



**Free Quality  
School  
Education**

Ministry of  
Basic and Senior  
Secondary  
Education

Lesson Plans for  
Senior Secondary  
*Mathematics*

**SSS  
I**

**Term  
III**

**STRICTLY NOT FOR SALE**



## **Foreword**

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

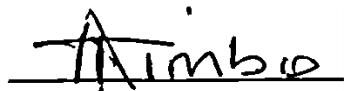
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.



**Mr. Alpha Osman Timbo**

Minister of Basic and Senior Secondary Education

**The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.**

**To achieve thus, DO NOT WRITE IN THE BOOKS.**

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# Introduction to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

- 1  The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
- 2  Teachers can use other textbooks alongside or instead of these lesson plans.
- 3  Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
- 4  Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
- 5  If there is time, quickly review what you taught last time before starting each lesson.
- 6  Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
- 7  Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
- 8  Use the board and other visual aids as you teach.
- 9  Interact with all pupils in the class – including the quiet ones.
- 10  Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.



Learning outcomes



Preparation

## **KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS<sup>1</sup>**

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

### **Common errors**

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

### **Suggested solutions**

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

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<sup>1</sup> This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

## FACILITATION STRATEGIES

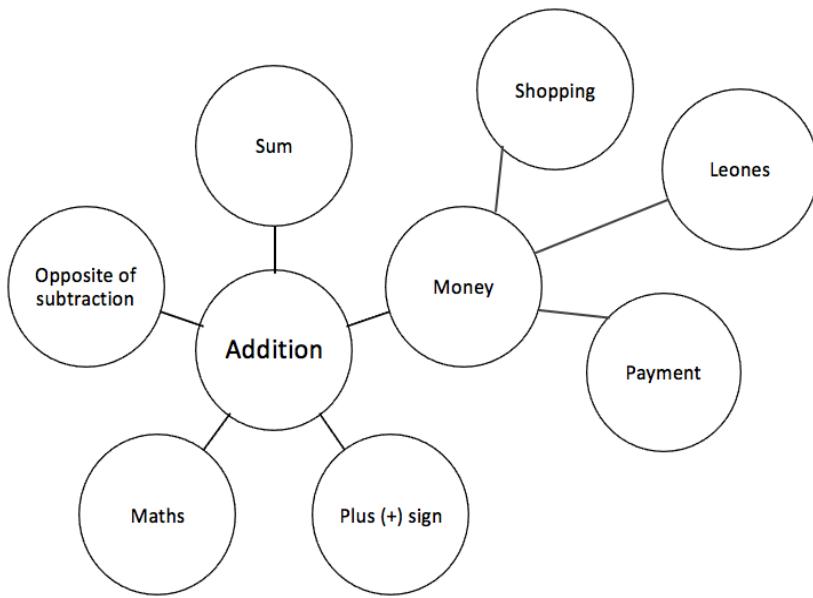
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

### Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I know' and 'What I want to know' about the topic. At the end of the lesson have pupils share 'What I learned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

### Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

### Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

## **Strategies for assessing learning with writing**

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

## **Strategies for engaging different kinds of learners**

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.
- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

<b>Lesson Title:</b> Relations and types of relations	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L097	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: 1. Identify and describe relations between sets. 2. Create arrow diagrams to show relations between sets.	 <b>Preparation</b> None	

### Opening (3 minutes)

1. Write the questions on the board:
  - a. What is a set?
  - b. List the members of the set of days in a week with an even number of letters.
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite volunteers to solve the problems simultaneously on the board.  
 (Answer: a. A set is a collection of things that have something in common or follow a rule; b. {Sunday, Monday, Thursday, Friday, Saturday})
4. Explain to the pupils that today's lesson is on identifying and describing relations between sets.

### Teaching and Learning (22 minutes)

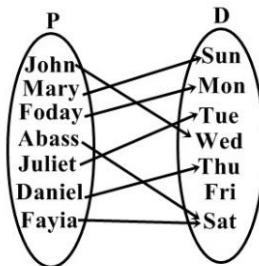
1. Write on the board:  
 Foday is the father of John.  
 Isha is the wife of Peter.
2. Discuss:
  - These two sentences express relationship between two objects.
  - In each case the first element is related to the second element.
  - The two elements named are from two separate sets.
3. Write on the board: Consider these two sets:

Name of Students (P)	Days of the week (D)
John	Wednesday
Mary	Sunday
Foday	Monday
Abass	Saturday
Juliet	Tuesday
Daniel	Thursday
Idrissa	Saturday

4. Discuss:

- This table shows the days of the week in which the students wash their clothes.
- The relation defined here is between the set of students {John, Mary, Foday, Abass, Juliet, Daniel, Idrissa} and the set of days of the week {Sunday, Monday, Tuesday, Wednesday, Thursday, Saturday}
- A **relation** is simply a connection between two sets.
- A relation can be represented by drawing arrows from the first set to the second set.

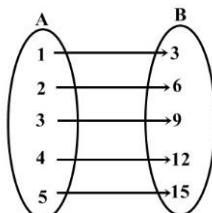
5. Draw the diagram on the board:



6. Explain:

- This is an **arrow diagram**. It shows the relation between people and the days of the week.
- The arrow shows on which day of the week each pupil washes their clothes.
- In symbols, we can write  $P \rightarrow D$

7. Draw the arrow diagram on the board:



8. Allow pupils 1 minute to discuss with seatmates what they notice about the above relation.

9. Ask for a volunteer to share their answer. (Answer: The arrow diagram suggest the relation “is 3 times” or “is thrice of”. That is, 3 is thrice of 1; 6 is thrice of 2, and so on).

10. Explain: In a relation, there are pairs where one element is from each set. These are the pairs connected by arrows.

11. Write one ordered pair from the relation on the board: (1, 3)

12. Ask pupils to write the remainder of the ordered pairs on the board.  
(Answer:{(1,3), (2,6), (3,9), (4,12), (5,15)}).

13. Explain the type of relations. There are four types of relations and these are one-to-one, one-to-many, many-to-one and many-to-many.

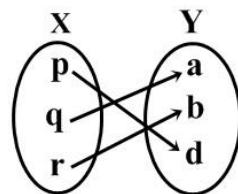
14. Discuss:

- A one-to-one relation is a relation in which each element in the domain has exactly one image in the co-domain.

15. Draw an illustrative arrow diagram of a one-to-one relation on the board (at right).

16. Discuss:

- A one-to-many relation is a relation in which one element in the domain has many elements in the co-domain.



17. Draw an illustrative arrow diagram of a one-to-many relation on the board (at right).

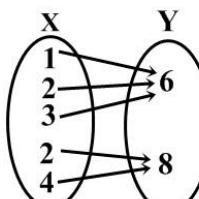
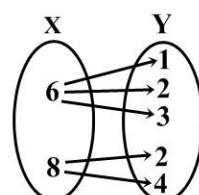
18. Discuss:

- A many-to-one relation is a relation in which more than one element in the domain has only one element in the co-domain.

19. Draw an illustrative arrow diagram of many-to-one relation on the board (at right).

20. Discuss:

- A many-to-many relation is a relation in which many elements in the domain have many elements in the co-domain.



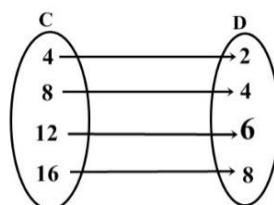
21. Draw an illustrative arrow diagram of a many-to-many relation on the board (at right).

22. Write two problems on the board.

- Set A of countries and set B of capital cities are given in the table below:

<b>Country (A)</b>	<b>Capital City (B)</b>
Ghana	Abuja
Liberia	Accra
Nigeria	Freetown
Sierra Leone	Monrovia

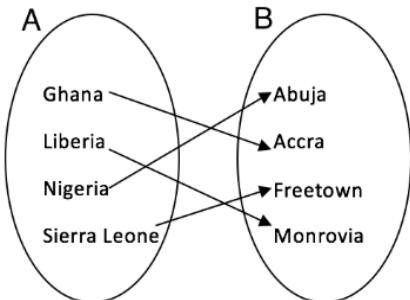
- i. Draw an arrow diagram showing the relation between the 2 sets.
- ii. List the ordered pairs in the relation.
- iii. Determine the type of relation.
- The relation between 2 sets C and D is given in the arrow diagram. Describe the relation in words.



23. Ask pupils to solve the problems with seatmates.
24. Walk around to check for understanding and clear misconceptions.
25. Invite volunteers to write their answers on the board.

**Answers:**

- a. i. Arrow diagram →
- ii. Ordered pairs: {(Ghana, Accra), (Liberia, Monrovia), (Nigeria, Abuja), (Sierra Leone, Freetown)}
- iii. One-to-one relation
- b. The arrow diagram suggest the relation “is half of”.



**Practice (14 minutes)**

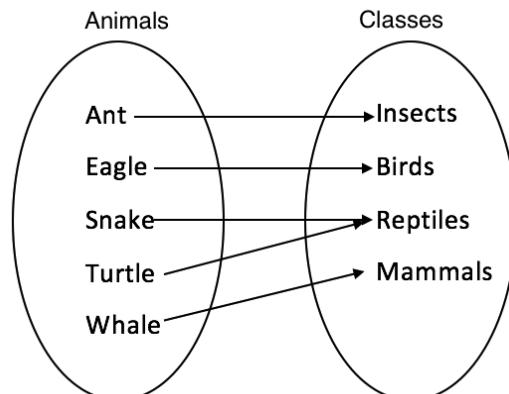
1. Write a problem on the board: Animals can be assorted with the classes they are in. Several animals and their classes are given in the table below.

Animal	Class
Ant	Insects
Eagle	Birds
Snake	Reptiles
Turtle	Reptiles
Whale	Mammals

- a. Describe this relation in words.
- b. Draw an arrow diagram for the relation.
- c. Write all of the ordered pairs in this relation.
- d. Determine the type of relation.
2. Ask pupils to solve the problem independently.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write their answers on the board and explain.

**Answers:**

- a. The relation shows the association "belongs to the class" between a set of animals and a set of classes.
- b. Arrow diagram →
- c. Ordered pairs: {(ant, insects), (eagle, birds), (snake, reptiles), (turtle, reptiles), (whale, mammals)}
- d. Many-to-one



**Closing (1 minute)**

For homework, have pupils do the practice activity PHM1-L097 in the Pupil Handbook.

<b>Lesson Title:</b> Mapping, including domain and range	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L098	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: 1. Determine the rule for a given mapping. 2. Distinguish between domain and range.	 <b>Preparation</b> None	

### Opening (3 minutes)

1. Write two questions on the board:
  - a. What is a relation?
  - b. Draw an arrow diagram for the following relation:  
 $\{(3,7), (0,4), (-1,0), (-5,-1)\}$ .
2. Allow 1 – 2 minutes for pupils to answer the questions independently.
3. Invite a volunteer to write the answers on the board and explain.

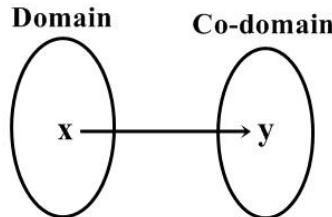
#### Answers:

- a. A relation is the connection between 2 sets; it is a collection of ordered pairs.
- b.
 

$x$	$y$
$3 \rightarrow 7$	
$0 \rightarrow 4$	
$-1 \rightarrow 0$	
$-5 \rightarrow -1$	
4. Explain that today's lesson is on determining the rule for a given mapping and to distinguish between domain and range.

### Teaching and Learning (22 minutes)

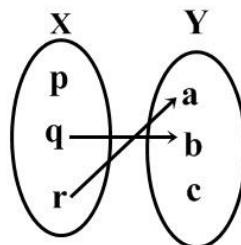
1. Explain: A relation between two sets can be described by a rule. Given the element of the domain (elements of the first set), we can use the rule to find the elements of the range (elements of the second set).
2. Draw on the board:



3. Explain:
  - In general, the relation between two sets can be illustrated by an arrow diagram.

- If an element  $x$  of the domain is related to an element  $y$  of the co-domain, then we say that  $y$  is the image of  $x$  under the given relation.
- The set of all possible images of the domain is called the range. The range is a subset of the co-domain.
- In other words, the co-domain is the set of all values that may possibly result from the relation. The range is the set of values that actually does result from the relation.

4. Draw on the board:

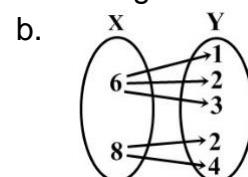
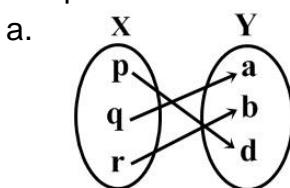


5. Ask pupils to list the elements in the domain. (Answer:  $p, q, r$ )
6. Ask pupils to list the elements in the co-domain. (Answer:  $a, b, c$ )
7. Discuss:
  - In the relation above, the domain is  $\{p, q, r\}$  and the co-domain is  $\{a, b, c\}$ . The range is  $\{a, b\}$ .
  - It can be seen that the element  $\{c\}$  of the co-domain have no elements of the domain associated with it.
8. Write on the board: Consider the ordered pairs of the relation  $\{(-2, 1), (-2, 3), (0, -3), (1, 4), (3, 1)\}$ . What are the elements in the domain and range?
9. Explain: When listing the elements of both domain and range, get rid of duplicates and write them in ascending order.
10. Ask volunteers to call out the elements of the domain. Write the domain on the board. (Answer:  $\{-2, 0, 1, 3\}$ )
11. Ask volunteers to call out the elements of the range. List the range on the board. (Answer:  $\{-3, 1, 3, 4\}$ ).

12. Explain **mapping**:

- A mapping is a relation in which each element in the domain maps onto only one element in the co-domain.
- Thus, mapping is either **one-to-one** or **many-to-one**.

13. Write a question on the board: Which of the following relations is/are mapping?



14. Allow pupils 1 minute to discuss with seatmates.

15. Ask volunteers to write the answers on the board. (Answer: a. Is a mapping; b. Is not a mapping.)

16. Ask another volunteer to give reasons for the answers. (Answer: a. It is a mapping because the relation is one-to-one relation; b. It is not a mapping because it is a one-to-many relation.)

17. Explain:

- A mapping can be described with a rule or formula.
- The mapping maybe linear ( $y = mx + b$ ) or exponential ( $y = ax^2$ ).

18. Explain **Linear mapping**:

- The mapping is linear, if the difference between the consecutive terms in both the domain and co-domain are constant.
- For example, consecutive terms in the domain may each increase by 1, and consecutive terms in the co-domain may each increase by 3. As long as the numbers remain constant for both the domain and co-domain, the mapping is linear.

19. Write on the board: Linear rule:  $y = mx + b$ ,  $m = \frac{\text{change in co-domain}}{\text{change in domain}}$  and  $b$  can be found by picking a value from a co-domain ( $y$ ) and a value from a domain ( $x$ ) and substitute them in the linear rule.

20. Write on the board: Find the rule of the mapping:

$x$	5	6	7	8
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$y$	17	21	25	29

21. Explain the solution on the board to pupils.

**Solution:**

Use 2 pairs, (5, 17) and (6, 21), to find the rule.

$$m = \frac{\text{difference in co-domain}}{\text{difference in domain}} = \frac{21-17}{6-5} = 4$$

Substitute  $m$  to solve for  $b$ :

$$\begin{aligned}y &= mx + b \\17 &= 4(5) + b \\17 - 20 &= b \\b &= -3\end{aligned}$$

Therefore, the rule of the mapping is  $y = 4x - 3$ .

22. Explain **exponential mapping**:

- The mapping is exponential if the ratios between the consecutive elements in the co-domain are the same.
- It is written as  $y = pr^{x-q}$  where  $p$  is the first element in the co-domain,  $q$  is the first element in the domain and  $r$  is the common ratio of the elements in the range.

23. Write the following problem on the board: Find the rule of the mapping:

$x$	0	1	2	3
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$y$	1	5	25	125

24. Explain the solution on the board to pupils.

**Solution:**

Note that for this mapping:

- $p = 1$ , the first term of the co-domain
- $q = 0$ , the first term of the domain
- $r = 5$ , the common ratio of the elements in the range (note that 5 is multiplied by each element to get the next element)

Substitute these values into the formula and simplify:

$$\begin{aligned}y &= pr^{x-q} && \text{Rule for exponential mapping} \\y &= 1 \times 5^{x-0} && \text{Substitute} \\&= 5^x && \text{Simplify}\end{aligned}$$

Therefore, the rule of the mapping is  $y = 5^x$ .

**Practice (14 minutes)**

1. Write two problems on the board: Find the rules of the following mappings:

•  $x$     0    1    2    3    4  
 $\downarrow$      $\downarrow$      $\downarrow$      $\downarrow$      $\downarrow$      $\downarrow$   
 $y$     1    3    9    27    81

•  $x$     3    4    5    6    7  
 $\downarrow$      $\downarrow$      $\downarrow$      $\downarrow$      $\downarrow$      $\downarrow$   
 $y$     7    5    3    1    -1

2. Ask pupils to solve the problems with seatmates.

3. Walk around to check for understanding and clear misconceptions.

4. Invite two volunteers from the class to write their solutions on the board.

**Solutions:**

- a. The general rule is  $y = pr^{x-q}$ . Note that  $p = 1, q = 0$  and  $r = 3$ .

$$\begin{aligned}y &= pr^{x-q} \\y &= 1 \times 3^x \\y &= 3^x\end{aligned}$$

Therefore, the rule is  $y = 3^x$ .

- b. The general rule is  $y = mx + b$

Find the value of  $m$  using ordered pairs (3, 7) and (4, 5):

$$m = \frac{5-7}{4-3} = \frac{-2}{1} = -2$$

Substitute  $m = -2$  and the ordered pair  $(3, 7)$  to solve for  $b$ :

$$\begin{aligned}y &= mx + b \\7 &= -2(3) + b \\7 &= -6 + b \\7 + 6 &= b \\13 &= b\end{aligned}$$

Therefore, the rule of the mapping is  $y = -2x + 13$ .

**Closing (1 minute)**

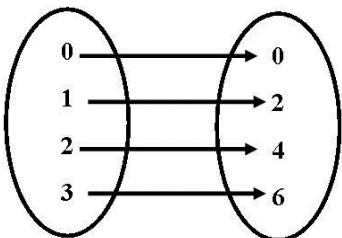
1. For homework, have pupils do the practice activity PHM1-L098 in the Pupil Handbook.

<b>Lesson Title:</b> Functions – Part 1	<b>Theme:</b> Algebraic Processes
<b>Lesson Number:</b> M1-L099	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
<b>Learning Outcomes</b>  By the end of the lesson, pupils will be able to: 1. Identify functions from certain relations. 2. Use function notation.	<b>Preparation</b>  Draw the mappings in the Opening on the board.

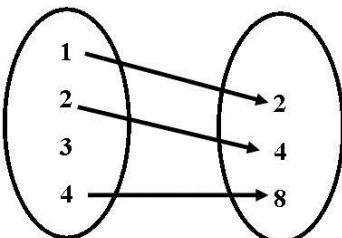
### Opening (3 minutes)

1. Write a problem on the board. Which of the following relations is/are mappings?

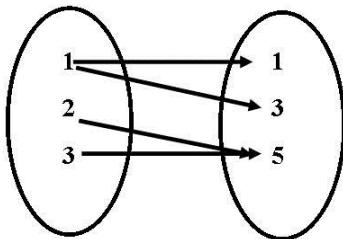
a.



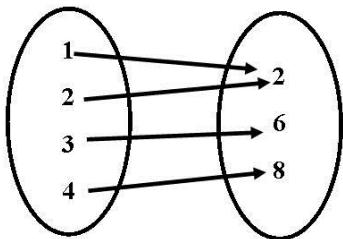
b.



c.



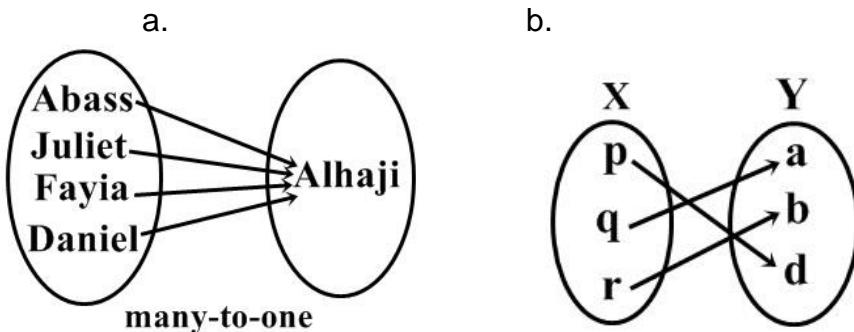
d.



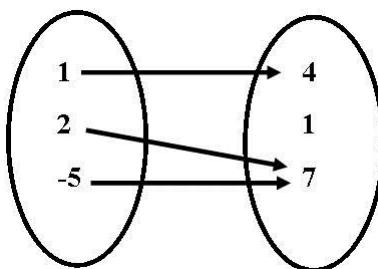
- Ask pupils to solve the problems independently in their exercise books.
- Ask volunteers to give their answers and explain. (Answer: The relations a. and d. are mappings since each member of the domain is mapped onto only one member of the co-domain).
- Explain to the pupils that today's lesson is on identifying functions and the use of function notation.

### Teaching and Learning (22 minutes)

- Explain:
  - Recall that a relation is a set of ordered pairs, and that a mapping can be one-to-one or many-to-one.
  - A **function** is a mapping in which each element in the domain is mapped onto one and only one member on the co-domain.
- Draw illustrative diagrams on the board of mapping functions.



3. Explain: In each diagram, each member of the domain is mapped onto exactly one member of the co-domain.
4. Write on the board: Are the following ordered pairs functions? Describe the mapping.
  - a.  $\{(2, -1), (5, 1), (-5, 1)\}$
  - b.  $\{(6, 3), (6, 5)\}$
  - c.



5. Ask pupils to solve the problem with seatmates.
6. Ask volunteers to share their answers with the class. Allow discussion.

**Answers:**

- a. Yes; each member of the domain (2, 5, -5) maps to exactly 1 member of the co-domain;
  - b. No; a member of the domain (6) maps to 2 members of the co-domain;
  - c. Yes; each member of the domain (1, 2, -5) maps to exactly 1 member of the co-domain. It is okay if 2 members of the domain map to the same member of the co-domain.
7. Write on the board:  $f(x) = 2x + 1$  is the same as  $y = 2x + 1$
  8. Explain **function notation**:
    - Function notation is another way of writing equations. This example is a linear equation.
    - You have probably seen equations written with  $y$ . In function notation,  $y$  is replaced with  $f(x)$ . In other words,  $y = f(x)$ .
    - It is read as "f of x".
    - "f of x" is not the multiplication of  $f$  times  $x$ .

- $f(x)$  is used to show functions of  $x$ . In other words, the function is in terms of  $x$ .

9. Write on the board:  $f: x \rightarrow y$

10. Explain:

- The notation  $f: x \rightarrow y$  tells us that the function's name is "f" and its ordered pairs are formed by elements  $x$  from the domain and elements  $y$  from the range.
- The arrow  $\rightarrow$  is read "is mapped to".

11. Write the following problem on the board: If  $f(x) = 3x + 4$ , find  $f(3)$  and  $f(0)$ .

12. Explain: This question asks us to find "f of 3" and "f of 0". This means that we should substitute  $x = 3$  and  $x = 0$  into the function and evaluate.

13. Write the solution of  $f(3)$  on the board.

**Solution:**

$$\begin{aligned} f(x) &= 3x + 4 && \text{Function} \\ f(3) &= 3(3) + 4 && \text{Substitute } x = 3 \\ &= 9 + 4 && \text{Simplify} \\ &= 13 \end{aligned}$$

14. Ask pupils to work with seatmates to find  $f(0)$ .

15. Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned} f(x) &= 3x + 4 \\ f(0) &= 3(0) + 4 && \text{Substitute } x = 0 \\ &= 0 + 4 \\ &= 4 \end{aligned}$$

16. Write another problem on the board: A function is defined by  $f: x \rightarrow 4x + 1$  on the domain  $\{-1, 0, 1, 2\}$ , find the range of the function.

17. Explain: For this problem, we need to substitute each value from the domain to find the corresponding value in the range.

18. Find the first 2 values of the range on the board (for  $x = -1$  and  $x = 0$ ).

$$f(-1) = 4(-1) + 1 = -4 + 1 = -3$$

$$f(0) = 4(0) + 1 = 0 + 1 = 1$$

19. Ask pupils to complete the problem with seatmates.

20. Walk around, if possible, to check what they are doing. Clear any misconceptions.

21. Invite three volunteers, one at a time, to write the solutions on the board.

**Solutions:**

$$f(1) = 4(1) + 1 = 4 + 1 = 5$$

$$f(2) = 4(2) + 1 = 8 + 1 = 9$$

Therefore, range =  $\{-3, 1, 5, 9\}$

### **Practice (14 minutes)**

1. Write the following two problems on the board:
  - a. If  $f(x) = 2x + 3$ , find: i.  $f(1)$       ii.  $f(2)$       iii.  $f(-3)$
  - b. A function  $f: x \rightarrow x^2 + 1$  is defined on the domain  $\{1, 2, 3\}$ . Find the range.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite any two volunteers to write their answers on the board.

#### **Answers:**

- a. i.  $f(1) = 2(1) + 3 = 2 + 3 = 5$   
ii.  $f(2) = 2(2) + 3 = 4 + 3 = 7$   
iii.  $f(-3) = 2(-3) + 3 = -6 + 3 = -3$
- b. Substitute each value of the domain into  $f(x) = x^2 + 1$ :

$$f(1) = (1)^2 + 1 = 1 + 1 = 2$$

$$f(2) = (2)^2 + 1 = 4 + 1 = 5$$

$$f(3) = (3)^2 + 1 = 9 + 1 = 10$$

Therefore, range = {2, 5, 10}

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L099 in the Pupil Handbook.

<b>Lesson Title:</b> Functions – Part 2	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L100	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to give reasons why a given relation is or is not a function.	 <b>Preparation</b> None	

### Opening (5 minutes)

1. Write the following problem on the board. A function  $f: x \rightarrow x^2 + 5$ , is defined on the domain  $\{0, 1, 2, 3\}$ . Find the range.
2. Ask pupils to solve the problem independently.
3. Invite volunteers to write the solution on the board.

#### Solution:

$$f(0) = (0)^2 + 5 = 0 + 5 = 5$$

$$f(1) = (1)^2 + 5 = 1 + 5 = 6$$

$$f(2) = (2)^2 + 5 = 4 + 5 = 9$$

$$f(3) = (3)^2 + 5 = 9 + 5 = 14$$

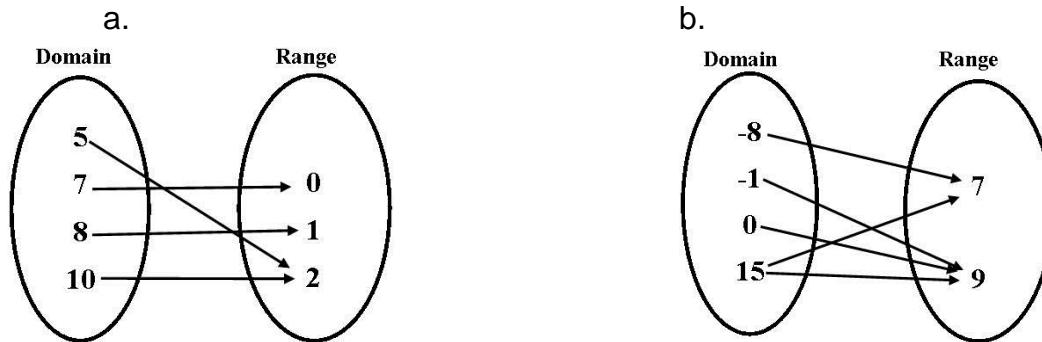
Therefore, the range is  $\{5, 6, 9, 14\}$ .

4. Discuss: Is the function one-to-one? Why or why not? (Answer: It is a one-to-one function. Each element in the domain maps to a different element in the range.)
5. Explain that today's lesson is also on functions. Pupils will be able to give reasons why a given relation is or is not a function.

### Teaching and Learning (20 minutes)

1. Explain:
  - Recall that a function is a relation between two sets: the domain and the co-domain, such that every element of the domain has **only one** image in the co-domain.
  - A function can be a one-to-one relation or a many-to-one relation.
    - Any other relation (i.e. one-to-many or many-to-many) is not a function.
2. Write a set of pairs on the board:  $\{(-3,7), (-1,5), (0, -2), (5, 9), (5, 3)\}$
3. Ask pupils what they notice about the set of ordered pairs. Allow 1 minute for them to discuss with seatmates.
4. Have 2-3 volunteers to tell their ideas to the class.
5. Explain:
  - Since we have repetitions or duplicates of  $x$ -values with different  $y$ -values, then this relation **is not** a function.

- Note that there are 2 5's in the domain. These map to different numbers, 9 and 3. A function maps a value in the domain to only 1 other value in the range.
6. Write another set of pairs on the board:  $\{(-2, 0), (-1, -2), (0, 3), (4, -1), (5, -3)\}$
  7. Ask pupils what they notice about the set of ordered pairs. Allow 1 minute for them to discuss with seatmates.
  8. Have 2-3 volunteers to tell their ideas to the class.
  9. Explain: This relation **is** a function because every  $x$ -value is unique and is associated to only one value of  $y$ .
  10. Write a problem on the board:  $\{(1, 5), (1, 5), (3, -8), (3, -8), (3, -8)\}$ .
  11. Ask pupils: Is this a function? Give reasons for your answer.
  12. Ask pupils to pair up with seatmates. Allow a few more moments for them to discuss the answer.
  13. Ask a volunteer to give the answer. (Answer: Yes, we have repeated values of  $x$  but they are being associated to the same values of  $y$ . The point  $(1, 5)$  shows up twice, while the point  $(3, -8)$  is within three times. This table can be cleaned up by writing a single copy of the repeating ordered pairs, i.e.  $\{(1, 5), (3, -8)\}$ . The relation is now clearly a function).
  14. Write on the board so it is clear:  $\{(1, 5), (1, 5), (3, -8), (3, -8), (3, -8)\} = \{(1, 5), (3, -8)\}$  is a function.
  15. Write two more problems on the board. Are the relations expressed in the mapping diagrams functions? Give your reasons.



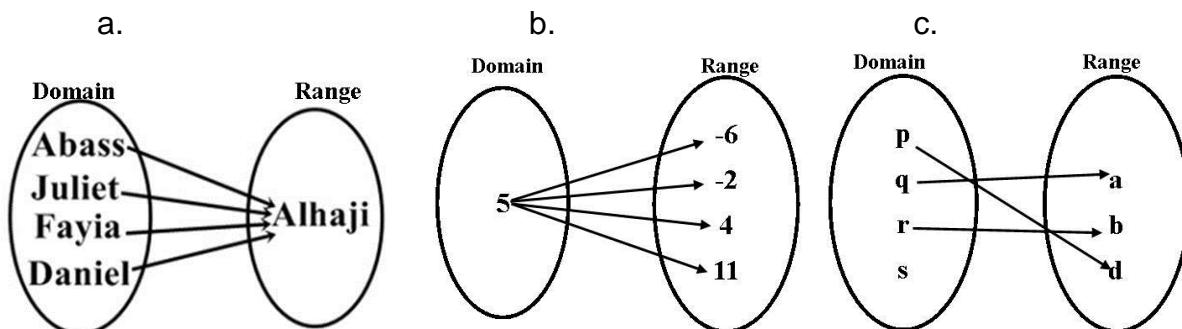
16. Ask pupils to solve the problem with seatmates.
17. Walk around to check answers and clear misconceptions.
18. Ask any two volunteers to give their answers and explain. Allow discussion.

#### Answers:

- Each element of the domain gives one and only element in the range. It is okay for two or more values in the domain to share a common value in the range. That is, even though the elements 5 and 10 in the domain share the same value of 2 in the range, this relation is still a function.
- The element 15 has two arrows, which point to both 7 and 9. Therefore, this relation is not a function.

### **Practice (14 minutes)**

1. Write three problems on the board: Are the relations expressed in the mapping diagrams functions? Give your reasons.



2. Ask pupils to solve the problems independently.
3. Walk around to check for understanding and clear misconceptions.
4. Ask three volunteers to give their answers and explain. Allow discussion.

#### **Answers:**

- a. This is a function. There is nothing wrong when four elements coming from the domain are sharing a common value in the range.
- b. This is not a function. A single element in the domain is being paired to four elements in the range. Remember, if an element in the domain is being associated to more than one element in the range, the relation is not a function.
- c. This is not a function. The element "s" in the domain is not being paired to any element in the range. Every element in the domain must have some kind of correspondence to the range for it to at least be considered a relation. Since this is not a relation, it follows that it can't be a function.

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L100 in the Pupil Handbook.

<b>Lesson Title:</b> Graphs of linear functions – Part 1	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L101	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: 1. Identify linear functions. 2. Make tables of values for given linear functions.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (4 minutes)

1. Write a problem on the board. Solve for  $x$  if  $5x - 3 = 3x + 7$ .
2. Allow 1 minute for pupils to solve the problem independently.
3. Invite any volunteer to write their answer on the board and explain. Ask other pupils to observe carefully to see if they agree with the calculation.
4. Correct any errors in the section on the board. Ask pupils to check their work.

#### Solution:

$$\begin{aligned}
 5x - 3 &= 3x + 7 \\
 5x - 3 - 3x &= 3x + 7 - 3x && \text{Subtract } 3x \text{ from both sides} \\
 2x - 3 &= 7 \\
 2x - 3 + 3 &= 7 + 3 && \text{Add 3 to both sides} \\
 2x &= 10 \\
 x &= \frac{10}{2} && \text{Divide throughout by 2} \\
 x &= 5
 \end{aligned}$$

5. Explain to the pupils that today's lesson is on identifying linear functions and to make tables of values for given linear functions.

### Teaching and Learning (21 minutes)

1. Write on the board:  $y = 2x + 1$        $f(x) = 5 - x$        $2y = 5x + 1$        $y = x$
2. Explain:
  - These are all examples of linear functions, or linear equations.
  - A linear function of  $x$  is one which contains  $x$  with a power of only 1.
  - Linear functions usually have 2 variables,  $x$  and  $y$ .
  - Remember that in function notation,  $y$  is replaced by  $f(x)$ .
3. Write on the board:  $y = mx + c$
4. Explain:
  - This is a common way of writing linear functions, which is called "slope-intercept form".  $m$  and  $c$  are constant numbers, and  $x$  and  $y$  are variables.

- In this function, the value of  $y$  will change depending on the value of  $x$ .  $y$  is called the dependent variable, and  $x$  is called the independent variable.
- It is best to write equations in this form before solving them.

5. Explain:

- In this lesson, we will fill tables of values with solutions to linear equations. A solution to a linear equation is an  $x$ -value and a corresponding  $y$ -value that satisfy the equation.
- A linear equation has infinitely many solutions.
- Linear equations can be graphed by finding solutions to the equation and plotting them on a Cartesian plane.

6. Write on the board: Create a table of values for the linear equation  $y = 2 - x$ .

7. Draw a table on the board with values of  $x$  from  $-4$  to  $+2$ :

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$
$y$							

8. As a class, substitute all the values of  $x$  from  $-4$  to  $2$  into the given relation and find the corresponding  $y$  values. Demonstrate the first few values, then invite volunteers to come to the board and find the others.

**Solution:**

$$\begin{aligned}y &= 2 - (-4) = 2 + 4 = 6 \\y &= 2 - (-3) = 2 + 3 = 5 \\y &= 2 - (-2) = 2 + 2 = 4 \\y &= 2 - (-1) = 2 + 1 = 3 \\y &= 2 - 0 = 2 \\y &= 2 - 1 = 1 \\y &= 2 - 2 = 0\end{aligned}$$

Completed table:

$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$
$y$	6	5	4	3	2	1	0

9. Explain: The table will be used in a future lesson to draw the graph of a straight line.

10. Write another problem on the board. Complete a table of values for  $f(x) = x + 2$  for the values of  $x$  ranging from  $-3$  to  $+3$ .

11. Invite volunteers to come to the board and complete the table with the help from the class.

**Solution:**

$$\begin{aligned}\text{When } x &= -3, & f(-3) &= -3 + 2 = -1 \\x &= -2, & f(-2) &= -2 + 2 = 0 \\x &= -1, & f(-1) &= -1 + 2 = 1 \\x &= 0, & f(0) &= 0 + 2 = 2 \\x &= 1, & f(1) &= 1 + 2 = 3\end{aligned}$$

$$\begin{array}{ll} x = 2, & f(2) = 2 + 2 = 4 \\ x = 3, & f(3) = 3 + 2 = 5 \end{array}$$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-1	0	1	2	3	4	5

12. Write the following problem on the board. Copy and complete the table for the relation

$$y = \frac{1}{2}x + 2.$$

$x$	-3	-2	-1	0	1	2	3
$y$	0.5			2		3	

13. Ask pupils to work with seatmates to complete the table.

14. Walk around to check for understanding and clear misconceptions.

15. Invite volunteers to write the solution on the board.

**Solution:**

$$\text{When } x = -2, \quad y = \frac{1}{2}(-2) + 2 = -1 + 2 = 1$$

$$\text{When } x = -1, \quad y = \frac{1}{2}(-1) + 2 = -0.5 + 2 = 1.5$$

$$\text{When } x = 1, \quad y = \frac{1}{2}(1) + 2 = 0.5 + 2 = 2.5$$

$$\text{When } x = 3, \quad y = \frac{1}{2}(3) + 2 = 1.5 + 2 = 3.5$$

$x$	-3	-2	-1	0	1	2	3
$y$	0.5	1	1.5	2	2.5	3	3.5

**Practice (14 minutes)**

1. Write two problems on the board: Draw and complete tables with values of  $x$  from  $-3$  to  $+3$  for the linear equations: a.  $f(x) = 3x + 5$    b.  $3x - y = 2$
2. Explain: To complete a table of values,  $y$  or  $f(x)$  should be by itself on one side of the equation. Rewrite equation b. before creating the table of values.
3. Ask pupils to work independently to answer the questions.
4. Walk around to check the answers and clear misconceptions.
5. Invite volunteers to write the solutions on the board.

**Solutions:**

a.  $f(x) = 3x + 5$

$$\text{When } x = -3, \quad f(-3) = 3(-3) + 5 = -9 + 5 = -4$$

$$x = -2, \quad f(-2) = 3(-2) + 5 = -6 + 5 = -1$$

$$x = -1, \quad f(-1) = 3(-1) + 5 = -3 + 5 = 2$$

$$x = 0, \quad f(0) = 3(0) + 5 = 0 + 5 = 5$$

$$x = 1, \quad f(1) = 3(1) + 5 = 3 + 5 = 8$$

$$x = 2, \quad f(2) = 3(2) + 5 = 6 + 5 = 11$$

$$x = 3, \quad f(3) = 3(3) + 5 = 9 + 5 = 14$$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-4	-1	2	5	8	11	14

b.  $3x - y = 2 \rightarrow$

$$y = 3x - 2$$

When  $x = -3$ ,  $y = 3(-3) - 2 = -9 - 2 = -11$

$$x = -2, \quad y = 3(-2) - 2 = -6 - 2 = -8$$

$$x = -1, \quad y = 3(-1) - 2 = -3 - 2 = -5$$

$$x = 0, \quad y = 3(0) - 2 = 0 - 2 = -2$$

$$x = 1, \quad y = 3(1) - 2 = 3 - 2 = 1$$

$$x = 2, \quad y = 3(2) - 2 = 6 - 2 = 4$$

$$x = 3, \quad y = 3(3) - 2 = 9 - 2 = 7$$

$x$	-3	-2	-1	0	1	2	3
$y$	-11	-8	-5	-2	1	4	7

### Closing (1 minute)

- For homework, have pupils do the practice activity PHM1-L101 in the Pupil Handbook.

<b>Lesson Title:</b> Graphs of linear functions – Part 2	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L102	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes	
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to use tables of values to draw straight line graphs within Cartesian axes.	 <b>Preparation</b> Bring rulers and ask pupils to bring rulers to draw axes. If rulers are not available, use any object (such as the side of a book) as a straight edge, and estimate lengths of 1 cm or 2 cm for the tick marks.	

### Opening (4 minutes)

1. Write a problem on the board: Draw a table of values of  $x$  from  $-2$  to  $+2$  of the relation  $f(x) = 4 - x$ .
2. Ask pupils to solve the problem independently.
3. Invite volunteers to write the solution on the board.

#### Solution:

Working:

$$f(-2) = 4 - (-2) = 4 + 2 = 6$$

$$f(-1) = 4 - (-1) = 4 + 1 = 5$$

$$f(0) = 4 - (0) = 4 - 0 = 4$$

$$f(1) = 4 - (1) = 4 - 1 = 3$$

$$f(2) = 4 - (2) = 4 - 2 = 2$$

Table of values:

$x$	-2	-1	0	1	2
$f(x)$	6	5	4	3	2

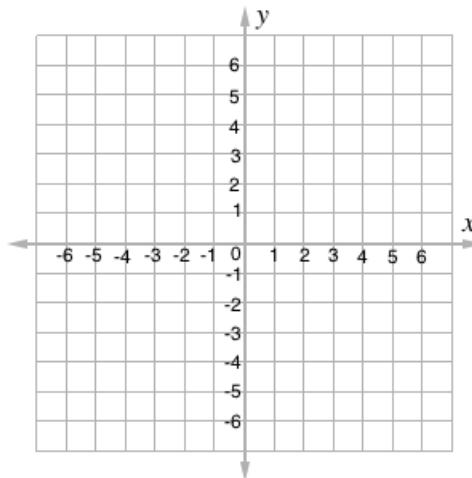
4. Explain that today's lesson is on drawing the graph of a straight line using table of values.

