TOON MUSKENS Onderwerp: HW ATTP GW

Datum:

1a) from the tutorial: $\dot{\omega}' = \frac{96 G^{5/3}}{5c^5} \mu M^{2/3} \omega^{11/3}$

$$\int_\omega^\infty d\omega' (\omega')^{-11/3} = \frac{96}{5c^5} G^{5/3} \mu M^{2/3} \int_t^{t_c} dt'$$

$$-\frac{3}{8} \omega^{-8/3} \Big|_\omega^\infty = \frac{96}{5c^5} G^{5/3} \mu M^{2/3} (t_c - t)$$

$$\frac{3}{8} \omega^{-8/3} = \frac{96}{5c^5} G^{5/3} \mu M^{2/3} \tau \Rightarrow \omega = \left(\frac{8}{3} \frac{96}{5c^5} G^{5/3} \mu M^{2/3} \tau\right)^{-\frac{3}{8}}$$

b) $m_1 = m_2$

$\Rightarrow \mu(m_1 = m_2) = \frac{1}{2} M$ $\qquad M = 2{,}8\, M_\odot$

$\omega = \frac{2\pi}{7{,}75 \times 3600} \approx 2{,}25 \cdot 10^{-4} \frac{rad}{s}$

$\frac{96 G^{5/3}}{5c^5} \mu M^{2/3} \simeq \frac{96 G^{5/3}}{5c^5} \frac{1}{2} \cdot 2{,}8\, M_\odot (2{,}8\, M_\odot)^{2/3} \approx 2{,}411 \cdot 10^{-58}\, M_\odot^{5/3}$

$\Rightarrow T_0 = \frac{(2{,}25 \cdot 10^{-4})^{-8/3}}{(2{,}411 \cdot 10^{-58} \cdot M_\odot^{5/3})} \cdot \frac{3}{8} \approx 830 \cdot 10^6\, y$

c) if $f = \omega/\pi \Rightarrow \omega_{GW} = \pi f$ $\qquad \omega_{orbit} = 2\,\omega_{GW}$

for $f = 10\, Hz \Rightarrow \omega = 10\pi$ and $T_{10} = T_0 \cdot \left(\frac{10\pi}{2{,}25 \cdot 10^{-4}}\right)^{-8/3} \approx 499$ sec

for $f = 100\, Hz \Rightarrow \omega = 100\pi$ and $T_{100} = T_{10} \cdot (10)^{-8/3} \approx 1{,}1\, s$

d) $\Delta T = (T_{10_{ch}} - T_{100_{ch}}) = (T_{10} - T_{100}) \cdot \left(\frac{1{,}4}{30}\right)^{5/3} \approx 3{,}01$

for leaving Lisa frequency and entering Ligo/virgo band

$\Rightarrow \Delta T = (T_{0,1} - T_{10}) \cdot \left(\frac{1{,}4}{30}\right)^{5/3} = T_{10} \cdot \left(\frac{1{,}4}{30}\right)^{5/3} \left[\left(\frac{0{,}1}{10}\right)^{-8/3} - 1\right] \approx 7{,}5$ days

$T_{0,1} = T_{10} \cdot \left(\frac{0{,}1}{10}\right)^{-8/3}$

e) $M_\odot \gg M_\oplus \Rightarrow \mu \approx \frac{M_\odot M_\oplus}{M_\odot} = M_\oplus \qquad M \approx M_\odot$

$f_{GW} = \frac{1}{3600 \cdot 365 \cdot 24} \approx 31{,}71 \cdot 10^{-9}\, Hz$

$\Rightarrow \frac{96 G^{5/3}}{5c^5} \mu M^{2/3} \approx 8{,}67 \cdot 10^{-59} \cdot M_\oplus \cdot M_\odot^{2/3}$

$\omega \approx 2 \cdot 10^{-7}\, s^{-1} \cdot rad$

$\Rightarrow T = \frac{(2 \cdot 10^{-7})^{-8/3}}{8{,}67 \cdot 10^{-59} \cdot M_\oplus M_\odot^{2/3}} \cdot \frac{3}{8} \approx 1{,}05 \cdot 10^{23}\, y$

2a) $\varphi = \int \omega \, dt = \int \left(\frac{96\, G^{5/3}}{5c^5} \mu M^{2/3} \tau \cdot \frac{8}{3} \right)^{-\frac{3}{8}} dt$

$\tau = t_c - t \Rightarrow \frac{d\tau}{dt} = -1 \Rightarrow -d\tau = dt$

$\Rightarrow \varphi(\tau) = -\int \left(\frac{96 G^{5/3}}{5c^5} \mu M^{2/3} \tau \frac{8}{3} \right)^{-3/8} d\tau$

