Logic Gates & Boolean Algebra

- Digital vs Analogue
- Computers and electricity
- Truth tables
- Logic gates

Learning Objectives

By the end of this topic you should understand:

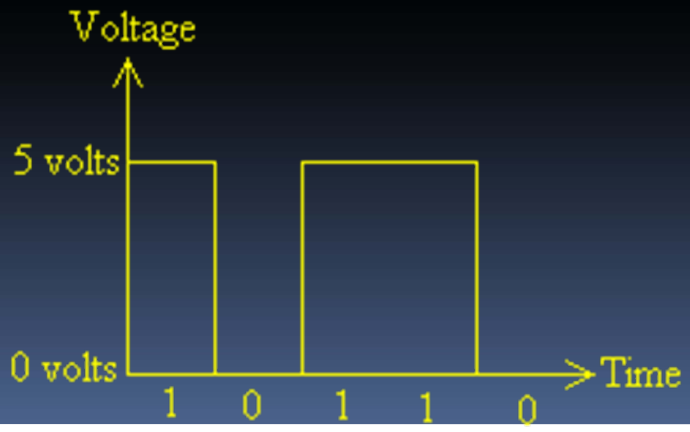
- **Truth tables** for NOT, AND, OR, XOR, NAND and NOR gates.
- **Logic gate diagrams** involving one or more of the logic gates.
- How to determine the **output of a simple logic circuit**

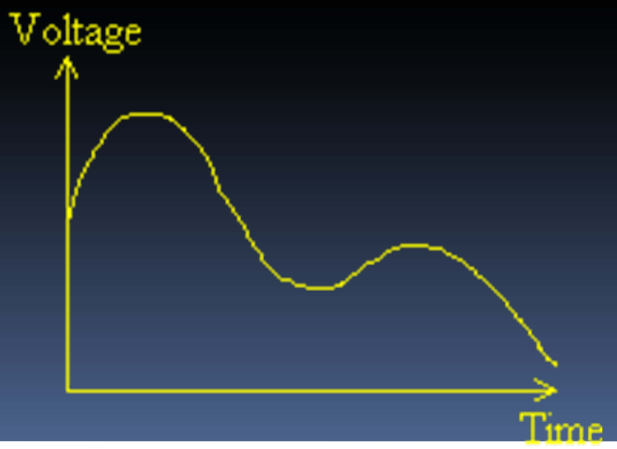
Digital vs Analogue

What does **DIGITAL** mean?

-> In general, digital signals are represented by only **two** possible voltages on a wire - 0 volts (which we call "binary 0", or just "0") and 5 volts (which we call "binary 1", or just "1"). We sometimes call these values "high" and "low", or "true" and "false".

The analogy that is often used is that of a light switch. It can be in just two positions - "on" or "off" .





"Boolean Algebra" = logic computation

- George Boole, 1847 – shorthand notation for a system of logic.
- Boolean variable – can have only 2 possible discrete values: true or false
- Eg the following are Boolean variables:
  - "it is raining today" = X
  - "today is my birthday" = Y. Truth tables:

X	Meaning
True (1)	It is raining today
False (0)	It is not raining today

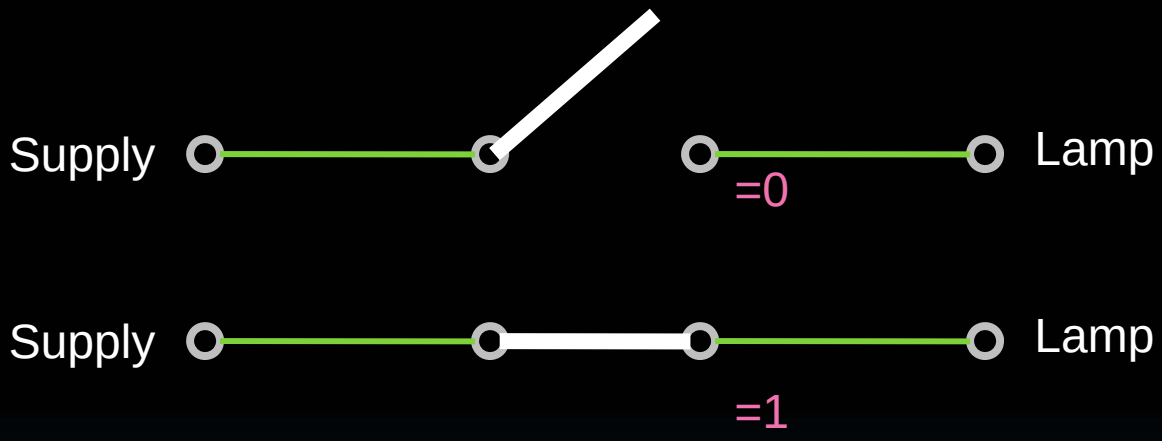
Y	Meaning
True (1)	Today is not my birthday.
False (0)	Today is my birthday.

