma_2 k) = \frac{1}{2}\sum_{k=n_0}^{n_1}\{\sin[2\pi k(\sigma_1+\sigma_2)] + \sin[2\pi k(\sigma_1-\sigma_2)]\}$$
$$= \frac{1}{2}\left(\frac{\sin[(n_1-n_0+1)\pi(\sigma_1+\sigma_2)]\sin[(n_1+n_0)\pi(\sigma_1+\sigma_2)]}{\sin\pi(\sigma_1+\sigma_2)} + \frac{\sin[(n_1-n_0+1)\pi(\sigma_1-\sigma_2)]\sin[(n_1+n_0)\pi(\sigma_1-\sigma_2)]}{\sin\pi(\sigma_1-\sigma_2)}\right).$$

With these substitutions made in Figure 1, we have the least squares matrix equation $B\mathbf{x} = \mathbf{y}$ generated in subroutine **SCFIT2**. Because B is symmetric it is sufficient to store only its upper triangle consisting of $2M^2+3M+1$ elements instead of the entire matrix of $(2M+1)^2$ elements.

Partitioning the matrix equation $B\mathbf{x} = \mathbf{y}$ into the form

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} \mathbf{c} \\ \mathbf{s} \end{pmatrix} = \begin{pmatrix} \mathbf{y}_c \\ \mathbf{y}_s \end{pmatrix},$$

where B_{11}, B_{12}, B_{21}, B_{22}, $\mathbf{c}$, $\mathbf{s}$, $\mathbf{y}_c$, $\mathbf{y}_s$ have dimensions $(M+1)\times(M+1)$, $(M+1)\times M$, $M\times(M+1)$, $M\times M$, $(M+1)\times 1$, $M\times 1$, $(M+1)\times 1$, $M\times 1$ respectively, it is easily seen when $n_0 = -n_1$ that B_{12} and B_{21} become zero matrices and two smaller matrix equations, $B_{11}\mathbf{c} = \mathbf{y}_c$ and $B_{22}\mathbf{s} = \mathbf{y}_s$, result. The combined computation time to solve these equations is less than that of the original (see Section 2.2.2) so it is desirable to attain this condition when possible. Since the time origin of the hourly observations is arbitrary provided it is consistent with that of the astronomical argument V, we can attain the desired condition for a record with no gaps by choosing the central hour of the record as the origin. (This requires that the total number of observations be odd and is satisfied by ignoring the last observation, if the total is even.) Although there is generally no corresponding matrix simplification in the case of a record with gaps, for consistency with the foregoing choice, it is convenient to choose the central hour of the record universally as the time origin.

2.2.2 Solution of the matrix equation, the condition number and statistical properties

Most of the discussion and development of the Cholesky factorization algorithm introduced in this section is taken directly from Forsythe and Moler (1967). Although all results and discussion are now stated only for the matrix B and the equation $B\mathbf{x} = \mathbf{y}$, they apply as well for the partitioned systems B_{11}, $B_{11}c = Y_c$ and B_{22}, $B_{22}s = Y_s$.

In addition to symmetry, a useful property of matrix B is its positive definiteness. This property requires that for all $(2M+1)\times 1$ dimensional vectors $\mathbf{x} \neq 0$, $\mathbf{x}^T B\mathbf{x} > 0$.

The positive definiteness of B can be demonstrated by considering the overdetermined matrix equation $\mathbf{y} = A\mathbf{x}+\mathbf{e}$ resulting from the system of equations $y(t_i) = C_0+\sum_{j=1}^{M}(C_j\cos 2\pi\sigma_j t_i + S_j \sin 2\pi\sigma_j t_i) + e_i$ for $i = 1, N$ where the vector $\mathbf{x}^T = (C_0, C_1, C_2, \ldots, C_M, S_1, S_2, \ldots, S_M)$, $\mathbf{y}^T = [y(t_1), \ldots, y(t_N)]$ and $\mathbf{e}$ is the vector of residuals. It is easily seen that $A^T A = B$, and so for any $x \neq 0$,

$$\mathbf{x}^T B\mathbf{x} = \mathbf{x}^T A^T A\mathbf{x} = \mathbf{z}^T\mathbf{z} = \sum_{i=1}^{N} z_i^2,$$

where $\mathbf{x}^T A^T = \mathbf{z}^T = (z_1, \ldots, z_N)$.

It is worth mentioning that the overdetermined system $\mathbf{y} = A\mathbf{x} + \mathbf{e}$ can be solved in many ways, depending on the criterion chosen for minimizing $\mathbf{e}$. For our purposes, those methods which solve the system without changing the form of the matrix are impractical from a storage, processing time and rounding error point of view because the first dimension of A (= the number of hourly observations) is commonly 9000. However, minimizing $\mathbf{e}^T\mathbf{e}$ is equivalent to the least squares criterion adopted here.

An important result for any positive definite symmetric matrix B is that it can be uniquely decomposed in the form $B = GG^T$, where G is a lower triangular matrix with positive diagonal elements.[4] Expanding this relationship leads to the matrix element equalities:

$$b_{jj} = \sum_{k=1}^{j} g_{jk}^2,$$

$$b_{ij} = \sum_{k=1}^{j} g_{ik} g_{jk} \qquad \text{for all } i > j.$$

The algorithm resulting from using these equations in the proper order to find the elements of G is known as Cholesky's square root method for factoring a positive definite matrix (also attributed to Banachiewicz; see Faddeev and Faddeeva (1963)). Unlike other matrix decomposition methods such as Gaussian elimination, it does not have to search for, and divide by pivots. Such techniques must insure that the reduced matrix elements are not too large so that rounding errors and loss of accuracy do not occur. In Cholesky's method however, we can see that $|g_{ij}| \le \sqrt{b_{ii}}$ for all i, j and so upper bounds for the elements of G always exist.

Once B has been decomposed into the upper and lower triangular matrices, it is a relatively easy matter to solve the matrix solution. This is done by breaking down the equation $GG^T\mathbf{x} = \mathbf{y}$ into $G\mathbf{b} = \mathbf{y}$ and $G^T\mathbf{x} = \mathbf{b}$. Because of the triangular nature of G, these equations can be solved by forward and backward substitution for $\mathbf{b}$ and $\mathbf{x}$ respectively.

The amount of arithmetic in a matrix algorithm is usually measured by the number of multiplicative operations (i.e. multiplications and divisions) used, since there are normally approximately the same number of additive operations. For a matrix of dimension $n \times n$, the Cholesky factorization algorithm requires n square roots and approximately $\frac{1}{6}n^3$ multiplications. This compares favourably with the $\frac{1}{3}n^3$ multiplications required by Gaussian elimination (Wilkinson, 1967) to produce a triangular matrix.

Wilkinson (1967) suggests a factorization of B into LDL^T, where L is a lower triangular matrix and D is a positive diagonal matrix, that involves no more multiplications than Cholesky and avoids the square roots. However, assuming that the time ratio of a square root operation to a multiplication is 15:1 (approximate ratio for the IBM 370-168) and that all 69 constituents in the data package are included in the analysis (i.e. $n = 137$) the time saved by eliminating the square roots in only 0.5%. Furthermore, some of this gain would be replaced by time required for storing and retrieving information from the additional matrix D, and for the n additional division operations each time a solution is calculated by forward and backward substitution. Hence the factorization was not adopted in the present program.

Because the time required for the factorization of B varies as the cube of the number of unknowns, an approximate four-fold time reduction should result when the tidal record has no

[4] If B is symmetric but not positive definite a similar decomposition exists. However, some elements of G may be complex or, in the degenerate case, zero along the diagonal.

gaps and the partitioned rather than the original matrix equations are solved. However, as the following table of execution times for sections of subroutine **SCFIT2** demonstrates, significant improvements can also be expected in the time required for matrix generation, and error calculation. The values shown in Table 7 were obtained on an IBM 370-168 computer with a 34-constituent analysis of a 38-day tidal record.

A rough indication of the round-off difficulties associated with solving the equation $Bx = y$ is given by the matrix condition number. Although several different definitions for a condition number exist, an appropriate one for our purposes, in the sense that it pertains to least squares matrices and is easily calculated, is specified by Davis and Rabinowitz (1961). Its development is as follows.

Table 7 Comparison of Processing Times between the Partitioned and Non-Partitioned Matrix Equation Solutions.

Components of Matrix Solution	Partitioned Matrix System Times (s)	Non-Partitioned Matrix System (s)
Parameter initializations and right-hand generation	0.347	0.346
Matrix generation	0.059	0.178
Matrix factorization	0.049	0.146
Solution	0.010	0.018
Error calculation	0.128	0.403

If $\{\mathbf{b}_1, \ldots \mathbf{b}_n\}$ are n-dimensional vectors such that the matrix

$$B = \begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \end{pmatrix} \begin{pmatrix} \mathbf{b}_1 & \ldots & \mathbf{b}_n \end{pmatrix} = \begin{pmatrix} \mathbf{b}_1 \cdot \mathbf{b}_1 & \ldots & \mathbf{b}_1 \cdot \mathbf{b}_n \\ \vdots & & \vdots \\ \mathbf{b}_1 \cdot \mathbf{b}_1 & \ldots & \mathbf{b}_1 \cdot \mathbf{b}_n \end{pmatrix},$$

then it can be shown that $0 \leq \det(B) \leq \|\mathbf{b}_1\|\,\|\mathbf{b}_2\|, \ldots, \|\mathbf{b}_n\|$ where if $\mathbf{b}_j = (b_{j1}, \ldots, b_{jn})$, the norm $\|\mathbf{b}_j\| = (\sum_{i=1}^{n} b_{ji}^2)^{1/2}$. Furthermore, $\det(B) = 0$ if and only if the vectors are linearly dependent, and $\det(B) = \|\mathbf{b}_1\|, \ldots, \|\mathbf{b}_n\|$ if and only if they are orthogonal (i.e. $\mathbf{b}_i \cdot \mathbf{b}_j = 0$ for $i \neq j$). This determinant is known as the Gram determinant of the system $\{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ and is the square of the n-dimensional volume of the parallelepiped whose edges are these vectors.

Since it can be shown that all least squares matrices can be expressed in this manner, this result can be applied to our situation. In particular when the vectors are normalized so that $\|\mathbf{b}_i\| = 1$, the actual value of $\det(B)$ will always be bounded and provide a measure of the linear independence of the system, and hence round-off difficulties encountered in solving the equation. A value close to 1 will mean near orthogonality, a virtually diagonal matrix for B, and thus an easy solution. On the other hand, a value close to 0 will mean that at least two rows are near scalar multiples of one another, and thus greater accuracy problems will occur when their difference is calculated during the equation solution.

For our particular case observe that $\det(B) = \det(GG^T) = (\det G)^2 = \prod_{i=1}^{n} g_{ii}^2$, and that B can be written as

$$GG^T = \begin{pmatrix} \mathbf{g}_1 \cdot \mathbf{g}_1 & \cdots & \mathbf{g}_1 \cdot \mathbf{g}_n \\ \vdots & & \vdots \\ \mathbf{g}_n \cdot \mathbf{g}_n & \cdots & \mathbf{g}_n \cdot \mathbf{g}_n \end{pmatrix},$$

where

$$G^T = \begin{pmatrix} g_{11} & g_{21} & \cdots & g_{n1} \\ 0 & \ddots\, g_{22} & \cdots & g_{n2} \\ \vdots & \ddots & \ddots & \\ 0 & \cdots & \ddots\, 0 & \ddots\, g_{nn} \end{pmatrix} = (\mathbf{g}_1, \mathbf{g}_2 \cdots \mathbf{g}_n).$$

Since $b_{jj} = \sum_{k=1}^{j} g_{jk}^2$, $\|\mathbf{g}_j\| = \sqrt{b_{jj}}$ and the determinant of the matrix resulting from normalizing the $\mathbf{g}_j$ vectors is $\prod_{i=1}^{n}(g_{ii/b_{ii}}^2)$. The square root of this value is the volume of the n-dimensional parallelepiped whose edges are these normalized vectors and is the quantity calculated as the condition number of the matrix B.

The statistical properties of the least squares fit solution can be found in any analysis of variance or regression model text. They are outlined briefly as follows.

Reverting to the overdetermined problem statement, the least squares objective can be stated as finding the vector $\mathbf{x}$ in $\mathbf{y} = A\mathbf{x} + \mathbf{e}$ such that $\mathbf{e}^T\mathbf{e}$ is minimized. This yields the solution $\hat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{y}$.

The total sum of squares is $\mathbf{y}^T\mathbf{y}$ and the sum of squares due to regression is $\hat{\mathbf{x}}^TA^Ty$. Their difference is the residual error sum of squares and this difference divided by the degrees of freedom in the fit is the residual mean square error (MSE). "Degrees of freedom" is the difference between the number of hourly observations (excluding gaps) and A the number of parameters fit in the analysis. If there were M constituents including Z_0 chosen for the analysis, the degrees of freedom would be $N - 2M + 1$.

If it is assumed, as is commonly done, that the vector $\mathbf{e}$ is distributed normally with 0 standard deviation and $\sigma^2 I$ variance, where I is the unit diagonal matrix, then the variance of $\hat{\mathbf{x}}$ is $(A^TA)^{-1}\sigma^2$. Since the mean square residual error is an unbiased estimator for σ^2, an estimate of the standard deviation of $\hat{x}_i$, the ith element of $\hat{\mathbf{x}}$, is

$$\sqrt{(\boldsymbol{\mu}_i^T(A^TA)^{-1}\boldsymbol{\mu}_i)\mathrm{MSE}}\,,$$

where $\boldsymbol{\mu}_i$ is the vector with one in the ith position of zeros elsewhere.

2.3 Modifications to the Least Squares Analysis Results

2.3.1 Astronomical argument and Greenwich phase lag

Instead of regarding each tidal constituent as the result of some particular component of the tidal potential, an artificial causal agent can be attributed to each constituent in the form of a fictitious star which travels around the equator with an angular speed equal to that of its corresponding constituent. Making use of this conceptual aid, the astronomical argument, $V(L,t)$, of a tidal constituent can then be viewed as the angular position of this fictitious star relative to longitude, L, and at time, t. Although the longitudinal dependence is easily calculated, for historical reasons L is generally assumed to be the Greenwich meridian, and V is reduced to a function of one variable.

The Greenwich phase lag, g, is the difference between this astronomical argument for Greenwich and the phase of the observed constituent signal. Its value is dependent upon the time zone in which the hourly heights of the record were taken. This means that when phases at various stations, not necessarily in the same time zone, are compared, they must be reduced to a common zone in order to avoid spurious differences due to difference relative times. Specifically, if σ is the constituent frequency and $g(j+\Delta_j)$ and $g(j)$ are the Greenwich phase lags evaluated for time zones $j+\Delta_j$ and j respectively (e.g. Pacific Standard Time is +8), then

$$g(j+\Delta_j) = g(j) - (\Delta_j)\sigma.$$

Although these adjustments are easily calculated, they can be tedious because each constituent must be handled individually. Therefore, to avoid possible misinterpretation of phases from nearby stations of subsequent phase alterations, it is suggested that all observations be recorded in, or converted to, GMT.

The calculation of g (see Section 2.3.3) requires that the astronomical argument need only be evaluated at one time, the central hour of the analysis period. For a particular main constituent, it is calculated as

$$V = i_0\tau + j_0 S + k_0 H + l_0 P + m_0 N' + n_0 P',$$

where $i_0, j_0, k_0, l_0, m_0, n_0$ are the Doodson numbers of the constituent and S, H, P, N', P' are the astronomical variables defined in Section 2.1.1. The variable, τ, the number of mean lunar days from an absolute time origin is calculated as sum of the local mean solar time from this origin and $(H-S)$, and so need not be read from the data cards.

