

Algorithm Design XIII

NP Problem I

Guoqiang Li School of Software



Efficient Problems, Difficult Problems

Efficient Algorithms



We have developed algorithms for

Efficient Algorithms



We have developed algorithms for

- FINDING SHORTEST PATHS IN GRAPHS,
- MINIMUM SPANNING TREES IN GRAPHS,
- MATCHINGS IN BIPARTITE GRAPHS,
- MAXIMUM INCREASING SUBSEQUENCES,
- MAXIMUM FLOWS IN NETWORKS,
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All these algorithms are efficient, since their time requirement grows as a polynomial function (such as n, n^2 , or n^3) of the size of the input.



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Are there "search problems" in which seeking a solution among an exponential chaos, and the fastest algorithms for them are exponential?

MINIMUM SPANNING TREES

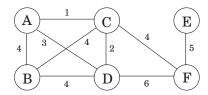
Build a Network



Suppose you are asked to network a collection of computers by linking selected pairs of them.

This translates into a graph problem in which

- nodes are computers,
- undirected edges are potential links, each with a maintenance cost.



A General Kruskal's Algorithm



```
\begin{split} X &= \{ \ \}; \\ \text{repeat until } |X| &= |V|-1; \\ \text{pick a set } S \subset V \text{ for which } X \text{ has no edges between } S \text{ and } V-S; \\ \text{let } e \in E \text{ be the minimum-weight edge between } S \text{ and } V-S; \\ X &= X \cup \{e\}; \end{split}
```

A Little Change of the MST



WHAT IF THE TREE IS NOT ALLOWED TO BRANCH?

SATISFIABILITY PROBLEM



The instances of SATISFIABILITY or SAT:

$$(x \vee y \vee z)(x \vee \overline{y})(y \vee \overline{z})(z \vee \overline{x})(\overline{x} \vee \overline{y} \vee \overline{z})$$

a Boolean formula in conjunctive normal form (CNF).



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Given a Boolean formula in conjunctive normal form, either find a satisfying truth assignment or else report that none exists.



Given a set of clauses, where each clause is the disjunction of two literals, looking for an assignment so that all clauses are satisfied.

$$(x_1 \lor x_2) \land (\overline{x}_1 \lor x_3) \land (x_1 \lor \overline{x}_2) \land (x_3 \lor x_4) \land (\overline{x}_1 \lor \overline{x}_4)$$



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Given an instance I of 2-SAT with n variables and m clauses, construct a directed graph $G_I = (V, E)$ as follows.

- G_I has 2n nodes, one for each variable and its negation.
- G_I has 2m edges: for each clause $(\alpha \vee \beta)$ of I, G_I has an edge from the negation of α to β , and one from the negation of β to α .



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Conclude that there is a linear-time algorithm for solving 2-SAT.

A Little Change of the 2-SAT



3-SAT, SAT?



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If no such solution exists, we must say so.



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ullet For SAT, it is easy to check whether the assignment specified by S satisfies every clause in I.



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We say S is a solution to I if and only if C(I, S) =true.



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There are two natural variants of SAT with good algorithms.

- 2-SAT can be solved in linear time.
- All clauses contain at most one positive literal, say Horn formula, can be found by the greedy algorithm.

TRAVELING SALESMAN PROBLEM





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To find a cycle that passes through every vertex exactly once, of total cost b or less - or to report that no such cycle.

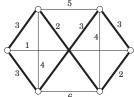


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A permutation $\tau(1), \ldots, \tau(n)$ of the vertices such that when they are toured in this order, the total distance covered is at most b:

$$d_{\tau(1),\tau(2)} + d_{\tau(2),\tau(3)} + \dots + d_{\tau(n),\tau(1)} \le b$$







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But why are we expressing the TSP in this way, when in reality it is an optimization problem, in which the shortest possible tour is sought?



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Any algorithm that solves the optimization also solves the search problem:

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Conversely, an algorithm for the search problem can also be used to solve the optimization problem:

- First suppose that we knew the cost of the optimum tour; then we could find this tour by calling
 the algorithm for the search problem, using the optimum cost as the budget.
- We can find the optimum cost by binary search.



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Given a potential solution to the TSP, it is easy to check the properties "is a tour" (just check that each vertex is visited exactly once) and "has total length $\leq b$."

But how could one check the property "is optimal"?



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This extra restriction on the structure of the tree results in a much harder problem.

