IMO Training
Homework, January 2018

This set of exercises is suitable for serious training for new people, and for warm up for the people who've been longer in the training.

Please hand in your solutions by Friday, February 23rd, in person at Päivölä, by email to `npalojar@abo.fi` or by postal mail to

Neea Palojärvi
Ratapihankatu 12 A 1
20100 Turku.

We have often been flexible about return dates, but this time the date is firm because of the EGMO team selection deadline. Owing to equality considerations, responses will not be accepted after February 23rd even from those who are not eligible to participate in EGMO.

Remember to write your name, your contact information and at which grade you are at school.

Problems

1. Paint the cells of a 10×10 board white and black in such a way that each cell on the board has exactly two black neighboring (having a common side) cells.

2. To celebrate a company jubilee, Pete was supposed to buy 1 cake, 3 bottles of champagne, and 20 crystal glasses. Instead, accidentally, Pete purchased 1 glass, 3 cakes, and 20 bottles of champagne, spending exactly the same amount of money as was originally planned. We know that a cake is cheaper than a bottle of champagne. What is more expensive: a bottle of champagne or a crystal glass?

3. 20 gentlemen met for a party, some of them came wearing neck-ties, some didn't. Time to time during the party, one of the gentlemen with a neck-tie would give it to another gentleman without one. In the end of the party, 10 gentlemen counted that each of them gave away neck-tie more times than received it. How many gentlemen came to the party with neck-ties?

4. There're 1000 cards with numbers 0, 1, 2, ..., 999 (one card with each number between 0 and 999). Alex took some cards and Michael took all the remaining ones. They both put all their cards in a row in some order, forming two long numbers. Is it possible that their two long numbers are equal?

5. Six segments are drawn in the plane, no two of those belong to the same straight line. All the segment intersection points are marked, and each marked point belongs to exactly two of the segments. The first segment contains 3 points, the second 4 points, the third, fourth, and fifth contain 5 points each. How many marked points are in the sixth segment?