For shallow water constituents, the astronomical argument is calculated as the linear combination of the coefficient number and the astronomical argument of the main constituents from which it is derived. For example,

$$V_{\mathrm{MSN}_2} = V_{\mathrm{M}_2} + V_{\mathrm{S}_2} - V_{\mathrm{N}_2} \quad \text{and} \quad V_{2\mathrm{MK}_5} = 2V_{\mathrm{M}_2} + V_{\mathrm{K}_1}.$$

2.3.2 Nodal corrections

Most of this section has been taken from the unpublished notes of G. Godin which were written subsequent to the Cartwright and Tayler (1971) and Cartwright and Edden's (1973) recalculation of the tide-generating potential. The material presented here is intended to give greater detail than that of Section 2.1.3.

Due to the presence of satellites in a given cluster, it is known from tidal potential theory that the analysed signal found at the frequency, σ_j, of the main constituent is actually the result of

$$a_j \sin(V_j - g_j) + \sum_k A_{jk} a_{jk} \sin(V_{jk} - g_{jk}) + \sum_l A_{jl} a_{jl} \cos(V_{jl} - g_{jl})$$

for the diurnal and terdiurnal constituents of direct gravitational origin, and

$$a_j \cos(V_j - g_j) + \sum_k A_{jk} a_{jk} \cos(V_{jk} - g_{jk}) + \sum_l A_{jl} a_{jl} \sin(V_{jl} - g_{jl})$$

for the slow and semidiurnal constituents. The variables, a, g and V, are the true amplitude, Greenwich phase and astronomical argument, respectively, at the central time of the record for all the constituents. Single j subscripts refer to the major contributor while jk and jl subscripts refer to satellites originating from tidal potential terms of the second and third order respectively. A is the element of the interaction matrix resulting from the interference of a satellite with the main constituent.

It is the convention in tides and an assumption for our least squares fit that all constituents arise through a cosine term and positive amplitude, i.e. the contribution for a constituent whose astronomical argument is V_j and whose Greenwich phase is g_j, is expected to be in the form $a_j \cos(V_j - g_j)$ for $a_j > 0$. However, the diurnal and terdiurnal constituents, assuming that they are due to second order terms in the tidal potential, actually arise through a $b_j \sin(V_j - g_j)$ term where b_j may be negative. Hence a phase correction (variable `SEMI` read in data input (iii) from logical unit 8) of either $-\frac{1}{4}$ or $-\frac{3}{4}$ cycles is necessary, i.e.

$$\begin{aligned} b_j \sin(V_j - g_j) &= |b_j| \cos\left(V_j - g_j - \tfrac{1}{4}\right) \qquad && b_j \geq 0, \\ &= |b_j| \cos\left(V_j - g_j - \tfrac{3}{4}\right) && b_j < 0. \end{aligned}$$

Similarly, an adjustment of $\frac{1}{2}$ cycle will only be necessary for slow and semidiurnal main constituents if the tidal potential amplitude is negative.

Making these changes, the combined result of a constituent cluster in the diurnal and terdiurnal cases is

$$|a_j| \cos(V_j' - g_j) + \sum_k A_{jk} a_{jk} \cos(V_{jk}' + \alpha_{jk} - g_k) + \sum_l A_{jl} a_{jk} \cos(V_{jl}' + \alpha_{jl} - g_{jl})$$

where if

$$a_j < 0, \qquad V' = V - \tfrac{3}{4}, \quad \alpha_{jk} = \tfrac{1}{2}, \quad \alpha_{jl} = \tfrac{3}{4},$$

and if

$$a_j > 0, \qquad V' = V - \tfrac{1}{4}, \quad \alpha_{jk} = 0, \quad \alpha_{jl} = \tfrac{1}{4}.$$

A further phase adjustment to satellite constituents can be made if we wish to ensure that their amplitudes are positive. This convention was adopted for the data package of Appendix 7.1 (variable `PH` read in data input (iv) from logical unit 8). Replacing a_{jk} and a_{jl} by their absolute values we now see that

$$\begin{aligned} \alpha_{jk} &= 0 \qquad && \text{if both } a_{jk} \text{ and } a_j \text{ have the same sign,} \\ &= \tfrac{1}{2} && \text{otherwise;} \\ \alpha_{jkl} &= \tfrac{1}{4} && \text{if both } a_{jl} \text{ and } a_j \text{ have the same sign,} \\ &= \tfrac{3}{4} && \text{otherwise.} \end{aligned}$$

Similarly, for the slow and semidiurnal constituents, the cluster contribution can be written as

$$|a_j| \cos(V_j' - g_j) + \sum_k A_{jk} |a_{jk}| \cos(V_{jk}' + \alpha_{jk} - g_{jk}) + \sum_l A_{jl} |a_{jl}| \cos(V_{jl}' + \alpha_{jl} - g_{jl}),$$

where

$$\begin{aligned} V' &= V + \tfrac{1}{2} \qquad && \text{if } a_j < 0, \\ & \;\; V && \text{otherwise;} \\ \alpha_{jk} &= 0 && \text{if } a_{jk} \text{ and } a_j \text{ have the same sign,} \\ & \;\; \tfrac{1}{2} && \text{otherwise;} \\ \alpha_{jl} &= -\tfrac{1}{4} && \text{if } a_{jl} \text{ and } a_j \text{ have the same sign,} \\ & \;\; \tfrac{1}{4} && \text{otherwise.} \end{aligned}$$

Special note should be made of the terdiurnal M_3 because both it and its only satellite are due to third-order terms in the tidal potential. Hence both contribute directly through a cosine term and so behave as if they were second order semidiurnals.

In order to determine the amplitude and phase of the major contributor, we assume that the result actually found in the analysis was $f_j a_j \cos(V_j^! - g_j + u_j)$, where f_j and u_j are called the nodal modulation corrections in amplitude and phase respectively. To avoid a possible misunderstanding, it is worth mentioning here that the term nodal modulation is actually a misnomer. It and the symbols f and u were first used before the advent of modern computers to designate corrections for the moon's nodal progression that were not incorporated into the calculations of the astronomical argument for the main constituent. However, now the term satellite modulation is more appropriate because our correction is due to the presence of satellite constituents differing not only in the contribution of the lunar node to their astronomical argument, but also in the lunar and solar perigee effect.

For the purpose of calculating f_j and u_j it is assumed that the admittance is very nearly a constant over the frequency range within a constituent cluster, and so $g_j = g_{jk} = g_{jl}$; and $r_{jk} = |a_{jk}|/|a_j|$, $r_{jl} = |a_{jl}|/|a_j|$ are equal to the ratio of the tidal equilibrium amplitudes of the satellite to the major contributor. These ratios are latitude dependent when satellites of the third order are involved, necessitating the correction factors mentioned in Section 2.1.3. However, the ratios are usually small and the correction is slight.

Dropping the 'prime' notation and grouping the second- and third-order terms in one summation, the relationship between the analysed results for a main constituent and the actual cluster contribution is

$$f_j|A_j|\cos(V_j + u_j - g_j) = |a_j|\Big[\cos(V_j - g_j) + \sum_k A_{jk} r_{jk} \cos(V_j - g_j + \Delta_{jk} + \alpha_{jk})\Big],$$

where $\Delta_{jk} = V_{jk} - V_j$.

Expanding this result and observing that it must be true for all $V_j(t)$, the following explicit formulae are found for f and u:

$$f_j = \left[\Big(1 + \sum_k A_{jk} r_{jk} \cos(\Delta_{jk} + \alpha_{jk})\Big)^2 + \Big(\sum_k A_{jk} r_{jk} \sin(\Delta_{jk} + \alpha_{jk})\Big)^2\right]^{1/2},$$

$$u_j = \arctan\left[\frac{\sum_k A_{jk} r_{jk} \sin(\Delta_{jk} + \alpha_{jk})}{1 + \sum_k A_{jk} r_{jk} \cos(\Delta_{jk} + \alpha_{jk})}\right].$$

For an analysis carried out over $2N+1$ consecutive observations, Δt time units apart, A_{jk} is given by

$$A_{jk} = \frac{\sin[(2N+1)\Delta t(\sigma_{jk} - \sigma_j)/2]}{(2N+1)\sin[\Delta t(\sigma_{jk} - \sigma_j)/2]},$$

where σ_j is the frequency of the main contributor and σ_{jk} is that of its satellite. However, A_{jk} is very nearly one, even for a one-year analysis, and in the program it is approximated by this value.

For a shallow water constituent whose frequency is calculated as $\sum_{j=1}^{N_0} c_j \sigma_j$, where σ_j is the frequency of the jth main constituent from which it is derived and c_j is the linear coefficient, the nodal modulation corrections for amplitude and phase are computed as

$$f = \prod_{j=1}^{N_0} f_j^{|c_j|} \quad \text{and} \quad u = \sum_{j=1}^{N_0} c_j u_j.$$

2.3.3 Final amplitude and phase results

The result of the least squares analysis was to find for a constituent with frequency σ_j, the optimal amplitude A_j and phase ϕ_j value for the tidal signal $A_j \cos 2\pi(\sigma_j t - \phi_j)$. However, due to nodal corrections, when the astronomical argument is calculated at the central time origin $t = 0$ of the record, we know that the actual contribution of the constituent cluster is $f_j a_j \cos 2\pi(V_j + u_j - g_j)$. Hence the amplitude and Greenwich phase lag of the constituent corresponding to frequency σ_j can be calculated as $a_j = A_j/f_j$ and $g_j = V_j + u_j + \phi_j$.

2.3.4 Inferred constituents

In accordance with previous notation, tidal signals in this section are assumed to be real in nature. However, an alternative presentation using complex numbers and the basis for the following development is given by Godin (1972).

If the length of a specific tidal record is such that certain important constituents will not be included directly in the analysis, provision is made via the data input on logical unit 4 to include these constituents indirectly by inferring their amplitudes and phases from neighbouring constituents that are included. If accurate amplitude ratios and phase differences are specified, inference has the effect of significantly reducing any periodic behaviour in the amplitudes and phases of the constituent used for the inference. This is due to the removal of interaction from the neighbouring inferred constituent. If it so happens that a constituent specified for inference is included directly in the analysis, the program will ignore the inference calculations.

The actual adjustments are as follows. Assume that the constituent with frequency, σ_2, is to be inferred from the constituent with frequency, σ_1, and that the least squares fit analysis found the latter's contribution to be $A_1^0 \cos 2\pi(\sigma_1 t - \phi_1^0)$, where A_1^0 and ϕ_1^0 are the amplitude and phase respectively (σ_1 and ϕ_1^0 are measured in cycles/h and cycles respectively). Letting

VU_1 be the astronomical argument + nodal modulation phase correction,
g_1 be the Greenwich phase lag,
f_1 be the nodal modulation amplitude correction factor,
and a_1 be the corrected amplitude.

then from Section 2.3.3 we know that

$$-\phi_1 = VU_1 - g_1$$

and

$$a_1 = A_1/f_1.$$

Assuming that A_1 and ϕ_1 are the post-inference amplitude and phase respectively for the constituent with frequency, σ_1,

$$r_{12} = \frac{a_2}{a_1} = \frac{(A_2/f_2)}{(A_1/f_1)}$$

and

$$\zeta = g_1 - g_2 = VU_1 + \phi_1 - VU_2 - \phi_2$$

(the latter two being data input variables `R` and `ZETA` respectively), then the presence of the inferred constituent in the analysed signal yields the relationship:

$$A_1^0 \cos 2\pi(\sigma_1 t - \phi_1^0) = A_1 \cos 2\pi(\sigma_1 t - \phi_1) + A_2 \cos 2\pi(\sigma_2 t - \phi_2)$$

$$
\begin{aligned}
&= A_1 \cos 2\pi(\sigma_1 t - \phi_1) \\
&\qquad \left\{ 1 + r_{12}\left(\frac{f_2}{f_1}\right) \cos 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta] \right\} \\
&\quad - A_1 \sin 2\pi(\sigma_1 t - \phi_1) \\
&\qquad \left\{ r_{12}\left(\frac{f_2}{f_1}\right) \sin 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta] \right\}.
\end{aligned}
$$

Since the constituent with frequency σ_2 was not chosen for inclusion in the least squares analysis, $|\sigma_2 - \sigma_1|N < RAY$, where N is the record length in hours and RAY is the Rayeigh criterion constant (usually 1.0). Assuming in general that $|\sigma_2 - \sigma_1|N$ is small, good approximations to $\cos 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta]$ and $\sin 2\pi[(\sigma_2 - \sigma_1)t + VU_2 - VU_1 + \zeta]$ are their average values over the interval $[-N/2, N/2]$, namely $\sin[\pi N(\sigma_2 - \sigma_1)] \cos[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)]$ and $\sin[\pi N(\sigma_2 - \sigma_1)] \sin[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)]$ respectively. Making these substitutions and setting

$$S = r_{12}\left(\frac{f_2}{f_1}\right) \sin[\pi N(\sigma_2 - \sigma_1)] \sin[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)]$$

and

$$C = 1 + r_{12}\left(\frac{f_2}{f_1}\right) \sin[\pi N(\sigma_2 - \sigma_1)] \cos[2\pi(VU_2 - VU_1 + \zeta)]/[\pi N(\sigma_2 - \sigma_1)],$$

we obtain

$$\frac{A_1^0}{A_1} \cos[2\pi(\sigma_1 t - \phi_1^0)] = C \cos[2\pi(\sigma_1 t - \phi_1)] - S \sin[2\pi(\sigma_1 t - \phi_1)].$$

Expanding and regrouping this result yields

$$
\begin{aligned}
&\cos 2\pi\sigma_1 t \left(\frac{A_1^0}{A_1} \cos 2\pi\phi_1^0 - C \cos 2\pi\phi_1 - S \sin 2\pi\phi_1 \right) \\
&\qquad\qquad = \sin 2\pi\sigma_1 t \left(-\frac{A_1^0}{A_1} \sin 2\pi\phi_1^0 + C \sin 2\pi\phi_1 - S \cos 2\pi\phi_1 \right).
\end{aligned}
$$

Now since this relationship must hold for all t, both terms in brackets are equal to zero. Hence

$$\frac{A_1^0}{A_1} \cos 2\pi\phi_1^0 = C \cos 2\pi\phi_1 + S \sin 2\pi\phi_1,$$

$$\frac{A_1^0}{A_1} \sin 2\pi\phi_1^0 = C \sin 2\pi\phi_1 - S \cos 2\pi\phi_1$$

and so

$$A_1 = \frac{A_1^0}{\sqrt{C^2 + S^2}},$$

$$\phi_1 = \phi_1^0 + \frac{\arctan(S/C)}{2\pi}.$$

The relative phase and amplitude of the inferred constituent are then calculated as

$$\phi_2 = VU_1 - VU_2 + \phi_1 - \zeta$$

and

$$A_2 = r_{12} A_1 \left(\frac{f_2}{f_1}\right).$$

3 USE OF THE TIDAL HEIGHTS PREDICTION COMPUTER PROGRAM

3.1 General Description

This program produces tidal height values at a given location for a specified period of time. Amplitudes and Greenwich phase lags of the tidal constituents to be used in the prediction are required as input and either equally spaced heights or all the high and low values can be produced.

3.2 Routines Required

(1) **MAIN** ...... reads in tidal station and time period information, amplitudes and Greenwich phases of constituents to be used in the prediction, and calculates the desired tidal heights.

(2) **ASTRO** ...... reads the standard constituent data package and calculates the frequencies, astronomical arguments, and nodal corrections for all constituents.

(3) **PUT** ...... controls the output for high–low predictions.

(4) **HPUT** ...... controls the output for equally spaced predictions.

(5) **GDAY** ...... returns the consecutive day number from a specific origin for any given date and vice versa.

(6) **ASTR** ...... calculates ephermides for the sun and moon.

3.3 Data Input

All input data required by the tidal heights prediction program is from logical unit 8. A sample set is given in Appendix 7.4. Although data types (i), (ii) and (iii) are identical to types (ii), (iii) and (iv) expected in logical unit 8 by the analysis program, for completeness they are repeated here.

(i) Two cards specifying values for the astronomical arguments `SO,HO,PO,ENPO,PPO,DS,DH,DP,DNP,DPP` in the format `(5F13.10)`.

