Cleaning robot:

We define dirty cell as d_o amount of dirt concentrated into area A_c. The amount of dirt satisfies

$$\frac{dD}{dt} = \text{constant} \doteq C$$

$$D = Ct + A$$

$$D = n \cdot d_o,$$

where n is number of dirty cells $\frac{dn}{dt} = \frac{1}{T}$

$$\Rightarrow \quad \frac{d_o}{T} = C$$

$$\Rightarrow \quad \underbrace{D(t) = \frac{d_o}{T} \cdot t + A}$$ (no cleaning occurring).

To clean d_o amount of dirt takes ε amount of energy and μ amount of time. So when cleaning

$$\frac{dD_c}{dt} = \sigma \quad \Rightarrow \quad D_c = \sigma t + \tilde{A}$$

$$\Rightarrow \quad -d_o = \sigma\mu \quad \Rightarrow \quad \sigma = -\frac{d_o}{\mu}$$

$$\Rightarrow \quad \underbrace{D_c(t) = -\frac{d_o}{\mu} t + \tilde{A}}$$ (cleaner reduces amount of dirt.)

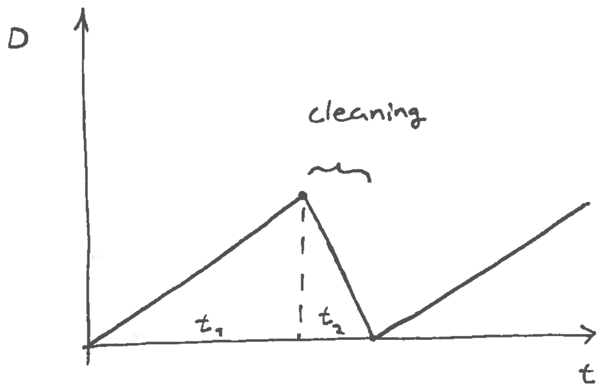
The $\frac{dD}{dt}$ now becomes

$$\frac{dD}{dt} = -\frac{d_0}{T}\begin{cases} -1, & \text{cleaner off} \\ +\frac{I}{\mu} - 1, & \text{cleaner on.} \end{cases}$$

This is machine limit.

Now for the cleaner to be succesful $1 \leq \mu \leq T$.

Assume we start from clean room. The amount of dirt. w.r.t., time is



The time average of this is

$$\frac{1}{2} D_{max} \cdot \cancel{(t_1+t_2)} \frac{1}{\cancel{t_1+t_2}} = \underbrace{\frac{1}{2}\frac{t_1}{T} d_0 = D_{av}} \qquad (\text{minimized by } t_1 \to 0)$$

The average amount of dirt can be minimized by minimizeng the rest time of the machine.

The amount of energy consumed when cleaning d_0 amount of dirt is ϵ. ~~and~~

Now the energy consumed when cleaning

$$dE = \delta\, dD_c \quad \Rightarrow \quad E = \delta D_c \qquad \begin{array}{l} D_c = -d_0 \\ E = \epsilon \end{array}$$

$$\Rightarrow \quad \delta = -\frac{\epsilon}{d_0}$$

$$dE = -\frac{\varepsilon}{d_0} dD_c$$

Now $D_c = -\frac{d_0}{\mu} t + \tilde{A} \Rightarrow dD_c = -\frac{d_0}{\mu} dt$

$$\Rightarrow \quad dE = \frac{\varepsilon}{\mu} dt$$

$$\therefore \quad E(t) = \frac{\varepsilon}{\mu} t$$

But this is only due to cleaning. We introduce term that consumes energy when robot moves $K = \delta t$

Total energ consumbtion is thus

$$\hat{E} \doteq E(t) + K(t) = \left(\frac{\varepsilon}{\mu} + \delta\right) t$$

Now energy spent on cleaning

$$\hat{E}(t_2) = \left(\frac{\varepsilon}{\mu} + \delta\right) t_1 \frac{\mu}{T-\mu}$$

$$\underline{P} = \frac{\hat{E}(t_2)}{t_1 + t_2} = \left(\frac{\varepsilon}{\mu} + \delta\right) \cancel{t_1} \frac{\mu}{T-\mu} \frac{1}{\cancel{t_1}\left(1 + \frac{\mu}{T-\mu}\right)}, \qquad t_1 + t_2 = t_1\left(\frac{T}{T-\mu}\right)$$

$$= \left(\frac{\varepsilon}{\mu} + \delta\right) \frac{\mu}{T}$$

$$= \underline{\underline{\frac{\varepsilon}{T} + \delta \frac{\mu}{T}}}$$

Now the energy consumption P is minimized if μ is minimized. μ measures the effectivines of the machine since it is the time taken to clean d_0 amount of dirt. ~~...~~ Time taken includes also the time it takes to find the dirt ☺

Now since Dirtyness would be minimized by making cleaning as frequent ass possible we notice that this would increase μ as the dirt would be sparsely scattered and thus moving from one dirty cell to another takes more time in comparison to very dirty room.

To solve the problem one needs information on how μ depends on t_1 i.e., the initial dirtyness.