Computers and electricity

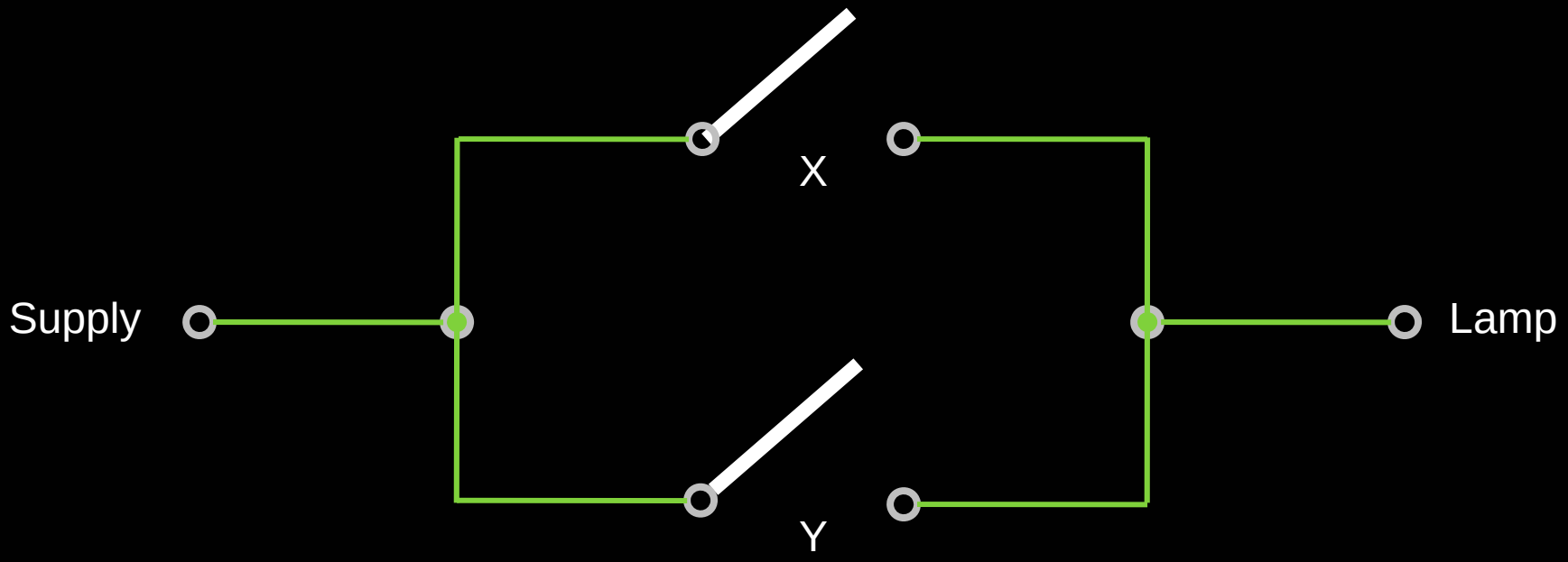
- A **gate** is a device that performs a basic operation on electrical signals
- Gates are combined into **circuits** to perform more complicated tasks

  - There are three different, but equally powerful, notational methods for describing the behavior of gates and circuits
    - Boolean expressions
    - logic diagrams
    - truth tables

Computers and electricity

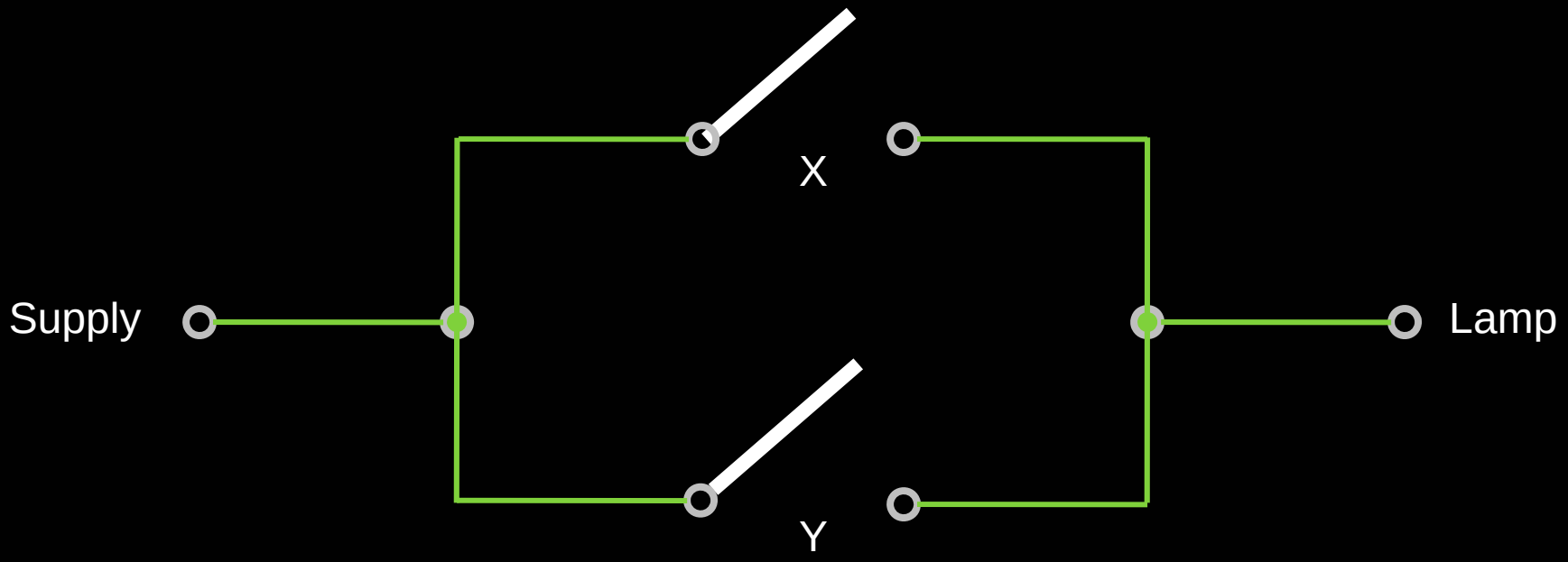


Truth tables



X	Y	Lamp (Q)

