

Question:

How often does the Bitcoin network see two consecutive blocks mined more than 2 hours apart from each other? We'd like to know your answer (it doesn't have to be precise) and your approach towards this solution using probability and statistics.

Solution:

In order to find the probability of a given number of events occurring in a fixed interval of time if events occur with a known constant mean rate and independently of the time since last event we should use Poisson's distribution:

$$P(k \text{ events in interval } t) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where:

- r is average rate at which event occur
- e is Euler's number ($e = 2.71828...$)
- $\lambda = rt$

Bitcoin expected block time is 10 minutes, that gives 6 blocks per hour. And given time should be 2 hours.

$$\lambda = rt = 6 * 2 = 12$$

We can calculate probability of interval time between two blocks mined to be greater than 2 hours by:

$$k = 0$$

$$\frac{12^0 e^{-12}}{0!} = e^{-12}$$

In order to find the frequency of this happening we can calculate the sum of events in a given timespan (6 per hour) with a probability of e^{-12} and divide by time period that goes to infinity:

$$\lim_{n \rightarrow \infty} \sum \frac{6ne^{-12}}{n} = 6e^{-12}$$

where:

- n is number of hours that goes to infinity
- $6n$ is the number of blocks in an hour

And then calculate reciprocal of received result:

$$\frac{1}{6e^{-12}} = \frac{e^{12}}{6} = 27125.79856983397$$

Answer:

The time between two consecutive blocks in the Bitcoin network will be greater than 2 hours should occur once every:

- ~97652874851.4 milliseconds
- ~97652874.85 seconds
- ~1627548.91 minutes
- **~27125.8 hours**
- ~1130.24 days
- ~37.67 months (30 days in a month)
- ~3.1 years (365 days in a year)