### Teaching and Learning (21 minutes)

1. Explain:
  - a. To graph  $f(x) = 4 - x$ , we want to plot points for the equation on the Cartesian plane.
  - b. The table gives ordered pairs that we plot.
2. Write on the board:  $(x, y) = (-2, 6)$
3. Explain:
  - a. This is the first ordered pair in the table. Each ordered pair has an  $x$ -value and a  $y$ -value.
  - b. The values are written in brackets, and the  $x$ -value always comes first.
4. Ask pupils to write in their exercise books all other ordered pairs in the table.
5. Invite a volunteer to write the answer on the board. (Answer:  $(-1, 5), (0, 4), (1, 3), (2, 2)$ )

6. Discuss: What is a Cartesian plane? What does it look like? (Answer: It has 2 intersecting axes, the x-axis and y-axis. They both have positive and negative directions.)

7. Draw a Cartesian plane on the board:



8. Explain:

- The x-axis is horizontal. The values increase as you go to the right. Negative values are on the left.
- The y-axis is vertical. The values increase as you go up. Negative values are below the x-axis.
- It is important that the tick marks (numbers) on the axes are the same distance apart from one another. The WASSCE exam often asks you to draw them 1 cm or 2 cm apart.

9. Explain:

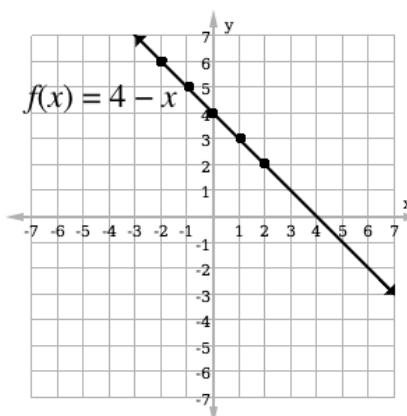
- We will plot the 5 points from the table on the Cartesian plane.
- We will draw a straight line through the points once we have plotted them.

10. Plot the points on the plane as a class. Make sure

pupils understand the following:

- a. Count to the right for a positive x-value
- b. Count to the left for a negative x-value
- c. Count up for a positive y-value
- d. Count down for a negative y-value

11. Connect the points with a straight line.



12. Write on the board: plot the graph of the table  $y = 1 - 5x$ . Use 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis.

$x$	-3	-2	-1	0	1	2
$y$	16	11	6	1	-4	-9

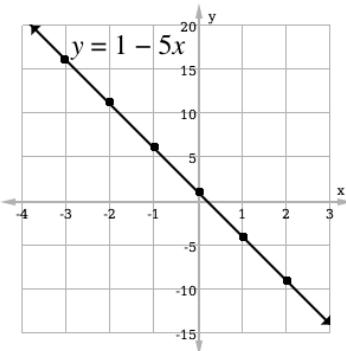
13. Explain:

- When drawing the axes, it is important that they extend beyond the values in the table.
- Our  $x$ -axis should extend beyond -3 and 2, and our  $y$ -axis should extend beyond 16 and -9.

14. Draw the axes on the board using the scales. Note that the graph on the right is not to scale. You may also draw the example on the board larger than the given scale of 2 cm. Make sure pupils understand any changes you make to the scale.

15. Invite volunteers from around the classroom plot the points on the plane.

16. Invite a volunteer connect the points with a line.



17. Write another problem on the board. Draw the graph of the relation  $f(x) = 6 - \frac{1}{2}x$  using the table of values. Use a scale of 2 cm to 1 unit on both axes.

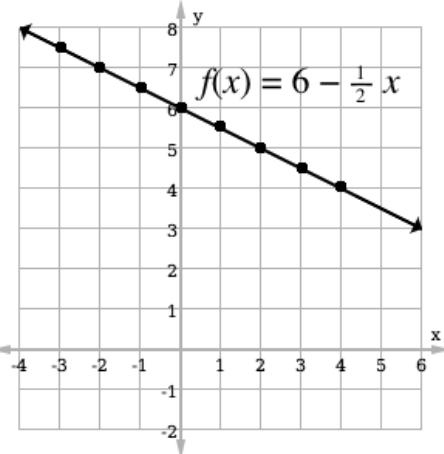
$x$	-3	-2	-1	0	1	2	3	4
$f(x)$	7.5	7	6.5	6	5.5	5	4.5	4

18. Ask pupils to solve the problem with seatmates.

Ask them to each draw their own individual graph then check with seatmates.

19. Walk around, if possible, to check their answers and clear misconceptions.

20. Invite volunteers draw the graph on the board.



### Practice (14 minutes)

1. Write the following two problems below on the board. If there is not enough time, choose only 1 problem.

Copy and complete the tables of values for  $-3 \leq x \leq 4$  for the relations:

a.  $y = 5x + 3$

$x$	-3	-2	-1	0	1	2	3	4
$y$	-12		-2	3		18	23	

Use a scale of 2 cm to 1 unit on the  $x$ -axis, and 2 cm to 5 units on the  $y$ -axis.

$$f(x) = \frac{1}{2}x - 5$$

$x$	-3	-2	-1	0	1	2	3	4
$y$	-6.5	-6			-4.5	-4		-3

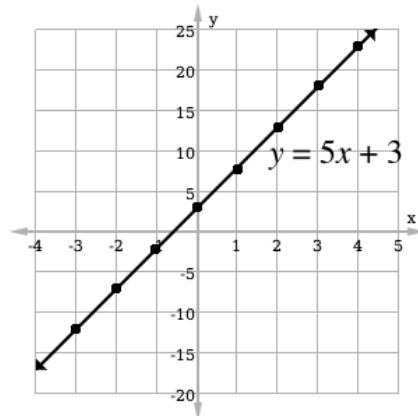
Use a scale of 2 cm to 1 unit on both the x-axis and the y-axis.

2. Ask pupils to work independently or with seatmates.
3. Walk around to check their answers.
4. Have two volunteers to draw the graph.

**Answers:**

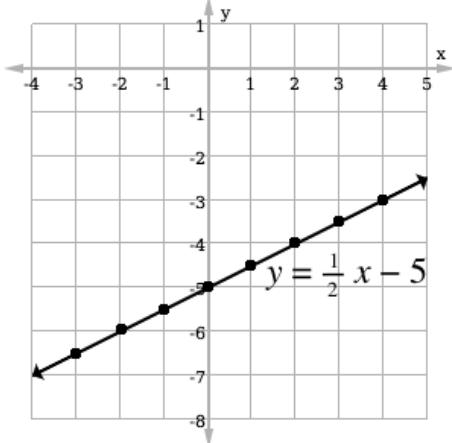
- a. Completed table and graph for  $y = 5x + 3$ :

$x$	-3	-2	-1	0	1	2	3	4
$y$	-12	-7	-2	3	8	13	18	23



- b. Completed table and graph for  $y = \frac{1}{2}x - 5$ :

$x$	-3	-2	-1	0	1	2	3	4
$y$	-6.5	-6	-5.5	-5	-4.5	-4	-3.5	-3



**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L102 in the Pupil Handbook.

<b>Lesson Title:</b> Quadratic functions	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L103	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to construct tables of values for given quadratic functions.	 <b>Preparation</b> None	

### Opening (3 minutes)

1. Write a problem on the board to revise substitution: Given that  $a = 2$ ,  $b = -1$ ,  $c = 3$  and  $x = 1$ , find the value of  $ax^2 + bx + c$ .
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board and explain.

#### Solution:

$$\begin{aligned}
 ax^2 + bx + c &= 2(1)^2 + (-1)(1) + (3) \\
 &= 2 - 1 + 3 \\
 &= 4
 \end{aligned}$$

4. Explain that today's lesson is on constructing tables of values for given quadratic functions.

### Teaching and Learning (22 minutes)

1. Write on the board:  $y = x^2 + 2x - 3$      $2x^2 = 1 + y$      $f(x) = 3x^2 + x + 2$
2. Explain:
  - a. These are all examples of quadratic functions.
  - b. A quadratic function of  $x$  is one which contains terms in  $x$  of power 2 only.
  - c. A quadratic function has 2 variables,  $x$  and  $y$ . The value of  $y$  will change depending on the value of  $x$ .  $y$  is called the dependent variable, and  $x$  is called the independent variable.
  - d. As with linear functions, we can create a table of values and graph a quadratic function.
3. Write a problem on the board: Create a table of values for the relation  $y = x^2$  for  $-3 \leq x \leq 3$ .
4. Draw an empty table with values of  $x$  from  $-3$  to  $+3$ .
5. Substitute the values of  $x$  into the given relation to have  $y$  value for each  $x$  value. Work the first few on the board, then invite volunteers to come to the board and work some.

#### Solution:

When $x = -3$ , $y = (-3)^2 = 9$ $x = -2$ , $y = (-2)^2 = 4$	When $x = 0$ , $y = (0)^2 = 0$ $x = 1$ , $y = (1)^2 = 1$
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$$x = -1, \quad y = (-1)^2 = 1$$

$$x = 2, \quad y = (2)^2 = 4$$

$$x = 3, \quad y = (3)^2 = 9$$

6. Write the values of  $x$  and  $y$  in the tabular form:

$x$	-3	-2	-1	0	1	2	3
$y$	9	4	1	0	1	4	9

7. Explain: The table will be used in a future lesson to draw the graph of the quadratic function.

8. Write another problem on the board. Find the values of  $y$  for the relation  $y = 2x^2 + x - 2$  for the values of  $x$  ranging from -3 to 2.

9. Explain to the pupils that there is another way of finding the values of the  $y$  for the relation  $y = 2x^2 + x - 2$ .

10. Discuss:

- The quadratic equation has three terms,  $2x^2$ ,  $x$  and  $-2$ .
- In the case of  $2x^2$ , we should substitute each value of  $x$  and square it before multiplying it by 2.
- In the case of  $x$ , we only substitute each value of  $x$ .
- In the case of  $-2$ , since it is a constant term, we share it to all the values of each column.
- After finding the values of the three terms, we add all of them to get the values of  $y$ .

11. Complete the table shown below on the board. The first row is for the given values of  $x$ . The next rows are for the terms of the quadratic function. The last row is for the  $y$ -values. These are found by adding the rows for the terms (in this case, rows 2-4).

**Solution:**  $y = 2x^2 + x - 2$

$x$	-3	-2	-1	0	1	2
$2x^2$	18	8	2	0	2	8
$x$	-3	-2	-1	0	1	2
$-2$	-2	-2	-2	-2	-2	-2
$y$	13	4	-1	-2	1	8

12. Explain: We can use either type of table to create our table of values. With either table, we have found the  $y$ -values that correspond to the given  $x$ -values.

13. Write another problem on the board: Find the values of  $y$  for the relation  $y = x^2 + 2x - 3$  for  $x$  ranging from -4 to +2.

14. Ask pupils to work with seatmates to complete the problem using the second method described above.

15. Walk around, if possible, to check their answers and clear misconceptions.

16. Remind the pupils that they could use either method in finding the values of  $y$ , but in this case they are asked to use the second method.

17. Invite volunteers to draw and fill the table on the board.

**Solution:**  $y = x^2 + 2x - 3$

$x$	-4	-3	-2	-1	0	1	2
$x^2$	16	9	4	1	0	1	4
$2x$	-8	-6	-4	-2	0	2	4
-3	-3	-3	-3	-3	-3	-3	-3
$y$	5	0	-3	-4	-3	0	5

### Practice ( 14 minutes)

1. Write the following two problems on the board:
  - a. Draw a table with values of  $x$  from -3 to 3 for the equation  $y = 2 - 3x - 2x^2$ .
  - b. Draw a table with values of  $x$  from 0 to 5 for the relation  $y = x^2 - 5x + 8$
2. Ask pupils to work independently to answer the questions.
3. Ask them to use the first method in solving for question a. and the second, tabular method for question b.
4. Walk around to check their answers and clear misconceptions.
5. Invite two volunteers one at a time to write their answers on the board.

#### Answers:

a.  $y = 2 - 3x - 2x^2$

$$\begin{aligned} \text{When } x = -3; \quad y &= 2 - 3(-3) - 2(-3)^2 = -7 \\ x = -2, \quad y &= 2 - 3(-2) - 2(-2)^2 = 0 \\ x = -1, \quad y &= 2 - 3(-1) - 2(-1)^2 = 3 \\ x = 0, \quad y &= 2 - 3(0) - 2(0)^2 = 2 \\ x = 1, \quad y &= 2 - 3(1) - 2(1)^2 = -3 \\ x = 2, \quad y &= 2 - 3(2) - 2(2)^2 = -12 \\ x = 3, \quad y &= 2 - 3(3) - 2(3)^2 = -25 \end{aligned}$$

$x$	-3	-2	-1	0	1	2	3
$y$	-7	0	3	2	-3	-12	-25

b.  $y = x^2 - 5x + 8$

$x$	0	1	2	3	4	5
$x^2$	0	1	4	9	16	25
$-5x$	0	-5	-10	-15	-20	-25
+8	+8	+8	+8	+8	+8	+8
$y$	8	4	2	2	4	8

### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L103 in the Pupil Handbook.

<b>Lesson Title:</b> Quadratic functions on the Cartesian plane – Part 1	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L104	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to use a table of values to draw the graph of quadratic functions on the Cartesian plane.	 <b>Preparation</b> Bring rulers and ask pupils to bring rulers to draw axes. If rulers are not available, use any object (such as the side of a book) as a straight edge, and estimate lengths of 1 cm or 2 cm for the tick marks.	

### Opening (4 minutes)

1. Write the following problem on the board: Draw a table of values with  $x$  from  $-1$  to  $4$  for the relation  $f(x) = 4 + 3x - x^2$ .
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board.

#### Solution:

$x$	-1	0	1	2	3	4
$+4$	+4	+	+4	+4	+4	+4
$3x$	-3	0	3	6	9	12
$-x^2$	-1	0	-1	-4	-9	-16
$f(x)$	0	4	6	6	4	0

4. Explain that today's lesson is on drawing the graphs of quadratic functions on the Cartesian plane using tables of values.

### Teaching and Learning (23 minutes)

1. Explain:
  - To draw a quadratic graph, the procedure is the same as that of a linear function.
  - Identify ordered pairs from a table of values, and plot them on the Cartesian plane. Connect them with a curve.
2. Write a question on the board: Draw the graph of  $y = x^2$  from the table below.

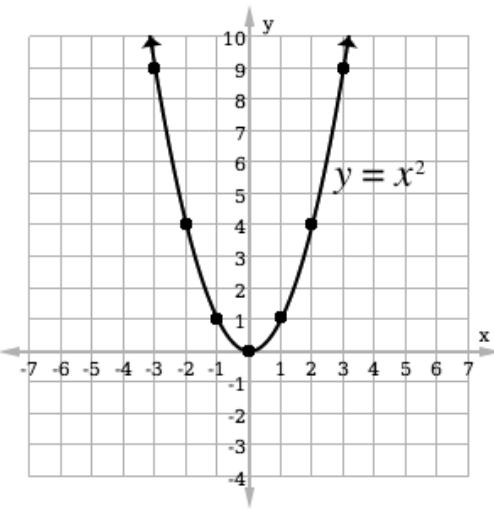
$x$	-3	-2	-1	0	1	2	3
$y$	9	4	1	0	1	4	9

3. Ask pupils to write down the ordered pairs from the table in their exercise books.
4. Invite volunteers to write the ordered pairs on the board. (Answers:  $(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)$ )
5. Discuss:
  - Remember  $x$  is the independent variable. We choose its value.
  - The scale we use to plot the points depends on the scale given in the question.

6. Ask a volunteer to give the smallest and the largest value of  $x$  in the table of  $y = x^2$  (Answer: Smallest  $-3$ , Largest  $+3$ )
7. Ask another volunteer to give the smallest and the largest value of  $y$  in the table. (Answer: Smallest  $0$ , Largest  $9$ )
8. Remind pupils that the axes should extend beyond the smallest and largest values.
9. Draw an empty Cartesian plane on the board.
10. Plot the points on the plane using the values in the table. Use a light chalk mark to move along the  $x$ -axis and  $y$ -axis to plot the required number.
11. Join the points to make a curve.

12. Explain:

- This curve is called a parabola. When graphed, quadratic functions always form parabolas.
- Parabolas are shaped like the letter u or the letter n. In other words, they can open up or down. This one opens up.



13. Ask pupils to copy this in their exercise books.

14. Write on the board. Plot the graph of the function  $f(x) = 2 + 3x - x^2$  for the interval  $-3 \leq x \leq 6$  using a scale of 2 cm to 1 unit on the  $x$ -axis and 2 cm to 5 units on the  $y$ -axis.

$x$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$	$4$	$5$	$6$
$f(x)$	$-16$	$-8$	$-2$	$2$	$4$	$4$	$2$	$-2$	$-8$	$-16$

15. Draw the axes of the Cartesian plane on the board using the required scale.

16. Invite volunteers to plot the 10 points on the graph.

17. Invite a volunteer to use free hand to draw a curve to join the points plotted on the graph.

18. The completed graph is shown on the right.

19. Explain: This is an example of a parabola that opens down.

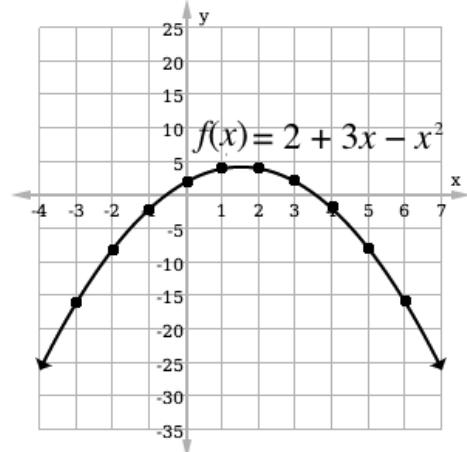
20. Write another problem on the board:

Draw the graph of the relation  $y = x^2 + 2x - 3$  for the interval  $-4 \leq x \leq 2$ . Using a scale of 2 cm to 1 unit on both axes.

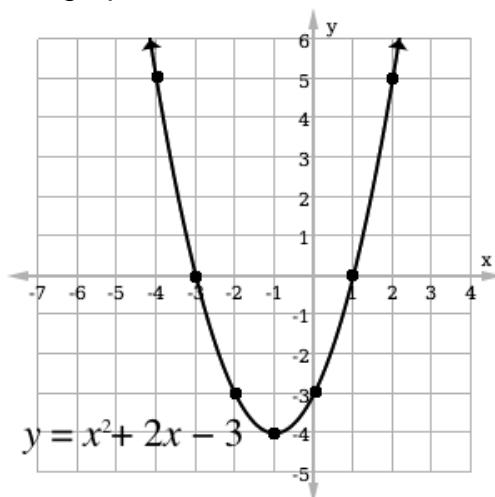
$x$	$-4$	$-3$	$-2$	$-1$	$0$	$1$	$2$
$y$	$5$	$0$	$-3$	$-4$	$-3$	$0$	$5$

21. Ask pupils to solve the problem with seatmates. Ask them to each draw their own individual graph then check with seatmates.

22. Walk around, if possible, to check their answers and clear misconceptions.



23. Invite volunteers to draw the graph on the board.



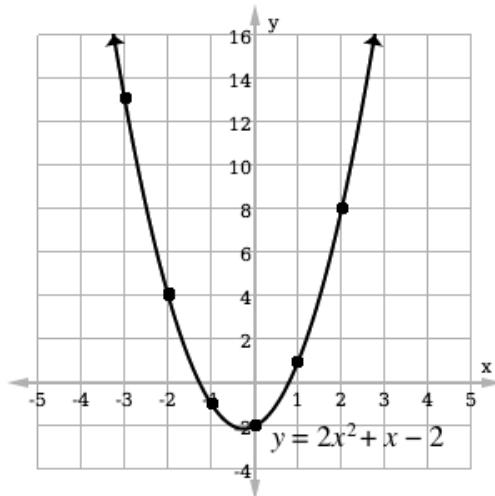
### Practice (12 minutes)

1. Write the following problem on the board: Draw a graph of the relation  $y = 2x^2 + x - 2$  using a scale of 2 cm to 1 unit on  $x$ -axis and 2 cm to 2 units on  $y$ -axis.

$x$	-3	-2	-1	0	1	2
$y$	13	4	-1	-2	1	8

2. Ask pupils to solve the problems independently.
3. Walk around to check their answers and clear misconceptions.
4. Invite volunteers to draw the graph on the board.

### Answer:



### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L104 in the Pupil Handbook.

<b>Lesson Title:</b> Quadratic functions on the Cartesian plane – Part 2	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L105	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to draw a smooth parabolic curve through plotted points.	 <b>Preparation</b> Bring rulers and ask pupils to bring rulers to draw axes. If rulers are not available, use any object (such as the side of a book) as a straight edge, and estimate lengths of 1 cm or 2 cm for the tick marks.	

### Opening (3 minutes)

1. Write the following problem on the board: Copy and complete the table of values below for the relation  $y = x^2 - x - 2$  for  $-3 \leq x \leq 2$ .

x	-3	-2	-1	0	1	2
y	10			-2		

2. Ask pupils to complete the table independently.
3. Invite volunteers to give their answers on the board.

**Answer:**

x	-3	-2	-1	0	1	2
y	10	4	0	-2	-2	0

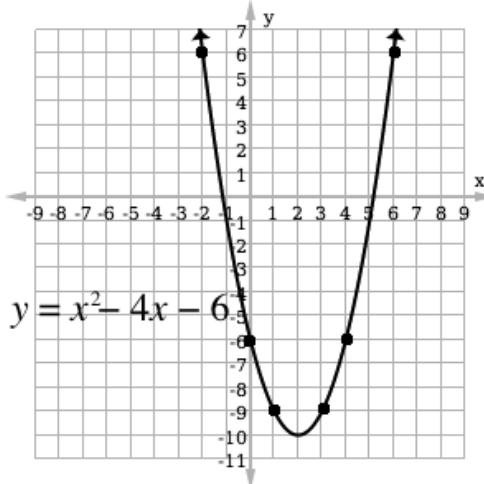
4. Explain that today's lesson is on drawing a smooth parabolic curve through plotted points.

### Teaching and Learning (22 minutes)

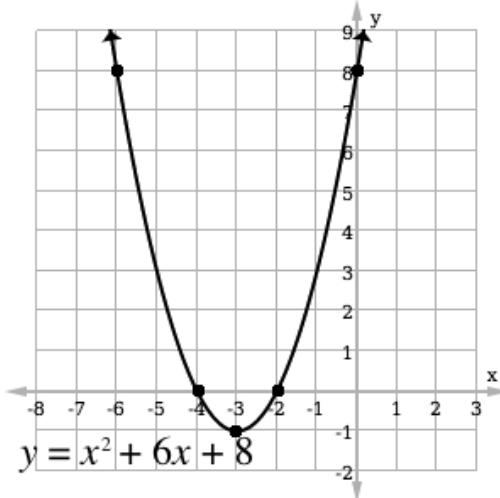
1. Discuss the previous lessons. Allow pupils to share their answers:
  - What form does a quadratic function have? (Example answers:  $ax^2 + bx + c$ , where  $a, b$  and  $c$  are real numbers and  $a \neq 0$ . It has a variable  $x$  with power 2.)
  - What do we call the curve formed by the graph of a quadratic function? (Answer: A parabola.)
  - What do you know about parabolas? (Answer: They can open up or down.)
2. Write some co-ordinates on the board:  
 $(-2, 6), (0, -6), (1, -9), (3, -9), (4, -6), (6, 6)$
3. Draw an empty Cartesian plane on the board with axes large enough for all of the points (at least  $-6$  to  $+6$  on the  $x$ -axis and  $-10$  to  $+8$  on the  $y$ -axis; see graph below).
4. Discuss:
  - Recall that ordered pairs can be given in a table of values, as in the one you completed at the start of the class.

- They can also be listed like this.
- Recall that the first value is the  $x$ -value and the second is the  $y$ -value.

5. Invite volunteers to come to the board and plot the points of the co-ordinates above.
6. Connect the points with a parabola and label it  $y = x^2 - 4x - 6$  (see graphed parabola below).



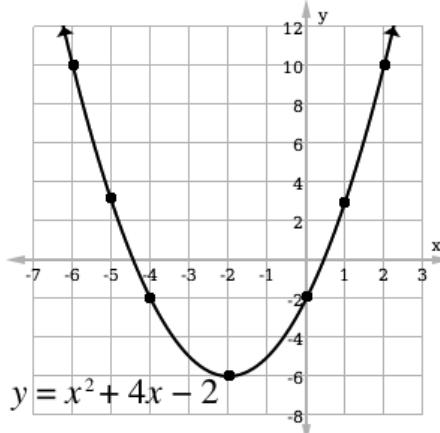
7. Write a question on the board: Draw a smooth parabolic curve from the following coordinate points:  $(-6, 8), (-4, 0), (-3, -1), (-2, 0), (0, 8)$ , which are solutions to  $y = x^2 + 6x + 8$ . Use a scale of 2 cm to 1 unit on the  $x$ -axis and 2 cm to 2 units on the  $y$ -axis.
8. Invite a volunteer to draw an empty Cartesian plane on the board using the scale given.
9. Invite volunteers, one at a time, to plot the four ordered pairs on the Cartesian plane.
10. Invite another volunteer to connect the points plotted on the Cartesian plane with free hand to draw the parabolic curve.
11. Invite a volunteer to label the graph as  $y = x^2 + 6x + 8$ . (see graphed parabola below)



12. Write another question on the board: The table below is for the function  $y = x^2 + 4x - 2$ . Use the table to draw the graph of  $y = x^2 + 4x - 2$ , using a scale of 2 cm to 1 unit on the  $x$ -axis and 2 cm to 2 units on the  $y$ -axis.

$x$	-6	-5	-4	-2	0	1	2
$y$	10	5	-2	-6	-2	3	10

13. Ask pupils to solve the problem with seatmates.  
 14. Walk around to check for understanding and clear misconceptions.  
 15. Invite volunteers to come to the board and plot the points, one at a time, and draw the curve.



### Practice (14 minutes)

1. Write on the board: Copy and complete the table below for the function  $f(x) = 2 + 3x - x^2$ . Use the table to draw the graph of  $f(x) = 2 + 3x - x^2$  using a scale of 2 cm to 1 unit on the  $x$ -axis and 2 cm to 2 units on the  $y$ -axis.

$x$	-2	-1	0	1	2	3	4	5
$f(x)$	-8		2		2	-2		

2. Ask pupils to solve the problems independently or with seatmates.  
 3. Walk around to check for understanding and clear misconceptions.  
 4. Invite two volunteers to come to the board to write the solution at the same time.

### Solution:

Find the missing values for the table:

$$f(-1) = 2 + 3(-1) - (-1)^2 = 2 - 3 - 1 = -2$$

$$f(1) = 2 + 3(1) - (1)^2 = 2 + 3 - 1 = 4$$

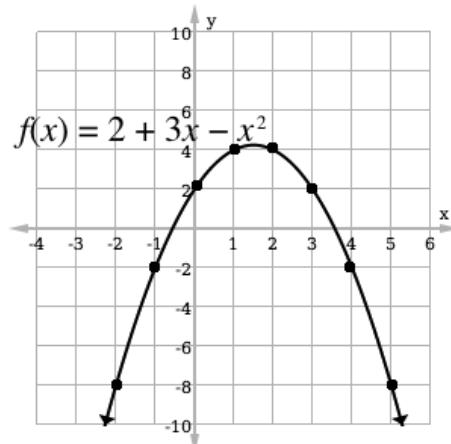
$$f(2) = 2 + 3(2) - (2)^2 = 2 + 6 - 4 = 4$$

$$f(5) = 2 + 3(5) - (5)^2 = 2 + 15 - 25 = -8$$

Complete the table:

$x$	-2	-1	0	1	2	3	4	5
$f(x)$	-8	-2	2	4	4	2	-2	-8

Plot and connect the points, and label the curve:



### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L105 in the Pupil Handbook.

<b>Lesson Title:</b> Values from the graphs of quadratic functions	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L106	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to read off values from the graphs of quadratic functions (including minimum and maximum values, and axis of symmetry).	 <b>Preparation</b> Write the problem in Opening on the board, and draw an empty Cartesian plane for its solution.	

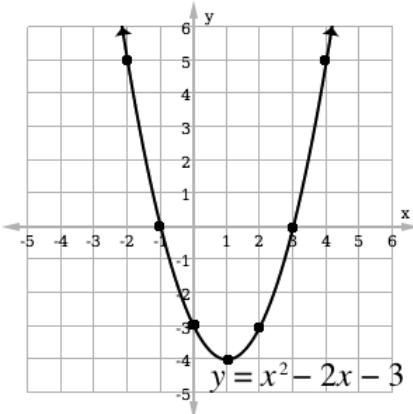
### Opening (5 minutes)

1. Write the following problem on the board. Graph the function of  $y = x^2 - 2x - 3$  using the table of values:

$x$	-2	-1	0	1	2	3	4
$y$	5	0	-3	-4	-3	0	5

2. Ask pupils to solve the problem independently.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to draw the graph on the board.

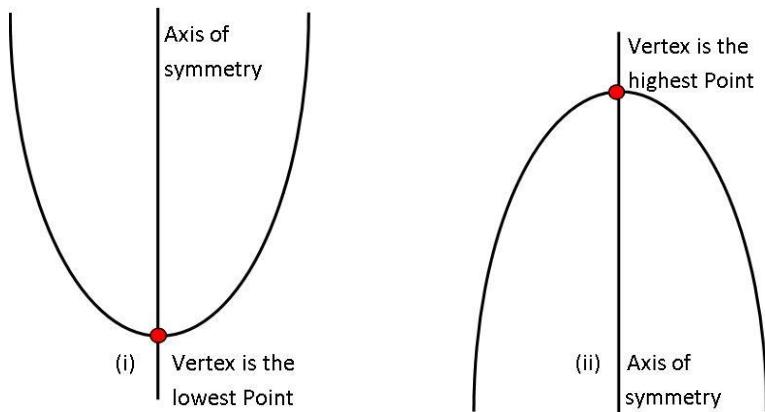
#### Answer:



5. Explain to pupils that today's lesson is on answering questions using the graphs of quadratic functions.

### Teaching and Learning (23 minutes)

1. Draw two curves on the board:

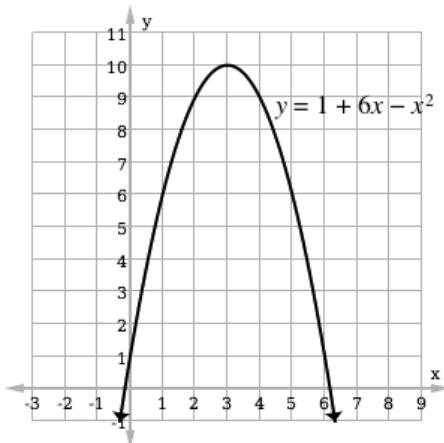


2. Explain:

- Figures (i) and (ii) are the graphs of quadratic functions, which we call **parabolas**.
- The graph on the left opens up and has a lowest point while the graph on the right opens down and has a highest point.
- The lowest or highest point of a parabola is called the **vertex**.
- The other name of the lowest point is **minimum** and the other name of the highest point is **maximum**.
- The vertical line passing through the vertex in each parabola is called the **axis of symmetry** or **line of symmetry**.

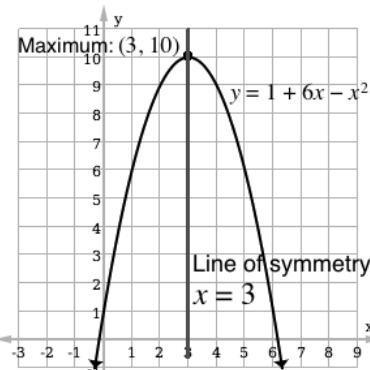
3. Write the following problem on the board. Use the graph below to find:

- a. The coordinates of the maximum of  $y = 1 + 6x - x^2$ .
- b. The equation of the line of symmetry of  $y = 1 + 6x - x^2$ .

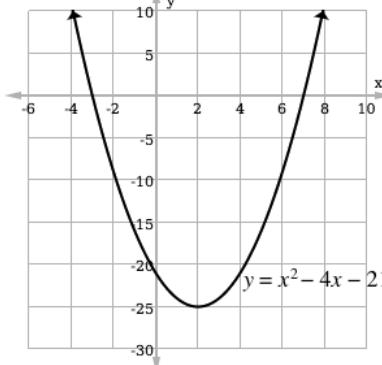


4. Ask volunteers to answer question a. by identifying the maximum point of  $y = 1 + 6x - x^2$ . (Answer: From the graph, the maximum is at (3,10)).

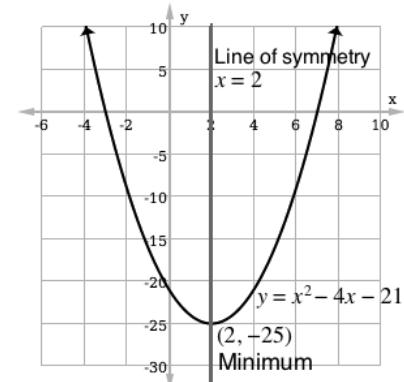
5. Label the maximum on the graph.
6. Remind pupils that the line of symmetry is the line that divides the graphs into two equal parts.
7. Invite a volunteer to draw the line of symmetry on the graph. (Answer:  $x = 3$ )
8. Label the axis of symmetry on the board.



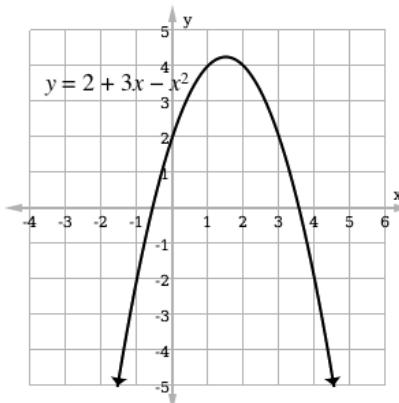
9. Write another question on the board: Use the graph to find the following:
  - a. The coordinates of the minimum point.
  - b. The equation of the line symmetry of  $y = x^2 - 4x - 21$ .



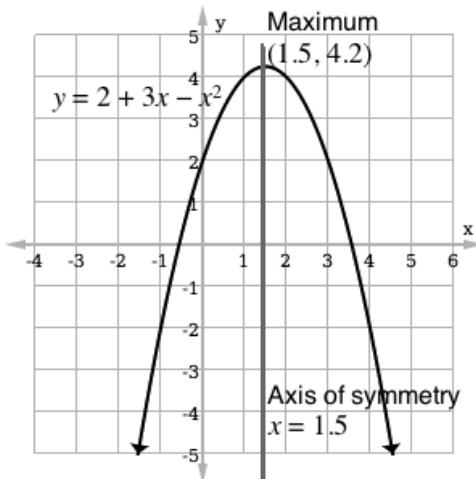
10. Invite a volunteer to identify the minimum point on the graph and write down its coordinates on the graph. (Answer:  $(2, -25)$ )
11. Invite another volunteer to find the equation of the line of symmetry and write it on the graph. (Answer:  $x = 2$ ).



12. Write another question on the board: Use the graph below to find the following for  $y = 2 + 3x - x^2$ :
  - a. The value of  $y$  when  $x = 2$ .
  - b. The value of  $x$  when  $y = -2$ .
  - c. The co-ordinates of the maximum.
  - d. The equation of the line of symmetry.
13. Ask pupils to solve the problem with seatmates.
14. Walk around to check for understanding and clear misconceptions.
15. Invite volunteers to come to the board and write their answers. Ask them to also label the graph. (Answers:



- a. When  $x = 2, y = 4$ ; b. When  $y = -2, x = -1$  and  $x = 4$ ; c. Maximum =  $(1.5, 4.2)$ ; d.  $x = 1.5$ )

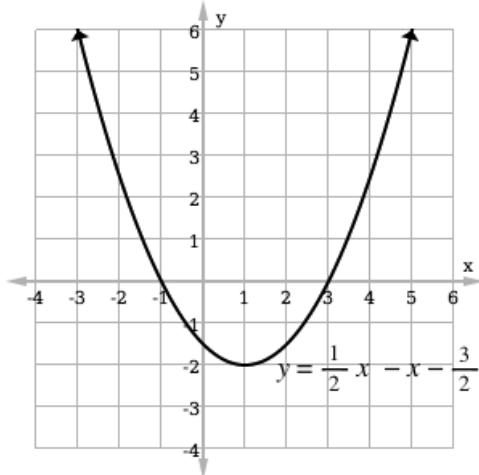


### Practice (11 minutes)

1. Write on the board: Use the graph of  $f(x) = \frac{1}{2}x^2 - x - \frac{3}{2}$  to find the following:
  - The value of  $f(x)$  when  $x = 4$ .
  - The value of  $x$  when  $f(x) = -2$ .
  - The coordinates of the minimum.
  - The equation of the line of symmetry.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite volunteers to write the answers on the board.

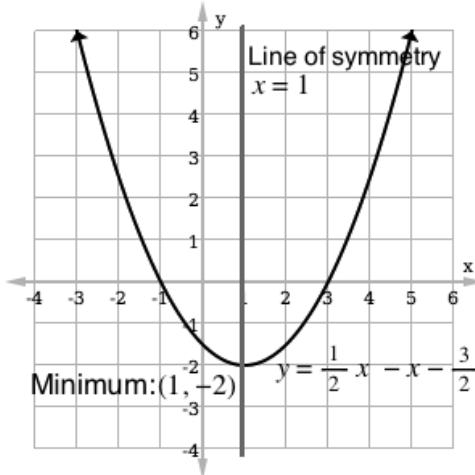
#### Answers:

- When  $x = 4, y = 2.5$
- When  $y = -2, x = 1$
- Minimum:  $(1, -2)$
- Line of symmetry:  $x = 1$



### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L106 in the Pupil Handbook.



<b>Lesson Title:</b> Factorising quadratic expressions	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L107	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to factorise quadratic expressions.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board. Expand  $(x + 2)(x + 5)$ .
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board.

**Solution:**  $(x + 2)(x + 5)$

$$\begin{aligned}
 &= x(x + 5) + 2(x + 5) \\
 &= x^2 + 5x + 2x + 10 \\
 &= x^2 + 7x + 10
 \end{aligned}$$

4. Explain that today's lesson is on factorising quadratic expressions. This is the opposite of expanding.

### Teaching and Learning (22 minutes)

1. Write on the board: Quadratic expression:  $ax^2 + bx + c$
2. Explain:
  - A quadratic expression has the general form  $ax^2 + bx + c$  where  $a, b$  and  $c$  are numbers.
  - It is very important to note that 2 is the highest power of  $x$  in a quadratic expression.
  - The number  $a$  is called the coefficient of  $x^2$ ,  $b$  is called the coefficient of  $x$ , and  $c$  is called the constant term.
  - These numbers can be positive or negative. The numbers  $b$  and  $c$  can also be zero.
3. We already know from our solution that  $x^2 + 7x + 10 = (x + 2)(x + 5)$ .
4. Explain that this is the same as  $x^2 + 7x + 10 = (x + 5)(x + 2)$  since we can multiply in any order.
5. Ask any volunteer to identify  $a, b$  and  $c$  from the expression  $x^2 + 7x + 10$ . (Answer:  $a = 1, b = 7$  and  $c = 10$ )
6. Ask pupils to look at both forms of the quadratic expressions on the left-hand side (LHS) and the right-hand side (RHS) of  $x^2 + 7x + 10 = (x + 2)(x + 5)$ .
7. Allow them to discuss for 1 minute anything they notice about the 2 expressions.

8. Ask any volunteer to share their ideas with the class. (Example answers: The  $x^2$  term is from  $x \times x$ ; the b term (7) is found by adding 2 and 5; the c term (10) is found by multiplying 2 and 5. Accept all reasonable answers).
9. Write on the board:  $ax^2 + bx + c = (x + p)(x + q)$  where  $p + q = b$  and  $p \times q = c$  when  $a = 1$ .
10. Discuss:
  - To factorise a quadratic expression, we want to find the 2 numbers that are added to  $x$  in the binomial factors.
  - When  $a = 1$ , it is relatively easy to find these 2 numbers.
  - If we add the 2 numbers together, we get the b term.
  - If we multiply the 2 numbers together, we get the c term.

11. Write a problem on the board: Factorise  $x^2 + 5x + 6$ .

12. Explain: We need to look for two numbers which multiply to give 6 and add to give 5.

13. Show how to solve the problem as shown below on the board.

$$x^2 + 5x + 6$$

$$a = 1, b = 5, c = 6$$

Identify the values of a, b and c in the given quadratic expression

$$x^2 + 5x + 6 = (x + p)(x + q)$$

Factors that give 6:	1	6
	2	3

Find all the factor pairs so that  $p \times q = 6$ .

Notice that 2 and 3 give  $p + q = 5$

$$x^2 + 5x + 6 = (x + 2)(x + 3) \quad \text{Factors of } x^2 + 5x + 6 \text{ are } (x + 2) \text{ and } (x + 3)$$

14. Write another problem on the board: Factorise  $x^2 - 7x + 12$

15. Ask pupils to find two numbers which multiply together to give 12.

16. Allow them to share with the class, and write the answers on the board. (Answers:  $-1$  and  $-12$ ,  $-3$  and  $-4$ ,  $-2$  and  $-6$ )

17. Ask a volunteer to choose the correct numbers to use, and explain why. (Answer:  $-3$  and  $-4$ , because these sum to  $b = -7$ .)