Truth tables



X	Y	Lamp (Q)
Open (0)	Open (0)	Off (0)
Open (0)	Closed (1)	On (1)
Closed (1)	Open (0)	On (1)
Closed (1)	Closed (1)	On (1)

Computers and electricity

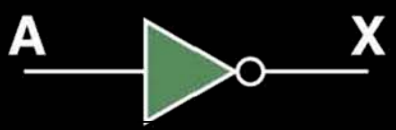
- Boolean expression (equation)
  - expresses a Boolean output Q in terms of the inputs X, Y, Z etc to which one or more Boolean functions (OR, AND, NOT...) are applied.
- Logic diagram / gate
  - an electronic circuit that performs a Boolean function
- Truth table
  - table that shows the result of applying the logical function to all possible combinations of inputs

NOT function & gate

Boolean Expression

$$X = \overline{A}$$

Logic Diagram Symbol



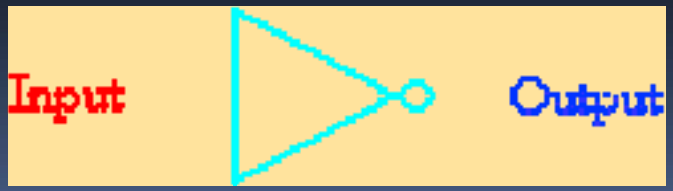
Truth Table

A	X
0	1
1	0

Notice that the NOT function inverts the input.

Equivalent Boolean expression: (I'm using the ! symbol for negation- you draw a line above the A!)

X = A!



AND function & gate

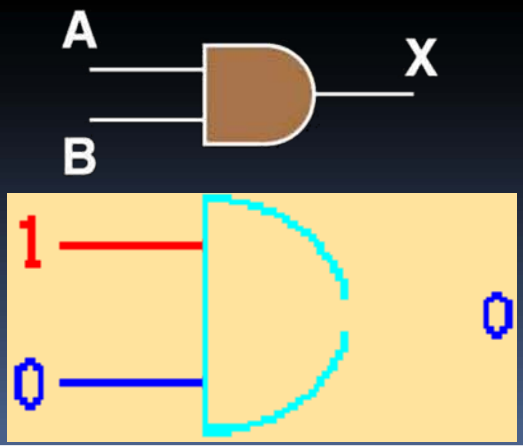
When will the lamp be on?

Only if both A and B switches are on.

