



Joe says that he lives 10 miles from Lake Michigan, that Beth lives 10 miles from Lake Michigan, and he and Beth live 100 miles apart. Is it possible that Joe is telling the truth?



To determine whether Joe's statement is possible, we can use the concept of a geometric locus. A locus is a set of points that satisfy a particular condition. In this case, the locus of points for both Joe and Beth is a circle with a radius of 10 miles, centered on Lake Michigan.

If Joe and Beth both live exactly 10 miles from Lake Michigan, they must each be on one of these circles. The question now is whether there are any points on these two circles that are exactly 100 miles apart.

There are a few possibilities:

1. **The Circles Do Not Overlap:** If the circles do not overlap or touch, the minimum distance between any two points on the circles will be less than 100 miles. In this case, it would be impossible for Joe and Beth to be 100 miles apart.
2. **The Circles Overlap:** If the circles overlap, the maximum distance between any two points on the circles (i.e., across the longest line through the overlap) could be 100 miles or more. This would make Joe's statement possible.
3. **The Circles Touch:** If the circles just touch each other, the maximum distance between any two points on the circles would be at the points where they touch. In this case, it would depend on the specific geometry.

To visualize this, imagine two circles on a piece of paper, each with a radius of 10 miles. As you move the centers of the circles closer or further apart, the distance between the farthest points on the circles changes.

To check the plausibility mathematically, we need to consider the distance between the centers of these two circles. Since both circles have a radius of 10 miles, if the centers are more than 20 miles apart, the circles won't overlap, and Joe and Beth cannot be 100 miles apart. If the centers are exactly 20 miles apart, the circles touch, and the maximum distance between points on the circles would be less than 100 miles. If the centers are less than 20 miles apart, the circles overlap, and it's theoretically possible for two points on the circles to be 100 miles apart.

Therefore, for Joe's statement to be true, the distance between the centers of the two 10-mile radius circles must be such that there exist points on each circle that are 100 miles apart. This seems unlikely, but without knowing the exact positions of Joe and Beth relative to Lake Michigan, we cannot definitively say it's impossible.