$= -\left(\frac{96}{5c^5} \mu M^{2/3} \right)^{-3/8} \cdot \left(\frac{8}{3}\right)^{-3/8} \cdot \frac{8}{5} \tau^{5/8} \cdot G\omega$

$= -\left(\frac{96}{5c^5} \frac{8}{3} \right)^{-3/8} \cdot \frac{8}{5} \cdot (G\mathcal{M})^{-5/8} \tau^{5/8}$

b) $h^{TT}_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow h^{TT}_{11} = -h^{TT}_{22} = h_+$

$\Rightarrow h_+ = \frac{1}{2}\left(h^{TT}_{11} - h^{TT}_{22} \right)$, $h_\times = \frac{1}{2}(h^{TT}_{12} + h^{TT}_{21}) = h^{TT}_{12} = h^{TT}_{21}$

$h^{TT}_+ = \frac{1}{2} \frac{1}{d} \frac{2G}{c^4} \left(\Lambda_{11kl} \ddot{Q}^{kl} - \Lambda_{22kl} \ddot{Q}^{kl} \right)$

$\Lambda_{11kl} = P_{1k} P_{1l} - \frac{1}{2} P_{11} P_{kl} = P_{1k} P_{1l} - \frac{1}{2} P_{kl}$

$\Lambda_{22kl} = P_{2k} P_{2l} - \frac{1}{2} P_{kl}$ $\boxed{P_{1k} = \delta_{1k} - n_1 n_k = \delta_{1k}}$ analogue for P_{1l} and P_{2k} and P_{2l} due to $\vec{n} = (0,0,1)$

$\Lambda_{11kl} \ddot{Q}^{kl} = (P_{1k} P_{1l} - \frac{1}{2} P_{kl}) 2\mu r^2 \omega^2 (\phi^k \phi^l - n^k n^l)$

$= 2\mu r^2 \omega^2 \left[\phi_1 \phi_1 - n_1 n_1 - \frac{1}{2}(\phi_l \phi^l - n^l n_l - 0 + 1) \right]$

$= 2\mu r^2 \omega^2 \left[\sin^2\varphi - \cos^2\varphi - \frac{1}{2} \right]$

$\Lambda_{22kl} \ddot{Q}^{kl} = 2\mu r^2 \omega^2 [\phi_2 \phi_2 - n_2 n_2 - \frac{1}{2}]$

$= 2\mu r^2 \omega^2 [\cos^2 \iota \cos^2\varphi - \cos^2 \iota \sin^2\varphi - \frac{1}{2}]$

$\Rightarrow -\Lambda_{22kl} \ddot{Q}^{kl} + \Lambda_{11kl} \ddot{Q}^{kl}$

$= 2\mu r^2 \omega^2 [\sin^2\varphi - \cos^2\varphi - \cancel{\tfrac{1}{2}} - \cos^2\iota \cos^2\varphi + \cos^2\iota \sin^2\varphi + \cancel{\tfrac{1}{2}}]$

$= 2\mu r^2 \omega^2 (1 + \cos^2 \iota) [\sin^2\varphi - \cos^2\varphi]$

$= -2\mu r^2 \omega^2 (1 + \cos^2\iota) \cos(2\varphi)$

$\boxed{h_+(\tau) = -\frac{2G}{c^4} \frac{1}{d} \mu r^2 \omega^2 (1 + \cos^2\iota) \cos(2\varphi)}$

Onderwerp:

Datum:

$P_{1k}P_{2l}(\phi^k\phi^l - n^k n^l) = (\phi_1\phi_2 - n_1 n_2)$ $\quad P_{12} = \delta_{12} - n_1 n_2 = 0$

$\Lambda_{12kl}\ddot{Q}^{kl} = (\phi_1\phi_2 - n_1 n_2) \times 2\mu r^2\omega^2$

$= 2\mu r^2\omega^2(-\cos i \sin\varphi\cos\varphi - \cos i\cos\varphi\sin\varphi)$

$= 2\mu r^2\omega^2(-2\cos i\sin\varphi\cos\varphi)$

$= -2\mu r^2\omega^2\cos i\sin 2\varphi$

$\Rightarrow \boxed{h_x = -2\mu r^2\omega^2\frac{2G}{dc^4}\cos i\sin 2\varphi}$

rewriting in form of chirp mass using $r = \left(\frac{GM}{\omega^2}\right)^{1/3}$