Boolean Expression

$$X = A \cdot B$$

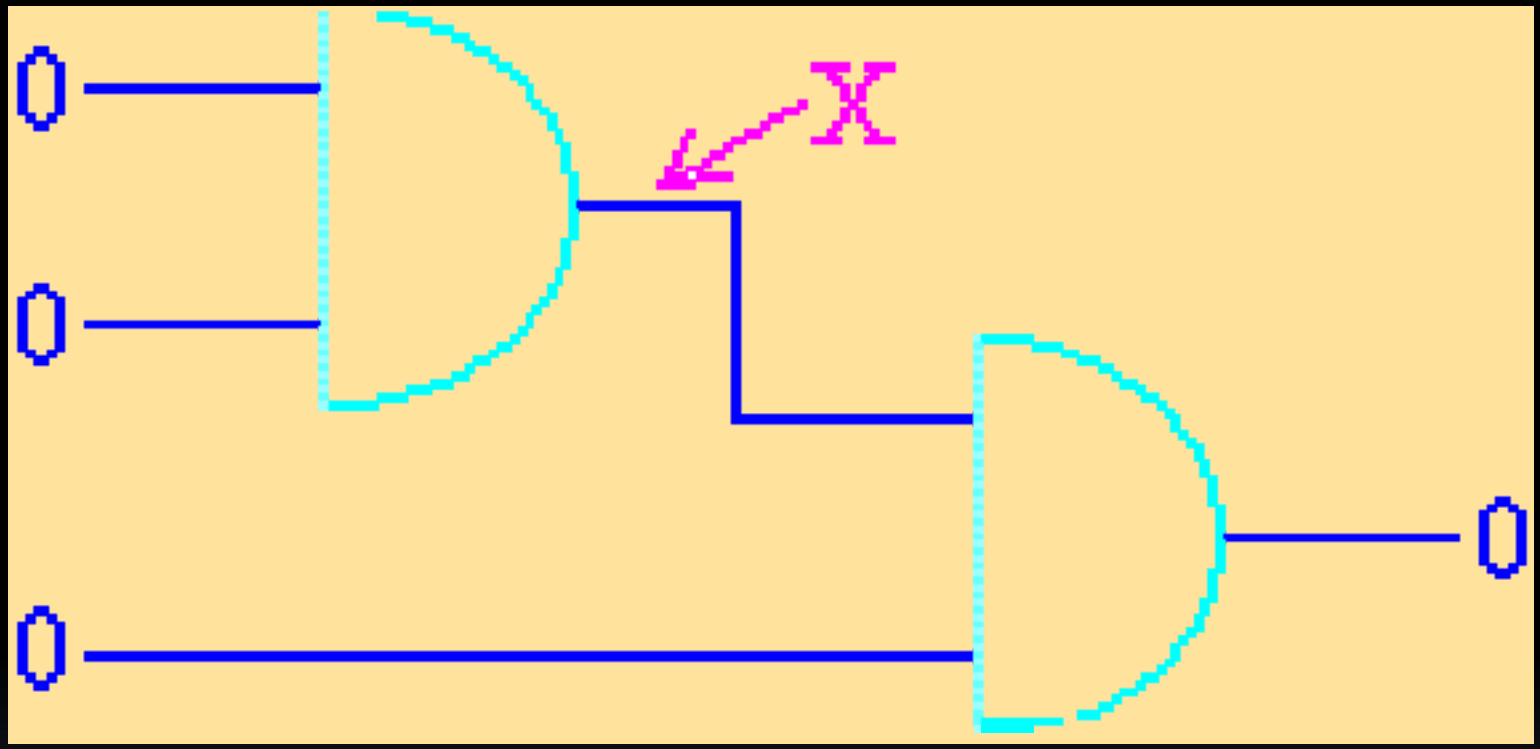
Logic Diagram Symbol



Truth Table

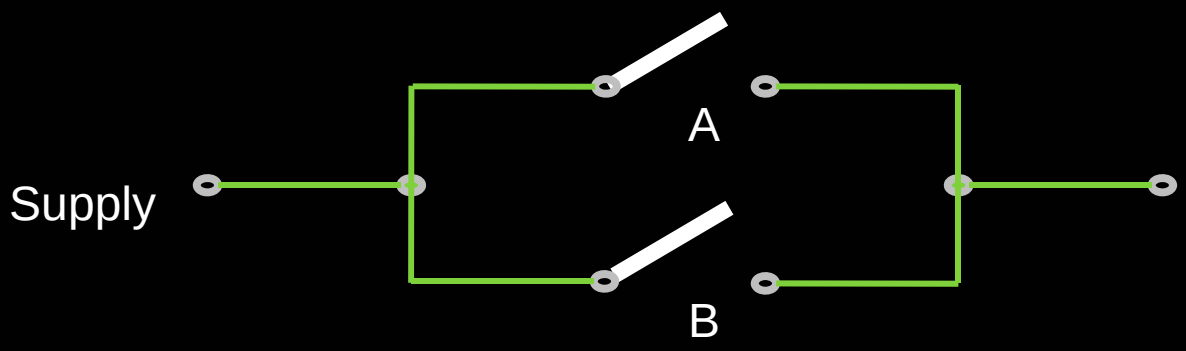
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Two AND gates



Draw the truth table for this circuit and its equivalent Boolean expression.

OR function & gate



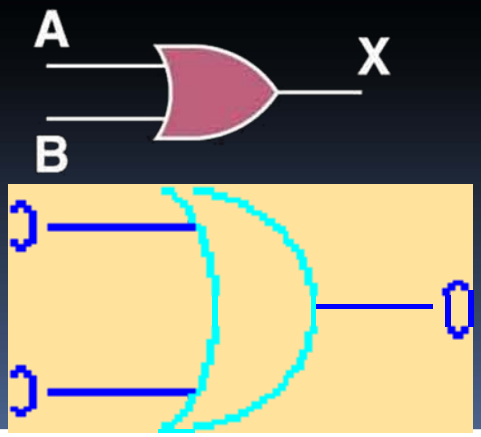
When will the lamp be on?

If either A or B switches are on.