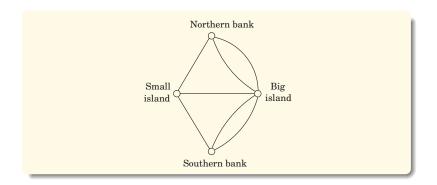
EULER AND RUDRATA

Euler Path



EULER PATH:

Given a graph, find a path that contains each edge exactly once



Euler Path



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The answer is yes if and only if

Euler Path



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- 1 the graph is connected and
- every vertex, with the possible exception of two vertices (the start and final vertices of the walk), has even degree.

Euler Path

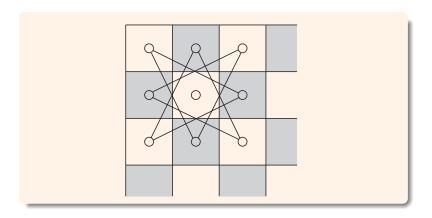


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There is a polynomial time algorithm for EULER PATH.







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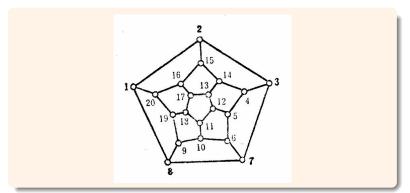
Given a graph, find a cycle that visits each vertex exactly once.



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In the literature this problem is known as the Hamilton cycle problem.



CUTS AND BISECTIONS



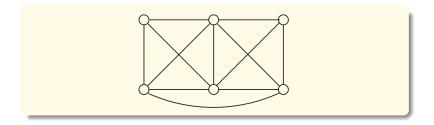


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MINIMUM CUT: given a graph and a budget b, find a cut with at most b edges.







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The smallest such flow will correspond (via the max-flow min-cut theorem) to the smallest cut.





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BALANCED CUT: Given a graph with n vertices and a budget b, partition the vertices into two sets S and T such that $|S|, |T| \ge n/3$ and such that there are at most b edges between S and T.

INTEGER LINEAR PROGRAMMING

Linear Programming



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Linear Programming



In a LINEAR PROGRAMMING problem we are given a set of variables, and to assign real values to them so as to

- 1 satisfy a set of linear equations and/or linear inequalities involving these variables, and
- 2 maximize or minimize a given linear objective function.

Linear Programming



$$\max x_1 + 6x_2 + 13x_3$$

$$x_1 \le 200$$

$$x_2 \le 300$$

$$x_1 + x_2 + x_3 \le 400$$

$$x_2 + 3x_3 \le 600$$

$$x_1, x_2, x_3 \ge 0$$



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We want to find a nonnegative integer *n*-vector *x* such that $A\mathbf{x} \leq b$ and $c \cdot \mathbf{x} \geq g$.



$$\begin{aligned} \max 2x_1 + 5x_2 \\ 2x_1 - x_2 &\leq 4 \\ x_1 + 2x_2 &\leq 9 \\ -x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

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2x_1 - x_2 \le 4
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But there is a redundancy here:

- the last constraint $c \cdot \mathbf{x} \ge g$ is itself a linear inequality and
- can be absorbed into $A\mathbf{x} \leq b$.



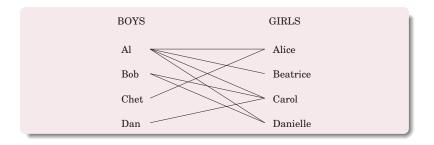
So, we define ILP to be following search problem:

Given A and b, find a nonnegative integer vector \mathbf{x} satisfying the inequalities $A\mathbf{x} \leq b$.

3D-MATCHING

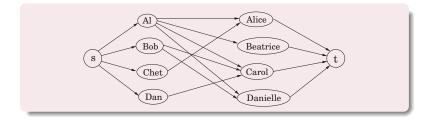
Bipartite Matching





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Three-Dimensional Matching



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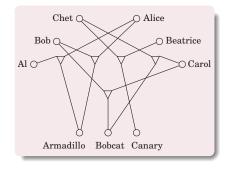
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A triple (b, g, p) means that boy b, girl g, and pet p get along well together.

To find n disjoint triples and thereby create n harmonious households.





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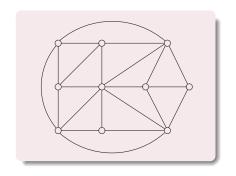
VERTEX COVER: Given a graph and an integer *b*, find *b* vertices cover (touch) every edge.



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LONGEST PATH





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To avoid trivial solutions we require that the path be simple, containing no repeated vertices.

KNAPSACK





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The problem is solvable in time O(nW) by dynamic programming.



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- Q: Could it be polynomial?

Subset Sum



Subset Sum



 $\begin{center} {\bf SUBSET~SUM} : Find a subset of a given set of integers that adds up to exactly W. \end{center}$