`SO` = mean longitude of the moon (cycles) at the reference time origin;
`HO` = mean longitude of the sun (cycles) at the reference time origin;
`PO` = mean longitude of the lunar perigee (cycles) at the reference time origin;
`ENPO` = negative of the mean longitude of the ascending node (cycles) at the reference time origin;
`PPO` = mean longitude of the solar perigee (perihelion) at the reference time origin.

DS,DH,DP,DNP,DP are their respective rates of change over a 365-day period at the reference time origin.

Although these argument values are not used by the program that was revised in October 1992, in order to maintain consistency with earlier programs, they are still required as input. Polynomial approximations are now employed to more accurately evaluate the astronomical arguments and their rates of change.

(ii) At least one card for all the main tidal constituents specifying their Doodson numbers and phase shift, along with as many cards as are necessary for the satellite constituents. The first card for each such constituent is in the format (6X,A5,1X,6I3,F5.2,I4) and contains the following information:

KON = constituent name;
II,JJ,KK,LL,MM,NN = the six Doodson numbers for KON;
SEMI = phase correction for KON;
NJ = number of satellite constituents.

A blank card terminates this data type.

If NJ>0, information on the satellite constituents follows, three satellites per card, in the format (11X,3(3I3,F4.2,F7.4,IX,I1,1X)). For each satellite the values read are:

LDEL,MDEL,NDEL = the last three Doodson numbers of the main constituent subtracted from the last three Doodson numbers of the satellite constituent;
PH = phase correction of the satellite constituent relative to the phase of the main constituent;
EE = amplitude ratio of the satellite tidal potential to that of the main constituent;
IR = 1 if the amplitude ratio has to be multiplied by the latitude correction factor for diurnal constituents,
= 2 if the amplitude ratio has to be multiplied by the latitude correction factor for semidiurnal constituents,
= otherwise if no correction is required to the amplitude ratio.

(iii) One card specifying each of the shallow water constituents and the main constituents from which they are derived. The format is (6X,A5,I1,2X,4(F5.2,A5,5X)) and the respective values read are:

KON = name of the shallow water constituent;
NJ = number of main constituents from which it is derived;
COEF,KONCO = combination number and name of these main constituents.

The end of these shallow water constituents is denoted by a blank card.

(iv) One card with the tidal station information ISTN,(NA(J),J=1,4),ITZONE,LAD,LAM,LOD, LOM in the format (5X,I4,1X,3A6,A4,A3,1X,I2,1X,I2,2X,I3,1X,I2).

ISTN = station number;
(NA(J),J=1,4) = station name;
ITZONE = time zone reference for the "Greenwich" phases;

LAD,LAM = station latitude in degrees and minutes;
LOD,LOM = station longitude in degrees and minutes.

(v) One card for each constituent to be included in the prediction with the constituent name (KON), amplitude (AMP) and phase lag (G) in the format (5X,A5,28X,F8.4,F7.2). (This format is compatible with the analysis program results produced on output device 2). The phase lag units should be degrees (measured in time zone ITZONE while the units of the predicted tidal heights will be the same as those of the input amplitudes. The last constituent is followed by a blank card.

(vi) One card containing the following information on the period and type of prediction desired. The format is (3I3,1X,3I3,1X,A4,F9.5,2X,2I3).

IDYO,IMOO,IYRO = first day, month and year of the prediction period;
IDYE,IMOE,IYRE = first day, month and year of the prediction period;
ITYPE = 'EQUI' if equally spaced predictions are desired,
= 'EXTR' if all the high and low tide times and heights are desired;
DT = time spacing of the predicted values if ITYPE='EQUI',
= time step increment used to initially bracket a high or low value if ITYPE='EXTR';
ICEO,ICEE = century number for the beginning and end of the prediction. (Blank values for ICEO or ICEE will be reset to 19.)

Equally spaced predictions begin at DT hours on the first day and extend to 2400 h (assuming 24 is a multiple of DT) of the last day. When ITYPE='EXTR', Godin and Taylor (1973) recommend using the following values for DT: 3 h for a semidiurnal tide, 6 h for a diurnal tide and 0.5 h for a mixed tide.

Type (vi) data may be repeated any number of times. One blank card following a type (vi) record will return the program to type (iv) input, while two blank cards will end the program execution.

3.4 Output

Two logical units are used for the output of results in the tidal heights prediction program. Device number 6 is the line printer and 10 is a data file. Both equally spaced and high–low predictions are put onto both devices with the same format. However the line printer also records the station name and location along with the amplitudes and phase lags of the constituents used in the prediction. Appendix 7.5 lists device 10 output resulting from the input of Appendix 7.4.

When daily high–low values are desired, the date, station number and a series of up to six heights and occurrence times are listed per record. Each record begins with the variable HL whose value is zero if the first height for that day is a high (i.e. larger than the second height) and one if the first height is a low. If there are less than six high–low values for a day, they are padded up to six with the values 9999 and 99.9 for the times and heights respectively. On device 10, the format used for the variables HL, the station number, the day, month, year, and the six pairs of times and heights is (1X,I1,I5,2I3,I2,6(I5,F5.1)).

When equally spaced heights are requested, 8 values are listed on each record preceded by the station number, the time, day, month and year of the first value, and followed by the time increment between heights. On device number 10, the format for these variables is `(1X,I4,F8.4,I3,2I2,8F6.3,F12.4)`

3.5 Program Conversion, Modifications, Storage and Dimension Guidelines

The source program and constituent data package described in this manual have been tested on various mainframe, PC and workstation computers at the Institute of Ocean Sciences, Patricia Bay. Although as much of the program as possible was written in basic FORTRAN, some changes may be required before the program and data package can be used on other installations. Please write or call the author if any problems are encountered.

The program in its present form requires approximately 33,000 bytes for the storage of its instructions and arrays respectively. As with the analysis program, changing the number or type of constituents in the standard data package may require alteration to the dimensions of some arrays. Restrictions on the minimum dimension of all arrays are now given.

Let

- `MTAB` be the total number of possible constituents contained in the data package (presently 146),
- `M` be the number of constituents to be included in the prediction,
- `MCON` be the number of main constituents in the standard data package (presently 45),
- `MSAT` be the sum of the number of satellites for these main constituents and the number of main constituents with no satellites (presently 162 plus 8 for the version of the constituent data package, listed in Appendix 7.4, that contains no third-order satellites for both N_2 and L_2),
- `MSHAL` be the sum for all shallow water constituents of the number of main constituents from which each is derived (presently 251),
- `NITER` be the iterations required to reduce the time interval within which it is known that a high or low tide exists, to a desired length (with the largest initial interval size of 6 h and a 6-min final interval, `NITER` is 6).

Then in the main program, arrays `SIGTAB,V,U` and `F` should have minimum dimension `MTAB`; array `KONTAB` should have minimum dimension `MTAB+1`; arrays `SIG,INDX,TWOC,CH,CHP,CHA,CHB,CHM,ANGO` and `AMPNC` should have minimum dimension `M`; arrays `KON,AMP` and `G` should have minimum dimension `M+1`; and the two-dimensional array `BTWDC` should have a minimum dimension of `M` by `NITER`. Array `COSINE` which stores pre-calculated cosine function values over the range of 0° to 360° and is used as a look-up table, presently has 2002 elements.

In subroutine **ASTRO**, the arrays `FREQ,V,U` and `F` should have minimum dimension `MTAB`; arrays `KON` and `NJ` should have minimum dimension `MTAB+1`; arrays `II,JJ,KK,LL,MM,NN` and `SEMI` should have minimum dimension `MCON+1`; arrays `EE,LDEL,MDEL,NDEL,IR` and `PH` should have minimum dimension `MSAT`; and arrays `KONCO` and `COEF` should have minimum dimension `MSHAL+4`.

In subroutine **PUT**, the dimensions of arrays `HGTK` and `ITIME` should be at least as large as the maximum number of high and low values per day (this is presently assumed to be 9).

In subroutine **HPUT**, the dimension of array `H` should be at least equal to the number of equally spaced tidal height values per output record of logical unit 10 or 6 (presently, this is 8).

In subroutine **CDAY**, both arrays `NDM` and `NDP` should have dimension 12.

4 TIDAL HEIGHTS PREDICTION PROGRAM DETAILS

4.1 Problem Formulation and the Equally Spaced Predictions Method

The tidal height, $h(t)$, at a particular station may be represented by the harmonic summation (see Section 2.3.3)

$$h(t) = \sum_{j=1}^{m} f_j(t) A_j \cos\left[2\pi(V_j(t) + u_j(t) - g_j)\right], \tag{1}$$

where

$$\begin{aligned} A_j,\, g_j &= \text{amplitude and phase lag of constituent, } j, \\ f_j(t),\, u_j(t) &= \text{nodal modulation amplitude and phase correction factors for constituent, } j, \\ V_j(t) &= \text{astronomical argument for constituent, } j. \end{aligned}$$

Expanding $V(t)$ as in Section 2.3.1 and using the first-order Taylor approximations for the astronomical arguments as in Section 2.1.1, $V(t)$ can be re-expressed as

$$\begin{aligned} V(t) &= i\tau(t) + jS(t) + kH(t) + lP(t) + mN'(t) + nP'(t) \\ &= i\tau(t_0) + jS(t_0) + kH(t_0) + lP(t_0) + mN'(t_0) + nP'(t_0) \\ &\qquad + (t - t_0)\frac{\partial}{\partial t}\left[i\tau(t) + jS(t) + kH(t) + lP(t) + mN'(t) + nP'(t)\right]_{t=t_0} \\ &= V(t_0) + (t - t_0)\sigma, \end{aligned}$$

where t_0 is the reference time origin and σ is the constituent frequency at this time origin. It follows from this result that $V(t_2) = V(t_1) + (t_2 - t_1)\sigma$ for arbitrary times, t_1, t_2, and so $V_j(t)$ can be replaced in (1) by $V_j(t_1) + (t - t_1)\sigma_j$ for some convenient time, t_1.

From Section 2.3.2 it is seen that $f(t)$ and $u(t)$ are time dependent only through the $\Delta_{jk}(t)$ variable. Since satellites differ from main constituents in only the last three Doodson numbers (see Section 2.1.3),

$$\begin{aligned} \Delta_{jk}(t) &= V_{jk}(t) - V_j(t) \\ &= \Delta lP(t) + \Delta mN'(t) + \Delta nP'(t). \end{aligned}$$

Using the first order Taylor approximations for P, N' and P', it follows that over a time period $[t_1, t_2]$ the change in $\Delta_{jk}(t)$ is

$$\begin{aligned} \Delta_{jk}(t_2) - \Delta_{jk}(t_1) &= \Delta l[P(t_2) - P(t_1)] + \Delta m[N'(t_2) - N'(t_1)] + \Delta n[P'(t_2) - P'(t_1)] \\ &= (t_2 - t_1)\frac{d}{dt}[\Delta lP(t) + \Delta mN'(t) + \Delta nP'(t)]_{t=t_0} \\ &= (t_2 - t_1)(\sigma_{jk} - \sigma_j). \end{aligned}$$

Since $d/dt[P(t) + N'(t) + P'(t)]_{t=t_0}$ is 0.16668884 cycles/356 days and $|\Delta l|$, $|\Delta m|$, $|\Delta n|$ are always less than or equal to 4, if $|t_2 - t_1| \leq 16$ days, $|\Delta_{jk}(t_2) - \Delta_{jk}(t_1)| \leq 0.03$ cycles. This small variation in $\Delta_j k(t)$ leads to a similar behaviour in $\cos[\Delta_{jk}(t)]$ and $\sin[\Delta_{jk}(t)]$, and hence $f(t)$ and $u(t)$. Thus only a small loss in accuracy but a considerable calculation time saving will result if $f(t)$ and $u(t)$ are approximated by a constant value throughout the period of a month. Consequently $f(t)$ and $u(t)$ are assumed to equal their value at 0000 h of the sixteenth day of the month for the entire monthly period; for convenience, $V(t)$ is set to $V(t_{16}) + (t - t_{16})\sigma$, where t_{16} is this same time.

The procedure for calculating a series of tidal heights is then as follows. Since the tidal prediction data package does not contain constituent frequencies, they must be calculated via the astronomical variable derivatives and the constituent Doodson numbers. The values f, u and V are then calculated for the sixteenth day of the first month of the desired prediction period and, as required, for subsequent months. Tidal heights for the desired values of t can then be calculated as

$$h(t) = \sum_{j=1}^{m} f_j(t_{16}) A_j \cos[2\pi(V_j(t_{16}) + (t - t_{16})\sigma_j + u_j(t_{16}) - g_j)]. \tag{2}$$

In order to avoid calling a trigonometric library function for each new value of t, when a sequence of equally spaced heights are required, the following Chebyshev iteration formula is used for each constituent contribution,

$$f(n+1) = 2\cos(\sigma\Delta t) f(n) - f(n-1), \tag{3}$$

where $f(n) = \cos(n\sigma\Delta t)$ or $\sin(n\sigma\Delta t)$.

4.2 The High and Low Tide Prediction Method

The material presented here is taken from Godin and Taylor (1973).

In Section 4.1 we saw that the tidal height at a given location can be represented by the harmonic sum

$$h(t) = \sum_{j=1}^{m} f_j(t_0) A_j \cos[2\pi(V_j(t_0) + (t - t_0)\sigma_j + u(t_0) - g_j)] \tag{1}$$

where

A_j, g_j, σ_j = amplitude, phase lag and frequency of constituent, j,
$f_j(t_0)$, $u_j(t_0)$ = nodal modulation amplitude and phase correction factors for constituent, j, at the time origin t_0,
$V_j(t_0)$ = astronomical argument for constituent j at the time origin t_0.

Letting $D(t)$ be the derivative of $h(t)$, i.e.

$$D(t) = -\sum_{j=1}^{m} f_j(t_0) A_j 2\pi\sigma_j \sin[2\pi(V_j(t_0) + (t - t_0)\sigma_j + u(t_0) - g_j)], \tag{2}$$

the high–low tide prediction method uses the following calculus results. If $D(t)$ is a continuous function on the interval $[t_1, t_2]$ and t_k is a point in this interval, then:

(i) $D(t_k) = 0$ if and only if t_k is an extreme point or saddle point,[5] or $h(t)$ is constant in the neighbourhood of t_k;

(ii) if $D(t_1)$ and $D(t_2)$ have opposite signs, then there exists a t_k in (t_1, t_2) with $D(t_k) = 0$.

[5] An example of a saddle point is $x = 0$ for the function $f(x) = x^3$.

Now for computational purposes we can assume that saddle points do not exist. That is to say, due to accuracy limitations of the computer, a zero derivative will be approximated by a number with a very small absolute value and thus perturb a saddle point so that it becomes either a maximum or minimum, or a near saddle point (in the neighbourhood of a "near saddle point", the derivative is of constant sign and almost assumes the value zero). And since, from its definition, we can reasonably assume that $h(t)$ is not constant over any arbitrarily small interval, the continuity of $D(t)$ everywhere implies that an interval $[t_1, t_2]$ with $D(t_1)$ and $D(t_2)$, having opposite signs, contains an extremum.

However, this result alone is not sufficient to guarantee the location of all extrema because it does not eliminate the possibility of having more than one extremum in an interval whose endpoints have different signs, nor does it imply that if the endpoints have the same derivative sign there is no extremum in the interval. In order to ensure these conditions and thus be assured of bracketing all extreme values, it is necessary that a minimum interval size be specified in which we can assume that there exists, at most, one high or low tide.

Clearly, the interval size, Δt, will be dependent upon the nature of the tide at a particular station. The time between successive high and low waters for predominantly semidiurnal and diurnal tides is approximately 6 and 12 h respectively. However, if the tide is mixed, the pattern of extremes is more complicated. Figure 2 shows the water level at Victoria, British Columbia between July 24 and 31, 1976. It is a mixed tide where the shorter period fluctuations override the major diurnal oscillations with a continuous shift in their position and amplitude.