Boolean Expression

$$X = A + B$$

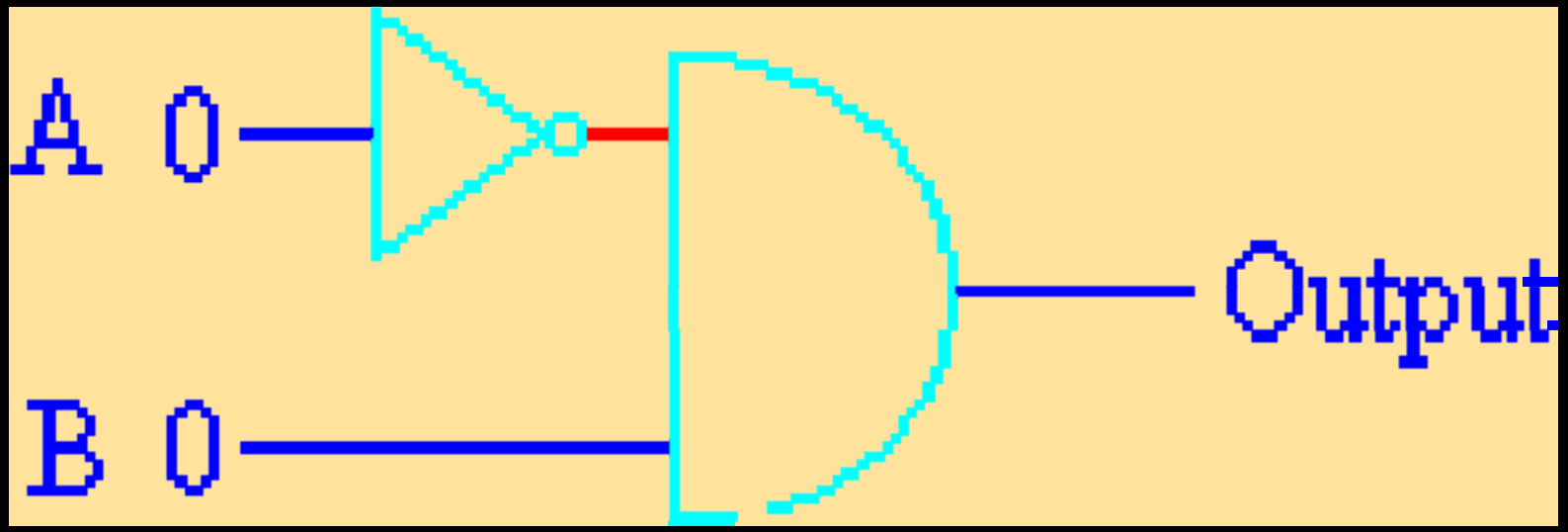
Logic Diagram Symbol



Truth Table

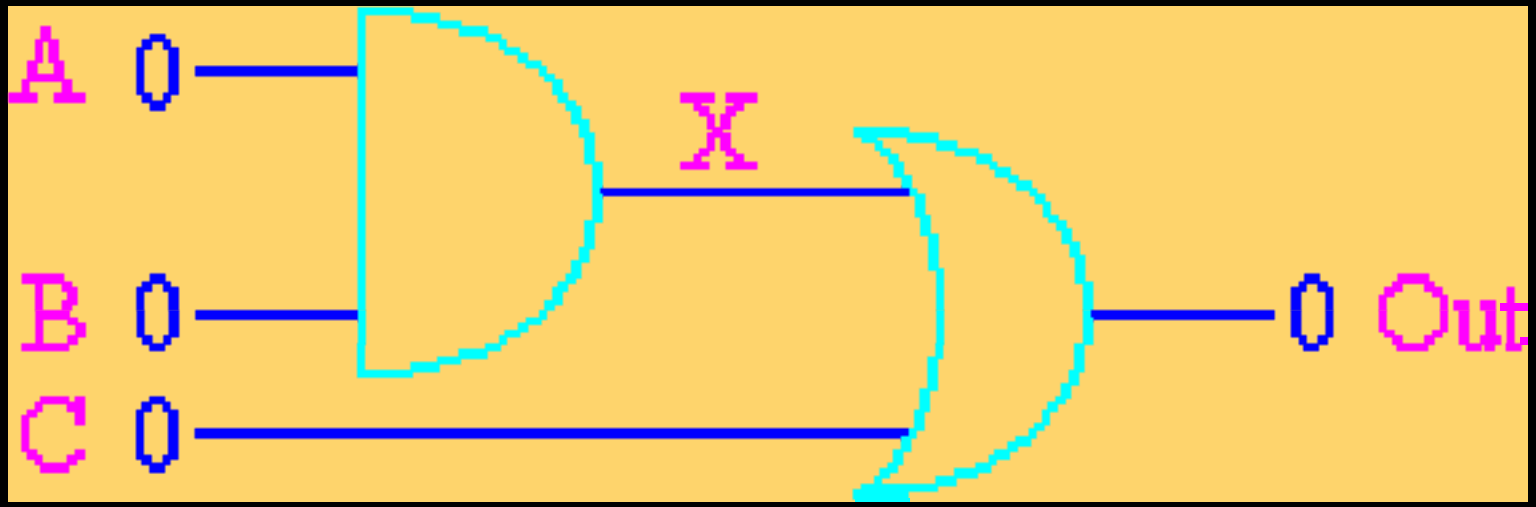
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

AND and NOT gates together



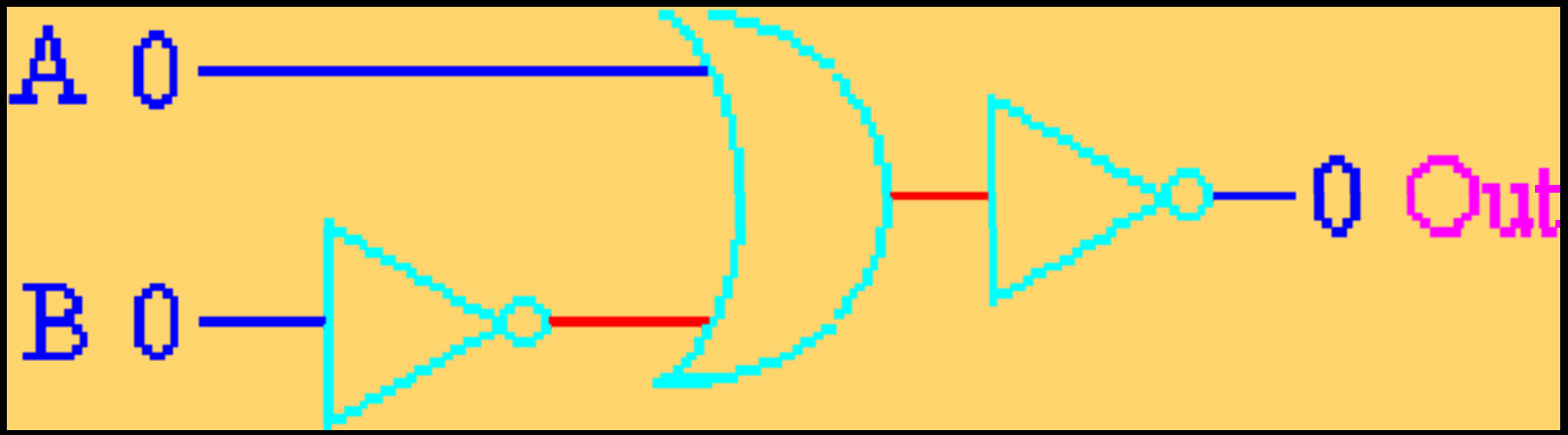
Draw the truth table for this circuit and its equivalent Boolean expression.

AND and OR gates together



Draw the truth table for this circuit and its equivalent Boolean expression.