18. Explain:

- Using the two numbers which add to give  $-7$ , we can split the  $-7x$  term into  $-3x$  and  $-4x$ .
- This will give us 4 terms in our expression instead of 3. We can factor the first 2 terms and the last 2 terms separately.

19. Write the solution on the board as shown, and explain:

$$\begin{aligned} x^2 - 7x + 12 &= x^2 - 3x - 4x + 12 \\ &= x(x - 3) - 4x + 12 && \text{Factorise the first two terms} \\ &= x(x - 3) - 4(x - 3) && \text{Factorise the last two terms} \\ &= (x - 3)(x - 4) && \text{Factorise the common factor of } x - 3. \end{aligned}$$

Therefore,  $x^2 - 7x + 12 = (x - 3)(x - 4)$ .

20. Write another problem on the board: Factorise  $x^2 + 6x + 8$ .

21. Ask pupils to work with seatmates to factorise the expression.

22. Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned}x^2 + 6x + 8 &= x^2 + 2x + 4x + 8 \\&= x(x + 2) + 4(x + 2) \quad \text{Factorise the first 2 and last 2 terms} \\&= (x + 2)(x + 4) \quad \text{Factorise the common factor of } x + 2.\end{aligned}$$

**Practice (14 minutes)**

1. Write two problems on the board: Factorise the following quadratic expressions:
  - a.  $x^2 + 2x - 8$
  - b.  $x^2 + 12x + 27$
2. Ask pupils to solve the problems independently. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite pupils from the front and the back of the classroom to volunteer to write the solutions on the board.

**Solutions:**

$$\begin{aligned}\text{a. } x^2 + 2x - 8 &= x^2 + 4x - 2x - 8 \\&= x(x + 4) - 2x - 8 \\&= x(x + 4) - 2(x + 4) \\&= (x + 4)(x - 2) \\ \text{b. } x^2 + 12x + 27 &= x^2 + 3x + 9x + 27 \\&= x(x + 3) + 9x + 27 \\&= x(x + 3) + 9(x + 3) \\&= (x + 3)(x + 9)\end{aligned}$$

**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L107 in the Pupil Handbook.

<b>Lesson Title:</b> Solving quadratic equations	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L108	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to solve quadratic equations using the principal that if $a \times b = 0$ , then either $a = 0$ or $b = 0$ , or both $a$ and $b$ are 0.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write a problem on the board: Factorise  $x^2 - 12x + 35$
2. Ask pupils to solve the problem in their exercise books.
3. Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned}
 x^2 - 12x + 35 &= x^2 - 5x - 7x + 35 \\
 &= x(x - 5) - 7(x - 5) \\
 &= (x - 5)(x - 7)
 \end{aligned}$$

4. Explain to the pupils that today's lesson is on solving quadratic equations.

### Teaching and Learning (20 minutes)

1. Write on the board:  $x^2 + 2x + 1 = 0$
2. Explain:
  - When a quadratic expression is set equal to 0, it forms a quadratic equation.
  - A quadratic equation can be solved by finding values of  $x$  which satisfy it.
  - A quadratic equation can have 0, 1, or 2 solutions.
3. Write on the board:  $x = -1$
4. Explain: This is the only solution to the quadratic equation on the board. We can substitute it into the equation to verify that it is a solution.
5. Verify the solution on the board:

$$\begin{array}{lll}
 x^2 + 2x + 1 &= 0 & \text{Given equation} \\
 (-1)^2 + 2(-1) + 1 &= 0 & \text{Substitute } x = -1 \\
 1 - 2 + 1 &= 0 & \text{Evaluate} \\
 0 &= 0 & \text{The solution is correct}
 \end{array}$$

6. Explain:
  - There are a few ways to solve quadratic equations. This can be done algebraically or graphically.
  - One way of solving a quadratic equation is to apply the following argument to a quadratic expression that has been factorised.
  - If the product of two numbers is 0, then one of the numbers (or possibly both of them) must be 0.

7. Write on the board:  $x^2 - 12x + 35 = (x - 5)(x - 7) = 0$
8. Explain:
- This is the quadratic equation that we have already factorised. We can use the factorisation to find the solutions.
  - The factorisation is 2 expressions multiplied together. We have set them equal to 0. We know that at least 1 of the expressions must equal 0, because their product is 0.
9. Write on the board: If  $(x - 5)(x - 7) = 0$  then either  $(x - 5) = 0$  or  $(x - 7) = 0$ , or both expressions are 0.
10. Solve the quadratic equation on the board:
- $$\begin{array}{l} x - 5 = 0 \\ x = 5 \end{array} \quad \text{or} \quad \begin{array}{l} x - 7 = 0 \\ x = 7 \end{array}$$

11. Explain: There are 2 solutions to this quadratic equation, 5 and 7.

12. Write on the board: Solve the equation  $(x + 1)(x - 2) = 0$

13. Ask pupils to discuss the problem with seatmates for 1 minute.

14. Solve the problem on the board as a class. Ask volunteers to give the steps.

**Solution:**

$(x + 1)(x - 2) = 0$  implies that either:

- $(x + 1) = 0$  because  $(0)(x - 2) = 0$ , or
- $(x - 2) = 0$  because  $(x + 1)(0) = 0$ .

This gives 2 solutions:

$$\begin{array}{l} x + 1 = 0 \\ x = -1 \end{array} \quad \text{or} \quad \begin{array}{l} x - 2 = 0 \\ x = 2 \end{array}$$

15. Write two problems on the board: Solve the following equations:

- $(x - 2)(x + 7) = 0$
- $x^2 + 6x + 8 = 0$

16. Ask pupils to solve the problems with seatmates. Remind them to factorise the equations first if needed.

17. Walk around to check for understanding and clear misconceptions.

18. Invite 2 volunteers to write their solutions on the board and explain.

**Solutions:**

- Either  $x - 2 = 0$  or  $x + 7 = 0$ . This gives  $x = 2$  or  $x = -7$ .
- Factorise the expression first:

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 2x + 4x + 8 \\ &= x(x + 2) + 4(x + 2) \\ &= (x + 2)(x + 4) \end{aligned}$$

Therefore, we have  $(x + 2)(x + 4) = 0$

Either  $x + 2 = 0$  or  $x + 4 = 0$ . This gives  $x = -2$  and  $x = -4$ .

### **Practice (16 minutes)**

1. Write three problems on the board: Solve the following equations:
  - a.  $(n - 12)(n + 4) = 0$
  - b.  $(3a + 2)(2a - 7) = 0$
  - c.  $x^2 - 3x - 10 = 0$
2. Ask pupils to solve the problems independently.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite volunteers to write their solutions on the board.

### **Solutions:**

c.  $(n - 12)(n + 4) = 0$   
Either  $(n - 12) = 0$   
Or  $(n + 4) = 0$

This gives  $n = 12$  or  $n = -4$ .

d.  $(3a + 2)(2a - 7) = 0$   
Either  $(3a + 2) = 0$   
Or  $(2a - 7) = 0$

Each expression can be solved for  $a$ , which gives  $a = \frac{-2}{3}$  or  $a = \frac{7}{2}$

e. First, factorise the expression:

$$\begin{aligned}x^2 - 3x - 10 &= x^2 + 2x - 5x - 10 \\&= x(x + 2) - 5(x + 2) \\&= (x - 5)(x + 2)\end{aligned}$$

This gives:

$$\begin{aligned}(x - 5)(x + 2) &= 0 \\ \text{Either } (x - 5) &= 0 \\ \text{Or } (x + 2) &= 0\end{aligned}$$

This gives  $x = 5$  and  $x = -2$ .

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L108 in the Pupil Handbook.

<b>Lesson Title:</b> Solving quadratic equations using factorisation	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L109	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to use factorisation to solve quadratic equations.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board: Solve the equation  $(d - 2)(d + 3) = 0$
2. Allow 1 minute for pupils to answer the question independently.
3. Ask volunteers to give the solution.

#### Solution:

Either  $d - 2 = 0$  or  $d + 3 = 0$ . This gives 2 solutions,  $d = 2, -3$

4. Explain to the pupils that today's lesson is on solving quadratic equations using factorisation.

### Teaching and Learning (22 minutes)

1. Write the following examples of quadratic equations on the board:

$$2y^2 - 5y - 3 = 0$$

$$n^2 + 50 = 27n$$

$$0 = (4b - 6)(2a + 4)$$

$$81 = k^2$$

2. Explain:

- All of these examples are quadratic equations.
- A quadratic equation is of the form  $ax^2 + bx + c = 0$ , where  $a, b$  and  $c$  are real numbers, and  $a \neq 0$ . However, it may be arranged differently, as in the examples on the board. The coefficients  $b$  and  $c$  may be equal to zero.
- A quadratic equation contains an equals sign and variable raised to the power 2.

3. Write the following problem on the board: Solve  $x^2 + 3x + 2 = 0$ .
4. Discuss: How would you solve this problem? (Example answer: Factor the expression and solve for  $x$ .)

5. Explain the steps required for solving a quadratic equation by factoring:

**Step 1.** Write the equation in standard form, equal to zero.

**Step 2.** Factor the expression.

**Step 3.** Use the zero product property and set each factor containing a variable equal to zero.

**Step 4.** Solve each factor that was set equal to zero by getting the  $x$  on one side and the answer on the other side.

6. Discuss:

In using factorisation method to solve quadratic equations, the following should be considered when finding the factor pairs.

Questions	Answers
+	+
-	-
+	-
-	+ or -

For example, if the standard equation has two positive signs, then the factor pairs should also follow with two positive signs; if it has a minus and a plus sign, then the factor pairs should follow with two negative signs, and so on.

7. Write the solution on the board for the pupils.

**Solution:**

**Step 1.** Standard form

$$x^2 + 3x + 2 = 0$$

**Step 2.** Factorise

$$x^2 + x + 2x + 2 = 0$$

$$x(x + 1) + 2(x + 1) = 0$$

$$(x + 1)(x + 2) = 0$$

**Step 3.** Zero product property

Either  $x + 1 = 0$  or  $x + 2 = 0$

**Step 4.** Solve for  $x$

$$x = 0 - 1 \text{ or } x = 0 - 2$$

$$x = -1 \text{ or } -2$$

**Answer:**  $x = -1$  or  $-2$ .

Check:

Let  $x = -1$

$$x^2 + 3x + 2 = 0$$

$$(-1)^2 + 3(-1) + 2 = 0$$

$$1 - 3 + 2 = 0$$

$$0 = 0$$

**True**

Let  $x = -2$

$$x^2 + 3x + 2 = 0$$

$$(-2)^2 + 3(-2) + 2 = 0$$

$$4 - 6 + 2 = 0$$

$$0 = 0$$

**True**

8. Write a problem on the board. Solve  $3f^2 - 10f - 8 = 0$

9. Ask any volunteer to identify the values of  $a$ ,  $b$  and  $c$ . (Answer:  $a = 3$ ,  $b = -10$  and  $c = -8$ )

10. Ask a volunteer to multiply the values of  $a$  and  $c$ . (Answer:  $a \times c = 3 \times (-8) = -24$ )

11. Ask another volunteer to give all the factor pairs  $-24$  to choose the right one which should give us the middle term  $-10$ .

**Answer:**

$$-24 = 1 \text{ and } -24$$

$$-24 = 2 \text{ and } -12$$

$$-24 = 3 \text{ and } -8$$

$$-24 = 4 \text{ and } -6$$

12. Ask a volunteer to choose the right factor pair that gives  $-10$ . (Answer: 2 and  $-12$ )

13. Write the factors 2 and  $-12$  to replace  $-10$  on the equation.

**Answer:**

$$3f^2 - 10f - 8 = 0$$

$$3f^2 + 2f - 12f - 8 = 0$$

14. Ask pupils to work with seatmates to complete the problem and find the solutions for  $f$ .

15. Invite a volunteer to write the solution on the board.

$$3f^2 - 10f - 8 = 0 \quad \text{Standard form}$$

$$3f^2 + 2f - 12f - 8 = 0 \quad \text{Factorise}$$

$$f(3f + 2) - 4(3f + 2) = 0$$

$$(3f + 2)(f - 4) = 0$$

Therefore, either:

$$3f + 2 = 0 \quad \text{or} \quad f - 4 = 0 \quad \text{Zero product property}$$

$$3f = -2 \quad \quad \quad f = 4 \quad \text{Solve for } f$$

$$f = \frac{-2}{3}$$

$$\text{Answer: } f = \frac{-2}{3}, 4$$

16. Write the following problem on the board: Solve the  $5x = 6 - 4x^2$

17. Ask pupils to work with seatmates to discuss and solve the problem.

18. Walk around to check for understanding and clear misconceptions.

19. Ask pupils from around the classroom to volunteer to give their answers.

20. Invite a volunteer with the correct answer to write the solution on the board. Ask pupils to check their work.

**Solution:**

$$4x^2 + 5x - 6 = 0 \quad \text{Write in standard form}$$

$$4x^2 + 8x - 3x - 6 = 0 \quad \text{Factorise}$$

$$4x(x + 2) - 3(x + 2) = 0$$

$$(x + 2)(4x - 3) = 0$$

Therefore, either:

$$x + 2 = 0 \quad \text{or} \quad 4x - 3 = 0 \quad \text{Zero product property}$$

$$x = 0 - 2 \quad \quad \quad 4x = 3 \quad \text{Solve for } x$$

$$x = -2 \quad \quad \quad x = \frac{3}{4}$$

$$\text{Therefore } x = -2 \text{ or } \frac{3}{4}$$

### **Practice (14 minutes)**

1. Write on the board: Solve the following quadratic equations:
  - a.  $x^2 = x + 6$
  - b.  $3n^2 - 15n + 18 = 0$
2. Ask pupils to work independently to answer the questions.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Encourage pupils to check their answers.
5. Ask pupils from around the classroom to volunteer to give their answers.

**Answer:**

f. 
$$\begin{aligned}x^2 - x - 6 &= 0 && \text{Standard form} \\x^2 + 2x - 3x - 6 &= 0 && \text{Factorise} \\x(x + 2) - 3(x + 2) &= 0 \\(x - 3)(x + 2) &= 0\end{aligned}$$

Therefore, either:

$$\begin{aligned}x - 3 = 0 \quad \text{or} \quad x + 2 = 0 && \text{Zero product property} \\x = 0 + 3 &\quad x = 0 - 2 && \text{Solve for } x\end{aligned}$$

**Answer:**  $x = 3$  or  $-2$

g. 
$$\begin{aligned}3n^2 - 15n + 18 &= 0 && \text{Standard form} \\3n^2 - 6n - 9n + 18 &= 0 && \text{Factorise} \\3n(n - 2) - 9(n - 2) &= 0 \\(n - 2)(3n - 9) &= 0\end{aligned}$$

Therefore, either:

$$\begin{aligned}n - 2 = 0 \quad \text{or} \quad 3n - 9 = 0 && \text{Zero product property} \\n = 0 + 2 &\quad 3n = 9 && \text{Solve for } n \\n = 2 &\quad n = \frac{9}{3} \\n = 2 &\quad n = 3\end{aligned}$$

**Answer:**  $n = 2$  or  $3$

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L109 in the Pupil Handbook.

<b>Lesson Title:</b> Finding a quadratic equation with given roots	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L110	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to form a quadratic equation given its roots.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board: Solve  $y^2 + 11y + 30 = 0$
2. Allow 1 minute for pupils to solve the problem.
3. Invite a volunteer to write the solution on the board and explain.

#### Solution:

$$\begin{aligned}
 y^2 + 11y + 30 &= 0 \\
 y^2 + 5y + 6y + 30 &= 0 \\
 y(y + 5) + 6(y + 5) &= 0 \\
 (y + 5)(y + 6) &= 0
 \end{aligned}$$

Either  $y + 5 = 0$  or  $y + 6 = 0$

$$y = -5 \text{ or } y = -6$$

4. Explain to pupils that today's lesson is on forming a quadratic equation given its roots.

### Teaching and Learning (22 minutes)

1. Explain:
  - The values of the unknown which satisfy a given quadratic equation are called the **roots** of the equation.
  - For the example on the board, the roots of the quadratic equation are  $y = -5$  and  $-6$ .
2. Write on the board:  $x^2 + bx + c = x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$
3. Explain:
  - We have factorised an equation to find its roots. Now we are going to work backwards. Given the roots, we will write the equation.
  - Recall that for a quadratic equation in standard form, the sum of the roots gives  $b$ , and the product of the roots gives  $c$ .
4. Write on the board: Find the quadratic equation that has the roots 2, 4.
5. Explain the solution on the board for pupils.

#### Solution:

$$\text{Sum of the roots} = 2 + 4 = 6$$

$$\text{Product of the roots} = 2 \times 4 = 8$$

$$\begin{aligned}\text{Equation} \Rightarrow \quad & x^2 - (6)x + (8) = 0 \\ & = x^2 - 6x + 8 = 0\end{aligned}$$

6. Write the following problem on the board: Find the quadratic equation that has the roots  $-2, -3$
7. Ask a volunteer to find the sum of roots. (Answer:  $-2 + (-3) = -2 - 3 = -5$ )
8. Ask another volunteer to find the product of roots. (Answer:  $-2(-3) = 6$ )
9. Ask another volunteer to substitute the values of the sum and product to find the equation.

**Answer:**  $x^2 - (-5)x + (6) = 0$   
 $x^2 - 5x + 6 = 0$

10. Write another problem on the board: The roots of a quadratic equation are  $\frac{2}{3}$  and  $-2$ . What is the general form of the equation?
11. Ask a volunteer to find the sum of the roots. (Answer:  $\frac{2}{3} + (-2) = \frac{2}{3} - \frac{2}{1} = \frac{2-6}{3} = \frac{-4}{3}$ )
12. Ask another volunteer to find the product of the roots. (Answer:  $\frac{2}{3} \times (-2) = \frac{-4}{3}$ )
13. Write the solution on the board and simplify to standard form.

**Solution:**

$$\begin{aligned}x^2 - \left(\frac{-4}{3}\right)x + \left(\frac{-4}{3}\right) &= 0 \\ x^2 + \frac{4}{3}x - \frac{4}{3} &= 0\end{aligned}$$

Multiply throughout by the LCM, 3

$$\begin{aligned}3(x^2) + 3\left(\frac{4}{3}\right)x - 3\left(\frac{4}{3}\right) &= 3(0) \\ 3x^2 + 4x - 4 &= 0\end{aligned}$$

14. Write the following two problems on the board: Find the quadratic equations that have these roots:

a.  $-7, 4$       b.  $\frac{-2}{3}, \frac{-1}{4}$

26. Ask pupils to work with seatmates.
27. Walk around to check for understanding and clear misconceptions.
28. Encourage pupils to check their answers.
29. Invite a volunteer to write the answers on the board.

**Answers:**

a.	Sum of roots	$-7 + 4 = -3$
	Product of roots	$-7 \times 4 = -28$
	Equation $\Rightarrow$	$x^2 - (-3)x + (-28) = 0$
		$x^2 + 3x - 28 = 0$
b.	Sum of roots	$\frac{-2}{3} + \left(\frac{-1}{4}\right) = \frac{-2}{3} - \frac{1}{4} = \frac{-11}{12}$
	Product of roots	$\frac{-2}{3} \times \left(\frac{-1}{4}\right) = \frac{2}{12} = \frac{1}{6}$
	Equation $\Rightarrow$	$x^2 - \left(\frac{-11}{12}\right)x + \left(\frac{1}{6}\right) = 0$

$$x^2 + \frac{11}{12}x + \frac{1}{6} = 0$$

Multiply throughout by LCM, 12

$$12(x^2) + 12\left(\frac{11}{12}\right)x + 12\left(\frac{1}{6}\right) = 12(0)$$

$$12x^2 + 11x + 2 = 0$$

### **Practice (14 minutes)**

1. Write the following two problems on the board: Find the quadratic equation for each set of roots:
  - a.  $\frac{1}{2}$  and  $\frac{3}{4}$
  - b.  $1\frac{2}{3}$  and  $-1\frac{2}{3}$
2. Ask pupils to work independently to answer the questions.
3. Walk around, if possible, to check for understanding and to clear misconceptions.
4. Have pupils volunteer to give their answers to the questions.
5. Ask pupils to exchange their exercise books. Encourage them to check the answers.

#### **Answers:**

a. Sum of the roots  $\frac{1}{2} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$

Product of the roots  $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Equation  $\Rightarrow x^2 - \left(\frac{5}{4}\right)x + \left(\frac{3}{8}\right) = 0$

$$x^2 - \frac{5}{4}x + \frac{3}{8} = 0$$

Multiply throughout by the LCM, 8

$$8(x^2) - 8\left(\frac{5}{4}\right)x + 8\left(\frac{3}{8}\right) = 8(0)$$

$$8x^2 - 10x + 3 = 0$$

b. Sum of the roots  $1\frac{2}{3} + \left(-1\frac{2}{3}\right) = \frac{5}{3} - \frac{5}{3} = 0$

Product of the roots  $1\frac{2}{3} \times \left(-1\frac{2}{3}\right) = \frac{5}{3} \left(-\frac{5}{3}\right) = \frac{-25}{9}$

Equation  $\Rightarrow x^2 - (0)x + \left(\frac{-25}{9}\right) = 0$

$$x^2 - \frac{25}{9} = 0$$

Multiply throughout by LCM, 9

$$9(x^2) - 9\left(\frac{25}{9}\right) = 9(0)$$

$$9x^2 - 25 = 0$$

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L110 in the Pupil Handbook

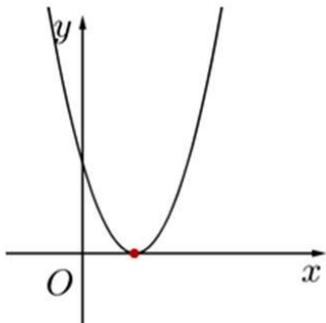
<b>Lesson Title:</b> Graphical solution of quadratic equations	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L111	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to use graphical methods to solve quadratic equations.	 <b>Preparation</b> None	

### Opening (3 minutes)

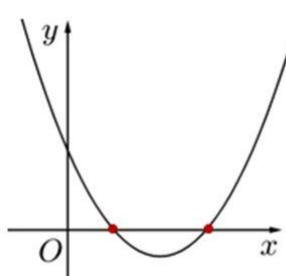
1. Tell pupils that they will be revising quadratic equations today.
2. Ask pupils to explain what quadratic equations are in their own words and allow them to discuss. (Example answers: They are equations with  $x^2$ . When graphed they make parabolas. They are equations that can be solved graphically.)
3. Explain to pupils that today's lesson is to use graphical method in solving quadratic equations.

### Teaching and Learning (22 minutes)

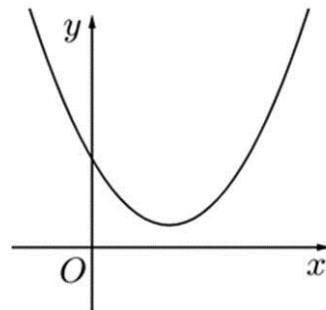
1. Explain: The graphical method only gives an estimated solution(s).
2. Sketch three different quadratic graphs on the board.



(i) One Solution



(ii) Two Solutions



(iii) No Solutions

3. Discuss:
  - The  $x$ -intercepts of a graph are the solutions to the equation.
  - In diagram (i), there is only one solution because the graph touches the  $x$ -axis at one point.
  - In diagram (ii), there are two solutions because the graph cuts the  $x$ -axis at two points.
  - In diagram (iii), there is no real solution because the graph does not intersect with the  $x$ -axis.
  - Thus, a quadratic equation can have 0, 1 or 2 solutions.
4. Write the following problem on the board: Solve the equation  $x^2 + 4x + 3 = 0$  using the graphical method.

5. Draw the table of values on the board.

$x$	-4	-3	-2	-1	0
$y$					

and a Cartesian plane

6. Explain:

- The equation  $x^2 + 4x + 3 = 0$  can be solved by graphing the corresponding function.
- Replace 0 with  $y$  or  $f(x)$ , and find points that satisfy the function.

7. Find one ordered pair and write it in the table of values:

$$\begin{aligned} y &= x^2 + 4x + 3 \\ &= (-4)^2 + 4(-4) + 3 \\ &= 16 - 16 + 3 \\ &= 3 \end{aligned}$$

8. Ask pupils to work with seatmates to find the other  $y$ -values and complete the table. As they finish, invite volunteers write the answers on the board:

$$\begin{aligned} y &= (-3)^2 + 4(-3) + 3 \\ &= 9 - 12 + 3 \\ &= 0 \end{aligned}$$

$x$	-4	-3	-2	-1	0
$y$	3	0	-1	0	3

$$\begin{aligned} y &= (-2)^2 + 4(-2) + 3 \\ &= 4 - 8 + 3 \\ &= -1 \end{aligned} \qquad \begin{aligned} y &= (0)^2 + 4(0) + 3 \\ &= 0 - 0 + 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} y &= (-1)^2 + 4(-1) + 3 \\ &= 1 - 4 + 3 \\ &= 0 \end{aligned}$$

9. Invite volunteers to come to the board and graph each of the 5 points from the table in the Cartesian plane.

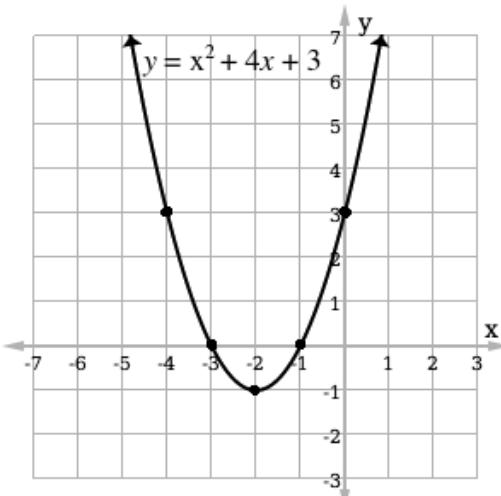
10. Connect the 5 points to make a smooth parabola.

11. Ask pupils to identify the solutions to the quadratic equation. If needed, remind them that the  $x$ -intercepts gives the solutions.  
(Answer:  $x = -3$  and  $x = -1$ ).

12. Tell pupils that the 2 solutions for the equation  $x^2 + 4x + 3$  are  $x = -3$  and  $x = -1$ .

13. Write another problem on the board:

- Using the given table of values, graph the quadratic function  $y = x^2 - x - 2$ .



$x$	-2	-1	0	1	2	3
$y$						

b. Based on your graph, what are the roots of the equation  $x^2 - x - 2 = 0$ ?

14. Ask pupils to work individually or with seatmates.

15. Walk around, if possible, to check for understanding and clear misconceptions.

16. Ask pupils to compare answers with seatmates and check their work. If time allows, invite volunteers to write the solution on the board.

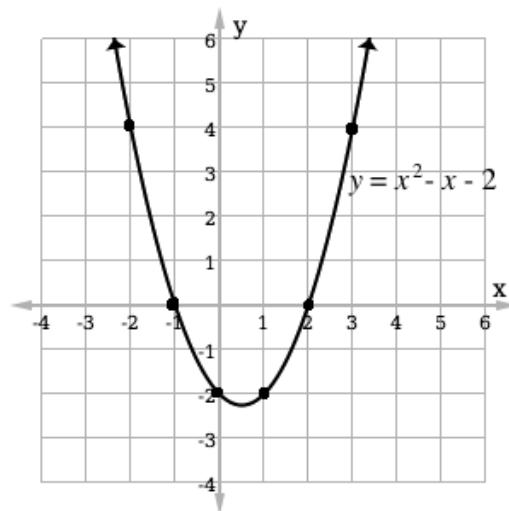
### Solutions:

a. Table of values and graph:

$$\begin{aligned} y &= (-2)^2 - (-2) - 2 \\ &= 4 + 2 - 2 \\ &= 4 \end{aligned}$$

$x$	-2	-1	0	1	2	3
$y$	4	0	-2	-2	0	4

$$\begin{aligned} y &= (-1)^2 - (-1) - 2 \\ &= 1 + 1 - 2 \\ &= 0 \\ y &= (0)^2 - (0) - 2 \\ &= 0 - 0 - 2 \\ &= -2 \\ y &= (1)^2 - (1) - 2 \\ &= 1 - 1 - 2 \\ &= -2 \end{aligned}$$



$$\begin{aligned} y &= (2)^2 - (2) - 2 & y &= (3)^2 - (3) - 2 \\ &= 4 - 2 - 2 & &= 9 - 3 - 2 \\ &= 0 & &= 4 \end{aligned}$$

b. The roots are  $x = -1$  and  $x = 2$  (the  $x$ -intercepts)

### Practice (14 minutes)

1. Write the following two problems on the board:

- a. Complete the table of values and solve the equation  $x^2 + x - 6 = 0$  using the graphical method.

$x$	-4	-3	-2	-1	0	1	2	3
$y$	0				-6		0	6

- b. Solve the equation of  $4 + 5x - 2x^2 = 0$  using the graphical method. Create and complete a table of values for  $-1 \leq x \leq 4$ .

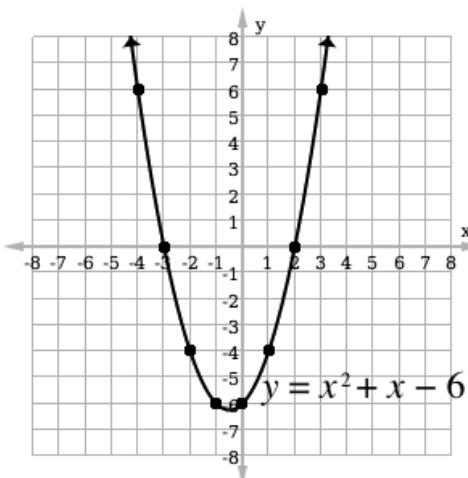
- Ask pupils to solve the problems independently.
- Walk around to check for understanding and clear misconceptions.
- Invite two volunteers, one at a time, to write the answers on the board.

**Answers:**

a Table of values:

x	-4	-3	-2	-1	0	1	2	3
y	6	0	-4	-6	-6	-4	0	6

Graph:

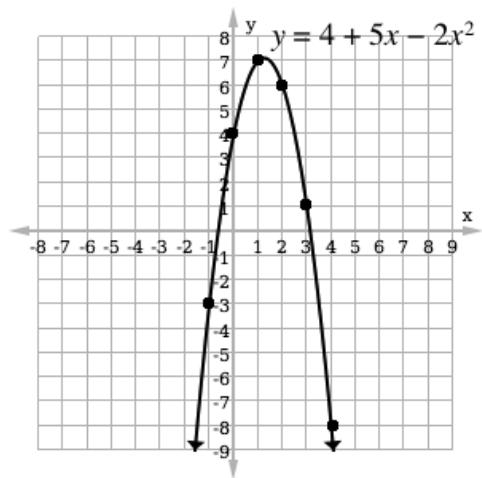


**Solutions:**  $x = -3, 2$

b Table of values:

x	-1	0	1	2	3	4
y	-3	4	7	6	1	-8

Graph:



**Solutions:**

approximately  $x = -0.7, 3.2$   
(accept reasonable estimates)

**Closing (1 minute)**

- For homework, have pupils do the practice activity PHM1-L111 in the Pupil Handbook.

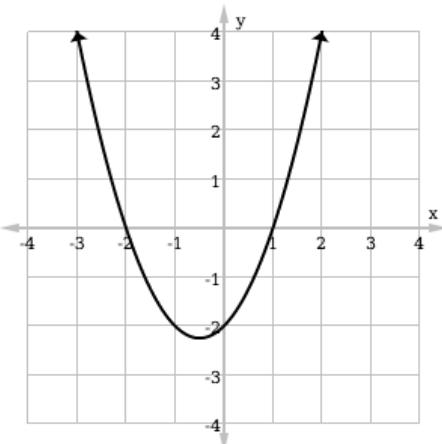
<b>Lesson Title:</b> Finding an equation from a given graph	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L112	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to form a quadratic equation from a given graph.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

1. Write on the board: Graph the quadratic function  $y = x^2 + 2x - 3$  for the interval  $-3 \leq x \leq 1$ . Use it to answer the following questions:
  - a. What is the minimum value of  $y = x^2 + 2x - 3$ ?
  - b. What is the solution set of the equation  $x^2 + 2x - 3 = 0$ ?
  - c. What is the equation of the line of symmetry?
2. Discuss:
  - a. How would you graph this equation? (Example answer: Find the  $y$ -values for  $-3 \leq x \leq 1$  and plot them on the Cartesian plane.)
  - b. What is the “solution set” of a quadratic equation? (Answer: The roots of the equation; where the parabola intersects with the  $x$ -axis.)
  - c. What is a line of symmetry? (Answer: It is the vertical line about which the parabola is symmetric.)
3. Explain that this lesson is on forming a quadratic equation from a given graph.

### Teaching and Learning (22 minutes)

1. Remind pupils that any quadratic equation can be written as a function in the form  $y = ax^2 + bx + c$ .
2. Explain:
  - To find the equation, we first need to find the roots from the graph. We need to estimate these points.
  - We will use the roots to find the equation.
3. Draw a graph on the board:

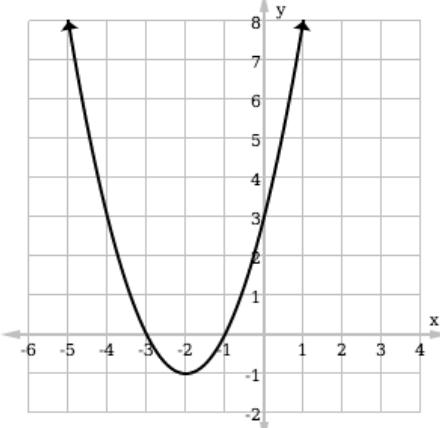


4. Ask volunteers to identify the roots of the graphed function. (Answer: The roots of the quadratic are  $x = -2$  and  $x = 1$ , the  $x$ -intercepts.)
5. Write on the board:  $f(x) = (x + 2)(x - 1)$
6. Explain:
  - This is the function that has  $x$ -intercepts at  $-2$  and  $1$ .
  - If we set this function equal to  $0$  and solve, we'll get our 2 intersection points,  $-2$  and  $1$ .
7. Ask pupils to work with seatmates to expand the bracket and write the function in standard form.
8. Invite a volunteer to expand the bracket on the board.

**Solution:**

$$\begin{aligned}
 f(x) &= (x + 2)(x - 1) \\
 &= x(x - 1) + 2(x - 1) \\
 &= x^2 - x + 2x - 2 \\
 &= x^2 + x - 2
 \end{aligned}$$

9. Explain:  $f(x) = x^2 + x - 2$  is the quadratic function which passes through the  $x$ -axis at the required points.
10. Write the following problem on the board: Find the quadratic function whose graph is shown below.



11. Ask a volunteer to give the values of  $x$  where the graph cuts the  $x$ -axis. (Answer:  $x = -3, -1$ )

12. Invite a volunteer to write the values in a bracket form. (Answer:  $(x + 3)(x + 1)$ )

13. Invite another volunteer to expand the bracket.

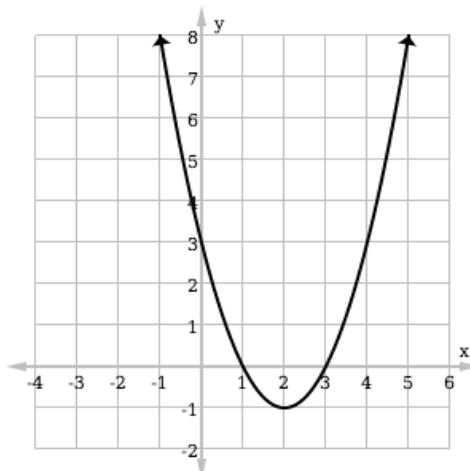
**Solution:**

$$\begin{aligned}f(x) &= (x + 3)(x + 1) \\&= x(x + 3) + 1(x + 3) \\&= x^2 + 3x + x + 3 \\&= x^2 + 4x + 3\end{aligned}$$

14. Invite another volunteer to label the parabola on the board with its quadratic function.

(Answer:  $f(x) = x^2 + 4x + 3$ )

15. Write another problem on the board: Find the quadratic function  $f$  whose graph is shown below.



16. Ask pupils to solve the problem with seatmates.

17. Walk around, if possible, to check for understanding and clear misconceptions.

18. Invite a volunteer to write the solution on the board.

**Solution:**

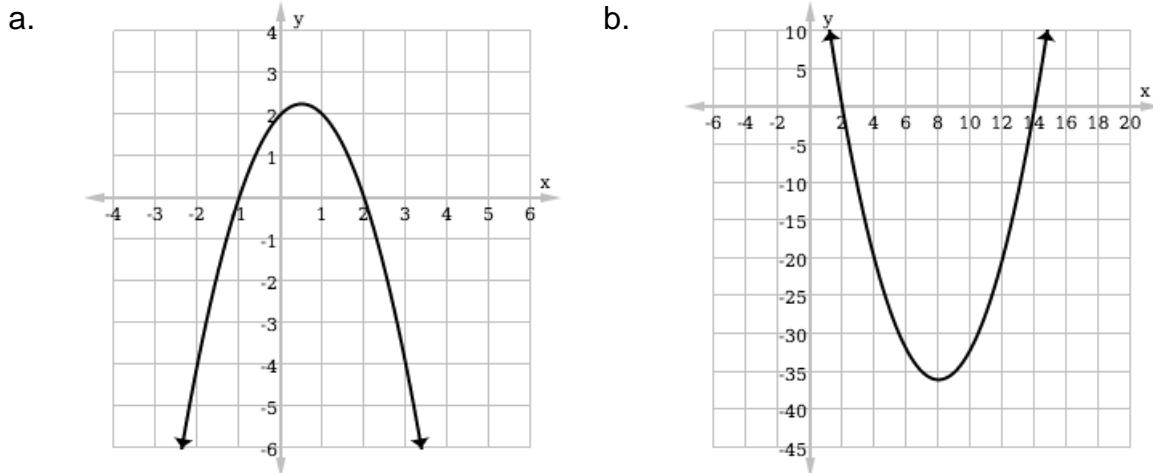
Roots:  $x = 1$  and  $3$

Function:

$$\begin{aligned}f(x) &= (x - 1)(x - 3) \\&= x(x - 3) - 1(x - 3) \\&= x^2 - 3x - x + 3 \\&= x^2 - 4x + 3\end{aligned}$$

### Practice (14 minutes)

1. Write the following problem on the board: Find the equation for the curve shown in each of the graphs below:



2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow the pupils to exchange their exercise books.
5. Invite any two volunteers, one at a time, to write the solutions on the board.

**Solutions:**

a. Roots:  $x = -1, 2$

Equation:

$$\begin{aligned}
 f(x) &= (x + 1)(x - 2) \\
 &= x(x - 2) + 1(x - 2) \\
 &= x^2 - 2x + x - 2 \\
 &= x^2 - x - 2
 \end{aligned}$$

b. Roots:  $x = 2, 14$

Equation:

$$\begin{aligned}
 f(x) &= (x + 2)(x + 14) \\
 &= x(x + 14) + 2(x + 14) \\
 &= x^2 + 14x + 2x + 28 \\
 &= x^2 + 16x + 28
 \end{aligned}$$

**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L112 in the Pupil Handbook.

<b>Lesson Title:</b> Completing the square and perfect squares	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L113	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to solve quadratic equations by using perfect squares and completing the square.	 <b>Preparation</b> Write the Opening questions on the board.	

### Opening (3 minutes)

1. Write the problems on the board: Expand: a.  $(x - 3)^2$       b.  $(x + 2)^2$
2. Allow 2 minutes for pupils to answer the question.
3. Invite volunteers to write the answers on the board.  
(Answer: a.  $(x - 3)^2 = x^2 - 6x + 9$ ; b.  $(x + 2)^2 = x^2 + 4x + 4$ ).
4. Explain to the pupils that today's lesson is on solving quadratic equation by using perfect squares and completing the square.

### Teaching and Learning (22 minutes)

1. Write on the board: Factorise  $x^2 + 4x + 2$
2. Ask pupils to discuss the problem with seatmates.
3. Ask any volunteer to say something about what they observe in factoring the expression. Acknowledge any answer given.
4. Explain:
  - We cannot use the method we learnt to factorise  $x^2 + 4x + 2$
  - We have to use process called "completing the square" to solve this problem.
5. Discuss: We started the lesson by expanding expressions such as  $(x + 2)^2$ . This expression is called a perfect square because it is a square of a binomial. It is easy to expand and easy to factorise.
6. Write on the board: Compare  $x^2 + 4x + 2$  with the perfect square  $(x + 2)^2 = x^2 + 4x + 4$ .
7. Ask any volunteer to tell the class why we choose  $(x + 2)^2$  and not any other perfect square?
8. Guide the pupils to say they have the same  $x$  term, that is,  $4x$ .
9. Show the 2 quadratics side by side on the board so pupils can see how the quadratic is factorised:

$x^2 + 4x + 4$ $x^2 + 4x + 4$ $(x + 2)^2$		$x^2 + 4x + 2$ $x^2 + 4x + 4 - 2$ $(x + 2)^2 - 2$
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10. Explain:

- We rewrite the given quadratic as a sum of a perfect square and a number.  
We work out the number as shown on the board. Since  $4 - 2 = 2$ , our quadratic remains unchanged.
- We are able to rewrite our quadratic in terms of a perfect square.

11. Discuss: There is a procedure we use called **completing the square**. This changes the quadratic to the sum of a perfect square and a number.

