$$p(x,z|c;\theta) = p(x|c;\theta)\, p(z|x,c;\theta)$$

$$\Rightarrow p(x|c;\theta) = \frac{p(x,z|c;\theta)}{p(z|x,c;\theta)}$$

$$\Rightarrow \log p(x|c;\theta) = \log p(x,z|c;\theta) - \log p(z|x,c;\theta)$$

Introduce an arbitrary distribution $q(z)$

$$\Rightarrow \int q(z) \log p(x|c;\theta)\, dz = \int q(z) \log p(x,z|c;\theta)\, dz - \int q(z) \log p(z|x,c;\theta)\, dz$$

$$= \int q(z) \log p(x,z|c;\theta)\, dz - \int q(z) \log q(z)\, dz$$

$$+ \int q(z) \log q(z)\, dz - \int q(z) \log p(z|x,c;\theta)\, dz$$

$$= \mathcal{L}(x,c,q,\theta) + KL(q(z) \| p(z|x,c;\theta))$$

$$\Rightarrow \log p(x|c;\theta) \geq \mathcal{L}(x,c,q,\theta)$$

$$\mathcal{L}(x,c,q,\theta) = \int q(z) \log p(x,z|c;\theta)\, dz - \int q(z) \log q(z)\, dz$$

$$= \int q(z) \log p(x|z,c;\theta)\, dz + \int q(z) \log p(z|c)\, dz - \int q(z) \log q(z)\, dz$$

$$= E_{z\sim q(z)} \log p(x|z,c;\theta)\, dz - KL(q(z) \| p(z|c))$$

let $q(z) = q(z|x,c;\phi)$

$$\Rightarrow \mathcal{L}(x,c,q,\theta) = E_{z\sim q(z|x,c;\phi)} \log p(x|z,c;\theta)\, dz - KL(q(z|x,c;\phi) \| p(z|c))$$