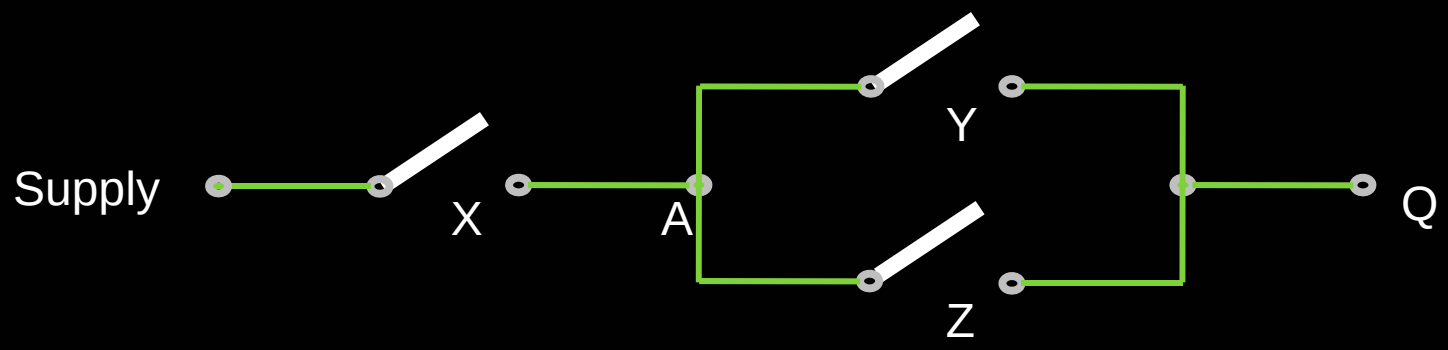
OR and NOT gates together



Draw the truth table for this circuit and its equivalent Boolean expression.

Task (10 mins)

1. Draw the truth tables for **AND**, **OR** and **NOT** functions.
2. What logic function is performed by this switch arrangement (write the Boolean expression):



3. Draw a switch arrangement for the AND function.
4. Draw the switch arrangement for output Q=X.Y+X.Z – what do you notice?

Answers

1.

X	Y	X.Y
1	1	1
1	0	0
0	1	0
0	0	0

X	Y	X+Y
1	1	1
1	0	1
0	1	1
0	0	0

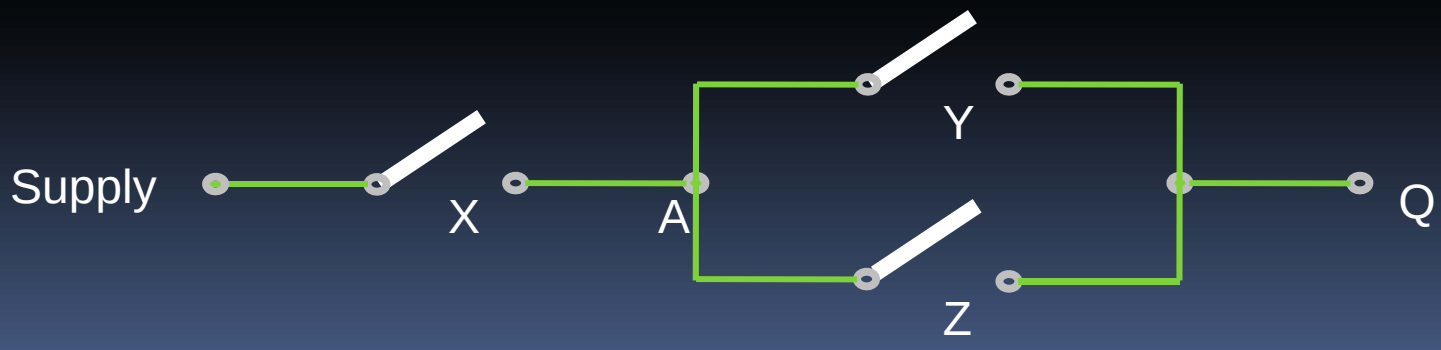
X	$\overline{X}$
1	0
0	1

2. X.(Y+Z)

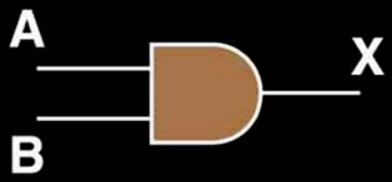
3. Supply

Lamp (X)

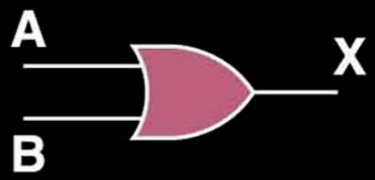
4.



Logic gates - recap



A.B=X (AND)



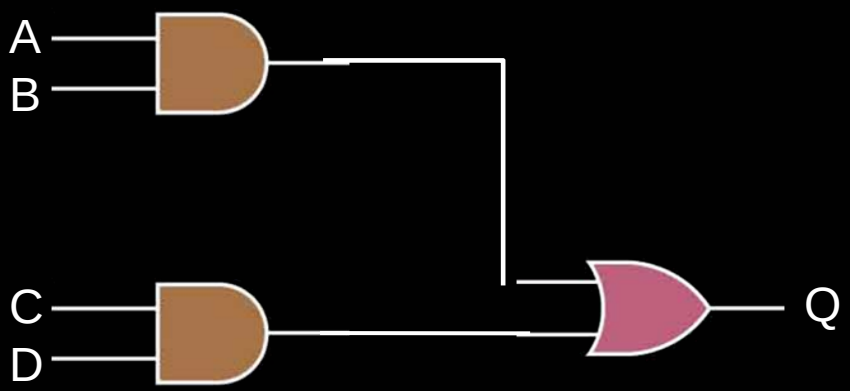
A+B=X (OR)

$\overline{A}$=X (NOT)

Starter - check Homework (5 mins)

Peer assessment – in pairs, check your homework answers.