$\Rightarrow h_+ = -\frac{2}{c^4}\frac{1}{d}\mu\left(\frac{GM}{\omega^2}\right)^{2/3}\omega^2(1+\cos^2 i)\cos(2\varphi)$ $\quad \mathcal{M} = \mu^{3/5}M^{2/5} \Rightarrow \boxed{\mathcal{M}^{5/3} = \mu M^{2/3}}$

$= -\frac{2}{dc^4}(G\mathcal{M})^{5/3}\omega^{2/3}(1+\cos^2 i)\cos(2\varphi)$

$h_x = -\frac{4}{dc^4}\mu\left(\frac{GM}{\omega^2}\right)^{2/3}G\cos i\sin 2\varphi\,\omega^2$

$= -\frac{4}{dc^4}(G\mathcal{M})^{5/3}\omega^{2/3}\cos i\sin 2\varphi$

c $\quad \mathcal{M}_{NS} = (0{,}5)^{3/5}\cdot 2^{2/5}\times 1{,}4M_\odot$ $\quad$ it is clear that $\omega_{GW} = 2\omega_{orb}$ $\Rightarrow f_{GW} = \frac{2}{\pi}\omega_{orb}$

$\mathcal{M}_{BH} = (0{,}5)^{3/5}\cdot 2^{2/5} + 35M_\odot$

for black holes @ either 2000 Mpc or 40 Mpc, then the only difference is the amplitude of the waves which are bigger in the 40 Mpc case than for the 2000Mpc case

SEE The Last Pages for the Plots

d $\quad \phi(t) = \omega t = \frac{1}{2}\pi f_{GW}\cdot t \Rightarrow f_{GW}\to f_{GW}/\varepsilon$ and $t\to t\varepsilon$

$\Rightarrow \phi'(t) = \frac{1}{2}\pi f'_{GW}t' = \frac{1}{2}\pi(f_{GW}/\varepsilon)\cdot t\varepsilon = \phi(t)$

$\frac{1}{d}\cdot\mathcal{M}^{5/3}\cdot(\frac{1}{2}\pi f_{GW})^{2/3} \longrightarrow \frac{1}{d\varepsilon}(\mathcal{M}\varepsilon)^{5/3}(\frac{1}{2}\pi f_{GW}/\varepsilon)^{2/3}$

$= \frac{1}{d}(\mathcal{M})^{5/3}(\frac{1}{2}\pi f_{GW})\underbrace{\varepsilon^{-1}\varepsilon^{5/3}\varepsilon^{2/3}}_{=1}$

$\Rightarrow$ the signals are invariant under these transformations wher h_x and h_+ in terms of f_{GW} are

$h_+ = -\frac{2}{dc^4}(G\mathcal{M})^{5/3}(\frac{1}{2}\pi)^{2/3}(f_{GW})^{2/3}(1+\cos^2 i)\cos(\pi f_{GW}T)$

$h_x = -\frac{2}{dc^4}(G\mathcal{M})^{5/3}(\frac{1}{2}\pi)^{2/3}(f_{GW})^{2/3}\cos i\sin(\pi f_{GW}T)$

Onderwerp:

Datum:

$$\mathcal{M}_{det}(z, \mathcal{M}_{source}) = \mathcal{M}_s \varepsilon = \mathcal{M}_s(1+z)$$

$$D_L = d\varepsilon = d(1+z)$$

3 a) E_1 : ~420 Mpc SNR = 23,7
E_2 : ~440 Mpc SNR = 13.0
E_3 : ~1 Gpc SNR = 9.7

$SNR_{treshold} = 12$

$h_+, h_\times \propto \frac{1}{d}$ $d \propto \frac{1}{SNR}$

event 1

$d_1 \propto \frac{1}{SNR_1}$

$d_{tresh} \propto \frac{1}{SNR_{tresh}}$

$$\Rightarrow \frac{d_1}{d_{tresh}} \propto \frac{SNR_{tresh}}{SNR_1} \Rightarrow d_{tresh} \approx \frac{SNR_1}{SNR_{tresh}} \times d_1 = 829,5 \text{ MPc}$$

$$V = \frac{4}{3}\pi d_{tresh}^3 = \frac{4}{3}\pi \cdot (0,829,5)^3 \approx 2,39 \text{ Gpc}^3$$

Event 2

$$d_{tresh} = \frac{13}{12} \cdot 440 \approx 476,7 \text{ MPc} \Rightarrow V \approx \frac{4}{3}\pi (476,7 \cdot 10^{-3})^3 \approx 0,454 \text{ Gpc}^3$$

