Yang-Mills Instantons

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1 Introduction : What is an Instanton?

What is instanton? Instanton is a solution to the classical field equations of motion in Euclidean space. Instanton solutions of Euclidean EL equation are localized in (Euclidean) space and time, and have finite (Euclidean) action. This is why it is called instanton. Instantons only appear in Euclidean space, and they are not solutions to the equations of motion in Minkowski space. So, they are not physical particles or fields, in Real spacetime. So, why do we care about instantons? The answer is that we essentially need wick rotation and Euclidean space to perform path integral calculations in (Minkowski) quantum field theory. So even we are interested in Minkowski space, we should consider instantons, which are classical solutions in Euclidean space. In detail, how do instantons affect the QFT in Minkowski space? We will discuss how the instanton effect appears in the path integral formulation, and how it is related to various physical phenomena.

2 Instanton effect in Path Integral : a Toy Model

2.1 $0+1$ Dimensional Toy Model : QM with a double well potential

2.2 Path Integral Formulation and Wick Rotation

2.3 Instanton Solutions in Euclidean Space

2.4 Instanton Contribution to the Path Integral

2.5 Tunneling Amplitude, Instanton Action and WKB Approximation

3 Yang-Mills Instanton

3.1 classical Yang-Mills theory: action and equations of motion

3.2 Hodge decomposition and self-dual equation

3.3 Instanton action of (anti-)self-dual solutions

3.4 Explicit construction of instanton solutions, and the Moduli space

4 Effects of Instantons in real world physics

4.1 Instanton effects in QCD

4.2 Vaccume structure of QCD and Tunneling effects

4.3 Easy analogy: QM on a circle

4.4 θ-vacuum and θ-angle

4.5 The $U_A(1)$ problem and the η' meson

4.6 Chiral Anomaly and θ-vacuum

5 Instanton effects in Axion phtsics

6 References