One characterization of the tide may be obtained by calculating the ratio of the amplitudes of the major harmonic constituents, M_2, S_2 K_1 and O_1. This value is called the form number (Dietrich, 1963) and is defined precisely as

$$F = \frac{\mathrm{K}_1 + \mathrm{O}_1}{\mathrm{M}_2 + \mathrm{S}_2}.$$

The tide is then said to be

(i) semidiurnal if $0 \le F \le 0.25$,
(ii) mixed if $0.25 < F \le 3.00$,
(iii) diurnal if $F > 3.00$.

For Victoria, $F = 2.1$.

In accordance with this determination, Godin suggests the following maximum time interval values in which it can be assumed that there exists at most one extremum:

(i) $\Delta t = 3$ h for semidiurnal tide,
(ii) $\Delta t = 0.5$ h for mixed tide,
(iii) $\Delta t = 6$ h for diurnal tide.

Although in fact, a mixed tide may have extrema closer than 0.5 h, he feels that for practical purposes it is sufficient to note just one of them.

With these values of Δt we can then bracket all extrema by moving forward in time with steps of size, Δt, and comparing signs of the interval endpoints. Once such upper and lower bounds have been found, the extreme point can be located exactly by any one of a number of search techniques. Because it requires a minimal amount of time, the one chosen is Bolzano's method of bisection coupled with linear interpolation. Although the bisection method does not take the minimal number of iterations when compared to more sophisticated search techniques,

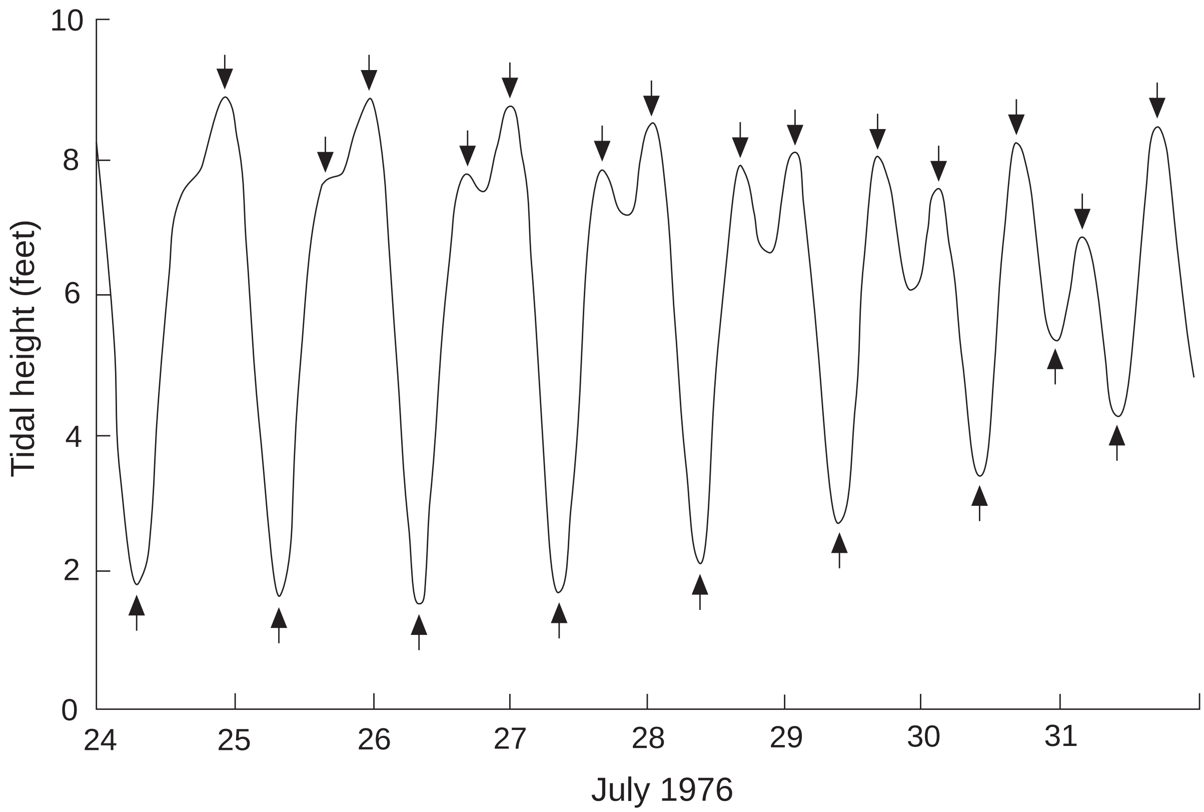

Figure 2 Synthesized water level at Victoria, British Columbia over the period July 24 to 31, 1976. The tide is of a mixed character with $F = 2.1$. The arrows indicate the time and height of the extrema predicted using the method described in Section 4.2. (Redrawn from C. Wallace)

it is able to make significant time savings by computing new sine function values as a linear combination of old ones and thus, unlike the other methods, avoid calls to the FORTRAN library function `SIN`.

In more detail, the search algorithm for an extremum is then as follows:

(i) Move forward in time from the origin, or the last extremum, in steps of Δt until either a change in sign exists between the derivative values at the endpoints of the interval (t_a, t_b), or t_b extends beyond the desired prediction period. Each constituent contribution in the summation $D(t)$ is evaluated by the Chebyshev iteration formula (3) of Section 4.1. When an interval containing an extremum is located, set $k = 1$ and proceed to (ii).

(ii) Calculate $t_k = t_a + \frac{1}{2^k}\Delta t$ and for each constituent in the sum evaluate $D(t_k)$ by using the formula

$$\sin(t_k) = \frac{\sin(t_a) + \sin(t_b)}{2\cos(1/2^k \Delta t)}.$$

If $|D(t_k)| \leq 10^{-16}$, set $D(t_k) = 10^{-16}$.

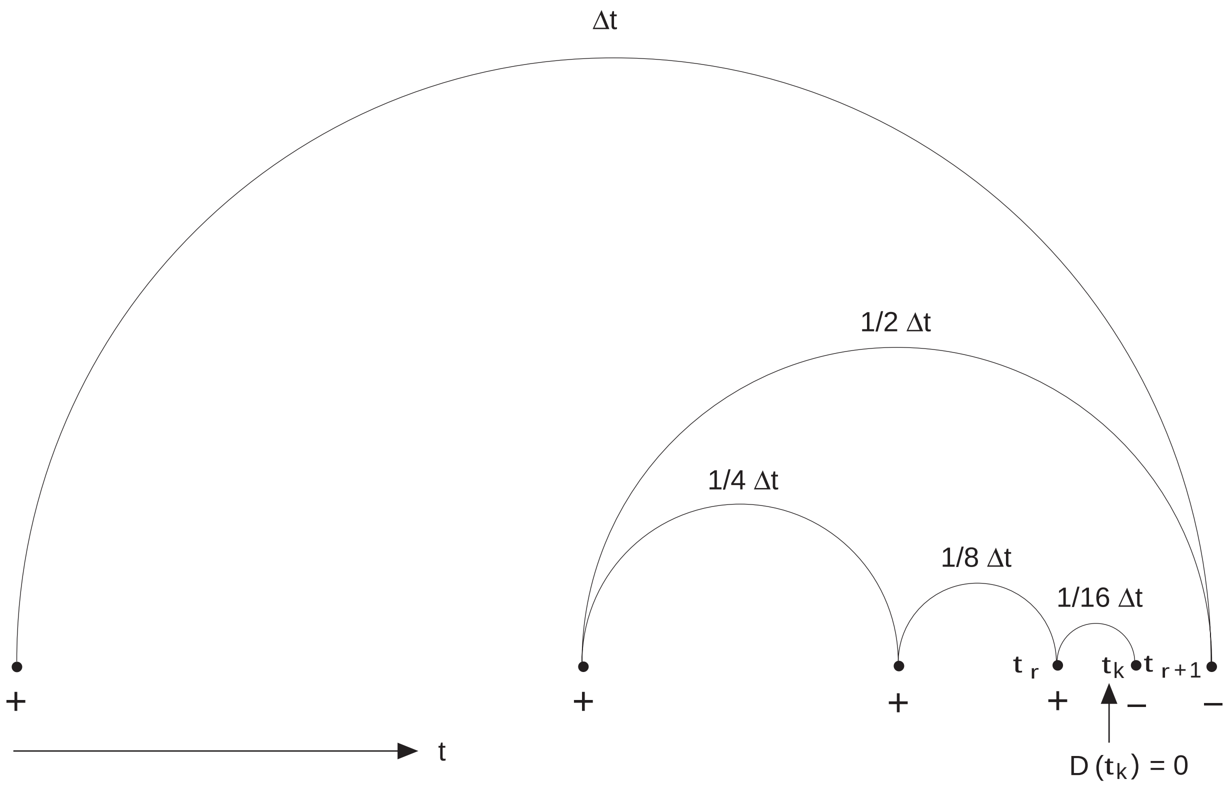

Figure 3 An example of the sequence of steps involved in locating a zero t_k of the derivative, $D(t)$. The sign of $D(t)$ at the various points tested is denoted by a plus or minus. After a step, Δt, the sign has changed; by a retrogression of $\frac{1}{2}\Delta t$, the sign has reverted to plus, forcing a forward step of $\frac{1}{4}\Delta t$ where the sign is still unchanged. Two further forward steps of $\frac{1}{8}\Delta t$ and $\frac{1}{16}\Delta t$ locate the minimum width interval (t_r, t_{r+1}) over which the position of t_k is determined by linear interpolation from the values of $D(t)$ at t_r and t_{r+1}. (Redrawn from C. Wallace)

(iii) Re-assign whichever of t_a or t_b has the same derivative sign as $D(t_k)$, by t_k. If the new interval length $t_b - t_a$ is less than 0.1 h, proceed to (iv). Otherwise set $k = k+1$ and return to (ii).

(iv) Use the following linear interpolation formula to find the extremum t_E,

$$t_E = t_a + [D(t_a)(t_b - t_a)]/[D(t_a) - D(t_b)],$$

and evaluate $h(t_E)$ via (1). For each constituent term in this sum, obtain the function value by using a pre-calculated stored table of 2002 cosine values with arguments in the range of 0° to 360°. Return to (i).

Figure 3 illustrates an example of the sequence of steps involved in the search for an extreme value. It is easily calculated that the number of iterations required to reduce the bracketing interval from Δt to 0.1 h is six for diurnal tides, three for mixed tides, and five for semidiurnal tides.

Arrows in Figure 2 indicate the extrema predicted for Victoria using the technique just described; the shaft of the arrow locates the time abscissa while the tip ends at the predicted height. The predicted hourly heights and the times and heights of all extrema are listed in Appendix 7.5.

5 CONSISTENCY OF THE ANALYSIS AND PREDICTION PROGRAMS

Although consistency between the tidal heights analysis program and the tidal heights prediction program was a major objective in their revision, they do have one difference. In particular, if a pseudo-tidal record were generated by the prediction program and analysed using the same constituents, the amplitude and phase results given by the analysis program would not be identical to those used as input for the prediction program.

In a small part, this discrepancy is due to round-off accumulated during the calculations. However, a test performed at the Institute of Ocean Sciences with a two-month period of synthesized hourly heights indicates that such errors occur no sooner than the fourth digit. The remainder of the difference (which is, at worst, in the third digit) can be attributed to different approximating assumptions for the calculation of f and u, the nodal modulation amplitude and phase correction factors. Whereas the prediction program calculates these values at the sixteenth day of each month in the desired time period and keeps them constant throughout the entire month, the analysis program assumes them to be constant over the entire analysis period and equal to their true values at the central hour of that period.

It is important to note, though, that significantly different results can be expected in a similar test run if there is at least one more constituent used in the synthesis than analysis. This is because the least squares fit technique will adjust the amplitudes and phases of constituents included in the analysis to partially account for contributions due to constituents included in the synthesis but not the analysis. In fact, this will occur even if the extra constituents are inferred (e.g. P_1 is included in the synthesis and in the analysis via inference from K_1) because of small inaccuracies in the approximating inference assumptions. However, except for round-off errors and the slightly different f and u values, having more constituents in the analysis than the synthesis will not affect the results.

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Appendix 7.1 Standard Constituent Input Data for the Tidal Heights Analysis Computer Program. This Data is Read by the Program from Logical Unit 8.

Z0	0.0	M2
SA	0.0001140741	SSA
SSA	0.0002281591	Z0
MSM	0.0013097808	MM
MM	0.0015121518	MSF
MSF	0.0028219327	Z0
MF	0.0030500918	MSF
ALP1	0.0343965699	2Q1
2Q1	0.0357063507	Q1
SIG1	0.0359087218	2Q1
Q1	0.0372185026	O1
RHO1	0.0374208736	Q1
O1	0.0387306544	K1
TAU1	0.0389588136	O1
BET1	0.0400404353	NO1
NO1	0.0402685944	K1
CHI1	0.0404709654	NO1
PI1	0.0414385130	P1
P1	0.0415525871	K1
S1	0.0416666721	K1
K1	0.0417807462	Z0
PSI1	0.0418948203	K1
PHI1	0.0420089053	K1
THE1	0.0430905270	J1
J1	0.0432928981	K1
2PO1	0.0443745198	
SO1	0.0446026789	OO1
OO1	0.0448308380	J1
UPS1	0.0463429898	OO1
ST36	0.0733553835	
2NS2	0.0746651643	
ST37	0.0748675353	
ST1	0.0748933234	
OQ2	0.0759749451	EPS2
EPS2	0.0761773161	2N2
ST2	0.0764054753	
ST3	0.0772331498	
O2	0.0774613089	
2N2	0.0774870970	MU2
MU2	0.0776894680	N2
SNK2	0.0787710897	
N2	0.0789992488	M2
NU2	0.0792016198	N2
ST4	0.0794555670	
OP2	0.0802832416	
GAM2	0.0803090296	H1
H1	0.0803973266	M2
M2	0.0805114007	Z0
H2	0.0806254748	M2
MKS2	0.0807395598	M2
ST5	0.0809677189	
ST6	0.0815930224	
LDA2	0.0818211815	L2
L2	0.0820235525	S2