12. Write on the board: Let us use  $(x + m)^2 = x^2 + 2mx + m^2$ , where  $x^2 + 2mx + m^2$  is a perfect square trinomial.

13. Write a problem on the board: Find the roots of  $x^2 + 4x + 1 = 0$  using completing square method.

14. Invite a volunteer to write down the values of  $a, b$  and  $c$  in this quadratic.

(Answer:  $a = 1, b = 4, c = 1$ )

15. Explain: We can write our quadratic as a perfect square and a number.

16. Write on the board:

$$x^2 + 4x + 1 = (x + m)^2 + n \quad x^2 + 4x + 1 = x^2 + 2mx + m^2 + n$$

17. Explain: We need to find the values of  $m$  and  $n$  to complete the square.

18. Show pupils that in the second line, the value of  $b$  in the quadratic equation is 4 on the left-hand side, and  $2m$  on the right-hand side.

19. Solve for  $m$  on the board:

$$\begin{aligned} 4 &= 2m \\ \frac{4}{2} &= \frac{2m}{2} \\ 2 &= m \end{aligned}$$

20. Show pupils that the value of  $c$  in the quadratic equation is 1 on the left-hand side, and  $m^2 + n$  on the right-hand side.

21. Solve for  $n$  on the board:

$$\begin{aligned} 1 &= m^2 + n \\ 1 &= 2^2 + n \\ 1 &= 4 + n \\ 1 - 4 &= n \\ -3 &= n \end{aligned}$$

22. Substitute the values of  $m$  and  $n$  on the board, and find the perfect square:

$$\begin{aligned} x^2 + 4x + 1 &= x^2 + 2mx + m^2 + n \\ &= x^2 + 2(2)x + (2)^2 - 3 \\ &= x^2 + 4x + 4 - 3 \\ &= (x + 2)^2 - 3 \end{aligned}$$

23. Solve the perfect square to find the roots of the equation on the board:

$$\begin{aligned} (x + 2)^2 - 3 &= 0 \\ (x + 2)^2 &= 3 \\ \sqrt{(x + 2)^2} &= \sqrt{3} \end{aligned}$$

$$\begin{aligned}x + 2 &= \pm\sqrt{3} \\x &= -2 \pm \sqrt{3}\end{aligned}$$

Therefore, the roots of the equation are  $-2 - \sqrt{3}$  and  $-2 + \sqrt{3}$ .

24. Write another problem on the board: Find the roots of  $x^2 + 6x + 1 = 0$  by completing the square. Use the equations  $m = \frac{b}{2}$  and  $n = c - m^2$ .

25. Ask any volunteer to find the values of  $a$ ,  $b$  and  $c$ . (Answer:  $a = 1$ ,  $b = 6$  and  $c = 1$ )

26. Ask pupils to work with seatmates to find the values of  $m$  and  $n$ .

27. Invite a volunteer to find the value of  $m$  on the board. (Answer:  $m = \frac{b}{2} = \frac{6}{2} = 3$ )

28. Invite a volunteer to find the value of  $n$  on the board. (Answer:  $n = c - m^2 = 1 - 3^2 = 1 - 9 = -8$ )

29. Ask pupils to work with seatmates to solve the problem.

30. Walk around to check for understanding and clear misconceptions.

31. Invite a volunteer to write the quadratic with a perfect square on the board:

$$\begin{aligned}x^2 + 6x + 1 &= x^2 + 2mx + m^2 + n \\&= x^2 + 2(3)x + (3)^2 + (-8) \\&= x^2 + 6x + 9 - 8 \\&= (x + 3)^2 - 8\end{aligned}$$

32. Invite another volunteer to find the roots of the equations:

$$\begin{aligned}(x + 3)^2 - 8 &= 0 \\(x + 3)^2 &= 8 \\\sqrt{(x + 3)^2} &= \sqrt{8} \\x + 3 &= \pm 2\sqrt{2} \\x &= -3 \pm 2\sqrt{2}\end{aligned}$$

The roots of the equation are  $-3 + 2\sqrt{2}$  and  $-3 - 2\sqrt{2}$ .

### Practice (14 minutes)

- Write two problems on the board: Find the roots of these equations by completing the square. Use the equation  $m = \frac{b}{2}$  and  $n = c - m^2$ 
  - $x^2 - 6x + 8 = 0$
  - $3x^2 + 2x - 4 = 0$
- Ask pupils to solve the problems independently in their exercise books.
- Walk around to check their answers and clear misconceptions.
- Encourage them to check the answers.
- Invite two volunteers to write their answers on the board.

#### Answers:

- a. Note that in  $x^2 - 6x + 8 = 0$ ,  $a = 1$ ,  $b = -6$  and  $c = 8$ .

Find the values of  $m$  and  $n$ :

$$\begin{aligned}m &= \frac{b}{2} = \frac{-6}{2} = -3 \\n &= c - m^2 = 8 - (-3)^2 = 8 - 9 = -1\end{aligned}$$

Write the quadratic equation with a perfect square:

$$\begin{aligned}x^2 - 6x + 8 &= x^2 + 2(-3)x + (-3)^2 + (-1) \\&= x^2 - 6x + 9 - 1 \\&= (x - 3)^2 - 1\end{aligned}$$

Find the roots:

$$\begin{aligned}(x - 3)^2 - 1 &= 0 \\(x - 3)^2 &= 1 \\\sqrt{(x - 3)^2} &= \sqrt{1} \\x - 3 &= \pm 1 \\x &= 3 + 1 \text{ and } 3 - 1 \\x &= 4 \text{ and } 2\end{aligned}$$

b. Divide throughout by 3 to find a, b and c:

$$\begin{aligned}3x^2 + 2x - 4 &= 0 \\x^2 + \frac{2}{3}x - \frac{4}{3} &= 0\end{aligned}$$

We have  $a = 1$ ,  $b = \frac{2}{3}$  and  $c = \frac{-4}{3}$

Find the values of m and n:

$$\begin{aligned}m &= \frac{b}{2} = \frac{2}{3} \div \frac{2}{1} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} \\n &= c - m^2 = \frac{-4}{3} - \left(\frac{1}{3}\right)^2 = \frac{-4}{3} - \frac{1}{9} = \frac{-13}{9}\end{aligned}$$

Write the quadratic equation with a perfect square:

$$\begin{aligned}x^2 - 6x + 8 &= x^2 + 2\left(\frac{1}{3}\right)x + \left(\frac{1}{3}\right)^2 + \left(\frac{-13}{9}\right) \\&= x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{13}{9} \\&= \left(x + \frac{1}{3}\right)^2 - \frac{13}{9}\end{aligned}$$

Find the roots:

$$\begin{aligned}\left(x + \frac{1}{3}\right)^2 - \frac{13}{9} &= 0 \\\left(x + \frac{1}{3}\right)^2 &= \frac{13}{9} \\\sqrt{\left(x + \frac{1}{3}\right)^2} &= \sqrt{\frac{13}{9}} \\x + \frac{1}{3} &= \pm \frac{\sqrt{13}}{3} \\x &= -\frac{1}{3} \pm \frac{\sqrt{13}}{3}\end{aligned}$$

Therefore, the roots of the equation are  $\frac{-1+\sqrt{13}}{3}$  and  $\frac{-1-\sqrt{13}}{3}$ .

### Closing (1 minute)

- For homework, have pupils do the practice activity PHM1-L113 in the Pupil Handbook.

<b>Lesson Title:</b> The quadratic formula	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L114	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to solve quadratic equations using the quadratic formula.	 <b>Preparation</b> Write the opening question on the board.	

### Opening (5 minutes)

1. Write the following problem on the board: Find the roots of the equation  $x^2 + 6x + 5 = 0$  by completing the square.
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board.

#### Solution:

$$\begin{aligned}
 x^2 + 6x + 5 &= 0 \\
 a = 1, \quad b = 6 \quad c = 5 \\
 \text{Therefore } m &= \frac{6}{2} = 3; \quad n = 5 - 3^2 = -4 \\
 x^2 + 2(3)x + (3)^2 + (-4) \\
 &\quad x^2 + 6x + 9 - 4 \\
 &\quad (x + 3)^2 - 4 = 0 \\
 \sqrt{(x + 3)^2} &= \sqrt{4} \\
 x + 3 &= \pm 2 \\
 x &= -3 \pm 2
 \end{aligned}$$

Therefore, the roots are  $-3 + 2 = -1$  and  $-3 - 2 = -5$

4. Explain to the pupils that today's lesson is on solving quadratic equation using the quadratic formula.

### Teaching and Learning (20 minutes)

1. Explain:
  - In addition to factoring and completing the square, there is another tool you can use to find the roots of a quadratic equation.
  - This other tool is called the quadratic formula.
2. Write on the board: Quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. Explain:
  - This formula is derived from the general form of quadratic equation  $ax^2 + bx + c = 0$  using the completing square method.
  - In solving problems using the formula, you have to write down the value of  $a, b$  and  $c$  and substitute those values to find the roots of the equation.

- Write on the board: Use the quadratic formula to solve the equation  $x^2 + 11x + 30 = 0$
- Ask a volunteer to determine the values of  $a, b$  and  $c$ . (Answer:  $a = 1, b = 11, c = 30$ )
- Write the solution on the board and explain.

**Solution:**

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Formula} \\
 &= \frac{-11 \pm \sqrt{(11)^2 - 4(1)(30)}}{2(1)} && \text{Substitute } a, b, \text{ and } c \\
 &= \frac{-11 \pm \sqrt{121 - 120}}{2} && \text{Simplify} \\
 &= \frac{-11 \pm \sqrt{1}}{2} && \\
 &= \frac{-11+1}{2} \text{ or } \frac{-11-1}{2} && \text{Note: } \sqrt{1} = 1 \text{ or } \sqrt{1} = -1 \\
 &= \frac{-10}{2} \text{ or } \frac{-12}{2} \\
 x &= -5 \text{ or } -6
 \end{aligned}$$

Therefore, the roots are  $-5$  and  $-6$ .

- Write another problem on the board: Use the quadratic formula to solve the equation  $x^2 + 8x + 16 = 0$ .
- Ask a volunteer to determine the values of  $a, b$  and  $c$ . (Answer:  $a = 1, b = 8, c = 16$ )
- Ask a volunteer from the class to substitute the values of  $a, b$  and  $c$  into the formula.

**Answer:**

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-8 \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}
 \end{aligned}$$

10. Ask pupils to work with seatmates to simplify the problem and find the roots.

11. Invite a volunteer to complete the solution on the board.

$$\begin{aligned}
 x &= \frac{-8 \pm \sqrt{64 - 64}}{2} \\
 &= \frac{-8 \pm \sqrt{0}}{2} \\
 &= \frac{-8 \pm 0}{2} \\
 &= \frac{-8+0}{2} \text{ or } \frac{-8-0}{2} \\
 &= \frac{-8}{2} \text{ or } \frac{-8}{2}
 \end{aligned}$$

$$x = -4 \text{ or } -4$$

Therefore, there is only one root to the equation,  $-4$  twice.

- Write a problem on the board: Use the quadratic formula to solve the equation  $5y^2 - 8y + 3 = 0$ .
- Ask pupils to solve the problem with seatmates.

14. Walk around, if possible, to check for understanding and clear misconceptions.
15. Allow pupils to discuss with others.
16. Invite a volunteer to write the solution on the board.

**Solution:**

Note that  $a = 5$ ,  $b = -8$  and  $c = 3$

$$\begin{aligned}
 y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(3)}}{2(5)} \\
 &= \frac{8 \pm \sqrt{64 - 60}}{10} \\
 &= \frac{8 \pm \sqrt{4}}{10} \\
 &= \frac{8 \pm 2}{10} \\
 &= \frac{8+2}{10} \text{ or } \frac{8-2}{10} \\
 &= \frac{-10}{10} \text{ or } \frac{6}{10} \\
 &= 1 \text{ or } \frac{3}{5}
 \end{aligned}$$

**Practice (14 minutes)**

1. Write two problems on the board: Use the quadratic formula to solve the following equations:
  - a.  $3x^2 + 2x - 7 = 0$
  - b.  $x^2 - 8x + 15 = 0$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Ask two volunteers to write the solutions on the board.

**Solutions:**

- a. Note that  $a = 3$ ,  $b = 2$  and  $c = -7$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-7)}}{2(3)} \\
 &= \frac{-2 \pm \sqrt{4+84}}{6} \\
 &= \frac{-2 \pm \sqrt{88}}{6} \\
 &= \frac{-2 \pm 9.38}{6} \\
 &= \frac{-2+9.38}{6} \text{ or } \frac{-2-9.38}{6} \\
 &= \frac{7.38}{6} \text{ or } \frac{-11.38}{6}
 \end{aligned}$$

$$= 1.23 \text{ or } -1.90$$

b. Note that  $a = 1$ ,  $b = -8$ , and  $c = 15$ .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)} \\&= \frac{8 \pm \sqrt{64 - 60}}{2} \\&= \frac{8 \pm \sqrt{4}}{2} \\&= \frac{8 \pm 2}{2} \\&= \frac{8+2}{2} \text{ or } \frac{8-2}{2} \\&= \frac{10}{2} \text{ or } \frac{6}{2} \\&= 5 \text{ or } 3\end{aligned}$$

### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L114 in the Pupil Handbook.

<b>Lesson Title:</b> Word problems leading to quadratic equations	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L115	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to form and solve word problems by forming and solving suitable quadratic equations.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write on the board: Write an expression for the following: A certain number minus fifteen is equal to nine times the number.
2. Explain that pupils should use a variable to represent the unknown number.
3. Ask pupils to write the expression with seatmates.
4. Invite a volunteer to write their expression on the board. If other pupils wrote something else, have them write it on the board too. Encourage all pupils, even if their expressions are incorrect.
5. Discuss the expressions as a class before determining the correct expression.  
(Answer:  $y - 15 = 9y$ )
6. Explain to the pupils that today's lesson is on solving quadratic equations from word problems.

### Teaching and Learning (22 minutes)

1. Explain: There are many types of word problems which involve relations among known and unknown numbers. These can lead to quadratic equations.
2. Explain the steps for solving quadratic equations from word problems. Ask pupils to look at this section in their Pupil Handbook; it is not necessary to write the steps on the board:
  - Read the problem carefully and note what is given and what is required.
  - Assign a variable to represent each unknown.
  - Identify any value that is multiplied by the variable (the coefficient).
  - Identify any value that is constant.
  - Write the quadratic expression representing the situation.
  - Solve the equation for the unknown variable.
  - Check whether the answer satisfies the conditions of the problem.
3. Write the problem on the board: The sum of two numbers is 12 and their product is 35. What are the two numbers?
4. Ask a volunteer to give the unknown in this problem. (Answer: There are 2 unknowns; the two numbers)

- Ask a volunteer to choose a variable to represent these unknowns. (Example answer:  $n$  and  $m$  for the two numbers)
- Invite volunteers to write down expressions from the given problem on the board.

**Answers:**

i.  $n + m = 12$       ii.  $n \times m = 35$

- Solve the problem on the board using substitution:

Solve equation (i) for  $n$ :  $n = 12 - m$

Substitute  $n$  into equation (ii) and evaluate:

$$(12 - m) \times m = 35 \quad \text{Substitute } n = 12 - m \text{ into (ii)}$$

$$12m - m^2 = 35 \quad \text{Removing bracket}$$

$$m^2 - 12m + 35 = 0 \quad \text{Transpose } 12m - m^2$$

$$m^2 - 5m - 7m + 35 = 0 \quad \text{Factor}$$

$$m(m - 5) - 7(m - 5) = 0$$

$$(m - 5)(m - 7) = 0$$

Either  $m - 5 = 0$  or  $m - 7 = 0$

$$m = 0 + 5 \quad m = 0 + 7$$

$$m = 5 \quad \text{or} \quad m = 7$$

Therefore, the 2 numbers are 5 and 7.

- Write another problem on the board: Find two consecutive integers that have a product of 42.
- Ask volunteers to explain what “consecutive integers”.
- Allow pupils to share ideas, then explain. (Answer: Consecutive integers come one after another in regular order.)
- Explain:
  - Let  $x$  represent the first integer and  $x + 1$  represent the second integer.
- Write on the board:
 

First integer:  $x$   
 Second integer:  $x + 1$   
 Product of the two integers:  $x(x + 1) = 42$
- Ask pupils to work with seatmates to finish the problem by solving for  $x$ .
- Walk around to check for understanding and clear misconceptions.
- Invite a volunteer to write the solution on the board.

**Solution:**

$$x(x + 1) = 42$$

$$x^2 + x = 42$$

$$x^2 + x - 42 = 0$$

$$x^2 + 7x - 6x - 42 = 0$$

$$x(x + 7) - 6(x + 7) = 0$$

$$(x - 6)(x + 7) = 0$$

Either  $x - 6 = 0$  or  $x + 7 = 0$

$$x = 0 + 6 \quad x = 0 - 7$$

$$x = 6 \quad \text{or} \quad x = -7$$

16. Remind pupils that this has only given us the first integer,  $x$ . We must find the next consecutive integer for each.
17. Ask a volunteer to find the consecutive integer of  $x = 6$  by substituting it into  $x + 1$ . (Answer: When  $x = 6$ ,  $x + 1 = 6 + 1 = 7$ . Therefore the two consecutive numbers are 6 and 7).
18. Ask another volunteer to find the consecutive integer of  $x = -7$  by substituting it into  $x + 1$ . (Answer: when  $x = -7$ ,  $x + 1 = -7 + 1 = -6$ . Therefore, the other consecutive numbers are -7 and -6.)
19. Write the answers on the board: The numbers are 6 and 7 or -7 and -6.
20. Write another problem on the board: Foday is 4 times older than his son. Five years ago, the product of their ages was 175. Find their present ages.
21. Ask pupils to write the equation with seatmates (they should not solve yet).
22. Walk around to check for understanding and clear misconceptions.
23. Invite a volunteer to write the equation on the board and explain how they found it.

**Solution:**

Let the child's age be  $p$  years, then Foday's age is  $4p$  years.

5 years ago, the child's age was  $(p - 5)$  years and Foday's age was  $(4p - 5)$  years.  
The product of their ages was  $(p - 5)(4p - 5)$ .

24. Ask pupils to work with seatmates to solve the problem. Remind them to find both ages.
25. Walk around to check for understanding and clear misconceptions.
26. Invite a volunteer to solve the problem on the board.

**Solution:**

$$\begin{aligned}(p - 5)(4p - 5) &= 175 \\ 4p^2 - 5p - 20p + 25 &= 175 \\ 4p^2 - 25p + 25 &= 175 \\ 4p^2 - 25p + 25 - 175 &= 0 \\ 4p^2 - 25p - 150 &= 0 \\ (4p + 15)(p - 10) &= 0 \\ \text{Either } 4p + 15 = 0 \text{ or } p - 10 &= 0 \\ 4p = -15 \text{ or } p &= 0 + 10 \\ p = \frac{-15}{4} \text{ or } p &= 10\end{aligned}$$

The child's age is 10 years. Therefore, Foday's age is:  $4p = 4(10) = 40$  years

27. Discuss:  $\frac{-15}{4}$  is not sensible for an age because ages cannot be negative. Therefore, the child is 10 years old and Foday is  $4 \times 10 = 40$  years old.

28. Check the result on the board:

$$(p - 5)(4p - 5) = 175$$

$$(10 - 5)(40 - 5) = 175$$

$$5 \times 35 = 175 \quad True$$

### Practice (14 minutes)

1. Write two problems on the board:
  - a. The product of two consecutive integers is 72. What are the integers?
  - b. Abass is 7 years older than Juliet. The product of their ages is 18 years. How old is Abass? How old is Juliet?
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers, one at a time, to write their answer on the board.

#### Solution:

- a. Let the numbers be  $y$  and  $y + 1$

$$y(y + 1) = 72$$

$$y^2 + y = 72$$

$$y^2 + y - 72 = 0$$

$$y^2 + 9y - 8y - 72 = 0$$

$$y(y + 9) - 8(y + 9) = 0$$

$$(y + 9)y - 8 = 0$$

Either  $y + 9 = 0$  or  $y - 8 = 0$

$$y = -9 \text{ or } 8$$

If  $y = -9$ , the other integer is  $-9 + 1 = -8$

If  $y = 8$ , the other integer is  $8 + 1 = 9$

- b. Let Juliet's age =  $x$  years and Abass' age =  $(7 + x)$  years

$$\text{Product } x(7 + x) = 18$$

$$7x + x^2 = 18$$

$$x^2 + 7x - 18 = 0$$

$$x^2 + 9x - 2x - 18 = 0$$

$$x(x + 9) - 2(x + 9) = 0$$

$$(x + 9)(x - 2) = 0$$

$$x = -9 \text{ or } 2$$

Therefore, Juliet's age is 2 years and Abass' age is  $7 + 2 = 9$  years.

### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L115 in the Pupil Handbook.

<b>Lesson Title:</b> Practice of quadratic equations	<b>Theme:</b> Algebraic Processes	
<b>Lesson Number:</b> M1-L116	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to apply various methods in solving quadratic equations.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write a problem on the board: Solve the equation  $(2x - 9)(x + 8) = 0$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite a volunteer to write the solution on the board.

#### Solution:

$$\begin{aligned}
 (2x - 9)(x + 8) &= 0 \\
 2x - 9 = 0 \quad \text{or} \quad x + 8 &= 0 \\
 2x = 9 \quad \text{or} \quad x &= 0 - 8 \\
 x = \frac{9}{2} = 4\frac{1}{2} \quad \text{or} \quad &-8
 \end{aligned}$$

4. Explain that today's lesson is on using various methods in solving quadratic equations.

### Teaching and Learning (20 minutes)

1. Remind pupils that they know several methods of solving quadratic equations: factorisation, completing the square, the quadratic formula, and graphing.
2. Write a problem on the board: Solve the equation  $y^2 - 13y + 30 = 0$
3. Discuss:
  - Which method would you use? Why? (Example answer: It is easiest to factorise because we can find numbers  $p$  and  $q$  which sum to  $-13$  and multiply to give  $30$ .)
  - What information would you use to solve this problem? (Example answer: Since the question has a minus and plus signs, then the pair factors of  $30$  should both have negative signs).
4. Ask pupils to work with seatmates to solve the problem using **factorisation**.
5. Walk around to check for understanding and clear misconceptions.
6. Invite any volunteer to write the solution on the board.

#### Solution:

$$\begin{aligned}
 y^2 - 13y + 30 &= 0 \\
 y^2 - 3y - 10y + 30 &= 0 \\
 y(y - 3) - 10(y - 3) &= 0 \\
 (y - 3)(y - 10) &= 0
 \end{aligned}$$

$$\begin{aligned} \text{Either } y - 3 &= 0 \quad \text{or} \quad y - 10 = 0 \\ y &= 0 + 3 \qquad \quad y = 0 + 10 \\ y &= 3 \quad \text{or} \quad 10 \end{aligned}$$

7. Write a problem on the board: Solve the equation  $3x^2 - 5x - 3 = 0$ . Give the roots correct to 2 decimal places.
8. Discuss: What method would you use to solve this problem? Why? (Example answer: The quadratic formula; it is not easy to factor this equation, but the values of  $a, b$  and  $c$  can be easily identified.)
9. Ask pupils to work with seatmates to solve the problem using the **quadratic formula**.
10. Walk around to check for understanding and clear misconceptions.
11. Invite any volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned} 3x^2 - 5x - 3 &= 0 \\ a = 3, \quad b = -5, \quad c = -3 \\ x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-3)}}{2(3)} \\ x &= \frac{5 \pm \sqrt{25+36}}{6} \\ x &= \frac{5 \pm \sqrt{61}}{6} \\ x &= \frac{5 \pm 7.81}{6} \\ x &= \frac{5+7.81}{6} \quad \text{or} \quad \frac{5-7.81}{6} \\ x &= 2.14 \quad \text{or} \quad -0.47 \end{aligned}$$

12. Write another problem on the board. Find the equation whose sum and product of roots are respectively  $1.5, -1.2$ .
13. Discuss: How would you solve this problem? (Answer: We only need to substitute these values into the equation  $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$ .)
14. Ask pupils to work with seatmates to solve the problem.
15. Walk around to check for understanding and clear misconceptions.
16. Invite any volunteer to write the solution on the board.

**Solution:**

$$\text{sum of the roots} = 1.5$$

$$\text{product of roots} = -1.2$$

Substitute these values into the equation formula

$$\begin{aligned} 0 &= x^2 - (\text{sum of roots})x + \text{product of roots} \\ 0 &= x^2 - (1.5)x + (-1.2) \\ 0 &= x^2 - 1.5x - 1.2 \\ 0 &= x^2 - \frac{15}{10}x - \frac{12}{10} && \text{Convert to fraction} \\ 0 &= x^2 - \frac{3}{2}x - \frac{6}{5} && \text{Simplify to its lowest term} \end{aligned}$$

$$10(0) = 10(x^2) - 10\left(\frac{3}{2}\right)x - 10\left(\frac{6}{5}\right) \quad \text{Multiply throughout by the LCM, 10}$$

$$0 = 10x^2 - 15x - 12$$

17. Write another problem on the board: Use the method of completing the square to solve the equation  $v^2 + 9v + 19 = 0$
18. Discuss: How would you solve this problem? (Answer: we will compare our quadratic equation with the perfect square  $(x + m)^2$  by finding  $m = \frac{b}{2}$  and  $n = c - m^2$ ).
19. Ask pupils to work with seatmates to solve the problem.
20. Walk around to check for understanding and clear misconceptions.
21. Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned} v^2 + 9v + 19 &= 0 \\ a = 1, b = 9, c = 19 \\ m = \frac{b}{2} &= \frac{9}{2} \quad n = c - m^2 = 19 - \left(\frac{9}{2}\right)^2 = 19 - \frac{81}{4} = \frac{-5}{4} \\ v^2 + 9v + 19 &= v^2 + 2\left(\frac{9}{2}\right)v + \left(\frac{9}{2}\right)^2 + \left(\frac{-5}{4}\right) \\ &= v^2 + 9v + \frac{81}{4} - \frac{5}{4} \\ \left(v + \frac{9}{2}\right)^2 - \frac{5}{4} &= 0 \\ \left(v + \frac{9}{2}\right)^2 &= \frac{5}{4} \\ \sqrt{\left(v + \frac{9}{2}\right)^2} &= \sqrt{\frac{5}{4}} \\ v + \frac{9}{2} &= \pm \frac{\sqrt{5}}{2} \\ v &= -\frac{9}{2} \pm \frac{\sqrt{5}}{2} \\ \text{The roots are } &\frac{-9+\sqrt{5}}{2} \text{ or } \frac{-9-\sqrt{5}}{2} \end{aligned}$$

### Practice (16 minutes)

1. Write 3 problems on the board:
  - a. Use factorisation to solve  $a(a - 1) = 6$
  - b. Use the quadratic formula to solve the equation  $3x^2 - 8x + 2 = 0$
  - c. Solve  $-x^2 + 4x + 5 = 0$  by graphing the related function on the Cartesian plane for  $-1 \leq x \leq 5$ .
2. Ask pupils to solve the problems independently in their exercise books. Allow discussions with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write their solutions on the board at the same time. Other pupils should check their work.

**Solutions:**

a.

$$\begin{aligned}
 a(a - 1) &= 6 \\
 a^2 - a &= 6 \\
 a^2 - a - 6 &= 0 \\
 a^2 - 3a + 2a - 6 &= 0 \\
 a(a - 3) + 2(a - 3) &= 0 \\
 (a + 2)(a - 3) &= 0 \\
 a + 2 = 0 \quad \text{or} \quad a - 3 &= 0 \\
 a = -2 \quad \text{or} \quad 3
 \end{aligned}$$

b.

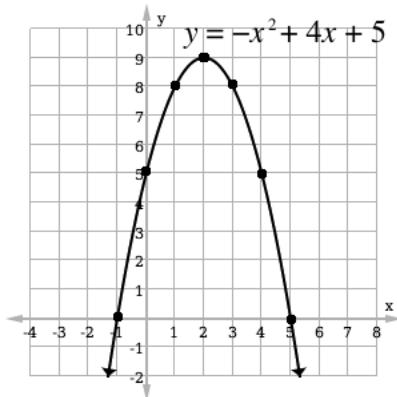
$$\begin{aligned}
 3x^2 - 8x + 2 &= 0 \\
 a = 3, \quad b = -8, \quad c = 2
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)} \\
 x &= \frac{8 \pm \sqrt{64 - 24}}{6} \\
 x &= \frac{8 \pm \sqrt{40}}{6} \\
 x &= \frac{8 \pm 6.33}{6} \\
 x &= \frac{8+6.33}{6} \quad \text{or} \quad \frac{8-6.33}{6} \\
 x &= 2.39 \quad \text{or} \quad 0.28
 \end{aligned}$$

c. Table of values:

$x$	-1	0	1	2	3	4	5
$y$	0	5	8	9	8	5	0

Graph:



**Solutions:**  $x = -1, 5$

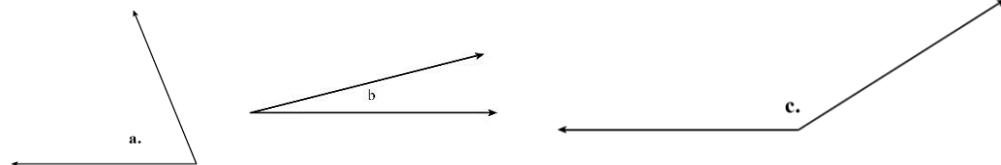
**Closing (1 minute)**

- For homework, have pupils do the practice activity PHM1-L116 in the Pupil Handbook.

<b>Lesson Title:</b> The degree as a unit of measure	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L117	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"> <li>Define the degree as a unit of measure.</li> <li>Describe how degree measurements are utilised in everyday life.</li> <li>Use a protractor to measure angles.</li> </ol>	 <b>Preparation</b> <ol style="list-style-type: none"> <li>Write the problems in Opening on the board.</li> <li>Bring a protractor to class (purchased or handmade), and ruler or any straight edge for drawing lines. See the note at the end of this lesson on making a protractor.</li> <li>Ask pupils to bring protractors if they already have them</li> </ol>	

### Opening (3 minutes)

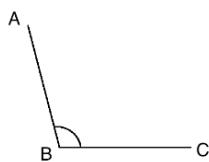
- Write on the board. Estimate the size of the following angles:



- Ask pupils to solve the problem independently in their exercise books.
- Ask any three volunteers to share their answers. Accept all answers and allow discussion. (Answers: a.  $70^\circ$  b.  $15^\circ$  c.  $160^\circ$ )
- Explain to the pupils that today's lesson is to define, describe and use a protractor to measure angles.

### Teaching and Learning (22 minutes)

- Draw an angle on the board:



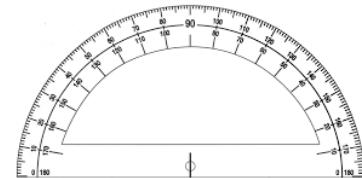
- Ask pupils to describe this angle with their own words.
- Allow pupils 1 minute to think and discuss with seatmates.
- Ask volunteers to share their ideas. (Example answers: There are two lines with a space in between; the space between the two lines is the angle.)
- Write on the board:  $\angle ABC$
- Explain:

- The corner point of an angle is called the vertex and the two straight lines are called rays. This is called "angle ABC" and it can be measured in degrees.
- Degrees are used to measure turn.
- There are 360 degrees in one full rotation (one complete circle).
- We use little circle ( $^\circ$ ) following the number to mean degrees.

7. Write on the board:  $\angle ABC = 105^\circ$
8. Ask a volunteer to pronounce it. (Answer: One hundred and five degrees)
9. Write on the board:  $45.12^\circ$ .
10. Ask a volunteer to pronounce it. (Answer: Forty-five point one two degrees).
11. Discuss:
  - Degrees may be further divided into minutes and seconds, but that division is not as universal as it used to be.
  - Each degree is divided into 60 equal parts called minutes.
  - However, the parts of a degree are now usually referred to decimals. For instance  $7\frac{1}{2}^\circ$  is now written as  $7.5^\circ$ .

12. Hold up a protractor (real or made with paper) and discuss:

- What is this tool? (Answer: a protractor)
- What is it used for? (Answer: To measure angles)
- How many degrees does it have? (Answer:  $180^\circ$ )

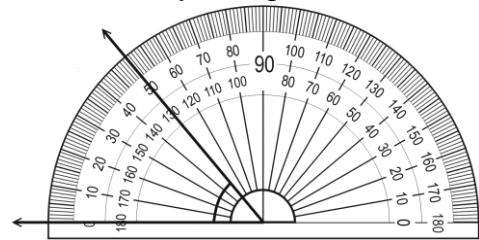


13. Draw an angle of any measure on the board.

14. Ask pupils to describe how to measure it in their own words.

15. Measure the angle using a protractor, and explain:

- To measure the angle, place a protractor over one ray so that its centre O is exactly over the vertex of the angle and the baseline is exactly along one line of the angle.
- The angle shown opens on the left. In this case, count the degrees using the outside numbers, starting on the left, from the baseline to where the other ray of the angle is pointing.  
The angle shown here is  $50^\circ$ .



16. Explain: An angle can open to the left or right. There are 2 sets of numbers on the protractor, so you can count in either direction depending on which direction it opens.

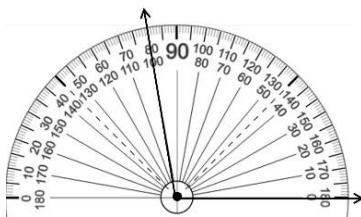
17. Write a problem on the board: Draw 2 angles and use a protractor to measure them. Write down their measures in degrees.

18. Ask pupils to solve the problems with seatmates.

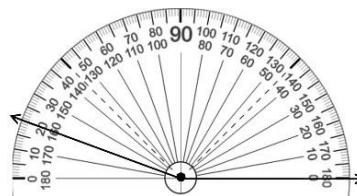
19. Walk around, if possible, to check for understanding and clear misconceptions.

20. Ask a few volunteers to share their angles with the class and explain how they measured them. Example answers:

a.  $100^\circ$



b.  $160^\circ$

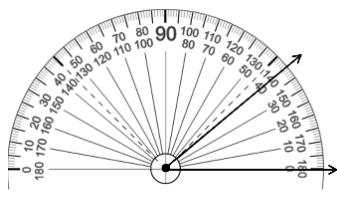


## Practice (10 minutes)

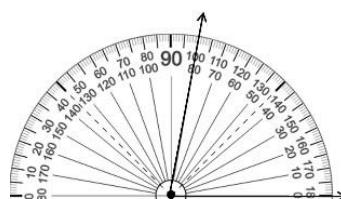
1. Write the following problem on the board: Draw 4 angles with any measure. Use a protractor to measure each angle, and write down its measure.
2. Ask pupils to solve the problems independently.
3. Walk around to check their answers and clear misconceptions.
4. Allow pupils to exchange their exercise books, and check the measures seatmates wrote down.

Example Answers:

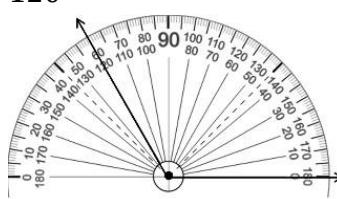
40°



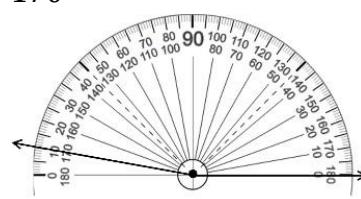
80°



120°



170°



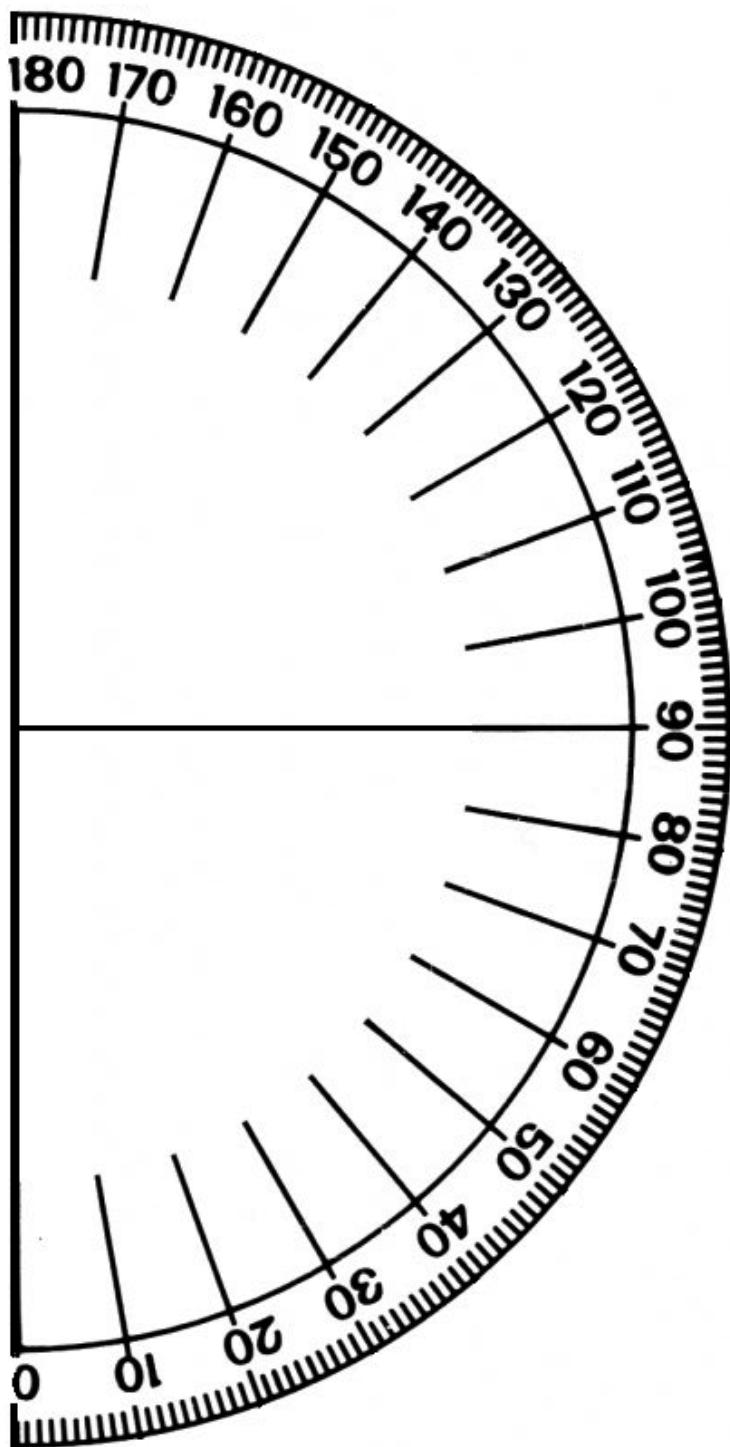
## Closing (5 minutes)

1. Allow pupils 2-3 minutes to discuss with seatmates how degree measurements are utilised in everyday life.
2. Have 1-2 volunteers share their ideas with the class.
3. Explain to them some real-life situations involving degrees:
  - In construction, angles are used to make buildings stable and sturdy. For example, the roof of a house has to be at least 39° and at a maximum 48° to prevent rain water and make sure the rain can slide off.
  - Carpenters use angles. For example, in making a book shelf, you would have to make sure the corner angles are all the same and to do that you'd have to find the degrees of all the angles.
  - Another place where angles are used is in football, not so much to do with the ball but the pitch itself. For example, the corner flag is 90°.
  - In engineering, the precise angle is required. For example, the wings of an airplane must be exactly angled at 15° upwards or otherwise the airflow over the wing will be compromised.

For homework, have pupils do the practice activity PHM1-L117 in the Pupil Handbook

[NOTE: MAKING A PROTRACTOR]

Teachers can use the large protractor below to show pupils how to measure angles on the board. This protractor can be traced with a pen onto a sheet of paper or photocopied. Then, cut it out with scissors. Pupils may do the same with the small protractors printed in their Pupil Handbooks.



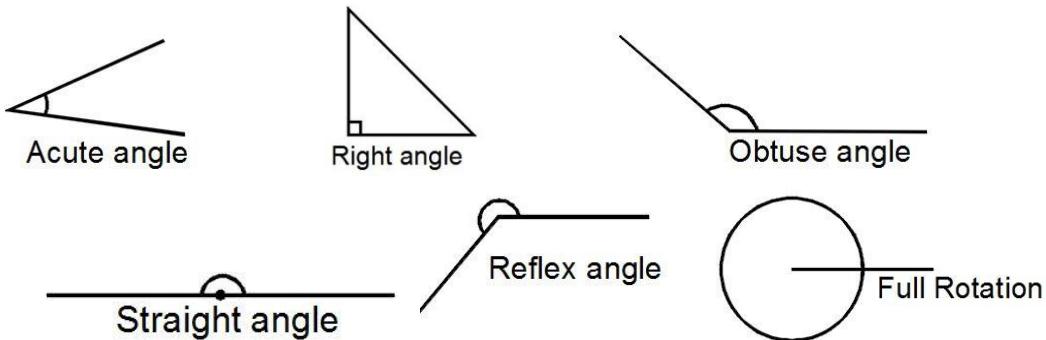
<b>Lesson Title:</b> Acute, obtuse, right, reflex, and straight angles	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L118	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"> <li>Identify and describe acute, obtuse, right, reflex, and straight angles.</li> <li>Classify angles as acute, obtuse, right, reflex, or straight.</li> </ol>	 <b>Preparation</b> Bring a protractor to class (real or paper), and ask pupils to do the same.