Event 3

$$d_{tresh} = \frac{9,7}{12} \cdot 1 = 0,81 \text{ Gpc} \Rightarrow V \approx 2,23 \text{ Gpc}^3$$

b $T = 49$ days $\sim 0,13$ y

$$\Rightarrow R_1 \sim \frac{1}{TV_1} \approx 3,22 \text{ Gpc}^{-3} \text{y}^{-1}$$

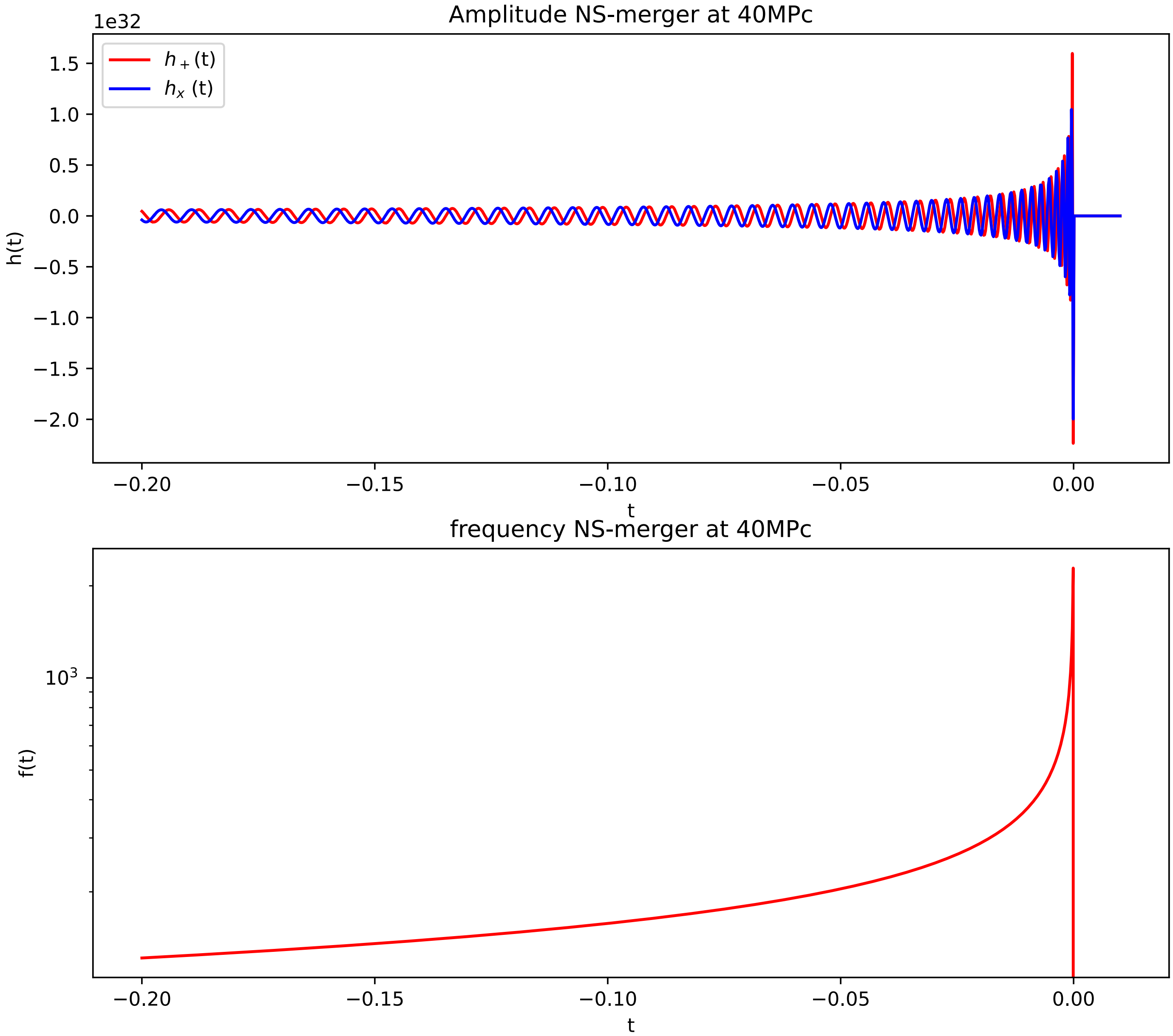
$$R_2 \sim \frac{1}{TV_2} \approx 16,94 \text{ Gpc}^{-3} \text{y}^{-1}$$