2SK2	0.0831051742	
T2	0.0832192592	S2
S2	0.0833333333	M2
R2	0.0834474074	S2
K2	0.0835614924	S2
MSN2	0.0848454852	ETA2
ETA2	0.0850736443	K2
ST7	0.0853018034	
2SM2	0.0861552660	
ST38	0.0863576370	
SKM2	0.0863834251	
2SN2	0.0876674179	
NO3	0.1177299033	
MO3	0.1192420551	M3
M3	0.1207671010	M2
NK3	0.1207799950	
SO3	0.1220639878	MK3
MK3	0.1222921469	M3
SP3	0.1248859204	
SK3	0.1251140796	MK3
ST8	0.1566887168	
N4	0.1579984976	
3MS4	0.1582008687	
ST39	0.1592824904	
MN4	0.1595106495	M4
ST9	0.1597388086	
ST40	0.1607946422	
M4	0.1610228013	M3
ST10	0.1612509604	
SN4	0.1623325821	M4
KN4	0.1625607413	
MS4	0.1638447340	M4
MK4	0.1640728931	MS4
SL4	0.1653568858	
S4	0.1666666667	MS4
SK4	0.1668948258	S4
MNO5	0.1982413039	
2MO5	0.1997534558	
3MP5	0.1999816149	
MNK5	0.2012913957	
2MP5	0.2025753884	
2MK5	0.2028035475	M4
MSK5	0.2056254802	
3KM5	0.2058536393	
2SK5	0.2084474129	2MK5
ST11	0.2372259056	
2NM6	0.2385098983	
ST12	0.2387380574	
2MN6	0.2400220501	M6
ST13	0.2402502093	
ST41	0.2413060429	
M6	0.2415342020	2MK5
MSN6	0.2428439828	
MKN6	0.2430721419	
ST42	0.2441279756	
2MS6	0.2443561347	M6

```
    2MK6     0.2445842938    2MS6
    NSK6     0.2458940746
    2SM6     0.2471780673    2MS6
    MSK6     0.2474062264    2SM6
    S6       0.2500000000
    ST14     0.2787527046
    ST15     0.2802906445
    M7       0.2817899023
    ST16     0.2830867891
    3MK7     0.2833149482    M6
    ST17     0.2861368809
    ST18     0.3190212990
    3MN8     0.3205334508
    ST19     0.3207616099
    M8       0.3220456027    3MK7
    ST20     0.3233553835
    ST21     0.3235835426
    3MS8     0.3248675353
    3MK8     0.3250956944
    ST22     0.3264054753
    ST23     0.3276894680
    ST24     0.3279176271
    ST25     0.3608020452
    ST26     0.3623141970
    4MK9     0.3638263489
    ST27     0.3666482815
    ST28     0.4010448515
    M10      0.4025570033
    ST29     0.4038667841
    ST30     0.4053789360
    ST31     0.4069168759
    ST32     0.4082008687
    ST33     0.4471596822
    M12      0.4830684040
    ST34     0.4858903367
    ST35     0.4874282766

  .7428797055  .7771900329  .5187051308  .3631582592  .7847990160  000GMT 1/1/76
13.3594019864  .9993368945  .1129517942  .0536893056  .0000477414  INCR./365DAYS
      Z0       0  0  0  0  0  0 0.0     0
      SA       0  0  1  0  0 -1 0.0     0
      SSA      0  0  2  0  0  0 0.0     0
      MSM      0  1 -2  1  0  0  .00    0
      MM       0  1  0 -1  0  0 0.0     0
      MSF      0  2 -2  0  0  0 0.0     0
      MF       0  2  0  0  0  0 0.0     0
      ALP1     1 -4  2  1  0  0 -.25    2
      ALP1   -1  0  0 .75 0.0360R1   0 -1  0 .00 0.1906
      2Q1      1 -3  0  2  0  0-0.25    5
      2Q1    -2 -2  0 .50 0.0063    -1 -1  0 .75 0.0241R1  -1  0  0 .75 0.0607R1
      2Q1     0 -2  0 .50 0.0063     0 -1  0 .0  0.1885
      SIG1     1 -3  2  0  0  0-0.25    4
      SIG1   -1  0  0 .75 0.0095R1   0 -2  0 .50 0.0061     0 -1  0 .0  0.1884
      SIG1    2  0  0 .50 0.0087
      Q1       1 -2  0  1  0  0-0.25  10
      Q1     -2 -3  0 .50 0.0007    -2 -2  0 .50 0.0039    -1 -2  0 .75 0.0010R1
```

```
Q1     -1 -1  0 .75 0.0115R1  -1  0  0 .75 0.0292R1   0 -2  0 .50 0.0057
Q1     -1  0  1 .0  0.0008     0 -1  0 .0  0.1884     1  0  0 .75 0.0018R1
Q1      2  0  0 .50 0.0028
RHO1     1 -2  2 -1  0  0-0.25   5
RHO1    0 -2  0 .50 0.0058     0 -1  0 .0  0.1882     1  0  0 .75 0.0131R1
RHO1    2  0  0 .50 0.0576     2  1  0 .0  0.0175
O1       1 -1  0  0  0  0-0.25   8
O1     -1  0  0 .25 0.0003R1   0 -2  0 .50 0.0058     0 -1  0 .0  0.1885
O1      1 -1  0 .25 0.0004R1   1  0  0 .75 0.0029R1   1  1  0 .25 0.0004R1
O1      2  0  0 .50 0.0064     2  1  0 .50 0.0010
TAU1     1 -1  2  0  0  0-0.75   5
TAU1   -2  0  0 .0  0.0446    -1  0  0 .25 0.0426R1   0 -1  0 .50 0.0284
TAU1    0  1  0 .50 0.2170     0  2  0 .50 0.0142
BET1     1  0 -2  1  0  0 -.75   1
BET1    0 -1  0 .00 0.2266
NO1      1  0  0  1  0  0-0.75   9
NO1    -2 -2  0 .50 0.0057    -2 -1  0 .0  0.0665    -2  0  0 .0  0.3596
NO1    -1 -1  0 .75 0.0331R1  -1  0  0 .25 0.2227R1  -1  1  0 .75 0.0290R1
NO1     0 -1  0 .50 0.0290     0  1  0 .0  0.2004     0  2  0 .50 0.0054
CHI1     1  0  2 -1  0  0-0.75   2
CHI1    0 -1  0 .50 0.0282     0  1  0 .0  0.2187
PI1      1  1 -3  0  0  1-0.25   1
PI1     0 -1  0 .50 0.0078
P1       1  1 -2  0  0  0-0.25   6
P1      0 -2  0 .0  0.0008     0 -1  0 .50 0.0112     0  0  2 .50 0.0004
P1      1  0  0 .75 0.0004R1   2  0  0 .50 0.0015     2  1  0 .50 0.0003
S1       1  1 -1  0  0  1-0.75   2
S1      0  0 -2 .0  0.3534     0  1  0 .50 0.0264
K1       1  1  0  0  0  0-0.75  10
K1     -2 -1  0 .0  0.0002    -1 -1  0 .75 0.0001R1  -1  0  0 .25 0.0007R1
K1     -1  1  0 .75 0.0001R1   0 -2  0 .0  0.0001     0 -1  0 .50 0.0198
K1      0  1  0 .0  0.1356     0  2  0 .50 0.0029     1  0  0 .25 0.0002R1
K1      1  1  0 .25 0.0001R1
PSI1     1  1  1  0  0 -1-0.75   1
PSI1    0  1  0 .0  0.0190
PHI1     1  1  2  0  0  0-0.75   5
PHI1   -2  0  0 .0  0.0344    -2  1  0 .0  0.0106     0  0 -2 .0  0.0132
PHI1    0  1  0 .50 0.0384     0  2  0 .50 0.0185
THE1     1  2 -2  1  0  0 -.75   4
THE1   -2 -1  0 .00  .0300    -1  0  0 .25 0.0141R1   0 -1  0 .50  .0317
THE1    0  1  0 .00  .1993
J1       1  2  0 -1  0  0-0.75  10
J1      0 -1  0 .50 0.0294     0  1  0 .0  0.1980     0  2  0 .50 0.0047
J1      1 -1  0 .75 0.0027R1   1  0  0 .25 0.0816R1   1  1  0 .25 0.0331R1
J1      1  2  0 .25 0.0027R1   2  0  0 .50 0.0152     2  1  0 .50 0.0098
J1      2  2  0 .50 0.0057
OO1      1  3  0  0  0  0-0.75   8
OO1    -2 -1  0 .50 0.0037    -2  0  0 .0  0.1496    -2  1  0 .0  0.0296
OO1    -1  0  0 .25 0.0240R1  -1  1  0 .25 0.0099R1   0  1  0 .0  0.6398
OO1     0  2  0 .0  0.1342     0  3  0 .0  0.0086
UPS1     1  4  0 -1  0  0 -.75   5
UPS1   -2  0  0 .00 0.0611     0  1  0 .00 0.6399     0  2  0 .00 0.1318
UPS1    1  0  0 .25 0.0289R1   1  1  0 .25 0.0257R1
OQ2      2 -3  0  3  0  0 0.0     2
OQ2    -1  0  0 .25 0.1042R2   0 -1  0 .50 0.0386
EPS2     2 -3  2  1  0  0 0.0     3
EPS2   -1 -1  0 .25 0.0075R2  -1  0  0 .25 0.0402R2   0 -1  0 .50 0.0373
```

```
2N2      2 -2  0  2  0  0 0.0    4
2N2    -2 -2  0 .50 0.0061    -1 -1  0 .25 0.0117R2  -1  0  0 .25 0.0678R2
2N2     0 -1  0 .50 0.0374
MU2      2 -2  2  0  0  0 0.0    3
MU2    -1 -1  0 .25 0.0018R2  -1  0  0 .25 0.0104R2   0 -1  0 .50 0.0375
N2       2 -1  0  1  0  0 0.0    4
N2     -2 -2  0 .50 0.0039    -1  0  1 .00 0.0008     0 -2  0 .00 0.0005
N2      0 -1  0 .50 0.0373
NU2      2 -1  2 -1  0  0 0.0    4
NU2     0 -1  0 .50 0.0373     1  0  0 .75 0.0042R2   2  0  0 .0  0.0042
NU2     2  1  0 .50 0.0036
GAM2     2  0 -2  2  0  0 -.50   3
GAM2   -2 -2  0 .00 0.1429    -1  0  0 .25 0.0293R2   0 -1  0 .50 0.0330
H1       2  0 -1  0  0  1-0.50   2
H1      0 -1  0 .50 0.0224     1  0 -1 .50 0.0447
M2       2  0  0  0  0  0 0.0    9
M2     -1 -1  0 .75 0.0001R2  -1  0  0 .75 0.0004R2   0 -2  0 .0  0.0005
M2      0 -1  0 .50 0.0373     1 -1  0 .25 0.0001R2   1  0  0 .75 0.0009R2
M2      1  1  0 .75 0.0002R2   2  0  0 .0  0.0006     2  1  0 .0  0.0002
H2       2  0  1  0  0 -1 0.0    1
H2      0 -1  0 .50 0.0217
LDA2     2  1 -2  1  0  0-0.50   1
LDA2    0 -1  0 .50 0.0448
L2       2  1  0 -1  0  0-0.50   5
L2      0 -1  0 .50 0.0366     2 -1  0 .00 0.0047     2  0  0 .50 0.2505
L2      2  1  0 .50 0.1102     2  2  0 .50 0.0156
T2       2  2 -3  0  0  1 0.0    0
S2       2  2 -2  0  0  0 0.0    3
S2      0 -1  0 .0  0.0022     1  0  0 .75 0.0001R2   2  0  0 .0  0.0001
R2       2  2 -1  0  0 -1-0.50   2
R2      0  0  2 .50 0.2535     0  1  2 .0  0.0141
K2       2  2  0  0  0  0 0.0    5
K2     -1  0  0 .75 0.0024R2  -1  1  0 .75 0.0004R2   0 -1  0 .50 0.0128
K2      0  1  0 .0  0.2980     0  2  0 .0  0.0324
ETA2     2  3  0 -1  0  0 0.0    7
ETA2    0 -1  0 .50 0.0187     0  1  0 .0  0.4355     0  2  0 .0  0.0467
ETA2    1  0  0 .75 0.0747R2   1  1  0 .75 0.0482R2   1  2  0 .75 0.0093R2
ETA2    2  0  0 .50 0.0078
M3       3  0  0  0  0  0 -.50   1
M3      0 -1  0 .50  .0564

2PO1 2   2.0 P1        -1.0 O1
SO1  2   1.0 S2        -1.0 O1
ST36 3   2.0 M2         1.0 N2        -2.0 S2
2NS2 2   2.0 N2        -1.0 S2
ST37 2   3.0 M2        -2.0 S2
ST1  3   2.0 N2         1.0 K2        -2.0 S2
ST2  4   1.0 M2         1.0 N2         1.0 K2        -2.0 S2
ST3  3   2.0 M2         1.0 S2        -2.0 K2
O2   1   2.0 O1
ST4  3   2.0 K2         1.0 N2        -2.0 S2
SNK2 3   1.0 S2         1.0 N2        -1.0 K2
OP2  2   1.0 O1         1.0 P1
MKS2 3   1.0 M2         1.0 K2        -1.0 S2
ST5  3   1.0 M2         2.0 K2        -2.0 S2
ST6  4   2.0 S2         1.0 N2        -1.0 M2        -1.0 K2
2SK2 2   2.0 S2        -1.0 K2
```

MSN2	3	1.0 M2	1.0 S2	-1.0 N2	
ST7	4	2.0 K2	1.0 M2	-1.0 S2	-1.0 N2
2SM2	2	2.0 S2	-1.0 M2		
ST38	3	2.0 M2	1.0 S2	-2.0 N2	
SKM2	3	1.0 S2	1.0 K2	-1.0 M2	
2SN2	2	2.0 S2	-1.0 N2		
NO3	2	1.0 N2	1.0 O1		
MO3	2	1.0 M2	1.0 O1		
NK3	2	1.0 N2	1.0 K1		
SO3	2	1.0 S2	1.0 O1		
MK3	2	1.0 M2	1.0 K1		
SP3	2	1.0 S2	1.0 P1		
SK3	2	1.0 S2	1.0 K1		
ST8	3	2.0 M2	1.0 N2	-1.0 S2	
N4	1	2.0 N2			
3MS4	2	3.0 M2	-1.0 S2		
ST39	4	1.0 M2	1.0 S2	1.0 N2	-1.0 K2
MN4	2	1.0 M2	1.0 N2		
ST40	3	2.0 M2	1.0 S2	-1.0 K2	
ST9	4	1.0 M2	1.0 N2	1.0 K2	-1.0 S2
M4	1	2.0 M2			
ST10	3	2.0 M2	1.0 K2	-1.0 S2	
SN4	2	1.0 S2	1.0 N2		
KN4	2	1.0 K2	1.0 N2		
MS4	2	1.0 M2	1.0 S2		
MK4	2	1.0 M2	1.0 K2		
SL4	2	1.0 S2	1.0 L2		
S4	1	2.0 S2			
SK4	2	1.0 S2	1.0 K2		
MNO5	3	1.0 M2	1.0 N2	1.0 O1	
2MO5	2	2.0 M2	1.0 O1		
3MP5	2	3.0 M2	-1.0 P1		
MNK5	3	1.0 M2	1.0 N2	1.0 K1	
2MP5	2	2.0 M2	1.0 P1		
2MK5	2	2.0 M2	1.0 K1		
MSK5	3	1.0 M2	1.0 S2	1.0 K1	
3KM5	3	1.0 K2	1.0 K1	1.0 M2	
2SK5	2	2.0 S2	1.0 K1		
ST11	3	3.0 N2	1.0 K2	-1.0 S2	
2NM6	2	2.0 N2	1.0 M2		
ST12	4	2.0 N2	1.0 M2	1.0 K2	-1.0 S2
ST41	3	3.0 M2	1.0 S2	-1.0 K2	
2MN6	2	2.0 M2	1.0 N2		
ST13	4	2.0 M2	1.0 N2	1.0 K2	-1.0 S2
M6	1	3.0 M2			
MSN6	3	1.0 M2	1.0 S2	1.0 N2	
MKN6	3	1.0 M2	1.0 K2	1.0 N2	
2MS6	2	2.0 M2	1.0 S2		
2MK6	2	2.0 M2	1.0 K2		
NSK6	3	1.0 N2	1.0 S2	1.0 K2	
2SM6	2	2.0 S2	1.0 M2		
MSK6	3	1.0 M2	1.0 S2	1.0 K2	
ST42	3	2.0 M2	2.0 S2	-1.0 K2	
S6	1	3.0 S2			
ST14	3	2.0 M2	1.0 N2	1.0 O1	
ST15	3	2.0 N2	1.0 M2	1.0 K1	
M7	1	3.5 M2			

```
ST16 3   2.0 M2         1.0 S2         1.0 O1
3MK7 2   3.0 M2         1.0 K1
ST17 4   1.0 M2         1.0 S2         1.0 K2         1.0 O1
ST18 2   2.0 M2         2.0 N2
3MN8 2   3.0 M2         1.0 N2
ST19 4   3.0 M2         1.0 N2         1.0 K2        -1.0 S2
M8   1   4.0 M2
ST20 3   2.0 M2         1.0 S2         1.0 N2
ST21 3   2.0 M2         1.0 N2         1.0 K2
3MS8 2   3.0 M2         1.0 S2
3MK8 2   3.0 M2         1.0 K2
ST22 4   1.0 M2         1.0 S2         1.0 N2         1.0 K2
ST23 2   2.0 M2         2.0 S2
ST24 3   2.0 M2         1.0 S2         1.0 K2
ST25 3   2.0 M2         2.0 N2         1.0 K1
ST26 3   3.0 M2         1.0 N2         1.0 K1
4MK9 2   4.0 M2         1.0 K1
ST27 3   3.0 M2         1.0 S2         1.0 K1
ST28 2   4.0 M2         1.0 N2
M10  1   5.0 M2
ST29 3   3.0 M2         1.0 N2         1.0 S2
ST30 2   4.0 M2         1.0 S2
ST31 4   2.0 M2         1.0 N2         1.0 S2         1.0 K2
ST32 2   3.0 M2         2.0 S2
ST33 3   4.0 M2         1.0 S2         1.0 K1
M12  1   6.0 M2
ST34 2   5.0 M2         1.0 S2
ST35 4   3.0 M2         1.0 N2         1.0 K2         1.0 S2
```

Appendix 7.2 Sample Tidal Station Input Data for the Analysis Program.