## Opening (2 minutes)

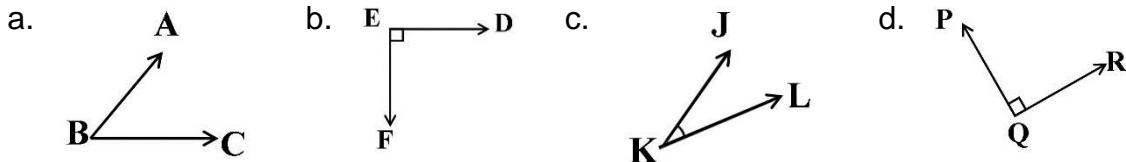
- Discuss:
  - What tool do we use to measure an angle? (Answer: protractor)
  - What units is an angle measured in? (Answer: degrees)
  - How many degrees make one complete circle? (Answer:  $360^\circ$ )
- Explain to the pupils that today's lesson is to identify and classify different angles.

## Teaching and Learning (18 minutes)

- Draw the different types of angles on the board:



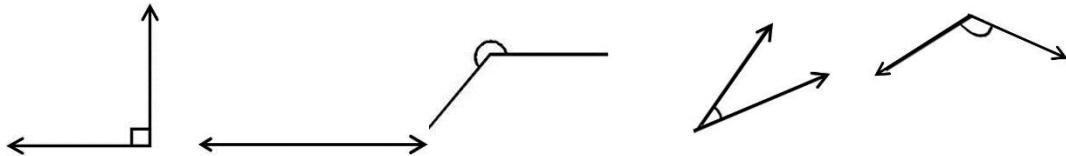
- Explain and make sure pupils understand each angle:
  - Acute angle is an angle less than  $90^\circ$ .
  - Right angle is an angle that is exactly  $90^\circ$ .
  - Obtuse angle is angle that is greater than  $90^\circ$  but less than  $180^\circ$ .
  - Straight angle is an angle that is exactly  $180^\circ$ .
  - Reflex angle is an angle greater than  $180^\circ$  but less than  $360^\circ$ .
  - Full rotation is exactly  $360^\circ$ .
- Explain that as the angle increases, the names change.
- Write a question on the board: Label each angle as acute, obtuse or right.



5. Ask pupils to solve the problems with seatmates.
6. Walk around to check for understanding and clear misconceptions.
7. Invite volunteers to write their answers on the board. (Answers: a. Acute; b. Right; c. Acute; d. Right)
8. Write on the board: Give 3 examples of: a. acute angles; b. obtuse angles; c. reflex angles.
9. Allow pupils to solve the problems with seatmates. They should write the degree measure of 3 angles of each type.
10. Invite volunteers to write their answers on the board. (Example answers: a. Acute angle =  $30^\circ, 5^\circ, 64^\circ$ ; b. Obtuse angle =  $92^\circ, 115^\circ, 165^\circ$ ; c. Reflex angle =  $184^\circ, 220^\circ, 335^\circ$ )

11. Draw 5 angles on the board:

- a. b. c. d. e.



12. Ask pupils to copy the angles into their exercise books and work with seatmates to identify the angles as right, acute, obtuse, straight, or reflex angles.
13. Walk around to check for understanding and clear misconceptions.
14. Ask any volunteer to share their answers. (Answers: a. Right; b. Straight; c. Reflex; d. Acute; e. Obtuse)

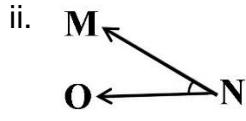
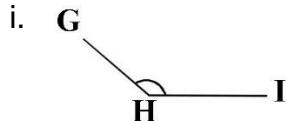
### Practice (19 minutes)

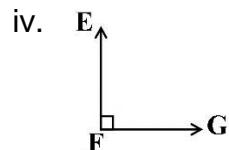
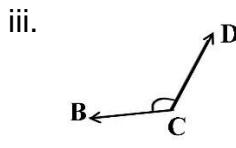
1. Write the problems on the board:

- a. Classify the type of each angle given its degree measure:

i.  $1^\circ$    ii.  $89^\circ$    iii.  $180^\circ$    iv.  $349^\circ$    v.  $179^\circ$

- b. Label each angle as acute, obtuse, or right:

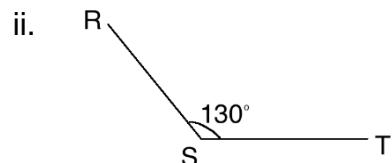
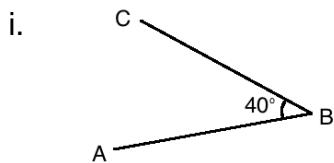




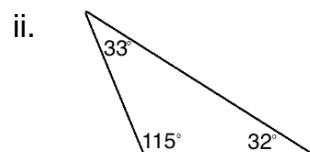
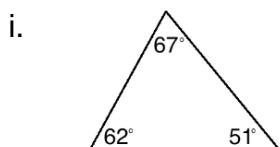
- c. Draw an acute angle ABC. Measure it with a protractor and label it with its measure.
  - d. Draw an obtuse angle RST. Measure it with a protractor and label it with its measure.
  - e. Draw a triangle with all of its angles acute. Measure and label its angles.
  - f. Draw a triangle with an obtuse angle. Measure and label its angles.
2. Ask pupils to solve the problems independently in their exercise books.
  3. Walk around to check for understanding and clear misconceptions.
  4. Allow pupils to exchange their exercise books.
  5. Invite volunteers to write their answers on the board. For parts c. and d., allow different volunteers to share their angles and shapes, and allow discussion.

**Answers:**

- a. i. acute; ii. acute; iii. straight; iv. reflex; v. obtuse;
- b. i. obtuse; ii. acute; iii. obtuse; iv. right
- c. Example answers:



- d. Example answers:



**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L118 in the Pupil Handbook.

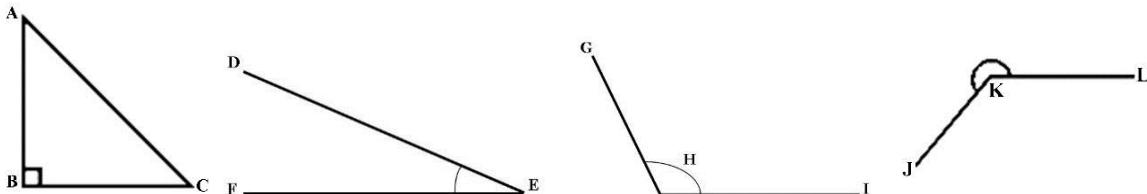
<b>Lesson Title:</b> Drawing of angles with specific measurements	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L119	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to draw angles with specific measurements.	 <b>Preparation</b> 1. Write the problems in Opening on the board. 2. Bring a protractor and ruler (or any other object with a straight edge) to class, and ask pupils to do the same.	

### Opening (3 minutes)

1. Write the following angle measures on the board.
  - a.  $202^\circ$
  - b.  $80^\circ$
  - c.  $4^\circ$
  - d.  $90^\circ$
  - e.  $45^\circ$
  - f.  $348^\circ$
2. Ask pupils to determine which angles are acute, obtuse, right or reflex angles.
3. Invite volunteers to write their answer on the board. (a. obtuse; b. acute; c. acute; d. right; e. acute; f. reflex)
4. Explain to pupils that today's lesson is on drawing of angles with specific measurements.

### Teaching and Learning (22 minutes)

1. Draw examples of right, acute, obtuse, and reflex angles as shown below.

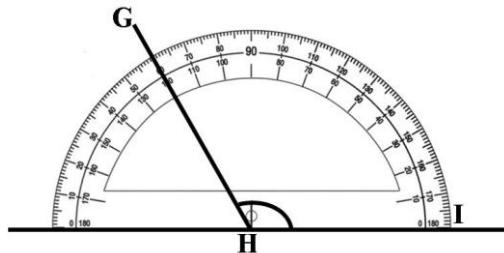


2. Explain:
  - Angle  $ABC$  is  $90$  degrees. It is a right angle. I know this because of the small square near the vertex  $B$ .
  - Angle  $DEF$  is less than  $90$  degrees. I know this because the angle is smaller than the right angle.
  - Angle  $GHI$  is more than  $90$  degrees, I know this because the angle is greater than the right angle.
  - Angle  $JKL$  is more than  $180$  degrees. I know this because the angle is greater than the obtuse angle.
3. Discuss: We can use a protractor to draw acute, obtuse and reflex angles
4. Show pupils how to draw the obtuse angle  $120^\circ$  on the board and explain the instructions given in the next step.
5. Explain:

- To draw an obtuse angle, I have to draw a base line.
- Place a protractor on the base line so that its centre 0, is exactly over the vertex of the angle and the base line is exactly along one line of the angle.
- Since the angle opens on the right, identify  $120^\circ$  from the numbers that start on the right, from the base line to where the other ray of the angle is pointing. These numbers are typically the inside set of numbers on a protractor.
- Mark  $120^\circ$  on your paper. Remove the protractor, and use a straight edge to connect the  $120^\circ$  mark to the vertex of the angle.

6. Write the angle measure on the board.

(Answer:  $\angle GHI = 120^\circ$ )



7. Write a question on the board: Draw an angle of  $75^\circ$

8. Remind pupils that when you are given measurement of an angle to draw, start by drawing a base line.

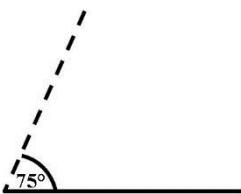
9. Invite a volunteer to draw the base line on the board with a ruler.

(Answer: \_\_\_\_\_ )

10. Invite another volunteer to use a protractor to draw an angle of  $75^\circ$ .

11. Remind pupils that the protractor should be placed over the angle so that its centre 0, is exactly over the vertex of the angle and the base line is exactly along the line of the angle.

12. Invite a volunteer to draw the angle:



13. Write a problem on the board: Draw the following angles: a. an acute angle of  $60^\circ$ ; b. an obtuse angle of  $110^\circ$ .

14. Ask pupils to draw the angles with seatmates.

15. Remind them that if an angle opens on the right, use the inside row of numbers on your protractor. If the angle opens on the left, use the outside row of numbers on your protractor.

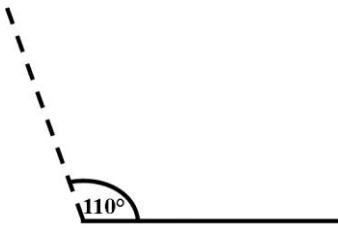
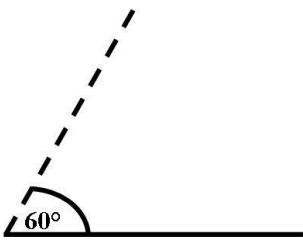
16. Walk around to check for understanding and assist pupils when necessary.

17. Invite two volunteers to draw the angles on the board one at a time.

Answer:

a.

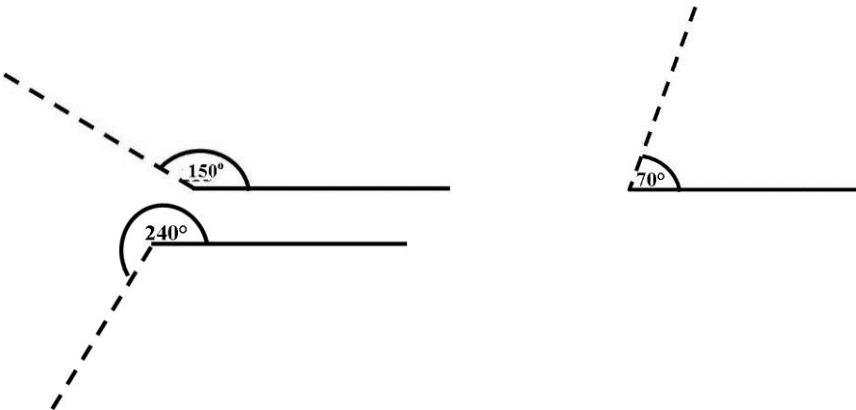
b.



### Practice (14 minutes)

1. Write the following problem on the board: Draw angles of  $150^\circ$ ,  $70^\circ$  and  $240^\circ$ .
2. Ask pupils to draw the angles independently in their exercise books.
3. Ask pupils to exchange exercise books, and to measure the angles. Ask them to write down the measurement in degrees.
4. Explain to pupils that once you have measured all three angles and written down the measurements, to exchange the books back.
5. Walk around to check for understanding and assist pupils when needed.
6. Invite volunteers to draw the angles on the board.

### Answers:



### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L119 in the Pupil Handbook.

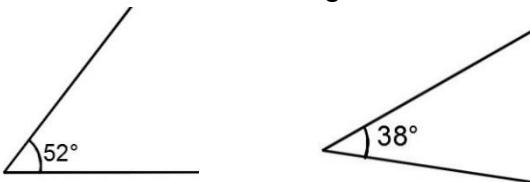
<b>Lesson Title:</b> Complementary and supplementary angles	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L120	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"> <li>1. Identify and describe complementary and supplementary angles.</li> <li>2. Classify angles as complementary or supplementary.</li> </ol>	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

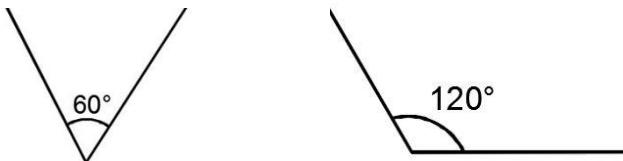
1. Write a problem on the board. Define the following angles and give two examples of each: a. Acute b. Obtuse c. Reflex
2. Ask pupils to solve the problem independently.
3. Invite three volunteers to write their answers on the board.  
Answers: a. Acute angle: Less than  $90^\circ$ ; eg.  $1^\circ$ ,  $45^\circ$ .  
b. Obtuse angle: Greater than  $90^\circ$  but less than  $180^\circ$ ; eg.  $92^\circ$ ,  $160^\circ$ .  
c. Reflex angle: Greater than  $180^\circ$  but less than  $360^\circ$ ; eg.  $194^\circ$ ,  $320^\circ$ .
4. Explain to the pupils that today's lesson is to identify, describe and classify angles as complementary or supplementary.

### Teaching and Learning (22 minutes)

1. Draw the two different angles on the board.



2. Explain **complementary angles**: Two angles are complementary when they add up to  $90^\circ$
3. Invite a volunteer to add the two angles drawn on the board. (Answer:  $52^\circ + 38^\circ = 90^\circ$ )
4. Discuss:
  - The two angles added are complementary angles because their sum is  $90^\circ$ .
  - The angles do not have to be together, but sometimes they do share a line.
5. Write on the board: What is the complementary angle of  $1^\circ$ ?
6. Allow the pupils to think carefully and discuss with seatmates.
7. Ask a volunteer to give their answer. (Answer: Complement of  $1^\circ$  =  $89^\circ$ ; because the sum of  $1^\circ$  and  $89^\circ$  is  $90^\circ$ .)
8. Draw two angles on the board:



9. Explain **supplementary angles**: Two angles are supplementary when they add up to  $180^\circ$ .

10. Invite a volunteer to add the two angles drawn on the board.

(Answer:  $60^\circ + 120^\circ = 180^\circ$ )

11. Discuss:

- The two angles added are supplementary because their sum is  $180^\circ$ .
- The angles again don't have to be together, but sometimes they do share a line.

12. Remind pupils any easy way to remember complementary and supplementary angles.

- 'C' of complementary stands for "corner"  $\angle$  (a Right Angle). If put together, 2 complementary angles form a right angle ( $90^\circ$ ).
- 'S' Supplementary stands for "straight" — (a Straight angle). If put together, 2 supplementary angles from a straight line ( $180^\circ$ ).

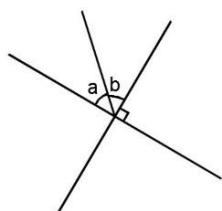
13. Write a question on the board: What is the supplementary angle of  $143^\circ$ ?

14. Invite any volunteer from the class to write the answer on the board.

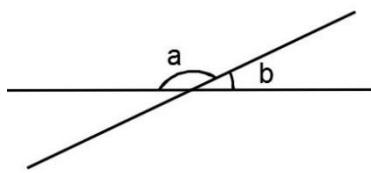
(Answer:  $180^\circ - 143^\circ = 37^\circ$ ; therefore the supplement of  $143^\circ$  is  $37^\circ$ ).

15. Draw two diagrams on the board:

a.



b.



16. Allow pupils 2-3 minutes to think and discuss with seatmates about the relationship between angle **a** and angle **b** in both diagrams.

17. Walk around to check for understanding and clear misconceptions.

18. Ask volunteers to give their answers, and allow discussion.

(Answers: (a)  $\angle a$  and  $\angle b$  form a right angle and their sum is  $90^\circ$ . Therefore they are complementary angles. (b)  $\angle a$  and  $\angle b$  form a straight angle and their sum is  $180^\circ$ . Therefore they are supplementary angles.)

19. Write on the board: Find the complement of the angle  $\frac{2}{3}$  of  $90^\circ$ .

20. Invite a volunteer to calculate the value of  $\frac{2}{3}$  of  $90^\circ$  on the board. (Answer:  $\frac{2}{3} \times 90^\circ = 60^\circ$ )

21. Invite another volunteer to find the complement of  $60^\circ$ . (Answer:  $90^\circ - 60^\circ = 30^\circ$ )

22. Explain that the complement of  $\frac{2}{3}$  of  $90^\circ$  is  $30^\circ$ .
23. Write another problem on the board: The measure of two supplementary angles are  $(3x + 15^\circ)$  and  $(2x + 5^\circ)$ . Find the value of  $x$ .
24. Ask pupils to solve the problem with seatmates.
25. Walk around, if possible, to check for understanding and clear misconceptions.
26. Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{array}{ll}
 (3x + 15^\circ) + (2x + 5^\circ) & = 180^\circ & \text{Sum of the angles is equal to } 180^\circ \\
 3x + 15^\circ + 2x + 5^\circ & = 180^\circ & \text{Solve for } x \\
 3x + 2x & = 180^\circ - 5^\circ - 15^\circ \\
 5x & = 160^\circ \\
 x & = \frac{160}{5} \\
 x & = 32^\circ
 \end{array}$$

**Practice (14 minutes)**

1. Write the following two problems on the board:
  - a. Find the supplementary of the angle  $\frac{4}{5}$  of  $90^\circ$ .
  - b. The measures of two complementary angles are  $(2x - 7^\circ)$  and  $(x + 4^\circ)$ . Find the value of  $x$ .
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite volunteers, one at a time, to write the solutions on the board.

**Solutions:**

$$\begin{array}{ll}
 \text{a. The angle that is } \frac{4}{5} \text{ of } 90^\circ \text{ is } \frac{4}{5} \times 90^\circ = 72^\circ. & \\
 \text{The supplement of this angle is } 180^\circ - 72^\circ = 108^\circ & \\
 \text{b. Set the sum of the angles equal to } 90^\circ, \text{ and solve for } x: & \\
 (2x - 7^\circ) + (x + 4^\circ) & = 90^\circ & \text{Sum of the angles is equal to } 90^\circ \\
 2x - 7^\circ + x + 4^\circ & = 90^\circ & \text{Solve for } x \\
 2x + x & = 90^\circ - 4^\circ + 7^\circ \\
 3x & = 93^\circ \\
 x & = \frac{93}{3} \\
 x & = 31^\circ
 \end{array}$$

**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L120 in the Pupil Handbook.

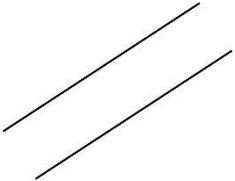
<b>Lesson Title:</b> Parallel lines	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L121	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: 1. Describe parallel lines. 2. Use a compass to draw a set of parallel lines.	 <b>Preparation</b> 1. Write the problems in Opening on the board. 2. Bring a geometry set to class, and ask pupils to do the same.	

### Opening (3 minutes)

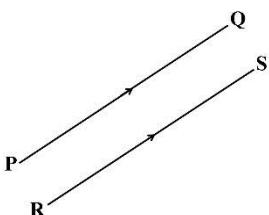
1. Write on the board: Define complementary and supplementary angles.
2. Ask pupils to work independently to write the definitions in their exercise books.
3. Have volunteers share their answers, and allow discussion. (Answer: Complementary angles are two angles whose sum is  $90^\circ$ . Supplementary angles are two angles whose sum is  $180^\circ$ .)
4. Explain that today's lesson is to describe parallel lines and use a pair of compasses to draw a set of parallel lines.

### Teaching and Learning (26 minutes)

1. Draw two parallel lines on the board:



2. Explain:
  - Parallel lines are two lines that are always the same distance apart and never touch.
  - In order for two lines to be parallel, they must be drawn in the same plane, a perfectly flat surface like a wall or sheet of paper.
3. Ask pupils what is an example of a parallel line.
4. Allow 1 – 2 minutes for pupils to discuss with seatmates and think of ideas.
5. Ask a volunteer share their answers with the class. (Example answers: the two sides of this page; the shelves of a bookcase)
6. Label the lines on the board as shown:

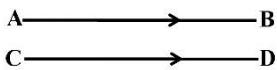


7. Discuss:

- To show that lines are parallel, we draw small arrow marks on them.
- In the diagram on the board, note the arrows on the line  $PQ$  and  $RS$ . This shows that those lines are parallel.

8. Invite a volunteer to draw parallel lines  $AB$  and  $CD$  on the board.

**Answer:**



9. Write on the board:  $\overline{PQ} \parallel \overline{RS}$

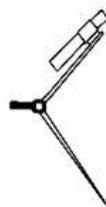
10. Explain:

- This is shorthand for writing about parallel lines.  $\overline{PQ} \parallel \overline{RS}$  is read as “the line segment  $PQ$  is parallel to the line segment  $RS$ ”.
- Recall that the horizontal bar over the letters indicates it is a line segment.

11. Hold up the pair of compasses.

12. Ask pupils what is the name of this tool? (Answer: A pair of compasses)

13. Explain that they will be using these compasses during this lesson and the following one.



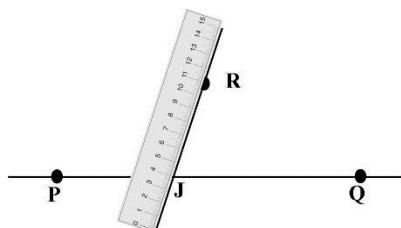
14. Draw a horizontal line segment across the board.

15. Label the line segment  $PQ$  and a point  $R$  above the line.

• R



16. Invite a volunteer to draw a transversal line through  $R$  and across  $PQ$  at any point  $J$  where it intersects the line  $PQ$ .

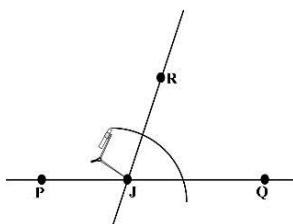


17. Explain:

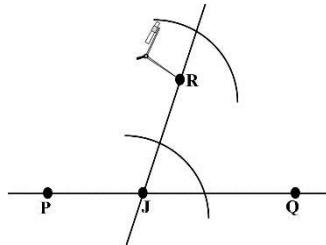
- We will construct a parallel line at point  $R$ .
- We will use  $J$  as a centre and choose a radius set to about half the distance  $JR$ .

18. Take the following steps on the board, explaining each one.

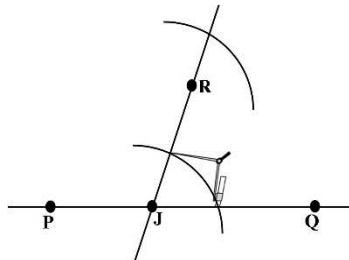
- Draw an arc across both lines.



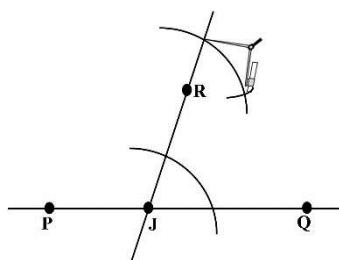
- Use the same radius, repeat at point  $R$ .



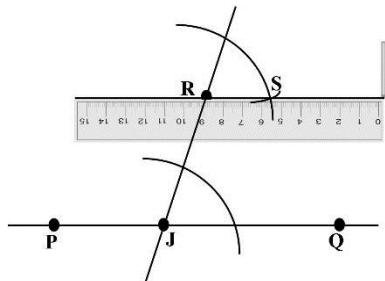
- Set the compass radius to the distance where the lower arc crosses the two lines.



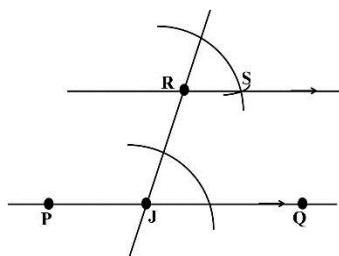
- Move the compasses to where the upper arc crosses the transversal line and draw an arc to make point  $S$ .



- Draw a straight line through  $R$  and  $S$  using a ruler.



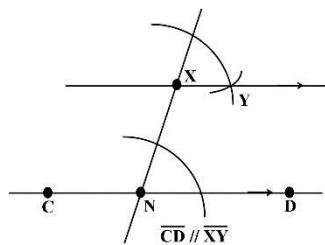
- The construction is complete. The line  $RS$  is parallel to  $PQ$



19. Explain the diagram:  $\overline{PQ}$  is parallel to  $\overline{RS}$ . This can be written as  $PQ \parallel RS$ .

20. Write on the board: Draw line segment  $\overline{CD}$ . Construct a parallel line  $\overline{XY}$ .
21. Explain: You can use any straight edge to draw a line. Try using the side of your exercise book, or a piece of paper or a pen.
22. Ask pupils to work with seatmates to draw the construction. Remind them that they will follow exactly the same steps as the ones demonstrated on the board.
23. Walk around to check for understanding and clear misconceptions.
24. Invite a volunteer from 1 group of seatmates to show the paper with their construction to the class. Ask them to explain the steps they took to draw it. Allow other pupils to ask questions and discuss.

**Answer:**

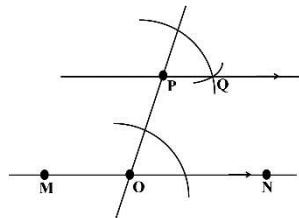


### Practice (10 minutes)

1. Write the following problems on the board:
  - a. Draw line segment  $\overline{MN}$ . Construct parallel line  $\overline{PQ}$ .
  - b. What are parallel lines?
2. Ask pupils to work independently to do the construction. If there are not enough pairs of compasses, encourage them to share but to each do their own construction.
3. Ask 1-2 volunteers to show their paper and explain how they did their construction. Allow discussion.
4. Ask volunteers to explain in their own words what parallel lines are.
5. If there is time, invite volunteers to write the answers on the board.

**Answers:**

a.



- b. Parallel lines move in the same direction but never cross one another.

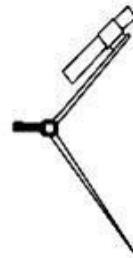
### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L121 in the Pupil Handbook.

<b>Lesson Title:</b> Perpendicular lines	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L122	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: 1. Describe perpendicular lines. 2. Use a compass to draw a set of perpendicular lines and label the angle measurements	 <b>Preparation</b> 1. Write the problems in Opening on the board. 2. Bring a geometry set to class, and ask pupils to do the same.

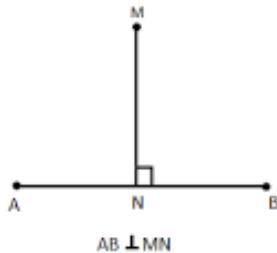
### Opening (3 minutes)

1. Hold up the pair of compasses if you have one.
2. Ask pupils:
  - a. What is the name of this tool?
  - b. What do we use the tool for?
3. Allow pupils to discuss and share their ideas with seatmates.
4. Ask a volunteer to answer the question. (Answers: a. A pair of compasses; b. It is used in geometry construction. For example, it is used to bisect lines or angles.)
5. Explain that today's lesson is on describing perpendicular lines and to use a compass to draw a set of perpendicular lines and label the angle measurements.



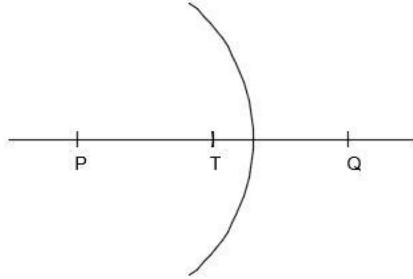
### Teaching and Learning (22 minutes)

1. Explain:
  - “Perpendicular” means that two lines meet at a right angle,  $90^\circ$ .
  - A line is said to be perpendicular to another line if the two lines intersect at a right angle.
2. Draw an illustrative diagram of perpendicular lines on the board:

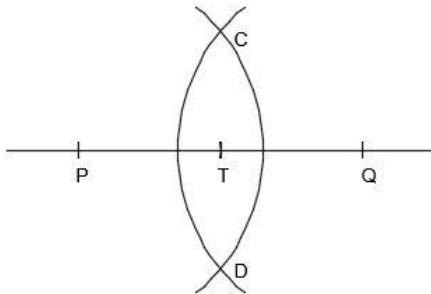


3. Tell pupils that line  $AB$  is perpendicular to line  $MN$
4. Explain **perpendicular lines**:
  - They are often shown with a small square on the right angle.
  - You can measure the angle with a protractor to see if the measure is  $90^\circ$ .
5. Ask pupils to find examples of perpendicular lines.

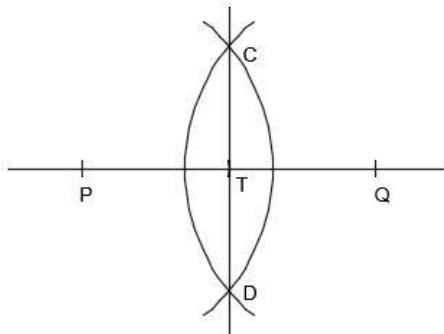
6. Allow pupils 1 -2 minutes to share ideas with seatmates.
7. Ask volunteers to share their answers and allow discussion.  
(Example answers: All right triangles are made from two perpendicular line segments. The corners of rectangles (and squares) are also made of perpendicular line segments. The sides of a cube are perpendicular planes).
8. Write on the board: Draw line segment  $\overline{PQ}$ . Construct its perpendicular bisector  $\overline{CD}$ .
9. Explain: A perpendicular bisector divides a line into 2 equal segments. So if you construct a perpendicular bisector of  $\overline{PQ}$ , you are drawing a perpendicular line that divides  $\overline{PQ}$  into 2 equal parts.
10. Draw a horizontal line segment across the board.
11. Ask a volunteer to choose any point around the middle of the line, and label it  $T$ .
12. Explain:
  - We will construct a perpendicular line at point  $T$ .
  - We will use  $T$  as a centre and choose any radius for our pair of compasses.
13. Take the following steps on the board, explaining each one:
  - Draw arcs to cut the line segment at 2 points the same distance from  $T$ , and label these points  $P$  and  $Q$  (It is important that  $\overline{PT} = \overline{TQ}$  ).
  - With point  $P$  as the centre, open your compass more than half way to point  $Q$ . Then draw an arc that intersects  $\overline{PQ}$ .



- Using the same radius and point  $Q$  as centre, draw an arc that intersects the first arc. Label the points where the 2 arcs intersect as  $C$  and  $D$ .



- Draw  $\overline{CD}$ .



14. Explain:

- $\overline{CD}$  is perpendicular to  $\overline{PQ}$  at point  $T$ . Point  $T$  is the middle point of  $\overline{PQ}$ .
- $\overline{CD}$  is called the perpendicular bisector of line segment  $\overline{PQ}$ .

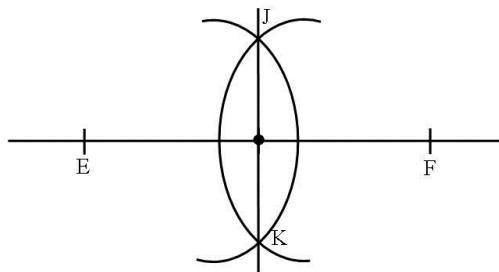
15. Write a problem on the board. Draw line segment  $\overline{EF}$ . Construct its perpendicular bisector  $\overline{JK}$ .

16. Ask pupils to work with seatmates to draw the construction. Remind them that they will follow exactly the same steps as the ones demonstrated on the board.

17. Walk around to check for understanding and clear misconceptions.

18. Ask volunteers to show their construction to the class and explain the steps they took to draw it. Allow other pupils to ask questions and discuss.

**Answer:**

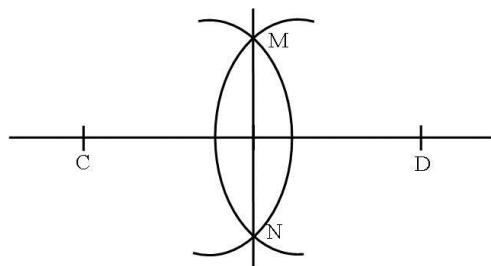


### Practice (14 minutes)

1. Write two problems on the board:
  - a. How do you know that two lines are perpendicular?
  - b. Draw line segment  $\overline{CD}$ . Construct its perpendicular bisector  $\overline{MN}$ .
2. Ask pupils to work independently to solve the problems. If there are not enough pairs of compasses, encourage them to share but to each draw their own construction.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Ask volunteers to share their answer to part a., and explain how they did their construction work.
5. If there is time, invite volunteers to write the answers on the board.

**Answers:**

- a. Two lines are perpendicular when they form right angles ( $90^\circ$ ) to each other.
- b. Construction:



**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L122 in the Pupil Handbook.

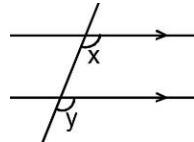
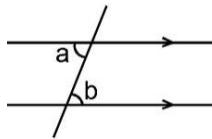
<b>Lesson Title:</b> Alternate and corresponding angles	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L123	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"> <li>1. Identify and describe alternate and corresponding angles.</li> <li>2. Classify angles as alternate or corresponding.</li> </ol>	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

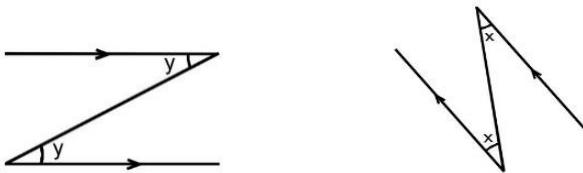
1. Write the following two problems on the board:
  - a. What is a parallel line?
  - b. What is a Perpendicular line?
2. Ask pupils to work independently and to write the answers in their exercise books.
3. Ask volunteers to share their answers and allow discussion. (Answers: a. Parallel lines are two lines that are always the same distance apart and never touch.  
b. A line is said to be perpendicular to another line if the two lines intersect at a right angle).
4. Explain to the pupils that today's lesson is to identify, describe and classify angles as alternate or corresponding angles.

### Teaching and Learning (22 minutes)

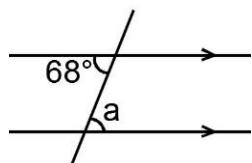
1. Draw the two different set of parallel lines on the board, as shown:
  - a. Alternate angles
  - b. Corresponding angles



2. Explain:
  - A **transversal line** is one that intersects two parallel lines.
  - When a transversal line intersects parallel lines, there are two pairs of alternate angles and there are four pairs of corresponding angles.
  - Alternate angles are equal, and corresponding angles are equal.
3. Explain **alternate angles**:
  - In diagram a., the labeled angles are alternate angles.
  - Angle  $a$  is equal to angle  $b$ . (Write on the board:  $\angle a = \angle b$ ).
  - Alternate angles form a Z pattern.
4. Draw illustrative diagrams of alternate angles within Z patterns on the board:



5. Write the following problem on the board: Find the value of  $a$  from the diagram:



6. Invite any volunteer to write the answer on the board. (Answer:  $a = 68^\circ$ )

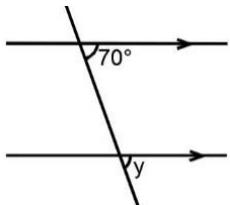
7. Explain **corresponding angles**:

- In the first diagram on the board (diagram b), angles  $x$  and  $y$  are corresponding angles.
- Angle  $x$  is equal to angle  $y$ . (Write on the board:  $\angle x = \angle y$ ).
- Corresponding angles form an F pattern.

8. Draw an illustrative diagram of F pattern on the board.

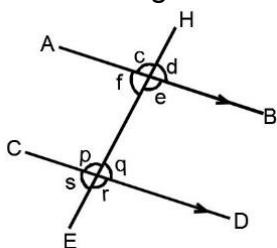


9. Write the following problem on the board: Find the value of  $y$  from the diagram:



10. Invite a volunteer to write the answer on the board. (Answer:  $y = 70^\circ$ ).

11. Write the following problem on the board. In the diagram below, name two pairs of alternate angles and four pairs of corresponding angles.

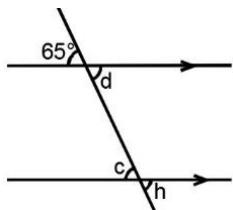


12. Ask pupils to solve the problem with seatmates.

13. Allow them to discuss it for 1-2 minutes.

14. Ask three volunteers one at a time to give their answers. (Answer: Pairs of alternate angles are  $f = q$  and  $e = p$  pairs of corresponding angles are  $c = p, f = s, d = q$  and  $e = r$ ).

15. Write another question on the board: Find the values of the unknown angles and give your reasons.



16. Ask pupils to solve the problem with seatmates.

17. Walk around, if possible, to check for understanding and clear misconceptions.

18. Ask 3 volunteers, one at a time, to write their answers on the board.

**Answers:**

$e = 65^\circ$  (Corresponding angle)

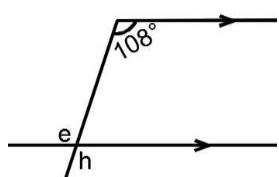
$d = 65^\circ$  (Alternate angle to  $e$ )

$h = 65^\circ$  (Corresponding angle to  $d$ )

**Practice (14 minutes)**

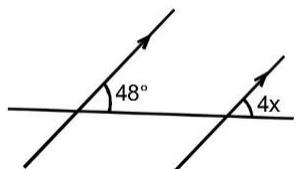
1. Write three problems on the board:

a. Find the values of  $e$  and  $h$  in the diagram:

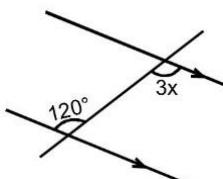


Find the value of  $x$  in each diagram:

b.



c.



2. Ask pupils to solve the problems independently in their exercise books.

3. Walk around, if possible, to check for understanding and clear misconceptions.

4. Allow pupils to exchange their exercise books.

5. Have 3 volunteers, one at a time, write the solutions on the board.

**Solutions:**

a.  $e = 108^\circ$  (alternate angle);  $h = 108^\circ$  (corresponding angle)

b. Set  $4x$  equal to  $48^\circ$  (corresponding angles), and solve for  $x$ .

$$\begin{aligned}4x &= 48^\circ \\x &= \frac{48^\circ}{4} \\x &= 12^\circ\end{aligned}$$

c. Set  $3x$  equal to  $120^\circ$  (alternate angles), and solve for  $x$ :

$$\begin{aligned}3x &= 120^\circ \\x &= \frac{120^\circ}{3} \\x &= 40^\circ\end{aligned}$$

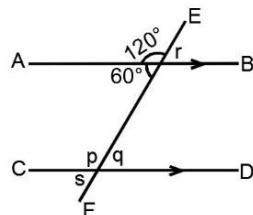
**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L123 in the Pupil Handbook.

<b>Lesson Title:</b> Adjacent and opposite angles	<b>Theme:</b> Geometry		
<b>Lesson Number:</b> M1-L124	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes	
<b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"><li>1. Identify and describe adjacent and opposite angles.</li><li>2. Classify angles as adjacent or opposite.</li></ol>	 <b>Preparation</b> Write the problems in Opening on the board.		

## **Opening** (*3 minutes*)

1. Write the following problem on the board: Find the values of  $p$ ,  $q$ ,  $r$  and  $s$ :



2. Ask pupils to solve the problem independently.
  3. Invite volunteers to write the answers on the board.

Answers:  $p = 120^\circ$  (Corresponding angles)

$q = 60^\circ$  (Alternate angles)

$r = 60^\circ$  (Corresponding angles)

$s = 60^\circ$  (Corresponding angles)

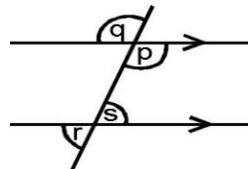
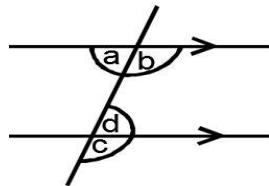
- Explain to pupils that today's lesson is to identify, describe and classify adjacent and opposite angles.

## **Teaching and Learning (22 minutes)**

1. Draw the two different set of parallel lines.

a. Adjacent angles

b. Opposite angles



- ## 2. Explain:

- When a transversal line intersects two lines, there are eight pairs of adjacent angles four pairs of opposite angles.
  - Adjacent angles sum up to  $180^\circ$ .
  - Opposite angle are equal.

