31. 10.

High School Mathematics Competition

First Round, Basic Level

2018

The problems are on two pages; the first six problems are multiple choice problems with zero to four correct answers.

1. The ratio between number of pupils in high schools A and B was a/b at the end of the year 2017. During the year 2017 the number of pupils in high school A had increased by 5 % and the number of pupils in high school B by 10 %. At the end of the year 2017, the ratio between the number of students was

a) $\dfrac{95a}{90b}$ b) $\dfrac{105a}{110b}$ c) $\dfrac{22a}{21b}$ d) $\dfrac{19a}{18b}$

2. What can we say about the exact value of the expression $k^{3x} - k^x$, for $k^{2x} = 16$ and $k > 0$.

a) The problem cannot be solved as the answer depends on the values of x and k.
b) Its value is 56.
c) Its value is 60.
d) Its value is an irrational number.

3. A square rotates 45° around its center forming a star-shaped hexadecagon (16-gon). The ratio between the perimeter of the hexadecagon to the perimeter of the square is

a) less than 1,5 b) $4 - 2\sqrt{2}$ c) $\frac{4}{2+\sqrt{2}}$ d) $\frac{2\sqrt{2}+1}{2}$

4. Consider the equation

$$\frac{2x + a^2 - 3a}{x - 1} = a,$$

where the unknown x is different from 1 and $a \in \mathbb{R}$ is a constant. What can we say about the number of solutions?

a) For a suitable parameter value a the equation can have infinitely many solutions.
b) For some parameter values a, the equation does not have a solution.
c) The equation always has solutions regardless of the value of a.
d) The equation always has three solutions.

5. Ten (distinct) lines divide a plane in different areas, the number of which can vary depending on how the lines are drawn. The number of areas can be

a) 20. b) 9. c) 56. d) 32.

6. A mathematically disturbed frogs leaps on the plane. The length of every leap is exactly $\sqrt{5}$. Furthermore, the frog suffers from a number-theoretical syndrome, and therefore it always lands in a point with integer coordinates. The frog starts in the origin, and returns to the origin after four leaps. In how many ways can frog perform a sequence of such four leaps? What can we say about the number of solutions?

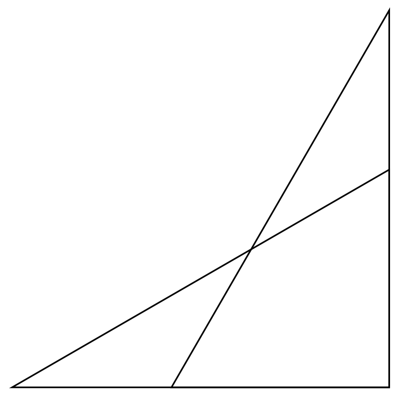
a) The frog can perform the sequence in over one hundred different ways.
b) The number of solutions is divisible by eight.
c) The number of solutions is divisible by five.
d) The frog can perform the sequence in at most 80 different ways.

7. Solve the Diophantine equation

$$x^2 - y^2 = 2018$$

(that is, find all pairs (x, y) of integers satisfying the equation).

8. In the following figure, we have two right triangles. In both of them, the length of the shorter catheti are 1 and the bigger ones of the sharp angles are equal to 60 degrees. Determine the common area.



31. 10. 2018

Basic Level Multiple Choice Answer Sheet

The first six problems are multiple choice problems. Their answers should be written in the table below. Each multiple choice problem has 0 to 4 correct answers. Put a "+" to the appropriate square, if the answer is right and a "−" if the answer is wrong. All correct marks give one point and incorrect or unintelligible marks give zero points. The answers to problems 7 and 8 can be written on a separate paper. For each of these problems, a maximum of 6 points is given.

The time allowed is 120 minutes. **The use of calculators and tables are not allowed.** *Please write your name and school with block letters on every paper you return.*

Name : ______________________

School : ______________________

Home address : ______________________

Email : ______________________

	a	b	c	d
1.				
2.				
3.				
4.				
5.				
6.				

31. 10.

High School Mathematics Competition
First Round, Intermediate Level

2018

The problems are on two pages; the first three problems are multiple choice problems with zero to four correct answers.

1. An arithmetic sequence has an even number of terms. The first term (the term with index 1) is 1. The sum of the terms with even indices is 210 and the sum of the terms with odd indices is 190. Then:

a) The sequence has in total 20 terms.
b) The difference between two consecutive terms is 4.
c) The last term is 38.
d) The last term is 39.