Combinations of logic gates

1. 

A.B+C.D=Q

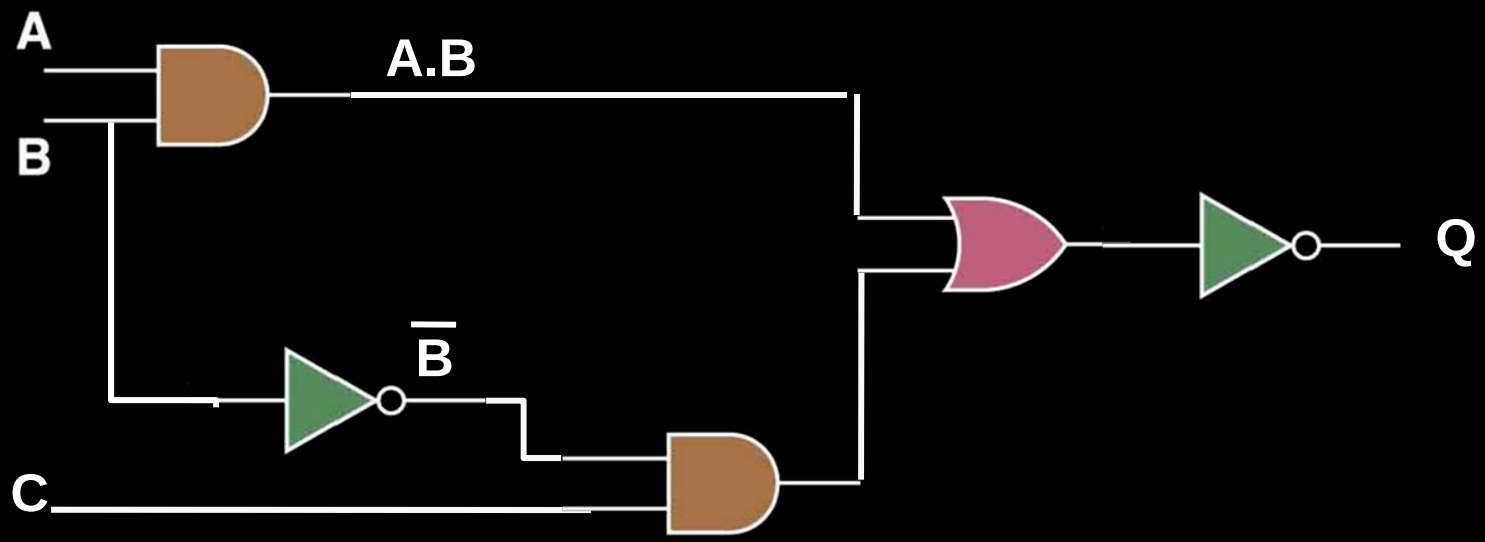
When is Q on?
If either A and B, or C and D are ON.

2. suppose we require an output Q from 3 inputs, A,B,C such as Q=0 when

- A is present (1) and B is present (1) or
- B is not present (0) but C is present (1).

Draw the logic diagram for this expression.