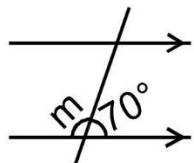
- ### 3. Explain adjacent angles:

- In diagram a., the labeled angles are adjacent.
- Adjacent angles are supplementary, since they form a straight line. (Write on the board:  $\angle a + \angle b = 180^\circ$  and  $\angle c + \angle d = 180^\circ$ )

4. Draw illustrative diagrams on the board of adjacent angles:



5. Write the following problem on the board: Find the value of  $m$  from the diagram:



6. Ask pupils to work with seatmates to find  $m$ .

7. Invite a volunteer to write the solution on the board.

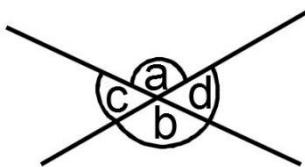
**Solution:**

$$\begin{aligned} m + 70^\circ &= 180^\circ && \text{Adjacent angles sum to } 180^\circ \\ m &= 180^\circ - 70^\circ \\ m &= 110^\circ \end{aligned}$$

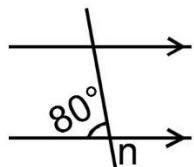
8. Explain **opposite angles**:

- In diagram b. on the board, the labeled angles are opposite angles.  $q$  and  $p$  are opposite;  $r$  and  $s$  are opposite.
- Opposite angles are equal.
- Opposite angles form a X pattern.

9. Draw an illustrative diagram of X pattern on the board.

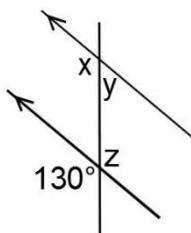


10. Write the following problem on the board: Find the value of  $n$  from the diagram:



11. Invite a volunteer to write the answer on the board. (Answer:  $n = 80^\circ$ ).

12. Write the following problem on the board: Find the values of  $x$ ,  $y$  and  $z$ :



13. Ask pupils to solve the problem with seatmates.

14. Allow 1-2 minutes for discussion.

15. Invite volunteers to write the solution on the board.

**Solution:**

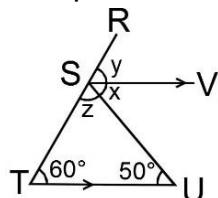
$x$  is a corresponding angle to the given angle; therefore,  $x = 130^\circ$

Find  $y$  using  $x$ , which is an adjacent angle:

$$\begin{aligned} x + y &= 180^\circ && \text{Adjacent angles sum to } 180^\circ \\ 130^\circ + y &= 180^\circ \\ y &= 180^\circ - 130^\circ \\ y &= 50^\circ \end{aligned}$$

$z$  is opposite the given angle; therefore,  $z = 130^\circ$

16. Write another question on the board: Find the values of  $x, y$  and  $z$ .



17. Ask pupils to solve the problem with seatmates.

18. Walk around, if possible, to check for understanding and clear misconceptions.

19. Invite volunteers to write the solution on the board.

**Solution:**

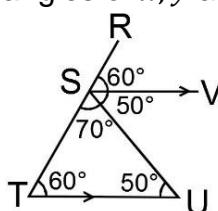
$x$  is an alternate angle to the given angle  $50^\circ$ ; therefore,  $x = 50^\circ$

$y$  is a corresponding angle to the given angle  $60^\circ$ ; therefore,  $y = 60^\circ$

$x, y$ , and  $z$  are angles on a straight line; therefore:

$$\begin{aligned} x + y + z &= 180^\circ && \text{Adjacent angles sum to } 180^\circ \\ 50^\circ + 60^\circ + z &= 180^\circ \\ 110^\circ + z &= 180^\circ \\ z &= 180^\circ - 110^\circ \\ z &= 70^\circ \end{aligned}$$

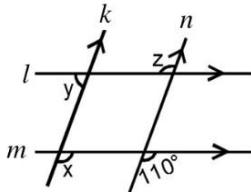
20. Label the angles of  $x, y$  and  $z$  with their measures:



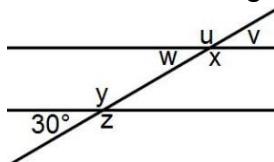
## Practice (14 minutes)

1. Write the following two problems on the board:

- a. In the figure below, line  $l$  is parallel to line  $m$  and line  $k$  is parallel to line  $n$ .  
Find the values of  $x, y, z$ .



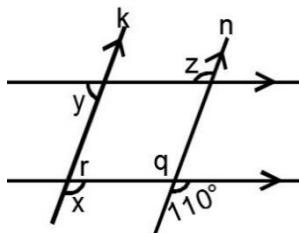
- b. Find the values of the lettered angles in the diagram below. Give reasons for your answers. All angles are in degrees.



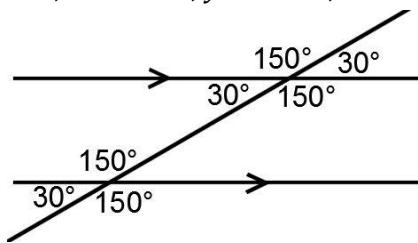
2. Ask pupils to solve the problems independently in their exercise books.  
3. Walk around, if possible, to check for understanding and clear misconceptions.  
4. Allow pupils to exchange their exercise books.  
5. Invite volunteers to write their answers on the board.

### Answers:

- c. Find additional angles (labeled  $q$  and  $r$  below) to do the calculations:  
d.  $y = 180^\circ - 30^\circ = 150^\circ$  (Adjacent angle)  
 $z = y = 150^\circ$  (Opposite angles)  
 $u = y = 150^\circ$  (Corresponding angles)  
 $w = 30^\circ$  (Corresponding angles)  
 $v = w = 30^\circ$  (Opposite angles)  
 $x = z = 150^\circ$  (Corresponding angles)  
Therefore,  $u = 150^\circ, v = 30^\circ, w = 30^\circ, x = 150^\circ, y = 150^\circ, z = 150^\circ$ .



$x = 110^\circ$  (Corresponding Angle)  
 $q = 110^\circ$  (Opposite angle)  
 $z = 110^\circ$  (Corresponding angle)  
 $r = 180^\circ - 110^\circ = 70^\circ$  (Adjacent angle)  
 $y = 70^\circ$  (Alternate angle)  
Therefore,  $x = 110^\circ, y = 70^\circ$  and  $z = 110^\circ$



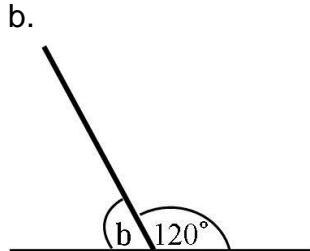
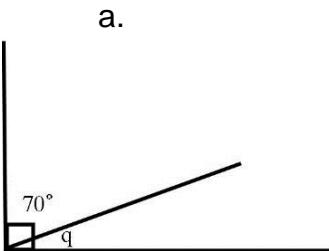
## Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L124 in the Pupil Handbook.

<b>Lesson Title:</b> Interior and exterior angles	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L125	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
<b>Learning Outcomes</b>  By the end of the lesson, pupils will be able to: 1. Identify and describe interior and exterior angles. 2. Classify angles as interior or exterior.	<b>Preparation</b>  Write the problems in Opening on the board.

### Opening (3 minutes)

1. Write the following problem on the board: Find the value of the unknown angles:



2. Ask pupils to work independently to find the answers in their exercise books.  
 3. Invite volunteers to write the solutions on the board.

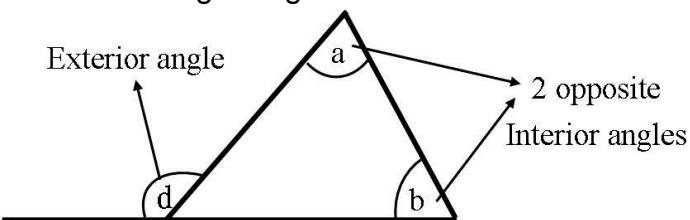
#### Solutions:

$$\begin{aligned} \text{a. } q + 70^\circ &= 90^\circ \\ q &= 90^\circ - 70^\circ \\ q &= 20^\circ \\ \text{b. } b + 120^\circ &= 180^\circ \\ b &= 180^\circ - 120^\circ \\ b &= 60^\circ \end{aligned}$$

4. Explain to the pupils that today's lesson is to identify, describe interior and exterior angles and classify angles as interior or exterior.

### Teaching and Learning (22 minutes)

1. Draw the following triangle on the board:

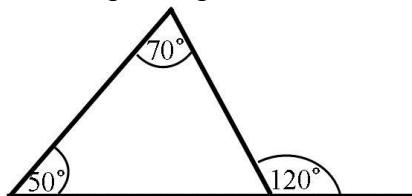


2. Explain:

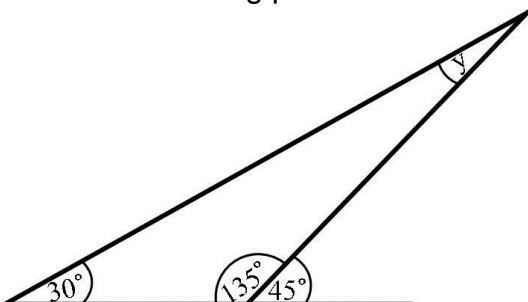
- In the diagram,  $a$  and  $b$  are called interior angles.
- An interior angle is an angle inside a shape.

- The angle  $d$  is an exterior angle
- An exterior angle of a shape is formed by any side of the shape and the extension of its adjacent sides.
- All types of polygons have interior and exterior angles.

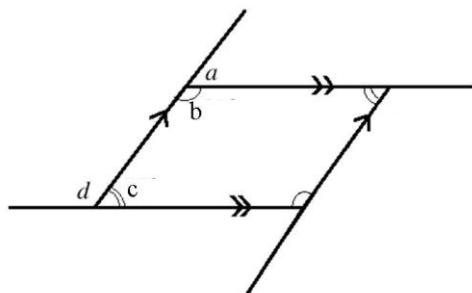
3. Draw the following triangle on the board:



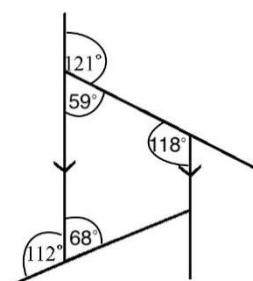
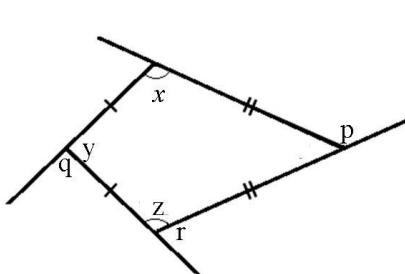
4. Ask volunteers to identify the interior angles. (Answers: 50°, 70°)
5. Ask any volunteer to identify the exterior angle. (Answer: 120°)
6. Write the following problem on the board.



7. Ask volunteers to identify the interior angles. (Answers: 30°, 135°,  $y$ )
8. Ask a volunteer to identify the exterior angle. (Answer: 45°)
9. Draw the following parallelogram on the board:



10. Ask a volunteer to identify all the angles that are interior angles. (Answer:  $\angle b$  and  $\angle c$ )
11. Ask another volunteer to identify the exterior angles. (Answer:  $\angle a$  and  $\angle d$ )
12. Write on the board. Identify the interior and exterior angles from the diagrams below:



13. Ask pupils to work with seatmates to identify the angles.
14. Walk around, if possible, to check for understanding and clear misconceptions.
15. Invite volunteers to write down the answers on the board.

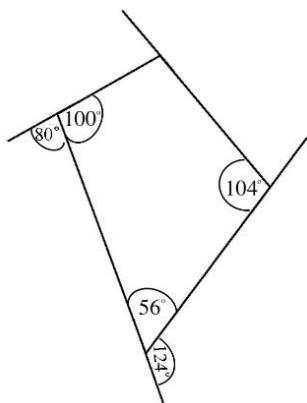
**Answers:**

- a. Interior angles=  $\angle x, \angle y$  and  $\angle z$   
Exterior angles=  $\angle p, \angle q$  and  $\angle r$
- b. Interior angles=  $59^\circ, 68^\circ$  and  $118^\circ$   
Exterior angles=  $121^\circ$  and  $112^\circ$

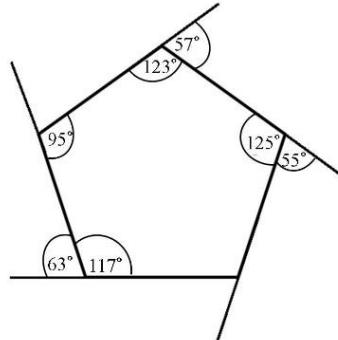
**Practice (14 minutes)**

1. Write the following problem on the board: Classify each angle in the diagrams below as an interior or exterior angle:

a.



b.



2. Ask pupils to solve the problems independently.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite 2 volunteers, one at a time, to write the answers on the board.

**Answers:**

- a. Interior angles=  $56^\circ, 104^\circ, 100^\circ$   
Exterior angles=  $80^\circ, 124^\circ$
- b. Interior angles=  $95^\circ, 117^\circ, 123^\circ, 125^\circ$   
Exterior angles=  $55^\circ, 57^\circ, 63^\circ$

**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L125 in the Pupil Handbook.

<b>Lesson Title:</b> Practical application of angle measurement	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L126	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to measure angles in real life.	 <b>Preparation</b> 1. Write the problems in Opening on the board. 2. Bring a ruler and protractor to class, and ask pupils to do the same.	

### Opening (3 minutes)

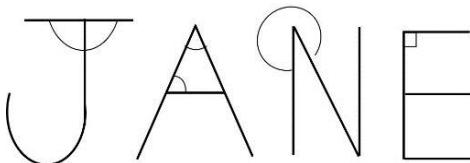
1. Write the following problem on the board: Define the following angles:
  - a. Acute
  - b. Obtuse
  - c. Reflex
  - d. Straight
  - e. Right
2. Ask pupils to write the definitions independently, in their own words.
3. Ask volunteers to share their definitions, and allow discussion.

#### Answers:

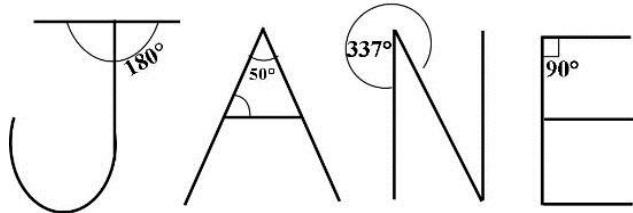
- a. An acute angle is an angle less than  $90^\circ$ .
  - b. An obtuse angle is an angle greater than  $90^\circ$  but less than  $180^\circ$ .
  - c. A reflex angle is an angle greater than  $180^\circ$  but less than  $360^\circ$ .
  - d. A straight angle is exactly  $180^\circ$ .
  - e. A right angle is exactly  $90^\circ$ .
4. Explain to pupils that today's lesson is to measure angles in real life.

### Teaching and Learning (23 minutes)

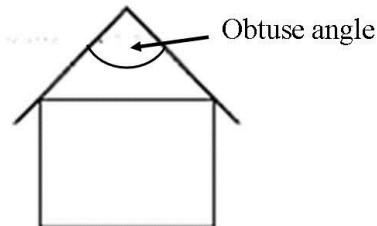
1. Ask pupils to use a ruler (or any object with a straight edge) to create the letters of their name.
2. Ask them to identify at least 5 angles in their names and measure each angle. Encourage them to share protractors if there are not enough.
3. Show them the example of "JANE" on the board, with the angles identified but not measured:



4. Ask volunteers to share their name drawings with the class and explain. For example:



5. Draw an illustration of a house with a roof showing an angle on the board.



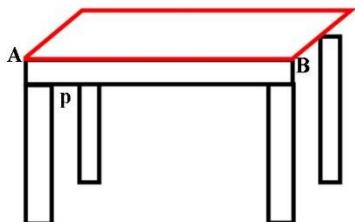
6. Ask pupils to discuss with seatmates for 2-4 minutes to give examples of obtuse, right, acute and straight angles in real life.

7. Ask a volunteer to share the answers with the class.

Example answers:

- The roof of a house
- The base of an open laptop
- A clothes hanger
- A flag pole
- A clock showing 8 o'clock

8. Draw a table on the board.



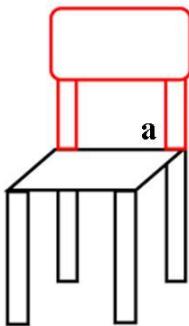
9. Ask pupils to measure the angle  $p$ .

10. Ask a volunteer to write the answer on the board (Answer:  $p = 90^\circ$ ).

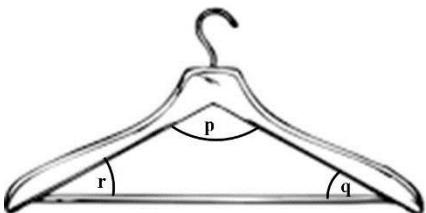
11. Ask pupils to write down the angle of  $\overline{AB}$ .

12. Invite a volunteer to write the answer on the board. (Answer:  $180^\circ$  because it is a straight line).

13. Draw a chair on the board.

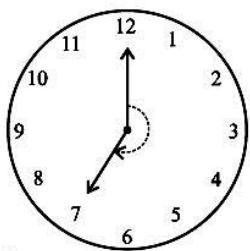


14. Ask pupils to measure the angles  $a$ .
15. Ask a volunteer to write the answer. (Answer:  $90^\circ$ ).
16. Draw a clothes hanger on the board.

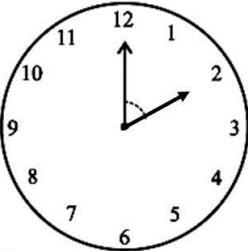


17. Ask pupils to draw their own clothes hanger and to measure the angles  $p$ ,  $q$  and  $r$ .
18. Allow pupils to solve the problem with seatmates.
19. Walk around to check for understanding and clear misconceptions.
20. Ask a volunteer to share their answer with the class. (Answer: the angles depend on their drawing; in this example, the angles are approximately  $p = 120^\circ$ ;  $q = 30^\circ$ ;  $r = 30^\circ$ ).
21. Draw two clocks and a laptop on the board.

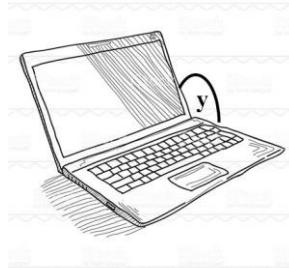
a.



b.



c.

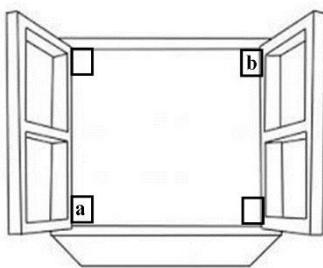


22. Ask pupils to copy the drawings in their exercise books, and to measure the angles of the hands on the clock and to find the value of  $y$  on the laptop.
23. Allow them to solve the problems with seatmates.
24. Walk around, if possible, to check for understanding and clear misconceptions.
25. Ask volunteers to share their answers with the class. Accept reasonable answers.  
(Answers for the diagrams above: a.  $210^\circ$  b.  $60^\circ$  c.  $130^\circ$ ).
26. Invite volunteers to come to the board to measure and label the angles in the drawings on the board.

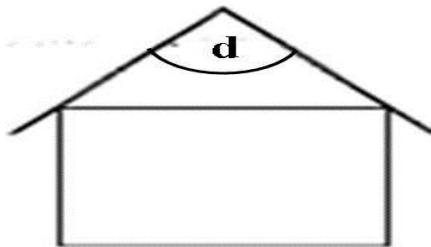
### **Practice (11 minutes)**

1. Write a problem on the board: Draw the following objects, and measure the marked angles in your drawings.

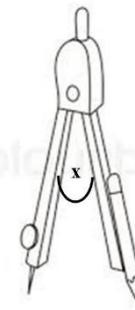
a.



b.



c.



2. Ask pupils to solve the problems independently.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books and to measure and check the angles drawn.
5. Ask some volunteers to share their drawings and angles with the class. (Example answers based on the diagrams above: a.  $a = 90^\circ$ ; b.  $x = 20^\circ$ ; c.  $d = 120^\circ$ )
6. Invite volunteers to come to the board to measure and label the angles in the drawings on the board.

### **Closing (3 minutes)**

1. Discuss and allow pupils to share their ideas:
  - a. Why is it useful to measure angles? What professions do you think use angle measurements in their jobs? (Example answers: carpenters and builders; engineers; tailors)
2. For homework, have pupils do the practice activity PHM1-L126 in the Pupil Handbook.

<b>Lesson Title:</b> Word problems involving angle measurement	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L127	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to solve word problems involving measurements of angles.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (5 minutes)

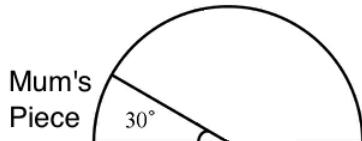
1. Write the following revision problems on the board:
  - a. Find the complement of the angle  $\frac{4}{9}$  of  $90^\circ$
  - b. Classify the angles of these degrees:  
i.  $43^\circ$    ii.  $179^\circ$    iii.  $359^\circ$ .
2. Ask pupils to solve the problems independently.
3. Invite volunteers to write the answers on the board.

#### Answers:

- a.  $\frac{4}{9} \times 90^\circ = 4 \times 10 = 40^\circ$   
 Complement =  $90^\circ - 40^\circ = 50^\circ$
  - b. i. Acute angle      ii. Obtuse angle      iii. Reflex angle.
4. Explain to the pupils that today's lesson is on solving word problems involving measurements of angles.

### Teaching and Learning (19 minutes)

1. Explain: There are many word problems which involve the measurement of angles. These can be written in the form of equations to be solved.
2. Remind pupils some basic steps for solving linear equations from word problems.
  - Assign a variable to represent each unknown angle.
  - Write the algebraic expression representing the situation.
  - Solve the equation for the unknown angles.
3. Write the following problem on the board: There is a half circle of bread left. You want to eat twice what your little brother eats but, you also need to save a slice for your mum. You cut her slice that is  $30^\circ$ . What is the measure of your piece of bread in degrees?
4. Ask a volunteer to explain which angle represents a half circle of bread? (Answer: Half bread =  $180^\circ$ )
5. Draw a diagram on the board to show the half circle of bread, and the portion you will save for your mum ( $30^\circ$ ):



6. Solve on the board, explain each step:

**Solution:**

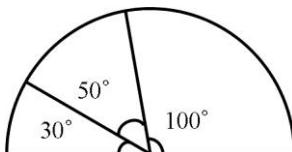
Let  $x$  equal to the degree measure of the brother's bread that he eats. Then  $2x$  is what you eat, and  $30^\circ$  is the piece for mum. In this situation,  $x + 2x + 30^\circ = 180^\circ$

Solve for  $x$ :

$$\begin{aligned} x + 2x + 30^\circ &= 180^\circ \\ 3x &= 180^\circ - 30^\circ \\ 3x &= 150^\circ \\ x &= \frac{150}{3} \\ x &= 50^\circ \end{aligned}$$

But what you eat is  $2x$ ; so  $2x = 2 \times 50^\circ = 100^\circ$

7. Draw on the board to illustrate the degrees of the bread.



8. Write another problem on the board: Two angles are complementary. If one of the angles is double the other angle, find the two angles.  
 9. Ask pupils to write the equation with seatmates (they should not solve yet).  
 10. Walk around to check for the understanding and clear misconceptions.  
 11. Invite a volunteer to write the equation on the board and explain.

**Solution:**

Let  $x$  be one of the angles then, the other angle  $2x$ .

Since  $x$  and  $2x$  are complementary, their sum is equal to  $90^\circ$ .

The equation is:  $x + 2x = 90^\circ$

12. Ask pupils to work with seatmates to solve the problem.  
 13. Walk around to check for understanding and clear misconceptions.  
 14. Invite a volunteer to solve the problem on the board.

**Solution:**

$$\begin{aligned} x + 2x &= 90^\circ \\ 3x &= 90^\circ \\ x &= \frac{90^\circ}{3} \\ x &= 30^\circ \end{aligned}$$

The other angle is  $2x = 2 \times 30^\circ = 60^\circ$ , hence the two angles are  $30^\circ$  and  $60^\circ$ .

### **Practice (15 minutes)**

1. Write 2 problems on the board:
  - a. Two angles are complementary. If one is two times the sum of the other angle and 3, find the two angles.
  - b. Two angles are supplementary. If one is 4 times the measure of the other, find the 2 angles.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite volunteers to solve the problems on the board.

### **Solutions:**

a. Let  $x$  and  $y$  be the two angles which are complementary.

So, we have  $x + y = 90^\circ$  .....Equation (i)

From the information, "one angle is two times the sum of the other angle and 3," we have:

$$x = 2(y + 3)$$

$$x = 2y + 6 \dots\dots\dots \text{Equation (ii)}$$

We have a system of 2 linear equations. Substitute  $x = 2y + 6$  into equation (i)

$$\begin{aligned} 2y + 6 + y &= 90^\circ \\ 2y + y &= 90^\circ - 6 \\ 3y &= 84 \\ y &= \frac{84}{3} \\ y &= 28^\circ \end{aligned}$$

Now, substitute  $y = 28^\circ$  into equation (ii)

$$\begin{aligned} x &= 2(28) + 6 \\ &= 56 + 6 \\ &= 62^\circ \end{aligned}$$

Hence the two angles are  $28^\circ$  and  $62^\circ$ .

- b. Let  $x$  be the smaller angle, then the other angle is  $4x$ . Their sum is  $180^\circ$ . This gives:

$$\begin{aligned} x + 4x &= 180^\circ \\ 5x &= 180^\circ \\ x &= \frac{180^\circ}{5} = 36^\circ \end{aligned}$$

The other angle is  $4x = 4(36^\circ) = 144^\circ$ .

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L127 in the Pupil Handbook.

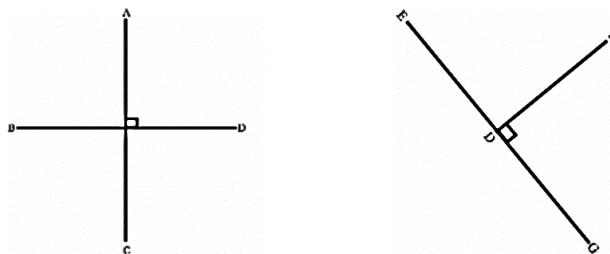
<b>Lesson Title:</b> Bisectors of angles and line segments	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L128	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to identify bisectors of angles and line segments.	 <b>Preparation</b> 1. Write the problems in Opening on the board. 2. Bring a protractor and ruler to class. Ask pupils to do the same.	

### Opening (3 minutes)

1. Write a problem on the board.
  - a. What are perpendicular lines.
  - b. Draw two examples of perpendicular lines.
2. Ask pupils to solve the problems independently.
3. Invite volunteers to write the answers on the board.

#### Answers:

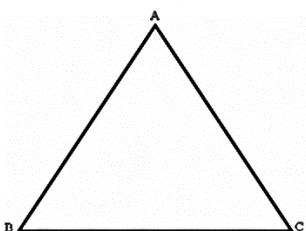
- a. A perpendicular line is when two lines meet at  $90^\circ$ .
- b.



4. Explain to the pupils that today's lesson is to identify bisectors of angles and line segments.

### Teaching and Learning (22 minutes)

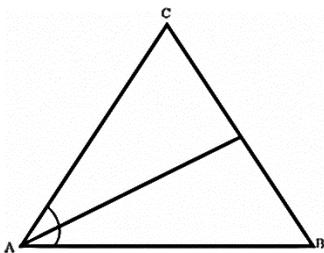
1. Explain:
  - An angle bisector is a line or ray that divides an angle into two equal angles.
  - The two types of angle bisector are interior and exterior.
2. Draw a triangle on the board.



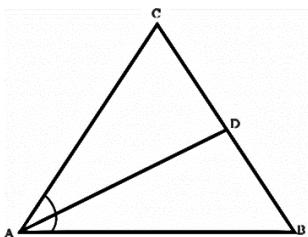
3. Ask any volunteer to use a protractor and measure angle A.  
 (Example answer: angle A =  $69^\circ$ ; this will depend on your triangle)

- Discuss: To bisect angle  $A$  simply means to divide angle  $A$  into two equal angles.
- Invite a volunteer to give half of  $69^\circ$ . (Answer:  $34.5^\circ$ )
- Invite any volunteer to divide angle  $A$  into two equal angles using a protractor. Note that there should be  $34.5^\circ$  on either side of their bisector.

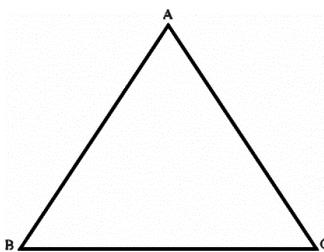
**Answer:**



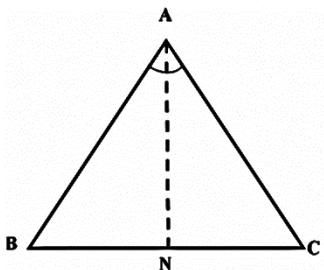
- Label the point  $D$  where the line meets  $|BC|$ :



- Write on the board:  $\angle CAD = \angle DAB$ .
- Write a problem on the board: Draw a bisector of angle  $A$  to meet line  $BC$  at  $N$ .



- Invite a volunteer to solve the problem on the board.



- Invite another volunteer to check their work by measuring  $\angle BAN$  and  $\angle CAN$ . Record the measure of each on the board, and make sure they are equal.

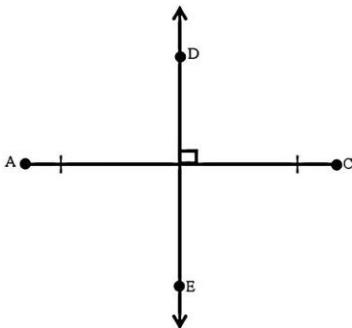
- Tell pupils that line  $AN$  bisects angle  $A$ .

- Explain:
  - A line bisector is the line that passes through the mid-point of a given segment.

- A line bisector is the line that passes through the mid-point of a given segment.

- A mid-point is a point on a line segment that divides it into two equal segments.

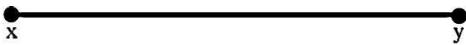
14. Draw a perpendicular bisector on the board.



15. Discuss:

- A line, segment, or ray that passes through a mid-point of another segment is called a segment bisector.
- A segment bisector is called a perpendicular bisector when the bisector intersects the segment at right angles.

16. Write on the board: Use a ruler to draw a bisector of the line segment below.

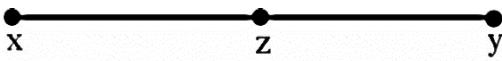


17. Ask a volunteer to first measure the line segment. (Example answer: 4 cm)

18. Explain: Since the line is 4 cm, the mid-point will be 2 cm from either end point, or half way between.

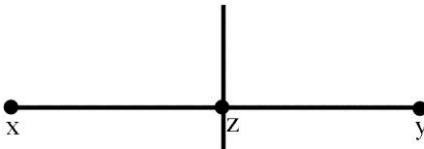
19. Invite a volunteer to measure 2 cm from one end point and draw the mid-point. Label the mid-point z.

**Answer:**



20. Ask another volunteer to draw a line that passes through the mid-point. It doesn't matter how the line intersects as long as it passes through.

**Answer:**



21. Write the following two problems on the board:

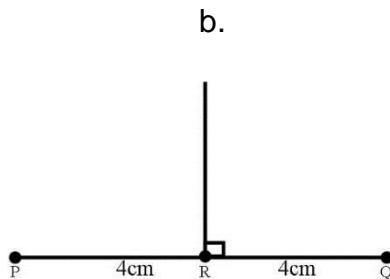
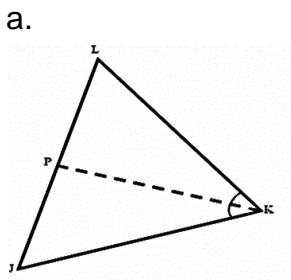
- a. Draw triangle JKL. Draw a bisector of angle  $JKL$  to meet line  $JL$  at P.
- b. Draw a line segment  $PQ$  with length 8 cm. Use a ruler to draw a bisector of the line segment.

22. Ask pupils to solve the problems with seatmates.

23. Walk around, if possible, to check for understanding and clear misconceptions.

24. Invite two volunteers, one at a time, to write their answers on the board.

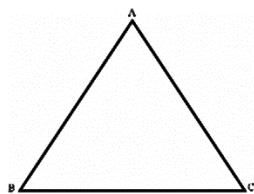
**Answers:**



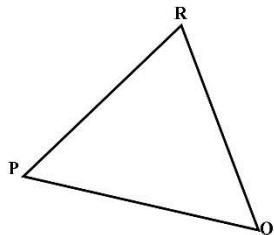
### Practice (14 minutes)

1. Write three problems on the board:

- a. Draw triangle  $ABC$  on your paper. Draw a bisector of angle  $ABC$  to meet  $|AC|$  at  $M$ . Measure and record the size of  $\angle ABM$  and  $\angle CBM$ .



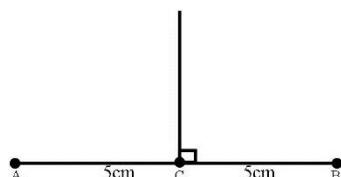
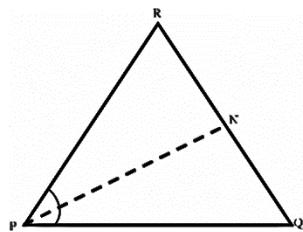
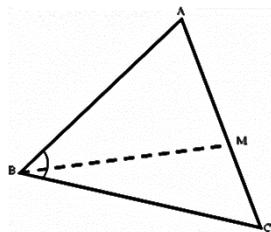
- b. Draw triangle  $PQR$  on your paper. Draw a bisector of angle  $P$  to meet  $|QR|$  at  $N$ . Measure and record the size of  $\angle RPN$  and  $\angle NPQ$ .



- c. Draw a line segment  $AB$  with length 10 cm. Use a ruler to draw a bisector of the line segment.

2. Ask pupils to solve the problems independently.
3. Walk around and check for understanding and clear misconceptions.
4. Ask pupils to exchange papers and check the bisectors drawn by seatmates.

Example Answers (measures of angles depend on pupils' drawings):



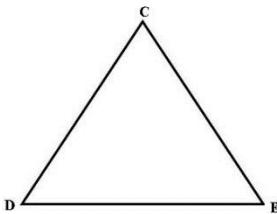
### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L128 in the Pupil Handbook.

<b>Lesson Title:</b> Intercept theorem	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L129	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
<b>Learning Outcome</b>  By the end of the lesson, pupils will be able to use the intercept theorem to calculate line segments.	<b>Preparation</b>  <ol style="list-style-type: none"> <li>1. Write the problems in Opening on the board.</li> <li>2. Bring a protractor to class and ask pupils to bring protractors.</li> </ol>

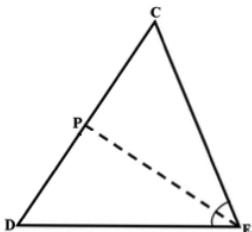
### Opening (3 minutes)

1. Write the following problem on the board. Copy triangle DEC in your exercise book. Use a protractor to bisect angle  $DEC$  to meet  $|CD|$  at  $P$ .



2. Ask pupils to solve the problem independently.
3. Invite a volunteer to bisect the angle of DEC on the board.

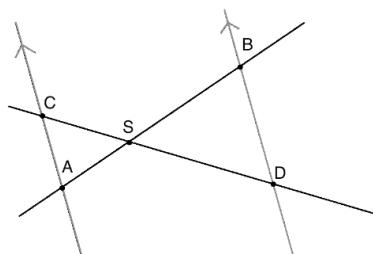
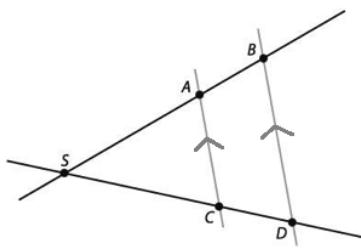
**Answer:**



4. Explain that today's lesson is on using the intercept theorem to calculate line segments.

### Teaching and Learning (22 minutes)

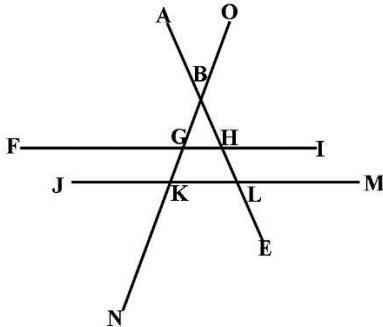
1. Draw the diagrams as shown below:



2. Explain:

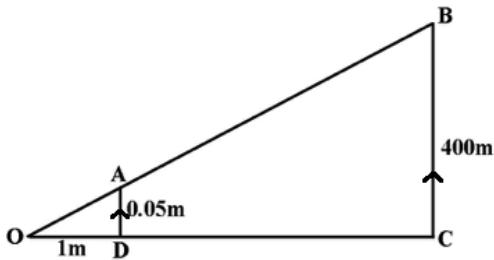
- The intercept theorem is about the ratio of line segments. We have two lines intersecting at point  $S$ .

- The same theorem applies to these 2 diagrams. In the first diagram, the lines intersect outside of the 2 parallel lines, and in the second diagram they intersect between the 2 parallel lines.
  - The lines intersect the parallel lines in points  $A, B, C$  and  $D$ .
  - The points make up various line segments such as  $\overline{SA}$  (the line from  $S$  to  $A$ );  $\overline{AC}$  (the line from  $A$  to  $C$ ) and so on.
  - The theorem tells us about the ratios of the length of these line segments.
3. Write the following ratios on the board:  $\frac{SA}{SB} = \frac{SC}{SD}$  and  $\frac{SA}{AB} = \frac{SC}{CD}$
4. Point out each side from the ratio in the diagram on the board. Make sure pupils understand.
5. Explain: The ratios of two segments on one line are equal to the ratio of the corresponding two segments on the other line.
6. Write the ratios on the board:  $\frac{SA}{AC} = \frac{SB}{BD}$ ;  $\frac{SC}{AC} = \frac{SD}{BD}$ ;  $\frac{SA}{SB} = \frac{AC}{BD}$ ;  $\frac{SC}{SD} = \frac{AC}{BD}$
7. Point out each side from the ratio in the diagram on the board. Make sure pupils understand.
8. Explain: This shows the ratio of two segments on one line is equal to the ratio of the parallel segments.
9. Draw the following diagram as shown on the board.



10. Explain:
- Two parallel lines  $FI$  and  $JM$  are cut by two self-intersecting lines  $AE$  and  $ON$ .
  - Two triangles are formed as a result of the intersections of the lines.
11. Ask pupils to discuss with seatmates to write down as many ratios as they can using the intercept theorem.
12. Invite volunteers to write the answers on the board. (Answers:  $\frac{BG}{GK} = \frac{BH}{HL}$ ;  $\frac{BG}{BK} = \frac{BH}{BL}$ ;  $\frac{BH}{GH} = \frac{BL}{KL}$ ,  $\frac{BG}{GH} = \frac{BK}{KL}$ ;  $\frac{BH}{BL} = \frac{GH}{KL}$ ;  $\frac{BG}{BK} = \frac{GH}{KL}$ )

13. Draw and label the diagram below on the board:



14. Explain: Here we have 2 triangles, where their sides are parallel. We can use the intercept theorem to find line  $OC$ .

15. Solve for  $OC$  on the board, explaining each step:

**Solution:**

If the straight lines  $AD$  and  $BC$  are parallel, then the ratios  $\frac{OB}{OA}$ ,  $\frac{OC}{OD}$  and  $\frac{BC}{AD}$  are equal. It is an intercept theorem.

Take  $\frac{OC}{OD} = \frac{BC}{AD}$ :

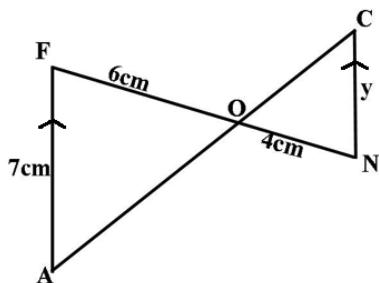
$$\frac{OC}{1} = \frac{400}{0.05} \quad \text{Substitute known sides}$$

$$OC = \frac{1 \times 400}{0.05} \quad \text{Cross multiply}$$

$$OC = \frac{400}{0.05} \quad \text{Solve for } OC$$

$$OC = 8,000 \text{ m}$$

16. Write another problem on the board: Find the length of  $y$ :



17. Ask volunteers to give the steps to solve for  $y$ . As they give the steps, solve on the board:

**Solution:**

$$\frac{ON}{OF} = \frac{CN}{FA} \quad \text{Set up the ratios}$$

$$\frac{4}{6} = \frac{y}{7} \quad \text{Substitute known sides}$$

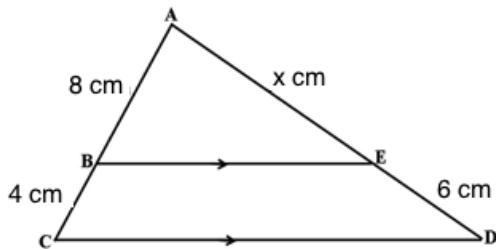
$$6y = 4 \times 7 \quad \text{Cross multiply}$$

$$6y = 28 \quad \text{Solve for } y$$

$$y = \frac{28}{6}$$

$$y = 4.7 \text{ cm}$$

18. Write another problem on the board. Find the length of  $x$  in the triangle using intercept theorem.



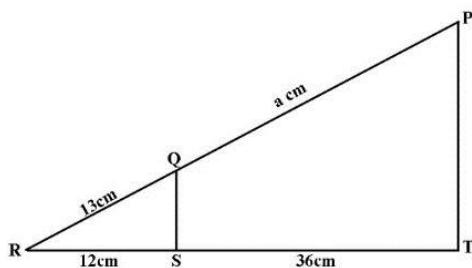
19. Ask pupils to work with seatmates to find the missing length of  $x$ .
20. Walk around, if possible, to check for understanding and clear misconceptions.
21. Invite a volunteer to write the solution on the board and explain.

