Stellar Interiors: Modeling A Star's Structure

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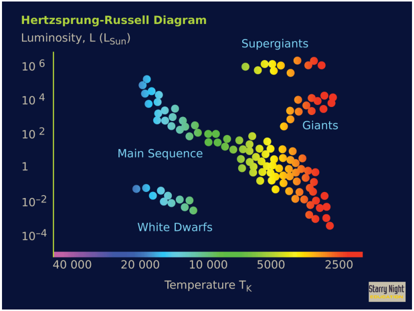
Project Goals

1. Solve equations of stellar structure numerically
2. Assess the model's behavior and accuracy
3. Solve the equations for a variety of initial compositions to recreate the Hertzsprung-Russell diagram



Equations of Stellar Structure

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \qquad (1)$$

$$\frac{dM_r}{dr} = 4\pi r^2\rho \qquad (2)$$

$$\frac{dL_r}{dr} = 4\pi r^2\rho\epsilon \qquad (3)$$

$$\frac{dT}{dr} = -\frac{3\bar{\kappa}\rho L_r}{4acT^3 4\pi r^2} \qquad (4)$$

$$\frac{dT}{dr} = -(1 - \frac{1}{\gamma})\frac{\mu m_H GM_r}{kr^2} \qquad (5)$$

- Convection criterion:

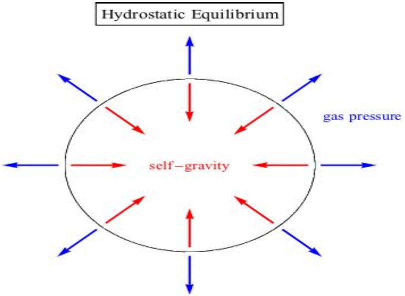
$$\frac{dlnP}{dlnT} > \frac{\gamma}{\gamma - 1} \approx 2.5 \qquad (6)$$

- Density is derived from the ideal gas law:

$$\rho = \frac{\mu m_H P}{kT}$$

Hydrostatic Equilibrium

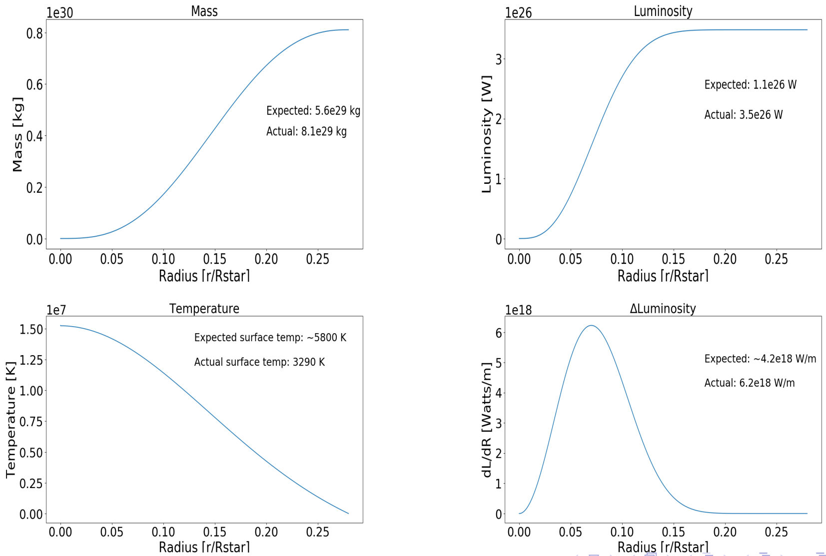
$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$$



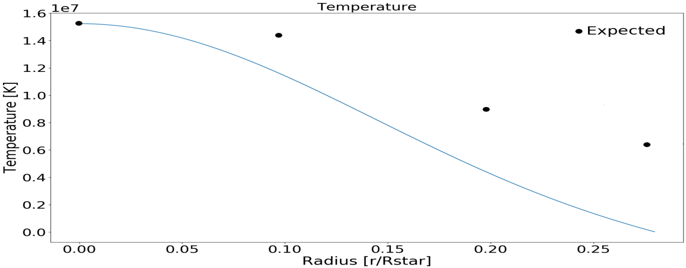
Numerical Techniques

- A Fourth-Order Runge-Kutta technique was written from scratch in Python
  - Ease of debugging
  - Greater manual control over integration range
- Stars are modeled as comprised of many spherical shells in hydrostatic equilibrium
  - Shell width was chosen to be $10^6 cm$ ($\sim .0001 R_\odot$)

Behavior & Results



Temperature Variation



Future Work

- Investigate temperature gradient
- Investigate energy density
- Run model with a variety of compositions to simulate the HR diagram

References

1 Carroll, Bradley W., Ostlie Dale A.. 2017. *An Introduction to Modern Astrophysics* (Ch. 9,10,11). Cambridge, UK: University Printing House

2 Maoz, Dan. 2016. *Astrophysics In A Nutshell* (pp.30-57). Princeton, NJ: Princeton University Press

3 Starry Night Education. Hertzsprung-Russell Diagram. Retrieved 1 May, 2019 from: *http://www.starrynighteducation.com/images/free_resources/hertzsprungRussell.png*

Appendix - Equations

- Energy Density: $\epsilon = \frac{.01125}{\sqrt{\frac{m_p}{2}}} \frac{\rho}{T^{2/3}} e^{\frac{-3.395e3}{T^{1/3}}}$