The following sample input for logical unit 4 will produce an analysis of Tuktoyaktuk, Northwest Territories data for the period 1600 MST July 6, 1975 to 1400 MST September 9, 1975 inclusive, with constituents P_1 and K_2 inferred, shallow water constituent M_{10} specifically designated for analysis inclusion and only line printer output of the results. The final analysis results are listed in Appendix 7.3.

```
 6   1.0         0.0
    K1        0.0417807462    P1        0.0415525871   0.33093   -7.07
    S2        0.0833333333    K2        0.0835614924   0.27215   -22.40

      M10       M8

8 16060775  14090975
     6485  TUKTOYUKTUK NWT      MST 6927 3302
1  6485        6 775
2  6485        6 775             215 224 215 202 215 227 234 242 238
1  6485        7 775 229 218 206 200 193 187 179 176 183 199 215 231
2  6485        7 775 252 263 260 244 210 176 154 145 153 162 182 203
1  6485        8 775 221 232 230 195 153 119 105 115 132 159 192 218
2  6485        8 775 246 262 264 252 228 197 166 154 159 178 201 229
1  6485        9 775 251 267 291 257 225 204 183 176 188 183 204 232
2  6485        9 775 255 272 285 298 296 253 199 152 112 111 132 167
1  6485       10 775 201 221 227 223 201 166 131  99  70  82 121 161
2  6485       10 775 209 264 302 321 329 303 254 205 168 148 163 180
1  6485       11 775 212 244 271 282 278 258 221 169 135 126 134 156
2  6485       11 775 182 219 249 257 262 243 205 168 135 110 105 118
1  6485       12 775 142 178 213 242 247 233 203 159 119  85  72  89
2  6485       12 775 116 148 180 205 223 222 186 147 105  66  43  55
1  6485       13 775  78 104 136 167 194 199 182 148 107  72  54  61
2  6485       13 775  87 108 139 165 182 190 185 158 125  89  59  49
1  6485       14 775  55  84 113 138 164 182 194 184 154 118  83  66
2  6485       14 775  66  87 117 146 165 180 181 164 134 100  62  40
1  6485       15 775  36  48  72 103 129 157 167 167 156 131 105  87
2  6485       15 775  71  72  80  95 114 128 137 144 132 110  83  63
1  6485       16 775  42  24  29  57  94 125 147 156 155 134 105  82
2  6485       16 775  70  64  63  74  95 116 131 135 130 115  93  75
1  6485       17 775  60  50  50  61  84 111 135 149 154 154 143 122
2  6485       17 775  99  81  72  73  85  99 115 127 127 115 108  94
1  6485       18 775  78  59  50  50  64  81 100 132 164 184 188 179
2  6485       18 775 159 137 133 135 137 143 147 148 153 161 170 161
1  6485       19 775 147 141 142 128 130 143 160 177 193 211 230 236
2  6485       19 775 226 204 181 165 157 161 178 178 178 186 196 198
1  6485       20 775 201 190 172 155 138 130 136 156 174 199 227 245
2  6485       20 775 254 256 245 199 162 141 129 134 157 183 206 220
1  6485       21 775 219 222 210 194 182 169 171 183 206 240 255 265
2  6485       21 775 288 296 292 282 262 238 212 190 186 198 220 240
1  6485       22 775 259 268 271 264 249 228 203 184 187 206 230 257
2  6485       22 775 283 293 295 282 261 232 204 182 165 171 192 218
1  6485       23 775 232 247 255 249 230 205 181 158 148 152 180 209
2  6485       23 775 234 260 272 261 231 196 160 130 111 109 125 157
1  6485       24 775 187 209 224 231 209 181 155 125 110 111 130 159
2  6485       24 775 195 227 249 250 233 200 161 123  94  87  97 123
```

1	6485	25	775	153	183	196	202	195	174	138	101	71	58	60	87
2	6485	25	775	122	159	185	202	199	179	144	103	66	40	35	48
1	6485	26	775	75	104	132	151	160	155	129	98	66	39	34	47
2	6485	26	775	79	113	144	163	172	167	151	117	85	50	20	19
1	6485	27	775	39	74	107	136	148	158	141	118	89	54	29	16
2	6485	27	775	41	76	105	143	189	202	196	185	185	162	160	163
1	6485	28	775	168	187	222	254	260	275	281	268	256	241	221	198
2	6485	28	775	208	230	258	264	285	301	291	270	247	212	188	176
1	6485	29	775	183	200	224	245	256	269	280	270	243	216	194	164
2	6485	29	775	163	177	201	232	263	282	281	290	259	238	202	179
1	6485	30	775	179	184	205	226	242	272	281	279	263	233	205	279
2	6485	30	775	168	184	210	235	247	253	263	259	244	221	193	183
1	6485	31	775	180	176	194	208	215	224	235	243	241	225	207	188
2	6485	31	775	176											
1	6485	1	875												
2	6485	1	875												
1	6485	2	875												
2	6485	2	875			104	95	93	95	103	112	118	118	116	108
1	6485	3	875	97	83	68	56	51	54	75	95	117	130	138	139
2	6485	3	875	133	120	103	87	71	56	52	66	81	98	109	107
1	6485	4	875	98	77	49	28	14	4	7	17	44	70	94	110
2	6485	4	875	117	116	107	88	71	55	46	44	60	84	108	125
1	6485	5	875	133	136	114	86	70	62	62	79	113	143	175	208
2	6485	5	875	238	256	266	240	203	179	143	117	118	146	167	186
1	6485	6	875	224	243	227	204	180	158	154	170	201	222	234	243
2	6485	6	875	254	260	247	231	211	188	160	143	137	145	167	195
1	6485	7	875	221	239	249	249	227	184	144	111	102	129	170	201
2	6485	7	875	233	255	260	252	227	195	156	123	107	118	149	180
1	6485	8	875	211	232	245	257	229	200	171	138	102	95	122	163
2	6485	8	875	207	253	295	338	369	353	318	285	221	184	165	175
1	6485	9	875	212	240	260	283	282	259	229	196	174	176	187	204
2	6485	9	875	244	288	329	356	369	370	324	281	289	294	293	287
1	6485	10	875	329	380	426	441	447	453	418	387	353	337	322	314
2	6485	10	875	342	365	404	438	470	482	487	456	441	423	438	448
1	6485	11	875	464	478	491	505	538	528	493	488	472	425	398	390
2	6485	11	875	393	408	421	438	444	433	412	379	337	300	262	247
1	6485	12	875	245	252	277	304	327	339	339	308	257	208	182	182
2	6485	12	875	203	235	260	281	319	315	297	273	237	198	168	158
1	6485	13	875	157	171	195	217	239	252	258	253	242	225	202	179
2	6485	13	875	167	172	190	217	242	257	266	263	244	217	187	155
1	6485	14	875	132	134	163	195	228	246	259	256	236	209	180	150
2	6485	14	875	129	122	136	161	184	200	207	205	195	177	158	136
1	6485	15	875	116	105	104	115	140	164	193	203	216	208	196	187
2	6485	15	875	159	142	147	164	175	183	197	202	202	202	192	176
1	6485	16	875	160	147	137	136	152	172	195	211	224	228	222	210
2	6485	16	875	199	186	171	165	163	169	180	190	201	203	200	193
1	6485	17	875	185	175	162	152	156	169	201	227	249	272	284	285
2	6485	17	875	295	280	259	241	225	211	211	226	247	268	286	297
1	6485	18	875	296	272	245	214	196	194	209	226	239	244	245	248
2	6485	18	875	246	239	229	218	201	183	165	158	160	183	207	221
1	6485	19	875	227	224	209	187	159	138	131	139	162	185	209	228
2	6485	19	875	239	242	233	212	183	152	129	119	132	167	193	218
1	6485	20	875	237	241	230	205	178	151	130	114	122	145	172	203
2	6485	20	875	226	237	237	223	197	165	131	108	103	118	144	173
1	6485	21	875	203	225	229	223	200	175	150	129	131	146	173	202
2	6485	21	875	236	258	263	256	233	198	165	137	127	133	159	190
1	6485	22	875	221	241	252	252	231	200	167	137	119	114	134	166
2	6485	22	875	201	234	256	264	249	212	176	140	111	103	115	140

1	6485	23	875	171	203	232	244	242	214	180	146	121	115	126	157
2	6485	23	875	187	211	235	249	247	229	194	151	114	88	87	110
1	6485	24	875	143	177	206	230	237	223	186	140	93	64	66	94
2	6485	24	875	129	165	197	215	220	205	177	144	112	84	80	100
1	6485	25	875	136	173	208	238	252	244	217	181	139	105	89	93
2	6485	25	875	121	159	188	199	200	185	155	121	84	64	45	28
1	6485	26	875	32	72	121	174	215	237	211	197	234	243	176	196
2	6485	26	875	250	219	272	361	391	376	355	389	370	321	300	285
1	6485	27	875	288	323	350	380	422	415	405	389	412	430	453	509
2	6485	27	875	557	559	548	560	576	557	513	489	462	422	388	383
1	6485	28	875	371	393	413	419	443	472	444	423	384	340	304	284
2	6485	28	875	280	286	300	309	312	324	319	299	270	227	203	181
1	6485	29	875	193	240	281	317	352	351	361	350	354	348	358	350
2	6485	29	875	326	317	316	324	327	313	298	283	264	244	230	215
1	6485	30	875	194	194	217	241	256	262	261	259	247	229	213	195
2	6485	30	875	175	163	168	177	187	198	203	204	191	171	144	119
1	6485	31	875	97	92	102	125	150	168	176	188	197	197	206	202
2	6485	31	875	191	186	192	200	197	199	206	205	207	208	205	198
1	6485	1	975	185	187	194	209	234	255	275	285	305	327	332	320
2	6485	1	975	301	295	291	275	277	294	312	328	344	335	321	328
1	6485	2	975	323	315	324	316	318	329	321	314	317	329	336	336
2	6485	2	975	327	316	301	284	263	245	236	231	233	240	250	262
1	6485	3	975	261	250	227	202	172	153	153	162	171	172	190	214
2	6485	3	975	226	228	214	186	160	143	142	155	173	201	236	255
1	6485	4	975	274	284	282	255	216	183	165	179	203	231	258	294
2	6485	4	975	327	364	353	332	299	262	227	207	219	240	256	275
1	6485	5	975	302	309	298	274	240	196	159	142	158	192	222	249
2	6485	5	975	270	280	282	269	239	197	154	110	99	125	159	187
1	6485	6	975	214	235	236	221	189	153	118	83	63	52	65	106
2	6485	6	975	132	151	165	175	169	148	115	74	42	18	5	30
1	6485	7	975	68	123	189	218	198	167	126	81	52	40	56	81
2	6485	7	975	121	173	203	211	199	173	137	100	72	58	57	92
1	6485	8	975	135	179	218	238	240	224	190	136	91	58	48	65
2	6485	8	975	99	140	173	194	195	175	141	96	57	33	30	50
1	6485	9	975	86	129	173	204	217	202	171	125	79	47	40	59
2	6485	9	975	88	121										

Appendix 7.3 Final Analysis Results Arising from the Input Data of Appendix 7.2 and the Standard Constituent Data Package of Appendix 7.1.

```
ANALYSIS OF HOURLY TIDAL HEIGHTS  STN  6485    16H  6/ 7/75 TO  14H  9/ 9/75
NO.OBS.=  1559    NO.PTS.ANAL.=  1559    MIDPT= 3H  8/ 8/75  SEPARATION =1.00
```

NO	NAME	FREQUENCY	STN	M-Y/ M-Y	A	G		AL	GL
1	Z0	0.00000000	6485	775/ 975	1.9806	0.00		1.9806	0.00
2	MM	0.00151215	6485	775/ 975	0.2121	263.34		0.2121	288.50
3	MSF	0.00282193	6485	775/ 975	0.1561	133.80		0.1561	115.15
4	ALP1	0.03439657	6485	775/ 975	0.0152	334.95		0.0141	180.96
5	2Q1	0.03570635	6485	775/ 975	0.0246	82.69		0.0226	246.82
6	Q1	0.03721850	6485	775/ 975	0.0158	65.74		0.0144	252.75
7	O1	0.03873065	6485	775/ 975	0.0764	74.23		0.0694	284.43
8	NO1	0.04026859	6485	775/ 975	0.0290	238.14		0.0380	275.85
9	P1	0.04155259	6485	775/ 975	0.0465	71.76	INF FR K1	0.0468	252.20
10	K1	0.04178075	6485	775/ 975	0.1406	64.69		0.1332	145.54
11	J1	0.04329290	6485	775/ 975	0.0253	7.32		0.0234	103.63
12	OO1	0.04483084	6485	775/ 975	0.0531	235.74		0.0463	358.47
13	UPS1	0.04634299	6485	775/ 975	0.0298	91.73		0.0233	239.12
14	EPS2	0.07617731	6485	775/ 975	0.0211	184.59		0.0216	109.98
15	MU2	0.07768947	6485	775/ 975	0.0419	83.23		0.0428	30.06
16	N2	0.07899925	6485	775/ 975	0.0838	44.52		0.0857	306.35
17	M2	0.08051140	6485	775/ 975	0.4904	77.70		0.5007	4.40
18	L2	0.08202355	6485	775/ 975	0.0213	35.21		0.0174	168.03
19	S2	0.08333334	6485	775/ 975	0.2195	126.65		0.2193	36.74
20	K2	0.08356149	6485	775/ 975	0.0597	149.05	INF FR S2	0.0515	131.15
21	ETA2	0.08507364	6485	775/ 975	0.0071	246.05		0.0059	235.38
22	MO3	0.11924206	6485	775/ 975	0.0148	234.97		0.0138	11.86
23	M3	0.12076710	6485	775/ 975	0.0123	261.57		0.0126	331.91
24	MK3	0.12229215	6485	775/ 975	0.0049	331.60		0.0048	339.15
25	SK3	0.12511408	6485	775/ 975	0.0023	237.69		0.0022	228.64
26	MN4	0.15951066	6485	775/ 975	0.0092	256.47		0.0096	85.00
27	M4	0.16102280	6485	775/ 975	0.0126	291.78		0.0131	145.17
28	SN4	0.16233259	6485	775/ 975	0.0083	270.85		0.0085	82.78
29	MS4	0.16384473	6485	775/ 975	0.0010	339.35		0.0011	176.14
30	S4	0.16666667	6485	775/ 975	0.0047	299.56		0.0047	119.75
31	2MK5	0.20280355	6485	775/ 975	0.0013	310.10		0.0013	244.34
32	2SK5	0.20844743	6485	775/ 975	0.0045	104.00		0.0043	5.04
33	2MN6	0.24002205	6485	775/ 975	0.0035	271.24		0.0038	26.46
34	M6	0.24153420	6485	775/ 975	0.0017	158.89		0.0018	298.97
35	2MS6	0.24435614	6485	775/ 975	0.0056	306.10		0.0059	69.59
36	2SM6	0.24717808	6485	775/ 975	0.0023	298.92		0.0023	45.80
37	3MK7	0.28331494	6485	775/ 975	0.0086	212.25		0.0086	73.20
38	M8	0.32204559	6485	775/ 975	0.0030	42.43		0.0033	109.22
39	M10	0.40255699	6485	775/ 975	0.0009	198.23		0.0010	191.71

Appendix 7.4 Sample Input for the Tidal Heights Prediction Program.