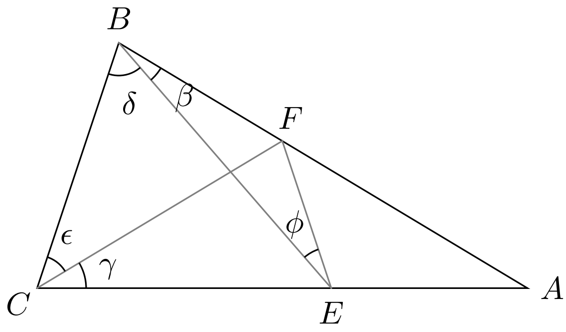
2. A square rotates $45°$ around its centre forming a star-shaped hexadecagon (16-gon). The ratio between the perimeter of the hexadecagon to the perimeter of the square is

a) less than 1.5. b) $4 - 2\sqrt{2}$. c) $\frac{4}{2+\sqrt{2}}$. d) $\frac{2\sqrt{2}+1}{2}$.

3. Assume that x and y are positive integers and that $x^2 + 4y^2 + 1$ is a prime which divides $8xy + 2$. Which of the following claims are true?

a) We know that x is even.
b) We know that $8xy + 2$ is divisible by five.
c) The quotient $\dfrac{8xy + 2}{x^2 + 4y^2 + 1}$ is smaller than four.
d) We know that y is even.

4. In the following figure, the triangle ABC and some lines have been drawn but the lengths of these are not necessarily accurate or up to scale in the figure. Furthermore, we know that $\beta = 20°$, $\gamma = 30°$, $\delta = 60°$ and $\epsilon = 50°$. Determine the angle ϕ.



5. Find all pairs of integers x and y satisfying the equation

$$x^2 + xy + 2x + y = 100.$$

6. We say that a 5×5 board has *five consecutive crosses*, if these crosses fill a full line, column, or one of the two diagonals. Determine the smallest possible number of crosses that can be put on the board in such a way that even if one of the crosses is wiped out, the remaining crosses still have some five consecutive crosses.

31. 10.

Intermediate Level Multiple Choice

Answer Sheet

2018

The first three problems are multiple choice problems. Their answers should be written in the table below. Each multiple choice problem has 0 to 4 correct answers. Put a "+" to the appropriate square, if the answer is right and a "−" if the answer is wrong. All correct marks give one point and incorrect or unintelligible marks give zero points. The answers to problems 4 to 6 can be written on a separate paper. For each of these problems, a maximum of 6 points is given.

The time allowed is 120 minutes. **Calculators and tables are not allowed.** *Please write your name and school with block letters on every paper you return.*

Name : ____________________

School : ____________________

Home address : ____________________

Email : ____________________

	a	b	c	d
1.				
2.				
3.				

31. 10.

High School Mathematics Competition
First Round, Open Level

2018

1. Solve the Diophantine equation

$$x^2 - y^2 = 2018$$

(that is, find all pairs (x, y) of integers satisfying the equation).

2. Let ABC be an acute triangle and assume that d is the distance between B and AC. Prove that $|AB| = |AC|$ if and only if for every point D on the side BC, we have $d = d_0 + d_1$ where d_0 is the distance between D and AB and d_1 is the distance between D and AC.

3. Let us consider n knights sitting around a round table. Everyone of them has a light and a light switch in front of them. When a knight presses the switch, it doesn't only affect his own light but also the lights of the knigths sitting next to him: if a light has been turned off, it will become turned on, and if it has been turned on, it will be turned off. In the beginning some lights are turned on and some of them are turned off. Find all integers $n > 3$ such that regardless of the initial state of the lights, the knights can turn off all the lights by a suitable combination of using the switches.

4. Assume $f\colon \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying the conditions $f(150) = 25$ och

$$f(x) + f(2f(x)) = 100$$

for all real numbers x. Find all possible values of $f(100)$.

Työaikaa on **120 minuuttia.**
Laskimet ja taulukkokirjat eivät ole sallittuja.
Tee kukin tehtävä omalle konseptiarkin sivulleen.
Merkitse koepaperiin selvästi tekstaten oma nimesi ja yhteystietosi (koulun nimi, kotiosoite ja sähköpostiosoite).