$$R_3 \sim \frac{1}{TV_3} \approx 3,45 \text{ Gpc}^{-3} \text{y}^{-1}$$

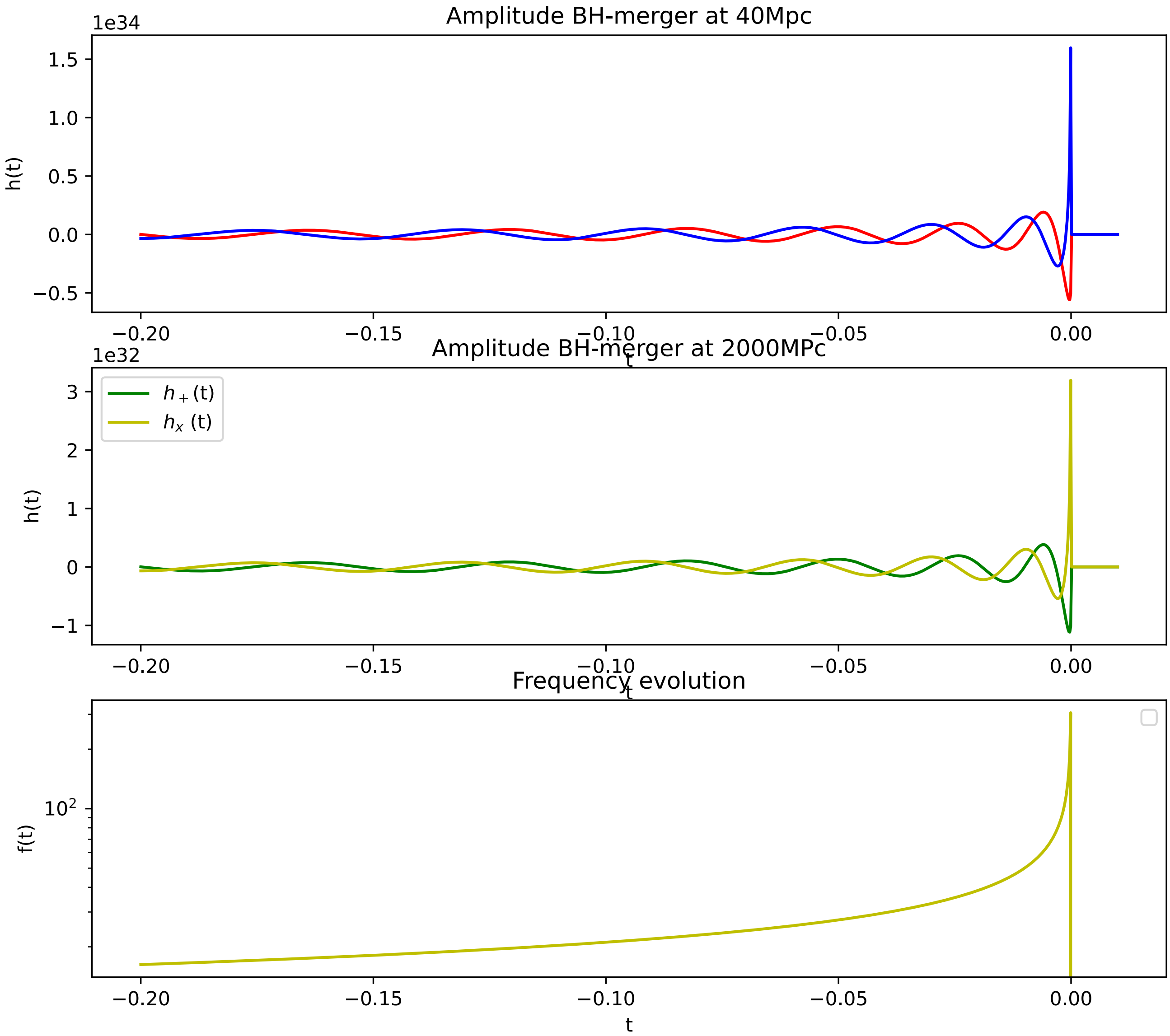
let the combined rate be the average of the values weighted by their $\frac{SNR_{signal}}{SNR_{treshold}}$ (their reliability)

$$\Rightarrow R_t \sim \frac{1,975 \cdot R_1 + 1,083 R_2 + 0,808 R_3}{1,975 + 1,083 + 0,808} \approx 7,11 \text{ Gpc}^{-3} \text{y}^{-1}$$

$\Rightarrow$ according to this data $\approx 7,11 \cdot 10^3$ black hole mergers happen per year (on average)



First figure shows the polarisation amplitudes of the NS merger with a mass of 1.4 solar masses
The second figure shows the frequency evolution during the inspiraling.



Plots of the BH merger with masses of 36 solar masses, the only difference that is evident from the 40Mpc distance and 2000Mpc distance is that the amplitude for 2000Mpc distance is two orders smaller but the behaviour of both polarisations looks the same.