The following sample input for logical unit 8 will synthesize hourly heights and the times and heights of all extrema at Victoria, British Columbia for the period 0100 PST July 1, 1976 to 2400 PST July 31, 1976 inclusive. The output results are listed in Appendix 7.5.

```
 .7428797055  .7771900329  .5187051308  .3631582592  .7847990160  000GMT 1/1/76
13.3594019864  .9993368945  .1129517942  .0536893056  .0000477414  INCR./365DAYS
    Z0       0  0  0  0  0  0 0.0     0
    SA       0  0  1  0  0 -1 0.0     0
    SSA      0  0  2  0  0  0 0.0     0
    MSM      0  1 -2  1  0  0  .00    0
    MM       0  1  0 -1  0  0 0.0     0
    MSF      0  2 -2  0  0  0 0.0     0
    MF       0  2  0  0  0  0 0.0     0
    ALP1     1 -4  2  1  0  0 -.25    2
    ALP1  -1  0  0 .75 0.0360R1   0 -1  0 .00 0.1906
    2Q1      1 -3  0  2  0  0-0.25    5
    2Q1   -2 -2  0 .50 0.0063     -1 -1  0 .75 0.0241R1  -1  0  0 .75 0.0607R1
    2Q1    0 -2  0 .50 0.0063      0 -1  0 .0  0.1885
    SIG1     1 -3  2  0  0  0-0.25    4
    SIG1  -1  0  0 .75 0.0095R1   0 -2  0 .50 0.0061      0 -1  0 .0  0.1884
    SIG1   2  0  0 .50 0.0087
    Q1       1 -2  0  1  0  0-0.25   10
    Q1    -2 -3  0 .50 0.0007     -2 -2  0 .50 0.0039     -1 -2  0 .75 0.0010R1
    Q1    -1 -1  0 .75 0.0115R1  -1  0  0 .75 0.0292R1   0 -2  0 .50 0.0057
    Q1    -1  0  1 .0  0.0008      0 -1  0 .0  0.1884      1  0  0 .75 0.0018R1
    Q1     2  0  0 .50 0.0028
    RHO1     1 -2  2 -1  0  0-0.25    5
    RHO1   0 -2  0 .50 0.0058      0 -1  0 .0  0.1882      1  0  0 .75 0.0131R1
    RHO1   2  0  0 .50 0.0576      2  1  0 .0  0.0175
    O1       1 -1  0  0  0  0-0.25    8
    O1    -1  0  0 .25 0.0003R1   0 -2  0 .50 0.0058      0 -1  0 .0  0.1885
    O1     1 -1  0 .25 0.0004R1   1  0  0 .75 0.0029R1   1  1  0 .25 0.0004R1
    O1     2  0  0 .50 0.0064      2  1  0 .50 0.0010
    TAU1     1 -1  2  0  0  0-0.75    5
    TAU1  -2  0  0 .0  0.0446     -1  0  0 .25 0.0426R1   0 -1  0 .50 0.0284
    TAU1   0  1  0 .50 0.2170      0  2  0 .50 0.0142
    BET1     1  0 -2  1  0  0 -.75    1
    BET1   0 -1  0 .00 0.2266
    NO1      1  0  0  1  0  0-0.75    9
    NO1   -2 -2  0 .50 0.0057     -2 -1  0 .0  0.0665     -2  0  0 .0  0.3596
    NO1   -1 -1  0 .75 0.0331R1  -1  0  0 .25 0.2227R1  -1  1  0 .75 0.0290R1
    NO1    0 -1  0 .50 0.0290      0  1  0 .0  0.2004      0  2  0 .50 0.0054
    CHI1     1  0  2 -1  0  0-0.75    2
    CHI1   0 -1  0 .50 0.0282      0  1  0 .0  0.2187
    PI1      1  1 -3  0  0  1-0.25    1
    PI1    0 -1  0 .50 0.0078
    P1       1  1 -2  0  0  0-0.25    6
    P1     0 -2  0 .0  0.0008      0 -1  0 .50 0.0112      0  0  2 .50 0.0004
    P1     1  0  0 .75 0.0004R1   2  0  0 .50 0.0015      2  1  0 .50 0.0003
    S1       1  1 -1  0  0  1-0.75    2
    S1     0  0 -2 .0  0.3534      0  1  0 .50 0.0264
```

```
K1       1  1  0  0  0  0-0.75  10
K1     -2 -1  0 .0  0.0002    -1 -1  0 .75 0.0001R1  -1  0  0 .25 0.0007R1
K1     -1  1  0 .75 0.0001R1   0 -2  0 .0  0.0001     0 -1  0 .50 0.0198
K1      0  1  0 .0  0.1356     0  2  0 .50 0.0029     1  0  0 .25 0.0002R1
K1      1  1  0 .25 0.0001R1
PSI1     1  1  1  0  0 -1-0.75   1
PSI1    0  1  0 .0  0.0190
PHI1     1  1  2  0  0  0-0.75   5
PHI1   -2  0  0 .0  0.0344    -2  1  0 .0  0.0106     0  0 -2 .0  0.0132
PHI1    0  1  0 .50 0.0384     0  2  0 .50 0.0185
THE1     1  2 -2  1  0  0 -.75   4
THE1   -2 -1  0 .00  .0300    -1  0  0 .25 0.0141R1   0 -1  0 .50  .0317
THE1    0  1  0 .00  .1993
J1       1  2  0 -1  0  0-0.75  10
J1      0 -1  0 .50 0.0294     0  1  0 .0  0.1980     0  2  0 .50 0.0047
J1      1 -1  0 .75 0.0027R1   1  0  0 .25 0.0816R1   1  1  0 .25 0.0331R1
J1      1  2  0 .25 0.0027R1   2  0  0 .50 0.0152     2  1  0 .50 0.0098
J1      2  2  0 .50 0.0057
OO1      1  3  0  0  0  0-0.75   8
OO1    -2 -1  0 .50 0.0037    -2  0  0 .0  0.1496    -2  1  0 .0  0.0296
OO1    -1  0  0 .25 0.0240R1  -1  1  0 .25 0.0099R1   0  1  0 .0  0.6398
OO1     0  2  0 .0  0.1342     0  3  0 .0  0.0086
UPS1     1  4  0 -1  0  0 -.75   5
UPS1   -2  0  0 .00 0.0611     0  1  0 .00 0.6399     0  2  0 .00 0.1318
UPS1    1  0  0 .25 0.0289R1   1  1  0 .25 0.0257R1
OQ2      2 -3  0  3  0  0 0.0    2
OQ2    -1  0  0 .25 0.1042R2   0 -1  0 .50 0.0386
EPS2     2 -3  2  1  0  0 0.0    3
EPS2   -1 -1  0 .25 0.0075R2  -1  0  0 .25 0.0402R2   0 -1  0 .50 0.0373
2N2      2 -2  0  2  0  0 0.0    4
2N2    -2 -2  0 .50 0.0061    -1 -1  0 .25 0.0117R2  -1  0  0 .25 0.0678R2
2N2     0 -1  0 .50 0.0374
MU2      2 -2  2  0  0  0 0.0    3
MU2    -1 -1  0 .25 0.0018R2  -1  0  0 .25 0.0104R2   0 -1  0 .50 0.0375
N2       2 -1  0  1  0  0 0.0    4
N2     -2 -2  0 .50 0.0039    -1  0  1 .00 0.0008     0 -2  0 .00 0.0005
N2      0 -1  0 .50 0.0373
NU2      2 -1  2 -1  0  0 0.0    4
NU2     0 -1  0 .50 0.0373     1  0  0 .75 0.0042R2   2  0  0 .0  0.0042
NU2     2  1  0 .50 0.0036
GAM2     2  0 -2  2  0  0 -.50   3
GAM2   -2 -2  0 .00 0.1429    -1  0  0 .25 0.0293R2   0 -1  0 .50 0.0330
H1       2  0 -1  0  0  1-0.50   2
H1      0 -1  0 .50 0.0224     1  0 -1 .50 0.0447
M2       2  0  0  0  0  0 0.0    9
M2     -1 -1  0 .75 0.0001R2  -1  0  0 .75 0.0004R2   0 -2  0 .0  0.0005
M2      0 -1  0 .50 0.0373     1 -1  0 .25 0.0001R2   1  0  0 .75 0.0009R2
M2      1  1  0 .75 0.0002R2   2  0  0 .0  0.0006     2  1  0 .0  0.0002
H2       2  0  1  0  0 -1 0.0    1
H2      0 -1  0 .50 0.0217
LDA2     2  1 -2  1  0  0-0.50   1
LDA2    0 -1  0 .50 0.0448
L2       2  1  0 -1  0  0-0.50   5
L2      0 -1  0 .50 0.0366     2 -1  0 .00 0.0047     2  0  0 .50 0.2505
L2      2  1  0 .50 0.1102     2  2  0 .50 0.0156
T2       2  2 -3  0  0  1 0.0    0
S2       2  2 -2  0  0  0 0.0    3
S2      0 -1  0 .0  0.0022     1  0  0 .75 0.0001R2   2  0  0 .0  0.0001
```

```
R2       2  2 -1  0  0 -1-0.50   2
R2      0  0  2 .50 0.2535     0  1  2 .0  0.0141
K2       2  2  0  0  0  0 0.0    5
K2     -1  0  0 .75 0.0024R2  -1  1  0 .75 0.0004R2   0 -1  0 .50 0.0128
K2      0  1  0 .0  0.2980     0  2  0 .0  0.0324
ETA2     2  3  0 -1  0  0 0.0    7
ETA2    0 -1  0 .50 0.0187     0  1  0 .0  0.4355     0  2  0 .0  0.0467
ETA2    1  0  0 .75 0.0747R2   1  1  0 .75 0.0482R2   1  2  0 .75 0.0093R2
ETA2    2  0  0 .50 0.0078
M3       3  0  0  0  0  0 -.50   1
M3      0 -1  0 .50  .0564

2PO1 2   2.0 P1        -1.0 O1
SO1  2   1.0 S2        -1.0 O1
ST36 3   2.0 M2         1.0 N2         -2.0 S2
2NS2 2   2.0 N2        -1.0 S2
ST37 2   3.0 M2        -2.0 S2
ST1  3   2.0 N2         1.0 K2         -2.0 S2
ST2  4   1.0 M2         1.0 N2          1.0 K2         -2.0 S2
ST3  3   2.0 M2         1.0 S2         -2.0 K2
O2   1   2.0 O1
ST4  3   2.0 K2         1.0 N2         -2.0 S2
SNK2 3   1.0 S2         1.0 N2         -1.0 K2
OP2  2   1.0 O1         1.0 P1
MKS2 3   1.0 M2         1.0 K2         -1.0 S2
ST5  3   1.0 M2         2.0 K2         -2.0 S2
ST6  4   2.0 S2         1.0 N2         -1.0 M2         -1.0 K2
2SK2 2   2.0 S2        -1.0 K2
MSN2 3   1.0 M2         1.0 S2         -1.0 N2
ST7  4   2.0 K2         1.0 M2         -1.0 S2         -1.0 N2
2SM2 2   2.0 S2        -1.0 M2
ST38 3   2.0 M2         1.0 S2         -2.0 N2
SKM2 3   1.0 S2         1.0 K2         -1.0 M2
2SN2 2   2.0 S2        -1.0 N2
NO3  2   1.0 N2         1.0 O1
MO3  2   1.0 M2         1.0 O1
NK3  2   1.0 N2         1.0 K1
SO3  2   1.0 S2         1.0 O1
MK3  2   1.0 M2         1.0 K1
SP3  2   1.0 S2         1.0 P1
SK3  2   1.0 S2         1.0 K1
ST8  3   2.0 M2         1.0 N2         -1.0 S2
N4   1   2.0 N2
3MS4 2   3.0 M2        -1.0 S2
ST39 4   1.0 M2         1.0 S2          1.0 N2         -1.0 K2
MN4  2   1.0 M2         1.0 N2
ST40 3   2.0 M2         1.0 S2         -1.0 K2
ST9  4   1.0 M2         1.0 N2          1.0 K2         -1.0 S2
M4   1   2.0 M2
ST10 3   2.0 M2         1.0 K2         -1.0 S2
SN4  2   1.0 S2         1.0 N2
KN4  2   1.0 K2         1.0 N2
MS4  2   1.0 M2         1.0 S2
MK4  2   1.0 M2         1.0 K2
SL4  2   1.0 S2         1.0 L2
S4   1   2.0 S2
SK4  2   1.0 S2         1.0 K2
```

```
MNO5 3   1.0 M2         1.0 N2         1.0 O1
2MO5 2   2.0 M2         1.0 O1
3MP5 2   3.0 M2        -1.0 P1
MNK5 3   1.0 M2         1.0 N2         1.0 K1
2MP5 2   2.0 M2         1.0 P1
2MK5 2   2.0 M2         1.0 K1
MSK5 3   1.0 M2         1.0 S2         1.0 K1
3KM5 3   1.0 K2         1.0 K1         1.0 M2
2SK5 2   2.0 S2         1.0 K1
ST11 3   3.0 N2         1.0 K2        -1.0 S2
2NM6 2   2.0 N2         1.0 M2
ST12 4   2.0 N2         1.0 M2         1.0 K2        -1.0 S2
ST41 3   3.0 M2         1.0 S2        -1.0 K2
2MN6 2   2.0 M2         1.0 N2
ST13 4   2.0 M2         1.0 N2         1.0 K2        -1.0 S2
M6   1   3.0 M2
MSN6 3   1.0 M2         1.0 S2         1.0 N2
MKN6 3   1.0 M2         1.0 K2         1.0 N2
2MS6 2   2.0 M2         1.0 S2
2MK6 2   2.0 M2         1.0 K2
NSK6 3   1.0 N2         1.0 S2         1.0 K2
2SM6 2   2.0 S2         1.0 M2
MSK6 3   1.0 M2         1.0 S2         1.0 K2
ST42 3   2.0 M2         2.0 S2        -1.0 K2
S6   1   3.0 S2
ST14 3   2.0 M2         1.0 N2         1.0 O1
ST15 3   2.0 N2         1.0 M2         1.0 K1
M7   1   3.5 M2
ST16 3   2.0 M2         1.0 S2         1.0 O1
3MK7 2   3.0 M2         1.0 K1
ST17 4   1.0 M2         1.0 S2         1.0 K2         1.0 O1
ST18 2   2.0 M2         2.0 N2
3MN8 2   3.0 M2         1.0 N2
ST19 4   3.0 M2         1.0 N2         1.0 K2        -1.0 S2
M8   1   4.0 M2
ST20 3   2.0 M2         1.0 S2         1.0 N2
ST21 3   2.0 M2         1.0 N2         1.0 K2
3MS8 2   3.0 M2         1.0 S2
3MK8 2   3.0 M2         1.0 K2
ST22 4   1.0 M2         1.0 S2         1.0 N2         1.0 K2
ST23 2   2.0 M2         2.0 S2
ST24 3   2.0 M2         1.0 S2         1.0 K2
ST25 3   2.0 M2         2.0 N2         1.0 K1
ST26 3   3.0 M2         1.0 N2         1.0 K1
4MK9 2   4.0 M2         1.0 K1
ST27 3   3.0 M2         1.0 S2         1.0 K1
ST28 2   4.0 M2         1.0 N2
M10  1   5.0 M2
ST29 3   3.0 M2         1.0 N2         1.0 S2
ST30 2   4.0 M2         1.0 S2
ST31 4   2.0 M2         1.0 N2         1.0 S2         1.0 K2
ST32 2   3.0 M2         2.0 S2
ST33 3   4.0 M2         1.0 S2         1.0 K1
M12  1   6.0 M2
ST34 2   5.0 M2         1.0 S2
ST35 4   3.0 M2         1.0 N2         1.0 K2         1.0 S2
```

```
     7120 VICTORIA HARBOUR BC   PST 48 23  123 22
     Z0                                 6.0670    .00
     Q1                                  .1970 130.30
     O1                                 1.2110 137.00
     NO1                                0.1120 120.80
     P1                                  .6740 148.50
     S1                                  .0980 154.10
     K1                                 2.0700 149.40
     J1                                  .1170 166.40
     N2                                  .2940  63.40
     M2                                 1.2130  87.00
     S2                                  .3320  93.90
001007076 031007076 EQUI  1.0
001007076 031007076 EXTR  0.5
```

Appendix 7.5 Tidal Heights Prediction Results Arising from the Input Data of Appendix 7.4. Figure 2 is the Plot of These Hourly Heights over the Period 0100 PST July 24, 1976 to 2400 PST July 31, 1976.