**Solution:**

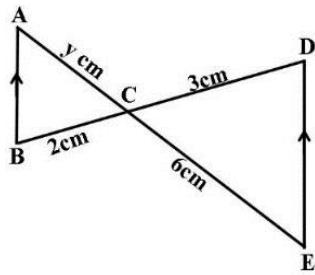
$$\begin{aligned}\frac{AB}{BC} &= \frac{AE}{ED} \\ \frac{8}{4} &= \frac{x}{6} \\ 4x &= 8 \times 6 \\ x &= \frac{48}{4} \\ x &= 12 \text{ cm}\end{aligned}$$

### Practice (14 minutes)

1. Write on the board: Find the missing sides marked with letters in each diagram.
- a.



b.



2. Ask pupils to work independently to find the missing sides.
3. Invite 2 volunteers to write the solutions on the board. They may come to the board at the same time.

**Solutions:**

a.

$$\begin{aligned}\frac{PQ}{QR} &= \frac{TS}{SR} \\ \frac{a}{13} &= \frac{36}{12} \\ 12a &= 13 \times 36 \\ a &= \frac{13 \times 36}{12} \\ a &= 39 \text{ cm}\end{aligned}$$

b.

$$\begin{aligned}\frac{BC}{CD} &= \frac{AC}{CE} \\ \frac{2}{3} &= \frac{y}{6} \\ 3y &= 2 \times 6 \\ y &= \frac{2 \times 6}{3} \\ y &= 4 \text{ cm}\end{aligned}$$

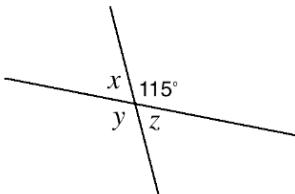
**Closing** (*1 minute*)

1. For homework, have pupils do the practice activity PHM1-L129 in the Pupil Handbook.

<b>Lesson Title:</b> Angle problem solving	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L130	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
<b>Learning Outcome</b>  By the end of the lesson, pupils will be able to apply angle theorems and properties to solve problems, including word problems.	<b>Preparation</b>  Write the problems in Opening on the board.

### Opening (3 minutes)

1. Write on the board: Find the measures of  $x$ ,  $y$  and  $z$  in the diagram:



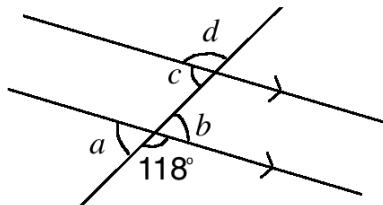
2. Ask pupils to solve the problem independently.
3. Invite volunteers to write the solution on the board and explain.

#### Solutions:

- $x$  and  $115^\circ$  are supplementary angles.  $x = 180^\circ - 115^\circ = 65^\circ$ .
  - $y$  and  $115^\circ$  are opposite angles, and are thus equal.  $y = 115^\circ$ .
  - $z$  is opposite  $x$ , so  $z = x = 65^\circ$ .  $z$  can also be calculated using the fact that it is supplementary to  $y$  and  $115^\circ$ .
4. Explain to pupils that today's lesson is on applying angle theorems and properties to solve problems, including word problems.

### Teaching and Learning (22 minutes)

1. Write the following problem on the board: Find the measures of  $a$ ,  $b$ ,  $c$  and  $d$  in the diagram below:



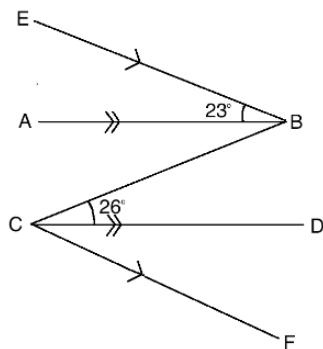
2. Ask pupils to work with seatmates to solve the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.

#### Solutions:

$a$  and  $b$  are both supplementary to  $118^\circ$ . Therefore,  $a = b = 180^\circ - 118^\circ = 62^\circ$ .  
 $c$  and  $a$  are corresponding angles. Therefore,  $c = a = 62^\circ$

$d$  and  $c$  are supplementary angles. Therefore,  $d = 180^\circ - 62^\circ = 118^\circ$

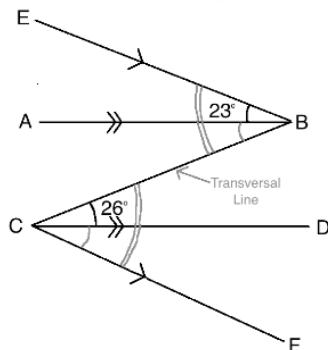
5. Write another problem on the board: In the diagram below,  $AB \parallel CD$  and  $EB \parallel CF$ . Find the measure of  $\angle BCF$ .



6. Ask volunteers to describe what they notice about the lines. (Example answer:  $BC$  is a transversal for both sets of parallel lines.)
7. Discuss: Which angles are equal in this diagram? (Answer:  $\angle EBC = \angle BCF$  and  $\angle ABC = \angle BCD$ )
8. Solve the problem on the board as a class. Make sure pupils understand.

**Solution:**

Draw arcs to show the equal angles formed by the transversal:

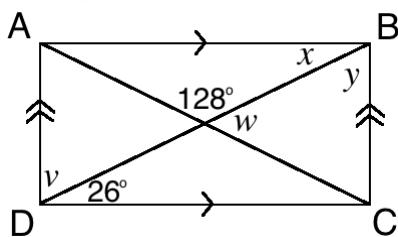


Since we are given  $\angle BCD = 26^\circ$ , we also have  $\angle ABC = 26^\circ$ . Add  $\angle EBA$  and  $\angle ABC$  to find  $\angle EBC$ :  $\angle EBC = \angle EBA + \angle ABC = 23^\circ + 26^\circ = 49^\circ$ .

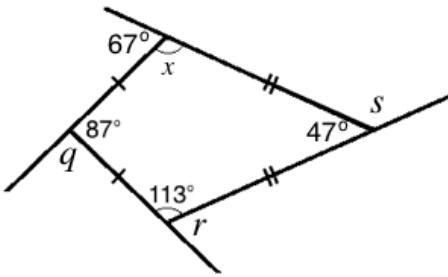
Now, since  $\angle EBC = \angle BCF$ , we have  $\angle BCF = \angle EBC = 49^\circ$ .

9. Write 2 more problems on the board:

- a. In the diagram,  $ABCD$  is a rectangle. Find the measures of  $v$ ,  $w$ ,  $x$  and  $y$ :



- b. Find the missing interior and exterior angles of the kite:



10. Ask pupils to work with seatmates to solve the problems.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write the solutions on the board.

**Solutions:**

- b. Note that the diagonals of the rectangle are transversal lines to each set of parallel lines. From this,  $x = 26^\circ$  because they are alternate angles. The other angles can be found using supplementary and complementary angles:

$$v = 90^\circ - 26^\circ = 64^\circ$$

$$w = 180^\circ - 128^\circ = 62^\circ$$

$$y = 90^\circ - 26^\circ = 64^\circ$$

- c. For each missing angle, a supplementary angle is given. Each can be found by subtracting from  $180^\circ$ :

$$q = 180^\circ - 87^\circ = 93^\circ$$

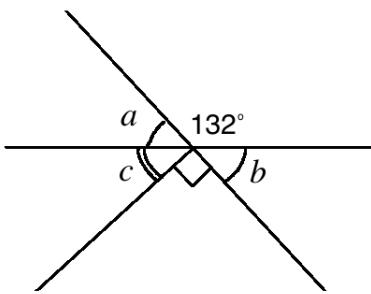
$$r = 180^\circ - 113^\circ = 67^\circ$$

$$s = 180^\circ - 47^\circ = 133^\circ$$

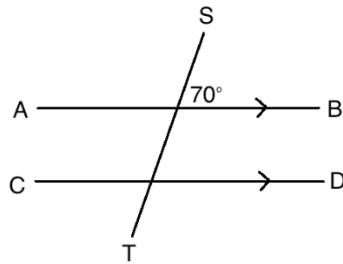
$$x = 180^\circ - 67^\circ = 113^\circ$$

**Practice (14 minutes)**

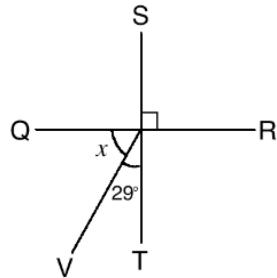
1. Write the following problems on the board:
  - a. In the diagram below, find the measures of  $a$ ,  $b$  and  $c$ :



- b. Find the measure of each angle in the diagram, and label them:



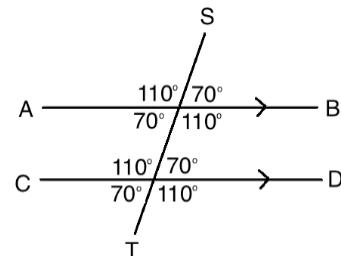
- c. Find the measure of  $x$ :



13. Ask pupils to solve the problem independently. Allow discussion with seatmates if needed.
14. Walk around to check for understanding and clear misconceptions.
15. Allow pupils to exchange their exercise books.
16. Invite volunteers to solve the problems on the board.

**Solutions:**

- a. Note that the following angles are supplementary:  $a$  and  $132^\circ$ , and  $b$  and  $132^\circ$ . Thus,  $a = b = 180^\circ - 132^\circ = 48^\circ$ . There are 2 ways to find  $c$ . One way is to subtract all of the known angles from  $360^\circ$  (one full rotation). Alternatively, subtract  $90^\circ$  and  $b$  from  $180^\circ$ . These angles form a straight line ( $180^\circ$ ). Thus,  $c = 180^\circ - 90^\circ - 48^\circ = 42^\circ$ .
- b. Note that the following angles are equal to the labeled angle,  $70^\circ$ : the opposite angle, and corresponding angles in the other intersection. Find the angles that are supplementary to the labeled angle by subtracting from  $180^\circ$ :  $180^\circ - 70^\circ = 110^\circ$ . Label the supplementary angles  $110^\circ$ . Note that the corresponding angles in the other intersection are also  $110^\circ$ .
- c. Note that  $x$  and  $29^\circ$  are complementary angles. Therefore,  $x = 90^\circ - 29^\circ = 61^\circ$



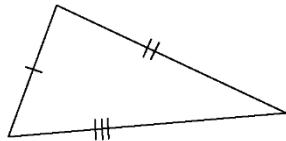
**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L130 in the Pupil Handbook.

<b>Lesson Title:</b> Classification of Triangles: Equilateral, isosceles, and scalene	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L131	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to classify illustrated triangles by their characteristics.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

1. Draw the triangle on the board:



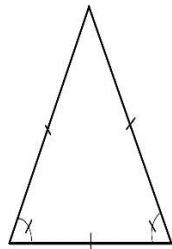
2. Allow pupils to write down everything they know about the triangle on the board.
3. Ask pupils to share their ideas with the class.
4. Invite a volunteer to write down the name of the triangle on the board.  
(Answer: Scalene triangle).
5. Explain to the pupils that today's lesson is to classify illustrated triangles by their characteristics.

### Teaching and Learning (22 minutes)

1. Explain **scalene triangles**:

- Scalene triangles are triangles with three sides of different lengths.
- When sides of a triangle are equal in length, we call them “congruent”. A scalene triangle has no congruent sides.
- All the angles inside a scalene triangle add up to  $180^\circ$ .
- All the angles of a scalene triangle have different measures.

2. Ask pupils to work with seatmates to write down any 3 angles of a scalene triangle.  
Remind them that they must sum up to  $180^\circ$  and all be different.
3. Ask volunteers to share their answers. (Example answers:  $40^\circ, 50^\circ, 90^\circ$ ).
4. Draw an isosceles triangle on the board:



5. Allow pupils 1 minute to write down everything they know. Ask them to share ideas.
6. Invite a volunteer to write down the name of the triangle on the board.

(Answer: Isosceles triangle)

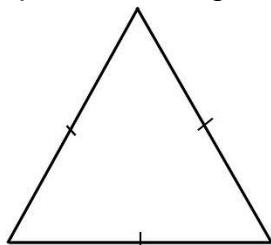
7. Explain **isosceles triangles**:

- Isosceles triangles have 2 sides equal. Two angles are also equal.
- The two equal sides can be called “congruent”.
- No matter which direction the triangles’ apex, or peak, points, it’s an isosceles triangle if two of its sides are equal.
- All the angles inside an isosceles triangle add up to  $180^\circ$ .

8. Write a problem on the board: Write a set of angles that are in an isosceles triangle.

9. Ask volunteers share to their answers. (Example answer:  $40^\circ, 40^\circ, 100^\circ$ )

10. Draw an equilateral triangle on the board:



11. Allow pupils 1 minute to write down everything they know. Ask them to share their ideas.

12. Invite a volunteer to write down the name of the triangle on the board. (Answer: Equilateral triangle)

13. Explain **equilateral triangles**:

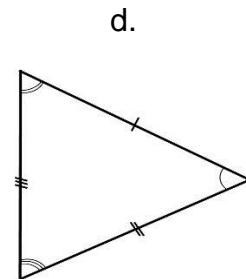
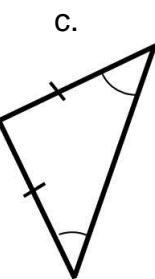
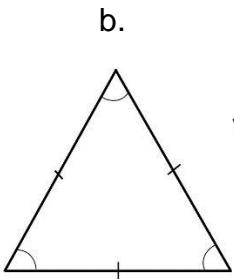
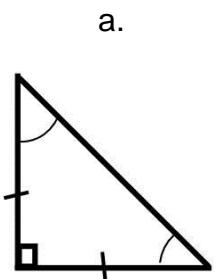
- Equilateral triangles have all three sides and all three angles that are equal, or congruent.
- The angles of an equilateral triangle add up to  $180^\circ$ .

14. Write a question on the board: Write a set of angles that are in an equilateral triangle.

15. Invite a volunteer to write the answer on the board. (Answer:  $60^\circ, 60^\circ, 60^\circ$ )

16. Explain: All 3 angles of an equilateral triangle are always  $60^\circ$ .

17. Write another problem on the board. Classify each triangle based on the labeled sides and angles.



18. Ask pupils to work with seatmates to answer the questions on the board.

19. Walk around, if possible, to check for understanding and clear misconceptions.

20. Ask volunteers from around the classroom to give their answers.

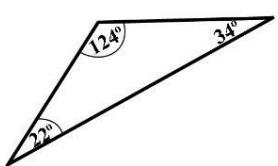
(Answer: a. Isosceles b. Equilateral c. Isosceles d. scalene)

### **Practice (14 minutes)**

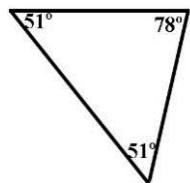
1. Write two problems on the board:

a. Classify each triangle by its angles.

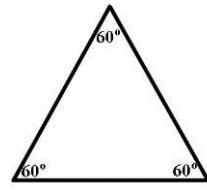
i.



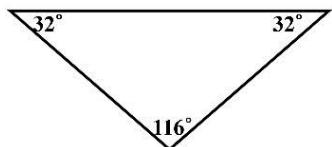
ii.



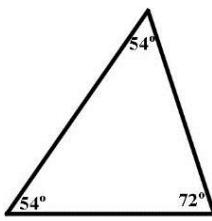
iii.



iv.



v.



b. Identify the type of triangle based on the following information.

i. A triangle with all sides and angles congruent: \_\_\_\_\_

ii. A triangle with no sides congruent: \_\_\_\_\_

iii. A triangle with sides of 11 cm, 15 cm, 11 cm: \_\_\_\_\_

iv. A triangle with angles 103°, 20°, 57°: \_\_\_\_\_

2. Ask pupils to solve the problems independently in their exercise books.

3. Walk around, if possible, to check for understanding and clear misconceptions.

4. Invite volunteers to write their answers on the board.

### **Answers:**

- a. i. Scalene triangle
  - ii. Isosceles triangle
  - iii. Isosceles triangle
  - iv. Isosceles triangle
  - v. Isosceles triangle
- b. i. Equilateral triangle
  - ii. Scalene triangle
  - iii. Isosceles triangle
  - iv. Scalene triangle

### **Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L131 in the Pupil Handbook.

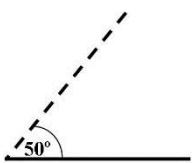
<b>Lesson Title:</b> Drawing of triangles	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L132	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
<b>Learning Outcome</b>  By the end of the lesson, pupils will be able to draw triangles based on numerical data.	<b>Preparation</b>  <ol style="list-style-type: none"> <li>1. Write the problems in Opening on the board.</li> <li>2. Bring a geometry set to class. Ask pupils to do the same.</li> </ol>

### Opening (3 minutes)

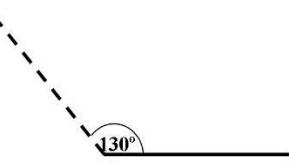
1. Write the following problem on the board: Draw the angles: a.  $50^\circ$       b.  $130^\circ$
2. Ask pupils to draw the angles independently in their exercise books.
3. Invite volunteers to write the answers on the board.

#### Answers:

a.



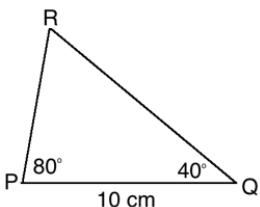
b.



4. Explain to the pupils that today's lesson is to draw triangles based on numerical data.

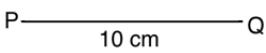
### Teaching and Learning (22 minutes)

1. Explain:
  - Triangles can be drawn using the measures of the triangle's sides and angles.
  - Triangles can be drawn accurately using only 2 angles and the line connecting them.
  - We use a protractor and ruler to draw an accurate triangle.
2. Write the following problem on the board and sketch the triangle: Make an accurate drawing of this triangle:



3. Discuss: When you are given measurements of two angles and one side, always start with the given side.
4. Invite a volunteer to draw a line of 10 cm on the board with a ruler.

#### Answer (not to scale):

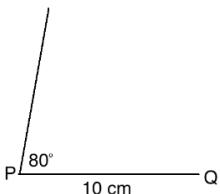


5. Invite a volunteer to use a protractor to draw an angle of  $80^\circ$  at P.

6. Discuss:

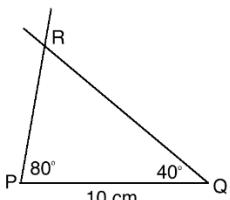
- Draw this line quite long so that when you draw  $RQ$  it will cut the line.
- Make sure the angle looks less than a right angle. Use the inner number of the protractor.

**Answer:**



7. Invite another volunteer to use the protractor to draw an angle of  $40^\circ$  from  $Q$ .

**Answer:**



8. Explain: Make sure the line crosses  $PR$ . You do not need to erase any parts of the line afterwards.

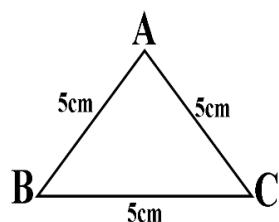
9. Discuss: What kind of triangle is  $PQR$ ? (Answer: scalene, acute)

10. Invite a volunteer to measure angle  $R$  of the triangle. (Answer:  $60^\circ$ )

11. Ask pupils to work with seatmates to verify that the angles of the triangle sum to  $180$ .

12. Invite a volunteer to write their work on the board. (Answer:  $80^\circ + 40^\circ + 60^\circ = 180^\circ$ )

13. Write another problem on the board: Make an accurate drawing of this triangle:



14. Discuss:

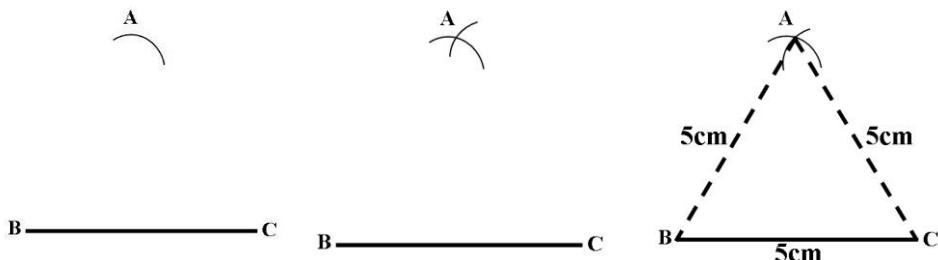
- What type of triangle is  $ABC$ ? (Answer: Equilateral)
- What do you know about equilateral triangles? (Example answers: All sides are equal, all angles are equal to  $60^\circ$ .)
- What step will we take first in drawing the triangle? (Answer: It is useful to start with  $BC$ , the horizontal line at the bottom of the triangle).

15. Invite a volunteer to draw the line  $BC$  on the board (line below is not to scale):

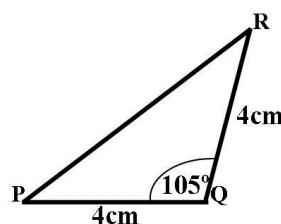


16. Invite volunteers to come to the board and draw angles of  $60^\circ$  from  $B$  and  $C$  to form  $AB$  and  $AC$ .

17. Label the point of intersection of the 2 lines as A.
18. Ask volunteers to measure  $|AB|$  and  $|AC|$  to verify that all of the sides are 5 cm in length.
19. If you have a pair of compasses available, you may also show pupils how to draw an equilateral triangle with the pair of compasses. Set your compass to 5 cm using a ruler, and draw intersecting arcs from B and C to identify point A:



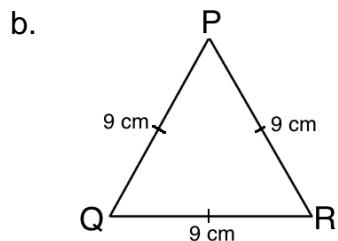
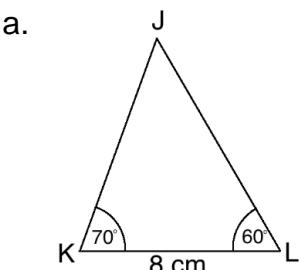
20. Write another problem on the board: Make an accurate drawing of triangle PQR:



21. Ask pupils to solve the problem with seatmates.
22. Walk around, if possible, to check for understanding and clear misconceptions.
23. Invite a volunteer to draw the triangle on the board using a protractor and ruler.

### **Practice (14 minutes)**

1. Write two problems on the board: Make accurate drawings of these triangles:



2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books and check their seatmates' triangles.
5. Invite two volunteers to draw the triangles on the board using a ruler and protractor.

### **Closing (1 minute)**

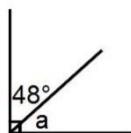
1. For homework, have pupils do the practice activity PHM1-L132 in the Pupil Handbook.

<b>Lesson Title:</b> Interior and exterior angles of a triangle	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L133	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to calculate the measurements of interior and exterior angles of a triangle.	 <b>Preparation</b> Write the problems in Opening on the board.	

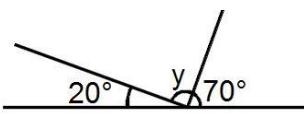
### Opening (3 minutes)

1. Revise complementary and supplementary angles. Draw on the board.

a.



b.



2. Ask volunteers to describe how to find the measures of angle  $a$  and  $y$ . Allow them to share their ideas, then explain.

- Angles  $a$  and  $48^\circ$  make a right angle. We know that together their measures make  $90^\circ$ .
- Angles  $y$ ,  $20^\circ$  and  $70^\circ$  make a straight line. We know that together their measures make  $180^\circ$ .

3. Ask pupils to work with seatmates to find  $a$  and  $y$ .

4. Invite two volunteers to write the solutions on the board.

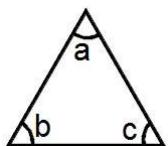
#### Solutions:

- $a + 48^\circ = 90^\circ; a = 90^\circ - 48^\circ = 42^\circ;$
- $y + 20^\circ + 70^\circ = 180^\circ; y = 180^\circ - 90^\circ = 90^\circ$

5. Explain to the pupils that today's lesson is to calculate the measurements of interior and exterior angles of triangles.

### Teaching and Learning (22 minutes)

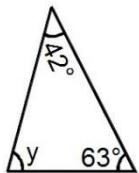
1. Draw a triangle on the board.



2. Explain:

- The angles  $a$ ,  $b$ , and  $c$  are called **interior** angles.
- An interior angle is an angle inside a shape.
- The sum of the interior angles of a triangle is equal to  $180^\circ$ .

3. Write a problem on the board. Find the value of  $y$  from the diagram:

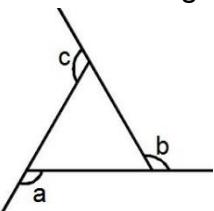


4. Solve the problem on the board, explaining each step.

**Solution:**

$$\begin{aligned}y + 42^\circ + 63^\circ &= 180^\circ \\y + 105^\circ &= 180^\circ \\y &= 180^\circ - 105^\circ \\y &= 75^\circ\end{aligned}$$

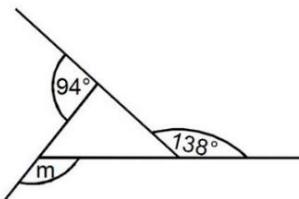
5. Draw the following triangle on the board:



6. Explain:

- The angles  $a$ ,  $b$  and  $c$  are called **exterior** angles.
- The exterior angles of a triangle are the angles that are supplementary and adjacent to the interior angles. They are made by extending the sides of a triangle.
- The sum of the exterior angles is equal to  $360^\circ$ .

7. Write the following problem on the board: Find the value of  $m$ :

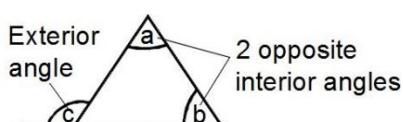


8. Solve the problem on the board.

**Solution:**

$$\begin{aligned}m + 94^\circ + 138^\circ &= 360^\circ \\m + 232^\circ &= 360^\circ \\m &= 360^\circ - 232^\circ \\m &= 128^\circ\end{aligned}$$

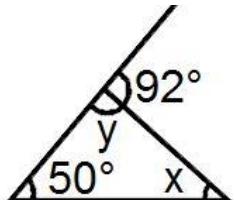
9. Draw a triangle on the board.



10. Explain:

- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.
- In the diagram,  $\angle a + \angle b = \angle c$  (write the equation on the board).

11. Write a problem on the board. Find the value of  $x$  and  $y$  in the following triangle:



12. Ask pupils to work with seatmates to calculate  $y$ .

13. Invite a volunteer to write the solution on the board.

$$\begin{aligned}y + 92^\circ &= 180^\circ && \text{Adjacent angles} \\y &= 180^\circ - 92^\circ \\y &= 88^\circ\end{aligned}$$

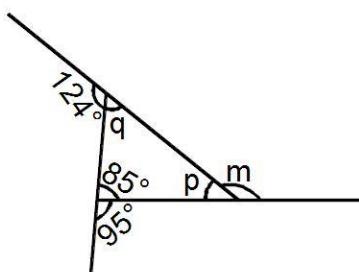
14. Ask pupils to work with seatmates to calculate  $x$ .

15. Invite a volunteer to write the answer on the board.

$$\begin{aligned}x + 50^\circ &= 92^\circ && \text{Two opposite interior angles are equal} \\x &= 92^\circ - 50^\circ && \text{to the exterior angle} \\x &= 42^\circ\end{aligned}$$

Note that pupils could also find  $x$  using the sum of the interior angles of a triangle.

16. Write another problem on the board. Find the values of  $m$ ,  $p$  and  $q$ .



17. Ask pupils to work with seatmates to find the measures of the unknowns.

18. Walk around, if possible, to check for understanding and clear misconceptions.

19. Invite 3 volunteers to write the solutions of each of the unknown letters on the board.

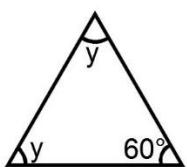
**Solution:**

$$\begin{array}{lll}m + 124^\circ + 95^\circ = 360^\circ & p + m = 180^\circ & q + 85^\circ + p = 180^\circ \\(\text{Sum of exterior angles of a triangle}) & (\text{adjacent angles}) & (\text{Sum of angles in a triangle}) \\m + 219^\circ = 360^\circ & p + 141^\circ = 180^\circ & q + 85^\circ + 39^\circ = 180^\circ \\m = 360^\circ - 219^\circ & p = 180^\circ - 141^\circ & q + 124^\circ = 180^\circ \\m = 141^\circ & p = 39^\circ & q = 180^\circ - 124^\circ = 56^\circ\end{array}$$

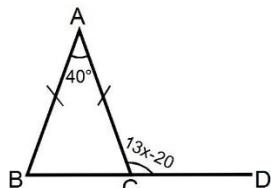
### Practice (14 minutes)

1. Write on the board: Find the value of each letter in the following diagrams:

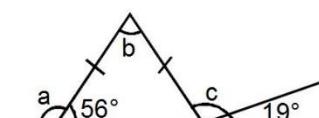
a.



b.



c.



2. Ask pupils to solve the problems independently to find the missing angles.  
3. Invite 3 volunteers to write the solutions on the board, and label the missing angles in the shapes. They may come to the board at the same time.

#### Solutions:

e.  $y + y + 60^\circ = 180^\circ$

(Sum of angles in a triangle)

$$2y + 60^\circ = 180^\circ$$

$$2y = 180^\circ - 60^\circ$$

$$2y = 120^\circ$$

$$y = \frac{120}{2}$$

$$y = 60^\circ$$

f. Triangle ABC is an isosceles triangle.

It means  $\angle ABC = \angle ACB$

$\angle ABC + \angle ACB + 40^\circ = 180^\circ$  (Sum of interior angles)

$$\angle ABC + \angle ACB = 180^\circ - 40^\circ$$

$$\angle ABC + \angle ACB = 140^\circ$$

$$\angle ABC = \angle ACB = \frac{140}{2} = 70^\circ$$

$70^\circ + (13x - 20^\circ) = 180^\circ$  (adjacent angles)

$$70^\circ + 13x - 20^\circ = 180^\circ$$

$$13x = 180^\circ - 70^\circ + 20^\circ$$

$$13x = 130^\circ$$

$$x = \frac{130}{13}$$

$$x = 10^\circ$$

- g. Since the triangle is isosceles, the other base angle is  $56^\circ$

$$a + 56^\circ = 180^\circ$$

(adjacent angles)

$$a = 180^\circ - 56^\circ$$

$$a = 124^\circ$$

$$b + 56^\circ + 56^\circ = 180^\circ$$

(Sum of interior angles in a triangle)

$$b + 112^\circ = 180^\circ$$

$$b = 180^\circ - 112^\circ$$

$$b = 68^\circ$$

$$56^\circ + c + 19^\circ = 180^\circ$$

(Sum of angles on a straight line)

$$c + 75^\circ = 180^\circ$$

$$c = 180^\circ - 75^\circ$$

$$c = 105^\circ$$

### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L133 in the Pupil Handbook.

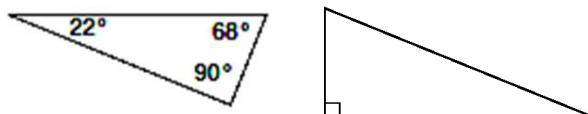
<b>Lesson Title:</b> Acute-, obtuse-, and right-angled triangles	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L134	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcomes</b> By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"> <li>Identify characteristics of acute-, obtuse-, and right-angled triangles.</li> <li>Classify triangles as acute, obtuse, or right.</li> </ol>	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

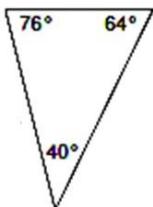
- Ask pupils to draw a right-angled triangle in their exercise books. Give them 30 seconds.
- Ask pupils to check the triangles seatmates drew.
- Explain to pupils that today's lesson is to identify and classify angles as acute-, obtuse- or right-angled.

### Teaching and Learning (22 minutes)

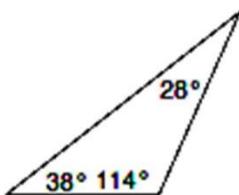
- Invite a volunteer to draw a right-angled triangle on the board. Examples:



- Allow pupils 1 minute to write everything they know about right-angled triangles in their exercise books.
- Ask volunteers to share what they wrote with the class. (Example answer: A right angle triangle has one angle equal to 90°)
- Explain:
  - One angle is equal to 90°, which can be marked with a square.
  - The other 2 angles are smaller than 90° (acute angles).
  - The longest side is called the hypotenuse.
  - The sum of the angles in the triangle is 180°. The 2 acute angles sum to 90°.
- Draw a triangle on the board an acute-angled triangle:

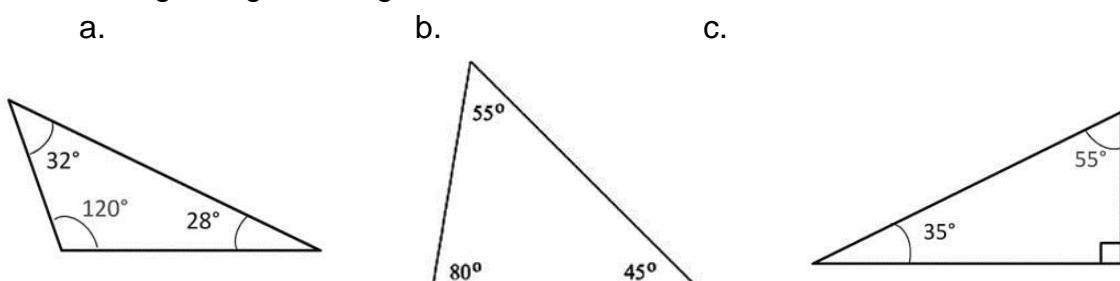


6. Allow pupils 1 minute to write down everything they know about acute-angled triangles.
7. Ask volunteers to share their ideas with the class. (Example answer: All the angle measures are less than  $90^\circ$ )
8. Explain:
  - An acute-angled triangle is a triangle with all three angles acute (less than  $90^\circ$ )
  - The sum of the angles in the triangle is  $180^\circ$ .
  - An equilateral triangle is always an acute-angled triangle because all of its angles are  $60^\circ$ . Some scalene and isosceles triangles are acute-angled triangles.
9. Draw an obtuse-angled triangle on the board:



10. Allow pupils 1 minute to write down everything they know about obtuse-angled triangles.
11. Ask volunteers to share their ideas with the class. (Example answer: An obtuse-angled triangle has exactly one obtuse angle.)

12. Explain:
  - An obtuse-angled triangle can only have one angle that measures more than  $90^\circ$ .
  - The sum of the angles in the triangle is  $180^\circ$ .
  - Scalene and isosceles triangles can be obtuse-angled triangles.
13. Write the following problem on the board. Classify each triangle as an acute-, obtuse- or right-angled triangle:



14. Ask pupil to work with seatmates to answer the question on the board.
15. Walk around, if possible, to check for understanding and clear misconceptions.
16. Ask volunteers to give their answers to the question.
17. Write down the correct answer on the board. Ask pupils to check their work.

(Answer: a. Obtuse-angled triangle; b. Acute-angled triangle; c. Right-angled triangle).

18. Write another question on the board: Identify the type of triangle based on the following information:

- a. A triangle with one angle  $90^\circ$ .
- b. A triangle with angles  $103^\circ$ ,  $20^\circ$ ,  $57^\circ$ .
- c. A triangle with angles  $49^\circ$ ,  $88^\circ$ ,  $43^\circ$ .
- d. A triangle with angles  $47^\circ$ ,  $90^\circ$ ,  $43^\circ$ .

19. Ask pupils to work with in pairs to answer the questions on the board.

20. Walk around, if possible, to check for understanding and clear misconceptions.

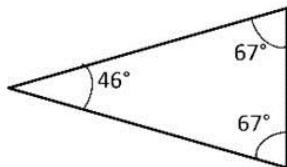
21. Invite 4 volunteers to give their answers, one at a time, on the board. (Answers: a. Right-angled triangle; b. Obtuse-angled triangle; c. Acute-angled triangle; d. Right-angled triangle)

### Practice (14 minutes)

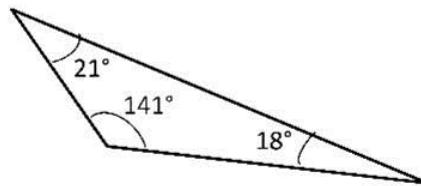
1. Write two problems on the board:

- a. Are the triangles acute-, obtuse- or right-angled triangles?

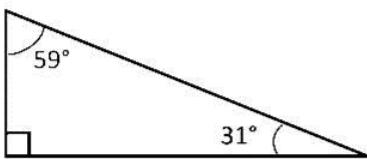
i.



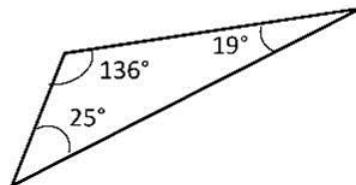
ii



iii.



iv



- b. Identify the types of triangles based on the following information.

i. A triangle with angles  $108^\circ$ ,  $54^\circ$ ,  $18^\circ$ .

ii. A triangle with angles  $1^\circ$ ,  $89^\circ$ ,  $90^\circ$ .

iii. A triangle with angles  $4^\circ$ ,  $89^\circ$ ,  $87^\circ$ .

c. Draw an obtuse-angled, isosceles triangle.

d. Draw an acute-angled isosceles triangle.

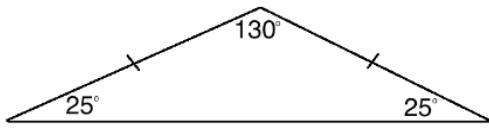
2. Ask pupils to solve the problems independently in their exercise books.

3. Walk around, if possible, to check for understanding and clear misconceptions.

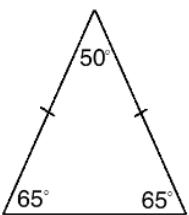
4. Invite volunteers to write their answers on the board.

**Answers:**

- a. i. Acute-angled triangle; ii. Obtuse-angled triangle; iii. right-angled triangle;  
iv. Obtuse-angled triangle.
- b. i. Obtuse-angled triangle; ii. Right-angled triangle; iii. Acute-angled triangle
- c. Example answer:



- d. Example answer:



**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L134 in the Pupil Handbook.

<b>Lesson Title:</b> Congruent and similar triangles	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L135	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to classify triangles as similar or congruent.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board. Identify the types of triangles based on the following information:
  - a triangle with angles  $30^\circ, 49^\circ, 101^\circ$ .
  - A triangle with one angle  $90^\circ$ .
  - A triangle with angles  $82^\circ, 49^\circ, 49^\circ$ .
2. Ask pupils to solve the problems independently in their exercise books.
3. Ask volunteers to share their answers with the class. (Answers: a. Obtuse-angled, Scalene triangle; b. Right-angled triangle; c. Acute-angled, isosceles triangle)
4. Explain to pupils that today's lesson is to classify triangles as similar or congruent.

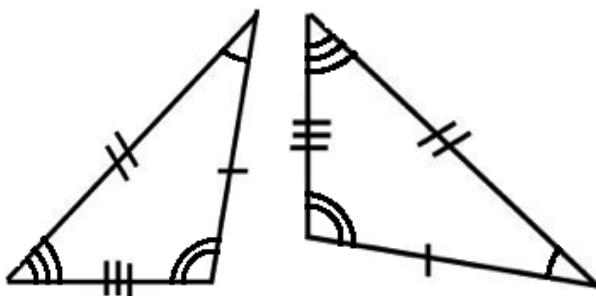
### Teaching and Learning (22 minutes)

1. Ask pupils when you hear the word "similar" what comes to mind?
2. Invite pupils from around the classroom to volunteer an answer. (Example answer: Things that look alike; Things that have something in common).
3. Draw on the board two triangles:



4. Explain:
  - Similar triangles have the same shape, but are of different sizes.
  - In the above diagram  $\angle B = \angle E$ ,  $\angle C = \angle F$ , and  $\angle A = \angle D$ .
5. Discuss: What do you notice about triangle  $ABC$  and triangle  $DEF$ ? (Example answer: The triangles are similar).
6. Explain:
  - Each side and angle in  $\triangle ABC$  has a corresponding side or angle in  $\triangle DEF$ .
  - For example, we can say that  $\angle B$  corresponds to  $\angle E$ .
  - Since  $\triangle ABC$  is similar to  $\triangle DEF$ , their corresponding angles are equal, and their corresponding sides are in the same ratio.
7. Ask pupils to write down the sides of the triangle that correspond to each other.

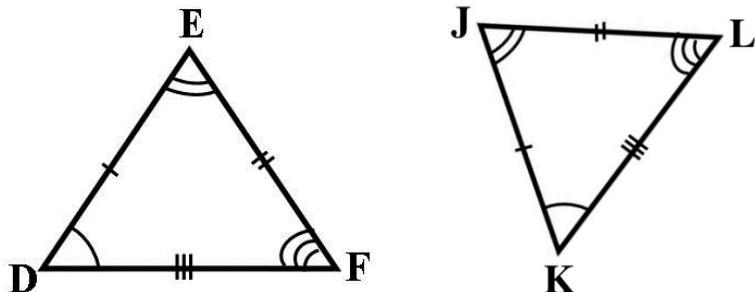
8. Allow them to work with seatmates.
9. Ask volunteers to give the corresponding sides. (Answer:  $|AB|$  and  $|DE|$ ;  $|BC|$  and  $|ED|$ ;  $|CA|$  and  $|FD|$ )
10. Write on the board:  $\frac{|AB|}{|DE|} = \frac{|BC|}{|ED|} = \frac{|CA|}{|FD|}$
11. Explain: The sides of similar triangles form ratios, which can be used to solve for missing sides in similar triangles.
12. Remind pupils that for any pair of similar shapes, corresponding sides are in the same ratio and corresponding angles are equal.
13. Draw two scalene triangles on the board.