Solution

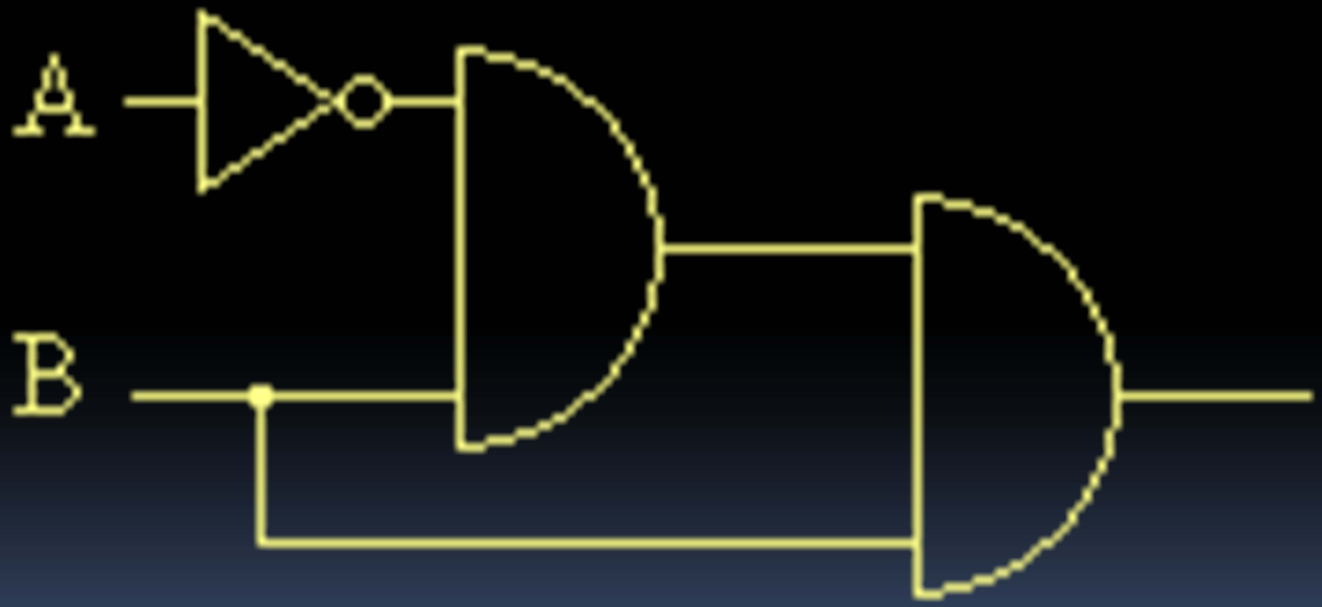


$$A.B+\overline{B}.C=\overline{Q}$$

$$\overline{A.B+\overline{B}.C}=Q$$

Question

In the following circuit, there is only one combination of inputs A and B that produces a 1 output. What is that combination?



A=..... B=.....

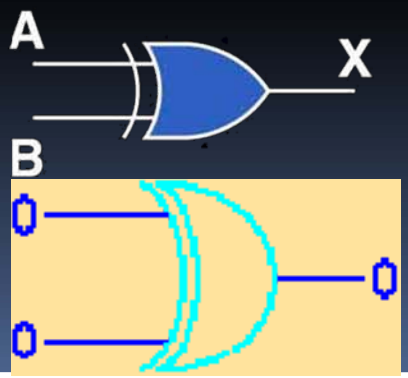
Exclusive OR function (XOR or EOR)

- An XOR gate produces 0 if its two inputs are the same, and a 1 otherwise
- Note the difference between the XOR gate and the OR gate; they differ only in one input situation
- When both input signals are 1, the OR gate produces a 1 and the XOR produces a 0

Boolean Expression

$$X = A \oplus B$$

Logic Diagram Symbol



Truth Table

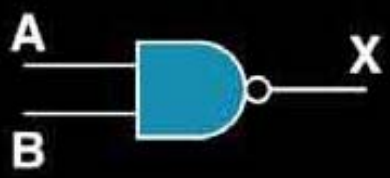
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

NAND and NOR functions

Boolean Expression

$$X = \overline{(A \cdot B)}$$

Logic Diagram Symbol



Truth Table

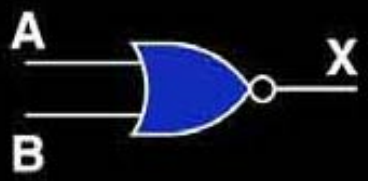
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Note the little extra circle added to the AND and OR gates

Boolean Expression

$$X = \overline{(A + B)}$$

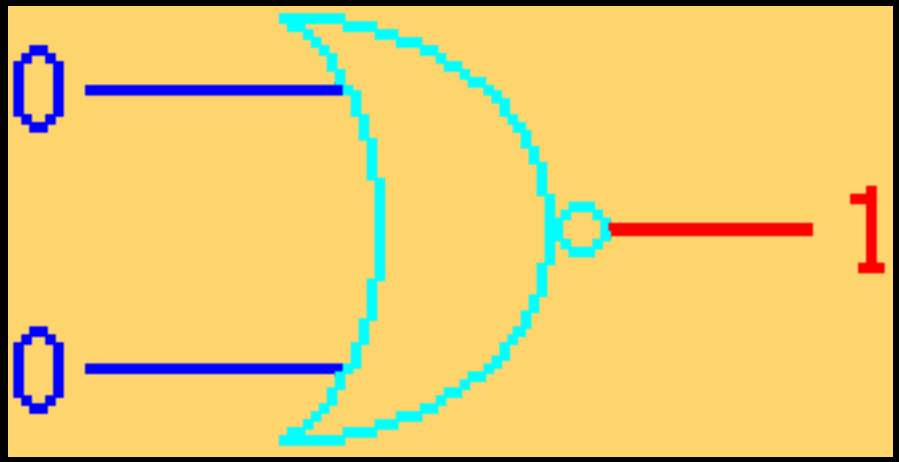
Logic Diagram Symbol

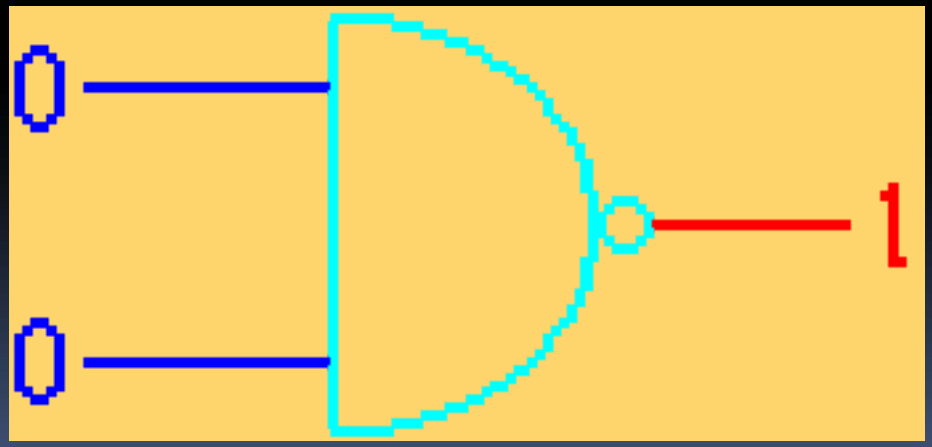


Truth Table

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

NAND and NOR functions





More practice

3.2.2 further logic gates.pdf (handout)