STN	1ST HR	DATE	1	2	3	4	5	6	7	8	DT HRS
7120	1.0000	1 776	7.459	7.736	7.926	7.886	7.518	6.799	5.797	4.658	1.0000
7120	9.0000	1 776	3.578	2.759	2.361	2.470	3.068	4.047	5.226	6.393	1.0000
7120	17.0000	1 776	7.356	7.979	8.211	8.092	7.732	7.283	6.896	6.676	1.0000
7120	1.0000	2 776	6.664	6.823	7.051	7.216	7.189	6.887	6.296	5.482	1.0000
7120	9.0000	2 776	4.581	3.766	3.210	3.044	3.323	4.012	4.995	6.097	1.0000
7120	17.0000	2 776	7.122	7.899	8.314	8.338	8.022	7.484	6.878	6.351	1.0000
7120	1.0000	3 776	6.011	5.900	5.988	6.186	6.375	6.436	6.289	5.914	1.0000
7120	9.0000	3 776	5.363	4.747	4.211	3.896	3.911	4.293	5.004	5.934	1.0000
7120	17.0000	3 776	6.919	7.782	8.370	8.586	8.407	7.893	7.164	6.375	1.0000
7120	1.0000	4 776	5.680	5.195	4.976	5.009	5.220	5.496	5.722	5.807	1.0000
7120	9.0000	4 776	5.712	5.462	5.136	4.851	4.727	4.857	5.277	5.957	1.0000
7120	17.0000	4 776	6.797	7.652	8.357	8.770	8.798	8.424	7.706	6.770	1.0000
7120	1.0000	5 776	5.778	4.899	4.267	3.958	3.975	4.255	4.688	5.143	1.0000
7120	9.0000	5 776	5.509	5.718	5.761	5.690	5.598	5.596	5.775	6.182	1.0000
7120	17.0000	5 776	6.798	7.539	8.271	8.836	9.093	8.948	8.381	7.454	1.0000
7120	1.0000	6 776	6.301	5.102	4.046	3.292	2.936	2.996	3.409	4.053	1.0000
7120	9.0000	6 776	4.775	5.432	5.923	6.211	6.325	6.350	6.399	6.574	1.0000
7120	17.0000	6 776	6.935	7.479	8.131	8.762	9.210	9.329	9.020	8.263	1.0000
7120	1.0000	7 776	7.127	5.764	4.376	3.180	2.357	2.019	2.187	2.787	1.0000
7120	9.0000	7 776	3.672	4.661	5.576	6.287	6.737	6.950	7.014	7.053	1.0000
7120	17.0000	7 776	7.188	7.494	7.976	8.561	9.114	9.465	9.458	8.991	1.0000
7120	1.0000	8 776	8.050	6.720	5.176	3.648	2.377	1.560	1.314	1.648	1.0000
7120	9.0000	8 776	2.465	3.589	4.803	5.903	6.743	7.259	7.482	7.516	1.0000
7120	17.0000	8 776	7.507	7.588	7.846	8.286	8.827	9.320	9.584	9.456	1.0000
7120	1.0000	9 776	8.838	7.730	6.241	4.572	2.979	1.719	0.993	0.906	1.0000
7120	9.0000	9 776	1.444	2.478	3.799	5.164	6.352	7.215	7.701	7.860	1.0000
7120	17.0000	9 776	7.821	7.744	7.777	8.005	8.422	8.932	9.371	9.548	1.0000
7120	1.0000	10 776	9.302	8.547	7.306	5.715	4.002	2.437	1.282	0.725	1.0000
7120	9.0000	10 776	0.846	1.596	2.812	4.258	5.675	6.844	7.632	8.010	1.0000
7120	17.0000	10 776	8.057	7.923	7.782	7.782	7.996	8.401	8.881	9.260	1.0000
7120	1.0000	11 776	9.345	8.986	8.121	6.800	5.186	3.522	2.086	1.121	1.0000
7120	9.0000	11 776	0.791	1.138	2.074	3.408	4.886	6.253	7.306	7.941	1.0000
7120	17.0000	11 776	8.163	8.077	7.850	7.660	7.643	7.851	8.240	8.679	1.0000
7120	1.0000	12 776	8.984	8.974	8.519	7.586	6.255	4.707	3.191	1.971	1.0000
7120	9.0000	12 776	1.261	1.184	1.740	2.808	4.176	5.591	6.816	7.681	1.0000
7120	17.0000	12 776	8.116	8.163	7.948	7.646	7.424	7.398	7.597	7.956	1.0000
7120	1.0000	13 776	8.333	8.553	8.452	7.933	6.991	5.729	4.337	3.054	1.0000
7120	9.0000	13 776	2.113	1.688	1.854	2.573	3.699	5.015	6.284	7.301	1.0000
7120	17.0000	13 776	7.935	8.158	8.038	7.714	7.356	7.115	7.078	7.253	1.0000
7120	1.0000	14 776	7.559	7.858	7.990	7.820	7.283	6.400	5.287	4.128	1.0000
7120	9.0000	14 776	3.136	2.502	2.353	2.722	3.537	4.642	5.830	6.894	1.0000
7120	17.0000	14 776	7.667	8.063	8.086	7.823	7.415	7.018	6.758	6.698	1.0000
7120	1.0000	15 776	6.827	7.064	7.279	7.338	7.138	6.641	5.886	4.988	1.0000
7120	9.0000	15 776	4.112	3.432	3.096	3.184	3.691	4.528	5.543	6.550	1.0000
7120	17.0000	15 776	7.379	7.906	8.080	7.931	7.553	7.081	6.649	6.360	1.0000
7120	1.0000	16 776	6.260	6.329	6.490	6.636	6.657	6.479	6.083	5.514	1.0000
7120	9.0000	16 776	4.873	4.295	3.914	3.832	4.092	4.665	5.459	6.333	1.0000
7120	17.0000	16 776	7.135	7.730	8.029	8.012	7.720	7.248	6.718	6.245	1.0000
7120	1.0000	17 776	5.915	5.762	5.769	5.871	5.981	6.016	5.919	5.677	1.0000
7120	9.0000	17 776	5.331	4.958	4.658	4.528	4.634	4.992	5.566	6.269	1.0000
7120	17.0000	17 776	6.981	7.578	7.959	8.062	7.883	7.469	6.910	6.316	1.0000

7120	1.0000	18	776	5.790	5.408	5.205	5.168	5.249	5.378	5.486	5.522	1.0000
7120	9.0000	18	776	5.471	5.355	5.226	5.154	5.207	5.426	5.817	6.345	1.0000
7120	17.0000	18	776	6.933	7.484	7.898	8.095	8.031	7.708	7.178	6.525	1.0000
7120	1.0000	19	776	5.855	5.266	4.835	4.602	4.564	4.681	4.892	5.128	1.0000
7120	9.0000	19	776	5.334	5.481	5.571	5.635	5.721	5.880	6.146	6.522	1.0000
7120	17.0000	19	776	6.980	7.456	7.867	8.127	8.167	7.952	7.489	6.833	1.0000
7120	1.0000	20	776	6.071	5.313	4.664	4.206	3.987	4.006	4.225	4.576	1.0000
7120	9.0000	20	776	4.980	5.366	5.689	5.935	6.122	6.291	6.489	6.752	1.0000
7120	17.0000	20	776	7.090	7.479	7.863	8.162	8.295	8.194	7.829	7.215	1.0000
7120	1.0000	21	776	6.415	5.531	4.686	3.997	3.556	3.409	3.554	3.935	1.0000
7120	9.0000	21	776	4.468	5.052	5.599	6.049	6.382	6.616	6.799	6.987	1.0000
7120	17.0000	21	776	7.223	7.524	7.865	8.185	8.399	8.420	8.179	7.652	1.0000
7120	1.0000	22	776	6.868	5.911	4.904	3.992	3.306	2.940	2.935	3.265	1.0000
7120	9.0000	22	776	3.851	4.579	5.324	5.983	6.491	6.835	7.049	7.196	1.0000
7120	17.0000	22	776	7.349	7.560	7.842	8.163	8.447	8.592	8.503	8.112	1.0000
7120	1.0000	23	776	7.407	6.440	5.326	4.217	3.280	2.656	2.433	2.628	1.0000
7120	9.0000	23	776	3.185	3.988	4.890	5.747	6.448	6.937	7.223	7.364	1.0000
7120	17.0000	23	776	7.450	7.567	7.770	8.062	8.390	8.654	8.735	8.528	1.0000
7120	1.0000	24	776	7.973	7.081	5.936	4.689	3.525	2.625	2.130	2.109	1.0000
7120	9.0000	24	776	2.545	3.339	4.336	5.360	6.255	6.917	7.315	7.488	1.0000
7120	17.0000	24	776	7.528	7.549	7.646	7.866	8.193	8.544	8.793	8.803	1.0000
7120	1.0000	25	776	8.469	7.746	6.677	5.386	4.057	2.903	2.111	1.808	1.0000
7120	9.0000	25	776	2.030	2.716	3.722	4.856	5.925	6.775	7.322	7.570	1.0000
7120	17.0000	25	776	7.597	7.529	7.495	7.593	7.854	8.229	8.607	8.835	1.0000
7120	1.0000	26	776	8.766	8.303	7.429	6.220	4.840	3.505	2.439	1.823	1.0000
7120	9.0000	26	776	1.756	2.232	3.142	4.301	5.494	6.520	7.241	7.611	1.0000
7120	17.0000	26	776	7.671	7.541	7.371	7.301	7.418	7.728	8.152	8.547	1.0000
7120	1.0000	27	776	8.744	8.597	8.023	7.038	5.755	4.368	3.113	2.211	1.0000
7120	9.0000	27	776	1.822	2.007	2.713	3.790	5.022	6.182	7.080	7.608	1.0000
7120	17.0000	27	776	7.758	7.616	7.332	7.074	6.982	7.123	7.477	7.934	1.0000
7120	1.0000	28	776	8.333	8.495	8.285	7.643	6.615	5.344	4.043	2.950	1.0000
7120	9.0000	28	776	2.270	2.131	2.551	3.435	4.600	5.815	6.856	7.559	1.0000
7120	17.0000	28	776	7.851	7.766	7.424	6.998	6.664	6.551	6.707	7.085	1.0000
7120	1.0000	29	776	7.556	7.946	8.084	7.847	7.201	6.216	5.052	3.924	1.0000
7120	9.0000	29	776	3.058	2.630	2.731	3.338	4.326	5.492	6.607	7.464	1.0000
7120	17.0000	29	776	7.931	7.972	7.656	7.128	6.573	6.166	6.022	6.169	1.0000
7120	1.0000	30	776	6.545	7.011	7.395	7.535	7.328	6.757	5.905	4.933	1.0000
7120	9.0000	30	776	4.045	3.439	3.258	3.557	4.283	5.293	6.383	7.336	1.0000
7120	17.0000	30	776	7.973	8.192	7.991	7.462	6.766	6.091	5.602	5.405	1.0000
7120	1.0000	31	776	5.516	5.866	6.321	6.716	6.903	6.793	6.379	5.738	1.0000
7120	9.0000	31	776	5.019	4.402	4.051	4.079	4.511	5.281	6.241	7.198	1.0000
7120	17.0000	31	776	7.957	8.368	8.355	7.938	7.223	6.375	5.578	4.992	1.0000

HL	STN	DATE		TIME	HGT	TIME	HGT	TIME	HGT	TIME	HGT	TIME	HGT	TIME	HGT
0	7120	1	776	322	7.9	1117	2.3	1907	8.2	9999	99.9	9999	99.9	9999	99.9
1	7120	2	776	33	6.6	424	7.2	1153	3.0	1933	8.4	9999	99.9	9999	99.9
1	7120	3	776	200	5.9	550	6.4	1228	3.9	2002	8.6	9999	99.9	9999	99.9
1	7120	4	776	321	5.0	759	5.8	1302	4.7	2034	8.8	9999	99.9	9999	99.9
1	7120	5	776	426	3.9	1047	5.8	1333	5.6	2110	9.1	9999	99.9	9999	99.9
1	7120	6	776	521	2.9	2148	9.3	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	7120	7	776	609	2.0	2230	9.5	9999	99.9	9999	99.9	9999	99.9	9999	99.9
1	7120	8	776	655	1.3	1548	7.5	1648	7.5	2313	9.6	9999	99.9	9999	99.9
1	7120	9	776	738	0.9	1611	7.9	1819	7.7	2358	9.5	9999	99.9	9999	99.9
1	7120	10	776	819	0.7	1640	8.1	1931	7.8	9999	99.9	9999	99.9	9999	99.9
0	7120	11	776	44	9.4	859	0.8	1709	8.2	2035	7.6	9999	99.9	9999	99.9
0	7120	12	776	129	9.0	937	1.1	1737	8.2	2137	7.4	9999	99.9	9999	99.9

0	7120	13	776	215	8.6	1013	1.7	1806	8.2	2240	7.1	9999	99.9	9999	99.9	
0	7120	14	776	300	8.0	1047	2.3	1833	8.1	2347	6.7	9999	99.9	9999	99.9	
0	7120	15	776	346	7.3	1118	3.1	1900	8.1	9999	99.9	9999	99.9	9999	99.9	
1	7120	16	776	102	6.3	438	6.7	1145	3.8	1926	8.1	9999	99.9	9999	99.9	
1	7120	17	776	226	5.7	549	6.0	1205	4.5	1951	8.1	9999	99.9	9999	99.9	
1	7120	18	776	345	5.2	755	5.5	1209	5.2	2016	8.1	9999	99.9	9999	99.9	
1	7120	19	776	442	4.6	2040	8.2	9999	99.9	9999	99.9	9999	99.9	9999	99.9	
1	7120	20	776	524	4.0	2106	8.3	9999	99.9	9999	99.9	9999	99.9	9999	99.9	
1	7120	21	776	559	3.4	2136	8.4	9999	99.9	9999	99.9	9999	99.9	9999	99.9	
1	7120	22	776	631	2.9	2210	8.6	9999	99.9	9999	99.9	9999	99.9	9999	99.9	
1	7120	23	776	701	2.4	2250	8.7	9999	99.9	9999	99.9	9999	99.9	9999	99.9	
1	7120	24	776	732	2.1	2333	8.8	9999	99.9	9999	99.9	9999	99.9	9999	99.9	
1	7120	25	776	804	1.8	1639	7.6	1850	7.5	9999	99.9	9999	99.9	9999	99.9	
0	7120	26	776	19	8.9	837	1.7	1644	7.7	1955	7.3	9999	99.9	9999	99.9	
0	7120	27	776	108	8.8	911	1.8	1657	7.8	2055	7.0	9999	99.9	9999	99.9	
0	7120	28	776	159	8.5	945	2.1	1714	7.9	2155	6.6	9999	99.9	9999	99.9	
0	7120	29	776	254	8.1	1019	2.6	1736	8.0	2259	6.0	9999	99.9	9999	99.9	
0	7120	30	776	356	7.5	1053	3.3	1800	8.2	9999	99.9	9999	99.9	9999	99.9	
1	7120	31	776	7	5.4	509	6.9	1126	4.0	1828	8.4	9999	99.9	9999	99.9	