31. 1.

High School Math Competition Finals

2014

1. Determine the value of
$$x^2 + y^2 + z^2$$
when
$$x + y + z = 13, \quad xyz = 72 \quad \text{and} \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{4}.$$

2. The center of the circle circumscribed to an acute triangle ABC is M, and the circle passing through A, B and M meets the sides BC and AC at P and Q, respectively. Prove that the extension of the line segment CM is perpendicular to the line segment PQ.

3. The points $P = (a, b)$ and $Q = (c, d)$ lie in the first quarter of the xy plane, and a, b, c and d are integers, satisfying $a < b$, $a < c$, $b < d$ and $c < d$. A *route* from P to Q is a broken line consisting of steps of unit length in the directions of the positive coordinate axes, and an *allowed route* is a route which does not meet or intersect the line $x = y$. Determine the number of allowed routes.

4. The radius r of a circle centered at the origin is an odd integer. The point (p^m, q^n) where p and q are prime numbers, and m and n are positive integers, lies on the circle. Determine r.

5. Determine the smallest number $n \in \mathbb{Z}_+$ which can be written in the form $n = \sum_{a\in A} a^2$ where A is a finite set of positive integers and $\sum_{a\in A} a = 2014$. In other words: What is the smallest positive integer that can be written as the sum of squares of different positive integers the sum of which is the number 2014?

Time allowed: **3 hours**.
Only writing and drawing instruments are allowed.
Write the solution of each problem on a separate sheet with your name on it.
On one of the papers, write your contact information (school, home address and email address).