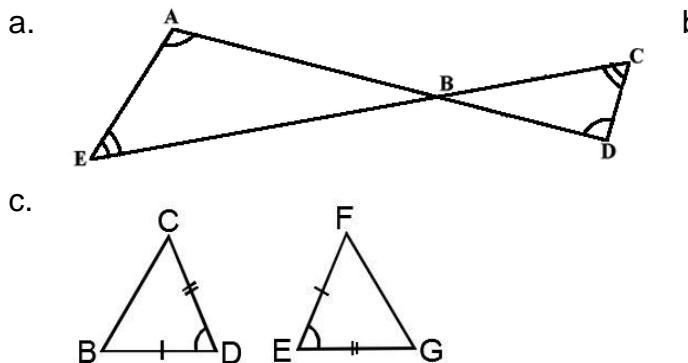
14. Ask pupils to discuss 1 minute what they know about the two triangles.
15. Allow them to discuss with seatmates.
16. Ask a volunteer to answer the question. (Answer: The two triangles have exactly the same shape and size; one is rotated.)
17. Explain:
  - The two triangles are **congruent**. Although at first sight, they may appear different, they are exactly the same.
  - Congruent shapes have exactly the same shape and size.
  - If one of these triangles were rotated and moved, it could be arranged to fit exactly over the other.
  - A triangle can be translated (moved), rotated, or flipped, and the result is still a congruent triangle.
18. Discuss:
  - When naming congruent triangles, give the letters in the correct order so that it is clear which letters of the triangles correspond to each other.
  - In geometry, the symbol  $\equiv$  means “is identically equal to” or “is congruent to”.
19. Explain: Two triangles are congruent if:
  - Two sides and the included angle of one are respectively equal to two sides and the included angle of the other (SAS).
  - Two angles and a side of one are respectively equal to two angles and the corresponding side of the other (ASA or AAS).
  - The three sides of one are respectively equal to the three sides of the other (SSS).

- They are right-angled, and have the hypotenuse and another side of one respectively equal to the hypotenuse and another side of the other (RHS).

20. Write a problem on the board: Show that the 2 triangles are congruent by naming the corresponding sides and angles.



21. Invite volunteers to write on the board the sides that are congruent. (Answer:  $|ED| = |JK|$ ,  $|EF| = |JL|$ ,  $|DF| = |KL|$ )
22. Ask another volunteer to write down the angles that are congruent. (Answer:  $\angle EDF = \angle JKL$ ,  $\angle DEF = \angle KJL$ ,  $\angle DFE = \angle KLJ$ ).
23. Write on the board:  $\Delta DEF \cong \Delta KJL$
24. Explain: This states that  $\Delta DEF$  is congruent to  $\Delta KJL$ . Remember to write the angles of the triangle in the correct order. For example, D and K are written first because they correspond to each other.
25. Write 3 problems on the board: Identify any similar or congruent triangles in the diagrams:



26. Ask pupils to work with seatmates to solve the problems.
27. Walk around to check for understanding and clear misconceptions.
28. Ask volunteers to give the answers and explain how they know. (Answers: a. ABE and DBC are similar; b. XNY and MNO are similar; c. BCD and FGE are congruent)

### **Practice (14 minutes)**

1. Write the following problem on the board:
  - a. In the diagram below, the lines  $DE$  and  $BC$  are parallel.

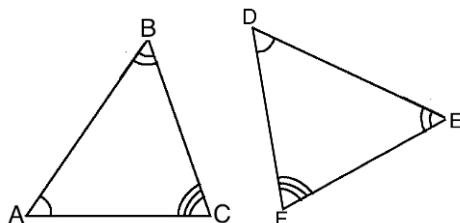
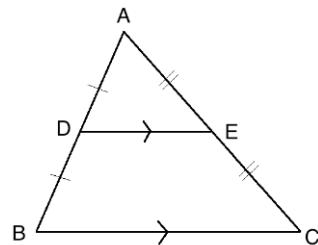
- i. Copy and complete the following statements with another angle in the figure:

$$\angle ADE = \angle \underline{\hspace{1cm}}$$

$$\angle AED = \angle \underline{\hspace{1cm}}$$

- ii. State whether there are any similar or congruent triangles in the diagram. Give a reason for your answer.

- b. Identify whether the triangles are congruent, and give your reason:



2. Ask pupils to solve the problems independently with seatmates.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Invite volunteers to write the answers on the board.

**Answers:**

- a. i.  $\angle ADE = \angle ABC$  and  $\angle AED = \angle ACB$ ; ii. The triangles are similar and not congruent because the angles are equal, but the corresponding lengths are not equal.
- b. There is not enough information to determine if the triangles are congruent. Note that this can only be determined from the following triangles: SAS, ASA, AAS, SSS.

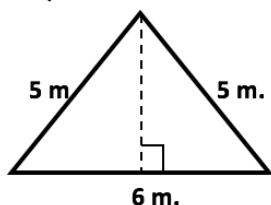
**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L135 in the Pupil Handbook.

<b>Lesson Title:</b> Area of triangles	<b>Theme:</b> Geometry
<b>Lesson Number:</b> M1-L136	<b>Class:</b> SSS 1 <b>Time:</b> 40 minutes
<b>Learning Outcomes</b>  By the end of the lesson, pupils will be able to: 1. Calculate the area of a triangle given the base and the height. 2. Calculate the area given the three sides.	<b>Preparation</b>  Write the problems in Opening on the board.

### Opening (3 minutes)

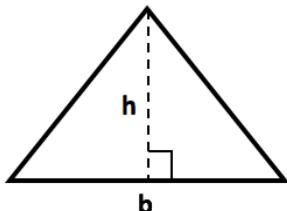
1. Write the following problem on the board: Calculate the perimeter of the triangle:



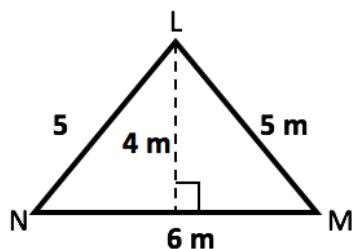
2. Ask pupils to solve the problem independently.  
 3. Invite volunteers to answer the question on the board. (Answer:  $P = a + b + c = 5 + 5 + 6 = 16$  m).  
 4. Explain to pupils that today's lesson is to calculate the area of a triangle given the base and height and also given three sides.

### Teaching and Learning (22 minutes)

1. Draw on the board a triangle.



2. Write the formula on the board: Area of a triangle =  $\frac{1}{2}$  base  $\times$  height =  $\frac{1}{2} bh$   
 3. Write on the board: Find the area of the triangle:

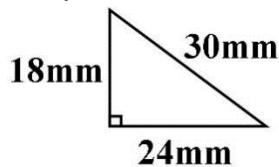


4. Ask a volunteer to give the base of the triangle. (Answer: Side  $NM$  which is 6 m in length)
5. Ask a volunteer to give the height of the triangle. (Answer: 4 m)
6. Discuss:
  - There are 2 numbers we need to find the area of a triangle. We will substitute them into the formula.
  - Base and height are always perpendicular to each other. You can take any side of the triangle as its base.
  - Then you can find the height of the triangle from that base. The height is a perpendicular line drawn from the base to the opposite angle.

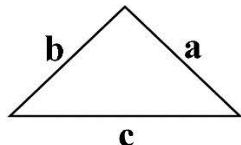
7. Solve the problem on the board:

$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 6 \times 4 \\
 &= 3 \times 4 \\
 &= 12 \text{ m}^2
 \end{aligned}$$

8. Explain: The final answer is in square units whenever you calculate an area.
9. Write another problem on the board. Find the area of the triangle.



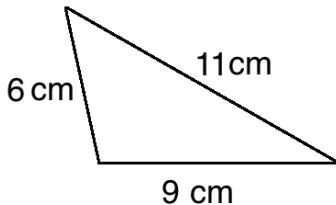
10. Ask a volunteer to give the base. (Answer: 24 mm)
11. Ask a volunteer to give the height. (Answer: 18 mm)
12. Remind pupils that the height is perpendicular to the base. It reaches from the base to the opposite angles.
13. Ask pupils to find the area with seatmates.
14. Invite a volunteer to write the solution on the board. (Answer:  $A = \frac{1}{2}bh = \frac{1}{2} \times 24 \times 18 = 216 \text{ mm}^2$ )
15. Draw on the board a triangle:



16. Explain: In some cases, we know the lengths of the 3 sides of a triangle, but we do not know the height.
  17. Write the formula on the board.
  18. Explain:
- Area of a triangle =  $\sqrt{s(s - a)(s - b)(s - c)}$ , where  $s = \frac{a+b+c}{2}$

- You can calculate the area of a triangle if you know the length of all three sides.
- It is called “Heron’s Formula” or “Hero’s Formula” after Hero of Alexandria.

19. Write the following problem on the board: Find the area of the triangle below:



20. Write the solution on the board and explain:

Let  $a = 6, b = 9$  and  $c = 11$

$$\text{Step 1. Find } s: s = \frac{a+b+c}{2} = \frac{6+9+11}{2} = \frac{26}{2} = 13$$

**Step 2.** Substitute  $s = 13; a = 6; b = 9; c = 11$  into Heron’s formula:

$$\begin{aligned} A &= \sqrt{13(13 - 6)(13 - 9)(13 - 11)} \\ &= \sqrt{13(7)(4)(2)} \\ &= \sqrt{728} \\ &= 27 \text{ cm}^2 \end{aligned}$$

21. Write the following problem on the board: What is the area of a triangle where every side is 4 m long? Give your answer correct to 1 decimal place.

22. Ask pupils to solve the problem with seatmates.

23. Walk around, if possible, to check for understanding and clear misconceptions.

24. Invite a volunteer to write the solution on the board.

**Solution:**

$a = 4; b = 4$  and  $c = 4$

$$\text{Step 1. } S = \frac{a+b+c}{2} = \frac{4+4+4}{2} = \frac{12}{2} = 6$$

$$\begin{aligned} \text{Step 2. } A &= \sqrt{6(6 - 4)(6 - 4)(6 - 4)} \\ &= \sqrt{6 \times 2 \times 2 \times 2} \\ &= \sqrt{48} \\ &= 6.928 \\ &= 6.9 \text{ m}^2 \end{aligned}$$

### Practice (14 minutes)

1. Write two problems on the board:

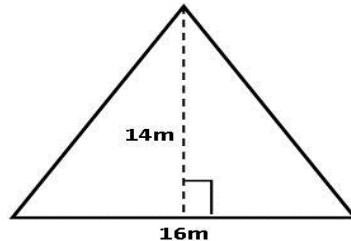
- If the base of a triangle is 16 m and the height is 14 m, what is the area of the triangle?
- Use Heron’s formula to find the area of a triangle of lengths 7 cm, 8 cm and 9 cm. Give your answer in surd form.

2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around, if possible, to check their answers and clear misconceptions.
4. Allow pupils to exchange their books.
5. Have 2 volunteers write the solutions on the board at the same time.

**Solutions:**

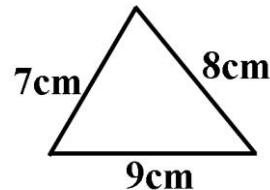
a.

$$\begin{aligned}
 \text{Area of triangle} &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 16 \times 14 \\
 &= 8 \times 14 \\
 &= 112 \text{ m}^2
 \end{aligned}$$



b.

$$\begin{aligned}
 a &= 7; b = 8; c = 9 \\
 s &= \frac{a+b+c}{2} = \frac{7+8+9}{2} = \frac{24}{2} = 12 \\
 A &= \sqrt{12(12-7)(12-8)(12-9)} \\
 &= \sqrt{12 \times 5 \times 4 \times 3} \\
 &= \sqrt{720} \\
 &= \sqrt{36 \times 20} \\
 &= 6 \times \sqrt{20} \\
 &= 6 \times 2\sqrt{5} \\
 &= 12\sqrt{5} \text{ cm}^2
 \end{aligned}$$



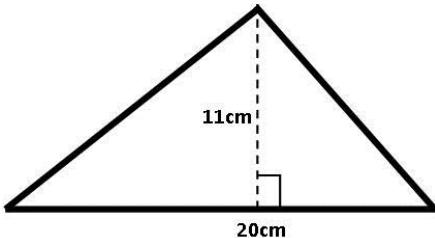
**Closing (1 minute)**

1. For homework, have pupils do the practice activity PHM1-L136 in the Pupil Handbook.

<b>Lesson Title:</b> Word problems involving triangles	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L137	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to solve word problems involving triangles.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

1. Write a problem on the board: Find the area of the triangle below:

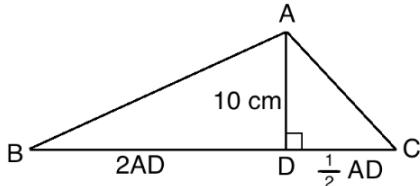


2. Ask pupils to solve the problem independently in their exercise books.
3. Invite a volunteer to write the answer on the board.  
 (Answer:  $A = \frac{1}{2}bh = \frac{1}{2} \times 20 \times 11 = 10 \times 11 = 110\text{cm}^2$ )
4. Explain to the pupils that today's lesson is to solve word problems involving triangles.

### Teaching and Learning (22 minutes)

1. Write the following problem on the board: In triangle  $ABC$ ,  $AD$  is its height, where  $D$  is a point on side  $BC$ .  $AD = 10$  cm, the length of  $CD$  is half the length of  $AD$ , and the length of  $BD$  is twice the length of  $AD$ . What is the area of triangle  $ABC$ ?
2. Ask pupils to work with seatmates to draw the shape.
3. Invite a volunteer to sketch the diagram on the board.

**Answer:**



4. Ask a volunteer to calculate length  $CD$ . (Answer:  $CD = \frac{1}{2}AD = \frac{1}{2} \times 10 = 5$  cm)
5. Ask a volunteer to calculate the length of  $BD$ . (Answer:  $BD = 2AD = 2 \times 10 = 20$  cm).
6. Ask another volunteer to calculate the length of  $BC$ . (Answer:  $BC = BD + CD = 20 + 5 = 25$ )
7. Explain to pupils that the length  $BC = 25$  cm is the base of the triangle.

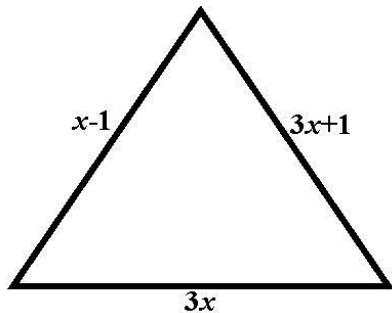
- Ask pupils to work with seatmates to calculate the area of triangle  $ABC$ .
- Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned} A &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 25 \times 10 \\ &= 25 \times 5 \\ &= 125 \text{ cm}^2 \end{aligned}$$

- Write the following problem on the board. The sides of a triangle are given as  $3x$ ,  $x - 1$  and  $3x + 1$ . If the perimeter is 56 cm, find the area.
- Invite a volunteer to sketch the diagram on the board.

**Answer:**



- Discuss:
  - What is a perimeter? (Answer: the distance around the object; the sum of its sides.)
  - How will you solve this problem? (Answer: Use the perimeter to find  $x$ , then use  $x$  to find the area.)

- Ask pupils to work with seatmates to find the value of  $x$ .

- Invite a volunteer to calculate  $x$  on the board.

**Solution:**

$$\begin{aligned} P &= x - 1 + 3x + 1 + 3x \\ 56 &= 7x - 1 + 1 \\ 56 &= 7x \\ x &= \frac{56}{7} \\ x &= 8 \end{aligned}$$

- Invite a volunteer to come to the board and find the lengths of the sides by substituting the value of  $x = 8$  into each.

**Solution:**

$$\begin{aligned} x - 1 &= 8 - 1 = 7 \\ 3x &= 3 \times 8 = 24 \\ 3x + 1 &= 3 \times 8 + 1 = 25 \end{aligned}$$

16. Ask pupils to work with seatmates to find the area of the triangle. Remind them to use Heron's formula.

17. Invite a volunteer to write the solution on the board.

**Solution:**

**Step 1.** Find  $s$ :  $s = \frac{a+b+c}{2} = \frac{7+24+25}{2} = \frac{56}{2} = 28$

**Step 2.** Calculate area:

$$\begin{aligned} A &= \sqrt{s(s-a)(s-b)(s-c)} && \text{Heron's Formula} \\ &= \sqrt{28(28-7)(28-24)(28-25)} \\ &= \sqrt{28 \times 21 \times 4 \times 3} \\ &= \sqrt{7056} \\ &= 84 \text{ cm}^2 \end{aligned}$$

18. Write the following problem on the board: Find the height of a triangle whose base is 11 cm and whose area is 88 cm<sup>2</sup>.

19. Ask pupils to solve the problem with seatmates.

20. Walk around, if possible, to check for understanding and clear misconceptions.

21. Invite a volunteer to write the solution on the board.

**Solution:**

$$A = \frac{1}{2} \times b \times h$$

$$88 = \frac{1}{2} \times 11 \times h$$

$$176 = 11h$$

$$h = \frac{176}{11}$$

$$h = 16$$

The height of the triangle is 16 cm.

### Practice (14 minutes)

1. Write two problems on the board:

- The height of a triangle is 4 m more than twice the length of the base. The area of the triangle is 35 m<sup>2</sup>. Find the height of the triangle.
- A triangle has a perimeter of 50. If two of its sides are equal and the third side is 5 more than the equal sides, what is the length of the third side?

2. Ask pupils to solve the problems independently.

3. Walk around, if possible, to check for understanding and clear misconceptions.

4. Allow pupils to exchange their exercise books.

5. Invite two volunteers, one at a time, to solve the problems on the board.

**Solutions:**

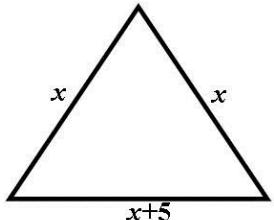
- If the base is  $b$ , then height is  $h = 2b + 4$ .

$$A = \frac{1}{2} \times b \times h \quad \text{Area formula}$$

$$\begin{aligned}
 35 &= \frac{1}{2}b(2b + 4) && \text{Substitute } h = 2b + 4 \\
 70 &= b(2b + 4) \\
 70 &= 2b^2 + 4b \\
 2b^2 + 4b - 70 &= 0 && \text{Quadratic equation} \\
 b^2 + 2b - 35 &= 0 && \text{Divide throughout by 2} \\
 b^2 + 7b - 5b - 35 &= 0 && \text{Factorise} \\
 b(b + 7) - 5(b + 7) &= 0 \\
 (b + 7)(b - 5) &= 0 \\
 \text{Either } b + 7 &= 0 \quad \text{or} \quad b - 5 = 0 \\
 b &= 0 - 7 && b = 0 + 5 \\
 b &= -7 && b = 5
 \end{aligned}$$

Therefore the height of the triangle is 5 metres, because height cannot be negative.

- b. First, draw a diagram. Let  $x$  be the length of the 2 equal sides.



$$\begin{aligned}
 P &= x + x + x + 5 && \text{Perimeter} \\
 50 &= 3x + 5 && \text{Solve for } x \\
 3x &= 50 - 5 \\
 3x &= 45 \\
 x &= \frac{45}{3} \\
 x &= 15
 \end{aligned}$$

Therefore the length of the third side is  $x + 5 = 15 + 5 = 20$ .

### Closing (1 minute)

- For homework, have pupils do the practice activity PHM1-L137 in the Pupil Handbook.

<b>Lesson Title:</b> Finding the hypotenuse of a right triangle	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L138	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to find the hypotenuse of a right-angled triangle using Pythagoras' theorem.	 <b>Preparation</b> Write the problems in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board. Write 2 similarities and 2 differences between a scalene right-angled triangle and an isosceles right-angled triangle.
2. Ask pupils to work independently to answer the question.
3. Have volunteers from around the classroom to give their answers.

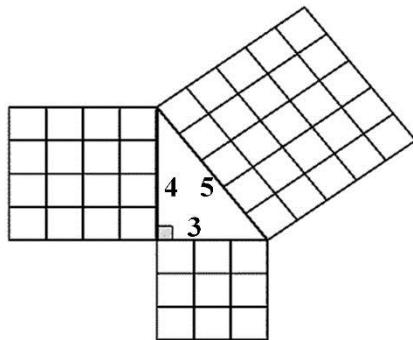
#### Answers:

SIMILARITIES	DIFFERENCES
Both have $90^\circ$ angles	Scalene has 3 different angles, isosceles has 2 angles the same size ( $45^\circ$ )
Both have one long side and 2 shorter sides	Scalene has 3 sides of different lengths; isosceles has 2 sides of same lengths.
Both have 2 acute angles which are complementary angles	

4. Explain to pupils that today's lesson is to find the hypotenuse of a right-angled triangle using Pythagoras' theorem.

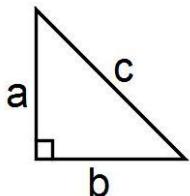
### Teaching and Learning (22 minutes)

1. Draw on the board: Illustration on the area of a square in the same 3-4-5 right-angled triangle:



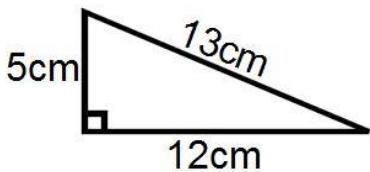
2. Explain: In Mathematics, the Pythagorean Theorem, also known as Pythagoras' theorem, is a relation among the three sides of a right-angled triangle.

3. Ask a volunteer to count the number of squares on the different sides in the diagram.  
(Answer: 9, 16 and 25).
4. Explain:
  - The diagram shows the formula for the Pythagorean Theorem.
  - In a right-angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides ( $9 + 16 = 25$ )
  - The longest side of the triangle is called the hypotenuse.
5. Point to the squares on the hypotenuse of the triangle and show how its area is equal to the sum of the areas of the other 2 sides. You may need to repeat this explanation more than once.
6. Write on the board. Find the square roots of 9, 16 and 25.
7. Invite volunteers to write the answers on the board. (Answer:  $\sqrt{9} = 3$ ;  $\sqrt{16} = 4$ ;  $\sqrt{25} = 5$ )
8. Write on the board:  $3^2 + 4^2 = 5^2$
9. Write the general formula on the board:



$$a^2 + b^2 = c^2$$

10. Explain: If we know the length of any 2 sides of a right-angled triangle, we can find the length of the missing side using Pythagoras' theorem.
11. Write on the board. Please verify Pythagoras' Theorem for the triangle below:



12. Write on the board:

$$a = 5, b = 12, \text{ and } c = 13$$

$$a^2 + b^2 = c^2$$

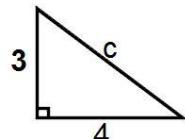
$$5^2 + 12^2 = 13^2$$

$$25 + 144 = 169$$

$$169 = 169$$

True

13. Write on the board: Find the length of c:



14. Ask a volunteer to give the values of a and b. (Answer:  $a = 3, b = 4$ )

15. Ask another volunteer to explain how to find the missing side  $c$ . (Example answer: substitute the values of the unknown sides,  $a = 3$  and  $b = 4$  into Pythagoras' Theorem).

16. Write the solution on the board:

**Solution:**

$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 4^2 \quad \text{Substitute the values}$$

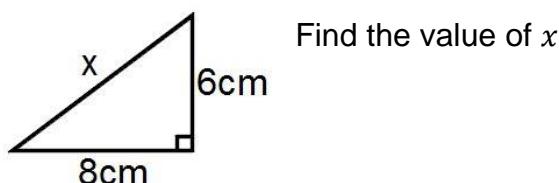
$$c^2 = 9 + 16 \quad \text{Simplify}$$

$$c^2 = 25$$

$$c = \sqrt{25} \quad \text{Take the square root of both sides}$$

$$c = 5$$

17. Write the following problem on the board:



18. Ask a volunteer to state the values of  $a$  and  $b$ . (Answer:  $a = 6$  cm,  $b = 8$  cm)

19. Ask volunteers to give the steps to solve. Solve on the board:

$$c^2 = a^2 + b^2$$

$$x^2 = 6^2 + 8^2 \quad \text{Substitute the values}$$

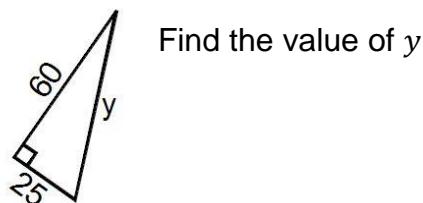
$$x^2 = 36 + 64 \quad \text{Simplify}$$

$$x^2 = 100$$

$$x = \sqrt{100} \quad \text{Take the square root of both sides}$$

$$x = 10 \text{ cm}$$

20. Write another problem on the board:



21. Ask pupils to work with seatmates to find the missing value of  $y$ .

22. Walk around to check for understanding and clear misconceptions.

23. Invite a volunteer to write the solution on the board.

**Solution:**

$$y^2 = 60^2 + 25^2$$

$$y^2 = 3,600 + 625$$

$$y^2 = 4,225$$

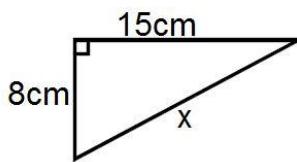
$$y = \sqrt{4,225}$$

$$y = 65$$

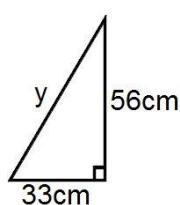
### Practice (14 minutes)

1. Write the following three problems on the board: Find the values of the missing sides in the following diagrams:

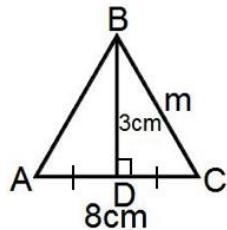
a.



b.



c.



2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their books.
5. Invite three volunteers, one at a time, to write their answers on the board.

### Solutions:

a. $c^2 = a^2 + b^2$	b. $c^2 = a^2 + b^2$	c. $ DC  = \frac{1}{2} AC $
$x^2 = 8^2 + 15^2$	$y^2 = 56^2 + 33^2$	$= \frac{1}{2} \times 8$
$x^2 = 64 + 225$	$y^2 = 3136 + 1089$	$= 4 \text{ cm}$
$x^2 = 289$	$y^2 = 4,225$	$m^2 = 3^2 + 4^2$
$x = \sqrt{289}$	$y = \sqrt{4,225}$	$m^2 = 9 + 16$
$x = 17 \text{ cm}$	$y = 65 \text{ cm}$	$m^2 = 25$
		$m = \sqrt{25}$
		$m = 5 \text{ cm}$

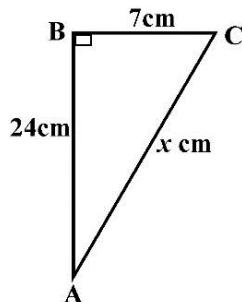
### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L138 in the Pupil Handbook.

<b>Lesson Title:</b> Finding the other sides of a right triangle	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L139	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to apply Pythagoras' theorem to find the length of the other two sides of a right-angled triangle.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board. In the triangle ABC below,  $|AB| = 24\text{ cm}$ ,  $|BC| = 7\text{ cm}$  and  $|AC| = x\text{ cm}$ . Find the value of  $x$ :



2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board.

#### Solution:

$$\begin{aligned}
 x^2 &= 7^2 + 24^2 \\
 x^2 &= 49 + 576 \\
 x^2 &= 625 \\
 x &= \sqrt{625} \\
 x &= 25\text{ cm}
 \end{aligned}$$

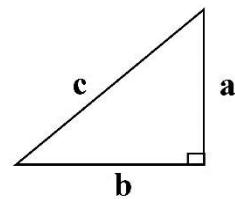
4. Explain that today's lesson is to apply Pythagoras' theorem to find the length of the other two sides of a right-angled triangle. We have found the length of the hypotenuse in the previous lesson.

### Teaching and Learning (22 minutes)

1. Remind pupils:
  - The hypotenuse is always opposite the right angle and it is always the longest side of the triangle.
  - Pythagoras' Theorem can be used to find the third side of a right-angled triangle if the other two sides are known.

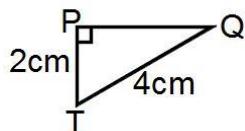
2. Write on the board:

- Pythagoras' Theorem:  $c^2 = a^2 + b^2$
- From  $c^2 = a^2 + b^2$ , we can also write the following:
  - $a^2 = c^2 - b^2$  or  $a = \sqrt{c^2 - b^2}$
  - $b^2 = c^2 - a^2$  or  $b = \sqrt{c^2 - a^2}$



3. Explain: We can use the formula for Pythagoras' theorem to find sides  $a$  or  $b$  of the triangle.

4. Write the problem on the board: Find  $|PQ|$  correct to 2 decimal places.



5. Invite a volunteer to write the values of the known sides on the board.

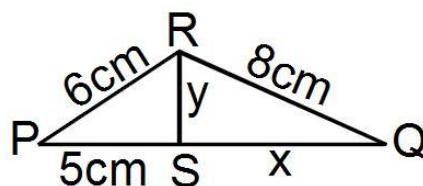
(Answers:  $|PT| = 2$  cm,  $|QT| = 4$  cm)

6. Write the solution on the board.

**Solution:**

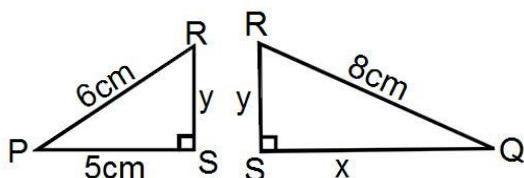
$$\begin{aligned} |PQ|^2 &= |QT|^2 - |PT|^2 \\ |PQ|^2 &= 4^2 - 2^2 \\ |PQ|^2 &= 16 - 4 \\ |PQ| &= \sqrt{12} \\ |PQ| &= 3.4641 \\ |PQ| &= 3.46 \text{ (2 decimal places)} \end{aligned}$$

7. Write on the board: Calculate  $x$  and  $y$ , correct to 2 decimal places:



8. Explain: In solving such a problem, we need to separate the triangles into two and solve them independently.

9. Draw on the board:



10. Discuss: First, we solve the triangle where two sides are known before solving for the other triangle.

11. Solve the problem on the board, explaining each step.

**Solution:**

**Step 1.** Take triangle  $PRS$  and solve for  $y$ :

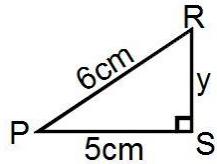
$$y^2 = 6^2 - 5^2$$

$$y^2 = 36 - 25$$

$$y^2 = 11$$

$$y = \sqrt{11}$$

$$y = 3.32 \text{ cm}$$



**Step 2.** Take triangle  $QRS$  and solve for  $x$ :

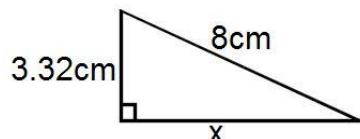
$$x^2 = 8^2 - (3.32)^2$$

$$x^2 = 64 - 11.02$$

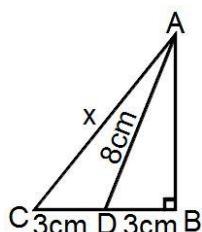
$$x^2 = 52.98$$

$$x = \sqrt{52.98}$$

$$x = 7.28 \text{ cm}$$



12. Write another problem on the board: Calculate the value of  $x$  correct to 2 s.f.

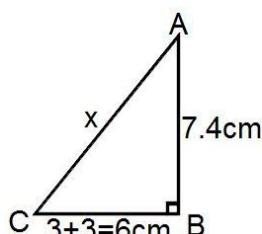
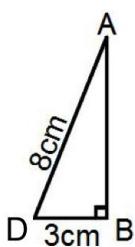


13. Allow pupils to discuss with seatmates for 1 minute on how to solve the problem.

14. Ask volunteers to give the steps to solve the problem. Solve on the board as they give the steps.

**Solution:**

Separate the triangle into two and first solve for the triangle whose two sides are given. Use the length of AB from the first triangle to find  $x$  in the second triangle.



$$|AB|^2 = 8^2 - 3^2$$

$$x^2 = 7.4^2 + 6^2$$

$$|AB|^2 = 64 - 9$$

$$x^2 = 54.76 - 36$$

$$|AB|^2 = 55$$

$$x^2 = 90.76$$

$$|AB| = \sqrt{55}$$

$$x = \sqrt{90.76}$$

$$|AB| = 7.416$$

$$x = 9.5268$$

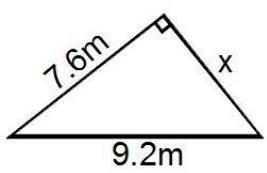
$$|AB| = 7.4$$

$$x = 9.5$$

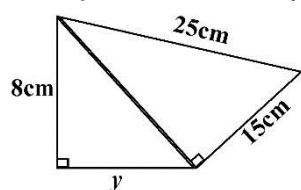
### Practice (14 minutes)

1. Write two problems on the board:

a. Find  $x$  to 2 decimal places:



b. Find  $y$  to 2 decimal places:



2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Allow pupils to exchange their books.
5. Invite two volunteers to write the answers on the board at the same time.

#### Solutions:

a.  $x^2 = 9.2^2 - 7.6^2$

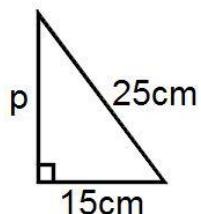
$$x^2 = 84.64 - 57.76$$

$$x^2 = 26.88$$

$$x = \sqrt{26.88}$$

$$x = 5.18 \text{ m}$$

b.



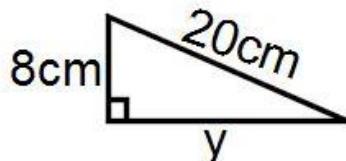
$$p^2 = 25^2 - 15^2$$

$$p^2 = 625 - 225$$

$$p^2 = 400$$

$$p = \sqrt{400}$$

$$p = 20 \text{ cm}$$



$$y^2 = 20^2 - 8^2$$

$$y^2 = 400 - 64$$

$$y^2 = 336$$

$$y = \sqrt{336}$$

$$y = 18.33 \text{ cm}$$

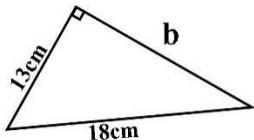
### Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L139 in the Pupil Handbook.

<b>Lesson Title:</b> Application of Pythagoras' Theorem	<b>Theme:</b> Geometry	
<b>Lesson Number:</b> M1-L140	<b>Class:</b> SSS 1	<b>Time:</b> 40 minutes
 <b>Learning Outcome</b> By the end of the lesson, pupils will be able to solve diagram and word problems involving Pythagoras' theorem.	 <b>Preparation</b> Write the problem in Opening on the board.	

### Opening (3 minutes)

1. Write the following problem on the board: Calculate the length of the missing side in the following triangle correct to 1 decimal place.



2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the solution on the board.

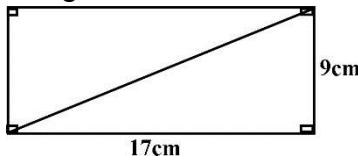
#### Solution:

$$\begin{aligned}
 b^2 &= c^2 - a^2 \\
 b^2 &= 18^2 - 13^2 \\
 b^2 &= 324 - 169 \\
 b^2 &= 155 \\
 b &= \sqrt{155} \\
 b &= 12.4 \text{ cm}
 \end{aligned}$$

4. Explain to the pupils that today's lesson is on solving word problems involving Pythagoras' theorem.

### Teaching and Learning (22 minutes)

1. Write a problem on the board: Find the length of the diagonal on a rectangle with length 9 cm and width 17 cm. Give your answer to 1 decimal place.
2. Ask pupils to work with seatmates to draw a diagram for the problem.
3. Invite a volunteer to draw the diagram on the board.



4. Ask pupils to think and share ideas as to what the shape now looks like.
5. Ask volunteers to share their ideas with the class. Encourage them to see that the diagram is 2 right-angled triangles.
6. Ask pupils how we can find the length of the diagonal.

- Ask any volunteer to answer. (Answer: By using Pythagoras' Theorem)
- Ask pupils to work with seatmates to find the length of the diagonal.
- Invite a volunteer to write the solution on the board.

**Solution:**

$$\begin{aligned}
 c^2 &= a^2 + b^2 \\
 c^2 &= 9^2 + 17^2 \\
 c^2 &= 81 + 289 \\
 c^2 &= 370 \\
 c &= \sqrt{370} \\
 c &= 19.2354 \\
 c &= 19.2 \text{ (1 decimal place)}
 \end{aligned}$$

Therefore, the length of the diagonal is 19.2 cm.

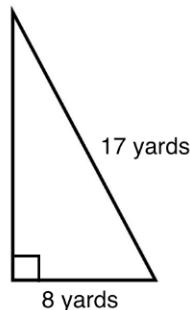
- Discuss:

- It is very important to draw a diagram to help with solving a problem.
- The diagram showed us that we needed to use Pythagoras' Theorem to solve the problem.

- Write the following problem on the board. A sail boat has a large sail in the shape of a right-angled triangle. The longest edge of the sail measures 17 yards, and the bottom edge of the sail is 8 yards. How tall is the sail?

- Ask pupils to draw a diagram for the problem in their exercise books.

- Invite a volunteer to draw the diagram on the board.



- Ask pupils to work with seatmates to solve the problem.

- Invite a volunteer to write the solution on the board.

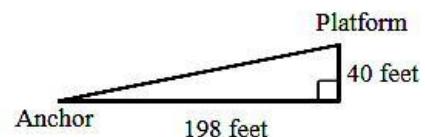
**Solution:**

Let the height of the sail be  $h$

$$\begin{aligned}
 h^2 &= 17^2 - 8^2 \\
 h^2 &= 289 - 64 \\
 h^2 &= 225 \\
 h &= \sqrt{225} \\
 h &= 15
 \end{aligned}$$

Therefore, the height of the sail is 15 yards.

16. Write another problem on the board and draw the diagram: A rope is tied to the top of a platform that is 40 feet above the ground. Its other end is anchored to the ground 198 horizontal feet from the base of the platform. How long is the rope/zip line?



17. Ask pupils to solve the problem with seatmates.  
18. Walk around, if possible, to check their answers and clear misconceptions.  
19. Invite a volunteer to write the solution on the board.

**Solution:**

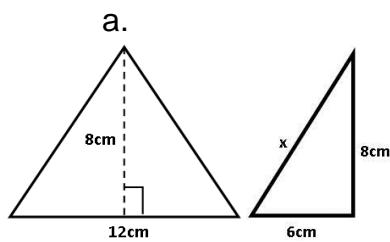
$$\begin{aligned} a^2 &= 198^2 + 40^2 \\ a^2 &= 39,204 + 1,600 \\ a^2 &= 40,804 \\ a &= \sqrt{40,804} \\ a &= 202 \end{aligned}$$

Therefore, the length of the rope/zip line is 202 feet.

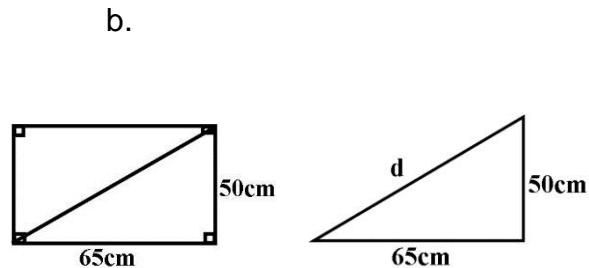
**Practice (14 minutes)**

1. Write the following two problems on the board:
  - a. An isosceles triangle has a base of 12 cm and a height of 8 cm. Calculate the length,  $x$ , of one of its equal sides.
  - b. A rectangle measures 60 cm long and 50 tall. What is the diagonal length of the rectangle to the nearest cm?
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around, if possible, to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite two volunteers, one at a time, to draw the diagrams and write the solutions on the board.

**Solutions:**



$$\begin{aligned} x^2 &= 6^2 + 8^2 \\ x^2 &= 36 + 64 \\ x^2 &= 100 \\ x &= \sqrt{100} \\ x &= 10 \end{aligned}$$



$$\begin{aligned} d^2 &= 65^2 + 50^2 \\ d^2 &= 4,225 + 2,500 \\ d^2 &= 6,725 \\ d &= \sqrt{6,725} \\ d &= 82.0 \end{aligned}$$

The length of  $x$  is 10 cm.

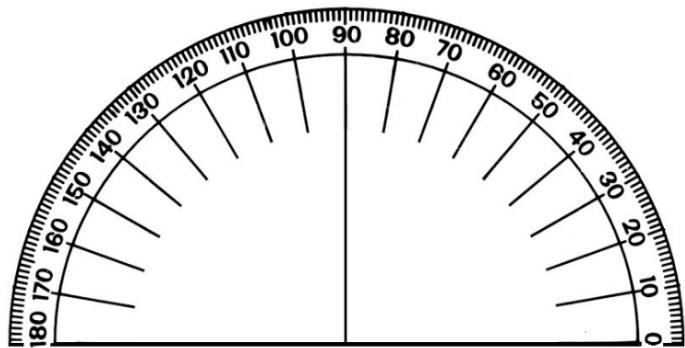
The length of the diagonal is 82.0 cm.

**Closing** (*1 minute*)

1. For homework, have pupils do the practice activity PHM1-L140 in the Pupil Handbook.

## **Appendix I: Protractor**

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.







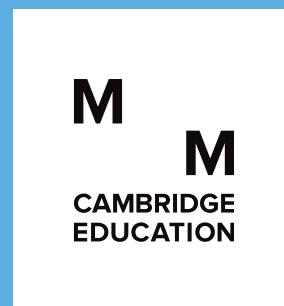


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