High School Mathematics Competition
First Round, Basic Level

The problems are on two pages; the first six problems are multiple choice problems with zero to four correct answers.

1. There are 40 % more conifers than broad-leaved trees in a forest. Logging reduces the number of conifers by 20 % and the number of broad-leaved trees by 12%. After logging, the proportion of conifers in the forest is

a) 46%	b) 56%	c) 58%	d) 14/25

2. A student is either sick or healthy. 95 % of the students who are healthy today, will also be healthy tomorrow. 55 % of those students who are sick today, will also be sick tomorrow. 20 % of the student population are sick today. How many percent of the student population will be sick tomorrow?

a) at most as many as today	b) at least as many as today
c) $22,5$ % of all	d) 15 % of all

3. The number x is a solution to the equation $x^2 + x - 2 = 0$ and the number y is a solution to the equation $y^2 - 3y + 2 = 0$. What do we know about numbers x and y?

a) $x \neq y$.	b) xy is an integer
c) $x + y > 0$.	d) $\lvert x + y\rvert \leq 3$.

4. A line intersects a circle in points A and B ($A \neq B$). We choose point C on the perimeter of the circle in such a way that we form an isosceles triangle that has area which is as large as possible and which has AB as its base. Which ones of the following claims are always true?

a) The triangle ABC has a right angle if and only if AB is a diameter of the circle.
b) The point C lies on the perpendicular bisector of AB.
c) The area of the triangle ABC is at least a fourth part of the area of the circle.
d) The perimeter of the triangle ABC is longer than the diameter of the circle.

5. If $|x| < \frac{1}{2}$ then $\left|\frac{x}{x-1}\right|$ is always

a) on the interval $[\frac{1}{2}, 1]$
b) smaller than 1
c) on the interval $[\frac{1}{2}, \frac{3}{2}]$
d) not necessarily any of the options given

6. The number of solutions to the equation $3 \cdot 3^x + 3^{-x} = 4$ is

a) 0 b) 1 c) 2 d) more than 2.

7. An art installation consists of circles with a common center and with radii 1 m, 2 m, ..., 100 m. The innermost circle is colored blue. The smallest annulus (the area between two consecutive circles) is colored red. Every other annuli is red, and every other blue. Determine the area of the blue annuli.

8. Two persons, Kari and Veera, play the following game: Veera has chosen an arbitrary element from the set $\{a, b, c\}$ consisting of three elements. Kari tries to find out which one it is. The questions allowed are only "Is it a?", "Is it b?" and "Is it c?". Veera answers questions "yes" or "no" but she is allowed to lie as long as out of three consecutive questions, at most one is a lie. Kari can repeat any question, but he is not allowed to ask any question thrice. Does Kari have a strategy so that he can find out which element Veera has chosen not using more than those six questions he is allowed?

30. 10.

Basic Level Multiple Choice Answer Sheet

2017

The first six problems are multiple choice problems. Their answers should be written in the table below. Each multiple choice problem has 0 to 4 correct answers. Put a "+" to the appropriate square, if the answer is right and a "−" if the answer is wrong. All correct marks give one point and incorrect or unintelligible marks give zero points. The answers to problems 7 and 8 can be written on a separate paper. For each of these problems, a maximum of 6 points is given.

The time allowed is 120 minutes. **The use of calculators and tables are not allowed.** *Please write your name and school with block letters on every paper you return.*

Name : ________________________________

School : ________________________________

Home address : ________________________________

Email : ________________________________

	a	b	c	d
1.				
2.				
3.				
4.				
5.				
6.				

High School Mathematics Competition
First Round, Intermediate Level

30. 10. 2017

1. If $|x| < \frac{1}{2}$ then $\left|\dfrac{x}{x-1}\right|$ is always

a) on the interval $[\frac{1}{2}, 1]$
b) smaller than 1
c) on the interval $[\frac{1}{2}, \frac{3}{2}]$
d) not necessarily any of the options given

2. The number of solutions to the equation $3 \cdot 3^x + 3^{-x} = 4$ is

a) 0 b) 1 c) 2 d) more than 2.

3. Consider the equation

$$x = a - \sqrt{a^2 - x\sqrt{x^2 + a^2}},$$

with $a > 0$. What can we say about the solutions to the equation?

a) The equation has only one solution.
b) The solutions are non-negative.
c) $x = a$ is one solution.
d) $x = \frac{3}{4}a$ is one solution.

4. An art installation consists of circles with a common center and with radii 1 m, 2 m, ..., 100 m. The innermost circle is colored blue. The smallest annulus (the area between two consecutive circles) is colored red. Every other annuli is red, and every other blue. Determine the area of the blue annuli.

5. Let us assume that the numbers $\dfrac{1}{a+b}, \dfrac{1}{a+c}, \dfrac{1}{b+c}$ form an arithmetic progression. Prove that the squares of the numbers a, b and c also form an arithmetic progression.

6. Two intersecting segments move on a plane in such a way that the angle between them stays the same. Prove that the area of the quadrilateral determined by the end-points of the segments, is a constant.

30. 10.

Intermediate Level Multiple Choice

Answer Sheet

2017

The first three problems are multiple choice problems. Their answers should be written in the table below. Each multiple choice problem has 0 to 4 correct answers. Put a "+" to the appropriate square, if the answer is right and a "−" if the answer is wrong. All correct marks give one point and incorrect or unintelligible marks give zero points. The answers to problems 4 to 6 can be written on a separate paper. For each of these problems, a maximum of 6 points is given.

The time allowed is 120 minutes. **Calculators and tables are not allowed.** *Please write your name and school with block letters on every paper you return.*

Name : ________________________________

School : ________________________________

Home address : ________________________________

Email : ________________________________

	a	b	c	d
1.				
2.				
3.				

High School Mathematics Competition
First Round, Open Level

30. 10. 2017

1. What is the last digit in the decimal representation of the number $2017^{2017} - 2016^{2016}$?

2. The students A, B, C, ..., J are planning to take final exams of the courses K_1, K_2, K_3, K_4, K_5 and K_6 according to the following list:

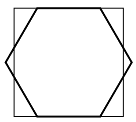
student	courses	student	courses
A	K_1, K_2	F	K_2, K_3
B	K_1, K_3	G	K_3, K_4
C	K_1, K_4	H	K_4, K_5
D	K_1, K_5	I	K_5, K_6
E	K_1, K_6	J	K_6, K_2

The school principal tries to organize the exam according to following regulations: It is possible to organize the exams of different courses at the same occasion but only one exam per course will be organized. At any given occasion, a student is allowed to take at most one exam.

a) How many different occasions will be needed?
b) If some student cancels his/her participation in some exam, does the minimal number of different exam occasions still stay the same?

3. Let us assume that the numbers $\frac{1}{a+b}$, $\frac{1}{a+c}$ $\frac{1}{b+c}$ form an arithmetic progression. Prove that the squares of the numbers a, b and c also form an arithmetic progression.

4. A regular hexagon and a square have a common center. Two of the sides of the hexagon are included in the sides of the squares, and the area of the square is 1 (see the figure). Calculate the area of the part that the hexagon and the square share together.



Time allowed: **120 minutes.**
Only writing and drawing equipments are allowed.
No calculators or tables!
Write your solutions of different problems on different sheets.
Mark every sheet with your name and provide
contact information (school, your own address and email).